Number systems:

Binary, Decimal, Octal and Hexadecimal

Conversion between various number systems

Definitions

- nybble = 4 bits
- byte = 8 bits
- (short) word = 2 bytes = 16 bits
- (double) word = 4 bytes = 32 bits
- (long) word = 8 bytes = 64 bits
- 1K (kilo or “kibi”) = 1,024
- 1M (mega or “mebi”) = (1K)*(1K) = 1,048,576
- 1G (giga or “gibi”) = (1K)*(1M) = 1,073,741,824

Number Systems

- ***Positional Notation***

  $N = (a_{n-1}a_{n-2} \ldots a_1a_0 \,.\, a_{-1}a_{-2} \ldots a_{-m})_r$

  where . = radix point

  r = radix or base

  n = number of integer digits to the left of the radix point

  m = number of fractional digits to the right of the radix point

  a_{n-1} = most significant digit (MSD)

  a_{-m} = least significant digit (LSD)

- ***Polynomial Notation*** (Series Representation)

  $N = a_{n-1} \times r^{n-1} + a_{n-2} \times r^{n-2} + \ldots + a_0 \times r^0 + a_{-1} \times r^{-1} \ldots + a_{-m} \times r^{-m} = \sum_{i=-m}^{n-1} a_i r^i$

- $N = (251.41)_{10} = 2 \times 10^2 + 5 \times 10^1 + 1 \times 10^0 + 4 \times 10^{-1} + 1 \times 10^{-2}$

Bases we will use

Binary:	**Base 2**
Octal:	**Base 8**
Decimal:	**Base 10**
Hexadecimal:	**Base 16**
Base-N Number System	**Base N**

Binary numbers

- Digits = {0, 1}
- $(11010.11)_2 = 1 \text{ x } 2^4 + 1 \text{ x } 2^3 + 0 \text{ x } 2^2 + 1 \text{ x } 2^1 + 0 \text{ x } 2^0 + 1 \text{ x } 2^{-1} + 1 \text{ x } 2^{-2}$

  $= (26.75)_{10}$
- 1 K (kilo) = 2^{10} = 1,024,
- 1 (mega) = 2^{20} = 1,048,576,
- 1G (giga) = 2^{30} = 1,073,741,824

Octal numbers

Digits = {0, 1, 2, 3, 4, 5, 6, 7}

$(127.4)_{8} = 1 \text{ x } 8^{2} + 2 \text{ x } 8^{1} + 7 \text{ x } 8^{0} + 4 \text{ x } 8^{-1} = (87.5)_{10}$

Hexadecimal numbers

Digits = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F}

$(B65F)_{16} = 11 \text{ x } 16^{3} + 6 \text{ x } 16^{2} + 5 \text{ x } 16^{1} + 15 \text{ x } 16^{0} = (46{,}687)_{10}$

Binary Number System

- Base 2
- Two Digits: 0, 1
- Example: 1010110_2
- Positional Number System

$$\begin{array}{cccccc} 2^{n-1} \cdots 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \\ b_{n-1} \cdots b_4 & b_3 & b_2 & b_1 & b_0 \end{array}$$

- **B**inary Dig**its** are called Bits
- Bit b_o is the least significant bit (LSB).
- Bit b_{n-1} is the most significant bit (MSB).

Decimal Number System

- Base 10
- Ten Digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
- Example: 1045_{10}
- Positional Number System

$$\begin{array}{cccccc} 10^{n-1}\cdots & 10^4 & 10^3 & 10^2 & 10^1 & 10^0 \\ d_{n-1}\cdots & d_4 & d_3 & d_2 & d_1 & d_0 \end{array}$$

- Digit d_0 is the least significant digit (LSD).
- Digit $d_{n\text{-}1}$ is the most significant digit (MSD).

Octal Number System

- Base 8
- eight Digits: 0, 1, 2, 3, 4, 5, 6, 7
- Example (127.4)$_8$
- Positional Number System

$$8^{n-1} \ldots 8^4 \quad 8^3 \quad 8^2 \quad 8^1 \quad 8^0$$

000	0
001	1
010	2
011	3

100	4
101	5
110	6
111	7

Hexadecimal Number System

- Base 16
- Sixteen Digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F
- Example: $EF56_{16}$
- Positional Number System $16^{n-1} \cdots 16^4 \;\; 16^3 \;\; 16^2 \;\; 16^1 \;\; 16^0$

0000	0
0001	1
0010	2
0011	3

0100	4
0101	5
0110	6
0111	7

1000	8
1001	9
1010	A
1011	B

1100	C
1101	D
1110	E
1111	F

Base-N Number System

- Base N
- N Digits: 0, 1, 2, 3, 4, 5, …, N-1
- Example: 1045_N
- Positional Number System

$$\begin{array}{cccccc} N^{n-1} \cdots & N^4 & N^3 & N^2 & N^1 & N^0 \\ d_{n-1} \cdots & d_4 & d_3 & d_2 & d_1 & d_0 \end{array}$$

- Digit d_o is the least significant digit (LSD).
- Digit d_{n-1} is the most significant digit (MSD).

Number Conversions

Decimal to Binary Conversion

Method I: Use repeated subtraction.
Subtract largest power of 2, then next largest, etc.

Powers of 2: 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2^n

Exponent: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 , n

2^0 2^1 2^2 2^3 2^4 2^5 2^6 2^7 2^8 2^9 2^{10} 2^n

Decimal to Binary Conversion

$$N = 1564_{10}$$

Subtract 1024: $1564-1024\ (2^{10}) = 540 \Rightarrow n=10$ or 1 in the (2^{10})'s position

Subtract 512: $540-512\ (2^{9}) = 28 \Rightarrow n=9$ or 1 in the (2^{9})'s position

$2^8=256$, $2^7=128$, $2^6 = 64$, $2^5=32 > 28$, so we have 0 in all of these positions

Subtract 16: $28-16\ (2^{4}) = 12 \Rightarrow n=4$ or 1 in (2^{4})'s position

Subtract 8: $12 – 8\ (2^{3}) = 4 \Rightarrow n=3$ or 1 in (2^{3})'s position

Subtract 4: $4 – 4\ (2^{2}) = 0 \Rightarrow n=2$ or 1 in (2^{2})'s position

Thus: $1564_{10} = (1\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0)_2$

Decimal –to– Binary Conversion

Method 2: Use repeated division by radix.

The Process : *Successive Division*

a) Divide the *Decimal Number* by 2; the remainder is the LSB of *Binary Number* .

b) If the quotation is zero, the conversion is complete; else repeat step (a) using the quotation as the Decimal Number. The new remainder is the next most significant bit of the *Binary Number.*

Decimal –to– Binary Conversion

Example: Convert the decimal number 6_{10} into its binary equivalent.

$$2\overline{)\,6\,}^{\,3} \qquad r = 0 \leftarrow \text{Least Significant Bit}$$

$$2\overline{)\,3\,}^{\,1} \qquad r = 1$$

$$2\overline{)\,1\,}^{\,0} \qquad r = 1 \leftarrow \text{Most Significant Bit}$$

$$\therefore \; 6_{10} = 110_2$$

Decimal to Binary Conversion

2 | 1564	
2 | 782	R = 0
2 | 391	R = 0
2 | 195	R = 1
2 | 97	R = 1
2 | 48	R = 1
2 | 24	R = 0
2 | 12	R = 0
2 | 6	R = 0
2 | 3	R = 0
2 | 1	R = 1
0	R = 1

Collect remainders in reverse order

1 1 0 0 0 0 1 1 1 0 0

Dec → Binary : Example 1

Convert the decimal number 26_{10} into its binary equivalent.

Solution:

$$2\,\overline{)\,26}^{\;13} \quad r = 0 \leftarrow \text{LSB}$$

$$2\,\overline{)\,13}^{\;6} \quad r = 1$$

$$2\,\overline{)\,6}^{\;3} \quad r = 0$$

$$2\,\overline{)\,3}^{\;1} \quad r = 1$$

$$2\,\overline{)\,1}^{\;0} \quad r = 1 \leftarrow \text{MSB}$$

$$\therefore\ 26_{10} = 11010_2$$

Dec → Binary : Example 2

Convert the decimal number 41_{10} into its binary equivalent.

Solution:

$$2\,\overline{)\,41}^{\,20} \quad r = 1 \leftarrow \text{LSB}$$

$$2\,\overline{)\,20}^{\,10} \quad r = 0$$

$$2\,\overline{)\,10}^{\,5} \quad r = 0$$

$$2\,\overline{)\,5}^{\,2} \quad r = 1$$

$$2\,\overline{)\,2}^{\,1} \quad r = 0$$

$$2\,\overline{)\,1}^{\,0} \quad r = 1 \leftarrow \text{MSB}$$

$$\boxed{\therefore\ 41_{10} = 101001_2}$$

Dec → Binary : More Examples

a) 13_{10} = $1\,1\,0\,1_{\,2}$

b) 22_{10} = $1\,0\,1\,1\,0_{\,2}$

c) 43_{10} = $1\,0\,1\,0\,1\,1_{\,2}$

d) 158_{10} = $1\,0\,0\,1\,1\,1\,1\,0_{\,2}$

Binary –to– Decimal Process

The Process : *Weighted Multiplication*

a) Multiply each bit of the *Binary Number* by it corresponding bit-weighting factor (i.e. Bit-0→2^0=1; Bit-1→2^1=2; Bit-2→2^2=4; etc).

b) Sum up all the products in step (a) to get the *Decimal Number*.

Example: Convert the decimal number 0110_2 into its decimal equivalent.

0	1	1	0	
2^3	2^2	2^1	2^0	Bit-Weighting Factors
8	4	2	1	

$$0 + 4 + 2 + 0 = 6_{10}$$

$$\therefore\ 0110_2 = 6_{10}$$

Binary → Dec : Example 1

Convert the binary number $\mathbf{10010_2}$ into its decimal equivalent.

Solution:

1		0		0		1		0		
2^4		2^3		2^2		2^1		2^0		
16		8		4		2		1		
16	+	0	+	0	+	2	+	0	=	18_{10}

$\therefore 10010_2 = 18_{10}$

Binary → Dec : Example 2

Convert the binary number 0110101_2 into its decimal equivalent.

Solution:

0	1	1	0	1	0	1
2^6	2^5	2^4	2^3	2^2	2^1	2^0
64	32	16	8	4	2	1

$$0 + 32 + 16 + 0 + 4 + 0 + 1 = 53_{10}$$

$$\therefore 0110101_2 = 53_{10}$$

Binary → Dec : More Examples

a) $0110_2 =$ 6_{10}

b) $11010_2 =$ 26_{10}

c) $0110101_2 =$ 53_{10}

d) $11010011_2 =$ 211_{10}

- ***Examples***
  - $(315)_{10} = (473)_8$

$8 \,\lfloor\, 315$	3	LSD
$8 \,\lfloor\, 39$	7	
$8 \,\lfloor\, 4$	4	MSD
0		

  - $(315)_{10} = (13B)_{16}$

$16 \,\lfloor\, 315$	B	LSD
$16 \,\lfloor\, 19$	3	
$16 \,\lfloor\, 1$	1	MSD
0		

- ***Examples***
  - $(0.479)_{10} = (0.3651\ldots)_8$

    MSD $\quad 3.832 \leftarrow 0.479 \times 8$

    $\quad\quad 6.656 \leftarrow 0.832 \times 8$

    $\quad\quad 5.248 \leftarrow 0.656 \times 8$

    LSD $\quad 1.984 \leftarrow 0.248 \times 8$

    …

  - $(0.479)_{10} = (0.0111\ldots)_2$

    MSD $\quad 0.9580 \leftarrow 0.479 \times 2$

    $\quad\quad 1.9160 \leftarrow 0.9580 \times 2$

    $\quad\quad 1.8320 \leftarrow 0.9160 \times 2$

    LSD $\quad 1.6640 \leftarrow 0.8320 \times 2$

    …

- ***Example***

$(18.6)_9 = (\ ?\ )_{11}$

(a) Convert to base 10 using series substitution method:

$$
\begin{aligned}
N_{10} &= 1 \times 9^1 + 8 \times 9^0 + 6 \times 9^{-1} \\
&= 9 + 8 + 0.666\ldots \\
&= (17.666\ldots)_{10}
\end{aligned}
$$

(b) Convert from base 10 to base 11 using radix divide and multiply method:

$$
\begin{aligned}
7.326 &\leftarrow 0.666 \times 11 \\
3.586 &\leftarrow 0.326 \times 11 \\
6.446 &\leftarrow 0.586 \times 11
\end{aligned}
$$

$N_{11} = (16.736\ \ldots)_{11}$

$$
\begin{array}{r|l l}
11 & 17 & 6 \\
11 & 1 & 1 \\
 & 0 &
\end{array}
$$

Counting . . . 2, 8, 10, 16

Decimal	Binary	Octal	Hexadecimal
0	00000	0	0
1	00001	1	1
2	00010	2	2
3	00011	3	3
4	00100	4	4
5	00101	5	5
6	00110	6	6
7	00111	7	7
8	01000	10	8
9	01001	11	9
10	01010	12	A
11	01011	13	B
12	01100	14	C
13	01101	15	D
14	01110	16	E
15	01111	17	F
16	10000	20	10
17	10001	21	11
18	10010	22	12
19	10011	23	13

Decimal ↔ Octal Conversion

The Process: Successive Division

- Divide the decimal number by **8**; the remainder is the LSB of the **octal** number .
- If the quotation is zero, the conversion is complete. Otherwise repeat step (a) using the quotation as the decimal number. The new remainder is the next most significant bit of the **octal** number.

Example:

Convert the decimal number 94_{10} into its octal equivalent.

$$8\overline{)94}^{\,11} \quad r = 6 \leftarrow \text{LSB}$$

$$8\overline{)11}^{\,1} \quad r = 3$$

$$8\overline{)1}^{\,0} \quad r = 1 \leftarrow \text{MSB}$$

$$\therefore \; 94_{10} = 136_8$$

Example: Dec → Octal

Convert the decimal number $\mathbf{189_{10}}$ into its octal equivalent.

Solution:

$$8\overline{)189}\ \ 23 \quad r = 5 \quad \leftarrow \text{LSB}$$

$$8\overline{)23}\ \ 2 \quad r = 7$$

$$8\overline{)2}\ \ 0 \quad r = 2 \quad \leftarrow \text{MSB}$$

$$\therefore \ 189_{10} = 275_8$$

Octal ↔ Decimal Process

The Process: Weighted Multiplication

- Multiply each bit of the **Octal** Number by its corresponding bit-weighting factor (i.e., Bit-0→8^0=1; Bit-1→8^1=8; Bit-2→8^2=64; etc.).
- Sum up all of the products in step (a) to get the decimal number.

Example:

Convert the octal number 136_8 into its decimal equivalent.

1	3	6		
8^2	8^1	8^0		Bit-Weighting Factors
64	8	1		
64 +	24 +	6	=	94_{10}

$\therefore\ 136_8 = 94_{10}$

Octal → Dec

Example:

Convert the octal number $\mathbf{134_8}$ into its decimal equivalent.

Solution:

1		3		4		
$\mathbf{8^2}$		$\mathbf{8^1}$		$\mathbf{8^0}$		
64		**8**		**1**		
64	+	**24**	+	**4**	=	92_{10}

$\therefore\ 134_8 = 92_{10}$

Decimal ↔ Hexadecimal Conversion

The Process: Successive Division

- Divide the decimal number by **16**; the remainder is the LSB of the **hexadecimal** number.
- If the quotation is zero, the conversion is complete. Otherwise repeat step (a) using the quotation as the decimal number. The new remainder is the next most significant bit of the **hexadecimal** number.

Example:

Convert the decimal number 94_{10} into its hexadecimal equivalent.

$$16 \overline{)\,94\,}^{\,5} \qquad r = E \quad \leftarrow \text{LSB}$$

$$16 \overline{)\,5\,}^{\,0} \qquad r = 5 \quad \leftarrow \text{MSB}$$

$$\therefore\ 94_{10} = 5E_{16}$$

Example: Dec $\rightarrow$ Hex

Convert the decimal number $\mathbf{429_{10}}$ into its hexadecimal equivalent.

Solution:

$$16 \overline{)\,429}^{\;26} \quad r = D\,(13) \;\leftarrow \text{LSB}$$

$$16 \overline{)\,26}^{\;1} \quad r = A\,(10)$$

$$16 \overline{)\,1}^{\;0} \quad r = 1 \qquad \leftarrow \text{MSB}$$

$$\therefore \boxed{429_{10} = 1AD_{16} = 1AD_{H}}$$

Hexadecimal ↔ Decimal Process

The Process: Weighted Multiplication

- Multiply each bit of the **hexadecimal** number by its corresponding bit-weighting factor (i.e., Bit-0→16^0=1; Bit-1→16^1=16; Bit-2→16^2=256; etc.).
- Sum up all of the products in step (a) to get the decimal number.

Example: Convert the octal number $\mathbf{5E_{16}}$ into its decimal equivalent.

5		E		
16^1		16^0		
16		1		Bit-Weighting Factors
80	+	14	=	94_{10}

$\therefore\ 5E_{16} = 94_{10}$

Example: Hex → Dec

Convert the hexadecimal number **$B2E_H$** into its decimal equivalent.

Solution:

B	2	E
16^2	16^1	16^0
256	16	1

2816 + 32 + 14 = 2862_{10}

$\therefore$ $B2E_H = 2862_{10}$

Example: Hex → Octal

Convert the hexadecimal number **$5A_H$** into its octal equivalent.

Solution:

First convert the hexadecimal number into its decimal equivalent, then convert the decimal number into its octal equivalent.

5		A		
16^1		16^0		
16		1		
80	+	10	=	90_{10}

$$8\,\overline{)\,90}^{\,11} \quad r = 2 \quad \leftarrow \text{LSB}$$

$$8\,\overline{)\,11}^{\,1} \quad r = 3$$

$$8\,\overline{)\,1}^{\,0} \quad r = 1 \quad \leftarrow \text{MSB}$$

$$\boxed{\therefore\ 5A_H = 132_8}$$

Example: Octal → Binary

Convert the octal number **132_8** into its binary equivalent.

Solution:

First convert the octal number into its decimal equivalent, then convert the decimal number into its binary equivalent.

1	3	2
8^2	8^1	8^0
64	8	1

64 + 24 + 2 = 90_{10}

$$
\begin{array}{llll}
2 \,\big)\, 90 = 45 & r = 0 & \leftarrow \text{LSB} \\
2 \,\big)\, 45 = 22 & r = 1 & \\
2 \,\big)\, 22 = 11 & r = 0 & \\
2 \,\big)\, 11 = 5 & r = 1 & \\
2 \,\big)\, 5 = 2 & r = 1 & \\
2 \,\big)\, 2 = 1 & r = 0 & \\
2 \,\big)\, 1 = 0 & r = 1 & \leftarrow \text{MSB}
\end{array}
$$

$\therefore\ 132_8 = 1011010_2$

Binary ↔ Octal ↔ Hex Shortcut

Because binary, octal, and hex number systems are all powers of two (which is the reason we use them) there is a relationship that we can exploit to make conversion easier.

$$1\,0\,1\,1\,0\,1\,0_2 = 132_8 = 5A_H$$

To convert directly between binary and octal, group the binary bits into sets of 3 (because $2^3 = 8$). You may need to pad with leading zeros.

$$\underbrace{0\,0\,1}_{1}\,\underbrace{0\,1\,1}_{3}\,\underbrace{0\,1\,0}_{2}{}_{2} = \underbrace{1}_{0\,0\,1}\quad\underbrace{3}_{0\,1\,1}\quad\underbrace{2}_{0\,1\,0}{}_{8}$$

To convert directly between binary and hexadecimal number systems, group the binary bits into sets of 4 (because $2^4 = 16$). You may need to pad with leading zeros.

$$\underbrace{0\,1\,0\,1}_{5}\,\underbrace{1\,0\,1\,0}_{A}{}_{2} = \underbrace{5}_{0\,1\,0\,1}\quad\underbrace{A}_{1\,0\,1\,0}{}_{16}$$

Example: Binary ↔ Octal ↔ Hex

Using the shortcut technique, convert the hexadecimal number $A6_{16}$ into its binary & octal equivalent.

Solution:

First convert the hexadecimal number into binary by expanding the hexadecimal digits into binary groups of (4).

$$\overbrace{1\ 0\ 1\ 0}^{A}\ \overbrace{0\ 1\ 1\ 0}^{6}{}_{16}$$

$$\therefore\ A6_{16} = 10100110_2$$

Convert the binary number into octal by grouping the binary bits into groups of (3).

$$\underbrace{0\ 1\ 0}_{2}\ \underbrace{1\ 0\ 0}_{4}\ \underbrace{1\ 1\ 0}_{6}$$

$$\therefore\ 10100110_2 = 246_8$$

Binary to Hex Conversion

1. Divide binary number into 4-bit groups

0 1 1 0 | 0 0 0 1 | 1 1 0 0

Pad with 0's
If unsigned number

2. Substitute hex digit for each group

Pad with sign bit
if signed number

$61C_{16}$

Decimal to Hex Conversion

Method 2: Use repeated division by radix.

16| 1564

16| 97 ______ R = 12 = C

16| 6 ______ R = 1

0 R = 6

$$N = 61C_{16}$$

Hexadecimal to Binary Conversion

Example

1. Convert each hex digit to equivalent binary

$$(1\ E\ 9\ C\ )_{16}$$

$$(0001\quad 1110\quad 1001\quad 1100)_2$$

- ***Important Number Systems***

Decimal	Binary	Octal	Hexadecimal
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	1	F
16	10000	20	10