Chapter One
Matrices

A matrix (plural matrices) is a rectangular array of real numbers, which is enclosed in large brackets like [ ]or( ). Matrices are generally denoted by boldface capital letters such as **A**, **B**, or **C**. The matrix is either rectangular or square. If the number of rows is not equal the number of columns, the matrix is called a rectangular matrix. If the number of rows is equal the number of columns, the matrix is called a square matrix. The real numbers which form the array are called **entries** or **elements** of the matrix. The elements in any horizontal line form a row and those in any vertical line form a column of the matrix.

If a matrix has **m** rows and **n** columns, it is said to be a matrix of size $m \times n$ (read m by n). for example

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ . & . & . & & . \\ & . & . & & . \\ . & . & . & & . \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix} \text{ or } A = \lfloor a_{i,j} \rfloor$$

If $m \neq n$, A is called a rectangular matrix, on the other hand, A is called a square matrix if $m = n$.

Some Types of Matrices

There many types of matrices (say: special matrices) which play an important role in matrices algebra.

Square Matrix

The square matrix is a matrix whose number of rows equals the number of columns, such as

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \\ 5 & 2 & 7 \end{bmatrix}$$

Trace of Square Matrix:

The sum of the diagonal of a square matrix A, is called the *trace* of A.

$$tr(A) = \sum_{i=1}^{n} a_{ii} = a_{11} + a_{22} + \cdots + a_{nn}$$

Example 1:

Find the trace of the following matrix.

$$B = \begin{pmatrix} 7 & 1 & 4 \\ 2 & 4 & 1 \\ 0 & 3 & 8 \end{pmatrix}$$

Solution:

$$tr(B) = \sum_{i=1}^{n} a_{ii} = 7 + 4 + 8 = 19$$

Some Properties of the Trace of matrix

1. $tr(A^T) = tr(A)$
2. $tr(A \pm B) = tr(A) \pm \mathrm{tr}(B)$
3. $tr(AB) = tr(BA)$

(in general $tr(AB) \neq tr(A)tr(B)$)

Rectangular Matrix

The rectangular matrix is a matrix whose number of rows does not equal the number of columns, such as

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

and

$$B = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

Where the matrix A is of size 3×2 and the matrix B is of size 2×3.

Zero Matrix

Zero matrix is a matrix whose all elements are zeros and it is denoted **O** where

$$O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ is a zero matrix of size } 3 \times 3,$$

$$O = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ is a zero matrix of size } 2 \times 4 \text{ and so}$$

on.

Diagonal Matrix

The diagonal matrix, which is denoted by **D** is a square matrix has scalar elements, not necessarily equal, on the main diagonal (North West – South East) and zeros on the off-diagonal positions.

$$D = \begin{bmatrix} a_{11} & 0 & 0 & \cdots & & & 0 \\ 0 & a_{22} & 0 & \cdots & & & 0 \\ \cdot & \cdot & a_{33} & \cdots & & & 0 \\ \cdot & \cdot & \cdot & \cdots & & & \\ \cdot & \cdot & \cdot & \cdots & & & \\ \cdot & \cdot & \cdot & \cdots & & & \\ 0 & 0 & 0 & 0 & \cdots & & a_{nn} \end{bmatrix} =$$

$$= \quad \operatorname{diag}\left[a_{11} \quad a_{22} \quad . \; . \; . \quad a_{nn}\right]$$

$$D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix} = diag[2 \quad 3 \quad 5]$$

Identity Matrix (Unit Matrix)

The identity matrix, which is denoted by **I**, is a matrix with units on principle diagonal and zeros on off- diagonal positions such as

$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is an identity matrix of second order (2×2).

$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is an identity matrix of third order (3×3).

$I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ is an identity matrix of fourth order (4×4).

Transpose Matrix

If $A = (a_{ij})_{m \times n}$ and $A' = (a_{ij})_{n \times m}$ then A' is said to be the transpose matrix of A.

Example 2:

$$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$ then the transpose of A is

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$$A'=\begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$$

Example 3:

$$A=\begin{bmatrix} 1 & 3 & 4 \\ 2 & -1 & 6 \\ 5 & -2 & 7 \end{bmatrix}$$

then the transpose of A is $A'=\begin{bmatrix} 1 & 2 & 5 \\ 3 & -1 & -2 \\ 4 & 6 & 7 \end{bmatrix}$

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Properties of Transpose

If A', B' *and* C' are transposes of A, B, and C respectively, and if k is a scalar, we have

(a) $(A')' = A$

(b) $(k\,A)' = k\,A'$

(c) $(A \pm B \pm c)' = A' \pm B' \pm C'$

(d) $(A\,B)' = B'\,A'$

(e) $(ABC)' = C'\,B'\,A'$

Example 4:

Given $A=\begin{bmatrix} -1 & 5 \\ 2 & 0 \\ 1 & 3 \end{bmatrix}$ and $B=\begin{bmatrix} 3 & 1 \\ -2 & 4 \\ -1 & 5 \end{bmatrix}$

(a) Find A' , B'

(b) Prove that: ﻳ أثبت

(i) $(kA)'=kA'$, $k=-2$

(ii) $(A+B)'=A'+B'$

Solution:

(a) $A'=\begin{bmatrix} -1 & 2 & 1 \\ 5 & 0 & 3 \end{bmatrix}$, $B'=\begin{bmatrix} 3 & -2 & -1 \\ 1 & 4 & 5 \end{bmatrix}$

(b) (i) $k\,A=-2\begin{bmatrix} -1 & 5 \\ 2 & 0 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 2 & -10 \\ -4 & 0 \\ -2 & -6 \end{bmatrix}$

$LHS = (kA)' = \begin{bmatrix} 2 & -4 & -2 \\ -10 & 0 & -6 \end{bmatrix}$

$RHS = k\,A' = -2\begin{bmatrix} -1 & 2 & 1 \\ 5 & 0 & 3 \end{bmatrix} = \begin{bmatrix} -2 & -4 & -2 \\ -10 & 0 & -6 \end{bmatrix}$

$$\therefore LHS = RHS$$

(ii) $A+B=\begin{bmatrix}-1 & 5\\ 2 & 0\\ 1 & 3\end{bmatrix}+\begin{bmatrix}3 & 1\\ -2 & 4\\ -1 & 5\end{bmatrix}=\begin{bmatrix}2 & 6\\ 0 & 4\\ 0 & 8\end{bmatrix}$

$LHS=(A+B)'=\begin{bmatrix}2 & 0 & 0\\ 6 & 4 & 8\end{bmatrix}$

$RHS=A'+B'=\begin{bmatrix}-1 & 2 & 1\\ 5 & 0 & 3\end{bmatrix}+\begin{bmatrix}3 & -2 & -1\\ 1 & 4 & 5\end{bmatrix}=\begin{bmatrix}2 & 0 & 0\\ 6 & 4 & 8\end{bmatrix}$

$$\therefore LHS=RHS$$

Triangle Matrix

The triangle matrix is either an upper triangle or a lower triangle. An upper triangle matrix exists if:

$$a_{ij}=0 \quad \forall\, i>j \quad and \quad \neq 0 \quad \forall\, i\leq j$$

Example 5:

$U=\begin{bmatrix}a_{11} & a_{12} & a_{13}\\ 0 & a_{22} & a_{23}\\ 0 & 0 & a_{33}\end{bmatrix}$ is an upper triangle matrix of order 3×3

On the other hand, the lower triangle matrix exists if:

$$a_{ij}=0 \quad \forall\, i\leq j \quad and \quad \neq 0 \quad \forall\, i>j$$

Example 6:

$$L = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$ is a lower triangle matrix of order 3×3

Symmetric Matrix

A is said to be symmetric if it equals to its transpose, that is if $A = A'$, then A is called a symmetric matrix. In other words if $a_{ij} = a_{ji} \Leftrightarrow \lfloor a_{ij} \rfloor$ is a symmetric matrix.

Example 7:

prove that: $A = \begin{bmatrix} 2 & 5 & 4 \\ 5 & 3 & 6 \\ 4 & 6 & 7 \end{bmatrix}$ is a symmetric matrix.

Proof:

$$A' = \begin{bmatrix} 2 & 5 & 4 \\ 5 & 3 & 6 \\ 4 & 6 & 7 \end{bmatrix} \Rightarrow A = A' \Rightarrow \text{A is a symmetric matrix.}$$

SKEW-SYMMETRIC MATRICES

A matrix A such that $A^T = -A$ is called skew-symmetric, i.e., $a_{ji} = -a_{ij}$ for all i and j.

The main diagonal elements of a skew-symmetric matrix must be zero.

Example 8:

$$A=\begin{bmatrix}0 & 2 & 3\\ -2 & 0 & 5\\ -3 & -5 & 0\end{bmatrix},\quad A^T=\begin{bmatrix}0 & -2 & -3\\ 2 & 0 & -5\\ 3 & 5 & 0\end{bmatrix}$$

We notice that $A^T = -A$

Since $A^T = -A$, the matrix A is a skew- symmetric matrix.

Non Singular Matrix

A matrix A is said to be a nonsingular if it has an inverse. If A does not have an inverse, then it is said to be a singular matrix. For simplicity, if $|A| \neq 0$ then A is a nonsingular matrix. In contrast, if $|A| = 0$ then A is a singular matrix. That is A is said to be a singular if and only if $|A| = 0$.

Example 9:

$A=\begin{bmatrix}1 & 2\\ 3 & 5\end{bmatrix}$ is a nonsingular matrix because

$|A| = 1(5)-2(3)=5-6=-1\neq 0$

Example 10:

$A=\begin{bmatrix}2 & 6\\ 1 & 3\end{bmatrix}$ is a singular matrix because

$|A| = 2(3)-6(1)=6-6=0$

Cofactors Matrix

The matrix of cofactor can be obtained by determine the cofactor of each element in the matrix. The cofactor of an element can be obtained by eliminate the row and the column in which this element stand (exists) with position sign.

Example 11:

$A=\begin{bmatrix}3 & 4 & -2\\ 1 & 6 & 5\\ 2 & 1 & -1\end{bmatrix}$, then the cofactor of each element will be:

$+\begin{vmatrix}6 & 5\\ 1 & -1\end{vmatrix} = 6\,(-1) - 5(1) = -11$ is a cofactor of element 3 ,

$-\begin{vmatrix}1 & 5\\ 2 & -1\end{vmatrix} = -1(-1) - 5(2) = -9$ is a cofactor of element 4 ,

$+\begin{vmatrix}1 & 6\\ 2 & 1\end{vmatrix} = 1(1) - 6(2) = -11$ is a cofactor of element -2 ,

$-\begin{vmatrix}4 & -2\\ 1 & -1\end{vmatrix} = -\,4(-1) - (-2)(1) = 6$ **is a cofactor of element 1 ,**

$+\begin{vmatrix}3 & -2\\ 2 & -1\end{vmatrix} = 3(-1) - (-2)(2) = 1$ **is a cofactor of element 6,**

$-\begin{vmatrix} 3 & 4 \\ 2 & 1 \end{vmatrix} = -3(1) - 4(2) = -11$ is a cofactor of element 5,

$+\begin{vmatrix} 4 & -2 \\ 6 & 5 \end{vmatrix} = 4(5) - (-2)(6) = 32$ is a cofactor of element 2,

$-\begin{vmatrix} 3 & -2 \\ 1 & 5 \end{vmatrix} = -3(5) - (-2)(1) = -13$ is a cofactor of element 1,

$+\begin{vmatrix} 3 & 4 \\ 1 & 6 \end{vmatrix} = 3(6) - 4(1) = 14$ is a cofactor of element -1

And the Cofactors Matrix will be:

$$\begin{bmatrix} -11 & -9 & -11 \\ 6 & 1 & -11 \\ 32 & -13 & 14 \end{bmatrix}$$

Row Matrix

A rectangular matrix having only one row is called a row matrix or row vector, such as

$A = [a_{11} \quad a_{12} \quad a_{13}]$ is a row matrix of size 1×3

Column Matrix

A rectangular matrix having only one column is called a column matrix or column vector, such as

$$A=\begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \\ a_{41} \end{bmatrix}$$ is a column matrix of size 4×1

Equality of Matrices

Two matrices A and B are said to be equal if:

(a) They are of the same size.

(b) Their corresponding elements are equal.

For example:

$A=\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$ *and* $B=\begin{bmatrix} 2 & x \\ y & 3 \end{bmatrix}$ are said to be equal if $x=1$ and $y=4$

Example 12:

Find the value of x, y an z if:

$$\begin{bmatrix} 4 & x & 3 \\ y & -1 & 2 \end{bmatrix}=\begin{bmatrix} y-1 & 2-x & 3 \\ 5 & z-1 & 2 \end{bmatrix}$$

Solution:

Since the two matrices are equal, then

$y = 5$

$x = 2 - x \quad \Rightarrow x + x = 2$

$\Rightarrow \quad 2x = 2$

$\Rightarrow \quad x = 1$

$Z - 1 = -1 \quad \Rightarrow \quad z = 1 - 1$

$\Rightarrow \quad z = 0$

Matrix-Operations

In this section, we will deal with some operations on matrices such as scalar multiplication of matrix, addition, subtraction and multiplication of matrices.

Scalar Multiplication of Matrix

Scalar Multiplication of Matrix refers to the operation of multiplying the matrix by a real number.

Example 13:

If $A=\begin{bmatrix} 4 & 3 & 2 \\ -6 & 1 & -1 \end{bmatrix}$, find 3 A.

Solution:

$$3A=3\begin{bmatrix} 4 & 3 & 2 \\ -6 & 1 & -1 \end{bmatrix}=\begin{bmatrix} 12 & 9 & 6 \\ -18 & 3 & -3 \end{bmatrix}$$

Addition and Subtraction of Matrices

Two matrices A and B of the same size can be added (or subtracted) by adding (or subtracting) their corresponding elements.

Example 14:

If $A=\begin{bmatrix} 2 & -4 & -5 \\ 3 & 6 & -1 \\ 7 & 4 & 5 \end{bmatrix}$ and $B=\begin{bmatrix} -1 & -5 & 4 \\ -2 & -7 & 3 \\ -6 & -1 & -3 \end{bmatrix}$

Find $3A-5B+4I$

Solution:

$3A - 5B + 4I$

$$= 3\begin{bmatrix} 2 & -4 & -5 \\ 3 & 6 & -1 \\ 7 & 4 & 5 \end{bmatrix} - 5\begin{bmatrix} -1 & -5 & 4 \\ -2 & -7 & 3 \\ -6 & -1 & -3 \end{bmatrix} + 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -12 & -15 \\ 9 & 18 & -3 \\ 21 & 12 & 15 \end{bmatrix} + \begin{bmatrix} 5 & 25 & -20 \\ 10 & 35 & -15 \\ 30 & 5 & 15 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 15 & 13 & -35 \\ 19 & 57 & -18 \\ 51 & 17 & 34 \end{bmatrix}$$

Multiplication of Matrices

Two matrices A and B can be multiplied if the number of column in the first is equal to the number of rows in the second.

Example 15:

Given $A=\begin{bmatrix} 3 & 4 \\ 0 & 2 \end{bmatrix}$ and $B=\begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 0 \end{bmatrix}$

Find AB and BA.

Solution:

$$AB=\begin{bmatrix}3 & 4\\0 & 2\end{bmatrix}\begin{bmatrix}3 & 2 & 1\\4 & 1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}3(3)+4(4) & 3(2)+4(1) & 3(1)+4(0)\\0(3)+2(4) & 0(2)+2(1) & 0(1)+2(0)\end{bmatrix}$$

$$=\begin{bmatrix}25 & 10 & 3\\8 & 2 & 0\end{bmatrix}$$

Since the number of column in B is not equal to the number of rows in A, then BA is not defined.

Inverse of a matrix

The inverse of a matrix which is denoted by A^{-1} for a matrix A, if exists, can obtained by using some methods such as cofactors method, row-reduction method, partitioning, …, etc.

In this section, we will obtain the inverse of a matrix by using the cofactors method. The procedure of obtaining the inverse of a matrix by this method is outlined in the following steps:

1. Find the determinant of the matrix $|A|$.
2. Obtain the matrix of cofactors.
3. Find the adjoint matrix by transpose the matrix of cofactors.

4. Divide the adjoint matrix by $|A|$.

Example 16:

If $A=\begin{bmatrix} 2 & 4 \\ -1 & 2 \end{bmatrix}$, find A^{-1}.

Solution:

1. $|A|=2\times2-4(-1)=4+4=8$

Since $|A|\neq0$, then A is a non singular matrix and its inverse can be obtained

2. Cofactors matrix $\begin{bmatrix} 2 & 1 \\ -4 & 2 \end{bmatrix}$

3. Adj (A) = $\begin{bmatrix} 2 & -4 \\ 1 & 2 \end{bmatrix}$

4. $A^{-1}=\frac{1}{8}\begin{bmatrix} 2 & -4 \\ 1 & 2 \end{bmatrix}$

Check!

$$A^{-1}A=\frac{1}{8}\begin{bmatrix} 2 & -4 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ -1 & 2 \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

That is $A^{-1}A=I$

Example 17:

If $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ 3 & 7 & 8 \end{bmatrix}$, find A^{-1}

Solution:

1. $|A| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ 3 & 7 & 8 \end{vmatrix} = 1\begin{vmatrix} 5 & 7 \\ 7 & 8 \end{vmatrix} - 2\begin{vmatrix} 2 & 7 \\ 3 & 8 \end{vmatrix} + 3\begin{vmatrix} 2 & 5 \\ 3 & 7 \end{vmatrix}$

$= 1(5\times 8 - 7\times 7) - 2(2\times 8 - 7\times 3) + 3(2\times 7 - 5\times 3)$

$= 1(-9) - 2(-5) + 3(-1)$

$= -9 + 10 - 3 = -2$

Since $|A| \neq 0$, then A is a non singular matrix and its inverse can be obtained.

2. Matrix of cofactors $= \begin{bmatrix} +\begin{vmatrix} 5 & 7 \\ 7 & 8 \end{vmatrix} & -\begin{vmatrix} 2 & 7 \\ 3 & 8 \end{vmatrix} & +\begin{vmatrix} 2 & 5 \\ 3 & 7 \end{vmatrix} \\ -\begin{vmatrix} 2 & 3 \\ 7 & 8 \end{vmatrix} & +\begin{vmatrix} 1 & 3 \\ 3 & 8 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 3 & 7 \end{vmatrix} \\ +\begin{vmatrix} 2 & 3 \\ 5 & 7 \end{vmatrix} & -\begin{vmatrix} 1 & 3 \\ 2 & 7 \end{vmatrix} & +\begin{vmatrix} 1 & 2 \\ 2 & 5 \end{vmatrix} \end{bmatrix}$

$$= \begin{bmatrix} -9 & 5 & -1 \\ 5 & -1 & -1 \\ -1 & -1 & 1 \end{bmatrix}$$

3. $Adj(A) = \begin{bmatrix} -9 & 5 & -1 \\ 5 & -1 & -1 \\ -1 & -1 & 1 \end{bmatrix}$

4. $A^{-1} = \frac{1}{|A|} Adj(A)$

$$A^{-1} = -\frac{1}{2} \begin{bmatrix} -9 & 5 & -1 \\ 5 & -1 & -1 \\ -1 & -1 & 1 \end{bmatrix}$$

Check!

$$A^{-1} A = -\frac{1}{2} \begin{bmatrix} -9 & 5 & -1 \\ 5 & -1 & -1 \\ -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ 3 & 7 & 8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Some Properties of Inverse

1. $AA^{-1} = A^{-1}A = I$
2. $(A^{-1})^{-1} = A$
3. $(A')^{-1} = (A^{-1})'$
4. $(AB)^{-1} = B^{-1}A^{-1}$

$AA^{-1} = A^{-1} = I$

Exercises One

1. Given $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ *and* $B = \begin{bmatrix} 2 & -1 \\ -3 & 1 \end{bmatrix}$

Prove that:

(a) $(AB)^{-1} = B^{-1}A^{-1}$

(b) $(A^{-1}B)^{-1} = B^{-1}A$

2. Find the value of each of the following determinants: $|A|$

(a) $\begin{vmatrix} 1 & -2 \\ 3 & -4 \end{vmatrix}$

(b) $\begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix}$

(c) $\begin{vmatrix} -5 & 1 \\ 2 & -3 \end{vmatrix}$

(d) $\begin{vmatrix} -1 & 6 & -3 \\ 2 & -5 & 1 \\ -2 & 3 & 4 \end{vmatrix}$

(e) $\begin{vmatrix} 5 & 3 & -1 \\ 4 & 7 & 1 \\ 2 & -2 & 7 \end{vmatrix}$

3. Given $A=\begin{bmatrix} 1 & 8 & 5 \\ 3 & 4 & 2 \\ 9 & -3 & -1 \end{bmatrix}$ and $B=\begin{bmatrix} 1 & 3 & -4 \\ -5 & -2 & 7 \\ 8 & -7 & 3 \end{bmatrix}$

Find:

(a) $5A - 3B + 4I$

(b) AB , BA. Is $AB \neq BA$?

(c) $A'B'$, $(BA)'$. What do you notice?

4. Given $A=\begin{bmatrix} 6 & 8 \\ 2 & 4 \end{bmatrix}$, $B=\begin{bmatrix} -3 & -4 \\ -1 & -5 \end{bmatrix}$, $C=\begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$

I. Find:

(a) A', B', C'

(b) A^{-1}, B^{-1} and C^{-1}

$A^{-1} = \frac{1}{|A|} \cdot adj(A)$

II. Prove that:

(c) $(ABC)' = C'B'A'$

(d) $(A')^{-1} = (A^{-1})'$

5. Find the inverse of each of the following matrices:

$A=\begin{bmatrix} 6 & 8 \\ 2 & 4 \end{bmatrix}$, $B=\begin{bmatrix} -3 & -4 \\ -1 & -5 \end{bmatrix}$, $C=\begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$

6. Find the inverse of each of the following matrices:

$A=\begin{bmatrix} 1 & 8 & 5 \\ 3 & 4 & 2 \\ 9 & -3 & -1 \end{bmatrix}$ $B=\begin{bmatrix} 1 & 3 & -4 \\ -5 & -2 & 7 \\ 8 & -7 & 3 \end{bmatrix}$

7. Given $A=\begin{bmatrix} 2 & 1 & 5 \\ 6 & 7 & -3 \\ 4 & 3 & -1 \end{bmatrix}$, $B=\begin{bmatrix} 9 & -7 & 3 \\ 8 & 6 & -1 \\ 5 & -2 & 1 \end{bmatrix}$,

$C=\begin{bmatrix} -2 & 1 & 4 \\ -4 & 7 & 2 \\ -1 & 3 & 2 \end{bmatrix}$

Carry out the following operations:

(a) find AB and BA. State your comment.

(b) Find A' , B' and C'

(c) Find A^{-1} , B^{-1} and C^{-1}

(d) Compute $10A-9B-8I$

(e) Prove that:

i. $(A-B)^2 = A^2 - 2AB + B^2$

ii. $(A+B)^2 = A^2 + 2AB + B^2$

iii. $(A+B-C)' = A' - B' - C'$

iv. $(AB)' = B'A'$

vi. $(ABC)' = C'B'A'$

v. $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

8. Solve the following systems of equations by finding the inverse of the coefficient matrix.

(a) $2x - 3y = 1$

$3x + 4y = 10$

(b) $3x + 2y = 1$

$2x - y = 3$

(c) $4x + 5y = 14$

$2x - 3y = 1$

(d) $2x - y + 3z = -3$

$x + y + z = 2$

$3x + 2y - z = 8$

(e) $x + 2y - z = 1$

$2z - 3x = 2$

$3y + 2z = 5$

(f) $x + 2y - 3z = 1$

$y - 2x + z = 3$

$2z + x - 2 = 0$