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## **Numerical Mathematics**

## 13) Newton-Raphson

a) Use the Newton-Raphson algorithm to solve the system of equations

$$e^{-x_1} - x_2 = 0 (1)$$

$$x_1 + x_2^2 - 3x_2 = 0 (2)$$

using initial guesses  $\vec{x}^{(0)} = 0$ . Plot the values  $x_1^{(k)}, x_2^{(k)}$  as a function of iteration number k.

**b)** Modify the program in a) for a switched order of equations and find the solution for the same initial guess.  $x_1^{(k)}, x_2^{(k)}$  as a function of iteration number k.

## 14) Implicit Euler, but don't bother with Newton

Apply the backward (implicit) Euler method to solve the IVP

$$y' = -y^2, \quad y(0) = 1$$

Calculate  $y_i$  for i = 1, 2, 3 on paper and plot the graph with more points on the computer. Use h = 0.1. Note that you do not have to employ a fixed-point method to solve for  $y_{i+1}$ , there is an easier way!

## 15) Explicit Euler?

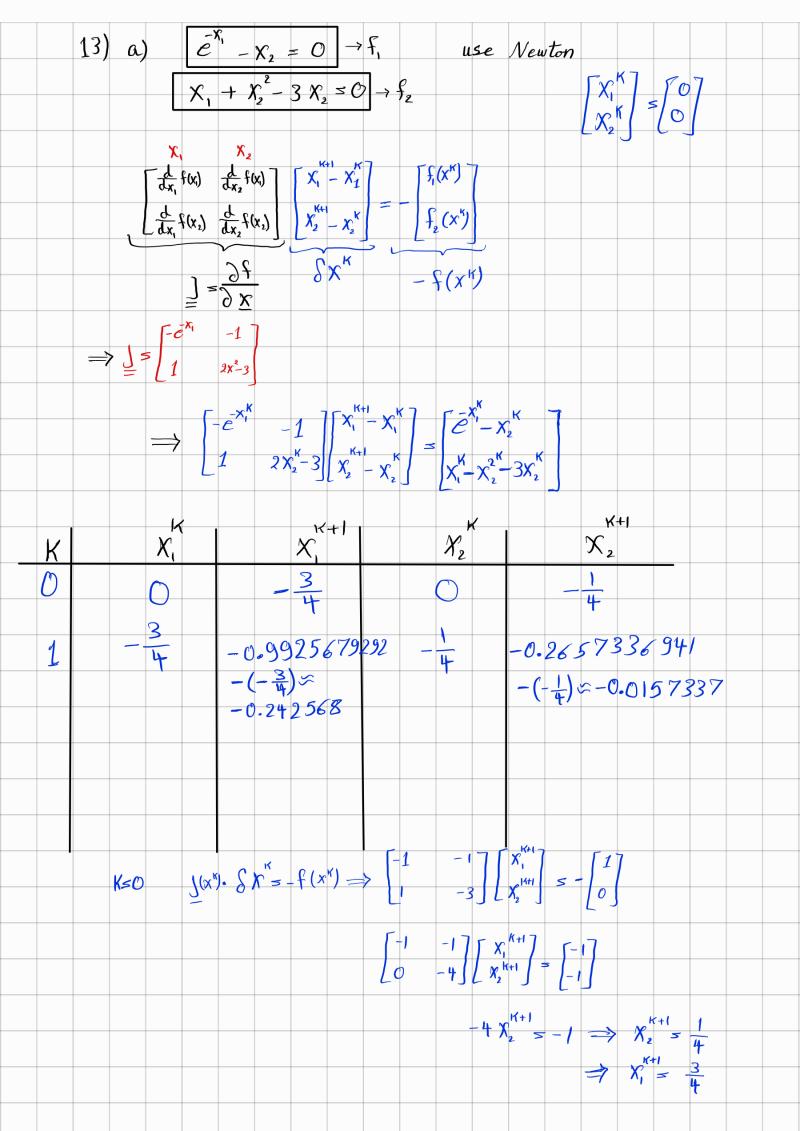
Consider the ODE

$$\frac{dy}{dx} = \sin\left(x\,\cos(y)\right) \tag{3}$$

with the IVP

$$y(x=0) = 0 (4)$$

- a) Apply the explicit Euler method to the IVP and write down the first three iterations on paper.
- **b)** Write a program to solve the IVP, using h = 0.01. Examine the solutions for  $x \in [0, 2.5]$ , for  $x \in [0, 250]$  and for  $x \in [0, 2500]$ . Discuss the results.
- c) Repeat the task in b) for h = 0.001. Compare and discuss the results.
- **d)** Repeat b) to d) with the implicit Euler method.



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