Wafa Gestion

Internship Report

A study on the validity of the Fama-French 3 factor Model : Moroccan stock Market as a use case

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1 A brieve Description of Wafa Gestion

Wafa Gestion is a leading asset management company in Morocco, operating as a subsidiary of Attijariwafa Bank. Established to provide expert investment management services, Wafa Gestion manages a diverse range of investment funds, catering to both institutional and individual investors. The company is known for its rigorous investment strategies, focusing on delivering consistent performance and managing risks effectively. With a strong emphasis on innovation and client satisfaction, Wafa Gestion leverages extensive market research and advanced financial models to offer tailored investment solutions that align with the evolving needs of its clients. The firm's commitment to excellence and adherence to ethical standards have positioned it as a trusted name in the Moroccan financial industry.

Figure 0 - WG's website

2 Abstract - Internship mission

This study examines the validity of the Fama-French Model in the context of the Moroccan stock market, using the MASI index as a representative benchmark. The analysis spans a period of three years, from April 2021 to April 2024, leveraging daily historical data sourced from the Bourse de Casablanca website. A total of 75 stocks listed on the MASI index were subjected to rigorous testing to evaluate the model's effectiveness in explaining stock returns within this market.

The study employs various statistical methods and tests to assess the model's validity, including tracking error analysis and backtesting to evaluate its predictive power. By scrutinizing the model's performance against empirical data from the Moroccan stock market, this research aims to provide insights into the applicability and reliability of the Fama-French Model within this specific financial landscape. The findings of this study contribute to the ongoing discourse on asset pricing models and offer practical implications for investors and financial practitioners operating in the Moroccan stock market.

Keywords : Fama and French, SMB, HML, BVC, Tracking-error, backtesting, statistic tests.

3 Summary

When studying financial variables such as asset prices, typically only one variable is considered : risk aversion. According to the perfect rationality assumption (Muth 1961), investors make decisions based on risk assessed by return volatility, given by the variance (standard deviation) of returns over a given period. According to Markowitz (1952), rational investor decisions are made considering two essential dimensions : expected return and variance. Expected return refers to future expected returns, while variance pertains to fluctuations around the mean. This seems to correspond to a perfect world where information flows freely and access to it is immediately available to all economic agents. Subsequently, these agents are capable of constructing their expectations by referring to price formation models and asset valuation models such as the Capital Asset Pricing Model (CAPM) developed following Markowitz's work, or models based on discounting future cash flows provided by holding a given asset. The idea conveyed by the rationality assumption is that securities always have their true values, called "fundamental value" or "intrinsic value." This assumption is closely linked to another fundamental assumption, namely informational efficiency of financial markets. A market is informationally efficient if all available information is immediately reflected in asset prices. According to Fama (1970), prices reflect all available information so that no one can beat the market by consistently achieving abnormally high returns. An observed price, according to Fama (1965), is a good estimator of intrinsic value.

Initially, nothing seemed to challenge the validity of these two assumptions. It wasn't until the late 1970s when a series of phenomena observed in global financial markets, particularly the U.S. market, began to question informational efficiency and rational expectation hypothesis. These phenomena, involving irregularities observed in the distribution of asset price (return) series, were termed anomalies. These anomalies are of various kinds. Indeed, the size effect, seasonal effects, excessive price (return) volatility, momentum effect, etc., are all pathological phenomena that contradict the informational efficiency hypothesis of markets.

To avoid questioning his hypothesis, Fama, in collaboration with French, introduced two factors, related to size and value, to show that under the assumption of efficient markets, a model that incorporates these two factors in addition to the market risk premium would be more suitable for explaining financial asset returns. In 1993, the two authors introduced the three-factor model as an alternative to the traditional model (Capital Asset Pricing Model CAPM).

The objective of this paper is to assess the applicability and effectiveness of the Fama-French model within the context of the Moroccan stock exchange. Specifically, the study aims to scrutinize whether this model serves as a reliable estimator of returns in the Moroccan market. By subjecting the Fama-French model to empirical testing within the unique dynamics of the Moroccan stock exchange, this research endeavors to provide insights into the model's suitability for capturing the complexities and nuances of this particular financial landscape.

The present paper is structured as follows : the first section provides a literature review on the three-factor model, followed by the second section dedicated to the study methodology. Subsequently, the third section presents the results of various estimations.

4 Litterature review

In a 1992 article, Eugene Fama and Kenneth French demonstrated the statistical insignificance of the Beta factor alone, which is theoretically considered a coefficient measuring the sensitivity of stock returns to market fluctuations, in what was supposed to be a linear relationship between portfolio excess returns and risk premium in Sharpe's model. Inspired by earlier works on anomalies such as the size effect (Banz, 1981) and the value effect (Basu, 1983), their research led to the development of the three-factor model in 1993. This model combines the market risk premium with two other factors related to market capitalization and the book-to-market ratio (BTM).

Numerous studies have been conducted to test the ability of Fama and French's model to predict and explain returns and/or examine if the three-factor model outperforms its traditional counterpart, the CAPM (Capital Asset Pricing Model). Financial anomalies, which have been the subject of empirical research since the late 1970s, were defined as observed irregularities that contradict the efficient market hypothesis (Fama, 1970). The informational efficiency hypothesis is a cornerstone of the CAPM. Thus, these anomalies argue for the construction of alternative models.

Faced with the limitations of the CAPM, Ross's Arbitrage Pricing Theory (APT) model (1976) incorporated multiple factors in a single linear combination to explain stock returns. This broadened researchers' perspectives, encouraging them to develop models that consider other factors in response to anomalies like the size effect, as well as macroeconomic factors as suggested by Ross's approach.

Eugene Fama, a founder of the efficient market theory, conducted this research to show the insufficiency of the Beta factor alone in explaining return variations, without contradicting the informational efficiency hypothesis. According to Fama, anomalies do not persist over time and are often due to measurement issues (Beuneau, 2014).

The three-factor model reveals the significance of factors related to a company's size and value. Fama and French identified two classes of firms : small-cap firms and those with high BTM ratios. Such companies are capable of generating high returns on their stocks compared to other classes, particularly large-cap firms, consistent with the size effect defined by Banz in 1981.

Researchers from different economic and geographical contexts have empirically tested the validity of the three-factor model to either confirm Fama and French's claims about the weak or non-existent statistical significance of the Beta factor alone in explaining stock price variations or to demonstrate the robustness of the model when size and BTM variables are included.

In less developed, emerging markets, financial market research has dedicated considerable attention to empirical studies of price formation and stock return evaluation models, particularly the CAPM, while seeking alternatives through ad-hoc models like Fama and French's 1993 model.

Al Mwalla and Karasneh (2011) examined whether the return rates of stocks listed on the Amman market could be accurately estimated by Fama and French's three-factor model over the period from 1999 to 2010. Their conclusion supported the model's validity and its resulting effects, namely the size effect and the value effect.

Kianpoor and Dehghani (2016) adopted Fama and French's methodology to select stocks according to the three-factor model in a study applied to the Iranian market between 2008 and 2012. Their research concluded that the model did not reliably estimate expected returns for certain portfolios due to observed forecast errors.

5 Methodology

For our analysis, we began by scraping historical data for 75 stocks over a three-year period (May 2021 to May 2024) from the Bourse de Casablanca website. We then proceeded to clean and prepare this data, adding necessary annotations and features. From our scraped data, we created a clean, structured dataframe ready for regression modeling.

After constructing our portfolios and defining all variables, we conducted a statistical analysis of our clean data. We then fit the data into our model, starting with linear regressions for each factor across all portfolios, followed by multiple regression analysis involving all factors for each portfolio. The models were evaluated using various statistical tests such as Fisher, Student's t-tests, R-squared, and AIC. Finally, we performed a diagnostic study of the residuals to assess the model's performance.

5.1 Data Scraping

To scrape and prepare historical stock data for our analysis, we implemented several custom functions using Python libraries such as `requests`, `BeautifulSoup`, and `pandas`. Here's a detailed description of the process we followed :

Scraping Historical Stock Data

We used the `wg_get_history` function to scrape historical price data for each stock from the Bourse de Casablanca website. This function constructs API requests to fetch stock data in batches of 250 records at a time, iterating until all available data from May 2021 to May 2024 is collected. The function then processes this data to extract relevant fields, such as adjusted closing prices, trading volumes, and capitalization values. It also computes daily returns and fills any missing dates to ensure a complete time series.

Scraping Financial Ratios

The `wg_ratios_emetteur` function retrieves financial ratios like Price-to-Book (PBR) and Price-to-Earnings (PER) for each company. This data is extracted from specific sections of the company's profile page on the Bourse de Casablanca website using `BeautifulSoup` to parse the HTML content.

Scraping Dividend Information

The `wg_dividendes_emetteur` function collects dividend data from the company pages. It parses tables containing dividend history, extracting and organizing this information into a `pandas` DataFrame.

Scraping General Company Information

The `wg_chiffres_emetteur` function gathers general financial information about each company, such as revenue and earnings. This function also uses `BeautifulSoup` to parse relevant sections of the company profile pages.

Compiling Instrument Data

The main script begins by setting up a `requests` session and iteratively fetching data about all listed instruments (stocks) from the Bourse de Casablanca API. It filters out non-equity instruments and joins additional data, such as sector and capitalization, by applying the `wg_instrument_data` function to each instrument's URL.

Creating a Comprehensive DataFrame

We combine all the scraped data into a single DataFrame (`df_instruments`), which includes stock symbols, ISIN codes, sector information, financial ratios, and more. Each stock's historical data, including daily returns, is linked to this master DataFrame, creating a comprehensive dataset ready for further analysis.

	libelleFR	symbol	codeISIN	instrument_url	drupal_internal__id	emetteur_url	Secteur	Capitalisation	PBR	PER
0	AFMA	AFM	MA0000012296	/fr/live-market/instruments/AFM	391	/fr/live-market/emetteurs/AFM151215	Assurances	1 227 000 000,00	21,85	20,00
1	AFRIC INDUSTRIES SA	AFI	MA0000012114	/fr/live-market/instruments/AFI	385	/fr/live-market/emetteurs/AFI050112	Bâtiment et Matériaux de Construction	93 280 000,00	2,22	21,91
2	AFRIQUIA GAZ	GAZ	MA0000010951	/fr/live-market/instruments/GAZ	498	/fr/live-market/emetteurs/GAZ030599	Pétrole et Gaz	14 265 625 000,00	4,63	28,56
3	AGMA	AGM	MA0000010944	/fr/live-market/instruments/AGM	491	/fr/live-market/emetteurs/AGM091198	Assurances	1 334 000 000,00	8,73	20,16
4	AKDITAL	AKT	MA0000012585	/fr/live-market/instruments/AKT	305583	/fr/live-market/emetteurs/AKT	Santé	8 501 868 904,00	4,41	31,76
...	...	...	...	...	...	...	...	...	...	...
117	TIMAR	TIM	MA0000011686	/fr/live-market/instruments/TIM	361	/fr/live-market/emetteurs/TIM170707	Transport	198 726 000,00	0,35	4,34
118	TOTALENERGIES MARKETING MAROC	TMA	MA0000012262	/fr/live-market/instruments/TMA	390	/fr/live-market/emetteurs/TMA290515	Pétrole et Gaz	12 615 680 000,00	4,49	206,84
119	UNIMER	UMR	MA0000012023	/fr/live-market/instruments/UMR	503	/fr/live-market/emetteurs/UMR290301	Agroalimentaire et Production	2 022 539 536,00	1,49	-
120	WAFA ASSURANCE	WAA	MA0000010928	/fr/live-market/instruments/WAA	488	/fr/live-market/emetteurs/WAA130798	Assurances	15 085 000 000,00	1,52	20,88
121	ZELLIDJA S.A	ZDJ	MA0000010571	/fr/live-market/instruments/ZDJ	485	/fr/live-market/emetteurs/ZDJ200655	Sociétés de Portefeuilles / Holdings	42 625 694,09	0,80	-

77 rows × 10 columns

Figure 1 - Scraped data into dataframe

5.2 Data Cleaning and Preparation

We began by converting all the string variables to numeric formats to facilitate calculations. We introduced the Book-to-Market (BTM) variable, which is calculated as $\text{BTM} = \frac{1}{\text{PBR}}$.

Constructing Portfolios

To construct our portfolios using size and the BTM ratio, we created four portfolios : Small Size, High BTM Ratio (SH) ; Big Size, High BTM Ratio (BH) ; Small Size, Low BTM Ratio (SL) ; and Big Size, Low BTM Ratio (BL). This was done by separating the stocks based on the median capitalization and the median BTM.

Calculating Daily Returns

We constructed four DataFrames, each corresponding to one of the four portfolios. For each stock within these portfolios, we calculated the daily returns using the function `get_history`. To handle missing values, we used the interpolate function, noting that there were not many NaN values.

The daily return (R_{daily}) for each stock was calculated using the formula :

$$R_{\text{daily}} = \frac{\text{Adjusted Closing Price}_t - \text{Adjusted Closing Price}_{t-1}}{\text{Adjusted Closing Price}_{t-1}}$$

where $\text{Adjusted Closing Price}_t$ is the adjusted closing price at day t.

Aggregating Portfolio Returns

We then calculated the daily return for each portfolio by taking the mean of the daily returns of its constituent stocks. This resulted in a new DataFrame containing the daily returns of the four portfolios (SH, BH, SL, BL) over the span of the trading days.

	R_SH	R_BH	R_SL	R_BL
0	NaN	NaN	NaN	NaN
1	0.001931	0.001543	0.000391	0.010676
2	-0.000136	-0.002327	0.001609	0.010915
3	-0.006196	-0.011038	-0.000045	-0.005028
4	-0.003267	0.013373	0.004585	0.007044
...	...	...	...	...
1089	0.004453	0.000067	-0.000162	0.005090
1090	0.001132	0.001987	0.002375	0.002919
1091	-0.002190	0.003907	0.004912	0.000748
1092	-0.005511	0.005827	0.007449	-0.001422
1093	-0.008832	0.007747	0.009986	-0.003593

Figure 2 - The returns Dataframe

5.3 The Fama French 3 factor model

The Fama-French Three-Factor Model is an asset pricing model developed by Eugene Fama and Kenneth French. This model expands on the Capital Asset Pricing Model (CAPM) by adding two factors to the market risk factor in CAPM : size risk and value risk. The model aims to provide a better explanation of the returns of diversified equity portfolios by incorporating these additional factors.

The formula for the Fama-French Three-Factor Model is :

$$R_i - R_f = \alpha + \beta_{1i}(R_m - R_f) + \beta_{2i}\text{SMB} + \beta_{3i}\text{HML} + \epsilon_i$$

Where :

- R_i is the return on portfolio i.
- R_f is the risk-free rate, specifically the daily return of 52-week Treasury bills.
- α is the intercept, representing the portfolio's excess return not explained by the model.
- β_{1i} is the sensitivity of the portfolio's return to the market return, or the market beta.
- R_m is the return on the market portfolio (MASI index).
- β_{2i} is the sensitivity to the size factor (SMB).
- SMB (Small Minus Big) is the return on small-cap portfolios minus the return on large-cap portfolios.
- β_{3i} is the sensitivity to the value factor (HML).
- HML (High Minus Low) is the return on high book-to-market (value) portfolios minus the return on low book-to-market (growth) portfolios.

— ϵ_i is the error term, representing the idiosyncratic risk of the portfolio.
The size and value factors, SMB and HML, are defined as follows :

$$\mathrm{SMB} = \frac{(\mathrm{SH} + \mathrm{SL}) - (\mathrm{BH} + \mathrm{BL})}{3}$$

$$\mathrm{HML} = \frac{(\mathrm{SH} + \mathrm{BH}) - (\mathrm{SL} + \mathrm{BL})}{3}$$

Where :

- SH is the portfolio of small-cap stocks with high book-to-market ratios.
- SL is the portfolio of small-cap stocks with low book-to-market ratios.
- BH is the portfolio of large-cap stocks with high book-to-market ratios.
- BL is the portfolio of large-cap stocks with low book-to-market ratios.

To effectively utilize the Fama-French Three-Factor Model in our analysis, it's crucial to incorporate the size and value factors (SMB and HML) into our existing dataset. These factors provide valuable insights into the performance of our portfolios beyond what the market factor alone can explain.

Adding SMB and HML to our dataframe allows us to capture the specific risk associated with size and value characteristics of the portfolios. By calculating these factors using the provided formulas and combining them with the daily excess returns of our portfolios (Ri-Rf), we can construct a more comprehensive model that accounts for the multidimensional sources of risk and return in equity markets.

We also scraped MASI Data, so we can fit our observations into our regression model.Here is the Dataframe containing all the Data cleaned, prepared.

	created	R_SH	R_BH	R_SL	R_BL	SMB	HML	R_i	R_f	Ri-Rf	MASI	Rm-Rf
1	2021-05-18	0.001931	0.001543	0.000391	0.010676	-0.003299	-0.002531	0.003635	0.000125	0.003510	-0.0008	-0.000925
2	2021-05-19	-0.000136	-0.002327	0.001609	0.010915	-0.002372	-0.004996	0.002515	0.000125	0.002390	0.0030	0.002875
3	2021-05-20	-0.006196	-0.011038	-0.000045	-0.005028	0.003275	-0.004054	-0.005577	0.000125	-0.005702	-0.0126	-0.012725
4	2021-05-21	-0.003267	0.013373	0.004585	0.007044	-0.006366	-0.000508	0.005434	0.000125	0.005309	0.0077	0.007575
7	2021-05-24	0.000929	0.004326	-0.003998	-0.001627	-0.001923	0.003627	-0.000092	0.000125	-0.000217	-0.0043	-0.004425
...	...	...	...	...	...	...	...	...	...	...	...	...
1087	2024-05-08	-0.000907	0.002173	0.002423	0.005937	-0.002198	-0.002365	0.002407	0.000125	0.002282	0.0032	0.003075
1088	2024-05-09	0.000491	0.010605	0.000324	-0.001208	-0.002861	0.003994	0.002553	0.000125	0.002428	0.0044	0.004275
1089	2024-05-10	0.004453	0.000067	-0.000162	0.005090	-0.000289	-0.000136	0.002362	0.000125	0.002237	0.0036	0.003475
1092	2024-05-13	-0.005511	0.005827	0.007449	-0.001422	-0.000822	-0.001903	0.001586	0.000125	0.001461	0.0011	0.000975
1093	2024-05-14	-0.008832	0.007747	0.009986	-0.003593	-0.001000	-0.002493	0.001327	0.000125	0.001202	-0.0029	-0.003025

Figure 3 - The Dataframe fit into the model

	R_SH	R_BH	R_SL	R_BL	SMB	HML	Rm-Rf
count	750.000000	750.000000	750.000000	750.000000	750.000000	750.000000	750.000000
mean	0.000169	0.000784	0.000176	0.000501	-0.000313	0.000092	0.000036
std	0.004229	0.007904	0.004871	0.005252	0.003789	0.002742	0.006753
min	-0.021683	-0.038887	-0.018684	-0.028336	-0.020351	-0.008554	-0.041225
25%	-0.002377	-0.003209	-0.003014	-0.002119	-0.002498	-0.001769	-0.003025
50%	0.000233	0.001101	-0.000104	0.000578	-0.000423	0.000061	0.000275
75%	0.002793	0.005001	0.003005	0.003289	0.001912	0.002061	0.003175
max	0.013598	0.047788	0.019026	0.035062	0.018952	0.009946	0.050675
skewness	-0.141243	-0.237379	0.209774	-0.308460	-0.021485	-0.009961	-0.314731

Figure 4 - Descriptive statistics of the daily returns dataframe

We conducted descriptive statistics to analyze our dataframe, which includes 750 daily observations of all the factors deployed in this study. Here are our main observations :

We notice that the mean of all portfolios is positive, except for SMB. This observation can be interpreted as an indication that, on average, small-cap stocks underperformed compared to large-cap stocks during the period studied. This underperformance could be due to various factors, such as unfavorable economic conditions for smaller companies or a heightened investor preference for more stable and established firms.

Regarding skewness, it is negative for all portfolios except SL. This means that the distribution of returns is skewed to the left, indicating that the magnitude of losses significantly exceeds the magnitude of gains. In other words, the tail of the distribution is much thicker than that of a normal distribution, reflecting a high probability of extreme events. This characteristic is particularly important for investors as it highlights the potential for significant losses, even though the average returns are positive.

5.4 Carhart four factors model

The Carhart Four Factors Model consists of a momentumaugmented Fama-French Three Factors Model. As such, it comprises all of the previously mentioned explanatory variables, in addition to the WML factor that stands for momentum.

The Winners Minus Losers (WML) factor, that is referred to in literature as the momentum premium, consists of the difference between difference between the monthly returns of an equallyweighted long position on the S/W and B/W portfolios on one hand, and the S/Lo and B/Lo portfolios on the other. More specifically, it can be expressed as follows :

$$R_i - R_f = \alpha + \beta_{1i}(R_m - R_f) + \beta_{2i}\text{SMB} + \beta_{3i}\text{HML} + \beta_{4i}\text{WML} + \epsilon_i$$

Where :

- β_{4i} is the sensitivity to the momentum factor (WML).
- WML (Winners Minus Losers) is the return on portfolios of past winners minus the return on portfolios of past losers.

Here, for the portfolio constructions, we construct six portfolios, adding the Winner (W) stocks above the daily return median value, and loser (Lc) stocks below it. Thus, we construct 8 portfolios :

S/L portfolio. contains small capitalization (S) stocks that are also low value (L) ones.

S/H portfolio. contains small capitalization (S) stocks that are also high value (H) ones.

B/L portfolio. contains big capitalization (B) stocks that are also low value (L) ones.

B/H portfolio. contains big capitalization (B) stocks that are also high value (H) ones.

S/W portfolio. contains small capitalization (S) stocks that are also winner (W) ones.

S/Lo portfolio. contains smalls capitalization (S) stocks that are also loser (Lo) ones.

B/W portfolio. contains big capitalization (B) stocks that are also winner (W) ones.

B/Lo portfolio. contains big capitalization (B) stocks that are also loser (Lo) ones.

5.5 Observations and corrections

After my initial work on the Fama-French three-factor model, my internship supervisor informed me that while the method I used was correct, it has several limitations that we need to address.

In the original code, the calculation of daily returns for the portfolio was performed by averaging the returns of each asset equally. However, to achieve a capitalization-weighted return, it is essential to weigh each asset's return by its market capitalization. The modified code now retrieves the capitalization value for each asset on each day and multiplies the daily return by this capitalization. The weighted sum of returns is then divided by the total capitalization of all assets to compute the weighted return. Specifically, for each day j, the capitalization-weighted return R_p is calculated using the formula :

$$R_p = \frac{\sum_{i=1}^{n} (R_{i,j} \cdot C_{i,j})}{\sum_{i=1}^{n} C_{i,j}}$$

where $R_{i,j}$ is the daily return of asset i on day j, and $C_{i,j}$ is the market capitalization of asset i on day j. This ensures that the returns are appropriately weighted according to the market significance of each asset.

Initially, the MASI index was used as R_m, the return on the market portfolio. The MASI index represents the overall performance of the Moroccan stock market. However, my supervisor suggested switching to the MASI Rentabilité index for this analysis. The MASI Rentabilité index includes dividends in its calculation, thus providing a more comprehensive measure of total returns, including both capital gains and dividend income. This change is crucial as it offers a more accurate representation of the investors' returns, considering the dividends which are an essential component of the total market performance.

In the original approach, the calculation of portfolio returns was based on the closing prices of stocks for each day. However, as part of the corrections made, I now utilize the adjusted closing prices ('cours ajusté'). The adjustment to these prices incorporates factors such as dividends, stock splits, and other corporate actions that affect the stock price's historical data. This adjustment is crucial because it provides a more accurate reflection of the true market value changes and ensures that the returns calculated are not distorted by factors unrelated to the stock's underlying performance. Therefore, by using adjusted closing prices, the analysis achieves a more precise evaluation of portfolio performance relative to market movements.

	date	weighted_return_BL	weighted_return_SL	weighted_return_SH	weighted_return_BH
0	2021-06-28	NaT	NaT	NaT	NaT
1	2021-06-29	-0.003626	0.000209	-0.003084	-0.002217
2	2021-06-30	NaN	-0.000989	-0.002292	-0.001714
3	2021-07-01	NaN	-0.002234	NaN	0.002949
4	2021-07-02	0.00097	-0.003644	NaN	-0.000176
...	...	...	...	...	...
1092	2024-06-24	0.021855	0.00328	-0.004901	0.015034
1093	2024-06-25	0.030785	0.001058	-0.006828	0.020301
1094	2024-06-26	-0.006965	0.004231	-0.001796	-0.001395
1095	2024-06-27	-0.001029	-0.00262	0.002894	-0.006689
1096	2024-06-28	-0.003792	-0.008569	-0.011544	0.002734

Figure - The portfolios cap-weighted returns

The **wg get MASIR history** function retrieves historical data for the MASI-R index from the Casablanca Stock Exchange API. It uses a series of requests to fetch data in batches, starting from January 1, 2021, and continuing until the most recent available data. The function aggregates this data into a Pandas DataFrame, extracting fields such as session date and index values. This approach ensures that the analysis incorporates up-to-date and accurate historical performance data of the MASI-R index, essential for evaluating portfolio returns and benchmarking against market performance trends.

	field_seance_date	field_index_value
0	2024-06-28	41697.3420000000
1	2024-06-27	41750.0803000000
2	2024-06-26	41850.7920000000
3	2024-06-25	42014.7922000000
4	2024-06-24	41118.3468000000
...	...	...
247	2021-07-02	35401.1500000000
248	2021-07-01	35380.5500000000
249	2021-06-30	35352.0500000000
0	2021-06-29	35360.0500000000
1	2021-06-28	35521.0600000000

Figure - The MASI-Rentabilité data

6 Results

6.1 Estimation of portfolio returns based on the Fama and French 3 factor model - 1st try

In our analysis of the four portfolios—Small Low (SL), Small High (SH), Big Low (BL), and Big High (BH)—using the French Fama model, we conducted a series of regressions to estimate the returns based on different factors. Initially, the portfolio returns were estimated using the market risk premium. The results indicated a significant impact of market risk on all four portfolios, with varying degrees of sensitivity. Next, we incorporated the Small Minus Big (SMB) factor, which highlighted the size effect. The inclusion of SMB revealed a substantial influence on the returns of the small-cap portfolios (SL and SH), underscoring the relevance of size in explaining their performance. Subsequently, we added the High Minus Low (HML) factor to account for value versus growth effects. The HML factor significantly affected the high book-to-market portfolios (SH and BH), indicating a strong value effect. Finally, combining all three factors—market risk premium, SMB, and HML—provided a comprehensive model. The three-factor model explained a substantial portion of the variability in returns across all portfolios, demonstrating the robustness of the Fama-French framework in capturing the nuances of portfolio performance. The regression coefficients and R-squared values confirmed that market risk, size, and value are crucial determinants of portfolio returns, with each factor contributing uniquely to the explanatory power of the model.

Portfeuilles	Alpha	Beta	T-stat(alpha)	prob	T-stat (Beta)	prob	R2	AIC	F-stati
SH	3.77*10^(-5)	0.17	0.254	0.799	7.772	0	0.075	-6126	60.41
BH	0.0006	0.7640	2.887	0.004	23.567	0	0.426	-5546	555.4
SL	4.441*10^(-5)	0.1886	0.258	0.796	7.410	0	0.068	-5908	54.91
BL	0.0004	0.5876	2.826	0.005	31.537	0	0.571	-6376	994.6

Figure 5 - Estimations of the portfolios based on the market risk premium

Given the individual significance of the t-test statistics, we observe that all probabilities corresponding to the t-statistic of Student for the Beta factor are below 5%, all of which are zero. Consequently, the market risk premium is significant and predictive of the returns for all the portfolios.

The overall significance is corroborated by the Fisher test, as all F-statistics are greater than their critical values at the 5% level.R squared is practically low, with a mean of 0.285 for all four portfolios. We conclude that the regression has an imperfect quality.

For the Akaike Information Criterion (AIC), it measures the predictive quality through the calculation of the prediction error. Indeed, according to this criterion, the best estimates are those that minimize it. Its value fluctuate between -6380 and -5500.

In conclusion, while the market risk premium is a significant predictor of portfolio returns, the overall explanatory power of the model is limited, as indicated by the low R squared values. The AIC values suggest that the model's predictive quality varies, but it generally provides reasonable estimates within the given range.

Portfeuilles	Alpha	Beta	T-stat(alpha)	prob	T-stat (Beta)	prob	R2	AIC	F-stati
SH	0.0001	0.2461	0.8	0.424	6.182	0	0.049	-6105	1.04*10^(-9)
BH	0.0002	-1,6055	0.842	0.4	-32,969	0	0.592	-5802	6.66*10^(-148)
SL	0.0001	0.2822	0.801	0.424	6.153	0	0.048	-5892	37.85
BL	0.0001	-0,8662	0.679	0.486	-21,888	0	0.390	-6113	1.86*10^(-82)

Figure 6 - Estimations of the portfolios based on SMB

Given the individual significance of the t-test statistics of Student, we observe that t-statistics for the Beta factor of all the portfolios are greater than their critical values at the 5% level, and all their associated probabilities are below 5%, and consequently, the (SMB) variable appears to be significant for the small portfolios

Regarding the direction of the relationships between portfolio returns and the SMB factor, we observe a negative relationship for the BH and BL portfolios, and a positive relationship for the two small-cap portfolios, SH and SL. This confirms the size effect as defined in the literature.

The overall significance is confirmed by the Fisher test of only the SL portfolio, as its F-stat is greater than 5%, and corroborated for SH portfolio.

However, R squared is low for the SL and SH portfolios, indicating that the model explains only a small portion of the variability in returns for these portfolios. This suggests that factors other than SMB might play a significant role in determining the returns of small-cap portfolios. Further investigation may be needed to capture additional factors influencing their returns effectively. The AIC values fluctuate between -6113 and -5802.

In conclusion, while the SMB variable appears to be significant for all the portfolios, the model's explanatory power is limited, especially for small-cap portfolios. Additional factors beyond SMB may be necessary to improve the model's performance in explaining the returns of these portfolios effectively.

Portfeuilles	Alpha	Beta	T-stat(alpha)	prob	T-stat (Beta)	prob	R2	AIC	F-stati
SH	-0,000005865	0.5417	-0,041	0.968	10.263	0	0.123	-6166	105.3
BH	0.0005	1.5739	2.125	0.034	17.830	0	0.298	-5395	317.9
SL	0.0001	-0,6661	0.68	0.497	-11,063	0	0.141	-5969	0
BL	0.0004	-0,2183	2.077	0.038	-3,138	0.002	0.013	-5752	9.845

Figure 7 - Estimations of the portfolios based on HML

The SH portfolio shows a non-significant alpha (p = 0.968) and a significant beta (p = 0), indicating that the portfolio's returns are significantly influenced by the HML factor. The model explains 12.3% of the variability in returns. The BH portfolio exhibits a significant positive alpha (p = 0.034) and a highly significant beta (p = 0) with HML. The model explains 29.8% of the variability in returns.The SL portfolio shows a non-significant alpha (p = 0.497) and a significant negative beta (p = 0) with HML. The model explains 14.1% of the variability in returns. The BL portfolio shows a significant positive alpha (p = 0.038) and a significant negative beta (p = 0.002) with HML. The model explains only 1.3% of the variability in returns.

We conclude that the HML factor seems to have a significant influence on the returns of all portfolios. BH portfolio has a significant positive alpha and beta, indicating strong exposure to the HML factor. SL portfolio shows a significant negative beta, indicating sensitivity to the HML factor. R-squared values are relatively low for all portfolios, suggesting that the model explains only a small portion of the variability in returns. AIC values indicate varying predictive quality across portfolios.

Overall, while the HML factor is significant for all portfolios, the model's explanatory power is limited, suggesting the presence of other factors influencing portfolio returns not captured by the model.

It's worth noting that the model's explanatory power using a single factor (either HML, PRM, or SMB) is limited, as indicated by relatively low R-squared values across all portfolios. This suggests that there are likely other factors influencing portfolio returns that are not captured by the single-factor model. Therefore, to enhance our understanding and improve the explanatory power of the model, we will proceed to employ the Fama-French three-factor model, which includes all the factors (market risk premium, SMB, and HML) known to have significant impacts on asset returns. This will provide a more comprehensive framework to better explain the variability in portfolio returns.

		BETA				
Portfeuilles	Alpha	SMB	HML	Rm-Rf	T-stat (Alpha)	prob
SH	0.0002	0.9186	0.8570	0.4295	2.316	0.021
BH	0.0003	-0,9234	0.9879	0.4064	1.836	0.067
SL	0.0003	0.5766	-0,5121	0.4064	1.836	0.067
BL	0.0002	-0,5814	-0,643	0.4295	2.316	0.021

T-stat(Beta)			Prob					
SMB	HML	Rm-Rf	SMB	HML	Rm-Rf	R2	AIC	F-stati
25.773	21.356	22.796	0	0	0	0.564	-6686	321.9
-18,471	17,552	15,381	0	0	0	0.755	-6179	0
11.535	-9,1	0.026	0	0	0	0.354	-6179	0
-16,311	-16,024	22.796	0	0	0	0.717	-6686	0

Figure 8 - Estimations of the portfolios based on HML,SMB and PRM

When conducting a joint estimation of excess returns on the three Fama and French factors, we observe that all three factors exhibit large t-statistics, with corresponding probabilities below 5%, implying that the independent variables are simultaneously significant in explaining the excess returns. Regarding overall significance, we find that the Fisher F-statistic for the SH portfolio is greater than its critical value at the 5% level (2.63), rejecting the null hypothesis of simultaneous coefficient insignificance and reinforcing the predictive quality of the model. The other F-statistics are null. The null F-statistics for the other portfolios suggest that they might not have enough explanatory power with the current model due to specific characteristics not captured by the factors.

The R-squared values are high (above 0.5), with an average of 0.597.

As a conclusion, the three-factor Fama and French model provides an explanation of around 60% of the variations in daily excess returns of Moroccan stocks. This indicates that the regression is of moderate quality and that the three-factor model can be considered as an evaluation model that fits the Moroccan data well.

6.2 Residual diagnostics study FF3F - 1st try

To continue our results, we did a residual diagnostics study using these statistics/tests :

- **Jarque-Bera Test :** Tests the skewness and kurtosis of the residuals to assess if they follow a normal distribution.
- **Probability (proba(JB)) :**If the probability is less than a significance level (e.g., 0.05), it indicates that the residuals are not normally distributed.
- **Omnibus Test :**Tests the overall normality of the residuals.
- **Probability (proba(omnibus)) :**Similar to the JB test, if the probability is less than a significance level (e.g., 0.05), it suggests non-normality.
- **Skewness :**Measures the asymmetry of the residuals. Ideally, it should be close to zero for normality.
- **Kurtosis :**Measures the heaviness of the tails of the distribution. For normality, it should be around 3.
- **Durbin-Watson Statistic :**Tests for autocorrelation in the residuals. If the statistic is around 2, it suggests no autocorrelation. Values significantly lower or higher than 2 indicate positive or negative autocorrelation, respectively.

Portfeuilles	Omnibus	Prob(Omnibus)	Skew	Kurtosis	Durbin-Watson	Jarque-Bera	Prob(JB)
SH	3,897	0,144	0,003	3,383	1,42	4,587	0,101
BH	43,985	0	0,273	4,842	1,657	115,399	0
SL	31,74	0	0,419	3,76	1,276	40,033	0
BL	142,15	0	0,649	8,5	1,696	997,836	0

Figure 9 -Residual diagnostics for the estimations of the four portfolios based on the market risk premium (PRM)

The first property we examined is normality. We found that all probabilities from the Jarque-Bera and Omnibus tests are below 5%, rejecting the null hypothesis that the residuals are normally distributed. Skewness is not close to zero except for SH, indicating that the residuals are asymmetric, except for SH. Similarly, kurtosis is around 3 only for SH and SL, suggesting heavy tails in residuals.

For all portfolios, Durbin-Watson statistics are less than 2, ranging from 1.4 to 1.7.

These results suggest that the residuals may not follow a normal distribution, and there are asymmetry and heavy-tailed distributions in residuals, especially for portfolios other than SH. Furthermore, Durbin-Watson statistics suggest positive autocorrelation in residuals for all portfolios.

In conclusion, the diagnostic results indicate that the residuals do not meet the assumptions of normality.

Portfeuilles	Omnibus	Prob(Omnibus)	Skew	Kurtosis	Durbin-Watson	Jarque-Bera	Prob(JB)
SH	49,275	0	-0,293	5,029	1,371	139,356	0
BH	30,195	0	-0,18	4,447	1,281	69,438	0
SL	28,739	0	-0,04	4,555	1,157	75,753	0
BL	33,533	0	-0,152	4,65	1,284	87,993	0

Figure 10 -Residual diagnostics for the estimations of the four portfolios based on SMB

The first property we examined is normality. We found that all probabilities from the Jarque-Bera and Omnibus tests are below 5%, rejecting the null hypothesis that the residuals are normally distributed. Skewness is not close to zero except for SL, indicating that the residuals are asymmetric, except for SL. Similarly, kurtosis is not around 3, suggesting heavy tails in residuals.

For all portfolios, Durbin-Watson statistics are less than 2, ranging from 1.1 to 1.4. These statistics suggest positive autocorrelation in residuals for all portfolios.

Portfeuilles	Omnibus	Prob(Omnibus)	Skew	Kurtosis	Durbin-Watson	Jarque-Bera	Prob(JB)
SH	15,926	0	-0,097	3,933	1,283	28,399	0
BH	93,564	0	-0,333	7,316	1,263	595,968	0
SL	32,49	0	-0,06	4,714	1,143	92,262	0
BL	132,367	0	-0,347	10,289	1,31	1675,521	0

Figure 11 -Residual diagnostics for the estimations of the four portfolios based on HML

We found that all probabilities from the Jarque-Bera and Omnibus tests are below 5%, rejecting the null hypothesis that the residuals are normally distributed. Skewness is not close to zero except for SL, indicating that the residuals are asymmetric, except for SL. Similarly, kurtosis is not around 3except for SH, suggesting heavy tails in residuals.

For all portfolios, Durbin-Watson statistics are less than 2, ranging from 1.1 to 1.3. These statistics suggest positive autocorrelation in residuals for all portfolios.

Conclusion : For the models relying on one factor, we conclude that the normality hypothesis isn't verified, positive autocorrelation in residuals for all portfolios, asymmetric residuals and heavy tails in residuals.

Portfeuilles	Omnibus	Prob(Omnibus)	Skew	Kurtosis	Durbin-Watson	Jarque-Bera	Prob(JB)
SH	64,021	0	0,225	5,986	1,546	284,873	0
BH	12,239	0,002	0,115	3,738	1,434	18,677	10^(-5)
SL	12,239	0,002	0,115	3,738	1,434	18,677	10^(-5)
BL	64,021	0	0,225	5,986	1,546	284,873	0

Figure 12 -Residual diagnostics for the estimations of the four portfolios based on the Fama-French factors

We found that all probabilities from the Jarque-Bera and Omnibus tests are below 5%, rejecting the null hypothesis that the residuals are normally distributed. Skewness is not that close to zero, indicating that the residuals are asymmetric. Similarly, kurtosis is not around 3 except for BH,SL, suggesting heavy tails in residuals.

For all portfolios, Durbin-Watson statistics are less than 2, ranging from 1.4 to 1.6. These statistics suggest positive autocorrelation in residuals for all portfolios.

Conclusion : Overall, the estimation results show a significant impact of size-related factors, more pronounced when this factor is associated with other factors related to value and market risk premium. Similarly, the negative relationship between portfolio size and profitability is confirmed, supporting the size effect as a market anomaly. In a comparative perspective between the CAPM and the three-factor model, the latter proves to be more appropriate for explaining portfolio returns. It extends the findings of previous research conducted in different economic and geographical contexts.

6.3 Estimation of portfolio returns based on the Fama and French 3 factor model - After corrections

Because this method seems more right, we conducted a deep analysis in our regressions.

The `perform_regression` function conducts linear regression analysis using the Ordinary Least Squares (OLS) method from the `statsmodels` library. First, it combines the independent variable x, dependent variable y, and a date column into a DataFrame, ensuring data integrity by dropping any NaN values. The function then cleans the data, converting columns to numeric types as necessary. It adds a constant term to the independent variable x to fit an intercept in the regression model. The function computes the regression using `sm.OLS` and prints a summary of the regression results, including coefficients, standard errors, and statistical significance.

To visually assess the model's assumptions and validity, the function generates three essential plots. The "Residuals vs Fitted" plot examines the relationship between the fitted values and residuals, checking for patterns that may indicate heteroscedasticity or non-linearity in the data. The "Q-Q Plot" (Quantile-Quantile plot) compares the distribution of residuals against a normal distribution, verifying if residuals are normally distributed, which is crucial for the validity of statistical inference. Finally, the "Regression Plot" displays the actual data points along with the regression line, providing a visual representation of how well the regression model fits the data and the relationship between x and y. These plots collectively help in diagnosing potential issues with the regression model and interpreting its results effectively.

Portfolio SH

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                 R_SH-Rf   R-squared:                       0.052
Model:                             OLS   Adj. R-squared:                  0.049
Method:                  Least Squares   F-statistic:                     20.42
Date:                 Sun, 30 Jun 2024   Prob (F-statistic):           8.36e-06
Time:                         16:19:58   Log-Likelihood:                 1455.3
No. Observations:                  377   AIC:                            -2907.
Df Residuals:                      375   BIC:                            -2899.
Df Model:                            1
Covariance Type:             nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0003      0.000      1.114      0.266      -0.000       0.001
SMB            0.2924      0.065      4.518      0.000       0.165       0.420
==============================================================================
Omnibus:                        4.304   Durbin-Watson:                   1.222
Prob(Omnibus):                  0.116   Jarque-Bera (JB):                4.765
Skew:                           0.127   Prob(JB):                       0.0923
Kurtosis:                       3.489   Cond. No.                         246.
==============================================================================
```

(a) Estimation based on SMB

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                 R_SH-Rf   R-squared:                       0.010
Model:                             OLS   Adj. R-squared:                  0.008
Method:                  Least Squares   F-statistic:                     5.301
Date:                 Sun, 30 Jun 2024   Prob (F-statistic):             0.0217
Time:                         16:17:52   Log-Likelihood:                 2087.1
No. Observations:                  541   AIC:                            -4170.
Df Residuals:                      539   BIC:                            -4162.
Df Model:                            1
Covariance Type:             nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const        7.49e-05      0.000      0.340      0.734      -0.000       0.001
MASIR-Rf      -0.0742      0.032     -2.302      0.022      -0.137      -0.011
==============================================================================
Omnibus:                        3.911   Durbin-Watson:                   1.452
Prob(Omnibus):                  0.141   Jarque-Bera (JB):                4.074
Skew:                           0.119   Prob(JB):                        0.130
Kurtosis:                       3.352   Cond. No.                         146.
==============================================================================
```

(b) Estimation based on the market risk premium

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                 R_SH-Rf   R-squared:                       0.319
Model:                             OLS   Adj. R-squared:                  0.318
Method:                  Least Squares   F-statistic:                     176.0
Date:                 Sun, 30 Jun 2024   Prob (F-statistic):           3.35e-33
Time:                         16:23:08   Log-Likelihood:                 1517.9
No. Observations:                  377   AIC:                            -3032.
Df Residuals:                      375   BIC:                            -3024.
Df Model:                            1
Covariance Type:             nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0003      0.000     -1.342      0.181      -0.001       0.000
HML            0.9555      0.072     13.266      0.000       0.814       1.097
==============================================================================
Omnibus:                        5.417   Durbin-Watson:                   1.246
Prob(Omnibus):                  0.067   Jarque-Bera (JB):                5.316
Skew:                           0.233   Prob(JB):                       0.0701
Kurtosis:                       3.349   Cond. No.                         323.
==============================================================================
```

(c) Estimation based on SMB

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                 R_SH-Rf   R-squared:                       0.410
Model:                             OLS   Adj. R-squared:                  0.405
Method:                  Least Squares   F-statistic:                     86.19
Date:                 Sun, 30 Jun 2024   Prob (F-statistic):           2.33e-42
Time:                         16:30:47   Log-Likelihood:                 1542.8
No. Observations:                  376   AIC:                            -3078.
Df Residuals:                      372   BIC:                            -3062.
Df Model:                            3
Covariance Type:             nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0002      0.000     -0.782      0.435      -0.001       0.000
SMB            0.3879      0.054      7.159      0.000       0.281       0.494
HML            0.9716      0.067     14.505      0.000       0.840       1.103
MASIR-Rf      -0.1427      0.038     -3.751      0.000      -0.218      -0.068
==============================================================================
Omnibus:                       17.873   Durbin-Watson:                   1.395
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               32.508
Skew:                           0.281   Prob(JB):                     8.73e-08
Kurtosis:                       4.326   Cond. No.                         324.
==============================================================================
```

(d) Estimation based on the 3 factors

Figure 13 - Estimations of the portfolio SH

Given the individual significance of the t-test statistics, we observe that all probabilities corresponding to the t-statistic of Student for the Beta factor are below 5%, all of which are zero. Consequently, all the factors are significant and predictive of the returns of SH.

The overall significance is corroborated by the Fisher test, as all F-statistics are greater than their critical values at the 5% level. R squared is practically low, with a maximum value concerning the Fama-French regression. We conclude that the regression has an imperfect quality, except Fama-French which is quite good.

For the Akaike Information Criterion (AIC), it measures the predictive quality through the calculation of the prediction error. Indeed, according to this criterion, the best estimates are those that minimize it. Its value fluctuate between -4000 and -2000.

Fama French best describes the return of the SH portfolio.

Portfolio BH

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                 R_BH-Rf   R-squared:                       0.528
Model:                             OLS   Adj. R-squared:                  0.526
Method:                  Least Squares   F-statistic:                     418.9
Date:                 Sun, 30 Jun 2024   Prob (F-statistic):           4.74e-63
Time:                         16:20:22   Log-Likelihood:                 1498.7
No. Observations:                  377   AIC:                            -2993.
Df Residuals:                      375   BIC:                            -2986.
Df Model:                            1
Covariance Type:             nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0004      0.000      1.496      0.136      -0.000       0.001
SMB           -1.1805      0.058    -20.467      0.000      -1.294      -1.067
==============================================================================
Omnibus:                       17.953   Durbin-Watson:                   1.267
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               38.287
Skew:                           0.213   Prob(JB):                     4.85e-09
Kurtosis:                       4.502   Cond. No.                         246.
==============================================================================
```

(a) Estimation based on SMB

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                 R_BH-Rf   R-squared:                       0.054
Model:                             OLS   Adj. R-squared:                  0.053
Method:                  Least Squares   F-statistic:                     42.68
Date:                 Sun, 30 Jun 2024   Prob (F-statistic):           1.19e-10
Time:                         16:18:37   Log-Likelihood:                 2590.9
No. Observations:                  750   AIC:                            -5178.
Df Residuals:                      748   BIC:                            -5169.
Df Model:                            1
Covariance Type:             nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0005      0.000      1.793      0.073   -4.76e-05       0.001
MASIR-Rf      -0.2696      0.041     -6.533      0.000      -0.351      -0.189
==============================================================================
Omnibus:                      121.180   Durbin-Watson:                   1.935
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             1720.509
Skew:                          -0.126   Prob(JB):                         0.00
Kurtosis:                      10.416   Cond. No.                         148.
==============================================================================
```

(b) Estimation based on the market risk premium

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                 R_BH-Rf   R-squared:                       0.092
Model:                             OLS   Adj. R-squared:                  0.090
Method:                  Least Squares   F-statistic:                     38.19
Date:                 Sun, 30 Jun 2024   Prob (F-statistic):           1.68e-09
Time:                         16:23:24   Log-Likelihood:                 1375.6
No. Observations:                  377   AIC:                            -2747.
Df Residuals:                      375   BIC:                            -2739.
Df Model:                            1
Covariance Type:             nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0006      0.000      1.777      0.076   -6.24e-05       0.001
HML            0.6491      0.105      6.180      0.000       0.443       0.856
==============================================================================
Omnibus:                       30.655   Durbin-Watson:                   1.124
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              101.226
Skew:                           0.261   Prob(JB):                     1.04e-22
Kurtosis:                       5.484   Cond. No.                         323.
==============================================================================
```

(c) Estimation based on SMB

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                 R_BH-Rf   R-squared:                       0.630
Model:                             OLS   Adj. R-squared:                  0.627
Method:                  Least Squares   F-statistic:                     211.3
Date:                 Sun, 30 Jun 2024   Prob (F-statistic):           5.35e-80
Time:                         16:31:45   Log-Likelihood:                 1540.3
No. Observations:                  376   AIC:                            -3073.
Df Residuals:                      372   BIC:                            -3057.
Df Model:                            3
Covariance Type:             nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const       5.905e-05      0.000      0.278      0.781      -0.000       0.000
SMB           -1.0651      0.055    -19.527      0.000      -1.172      -0.958
HML            0.5613      0.067      8.324      0.000       0.429       0.694
MASIR-Rf      -0.2080      0.038     -5.431      0.000      -0.283      -0.133
==============================================================================
Omnibus:                       18.510   Durbin-Watson:                   1.486
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               38.806
Skew:                           0.234   Prob(JB):                     3.74e-09
Kurtosis:                       4.502   Cond. No.                         324.
==============================================================================
```

(d) Estimation based on the 3 factors

Figure 14 - Estimations of the portfolio BH

Given the individual significance of the t-test statistics, we observe that all probabilities corresponding to the t-statistic of Student for the Beta factor are below 5%, all of which are zero. Consequently, all the factors are significant and predictive of the returns of BH.

The overall significance is corroborated by the Fisher test, as all F-statistics are greater than their critical values at the 5% level. R squared is practically low, with a maximum value concerning the Fama-French and SMB regression. We conclude that the regressions have an imperfect quality, except Fama-French and SMB which are quite good.

For the Akaike Information Criterion (AIC), it measures the predictive quality through the calculation of the prediction error. Indeed, according to this criterion, the best estimates are those that minimize it. Its value fluctuate between -4000 and -2000.

Fama French and SMB best describe the return of the BH portfolio.

Portfolio SL

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                 R_SL-Rf   R-squared:                       0.084
Model:                             OLS   Adj. R-squared:                  0.081
Method:                  Least Squares   F-statistic:                     34.36
Date:                 Sun, 30 Jun 2024   Prob (F-statistic):           1.00e-08
Time:                         16:21:41   Log-Likelihood:                 1456.9
No. Observations:                  377   AIC:                            -2910.
Df Residuals:                      375   BIC:                            -2902.
Df Model:                            1
Covariance Type:             nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0003      0.000     -1.313      0.190      -0.001       0.000
SMB            0.3777      0.064      5.862      0.000       0.251       0.504
==============================================================================
Omnibus:                       19.694   Durbin-Watson:                   1.614
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               35.144
Skew:                           0.323   Prob(JB):                     2.34e-08
Kurtosis:                       4.349   Cond. No.                         246.
==============================================================================
```

(a) Estimation based on SMB

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                 R_SL-Rf   R-squared:                       0.019
Model:                             OLS   Adj. R-squared:                  0.018
Method:                  Least Squares   F-statistic:                     14.48
Date:                 Sun, 30 Jun 2024   Prob (F-statistic):           0.000153
Time:                         16:19:02   Log-Likelihood:                 2904.2
No. Observations:                  750   AIC:                            -5804.
Df Residuals:                      748   BIC:                            -5795.
Df Model:                            1
Covariance Type:             nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const       9.539e-06      0.000      0.052      0.959      -0.000       0.000
MASIR-Rf      -0.1034      0.027     -3.806      0.000      -0.157      -0.050
==============================================================================
Omnibus:                       33.098   Durbin-Watson:                   1.760
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               87.868
Skew:                           0.137   Prob(JB):                     8.31e-20
Kurtosis:                       4.654   Cond. No.                         148.
==============================================================================
```

(b) Estimation based on the market risk premium

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                 R_SL-Rf   R-squared:                       0.309
Model:                             OLS   Adj. R-squared:                  0.307
Method:                  Least Squares   F-statistic:                     167.3
Date:                 Sun, 30 Jun 2024   Prob (F-statistic):           6.63e-32
Time:                         16:23:41   Log-Likelihood:                 1509.9
No. Observations:                  377   AIC:                            -3016.
Df Residuals:                      375   BIC:                            -3008.
Df Model:                            1
Covariance Type:             nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const      -6.024e-05      0.000     -0.261      0.794      -0.001       0.000
HML           -0.9515      0.074    -12.936      0.000      -1.096      -0.807
==============================================================================
Omnibus:                        6.046   Durbin-Watson:                   1.455
Prob(Omnibus):                  0.049   Jarque-Bera (JB):                7.568
Skew:                           0.139   Prob(JB):                       0.0227
Kurtosis:                       3.636   Cond. No.                         323.
==============================================================================
```

(c) Estimation based on SMB

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                 R_SL-Rf   R-squared:                       0.422
Model:                             OLS   Adj. R-squared:                  0.417
Method:                  Least Squares   F-statistic:                     90.47
Date:                 Sun, 30 Jun 2024   Prob (F-statistic):           5.54e-44
Time:                         16:32:06   Log-Likelihood:                 1540.3
No. Observations:                  376   AIC:                            -3073.
Df Residuals:                      372   BIC:                            -3057.
Df Model:                            3
Covariance Type:             nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const       5.905e-05      0.000      0.278      0.781      -0.000       0.000
SMB            0.4349      0.055      7.973      0.000       0.328       0.542
HML           -0.9387      0.067    -13.920      0.000      -1.071      -0.806
MASIR-Rf      -0.2080      0.038     -5.431      0.000      -0.283      -0.133
==============================================================================
Omnibus:                       18.510   Durbin-Watson:                   1.486
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               38.806
Skew:                           0.234   Prob(JB):                     3.74e-09
Kurtosis:                       4.502   Cond. No.                         324.
==============================================================================
```

(d) Estimation based on the 3 factors

Figure 15 - Estimations of the portfolio SL

Given the individual significance of the t-test statistics, we observe that all probabilities corresponding to the t-statistic of Student for the Beta factor are below 5%, all of which are zero. Consequently, all the factors are significant and predictive of the returns of SL.

The overall significance is corroborated by the Fisher test, as all F-statistics are greater than their critical values at the 5% level. R squared is very low, with a maximum value concerning the Fama-French and HML regression. We conclude that the regressions have an imperfect quality, except Fama-French which is quite good.

For the Akaike Information Criterion (AIC), Its value fluctuate between -4000 and -2000.

Fama French best describes the return of the SL portfolio.

Portfolio BL

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                 R_BL-Rf   R-squared:                       0.529
Model:                             OLS   Adj. R-squared:                  0.528
Method:                  Least Squares   F-statistic:                     421.3
Date:                 Sun, 30 Jun 2024   Prob (F-statistic):           2.70e-63
Time:                         16:22:14   Log-Likelihood:                 1509.9
No. Observations:                  377   AIC:                            -3016.
Df Residuals:                      375   BIC:                            -3008.
Df Model:                            1
Covariance Type:             nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0004      0.000     -1.764      0.079      -0.001    4.64e-05
SMB           -1.1493      0.056    -20.525      0.000      -1.259      -1.039
==============================================================================
Omnibus:                       14.855   Durbin-Watson:                   1.417
Prob(Omnibus):                  0.001   Jarque-Bera (JB):               26.338
Skew:                           0.228   Prob(JB):                     1.91e-06
Kurtosis:                       4.212   Cond. No.                         246.
==============================================================================
```

(a) Estimation based on SMB

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                 R_BL-Rf   R-squared:                       0.066
Model:                             OLS   Adj. R-squared:                  0.064
Method:                  Least Squares   F-statistic:                     33.22
Date:                 Sun, 30 Jun 2024   Prob (F-statistic):           1.49e-08
Time:                         16:19:23   Log-Likelihood:                 1689.0
No. Observations:                  472   AIC:                            -3374.
Df Residuals:                      470   BIC:                            -3366.
Df Model:                            1
Covariance Type:             nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0001      0.000     -0.431      0.666      -0.001       0.000
MASIR-Rf      -0.3117      0.054     -5.763      0.000      -0.418      -0.205
==============================================================================
Omnibus:                       90.912   Durbin-Watson:                   1.658
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             1236.390
Skew:                          -0.336   Prob(JB):                    3.32e-269
Kurtosis:                      10.900   Cond. No.                         174.
==============================================================================
```

(b) Estimation based on the market risk premium

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                 R_BL-Rf   R-squared:                       0.046
Model:                             OLS   Adj. R-squared:                  0.043
Method:                  Least Squares   F-statistic:                     17.97
Date:                 Sun, 30 Jun 2024   Prob (F-statistic):           2.83e-05
Time:                         16:23:55   Log-Likelihood:                 1376.7
No. Observations:                  377   AIC:                            -2749.
Df Residuals:                      375   BIC:                            -2742.
Df Model:                            1
Covariance Type:             nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0003      0.000      1.043      0.298      -0.000       0.001
HML           -0.4439      0.105     -4.239      0.000      -0.650      -0.238
==============================================================================
Omnibus:                       51.999   Durbin-Watson:                   1.266
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              246.904
Skew:                           0.451   Prob(JB):                     2.43e-54
Kurtosis:                       6.861   Cond. No.                         323.
==============================================================================
```

(c) Estimation based on SMB

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                 R_BL-Rf   R-squared:                       0.614
Model:                             OLS   Adj. R-squared:                  0.611
Method:                  Least Squares   F-statistic:                     196.9
Date:                 Sun, 30 Jun 2024   Prob (F-statistic):           1.83e-76
Time:                         16:32:25   Log-Likelihood:                 1542.8
No. Observations:                  376   AIC:                            -3078.
Df Residuals:                      372   BIC:                            -3062.
Df Model:                            3
Covariance Type:             nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0002      0.000     -0.782      0.435      -0.001       0.000
SMB           -1.1121      0.054    -20.524      0.000      -1.219      -1.006
HML           -0.5284      0.067     -7.889      0.000      -0.660      -0.397
MASIR-Rf      -0.1427      0.038     -3.751      0.000      -0.218      -0.068
==============================================================================
Omnibus:                       17.873   Durbin-Watson:                   1.395
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               32.508
Skew:                           0.281   Prob(JB):                     8.73e-08
Kurtosis:                       4.326   Cond. No.                         324.
==============================================================================
```

(d) Estimation based on the 3 factors

Figure 16 - Estimations of the portfolio BL

Given the individual significance of the t-test statistics, we observe that all probabilities corresponding to the t-statistic of Student for the Beta factor are below 5%, all of which are zero. Consequently, all the factors are significant and predictive of the returns of BL.

The overall significance is corroborated by the Fisher test, as all F-statistics are greater than their critical values at the 5% level. R squared is very low, with a maximum value concerning the Fama-French and SMB regression. We conclude that the regressions have an imperfect quality, except Fama-French which is quite good.

For the Akaike Information Criterion (AIC), Its value fluctuate between -4000 and -2000.

Fama French best describes the return of the BL portfolio.

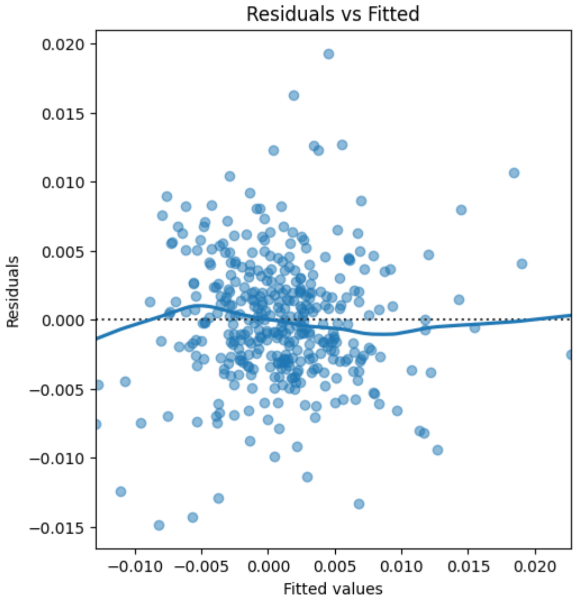
6.4 Residual diagnostics study FF3F - After corrections

For all portfolios, and The first property we examined is normality. We found that all probabilities from the Jarque-Bera and Omnibus tests are below 5%, rejecting the null hypothesis that the residuals are normally distributed. Also , the Q-Q plots deviate noticeably from the straight line, suggesting that the data deviates from a normal distribution. Skewness is not close to zero, indicating that the residuals are asymmetric. Similarly, kurtosis is around 3 , suggesting heavy tails in residuals.

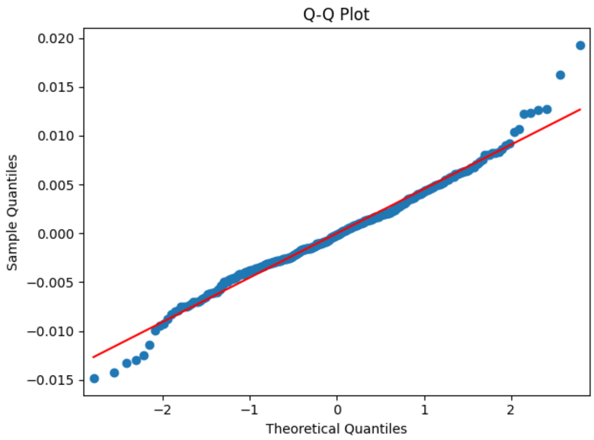
For all portfolios, Durbin-Watson statistics are less than 2, ranging from 1.1 to 1.8.

These results suggest that the residuals may not follow a normal distribution, and there are asymmetry and heavy-tailed distributions in residuals, especially for portfolios other than SH. Furthermore, Durbin-Watson statistics suggest positive autocorrelation in residuals for all portfolios.

In conclusion, the diagnostic results indicate that the residuals do not meet the assumptions of normality.



(a) Residuals vs Fitted



(b) Q-Q Plot

(c) Regression Plot

Figure 17 - Residual plots for the regression by SMB of the portfolio BH (example)

The other figures are present in the Colab Notebook, not put here because an example is sufficient.

7 Tracking-error Ex-ante

Tracking error ex-ante is a crucial measure for portfolio managers who aim to replicate or closely follow a benchmark index. It represents the predicted divergence between the portfolio's returns and the benchmark's returns over a specified future period. This measure is vital for understanding the potential risk and performance consistency relative to the benchmark.

The formula for tracking error ex-ante is typically given by :

$$\text{Tracking Error} = \sqrt{w^T C w}$$

where :
- w is the vector of excess weights relative to the benchmark.
- C is the forecasted covariance matrix of the returns.

Using the Fama-French Model

The Fama-French three-factor model is a robust approach to assess tracking error due to its ability to capture various dimensions of systematic risk that affect stock returns. The model includes three factors :

1. **Market Risk Premium (MKT-Rf)** : The excess return of the market over the risk-free rate.
2. **Small Minus Big (SMB)** : The return difference between portfolios of small-cap stocks and large-cap stocks.
3. **High Minus Low (HML)** : The return difference between portfolios of high book-to-market stocks and low book-to-market stocks.

By incorporating these factors, the Fama-French model provides a more comprehensive understanding of the sources of return, beyond just the market risk, thus making it an effective tool for evaluating and managing tracking error.

To calculate the ex-ante tracking error using the Fama-French model, follow these steps :

1. Determine Factor Exposures (Betas)

Perform a regression analysis of the portfolio returns against the Fama-French factors to obtain the factor loadings (betas). The regression model is :

$$R_p - R_f = \alpha + \beta_{MKT}(R_{MKT} - R_f) + \beta_{SMB} \cdot SMB + \beta_{HML} \cdot HML + \epsilon$$

where R_p is the portfolio return, R_f is the risk-free rate, and β_{MKT}, β_{SMB}, and β_{HML} are the factor loadings.

2. Construct the Covariance Matrix

Use historical data to estimate the covariance matrix C of the Fama-French factor returns. An example covariance matrix might look like this :

$$C = \begin{bmatrix} \sigma^2_{MKT} & \sigma_{MKT,SMB} & \sigma_{MKT,HML} \\ \sigma_{MKT,SMB} & \sigma^2_{SMB} & \sigma_{SMB,HML} \\ \sigma_{MKT,HML} & \sigma_{SMB,HML} & \sigma^2_{HML} \end{bmatrix}$$

3. Calculate Excess Weights

Determine the excess weights of the portfolio relative to the benchmark. If w_p represents the portfolio weights and w_b the benchmark weights, the excess weights w are :

$$w = w_p - w_b$$

4. Compute the Tracking Error

Using the excess weights and the covariance matrix, calculate the tracking error ex-ante :

$$\text{Tracking Error} = \sqrt{w^T C w}$$

By leveraging the Fama-French factors, this method captures a broader spectrum of systematic risks, providing a more accurate and detailed assessment of tracking error. This approach helps portfolio managers to better manage and predict the potential deviation from the benchmark, ultimately leading to more informed investment decisions and risk management strategies.

Our Methodology

To apply this methodology, we first construct a portfolio comprising 10 stocks with a certain allocation. For example, we might assign more weight to Small (S) stocks to observe how the betas change with varying weights and identify which factor explains the performance of a portfolio dominated by S stocks.

Then, we calculate the covariance matrix C using historical data. The portfolio weights vector w is represented through the three betas. The weights for each factor are calculated as follows :

$$w_{MKT} = \frac{\beta_{MKT}}{\beta_{MKT} + \beta_{SMB} + \beta_{HML}}, \quad w_{SMB} = \frac{\beta_{SMB}}{\beta_{MKT} + \beta_{SMB} + \beta_{HML}}, \quad w_{HML} = \frac{\beta_{HML}}{\beta_{MKT} + \beta_{SMB} + \beta_{HML}}$$

by absolute value.

Using these weights, we compute the tracking error :

$$\text{Tracking Error} = \sqrt{w^T C w}$$

This approach helps us understand the contribution of each factor to the tracking error and manage the portfolio accordingly to align closely with the benchmark.

In the Colab Notebook, we constructed a portfolio of 10 random S stocks listed below :

	libelleFR	symbol	codeISIN	instrument_url	drupal_internal__id	emetteur_url	Secteur	Capitalisation	PBR	PER	BTM	Category	BTM_Category
0	IMMORENTE INVEST	IMO	MA0000012387	/fr/live-market/instruments/IMO	388	/fr/live-market/emetteurs/IMO110518	Sociétés de placement immobilier	8.340482e+08	1.23	527,16	0.813008	S	H
1	ENNAKL	NKL	MA0000011942	/fr/live-market/instruments/NKL	381	/fr/live-market/emetteurs/NAKL130710	Distributeurs	1.056000e+09	1.14	6,92	0.877193	S	H
2	SAMIR	SAM	MA0000010803	/fr/live-market/instruments/SAM	492	/fr/live-market/emetteurs/SAM190396	Pétrole et Gaz	1.520777e+09	0.41	-	2.439024	S	H
3	DISWAY	DWY	MA0000011637	/fr/live-market/instruments/DWY	536	/fr/live-market/emetteurs/MAR280207	Matériels, Logiciels et Services Informatiques	1.314376e+09	1.83	18,71	0.546448	S	H
4	SNEP	SNP	MA0000011728	/fr/live-market/instruments/SNP	367	/fr/live-market/emetteurs/SNP071107	Chimie	1.223760e+09	1.64	-	0.609756	S	H
5	ALUMINIUM DU MAROC	ALM	MA0000010936	/fr/live-market/instruments/ALM	490	/fr/live-market/emetteurs/ALM271098	Bâtiment et Matériaux de Construction	7.031246e+08	2.38	0,96	0.420168	S	L
6	S.M MONETIQUE	S2M	MA0000012106	/fr/live-market/instruments/S2M	386	/fr/live-market/emetteurs/S2M271211	Matériels, Logiciels et Services Informatiques	1.436958e+08	2.02	6,78	0.495050	S	L
7	COLORADO	COL	MA0000011934	/fr/live-market/instruments/COL	530	/fr/live-market/emetteurs/COL271006	Bâtiment et Matériaux de Construction	8.537499e+08	2.30	19,40	0.434783	S	L
8	DIAC SALAF	DIS	MA0000010639	/fr/live-market/instruments/DIS	466	/fr/live-market/emetteurs/DIS010662	Sociétés de financement et Autres Activités Fi...	2.765186e+07	NaN	-	NaN	S	L
9	AGMA	AGM	MA0000010944	/fr/live-market/instruments/AGM	491	/fr/live-market/emetteurs/AGM091198	Assurances	1.365000e+09	8.73	20,16	0.114548	S	L

Figure 17 - The dataframe of the portfolio selected

After that we calculated the weighted return on a daily basis, then conducted the Fama French 3 factor regression to have the 3 betas (Red cercle).

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                  Ri-Rf   R-squared:                       0.018
Model:                            OLS   Adj. R-squared:                  0.006
Method:                 Least Squares   F-statistic:                     1.479
Date:                Sun, 30 Jun 2024   Prob (F-statistic):              0.221
Time:                        19:02:47   Log-Likelihood:                 935.38
No. Observations:                 252   AIC:                            -1863.
Df Residuals:                     248   BIC:                            -1849.
Df Model:                           3
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const      -7.179e-05      0.000     -0.185      0.853      -0.001       0.001
SMB            0.1669      0.102      1.630      0.104      -0.035       0.369
HML            0.1681      0.123      1.369      0.172      -0.074       0.410
MASIR-Rf      -0.0631      0.064     -0.991      0.323      -0.189       0.062
==============================================================================
Omnibus:                        3.913   Durbin-Watson:                   1.961
Prob(Omnibus):                  0.141   Jarque-Bera (JB):                4.862
Skew:                           0.007   Prob(JB):                       0.0879
Kurtosis:                       3.680   Cond. No.                         331.
==============================================================================
```

	HML	SMB	MASIR-Rf
HML	9.632313e-06	-6.349924e-07	-8.030231e-07
SMB	-6.349924e-07	1.653509e-05	7.858923e-06
MASIR-Rf	-8.030231e-07	7.858923e-06	3.351117e-05

Figure 18 - The betas of the regression and the cov Matrix

Then, using the formulas above, we calculated the vector w, and using the tracking error formula we found that :

$$\text{Tracking Error} = 0.0024906926163459787 \approx 0.25\%$$

8 Future perspectives

As a future perspective, the study aims to extend the analysis by incorporating the Carhart 4-Factor Model and the Fama-French 5-Factor Model to explore additional factors like momentum. These extended models could potentially provide deeper insights into the drivers of stock returns in the Moroccan market. Additionally, there is an interest in applying these models in trading strategies and conducting backtesting to evaluate which model offers the best return over short and long terms in algorithmic trading. This approach will help in identifying the most effective model for practical trading applications and improving investment strategies.

9 Contributions of the internship

This internship at Wafa Gestion has significantly enhanced my understanding of financial models and their application. Through rigorous statistical analysis and quantitative research, I have gained proficiency in evaluating the performance of different financial models such as the CAPM and Fama-French factors. I have also learned how to apply these models to real market data, improving my skills in data handling, statistical analysis, and financial modeling.

From an ethical perspective, the internship has instilled the importance of integrity and transparency in financial analysis. It has taught me to approach financial research with a critical mindset, ensuring that all analyses are conducted rigorously and ethically. The experience has also emphasized the importance of responsible financial decision-making and the impact of accurate financial modeling on investment strategies and market behavior.

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