Period of Examinations
Winter Semester 2020/21 - January
Test-Exam

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Module: Modelling and Simulation

Examination

Please write legibly!

Date: ____________________

Name: ____________________

Register No.: ____________________

Course of Study: ____________________

Please put your name and your matriculation number in the following declaration and sign it:

I, _______________________________________ ,
full name, matriculation number

hereby confirm in lieu of an oath that I am the person who was admitted to this examination. Further, I confirm that the submitted work is my own and was prepared without the use of any unauthorised aid or materials.

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Signature

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1.) A ball is rolling from a table with a height of *h* measured from the ground. The equations of the vertical motion are

$$m \cdot \ddot{x} + d_{fall} \cdot \dot{x} = -m \cdot g$$

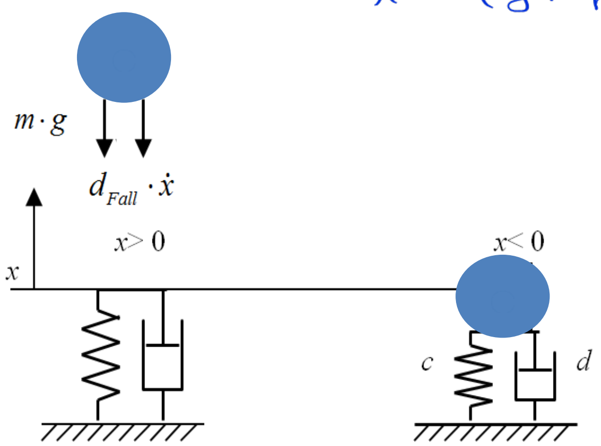
Therein *m* is the mass of the ball, d_{fall} is the drag coefficient and *g* is the gravitational constant. The position over ground is measured by the coordinate *x*. Complete the initial value problem.

$$\ddot{x} = -\left(g + \frac{d_{fall}}{m}\dot{x}\right)$$

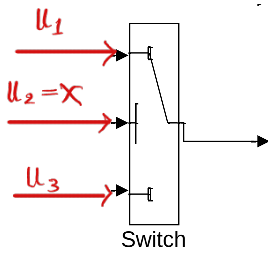
2.) When the ball of problem 1.) hits the ground, the equations of motion during the contact phase are modelled by

$$m \cdot \ddot{x} + d \cdot \dot{x} + c \cdot x = -m \cdot g$$

$$\ddot{x} = -\left(g + \frac{d}{m}\dot{x} + \frac{c}{m}x\right)$$



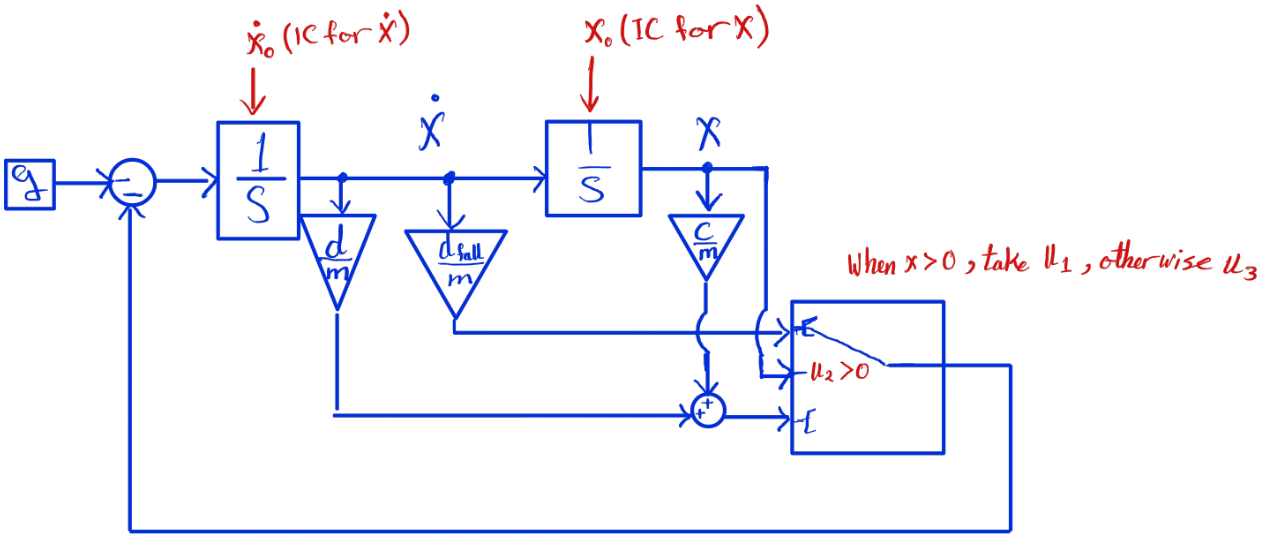
Therein *d* is is the damping coefficient during the contact phase and *c* models the stiffness during the contact phase. Design a Simulink-block-diagram in which the flight phase (x>0) as well as the contact phase (x<0) are modelled. The active part of the model should be triggered by a switch.



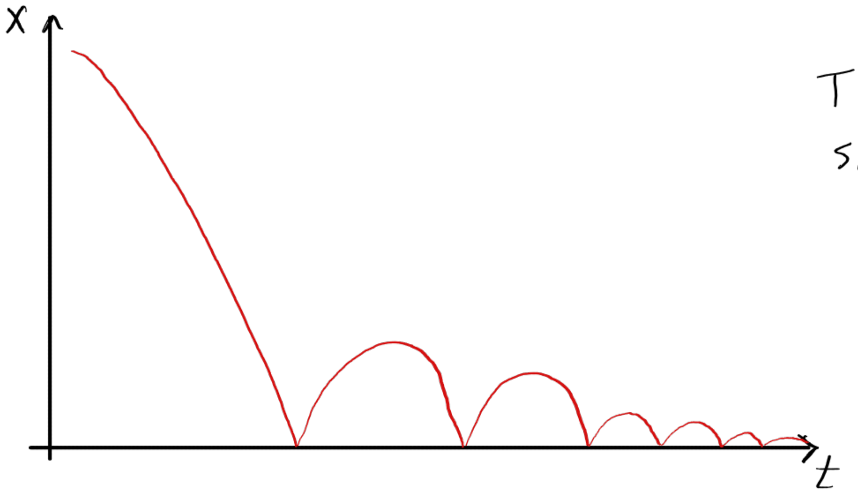
$$\ddot{x} = -\left(g + \overbrace{\frac{d_{fall}}{m}\dot{x}}^{u_1}\right)$$

$$\ddot{x} = -\left(g + \overbrace{\frac{d}{m}\dot{x} + \frac{c}{m}x}^{u_3}\right)$$

Indicate where the initial conditions have to be set.



3.) Draw a qualitative diagram with the expected simulation results of height x verses time t of the problem described in problems 1.) and 2.). What is the expected steady state?



The expected steady state is x=0

4.) A vibration problem is described by the following equation:

$$\ddot{x} + d \cdot sign(\dot{x}) \cdot \dot{x}^2 + {\omega_0}^2 \cdot x = \sin(\omega \cdot t)$$

The parameters ω, ${\omega_0}^2$and d are known. Is the problem linear or non-linear? How many integrators are necessary in Simulink in order to model the vibration problem?

Non-linear, because of $(\dot{x}^2)$

2 integerators

5.) Transform the system given in 4.) into a system of first order differential equations (ODEs).

$\ddot{x} = -d\,\mathrm{sign}(\dot{x}) \cdot \dot{x}^2 - w_0^2 \cdot x + \sin(w \cdot t)$

$x_1 = x$

$x_2 = \dot{x}$

$$\boxed{\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -d\,\mathrm{sign}(x_2) \cdot x_2^2 - w_0^2 \cdot x_1 + \sin(w t \end{aligned}}$$

6.) In the following prey-predator system the amount of prey is modelled by x_1 and the amount of predator by x_2. The coefficients $\theta, \mu, \varepsilon,$and ω are known constants with positive values (>0). In the model it is assumed that only the two modelled species interact and that there is an infinite food resource for the prey. However, the only food resource for the predators is assumed to be the prey.

$$\dot{x}_1 = \theta x_1 - \mu x_1 x_2$$

$$\dot{x}_2 = -\varepsilon x_2 + \omega x_1 x_2$$

Linearize the system given above about $\underline{x}_e = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. **(8 points)**

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Calculations Task 6.)

$\underline{x}_0 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$$\underline{f}(\underline{x}) = \dot{\underline{x}} = \begin{bmatrix} \theta x_1 - \mu x_1 x_2 \\ -\varepsilon x_2 + \omega x_1 x_2 \end{bmatrix}$$

$$\underline{f}(\underline{x}) \simeq \underline{f}(\underline{x}_0) + \left.\frac{\partial f}{\partial x}\right|_{\underline{x}=\underline{x}_0} \cdot \Delta \underline{x}$$

$$\underline{f}(\underline{x}_0) = \begin{bmatrix} \theta - 2\mu \\ -2\varepsilon + 2\omega \end{bmatrix} \quad , \quad \Delta \underline{x} = \underline{x} - \underline{x}_0 = \begin{bmatrix} x_1 - 1 \\ x_2 - 2 \end{bmatrix}$$

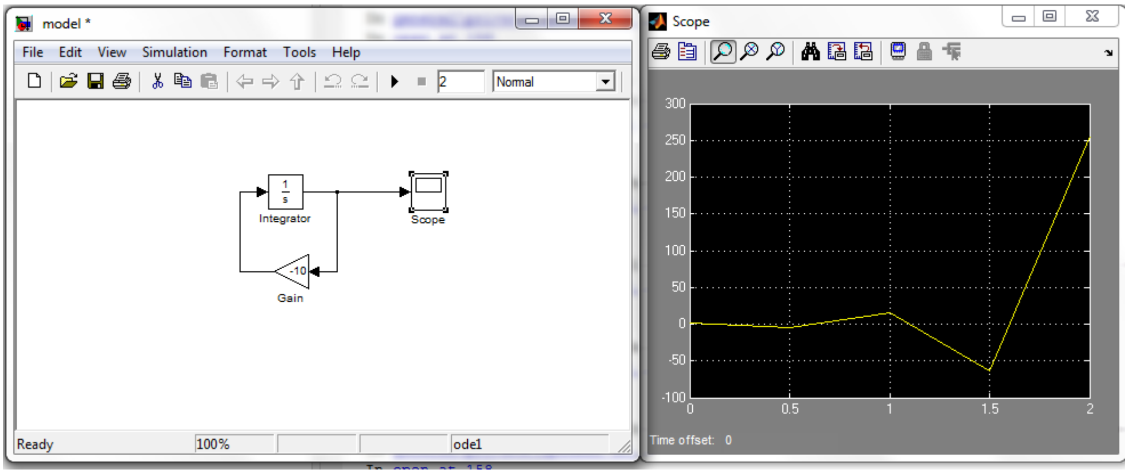
$$\underline{\underline{J}} = \frac{\partial f}{\partial x} = \begin{bmatrix} \frac{\partial f(x_1)}{\partial x_1} & \frac{\partial f(x_1)}{\partial x_2} \\ \frac{\partial f(x_2)}{\partial x_1} & \frac{\partial f(x_2)}{\partial x_2} \end{bmatrix} = \begin{bmatrix} \theta - \mu x_2 & -\mu x_1 \\ \omega x_2 & -\varepsilon + \omega x_1 \end{bmatrix}$$

$$\left.\frac{\partial f}{\partial x}\right|_{\underline{x}=\underline{x}_0} = \begin{bmatrix} \theta - 2\mu & -\mu \\ 2\omega & -\varepsilon + \omega \end{bmatrix}$$

$$\underline{f}(\underline{x}) \simeq \begin{bmatrix} \theta - 2\mu \\ -2\varepsilon + 2\omega \end{bmatrix} + \begin{bmatrix} \theta - 2\mu & -\mu \\ 2\omega & -\varepsilon + \omega \end{bmatrix} \cdot \begin{bmatrix} x_1 - 1 \\ x_2 - 2 \end{bmatrix}$$

$$\underline{f}(\underline{x}) \simeq \underbrace{\begin{bmatrix} \theta - 2\mu & -\mu \\ 2\omega & -\varepsilon + \omega \end{bmatrix}}_{\underline{\underline{A}}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\underline{x}} + \underbrace{\begin{bmatrix} 2\mu \\ -2\omega \end{bmatrix}}_{\underline{\underline{B}} \cdot \underline{u}}$$

7.) The following simulation result is given.



a. Give a potential reason for the growing simulation results. **(2 points)**

Numerically unstable.
Bad combination between solver & step-size.
(too big in our case)

b. What can be changed in Simulink to optimize the simulation results? **(3 points)**

Change the step size to $h < \frac{1}{\alpha} = \frac{1}{10}$
a step size smaller than 0.1 will be an ustable.
or change the solver type.

c. Is the physical system unstable? Explain! **(3 points)**

The system it self is stable, since $\dot{x} = -10x$
doesn't have any stability problem.

d. What could be an advantage in using a variable step size solver such as ode45 in comparison to a "fixed step size" solver in Matlab Simulink? Explain! **(2 points)**

A fixed step size won't react to any changes in the system, it will be slow in case of chosing a very small step size, and inaccurate if we choose bigger stepsize.
On the other hand the variable step size solver can react with the changes of the system.

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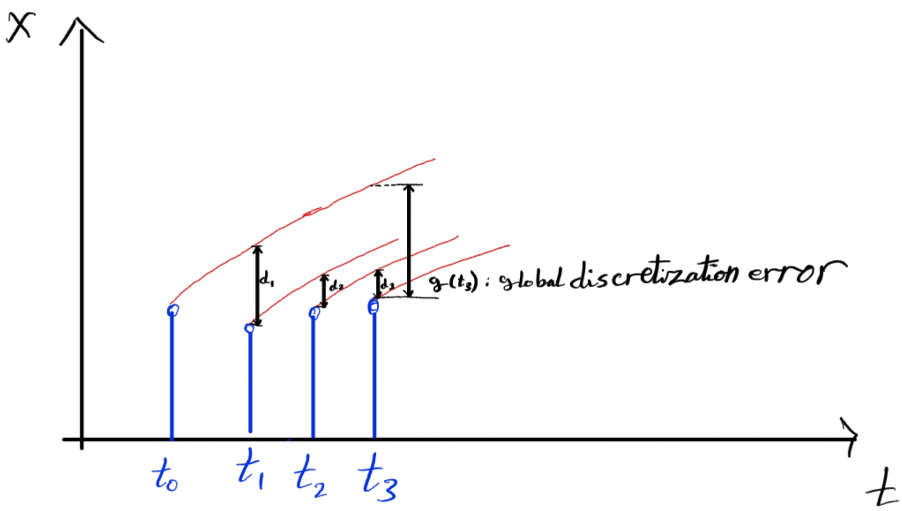
8.) Characterize:

Find all linear ODEs and give the order of all ODEs! **(2 points)**

$\dddot{y} = \ddot{y} + +y^0$	linear, 3rd-order ODE
$y + y^1 = \dot{y}$	linear, 1st-order ODE
$3(\dot{y} + 4\ddot{y}) = 9a^2 * \dot{y}$; with $a = 100$	linear, 2nd order ODE
$y * y = \ddot{y}$	non-linear, 2nd order ODE

9.) Explain the term global discretization error. **(3 points)**



$d_1, d_2, \& d_3$ are local discretization error between each step and the previous one. Where the global discretization error $g(t_3)$ is between last step at $t = t_3$, and the true function $X(t_3)$.