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**Research and** **article**A picture containing text, font, logo, graphics

Description automatically generated some mathematical models for the interpretation of Electromagnetic wave propagation

Under the supervision of

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Specialization: - Applied Mathematics

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We bow our heads with the deepest gratitude to Almighty Allah, who enabled us to complete this piece of work. We offer my deepest words of thanks to the holy prophet Muhammed (peace be upon him) who is forever a torch of guidance for humanity. Also, He showed the light of knowledge to the humanity as a whole.

We express our deepest gratitude and our sincere thanks to ouir main supervisor **Dr. Ahmed Refaie Ali Yousof,** lecturer of applied Mathematics, Mathematics and Computer Science department, Faculty of Science, Menoufia University, who suggested the subject and suggesting the ideas and for guidance in this research& article. We would like to express our deep appreciation to **Dr. Ahmed Refaie Ali Yousof** for his’ inspiring guidance, supervision, appreciable help, and for provided us a supportive environment to work comfortably and confidently. As well as his kind facilities offered through this investigation. His guidance, devotion and keen interest enabled us to develop a deep understanding of the subject; therefore, we are grateful to him. His genuine concern and his sincere are irreplaceable.

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Last but not the least, we are deeply indebted to my parents and my Family for their care, kindness, for their serious help, inspiration, continuous encouragement, moral support, their assurance and good-will in every way.

***All participants***

(2023)

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**English Summary**

In this research and article, some mathematical models of electromagnetic wave propagation were studied. In the first chapter, the necessary mathematical foundations were studied to derive the wave equation from Maxwell's equations. A survey was conducted on the subject of the study for some scientific references related to the subject of the study. In the second chapter, the focus was on solving a mathematical problem, and the general solution to the wave equation in the fractional space was found, and this is the new addition in this research and article. The solution was compared to obtain the general solution in the right space as a special case. It turns out that the propagation of the electromagnetic wave is related to the values of the parameters of the fractional space.

|  |  |
| --- | --- |
|  |  |
|  |  |
| **Chapter one: General Introduction** |  |

##### classical electrodynamics:

* Electro statics

Dynamics

* Magneto statics

We are planning to discuss the condition where the electrostatics and magnetostatics how the discussion on the electrostatics and magnetostatics can be taken to the case of dynamics so which means that in all the previous lectures we have been discussing the case where the charge density is independent of time and the same is the case of the magnetostatics where we say that the current density is independent of time so how the current density will be independent of time is that when the charge density does not change with time charge can move that is not a problem only when the charge is moving you will get the current so when the charge moves we don’t say that the charge density is the function of time so there the word density is what you have to pay the tension.

Charge density means number of charges per unit length so that will not change when the charges are moving so that is what we mean to say the magnetostatics so charge density should not fluctuate with time so if that is the case, we say that the current density is uniform and in which case the magnetic field generated is also uniform.

Uniform means constant so similar situation so all these conditions we have done already which means that we know how to write down the experimental observations into the corresponding vector form and how to exploit the vector calculus that means some of the product rules and some of the properties of the vector functions and theorems we have been using to establish certain equations what are known as the maxwell’s equations so by now writing down all the equations together in one place we will be able to understand the whole concept together in a single place. Now that we have seen different equations scattered in different lectures so if you can write down all the equations in one place your understanding will be better so once we do that is also going to be helpful for you to make a transition from the static case to the dynamic case where in we consider the charge density that can fluctuate with respect to time so that we will be having time dependent electric field and time dependent magnetic field so we can we can then naturally ask the question what is the in what is the methodology that we can make a transition from static to the dynamics so when you are making a transition we do not use the word two different words here so one is the electrodynamics and second one is magneto dynamics so we do not require two terms for that electrodynamics naturally involves magnetism.

Naturally so you don’t require an additional terminology called magneto dynamics so that is the reason why we have a common name only electrodynamics is sufficient or instead of electrodynamics you can say electromagnetic theory so we are going to see later that the phenomenon of electricity or we say the phenomenon of electric field and the magnetic field they are indistinguishable from a relativistic point of view so that will be understood later when discuss the relativity so because of this reason what happens is you don’t require two names magneto dynamics is not required so that is the reason only one name is enough electrodynamics so what we will do is that whatever equation that we had in the last several of these classes let us write down in a proper way so that the entire thing becomes simple in one place so I will now show shat are the maxwell’s equations that we have derived till now so here it is there so what are the equations that we have

|  |  |
| --- | --- |
| **What we have till now:**  --------- (1)  ------- (2)    (Valid only for electrostatics) | --------- (3)  ---- (4)      (Valid only for magnetostatics) |
| Same equations in integral form  ---- (1)  ----------- (2) | ---- (3)  ----------- (4) |

|  |
| --- |
| Equation (1) - - - - - Gauss law    (2) - - - No name  Maxwell’s law equations  (3) - - - No name  (4) - - -Ampere’s law |

|  |
| --- |
| In addition, we have the following additional information.  ------ (5)  Constitutive relations  ------ (6)  q [+()] ------ (7) Lorentz force law    ------ (8) continuity equation  This equation (8) in not  Independent of maxwell’s equations |
| Conclusion: Equation, (1) – (4) with (5) –(7) explains complete electrostatics and magnetostatics  Note that equation (8) is not included, because it can be derived from the set (1) – (4) |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| In linear media (also homogeneous and isotropic)   |  |  | | --- | --- | | --------- (1)  --------- (2) | --------- (3)  --------- (4) |   Integral form–linear media   |  |  | | --- | --- | | ---- (1)  ----------- (2) | ---- (3)  ----------- (4) | |

|  |  |
| --- | --- |
| Electrostatics | Magnetostatics |
| (), (), V ()  Given any one quantity, find the other tw    (1) (3)  (4) (6)      (5)    V E  (2)   1. V = 2. V = 3. E = 4. E = V | (), (), ()  Given any one quantity, find the other tw  J    (7) (8)  (9) (10)  (11)  A B     1. A = 2. B = 4. E = A; A= |

|  |
| --- |
| A=  Related to Gauge  Transformations |

Why the continuity equation is not an independent equation?

--------- (cont., equation)

Consider the maxwell’s equation: ---- Ampere’s law.

Take divergence: . ( )

0 =

This is equal to if is not a fn., of time.

|  |
| --- |
| that is, we’ll look back into the  derivation of derivation of  = |

This means, if p is a fn., of

time, then, it should be reflected derivation of in the Ampere’s law.

Static Dynamics

Information about time – dependence is required.

Faraday’s law continuity equ.

|  |  |
| --- | --- |
| flux  V=    =    Let’s apply Stoke’s theorem to LHS.  . =  Here is a fn., of and time.    Since the surface integration is same /identical on both sides, we can write.  (Faraday’s law)  In static situation, | Hence, = Gauss law :    = {}    = .{}  .{ } = --------(A)  But we used = ---------(B)  Comparing (A) and (B)  for  statics Dynamics.  Ampere’s law  Hence, the ampere’s law has to be modified with this new . that is,  { }  --- Final maxwell’s equ.  Maxwell’s correction to Ampere’s law |

**Chapter two:**

**Response above a plane – conducting earth to a pulsed vertical magnetic dipole at the surface**

Z

observation point

source

**air**

**d earth**

**( the location of source and observation point )**

1 – **Mathematical formulation and it’s integral representation:**

The radiation field of this vertical dipole can be derived from magnetic hertz .

The electric field vector E and magnetic field H are expressed in terms of through the relations:

In the region , we write

,

where represent the incident wave, while account for the reflected wave Similarly in the region we write:

Where denotes the transmitted filed. at any interior point of the appropriate half space and are assumed to be continuous together with there first and second order partial derivatives. In the region the function satisfies the homogenous wave equation:

and in the region the function satisfies the differential equation:

In these equations

Denotes the three-dimensional Laplacian. Further

**2- Mathematical formulation and it’s fractional representation**:

The Laplacian operator in D-dimensional fractional space in three spatial coordinates is given as:

Where three parameters: ( , )

and according to Caputo law:

,

**The electromagnetic wave equation in fractional space are:**

**Part 1**

**Let**

**1- Derivative**

**2- Derivative**

**3- Derivative**

**4- Derivative**

**From Caputo rule:**

Putting 1, 2 ,3 and 4 in the electromagnetic wave equation we get:

By dividing with

**By using separation of variables:**

For x we get:

By multiplying with u(x) we get:

The equation has the form:

For y we get:

By multiplying with v(y) we get:

The equation has the form:

For z we get:

By multiplying with w(z) we get:

The equation has the form:

For t we get:

By multiplying with we get:

By multiplying with we get:

The equation has the form:

**From . The general wave equation in fractional space are:**

Where in addition to as constraint equation. And are known as wave constants in x, y, z directions.

**Part 2**

Similarly, from the mathematical representation in **part 1:**

**Let**

**1- Derivative**

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The equation has the form:

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The equation has the form:

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**Part 3:**

Bessel’s Formula

for the electromagnetic wave equations

from the wave equation:

The four equations have the same form solution for any one of them can be replicated for others by inspection:

**First equation:**

By multiplying with(x) we get:

Putting 1,2 and 3 in equ1 we get:

Put

Putting:

The equation has the form:

The solution of Bessel’s equation is given as:

,

Is the general solution of Bessel’s equation for all and is the first order and is second orders.

**Similarly: Second equation:**

By multiplying with(y) we get:

Putting 1,2 and 3 in equ1 we get:

Put

Putting:

The equation has the form:

The solution of Bessel’s equation is given as:

Is the general solution of Bessel’s equation for all as an integer number and is the first order and is second order.

**Similarly: third equation:**

By multiplying with(z) we get:

Putting 1,2 and 3 in equ1 we get:

Put

Putting:

The equation has the form:

The solution of Bessel’s equation is given as:

Is the general solution of Bessel’s equation for all as an integer number and is the first differential order and is second differential order

**Similarly: forth equation**

The solution of is:

Where

Where

**Finally the general solutions of Bessel’s equation is:**

,

Where

Where

Since from of order(n)

is the Bessel’s function of the second order of (n).

**The general solution of wave propagation is:**

=

=

**Chapter three:**

**Response above loss less dielectric Medium to a pulsed vertical Magnetic dipole the surface.**

vertical Magnetic dipole Z

observation point

source

**air**

**d loss less dielectric**

**=**

**where**

**( the location of source and observation point )**

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References

[1] C. G. Moschovitis, K. T. Karakatselos, E. G. Papkelis et al., “Scattering of electromagnetic waves from a rectangular plate using an enhanced stationary phase method approximation,”IEEE Transactions on Antennas and Propagation, vol. 58, no. 1, pp. 233-238, (2010).

[2] O. M. Abo-Seida and S. T. Bishay, “Propagation of electromagnetic field from a pulsed electric dipole in dielectric medium,”Chin. J. Phys., vol.39, pp.177-181, (2001).

[3] M. Zubair et al., “The wave equation and general plane wave solutions in fractional space”, Prog. Electromagn. Res. Lett. , vol. 19, 137-146,(2010).

[4] G. Arfken, and H. J. Weber, “Mathematical Methods for Physicists”, Academic Press, (2001).

[5] O. M. Abo-Seida, N. T. M. El-dabe, A. Refaie Ali and G. A. Shalaby, "Cherenkov FEL Reaction With Plasma-Filled Cylindrical Waveguide in Fractional D-Dimensional Space," in *IEEE Transactions on Plasma Science*, vol. 49, no. 7, pp. 2070-2079, July 2021, doi: 10.1109/TPS.2021.3084904.

الملخص العربى Arabic summary

**فى هذا البحث والمقال، تم دراسة بعض النماذج الرياضية لانتشار الموجة الكهرومغناطيسية. فى الفصل الاول تم دراسة الاسس الرياضية اللازمة لاستنباط المعادلة الموجية من معادلات ماكسويل. تم عمل مسح لموضوع الدراسة لبعض المراجع العلمية المرتبطة بموضوع الدراسة. فى الشابتر الثانى تم التركيز على حل مسالة رياضية وتم ايجاد الحل العام للمعادلة الموجية فى الفراغ الكسرى وهذا هى الاضافة الجديدة فى هذا البحث والمقال. تم مقارنة الحل للحصول على الحل العام فى الفراغ الصحيح كحالة خاصة. يتضح ان انتشار الموجة الكهرومغناطيسية مرتبط بقيم البارامترات للفراغ الكسرى**.

**"قَالُوا سُبْحَانَكَ لَا عِلْمَ لَنَا إِلَّا مَا عَلَّمْتَنَا ۖ إِنَّكَ أَنْتَ الْعَلِيمُ الْحَكِيمُ"**

الأية 32 البقرة

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الاية 11 المجادلة

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بحث ومقال Research and Article   
بعض النماذج الرياضية لتفسير انتشار الموجة الكهرومغناطيسية

Some Mathematical models for interpretation of Electromagnetic wave propagation

تحت إشراف

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