AI 330: Machine Learning

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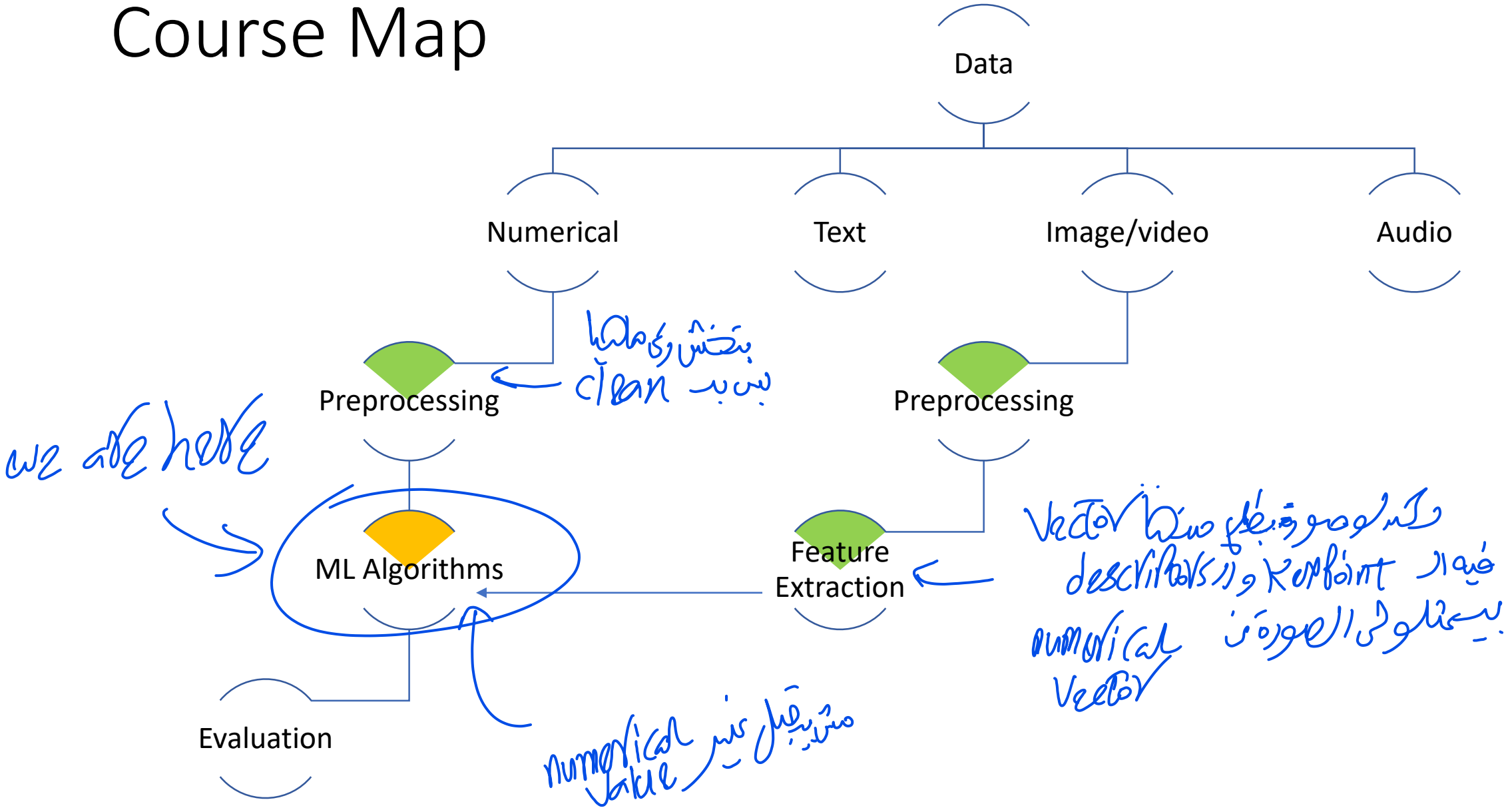
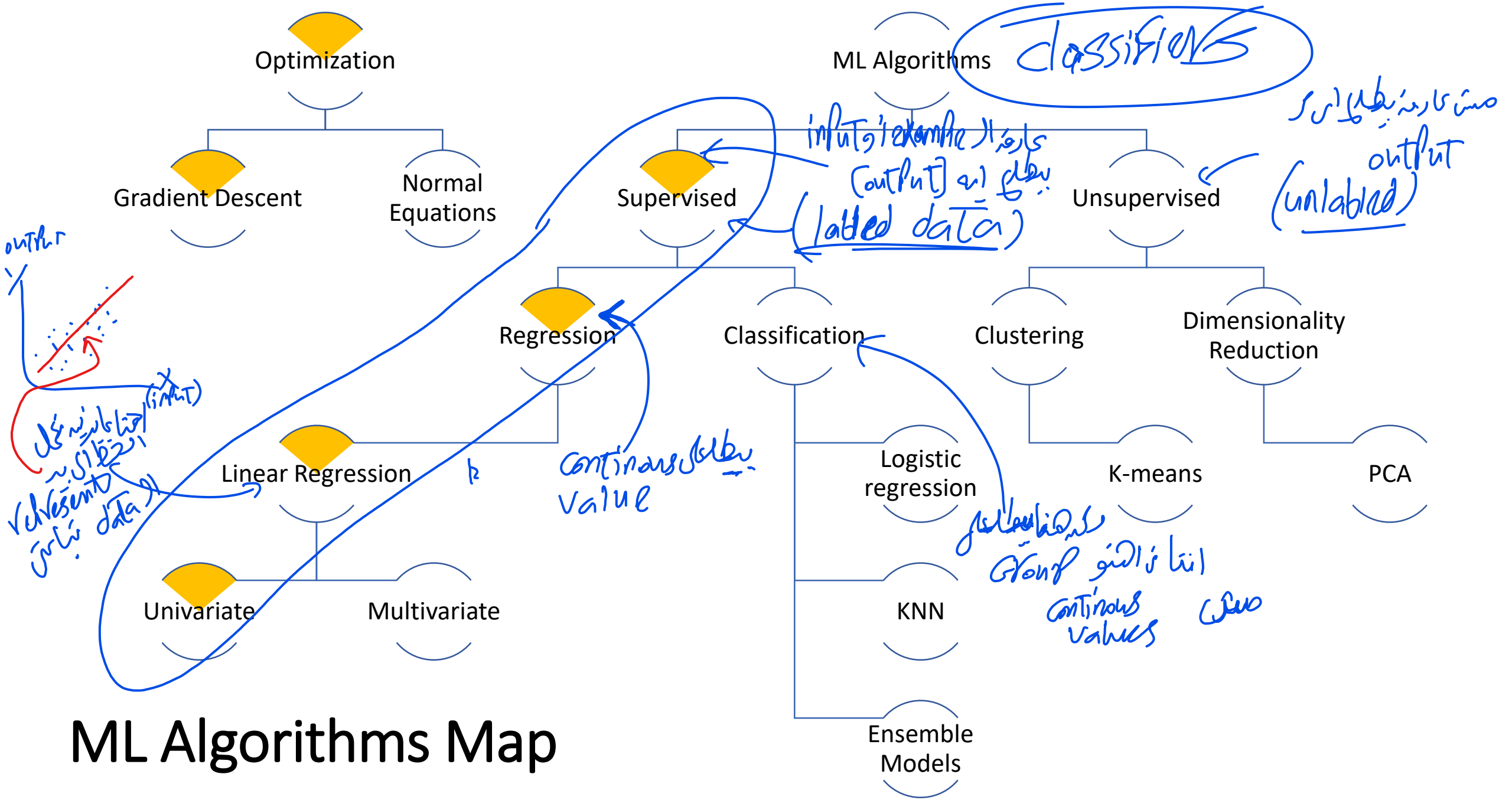
Course Map


Data
Numerical
Text
Image/video
Audio
Preprocessing
Preprocessing
ML Algorithms
Feature Extraction
Evaluation



ML Algorithms Map

Lecture 4
Univariate Linear Regression
&
Gradient Descent

Hypothesis – Cost function – Optimization (Gradient Descent)

Slides of:
Machine Learning Specialization https://www.coursera.org/specializations/machine-learning-introduction
at Stanford University (**Andrew Ng**)

CLASSICAL MACHINE LEARNING

Data is pre-categorized or numerical

SUPERVISED

Predict a category

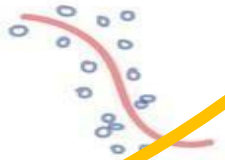
CLASSIFICATION

«Divide the socks by color»

Predict a number

REGRESSION

«Divide the ties by length»



Data is not labeled in any way

UNSUPERVISED

Divide by similarity

CLUSTERING

«Split up similar clothing into stacks»

Find hidden dependencies

DIMENSION REDUCTION (generalization)

«Make the best outfits from the given clothes»

Identify sequences

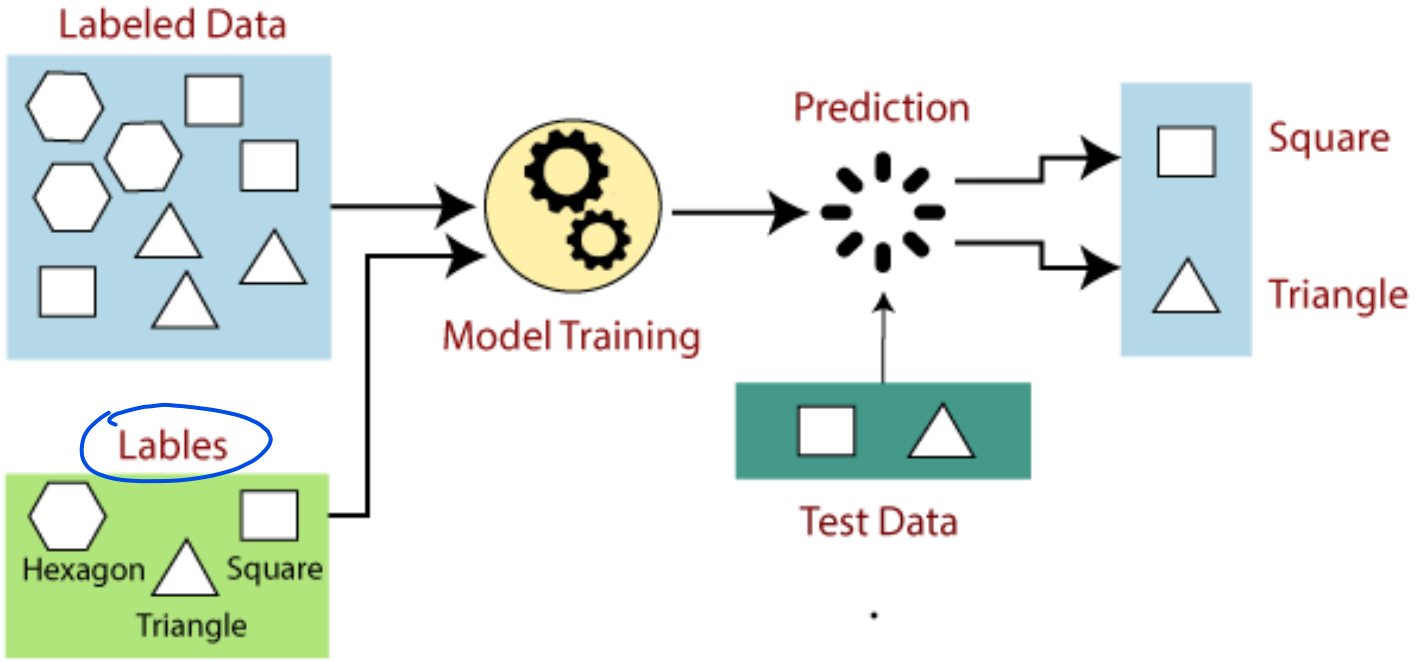
ASSOCIATION

«Find what clothes I often wear together»

https://medium.com/@dkatzman_3920/supervised-vs-unsupervised-learning-and-use-cases-for-each-8b9cc3ebd301

Supervised Learning

- **In Supervised learning problems,** We are given a (**labelled or annotated**) dataset that we already know what our correct output should look like (**true label**), having the idea that there is a relationship between the input and the output.

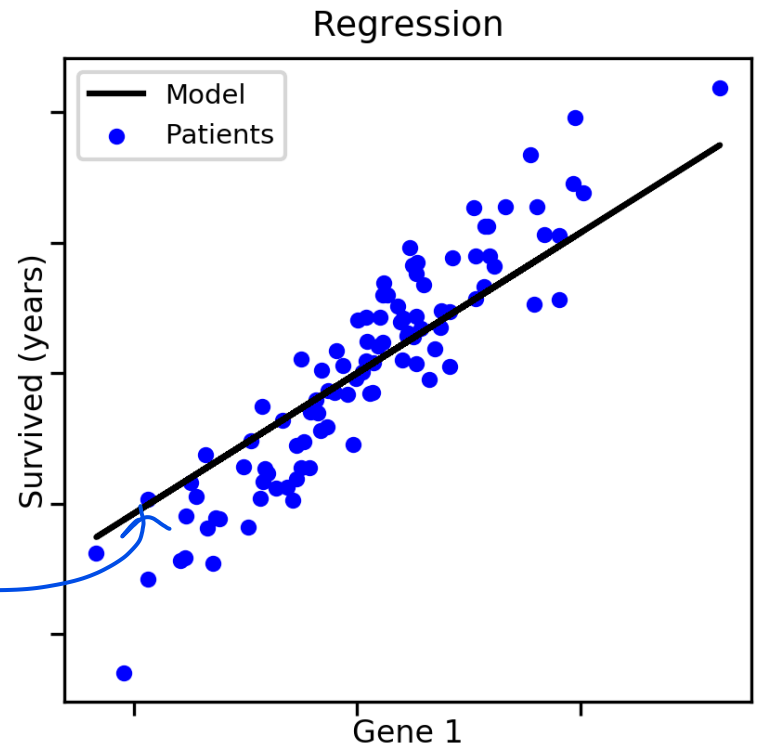


Supervised Learning

- **Supervised learning problems are categorized into:**
  - Regression Problem. → Predict a number
  - Classification Problem. → in a category

Regression Problem

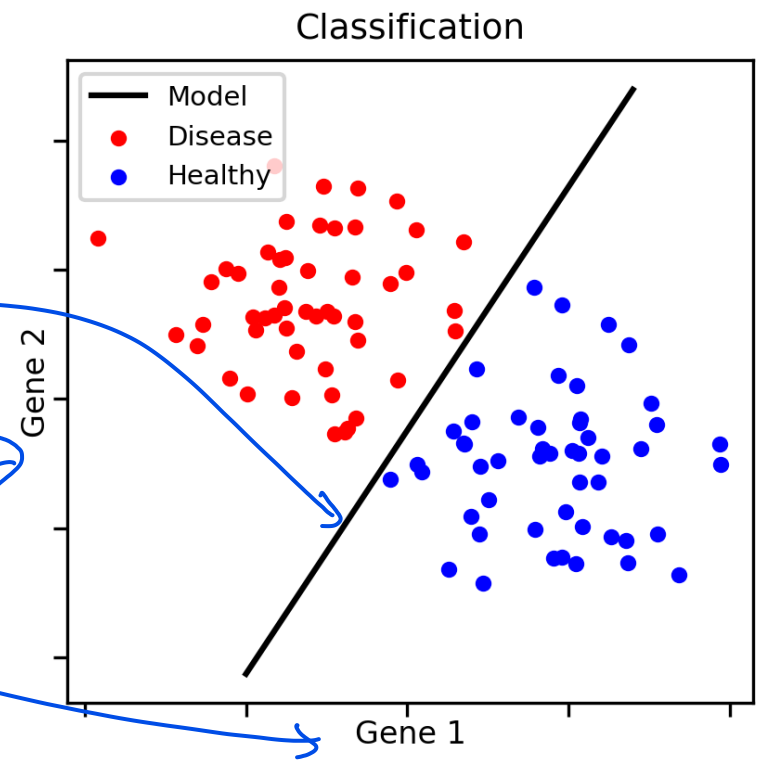
- In a **regression problem**,
  - we are trying to predict results within a continuous output, meaning that we are trying to map input variables to some continuous function.



https://aldro61.github.io/microbiome-summer-school-2017/sections/basics/

Classification Problem

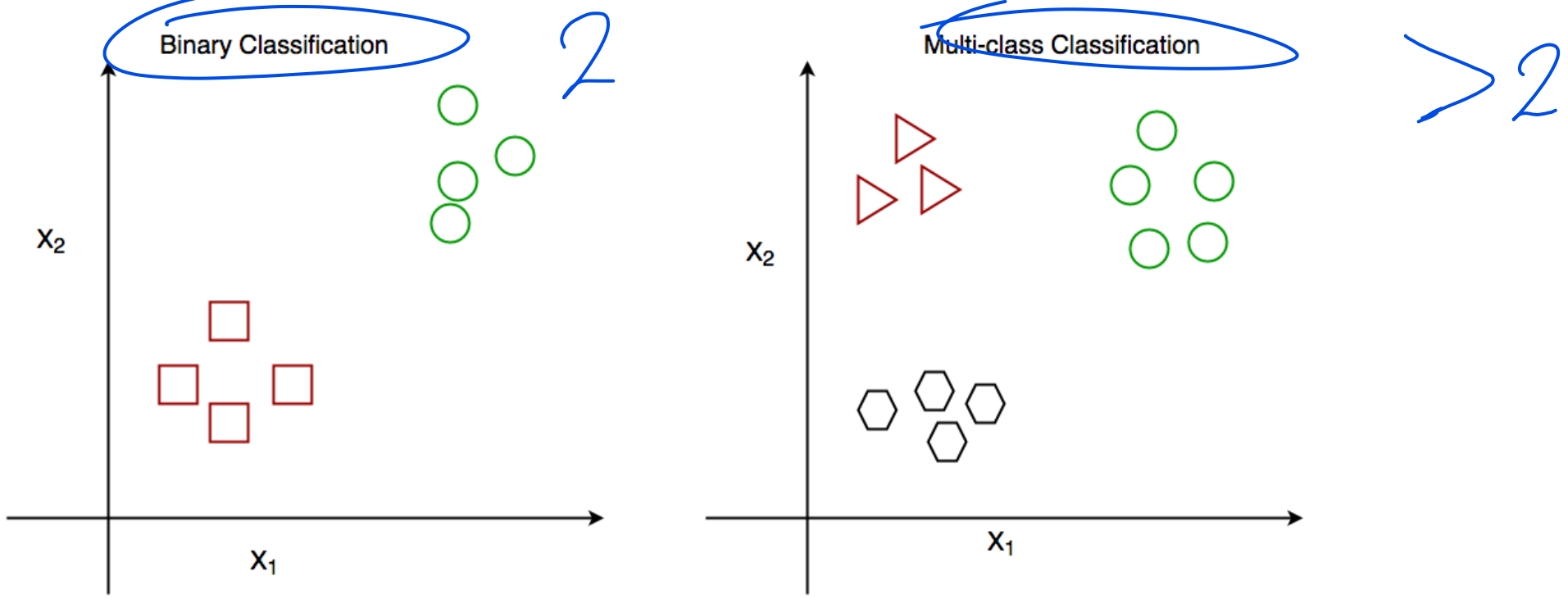
- In a **classification problem**,
  - we are trying to predict results in a discrete output.



https://aldro61.github.io/microbiome-summer-school-2017/sections/basics/

Classification Problem

- **Classification problems** can be:
  - **Binary** classification has only 2 outputs
  - **Multi-class** classification has more than 2 outputs (3 or more).



https://www.geeksforgeeks.org/ml-classification-vs-regression/

Regression or Classification?

- Given data about the size of houses on the real estate market, try to predict their price. — R
- Given a picture of Male/Female, We have to predict his/her age on the basis of given picture. — R
- Given a picture of Male/Female, We have to predict Whether She is of High school, College, Graduate age. — C
- Banks have to decide whether or not to give a loan to someone on the basis of his credit history. — C

the first classifier (algorithm)

Regression Problem:

Univariate Linear Regression

Linear Regression with One Variable

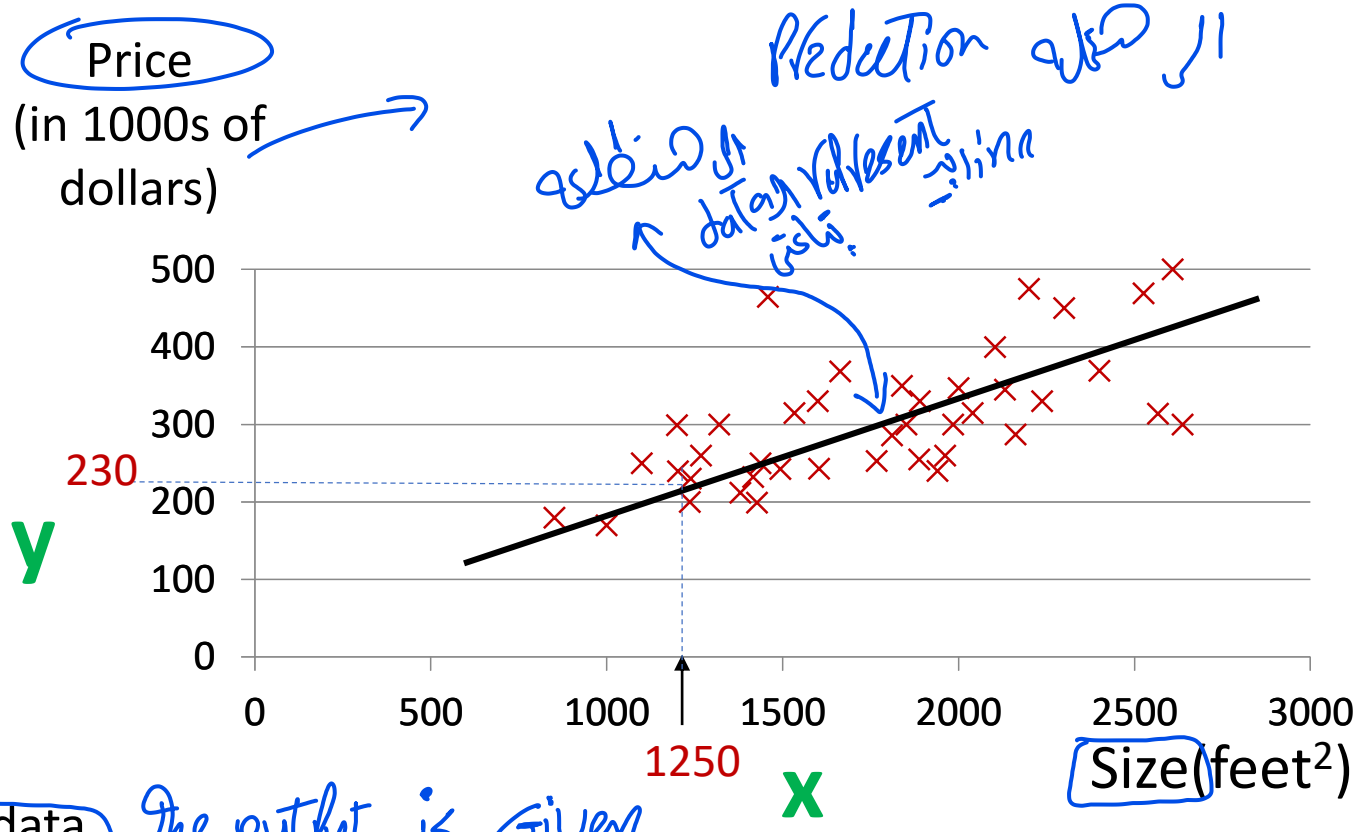
يعني Variable واحد بس وعلى حسب قيمته هعمل Prediction

Linear Regression with One Variable

- Linear regression with one variable is also known as "**univariate**"
- We want to predict a **single output** value **y** from a **single input** value **x**.

Housing Prices (Portland, Oregon)

Living area ($feet^2$)	Price (1000$s)
2104	400
1600	330
2400	369
1416	232
3000	540
⋮	⋮



Supervised Learning

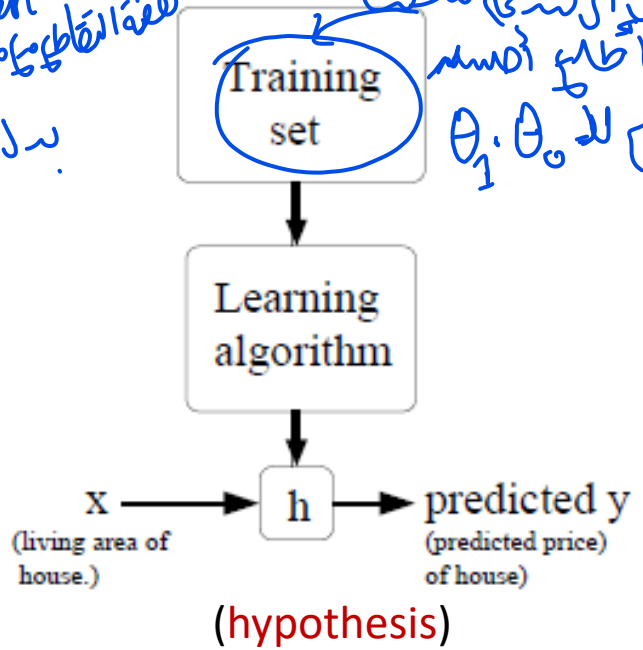
Given the "right answer" for each example in the data.

The Hypothesis Function

- Our **hypothesis function** has the general form:

$$\hat{y} = h_\theta(x) = \theta_0 + \theta_1 x$$

  - like the equation of a straight line
  - we are trying to create a function called h_θ that is trying to map our input data (the x's) to our output data (the y's).



Parameters/ weights

Notation:

x's = "input" variable / features

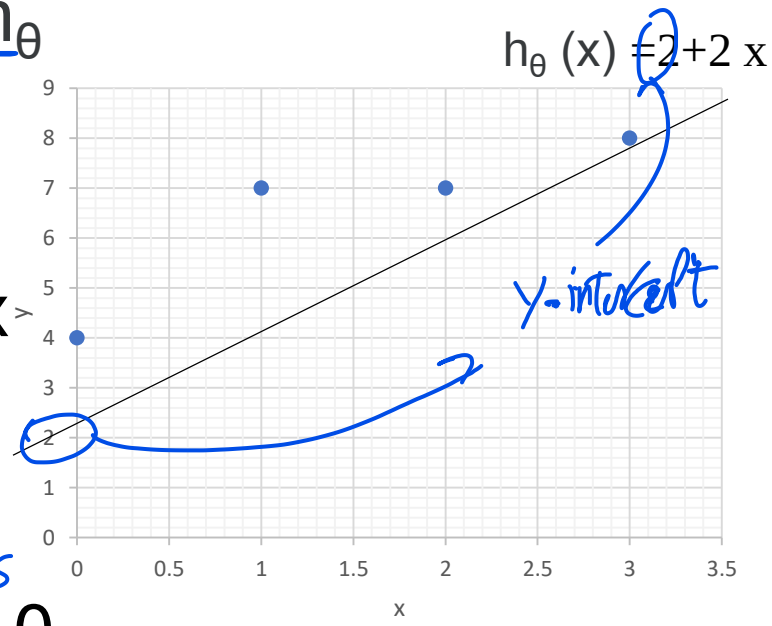
y's = "output" variable / "target" variable

Example

input x	output y
0	4
1	7
2	7
3	8

- Suppose we have the following set of training data:
- Now we can make a random guess about our h_θ

  For example: θ_0 , θ_1 =2



- The hypothesis function becomes h_θ (x) =2+2 x
- So for input of x=1 to our hypothesis $h_\theta(1)$,

  $\hat{y} = 4$. This is off by 3.

Note that we will be trying out various values θ_0 , θ_1

- to try to find values which provide the best possible "**fit**" or the most representative "**straight line**" through the data points mapped on the x-y plane.

Cost Function

- We can measure the accuracy of our hypothesis function by using a **cost function**.
- This takes an average of all the results of the hypothesis with inputs from x's compared to the actual output y's.

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right)^2$$

Squared error function

Notation

m = Number of training examples

$(x^{(i)}, y^{(i)}) = i^{th}$ training example

Objective function (Our goal) : $minimize_{\theta_0,\theta_1} J(\theta_0, \theta_1)$

Choose θ_0, θ_1 so that $h_\theta(x)$ is close to output y for our training examples (x,y)

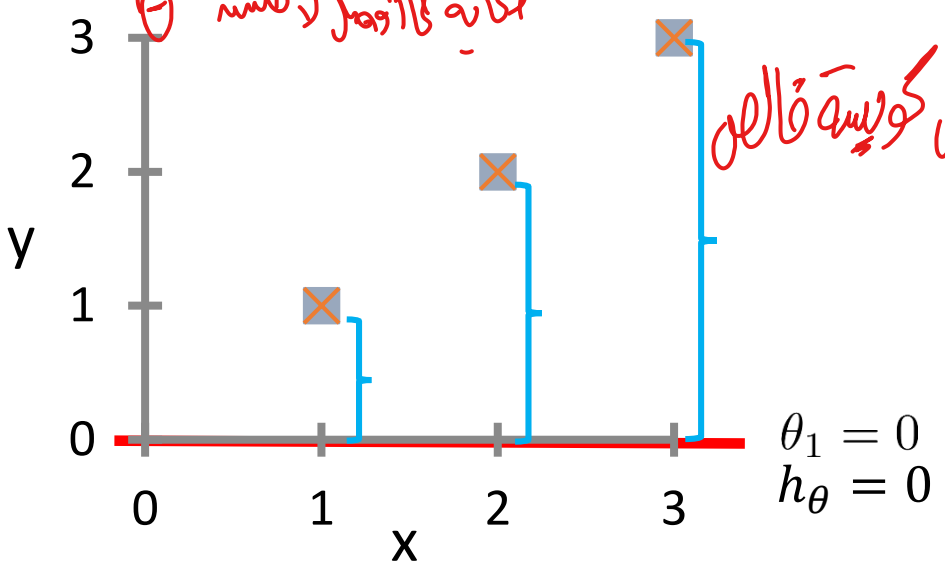
Simplified Example

$$h_\theta = \theta_1 x$$

input x	output y
1	1
2	2
3	3

$h_\theta(x)$

(for fixed θ_1 , this is a function of x)



Cost function $J(\theta_1) = \frac{1}{2m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2$

Where m (# of samples)= 3

$$J(0) = \frac{1}{6}((0-1)^2 + (0-2)^2 + (0-3)^2)$$

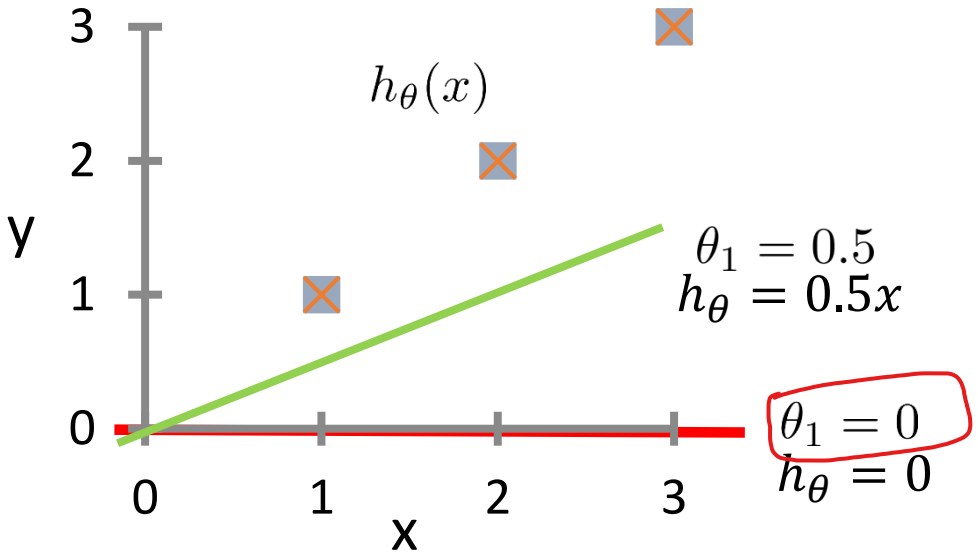
$$= \frac{1}{6}(1+4+9) \approx 2.3$$

$J(\frac{1}{2})$

Simplified Example

$h_\theta(x)$

(for fixed θ_1 , this is a function of x)



$$h_\theta = \theta_1 x$$

input x	output y
1	1
2	2
3	3

Cost function $J(\theta_1) = \frac{1}{2m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2$

Where m (# of samples)= 3

$$J(0) = \frac{1}{6}((0-1)^2 + (0-2)^2 + (0-3)^2)$$

$$= \frac{1}{6}(1+4+9) \approx 2.3$$

$$J(0.5) = \frac{1}{6}((0.5-1)^2 + (1-2)^2 + (1.5-3)^2)$$

$$= \frac{1}{6}(0.25+1+2.25) \approx 0.58$$

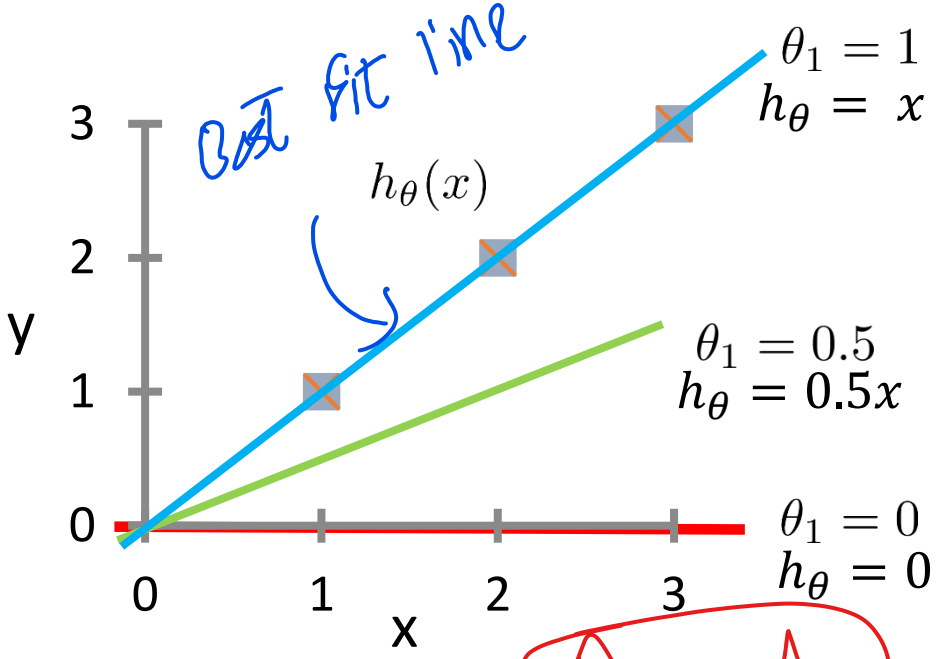
Simplified hypothesis

$$h_\theta = \theta_1 x$$

input x	output y
1	1
2	2
3	3

$h_\theta(x)$

(for fixed θ_1 , this is a function of x)



Cost function $J(\theta_1) = \frac{1}{2m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2$

Where m (# of samples)= 3

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$$= \frac{1}{6}(0.25+1+2.25) \approx 0.58$$

$$J(1) = \frac{1}{6}((1-1)^2 + (2-2)^2 + (3-3)^2)$$

$$= \frac{1}{6}(0+0+0) = 0$$

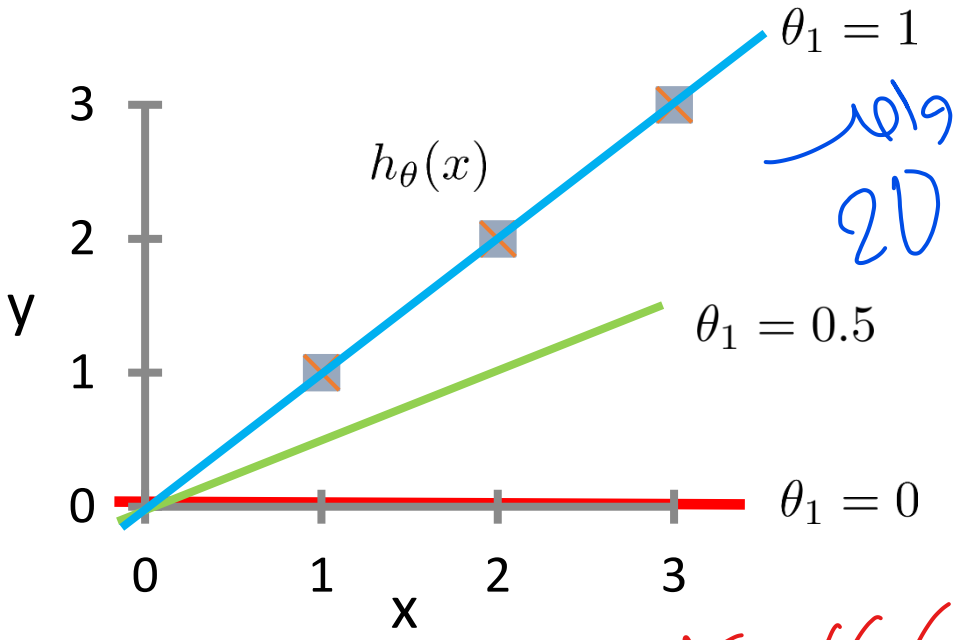
Simplified Example

$$h_\theta(x) = \theta_1 x$$

J(0) ≈ 2.3
J(0.5) ≈ 0.58
J(1)=0

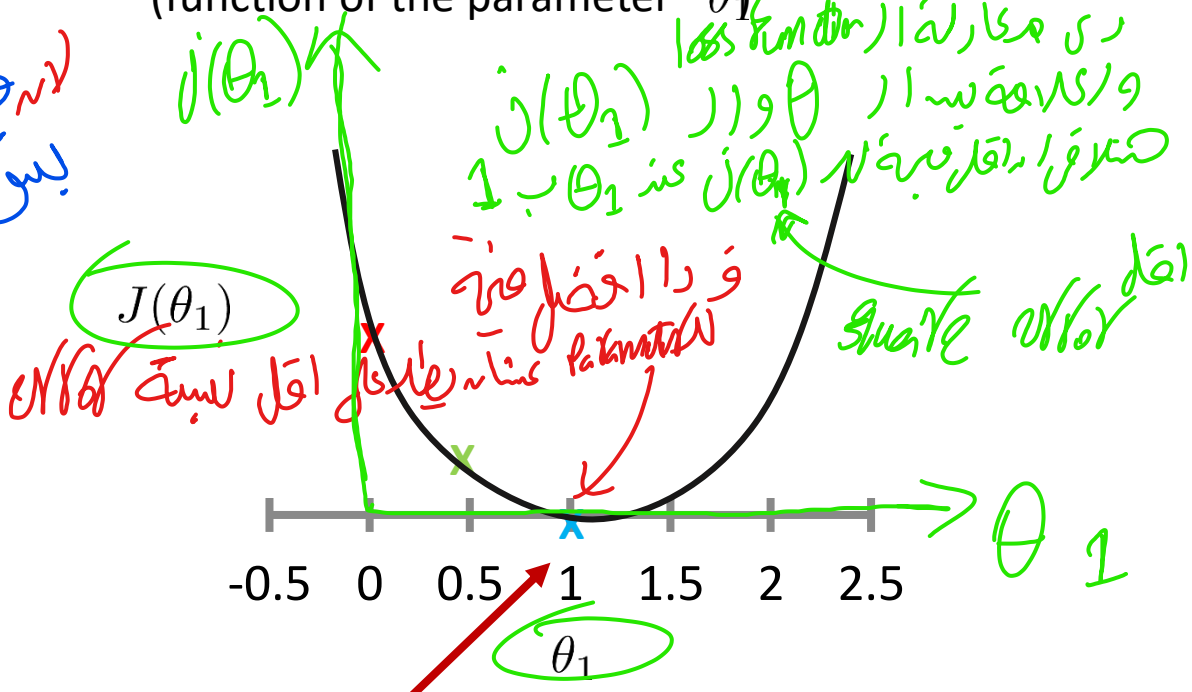
$h_\theta(x)$

(for fixed θ_1 , this is a function of x)



$J(\theta_1)$

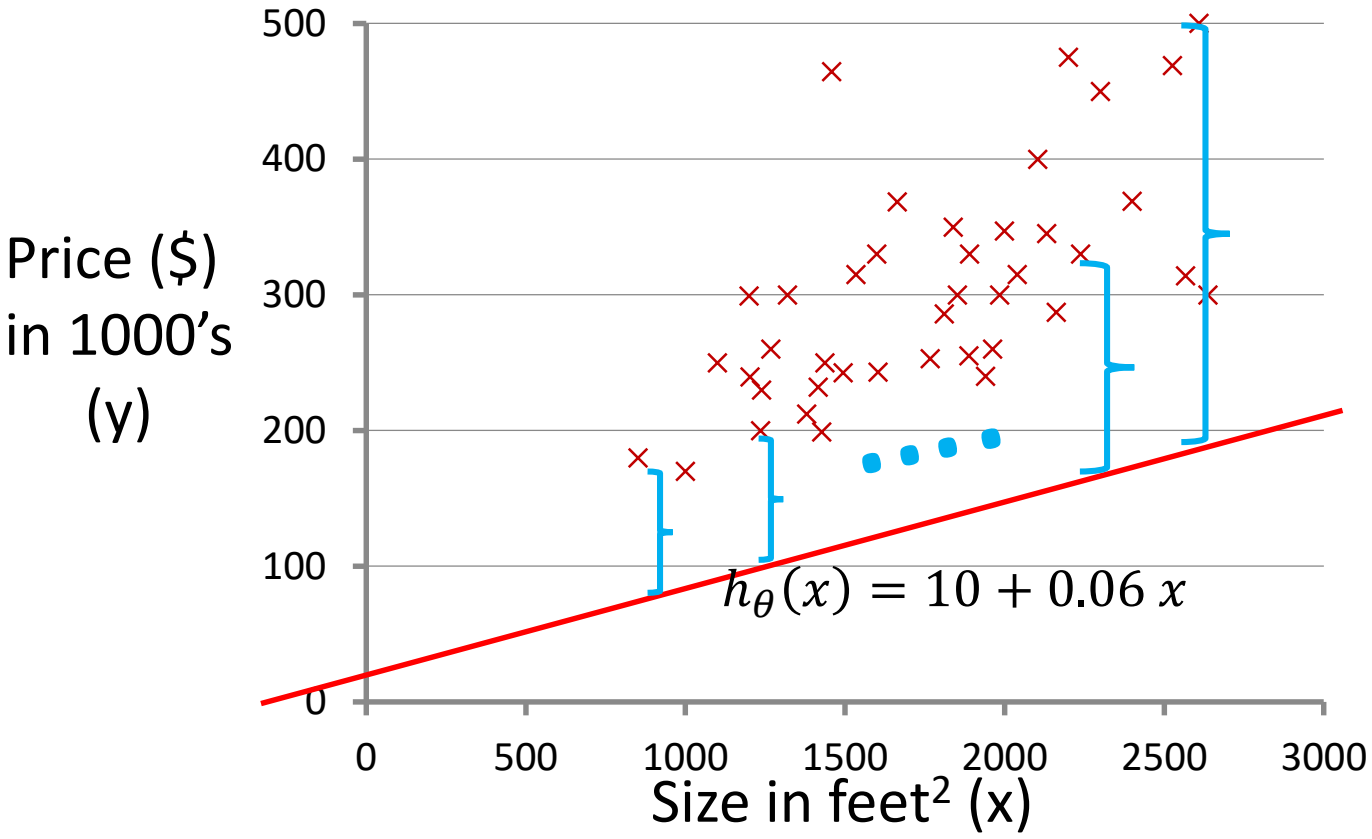
(function of the parameter θ_1)



Minimum cost = Best fit line (our objective)

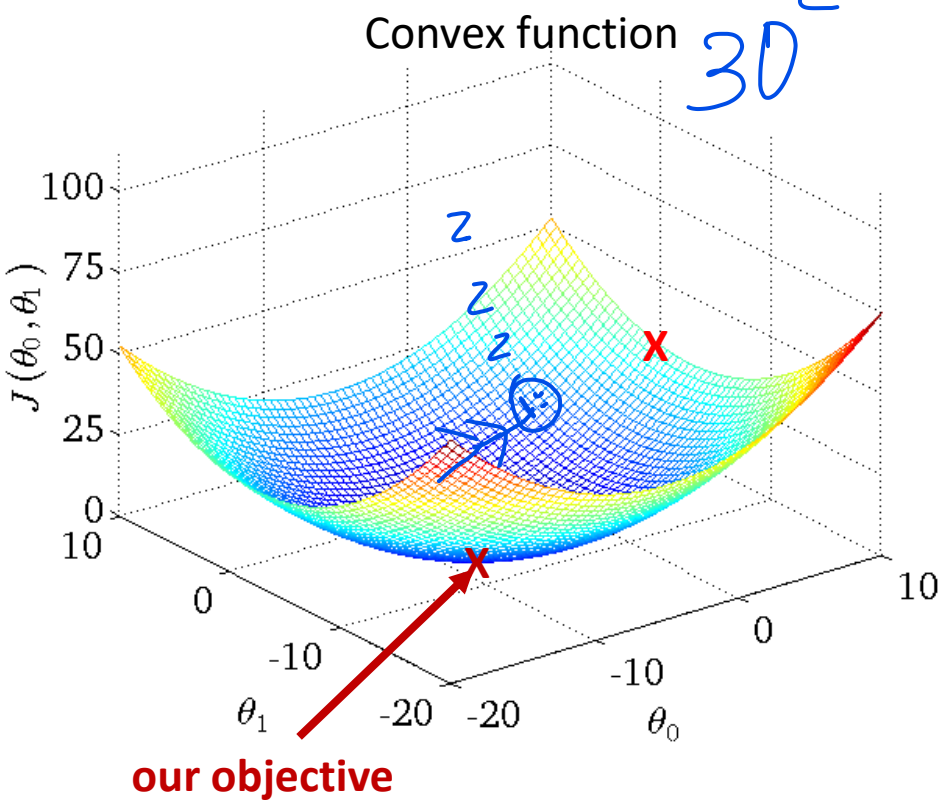
Hypothesis: $h_\theta(x) = \theta_0 + \theta_1 x$

(for fixed θ_0, θ_1 this is a function of x)



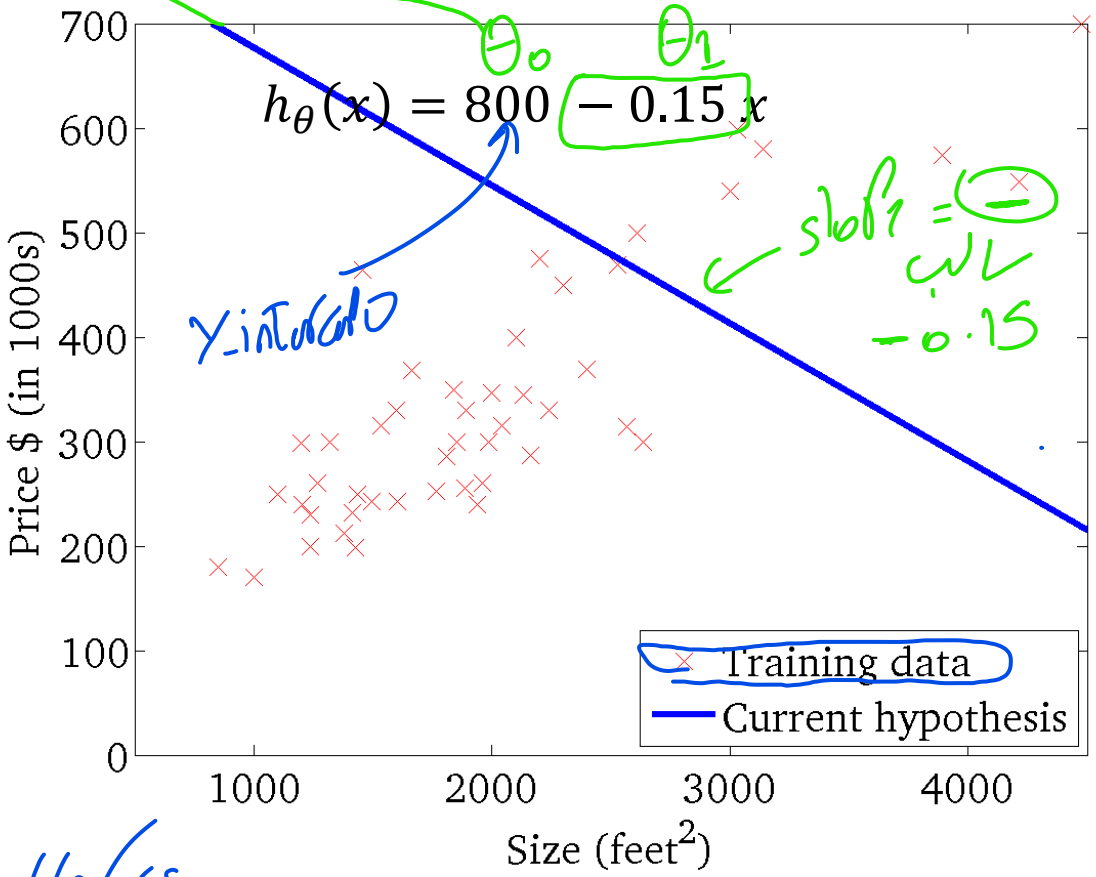
$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



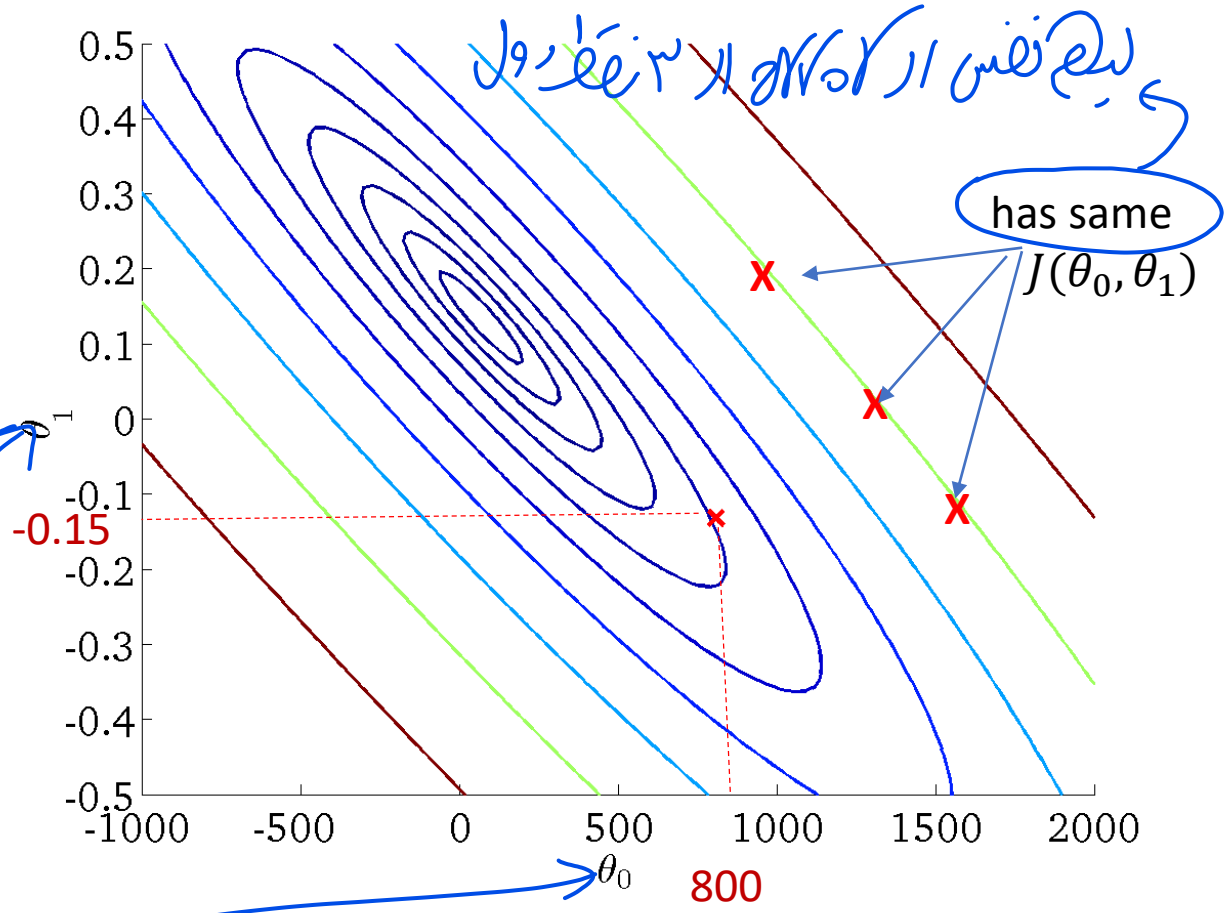
$h_\theta(x)$

(for fixed θ_0, θ_1 this is a function of x)



$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



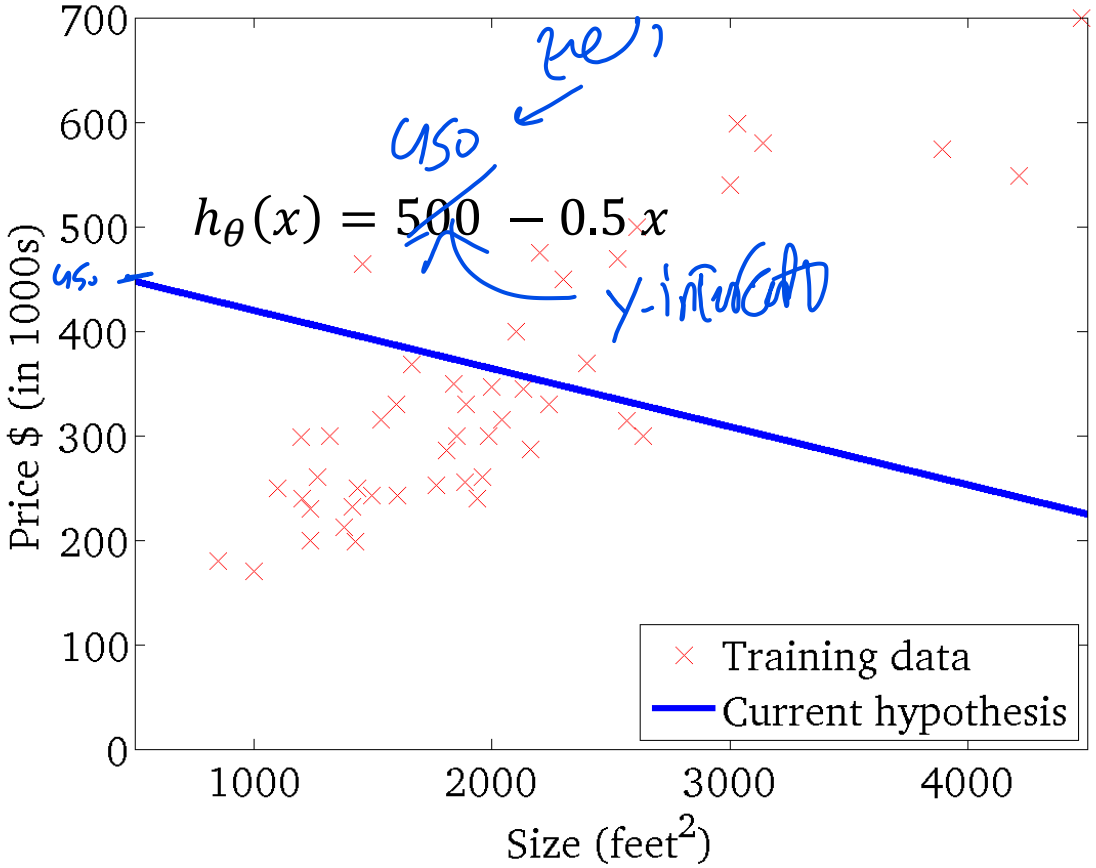
"Contour plots" or "Contour figures"

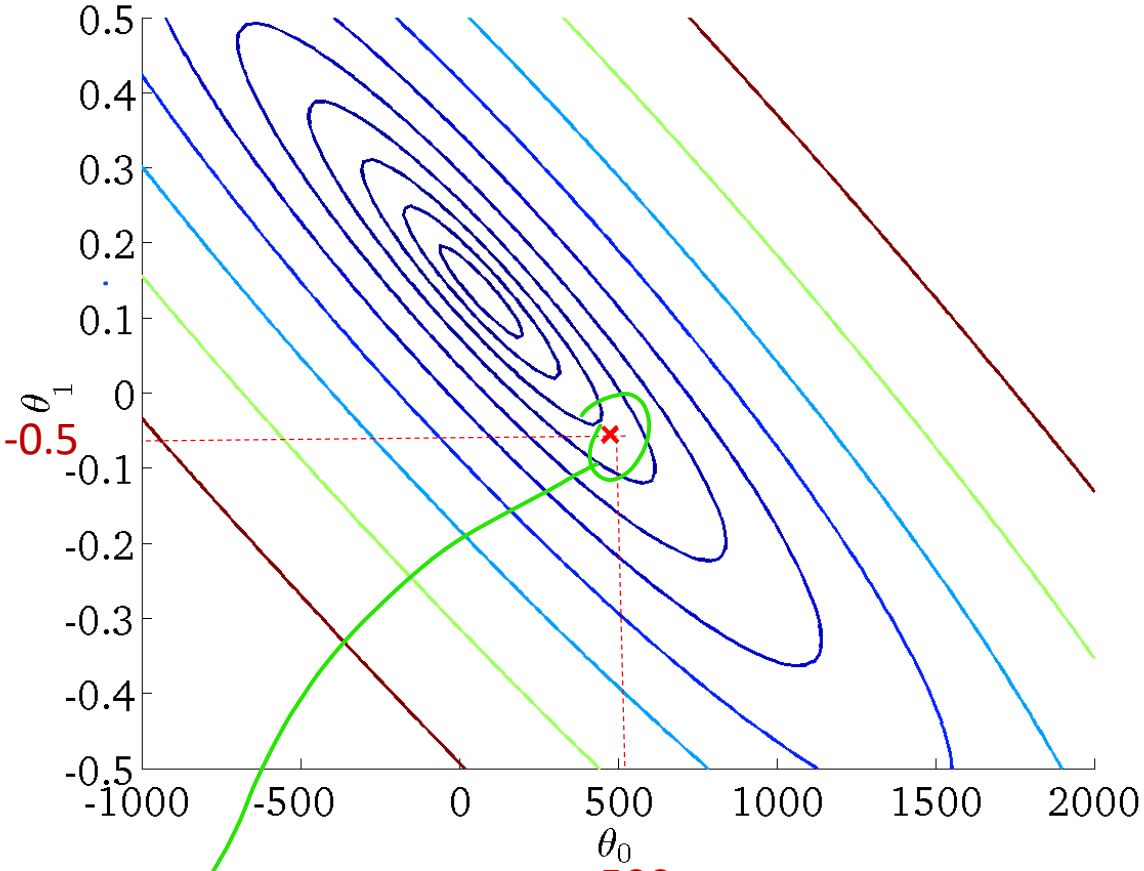
$h_\theta(x)$

(for fixed θ_0, θ_1 this is a function of x)

$J(\theta_0, \theta_1)$

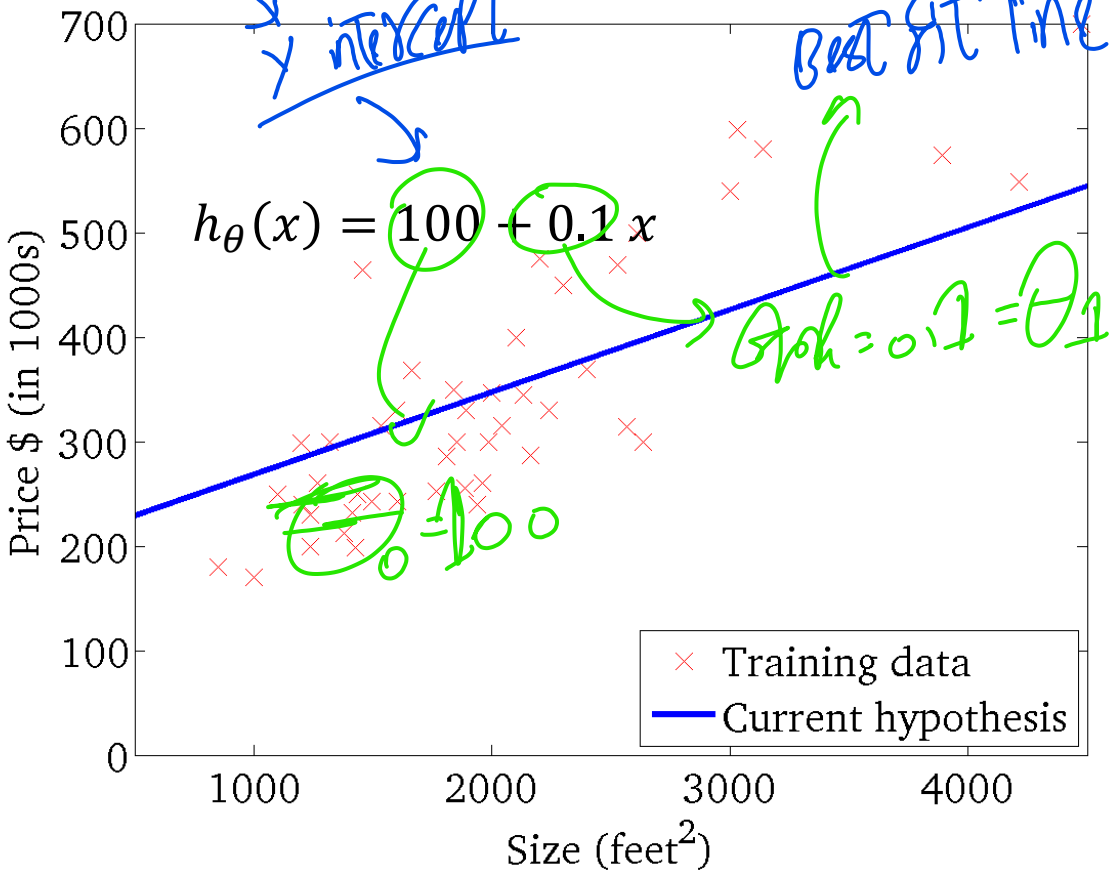
(function of the parameters θ_0, θ_1)





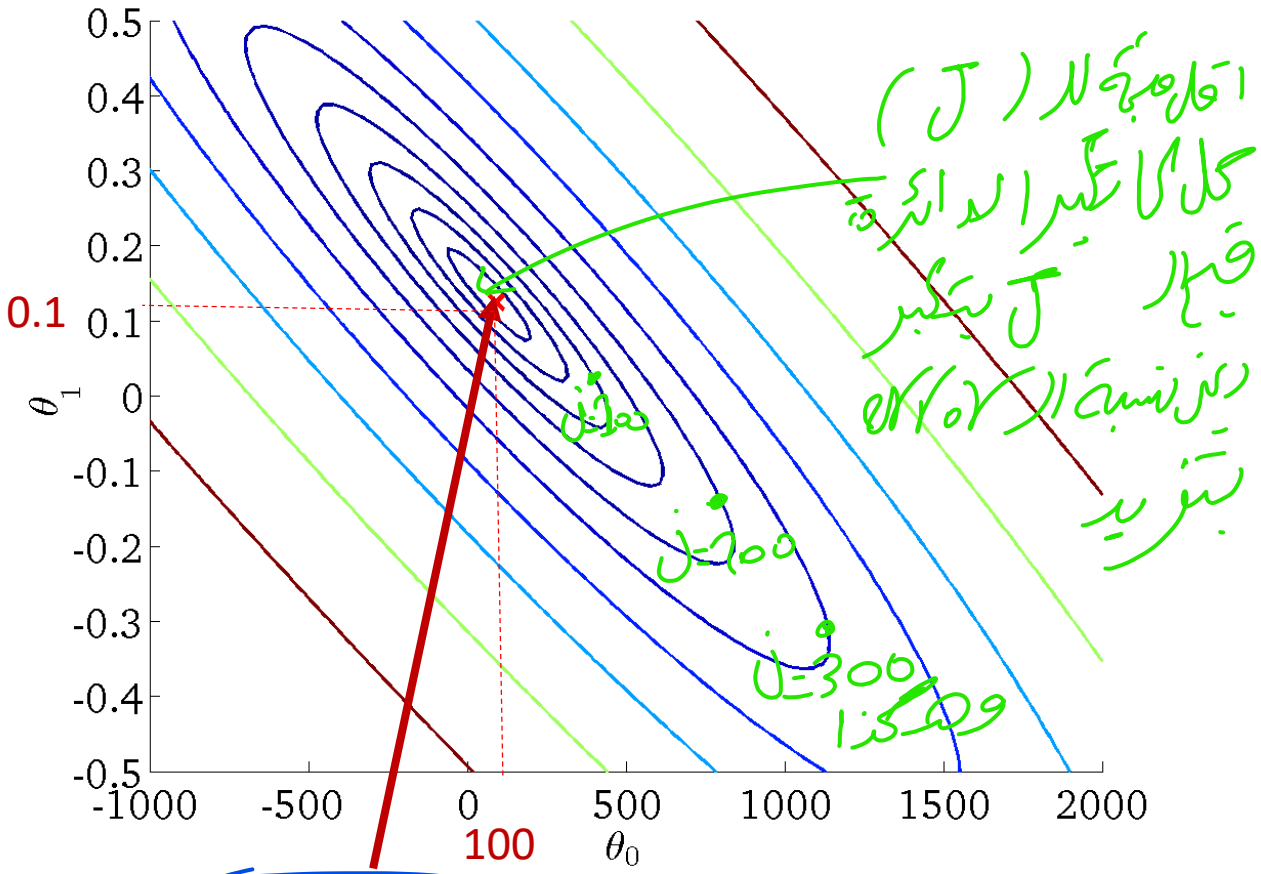
$h_\theta(x)$

(for fixed θ_0, θ_1 this is a function of x)

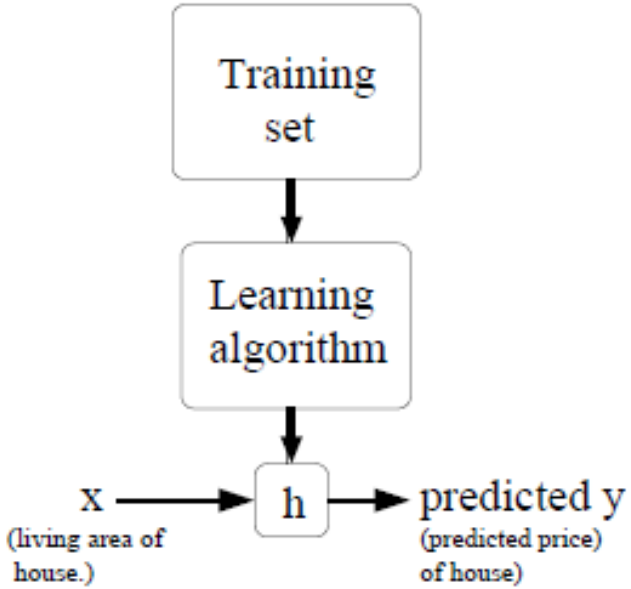


$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



Quick Summary until now



Hypothesis: $h_\theta(x) = \theta_0 + \theta_1 x$

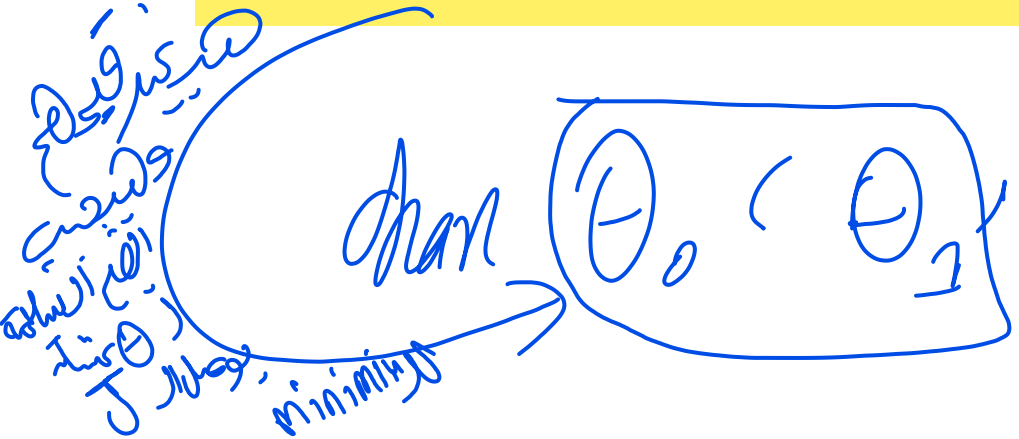
Cost Function: $J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right)^2$

Goal: $\underset{\theta_0, \theta_1}{\text{minimize}}\ J(\theta_0, \theta_1)$

Our strategy until now: Keep changing θ_0, θ_1 to reduce $J(\theta_0, \theta_1)$ until we hopefully end up at a minimum!!

Optimization Technique:

Gradient Descent



Optimization

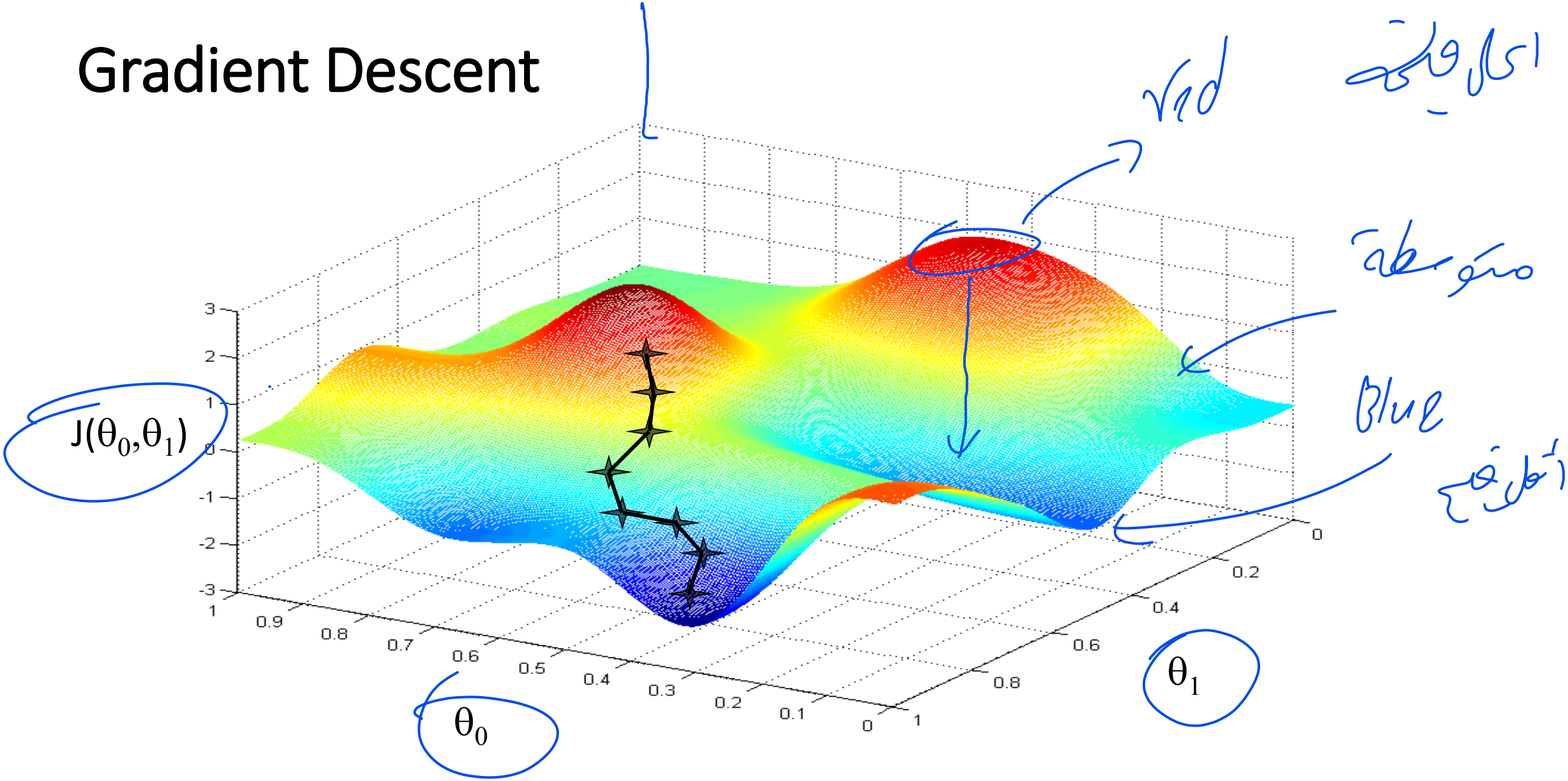
- **Optimization** is the process of finding the set of parameters/weights θ_0, θ_1 that minimize the **cost function**.

- **Strategy 1:** A first very bad idea solution: RANDOM SEARCH
  - is to simply try out many different random parameters and keep track of what works best. (iterative refinement.)
  - start with random weights and iteratively refine them over time to get lower cost

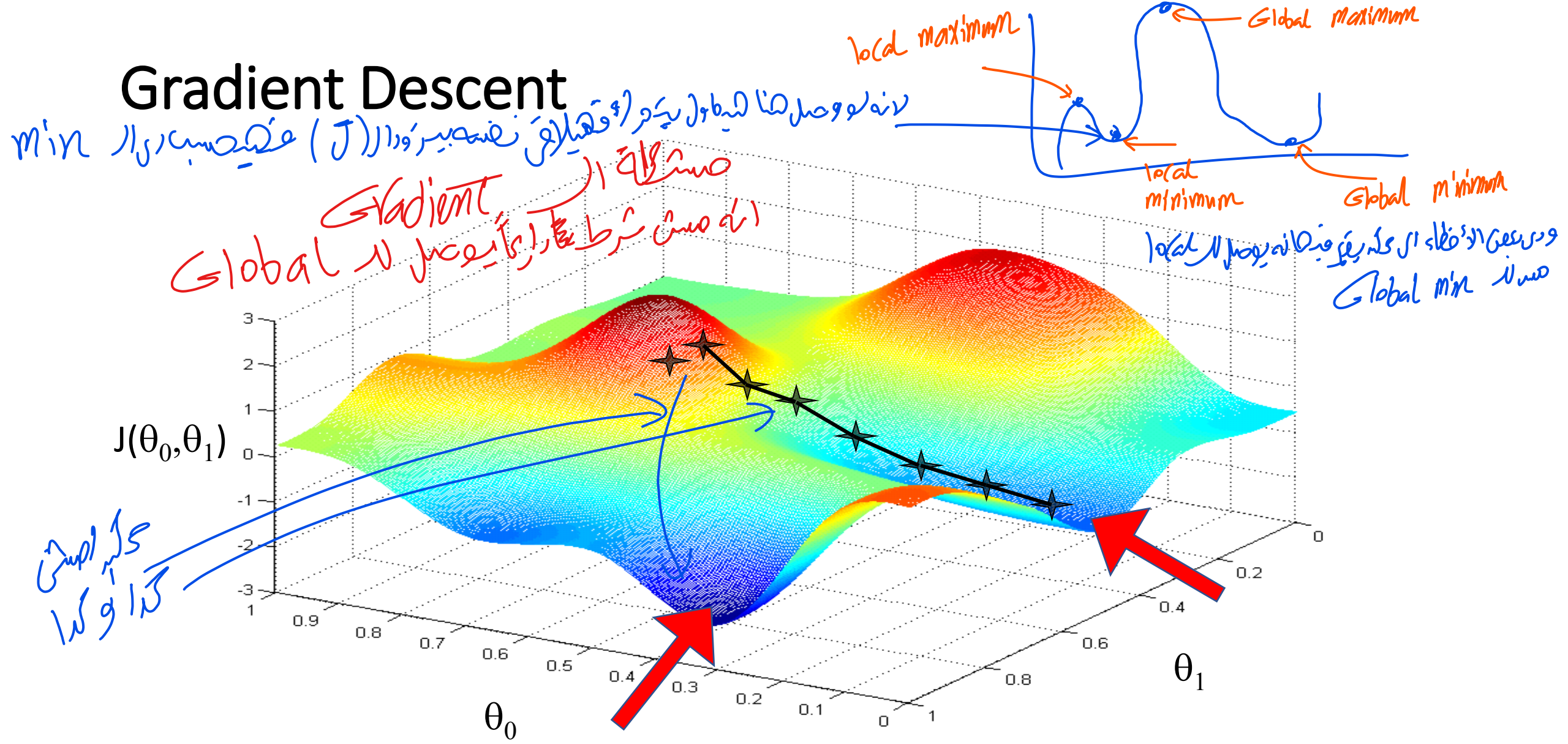
Optimization

- **Strategy 2: Following the Gradient**
  - Compute the best direction along which we should change our parameter (weight) vector that is mathematically guaranteed to be the direction of the **steepest descend.**
  - This direction will be related to the **gradient** of the cost function.

Gradient Descent

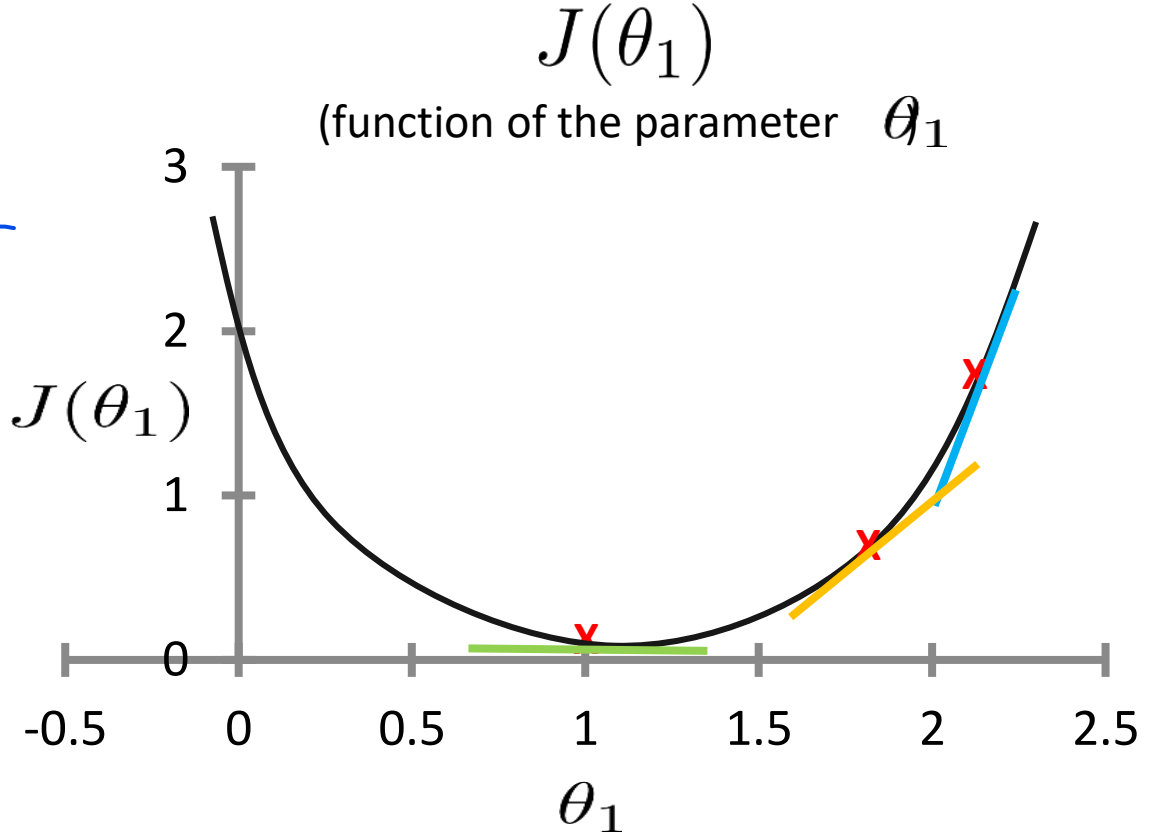


Gradient Descent



Gradient Descent

- The way we do this is by taking the derivative (the tangential line to a function) of our cost function.
- The **slope of the tangent** is the derivative at that point and it will give us a direction to move towards.
- We will know that we have succeeded when our cost function is at the very bottom of the pits in our graph, i.e. when its value is the minimum.



Gradient Descent

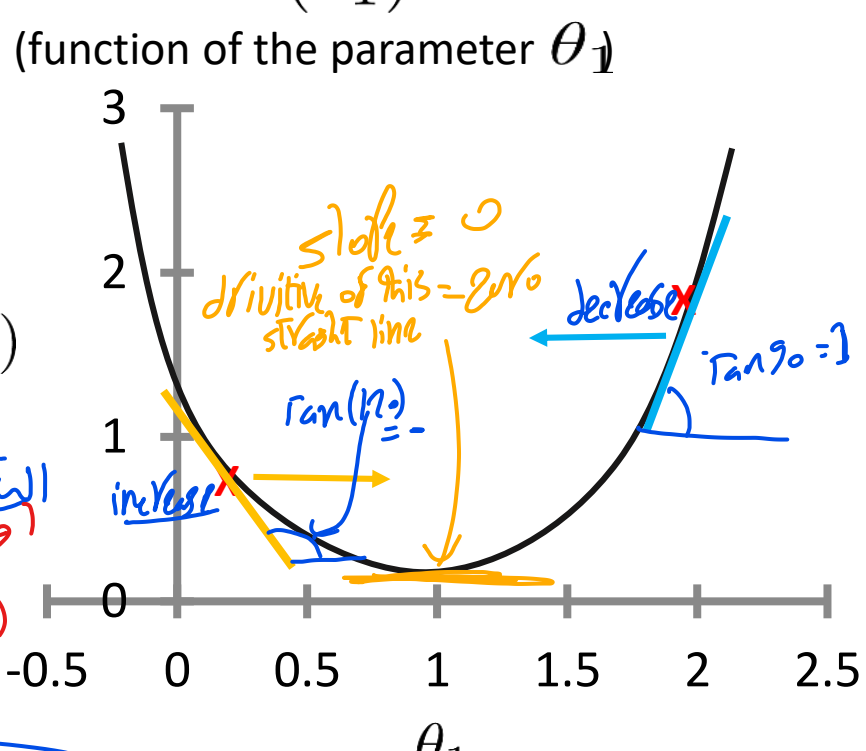
- We make steps down the cost function in the direction with the steepest descent, and the size of each step is determined by the parameter **α**, which is called the **learning rate.**
- The gradient descent algorithm is:
- Repeat until **convergence** {

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1)$$

(for $j = 0$ and $j = 1$)

}

Learning rate (step size)



Positive slope (positive number)→ Θ will decrease

Negative slope (negative number)→ Θ will increase

Learning Rate

- The gradient tells us the direction, but it does not tell us how far along this direction we should step.

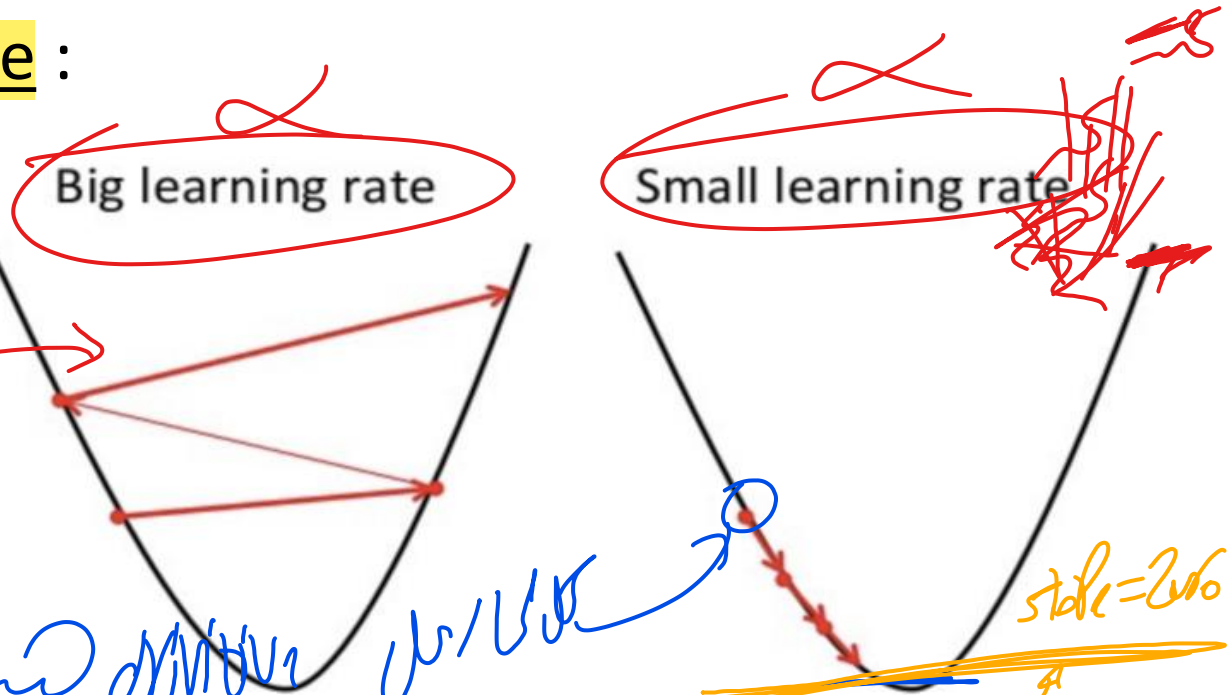
- The **learning rate (step size)** determines how big the step would be on each iteration. It determines how fast or slow we will move towards the optimal weights.

Learning Rate

- If **learning rate is large**, it may fail to converge and overshoot the minimum.
- If **learning rate is very small**, it would take long time to converge and become computationally expensive.
- The most commonly used rates are :

  0.001, 0.003, 0.01 (default),

  0.03, 0.1, 0.3



Gradient Descent for Linear Regression

When specifically applied to the case of **linear regression**, a new form of the gradient descent equation can be derived. We can substitute our actual cost function and our actual hypothesis function and modify the equation to:

- Start by initializing the parameters θ_0, θ_1 randomly
- Repeat until convergence { // until error is small
  - Predicted values with linear regression hypothesis.
  - Calculate the cost function.
  - If cost is large, update parameters using GD

$$\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right) \quad \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)$$

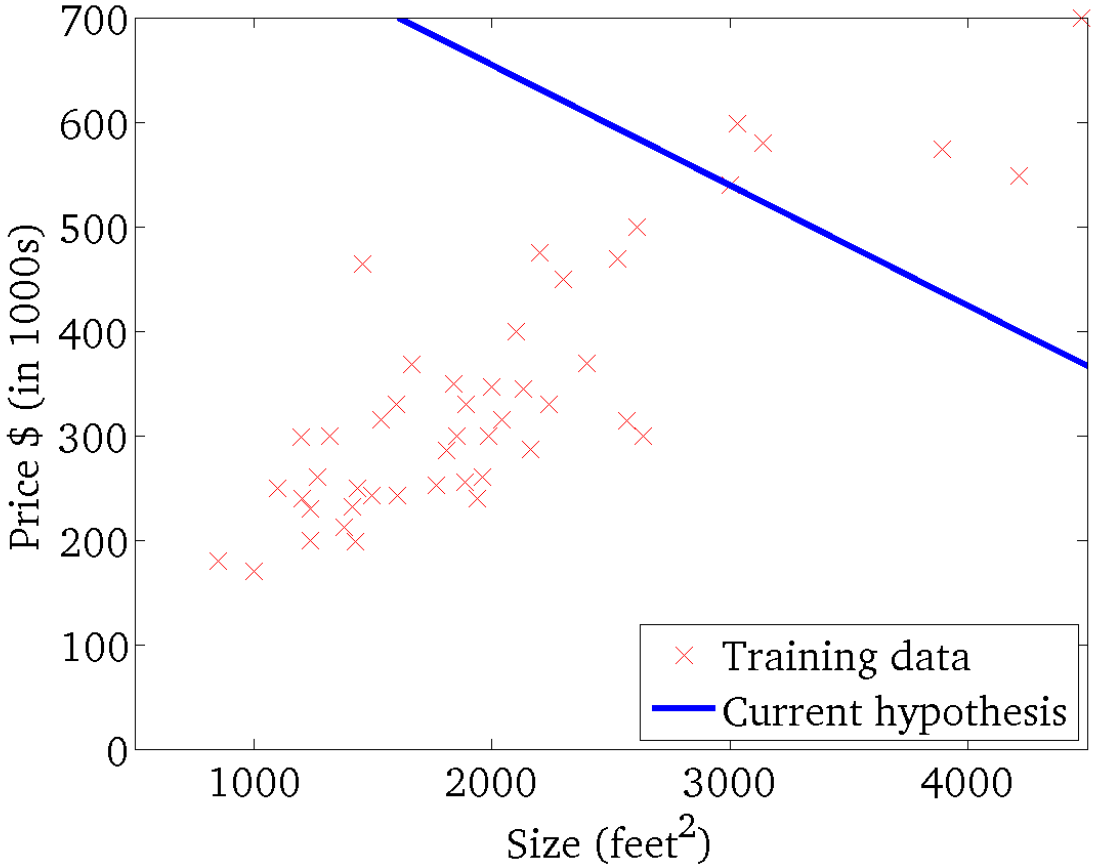
$$\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right) \cdot x^{(i)} \quad \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1)$$

Should be done simultaneously

}

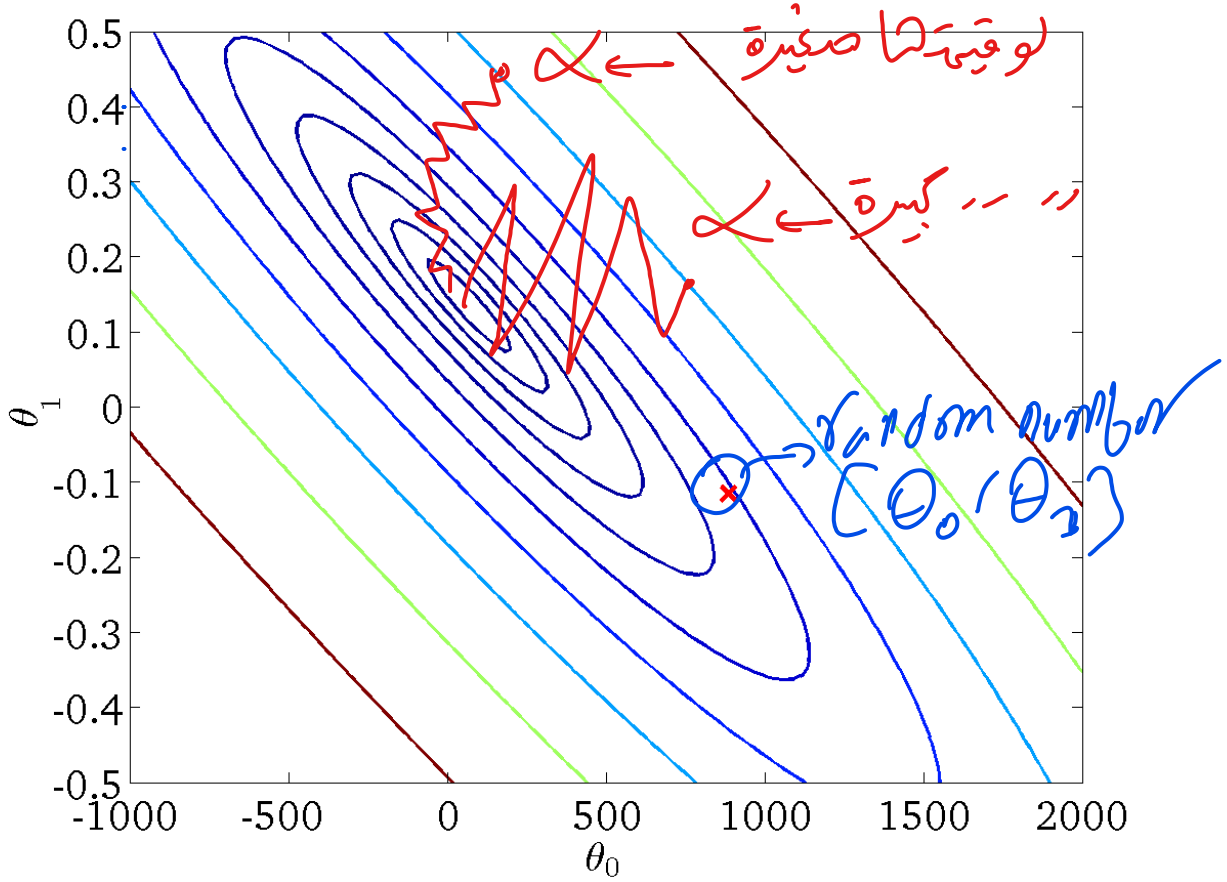
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



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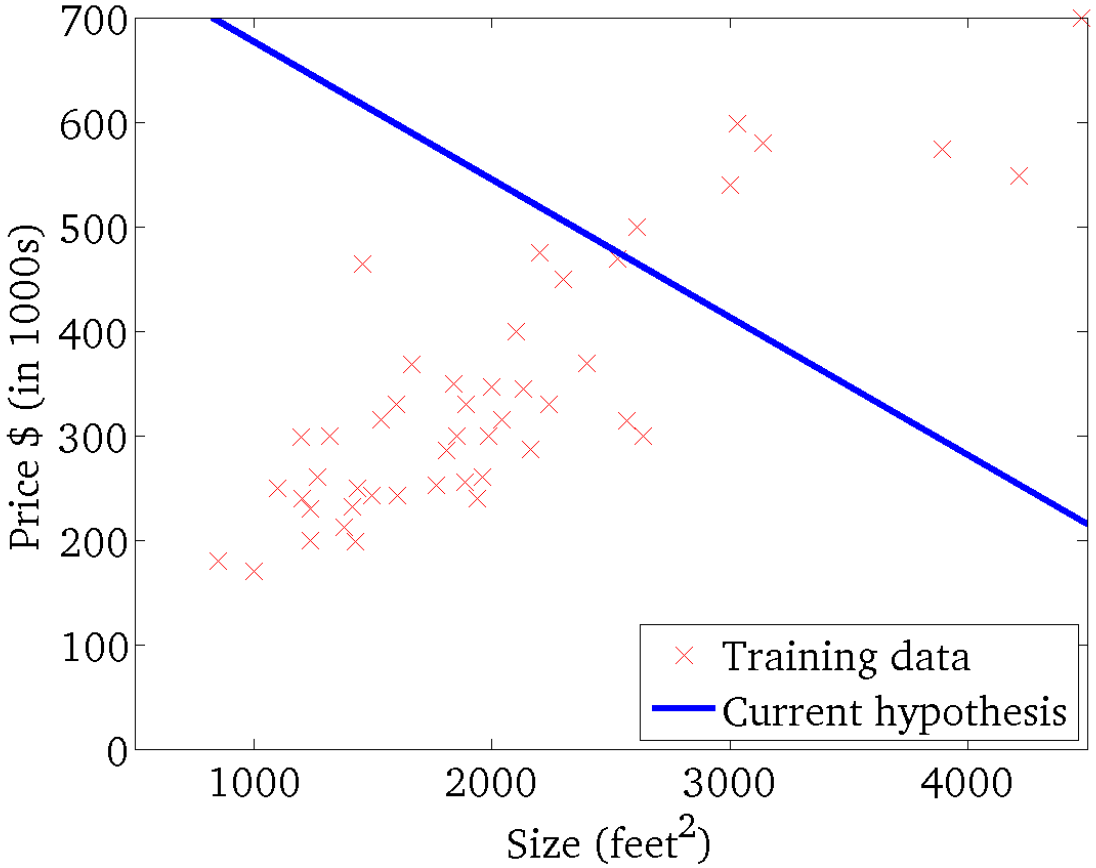

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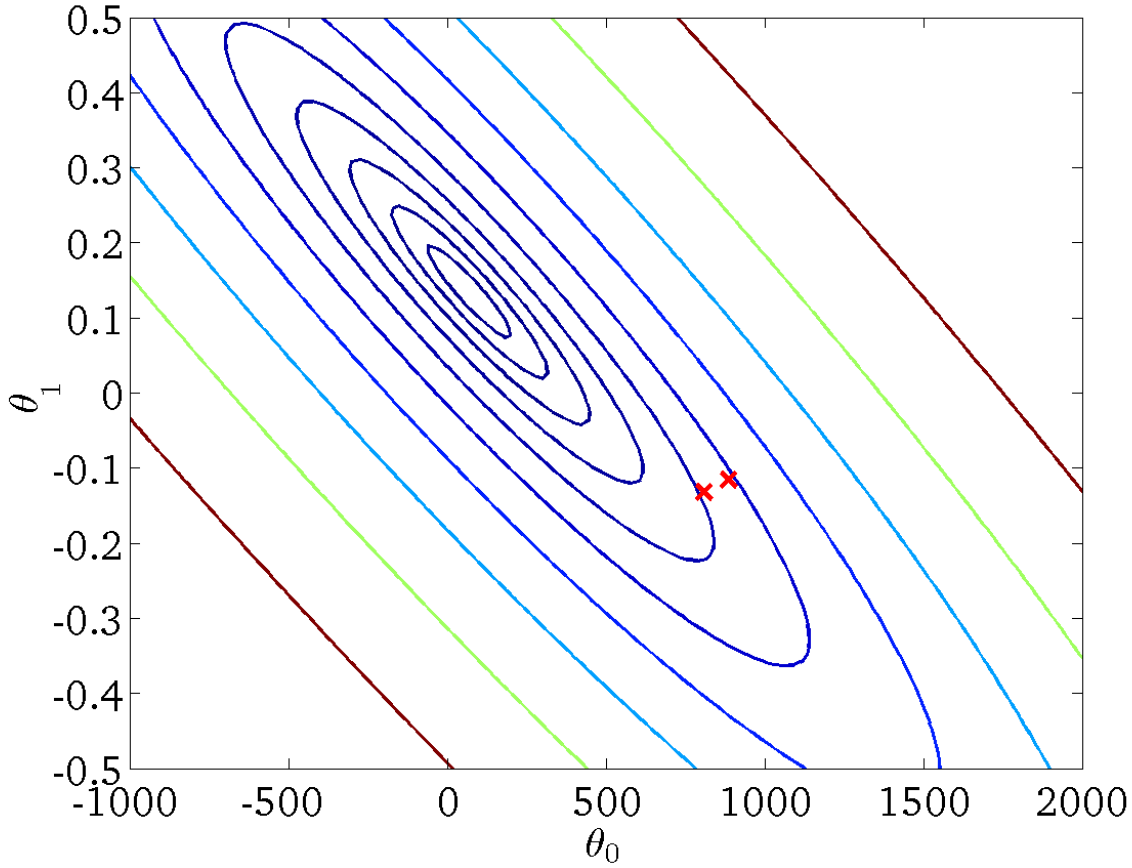

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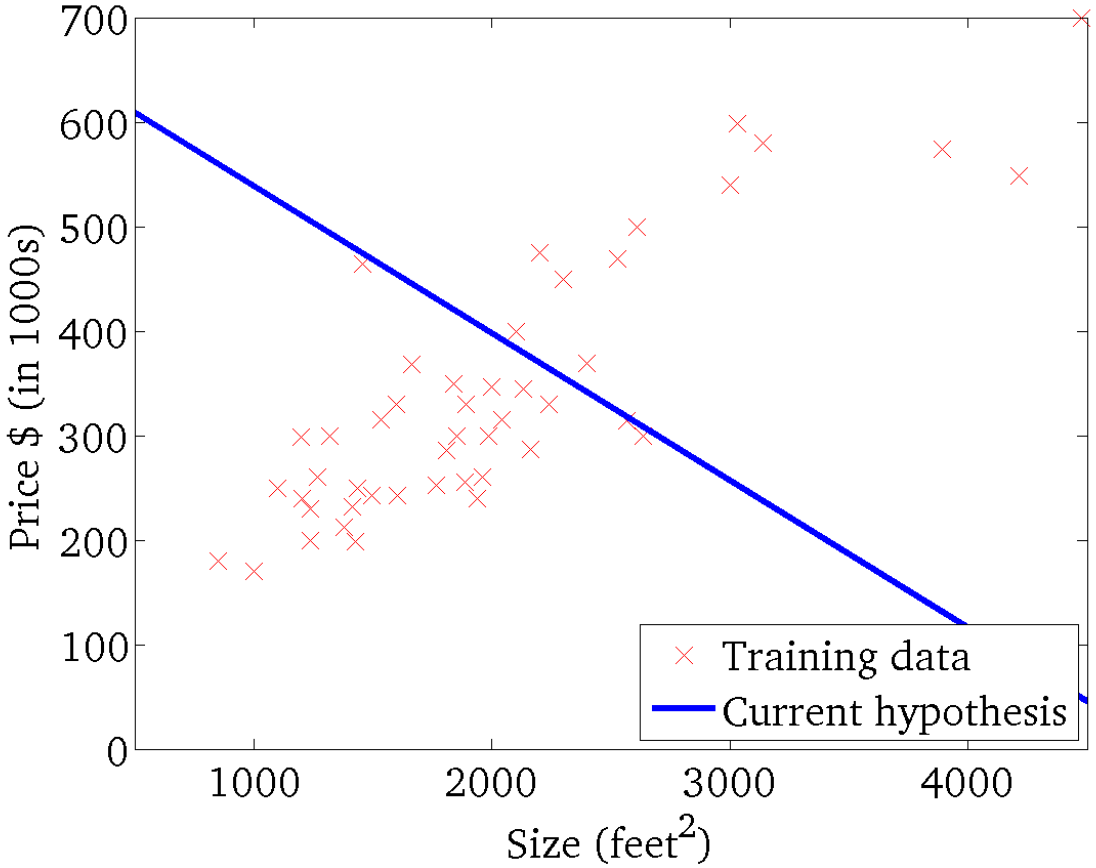

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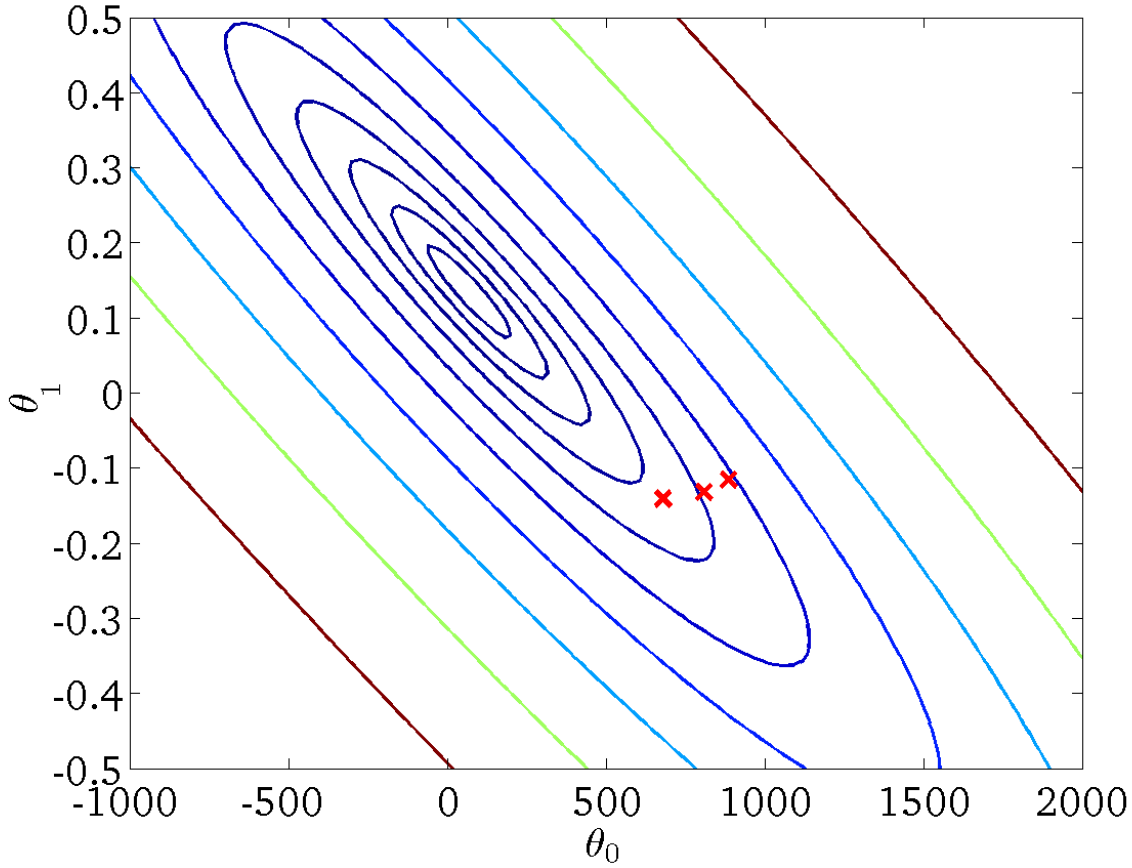

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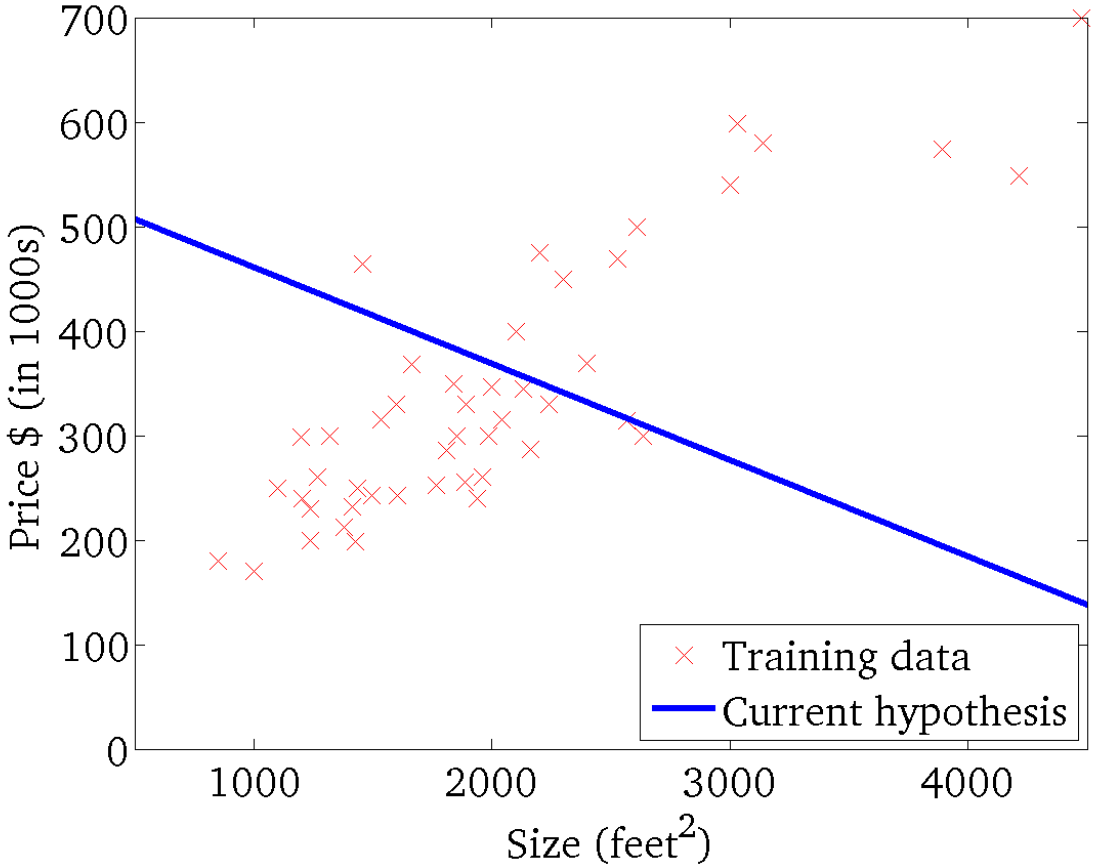

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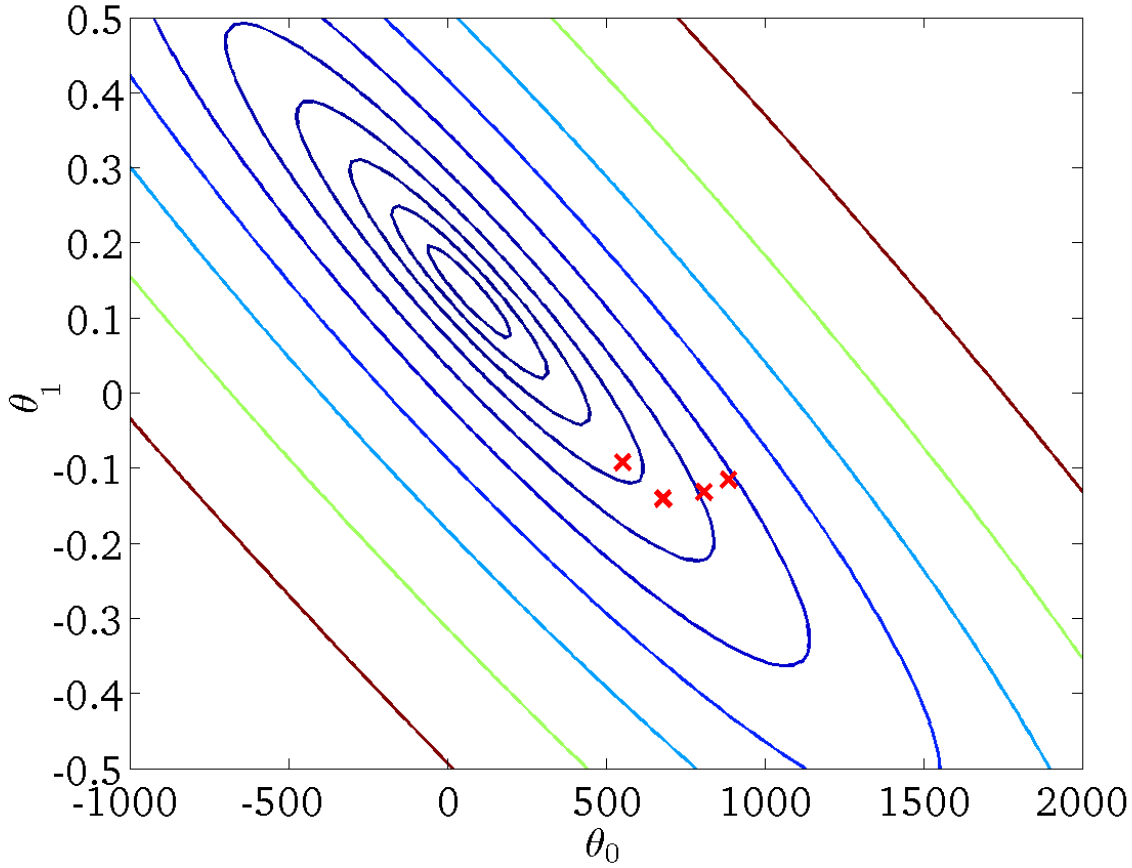

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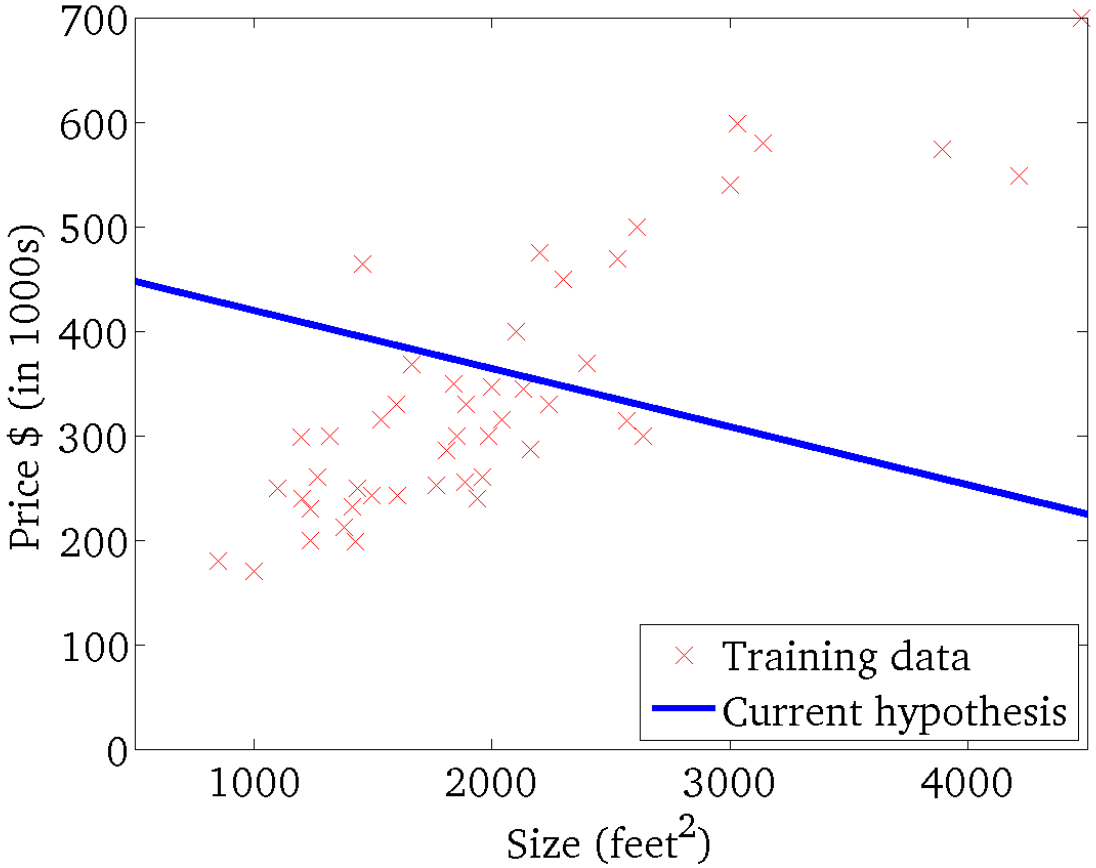

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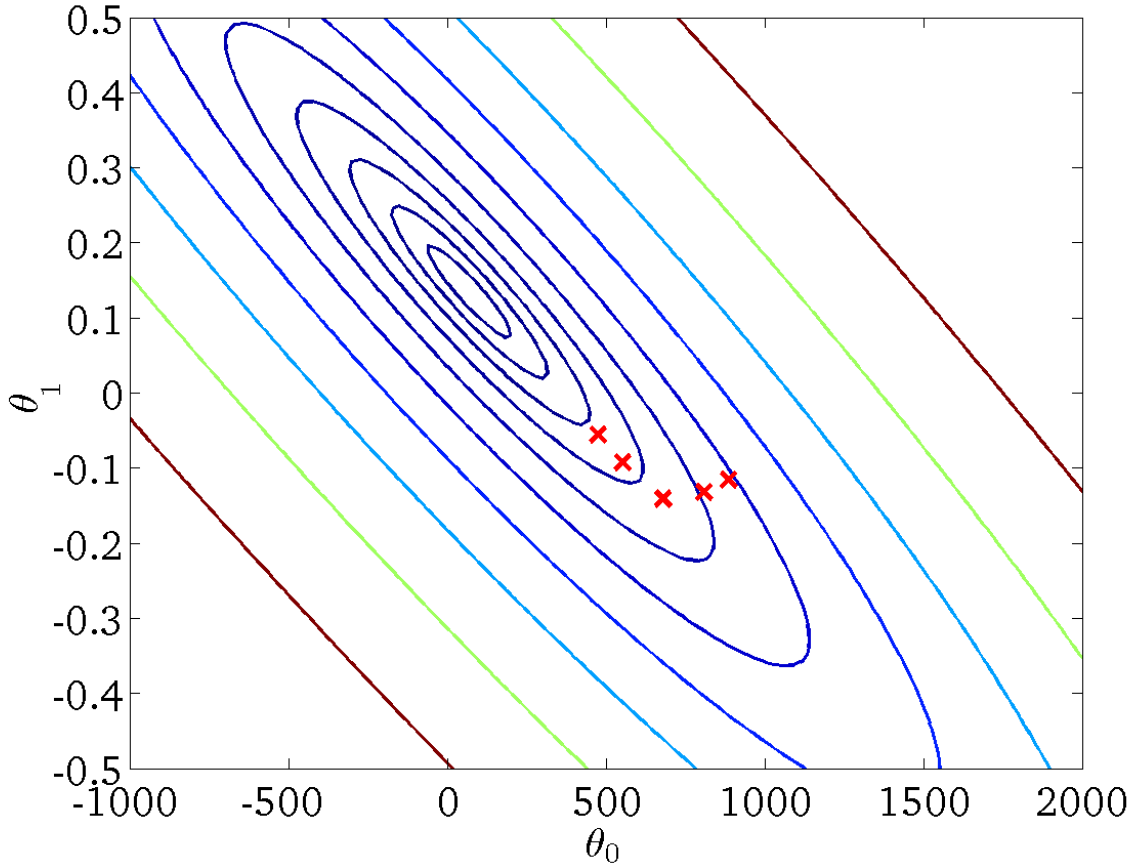

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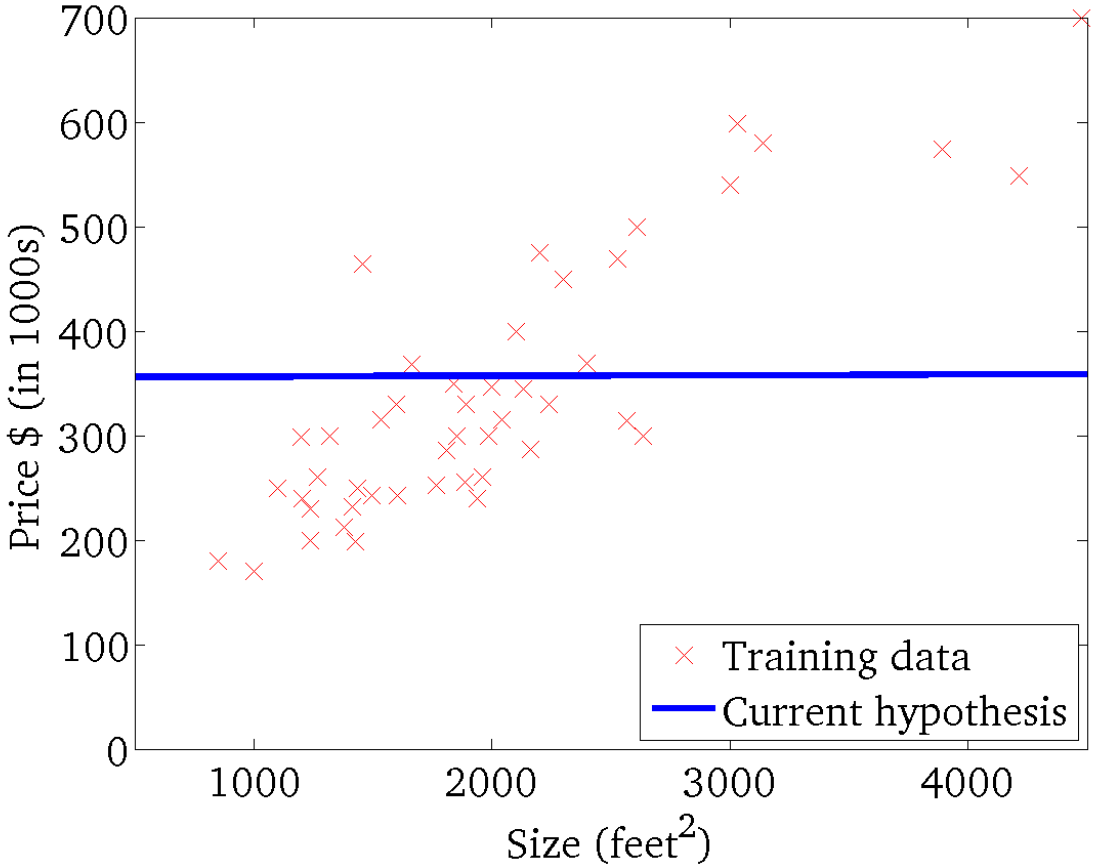

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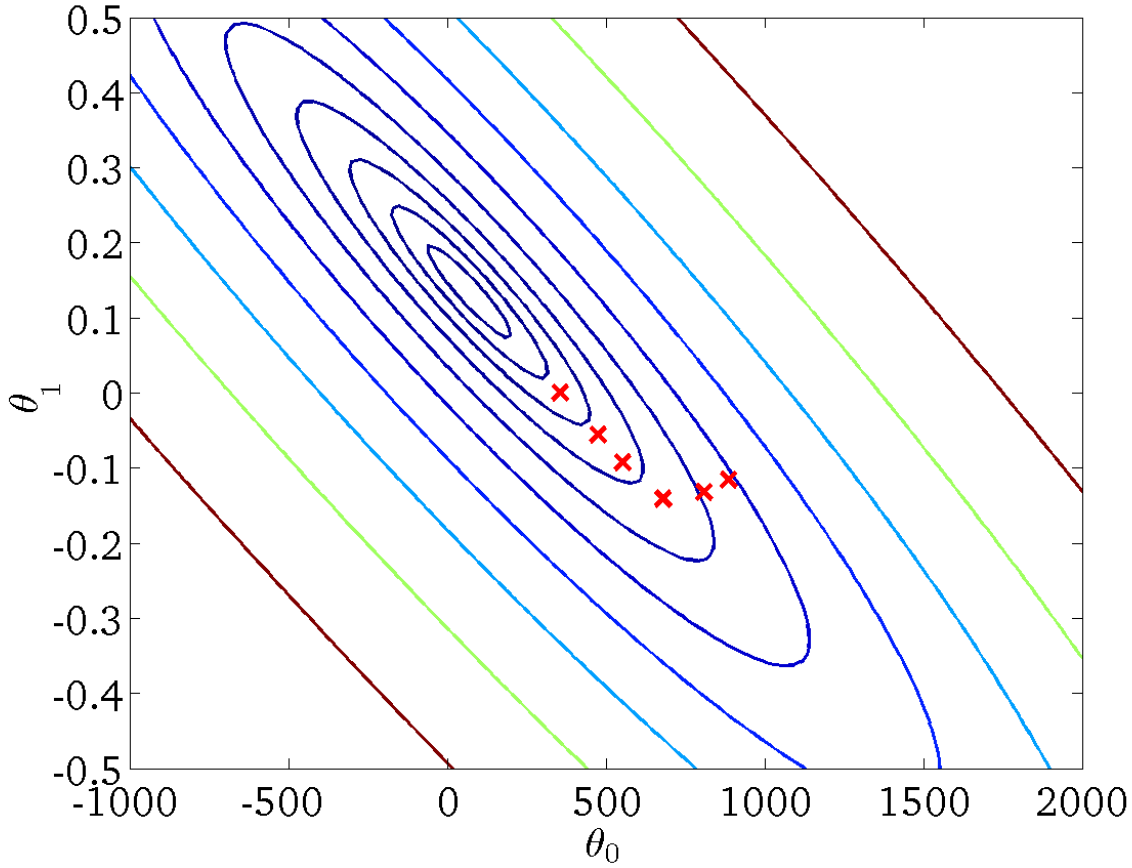

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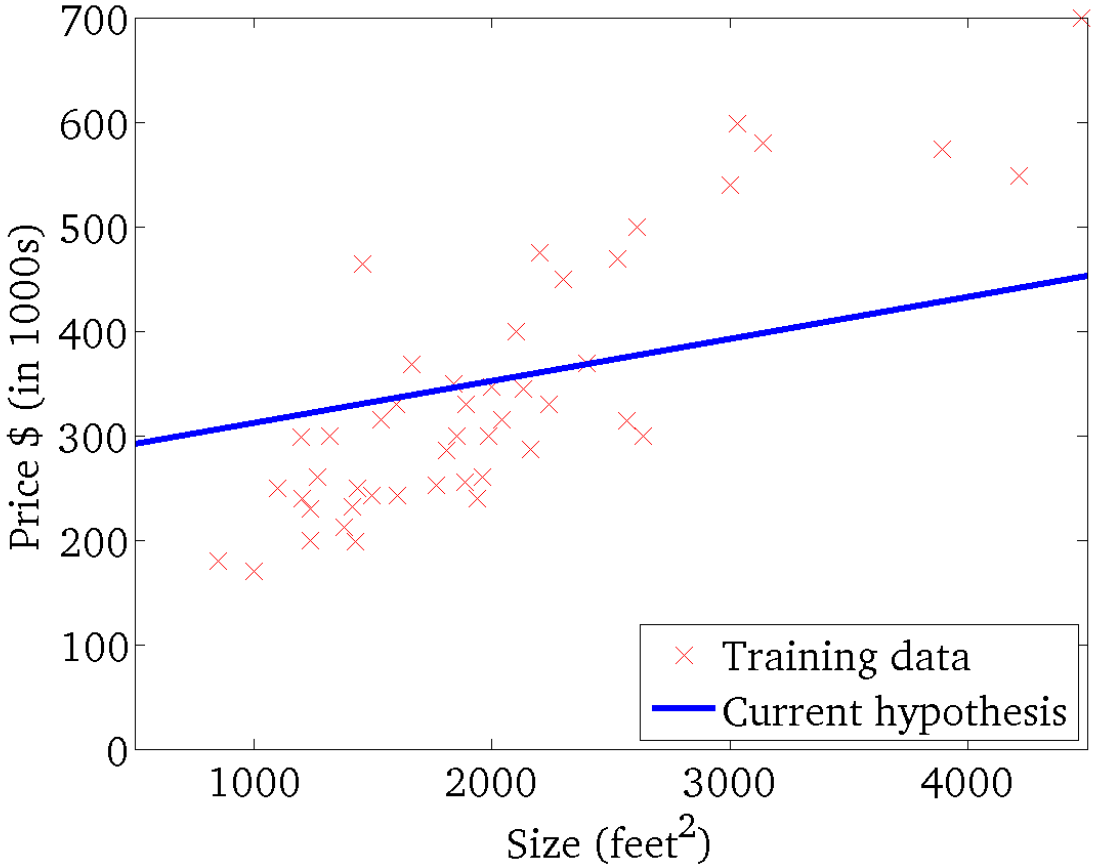

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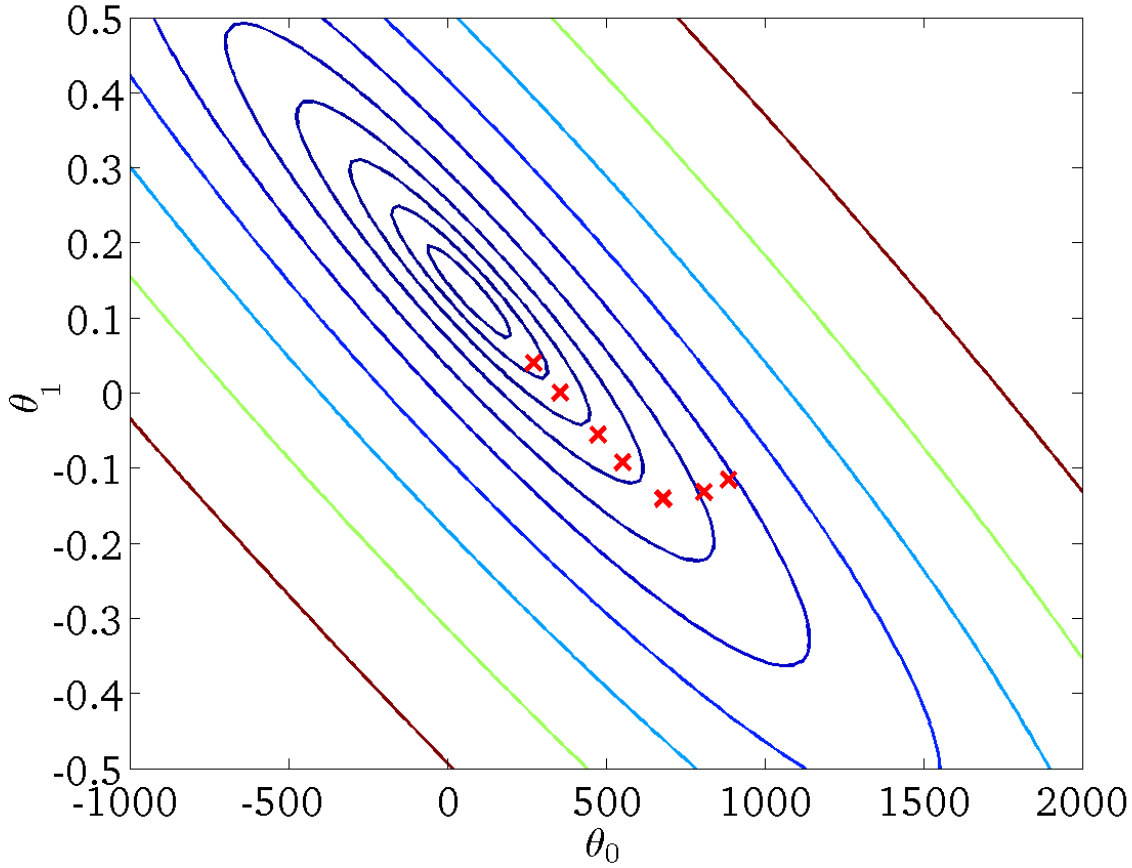

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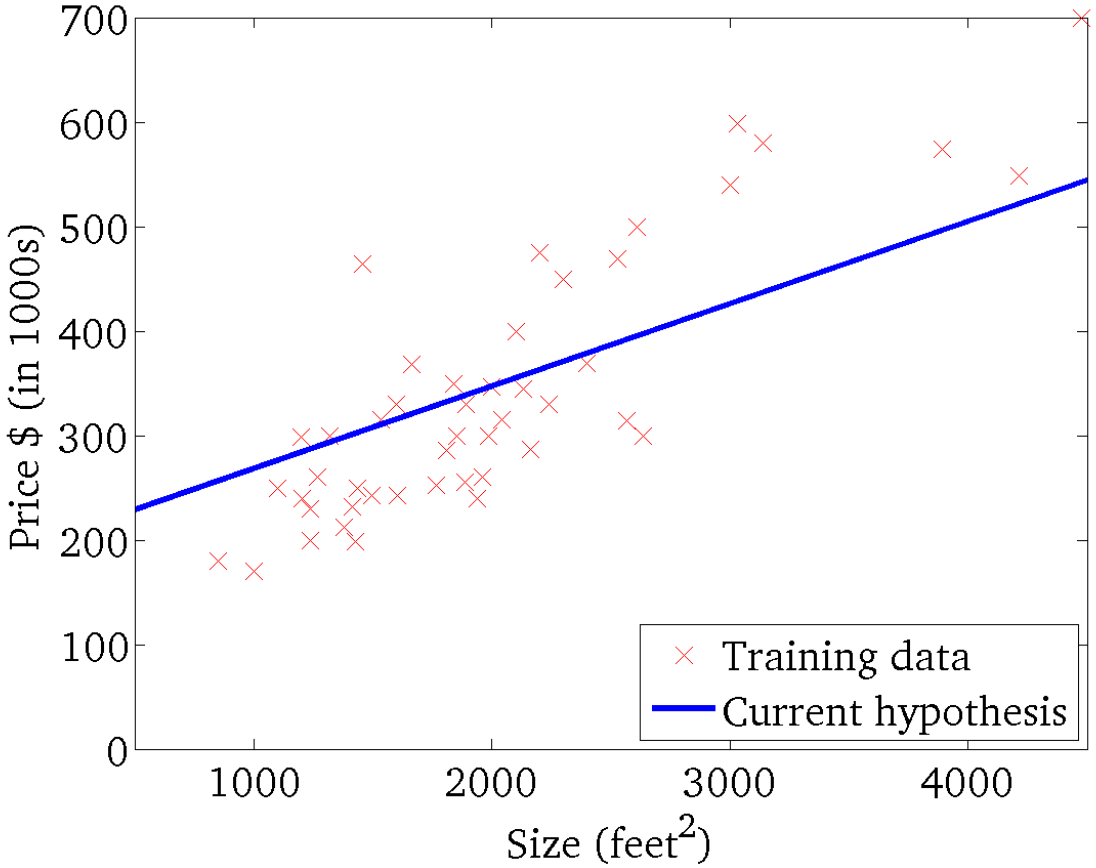

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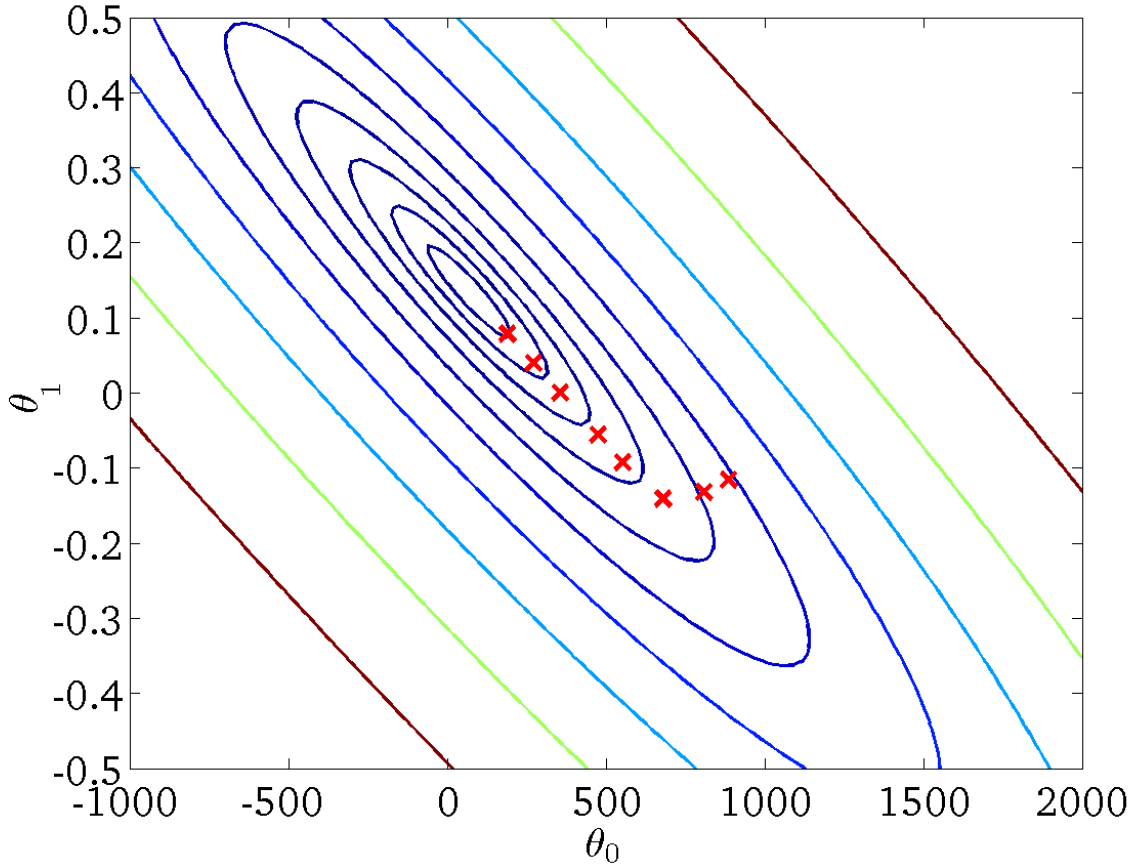

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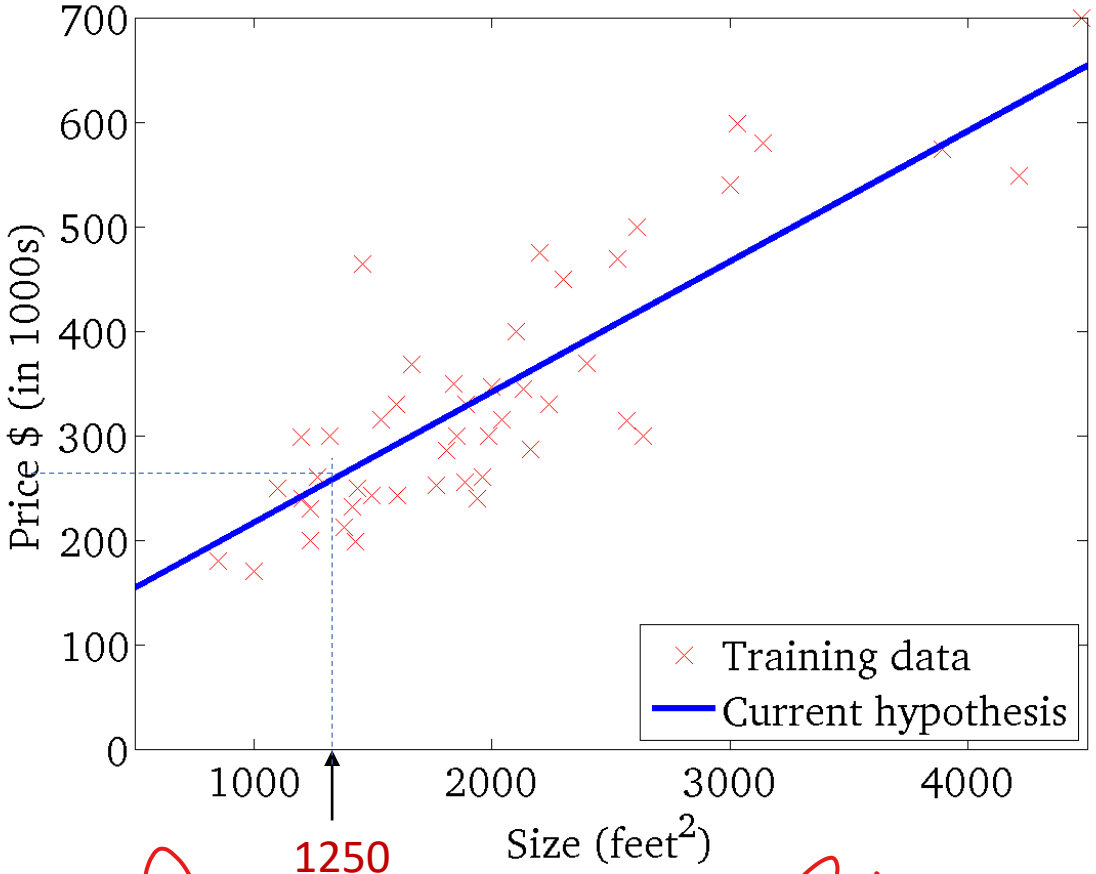

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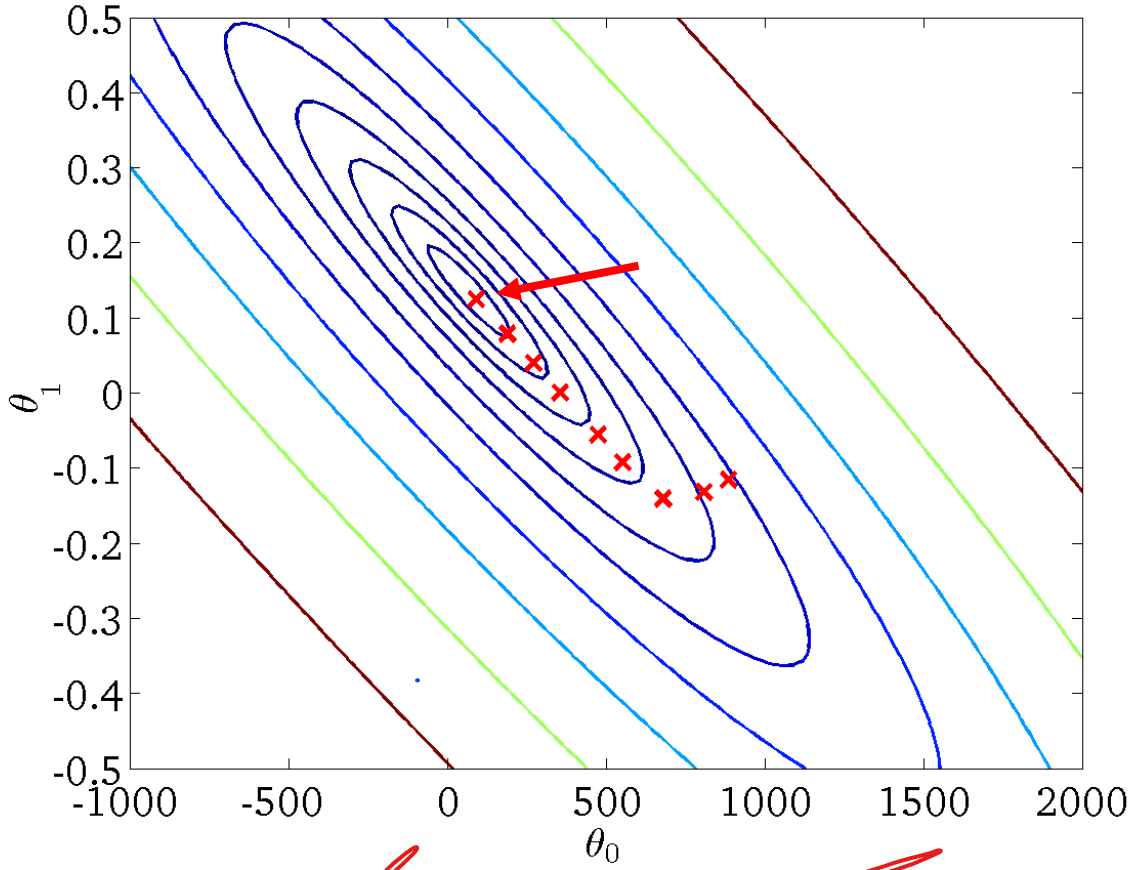


یک تابع هزینه از پارامترهای θ_0 و θ_1

Gradient Descent variants

- There are three variants of gradient descent based on the amount of data used to calculate the gradient:

1. Batch gradient descent
2. Stochastic gradient descent
3. Mini-batch gradient descent

Batch Gradient Descent

- Batch Gradient Descent, aka **Vanilla gradient descent**, calculates the error for each observation in the dataset but performs an update only after all observations have been evaluated.
- One cycle through the entire training dataset is called a training epoch. Therefore, it is often said that batch gradient descent performs model updates at the end of each training epoch.
- Batch gradient descent is not often used, because it represents a huge consumption of computational resources, as the entire dataset needs to remain in memory.

Stochastic Gradient Descent (SGD)

- Stochastic gradient descent, often abbreviated **SGD**, is a variation of the gradient descent algorithm that calculates the error and updates the model for each example in the training dataset.

- SGD is usually faster than batch gradient descent, but its frequent updates cause a higher variance in the error rate, that can sometimes jump around instead of decreasing.

- The noisy update process can allow the model to **avoid local minima** (e.g. premature convergence).

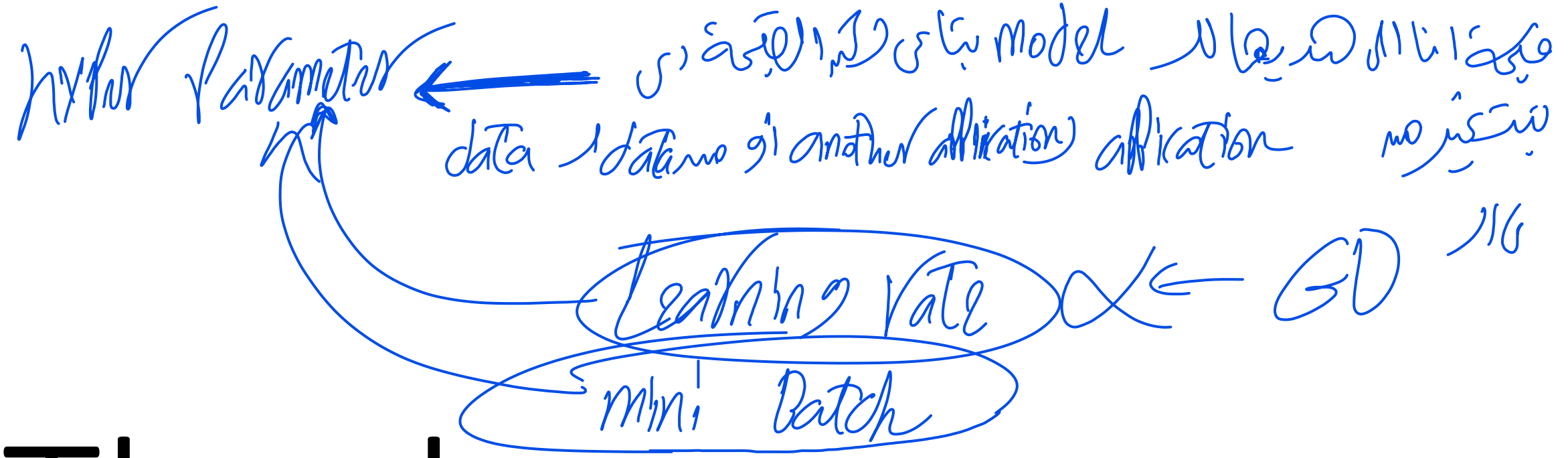
Mini-Batch Gradient Descent

- Mini-batch gradient descent seeks to find a balance between the robustness of stochastic gradient descent and the efficiency of batch gradient descent.

- It is the *most common implementation* of gradient descent used in the field of deep learning.

- It splits the training dataset into **small batches** that are used to calculate model error and update model coefficients.

- **"Batch size" commonly used as power of 2: 32, 64, 128, 256, and so on.**



Thanks