Convert fractional Binary number to Fractional Decimal number

- Write out the binary number as (-)ve power of two. The various digits positions after binary points are 1,2,3,4…..and so on.
- Convert each power of two into its decimal equivalent
- Add these to give the decimal number

Fractional Decimal number to binary

The fractional decimal number is converted in to binary number by using successive fraction multiplications method.

1. The fractional decimal number is multiplied with 2 by successive fraction multiplications method.
2. If ‘1’ or ‘0’ occurs in units place in the product, transfer that ‘1’ or ‘0’ to the binary record.
3. The multiplication is continued with the remaining fraction.
4. The same procedure is followed in each multiplication.
5. The first transferred number (1 or 0) to binary record is taken as most significant bit (MSB).
6. The last transferred number (1 or 0) to binary record is taken as least significant bit (LSB).
7. If the multiplication does not end, it can be stopped at any of our desired level.

Ex : - Convert the fractional decimal number $(0.638)_{10}$ in to fractional binary number.

Sol :-

Successive multiplications	**Binary**	
$0{\cdot}638 \times 2 = 1{\cdot}276 \longrightarrow$	1	MSB
$0{\cdot}276 \times 2 = 0{\cdot}552 \longrightarrow$	0	
$0{\cdot}552 \times 2 = 1{\cdot}104 \longrightarrow$	1	
$0{\cdot}104 \times 2 = 0{\cdot}208 \longrightarrow$	0	
$0{\cdot}208 \times 2 = 0{\cdot}416 \longrightarrow$	0	
$0{\cdot}416 \times 2 = 0{\cdot}832 \longrightarrow$	0	
$0{\cdot}832 \times 2 = 1{\cdot}664 \longrightarrow$	1	
$0{\cdot}664 \times 2 = 1{\cdot}328 \longrightarrow$	1	
$0{\cdot}328 \times 2 = 0{\cdot}656 \longrightarrow$	0	
$0{\cdot}656 \times 2 = 1{\cdot}312 \longrightarrow$	1	LSB

So, $(0.638)_{10} = (0.1010001101)_2$

Example

- Decimal number is (0.625)

Fractional decimal number	Operation	Product	Fractional part of product	Integer part of product
0.625	Multiply by 2	1.250	.250	1
0.250	-do-	0.500	.500	0
0.500	-do-	1.000	0	1

Ans: $(0.625)_{10} = (0.101)_2$

Fractional Binary number to Decimal number

For the fraction (after the decimal point) the place value starts with negative power of 2. This negative power value increases from left to right.

The place value of the first left digit in fraction is (2^{-1})

The place value of the next right digit in fraction is (2^{-2})

The place value of the next right digit in fraction is (2^{-3})

……. so on

Ex :- Convert the binary number $(0.1101)_2$ into decimal number.

$$(0.1101)_2 = (1 \text{ x } \frac{1}{2}) + (1 \text{ x } \frac{1}{4}) + (0 \text{ x } \frac{1}{8}) + (1 \text{ x } \frac{1}{16})$$

$$= 1 \text{ x } 0.5 + 1 \text{ x } 0.25 + 0 \text{ x } 0.125 + 1 \text{ x } 0.0625$$

$$= 0.5 + 0.25 + 0 + 0.0625$$

$$= (0.8125)_{10}$$

Ex :- Convert the binary number $(1010.0101)_2$ into decimal number.

Sol :- <u>Integer part</u> $(1010)_2$ $= (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (0 \times 2^0)$

$= (1 \times 8) + (0 \times 4) + (1 \times 2) + (0 \times 1)$

$= (10)_{10}$

<u>Fractional part</u> $(0.0101)_2 = (0 \times 2^{-1}) + (1 \times 2^{-2}) + (0 \times 2^{-3}) + (1 \times 2^{-4})$

$= (0 \times \frac{1}{2}) + (1 \times \frac{1}{4}) + (0 \times \frac{1}{8}) + (1 \times \frac{1}{16})$

$= 0 \times 0.5 + 1 \times 0.25 + 0 \times 0.125 + 1 \times 0.0625$

$= 0 + 0.25 + 0 + 0.0625$

$= (0.3125)_{10}$

Questions

- Convert decimal 89 into equivalent binary number by using Double-Dabble Method

  $(89)_{10}= (1011001)_2$

- Convert decimal 89 into equivalent binary number by using Direct Method

  $(89)_{10}= (1011001)_2$

- Convert decimal 0.8125 into fractional binary number

  $(0.8125)_{10} = (0.1101)_2$

Example

. 1		0		1		1
$1*2^{-1}$		$0*2^{-2}$		$1*2^{-3}$		$1*2^{-4}$
0.5	+	0	+	0.125	+	0.0625

= 0.6875

Questions

- Convert the fractional binary number to decimal number
- (0.1101)

ans= 0.8125

- (0.1011)

ans= 0.6875

Important Number Systems

Decimal	Binary	Octal	Hexadecimal
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	1	F
16	10000	20	10

Number systems

- How do we write negative binary numbers?
- Historically: 3 approaches
  - Sign-and-magnitude
  - Ones-complement
  - Twos-complement
- For all 3, the most-significant bit (MSB) is the sign digit
  - $0 \equiv$ positive
  - $1 \equiv$ negative
- twos-complement is the important one
  - Simplifies arithmetic
  - Used almost universally

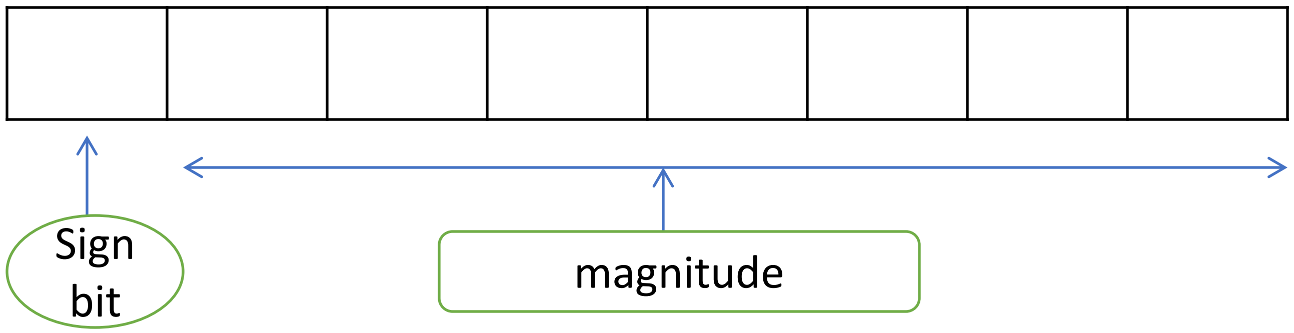
Integer Representation

- An integer can be represented by fixed point representation
- The left most bit is considered as sign bit.
- The magnitude of the number can be represented in following three ways:

1. Signed magnitude representation.
2. Signed 1’s complement representation.
3. Signed 2’s complement representation.

Signed Magnitude

- In this representation , if n bit of storage is available then 1 bit is reserved for sign and n-1 bits for the magnitude.



- The Disadvantage of this representation is that during addition and Subtraction, the sign bit has to be considered along with the magnitude.

0	0	0	0	0	0	0	0	$(+0)_{10}$
1	0	0	0	0	0	0	0	$(-0)_{10}$

Signed 1’s Compliment

- The 1’s Compliment of a binary integer can be obtained by simply replacing the digit 0 by 1 and digit 1 by 0
- Example: <u>00001100</u> is <u>11100111</u>

0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1

Signed 2's Compliment

- The 2's Compliment of a binary number is obtained by adding 1 to 1's Compliment.
- Example: $(+12)_{10}$= 1100

0	0	0	0	1	1	0	0	
1	1	1	1	0	0	1	1	1's

- 11110011 1's Compliment

$$\begin{array}{r} 1 \\ \hline 11110100 \end{array}$$

11110100 2,s Compliment

1	1	1	1	0	1	0	0	$(-12)_{10}$

Therefore, Positive integer 2's compliment is the negative integer

Question

- Express the following in signed magnitude form, 1’s Compliment, 2’s Compliment:
- $(35)_{10}$ = 100011

Arithmetic Operation

Arithmetic operations

- Basic arithmetic operations include addition, subtraction, multiplication and division .

Operation of Binary

- There are four rules for binary subtraction
- 0-0 = 0
- 1-1 = 0
- 1-0 = 1
- 0-1 = 1 borrow 1

Borrow 1 is required from the next higher order bit to subtract 1 from 0. So, the result became 0.

Operation of Binary

- Subtract Operation
- 11000100 – 00100101
-

		1	**1**	**1**	**1**	**1**	**1**	
	1	**0**	**1**	**1**	**1**	**0**	**1**	
	1	**1**	**0**	**0**	**0**	**1**	**0**	**0**
-	**0**	**0**	**1**	**0**	**0**	**1**	**0**	**1**
	1	**0**	**0**	**1**	**1**	**1**	**1**	**1**

Operation of Binary

There are four rules for binary multiplication

- 0*0 = 0
- 0*1 = 0
- 1*0 = 0
- 1*1 = 1

Operation of Binary

- Multiply Operation 1111 * 1101

```
          1 1 1 1
      *   1 1 0 1
  ---------------------
          1 1 1 1
        0 0 0 0
      1 1 1 1
    1 1 1 1
  ---------------------
    1 0 0 0 0 1 1
```

Basic Rules of Addition:

1. Binary Addition:

the basic rules of binary addition as follows:

1. $0 + 0 = 0$.

2. $0 + 1 = 1$.

3. $1 + 0 = 1$.

4. $1 + 1 = 0$ with a carry of ‘1’ to the next more significant bit.

5. $1 + 1 + 1 = 1$ with a carry of ‘1’ to the next more significant bit.

Example: Add the following binary number:

a)11+11

b)11+111

Solution:

a)

$$\begin{array}{r} 11 \\ 11+ \\ \hline 110 \end{array}$$

b)

$$\begin{array}{r} 011 \\ 111\ + \\ \hline 1010 \end{array}$$

Operation of Binary

- Add Operation
- 1101 + 1011

	1 1 1 1	**carry bit**
	1 1 0 1	**add bit**
+	1 0 1 1	**added bit**
	1 1 0 0 0	**result**

2. Octal Addition:

If the sum result >= 8 , subtract 8 and carry 1

Example: Add the following octal number:

57+432

057

432+

511

3. Hexadecimal Addition:

If the sum result >= 16 , subtract 16 and carry 1

Example: Add the following Hexadecimal number:

58+4B

$$\begin{array}{r} 58 \\ 4B+ \\ \hline A3 \end{array}$$

Complement :

Complement are used in digital computer for simplifying the **subtraction** operation and for logical manipulation. There are two types of complement for each base system the **r’s and (r-1)’ complement.**

1. Binary Number Complement

In binary number system we have the **1’s and 2’s** complement the 1’s is obtained by replacing 0s with 1s and 1s with 0s.

2’s complement = 1’s complement +1.

Example : Find the 1’s and 2’s complement of the following number : 1011000.

Solution :

1’s comp.=0100111 ; 2’s =0100111+1=0101000

2. Decimal Number Complement

In decimal number system we have the **9's and 10's** complement the 9's is obtained by subtracting each digit from 9

10's complement = 9's complement +1.

Example : Find the 9's and 10's complement of the following number :2496.

Solution :

9's comp.= 9999-2496 = 7503; 10's =7503+1=7504

3. Octal Number Complement:

In octal number system we have the 7's and 8's complement the 7's is obtained by subtracting each digit from 7

8's complement = 7's complement +1.

Example : Find the 7's and 8's complement of the following number :562.

Solution :

7's comp.= 777- 562 = 215; 8's =215+1=216

4. Hexadecimal Number Complement:

In hexadecimal number system we have the 15’s and 16’s complement the

15’s is obtained by subtracting each digit from 15

16’s complement = 15’s complement +1.

Example : Find the 15’s and 16’s complement of the following number :3BF.

Solution :

15’s comp.= 15 15 15 – 3 B F = C 4 0; 16’s =C 4 0+1=C 4 1

- **Subtraction with Complements**

The efficient method for subtraction is used complement . The subtraction of two n-digit numbers M-N can be :

1- If use (r-1)’s complement [1’s 9’s 7’s 15’s]:

a. If the sum produce an end carry which can be added to the sum.
b. If the sum does not produce an end carry take the (r-1)’s comp. of the sum and place – sign.

2- If use (r)’s complement [2’s 10’s 8’s 16’s]:

a. If the sum produce an end carry which can be discarded.
b. If the sum does not produce an end carry take the (r)’s comp. of the sum and place – sign.

Example : Subtract the following binary number :

a) 1010100-1000011 b)1000011-1010100

Solution :

1)Using 1’s comp.		2)Using 2’s comp.	
a)1010100	b)1000011	a)1010100	b) 1000011
1000011-	1010100-	1000011-	1010100-
1010100	1000011	1010100	1000011
0111100+	0101011+	0111101+	0101100+
10010000	1101110	10010000	1101111
1+	1’s=0010001		2’s=0010001
0010001	-(0010001)	discarded	-(0010001)

Example : Subtract the following decimal number : 72532-3250

Solution:

1)Using 9's comp.

```
 72532
 03250-
-------
 72532
 96749+
-------
169281
     1+
-------
 69282
```

2)Using 10's comp.

```
 72532
 03250-
-------
 72532
 96750
-------
169282
```

discarded

Example : Subtract the following octal number : 256 -341

Solution :

1)Using 7's comp.	2)Using 8's comp.
256	256
341-	341-
256	256
436	437
714	715
7's comp. = 063	8's comp.= 063
-(63)	–(63)

- ***Binary Arithmetic***
  - ***Addition***

      111011 Carries

    101011 Augend

  <u>+ 11001</u> Addend

   1000100

  - ***Subtraction***

    0 1 10 0 10   Borrows

  1 0 0 1 0 1 Minuend

<u>-   1 1 0 1 1</u> Subtrahend

        1 0 1 0

- ***Multiplication***

```
        1 1 0 1 0    Multiplicand
      x   1 0 1 0    Multiplier
      -----------
        0 0 0 0 0
      1 1 0 1 0
    0 0 0 0 0
  1 1 0 1 0
 ----------------
1 0 0 0 0 0 1 0 0    Product
```

Division

```
                    1 1 0    Quotient
                ------------
Divider  1 0 0 1 ) 1 1 1 1 0 1    Dividend
                   1 0 0 1
                   ---------
                     1 1 0 0
                     1 0 0 1
                     -------
                       1 1 1    Remainder
```

- ***Octal Arithmetic*** (Use Table 1.4)
  - ***Addition*** ***Subtraction***

```
   1 1 1     Carries
   5 4 7 1   Augend
 + 3 7 5 4   Addend
 ---------
 1 1 4 4 5   Sum
```

```
 6 10  4 10  Borrows
     7   4   5   1   Minuend
   - 5   6   4   3   Subtrahend
   ----------------
   1   6   0   6   Difference
```

- ***Multiplication***

326	Multiplicand
x 67	Multiplier
2732	Partial products
2404	
26772	Product

Division

		114	Quotient
Divider	63	7514	Dividend
		63	
		114	
		63	
		364	
		314	
		50	Remainder

Hex Digit Addition Table

+	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
1	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F	10
2	2	3	4	5	6	7	8	9	A	B	C	D	E	F	10	11
3	3	4	5	6	7	8	9	A	B	C	D	E	F	10	11	12
4	4	5	6	7	8	9	A	B	C	D	E	F	10	11	12	13
5	5	6	7	8	9	A	B	C	D	E	F	10	11	12	13	14
6	6	7	8	9	A	B	C	D	E	F	10	11	12	13	14	15
7	7	8	9	A	B	C	D	E	F	10	11	12	13	14	15	16
8	8	9	A	B	C	D	E	F	10	11	12	13	14	15	16	17
9	9	A	B	C	D	E	F	10	11	12	13	14	15	16	17	18
A	A	B	C	D	E	F	10	11	12	13	14	15	16	17	18	19
B	B	C	D	E	F	10	11	12	13	14	15	16	17	18	19	1A
C	C	D	E	F	10	11	12	13	14	15	16	17	18	19	1A	1B
D	D	E	F	10	11	12	13	14	15	16	17	18	19	1A	1B	1C
E	E	F	10	11	12	13	14	15	16	17	18	19	1A	1B	1C	1D
F	F	10	11	12	13	14	15	16	17	18	19	1A	1B	1C	1D	1E

Hex Addition

- 4-bit Addition

  4 + 4 = 8

  4 + 8 = C

  8 + 7 = F

  F + E = 1D   Note “carry”

Example : Subtract the following hexadecimal number :

592-3A5

Solution :

1)Using 15's comp.

```
   592
   3A5
  -----
   592
   C5A
 -----
  11EC
     1+
  -----
   1ED
```

2) Using 16's comp.

```
   592
   3A5
  -----
   592
   C5B
  -----
  11ED
```

discarded

- ***Hexadecimal Arithmetic*** (Use Table 1.5)
  - ***Addition***

```
 1 0 1 1     Carries
   5 B A 9   Augend
+  D 0 5 8   Addend
-------------------
 1 2 C 0 1   Sum
```

  - ***Subtraction***

```
   9 10  A 10  Borrows
   A  5  B  9  Minuend
+  5  8  0  D  Subtrahend
------------------------
   4  D  A  C  Difference
```

• ***Multiplication*** ***Division***

```
   B9A5   Multiplicand
 x  D50   Multiplier
 3A0390   Partial products
96D61
9A76490   Product
```

```
               79B    Quotient
Divider  B9 ) 57F6D   Dividend
              50F
               706
               681
                85D
                7F3
                 6A   Remainder
```

There is an alternative method also to find addition and subtraction of hexadecimal numbers. The steps are as follows:

Convert each hexadecimal numbers into decimal numbers.

Add or subtract the decimal numbers obtained from step one.

Convert the decimal number obtained from step two into hexadecimal number.

The hexadecimal number obtained from step three is the final answer.

Example

Add the hexadecimal numbers $A21_{16}$ and $2B1_{16}$?

Solution: To add the above two hexadecimal numbers we have to follow the above steps:

Convert $A21_{16}$ and $2B1_{16}$ into decimal numbers:

$$A21_{16} = Ax16^2+2\times16^1+1\times16^0$$

$$= 10\times256+2\times16+1\times1$$

$$= 2560+32+1 \qquad = 2593_{10}$$

$$2B1_{16} = 2\times16^2+Bx16^1+1\times16^0$$

$$= 2\times256+11\times16+1\times1$$

$$= 512+176+1 \qquad = 689_{10}$$

$2593_{10} + 689_{10} = 3282_{10}$

Convert 3282_{10} into hexadecimal number:

16|3282	Remainder
16|205	2
16|12	13 (D)
16|0	12 (C)

So the sum is $CD2_{16}$.

Subtraction of hexadecimal numbers

Subtraction of hexadecimal numbers can be performed by using complement methods or simply as decimal subtractions.

The rule of simple hexadecimal subtraction is the digit borrowed from the immediate higher place is counted as 16.

Subtract the hexadecimal numbers ABC_{16} and $A3B_{16}$?

Solution: here we will find the subtraction of the two given

numbers

using 15's complement. So the solution for 15's complement

of $A3B_{16}$ is as follows:

15 – A = 5 ,15 – 3 = C (in hexadecimal number system 12 = C)

15 – B = 4,So the 15's complement of **$A3B_{16}$ is $5C4_{16}$.**

Now add ABC_{16} and $5C4_{16}$.

$$\begin{array}{r} A\ B\ C \\ 5\ C\ 4 \\ \hline 1\ 0\ 8\ 0 \end{array}$$

The result is 1080. Now we have to discard the left most end carry 1 and then we have to add 1 to the right most digit.

Thus 1080 becomes 80 + 1 = 81

Subtract the hexadecimal numbers $67A_{16}$ and 549_{16}

using 16’s complement

Ans 131_{16}.

Multiplication and Division of Hexadecimal Numbers

The alternative method of finding multiplication or division of hexadecimal numbers is shown below:

1- Change the hexadecimal numbers into decimal numbers.

2- Multiply or divide the hexadecimal numbers.

3- The decimal number obtained in the second step has to be changed into hexadecimal number and that is the final value.

Signed Numbers – 4-bit example

Decimal	2's comp	Sign-Mag
7	<u>0</u>111	0111
6	<u>0</u>110	0110
5	<u>0</u>101	0101
4	<u>0</u>100	0100
3	<u>0</u>011	0011
2	<u>0</u>010	0010
1	<u>0</u>001	0001
0	<u>0</u>000	0000

Signed Numbers-4 bit example

Decimal	2's comp	Sign-Mag
-8	<u>1</u>000	N/A
-7	<u>1</u>001	1111
-6	<u>1</u>010	1110
-5	<u>1</u>011	1101
-4	<u>1</u>100	1100
-3	<u>1</u>101	1011
-2	<u>1</u>110	1010
-1	<u>1</u>111	1001
-0	<u>0</u>000 (= +0)	1000

Binary Subtraction - Example

- Let n=4, A=0100_2 (4_{10}), and B=0010_2 (2_{10})
- Let's find A+B, A-B and B-A

A+B

$$
\begin{array}{r l}
0\,1\,0\,0 & \Rightarrow (4)_{10} \\
+\;\;0\,0\,1\,0 & \Rightarrow (2)_{10} \\
\hline
0\,1\,1\,0 & \quad 6
\end{array}
$$

Binary Subtraction - Example

A-B

$$\begin{array}{r} 0\,1\,0\,0 \\ -\;\;0\,0\,1\,0 \\ \hline \end{array} \qquad \begin{array}{l} \Rightarrow (4)_{10} \\ \Rightarrow (2)_{10} \end{array}$$

A+ (-B)

$$\begin{array}{r} 0\,1\,0\,0 \\ +\;\;1\,1\,1\,0 \\ \hline 1\,0\,0\,1\,0 \end{array} \qquad \begin{array}{l} \Rightarrow (4)_{10} \\ \Rightarrow (-2)_{10} \\ \quad 2 \end{array}$$

"Throw this bit" away since n=4

Binary Subtraction - Example

B-A

$$\begin{array}{rl} 0\,0\,1\,0 & \Rightarrow (2)_{10} \\ -\quad \underline{0\,1\,0\,0} & \Rightarrow (4)_{10} \end{array}$$

B + (-A)

$$\begin{array}{rl} 0\,0\,1\,0 & \Rightarrow (2)_{10} \\ +\ \underline{1\,1\,0\,0} & \Rightarrow (-4)_{10} \\ 1\,1\,1\,0 & \quad -2 \end{array}$$

$$1\,1\,1\,0_2 = -\,0\,0\,1\,0_2 = -2_{10}$$