First period of examinations
Summer semester 2021 / July

HOCHSCHULE RHEIN-WAAL
Rhine-Waal University of Applied Sciences

Study Course: _________ **(Use abbreviation ME or IE or SE respectively)**

Module Title: **27101 Fluid Mechanics B.Sc. (Prof. Gebel)**

Family Name: ____________________

First Name: ____________________

Student No.: ____________________

I hereby confirm in lieu of an oath that I am the person who was admitted to this examination. Further, I confirm that the submitted work is my own and was prepared without the use of any unauthorised aides or materials*. I am fully aware of the legal consequences of making a false declaration.

Date: 16.07.2021 **Signature:** ______________________________

* Authorised aides and materials are:

- Non-programmable pocket calculator
- Geometrical tools (no coloured pencils)
- All lectures slides and exercises available in the Moodle course ‘Fluid Mechanics B.Sc. SoSe21’
- Your personal handwritten notes
- Textbooks neither as hardcopy nor as e-book are not permitted!

Examination form: Written exam

Points: 84

Duration: 120 Minutes

All calculations and sketches can be made on your own sheets of paper.

Each sheet must contain your name, your student number and your signature.

Tablets or comparable devices are not permitted.

Please write legibly. Thank you very much and good luck.

Register No.: ____________

<u>FOR INTERNAL USE ONLY:</u>

	Q1	Q2	Q3	Q4	Q5	Q6	Written exam	Practical Training	Total
Max. Points	10	16	12	20	14	12	84	16	100
Achieved									
							$\geq$ 42 Yes | No	Grade	

Graded by		Checked by

Final Grade

Regular grading key.	
Adjusted grading key. (Please add the adjusted grading key to the exam-results)	×

Question 1 … Hydrostatics **Points: 10**

For the inclined manometer containing mercury, shown in the figure below, determine the pressure at pipe B if the pressure in pipe A is 10 kPa.

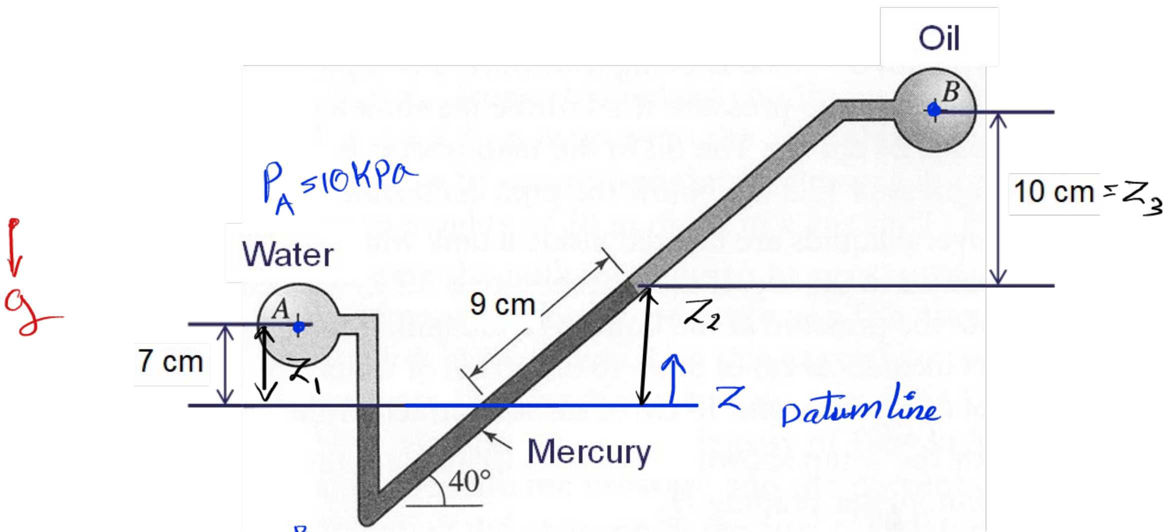
Pipe A has water flowing through it, and oil is flowing through pipe B.

Given:

Density of mercury: $\rho_{Hg} = 13,600 \frac{kg}{m^3}$

Density of oil: $\rho_{Oil} = 870 \frac{kg}{m^3}$

Density of water: $\rho_{Water} = 1,000 \frac{kg}{m^3}$



$$\int_A^B dp = -\int_A^B \rho g \, dz$$

inCompressable fluids ⇒ ρ = cons.

Solution:

p_B ~~= 886 Pa~~

$$P_A + \rho_w g Z_1 = \rho_{Hg} g Z_2 + \rho_{oil} g Z_3 + P_B$$

$$P_B = 10^4 Pa - 9.81 \frac{m}{s^2} \cdot 10^3 \frac{Kg}{m^3} \left(13.6(0.09m)\sin(40°) + 0.87 * 0.1m - 1(0.07m)\right)$$

$$= 2115\ Pa = 2.115\ KPa$$

Register No.: ____________

Question 2 … Momentum equation **Points: 16**

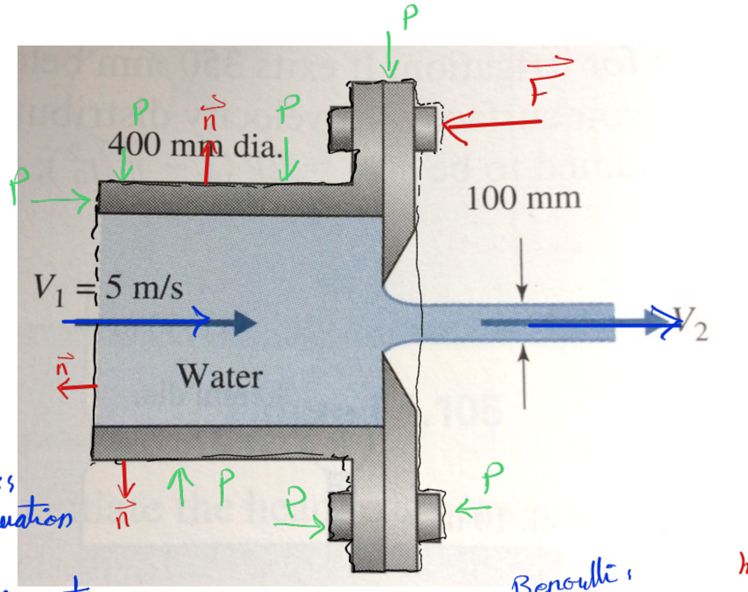
Water flows through an orifice into the environment as shown in the figure:

Continuity equ.

$V_1 d_1^2 = V_2 d_2^2$

$V_2 = 5\frac{m}{s}\left(\frac{0.4}{0.1}\right)^2 = 80\frac{m}{s}$

$\dot{m} = \rho_w V_1 A_1 = 200\pi \frac{kg}{s}$



Hydrodynamics: Momentum equation

X-direction:

$-F + P_1 A_1 = \dot{m} V_2 - \dot{m} V_1$

$F = P_1 A_1 - \dot{m}(V_2 - V_1)$

Benoulli:

horezontal arrangment

$P_1 + \frac{\rho}{2}V_1^2 + \cancel{\rho g z_1} = \cancel{P_2} + \frac{\rho}{2}V_2^2 + \cancel{\rho g z_2}$

open to atm ⇒ gage = 0

$\boxed{P_1 = \frac{\rho}{2}(V_2^2 - V_1^2)}$

Calculate the net force needed to hold the orifice plate onto the pipe.

$F = \frac{\rho}{2}A_1(V_2^2 - V_1^2) - \dot{m}(V_2 - V_1)$

$= 353429.17\ N$

$= 353.43\ kN$

Neglect viscous effects and losses.

Given:

Velocity v_1	$v_1 = 5\ \frac{m}{s}$
Diameter of the pipe	$d_1 = 400\ mm$
Diameter of the jet	$d_2 = 100\ mm$
Density of water:	$\rho = 1{,}000\frac{kg}{m^3}$

Solution:

$F = 353\ kN$

Register No.: ____________

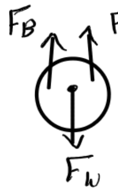
Question 3 … Drag and terminal velocity **Points: 12**

Consider a 5-μm-diameter volcanic dust particle falling down to earth.

a) Approximately how many days will it take the particle to reach the earth from an altitude of 8 km.

Provide an explicit step-by-step description of the procedure for the determination of the terminal velocity.

b) Determine the Reynolds number in order to crosscheck whether Stokes' law can be applied.



Assumptions:

(1) The particle is a sphere.

$$V_\infty = \sqrt{\frac{2g(\rho_s-\rho_{air})4d}{6 C_D \rho_{air}}}$$

(2) Assume average values for density of air and viscosity of air.

$$C_D = \frac{24}{Re}$$

(3) Neglect the buoyancy.

$$V_\infty = \frac{g(\rho_s-\rho_{air})d^2}{18\,\eta_{air}}$$

(4) The drag coefficient c_D can be expressed by Stokes' drag formula:

$$c_D = \frac{24}{Re_d} \quad \text{if} \quad Re_d < 1$$

$$V_\infty = 1.914 \frac{mm}{s}$$

Given:

$$V_\infty = \frac{distance}{time}$$

Density of sphere: $\rho_S = 2{,}600 \frac{kg}{m^3}$

$$time = \frac{distance}{V_\infty}$$

Density of air: $\rho_{Air} = 1.2 \frac{kg}{m^3}$

$$time = \frac{8\times10^{3}\,m}{1.914\times10^{-3}\,\frac{m}{s}} * \frac{1\,h}{3600\,s} \cdot \frac{1\,day}{24\,h}$$

$$\approx 48.377\ days$$

Viscosity of air: $\eta_{Air} = 1.85 \times 10^{-5} \frac{kg}{m\,s}$

$$Re = \frac{\rho_{air} V_\infty d}{\eta_{air}} = 6.2\times10^{-4}$$

Gravity $g = 9.81 \frac{m}{s^2}$

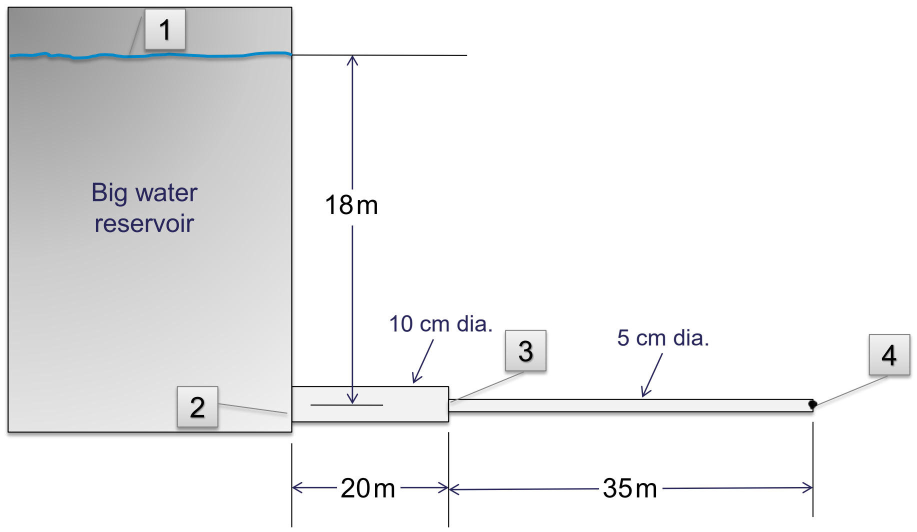
<u>Solution:</u>

a) $v_\infty = 0.2 \frac{cm}{s}$ $\quad t = 48$ days

b) $Re = 6.2 \times 10^{-4}$

Question 4 … Water flow from a tank **Points: 20**

Water is to be discharged from an open reservoir using two horizontal pipes connected in series. The first pipe is 20 m long and has a 10-cm diameter, while the second pipe is 35 m long and has a 5-cm diameter.



a) Ignore all losses and calculate the velocity of the outflowing water at point 4 using Bernoulli equation.

b) What is then the velocity of the water in the 20-m pipe?

c) Check for the 20-m pipe and the 35-m pipe if the flow of water is turbulent or laminar?

d) Now calculate the outflow velocity v_4 once again <u>including</u> all losses.

Given:

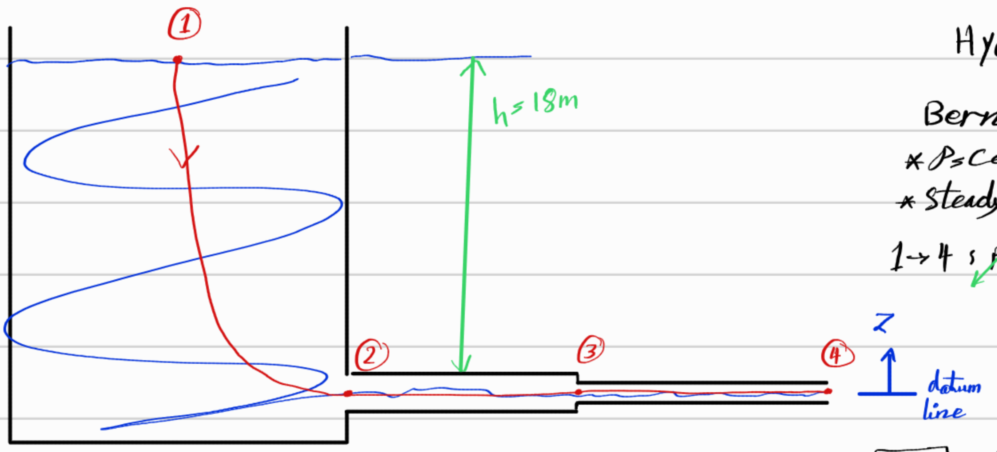
Density of water $\rho = 1{,}000\ \frac{kg}{m^3}$

Dynamic viscosity of water $\eta = 1\ \text{mPas}$

Material of the pipes: Galvanized iron

Minor losses: $\zeta_{inlet} = 0.1$

$\zeta_{Contraction} = 0.35$ with respect to the velocity after the contraction



Hydrodynamics:

Bernoulli equation

* ρ=Cons. * No viscous forces
* Steady-state * Along streamline

$$1\to 4:\ \cancel{P_1}+\frac{\rho}{2}\cancel{V_1^2}+\cancel{\rho} g z_1=\cancel{P_4}+\frac{\rho}{2}V_4^2+\rho g \cancel{z_4}$$

assumptions
* $z_4=0$ datum line
* $P_1=P_{atm}\Rightarrow P_{1gage}=0$
* $P_4=P_1=0$
* $V_1=0$ at rest

$$\Rightarrow V_4=\sqrt{2gh}=\sqrt{2\times 9.81\,\tfrac{m}{s^2}(18\,m)}$$

$$\boxed{V_4\simeq 18.8\ m/s}$$

continuity equ: $\dot m_{in}=\dot m_{out}$

$$\cancel{\rho}V_2A_2=\cancel{\rho}V_4A_4\Rightarrow\boxed{V_2=V_4\left(\frac{d_4}{d_2}\right)^2=18.8\tfrac{m}{s}\left(\frac{5}{10}\right)^2=4.7\,m/s}$$

$$Re_2=\frac{\rho V_2 d_2}{\eta}=\frac{10^3\frac{Kg}{m^3}(4.7\frac{m}{s})(0.1\,m)}{10^{-3}\,Pa.s}=4.7\times10^5\gg 2300$$

Turbulent flow

$$Re_4=\frac{\rho V_4 d_4}{\eta}=9.4\times10^5\gg 2300$$ Turbulent flow

Extended Bernoulli equation

$$1\to4:\ \cancel{P_1}+\frac{\rho}{2}\cancel{V_1^2}+\rho g z_1=\cancel{P_4}+\frac{\rho}{2}V_4^2+\rho g\cancel{z_4}+\Delta p_{losses}=$$

$$\Delta p_{losses}=\Delta p_\lambda+\Delta p_\zeta\Rightarrow\cancel{\rho}gh=\frac{\cancel{\rho}}{2}V_4^2+\lambda_2\frac{L_2}{d_2}\frac{\cancel{\rho}}{2}V_2^2+\lambda_4\frac{L_4}{d_4}\frac{\cancel{\rho}}{2}V_4^2+\zeta_{inlet}\cancel{\rho}\frac{V_2^2}{2}+\zeta_{con}\cancel{\rho}\frac{V_4^2}{2}$$

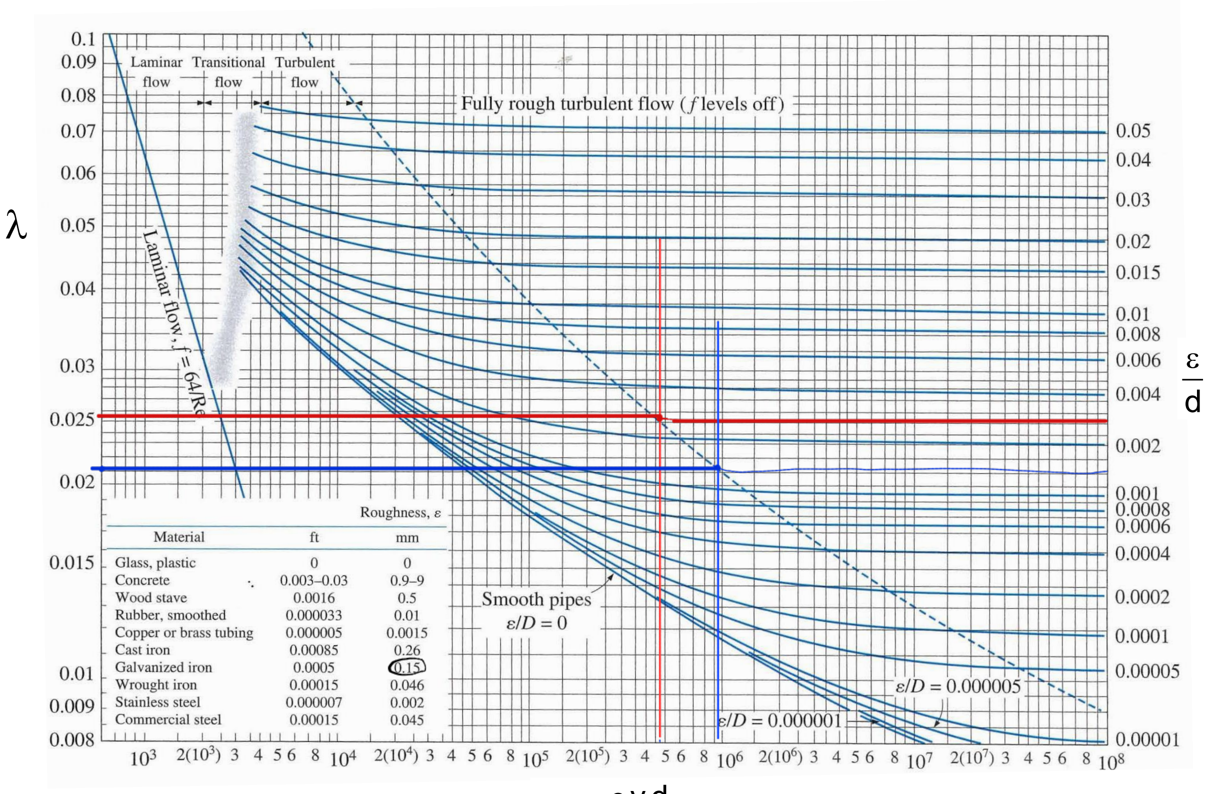
$$gh=V_4^2\left(\lambda_4\frac{L_4}{2d_4}+\frac{\zeta_{con}}{2}+\frac{1}{2}\right)+V_2^2\left(\lambda_2\frac{L_2}{2d_2}+\frac{\zeta_{inlet}}{2}\right)$$

Continuity equ. $\boxed{V_2=V_4\left(\frac{d_4}{d_2}\right)^2=V_4\cdot\frac{1}{4}}\Longrightarrow gh=\frac{V_4^2}{2}\left(\lambda_4\frac{L_4}{d_4}+\zeta_{con}+1+\lambda_2\cdot\frac{L_2}{16d_2}+\frac{\zeta_{inlet}}{16}\right)$

$$\boxed{V_4=\sqrt{\frac{2gh}{\lambda_4\frac{L_4}{d_4}+\zeta_{con}+1+\lambda_2\cdot\frac{L_2}{16d_2}+\frac{\zeta_{inlet}}{16}}}}$$

Moody diagram
- at $Re_2=4.7\times10^5$ & $\frac{e}{d_2}=\frac{0.15\,mm}{50\,mm}=0.003\longrightarrow\lambda_2=0.0255$
- at $Re_4=9.4\times10^5$ & $\frac{e}{d_4}=0.0015\longrightarrow\lambda_4=0.021$

$$\boxed{V_4=\sqrt{21.62}\ m/s\simeq 4.65\ m/s}$$ with Losses



$$Re = \frac{\rho\, v\, d}{\eta}$$

<u>Solution:</u>

a) $v_4 = 18.8 \ \frac{m}{s}$ without losses

b) $v_3 = 4.7 \ \frac{m}{s}$

c) $Re_3 = 469{,}750$

$Re_4 = 939{,}500$

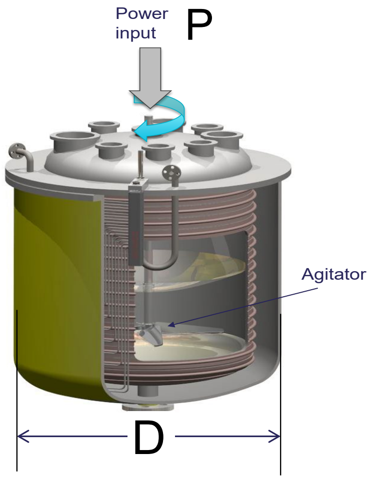
d) $v_4 = 4.42 \ \frac{m}{s}$ with losses

Register No.: ____________

Question 5 … Dimensional analysis **Points: 14**

The mixing time t to homogenize a liquid substance, for instance motor oil or paint, depends on:

- Power input P
- Diameter of mixing vessel D
- Density of the two liquids ρ
- Dynamic viscosity of the two liquids η



a) Determine the number of dimensionless π - terms for the problem.

b) Define the repeating variables.

$N = Kg\,m/s^2$

c) Provide an explicit step-by-step description of the procedure for the determination of the relevant π - terms $n = 5$, $m = 3$

$n - m = 2$

$P = W = J/s = Nm/s = \frac{Kg \cdot m^2}{s^3}$

Use the following table for your solution as template.

remaining variables | repeating variables

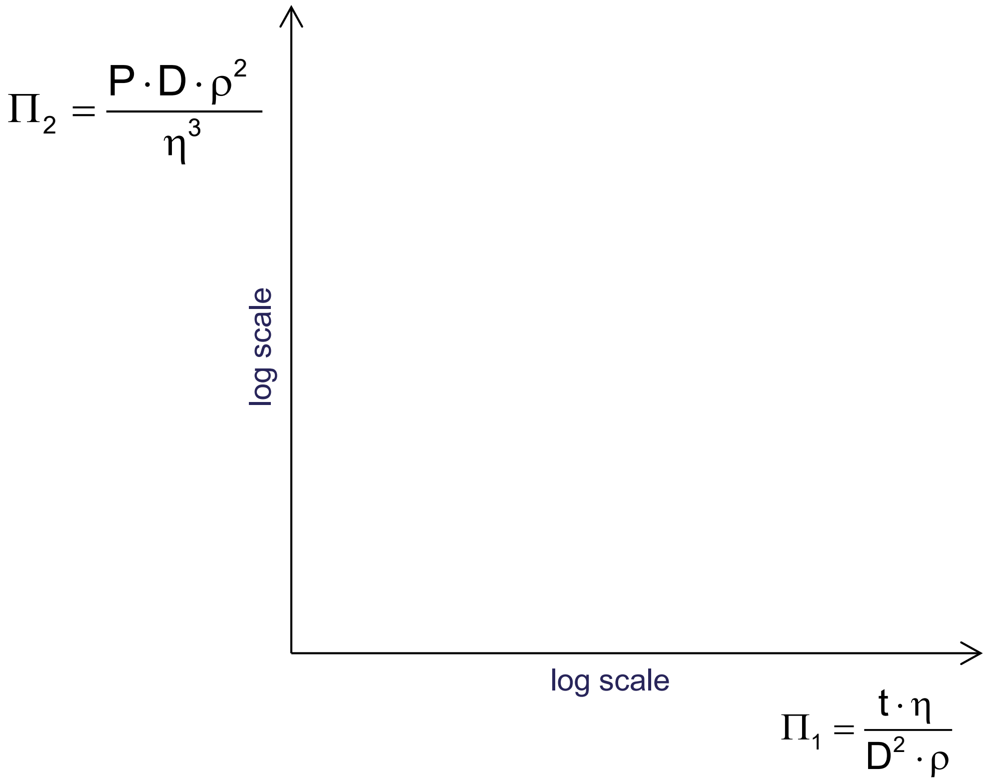
	t	P	D	ρ	η	
M	0	1	0	1	1	
L	0	2	1	-3	-1	
T	-1	-3	0	0	-1	

d) Sketch the curve Π_2 versus Π_1 in a double logarithmic reference frame (see template) if the development is supposed to follow a power law.

$\pi_1 = f(\pi_2)$

$\pi_1 = t$

$\pi_2 = P$


$\Pi_2 = \frac{P \cdot D \cdot \rho^2}{\eta^3}$
log scale
log scale
$\Pi_1 = \frac{t \cdot \eta}{D^2 \cdot \rho}$

Question 6 … Wind energy **Points: 12**

The torque of a windmill can be generated by rotating blades at a certain speed.

a) Assuming the wind is moving from left to right, indicate the direction of rotation of the windmill shown in the picture. Is it clockwise or counter clockwise?

b) Why are the blades twisted from the root to the tip? Use three velocity polygons at different radii for your explanations. Use the template shown below for the drawings.

b) Determine the diameter of a wind turbine under the following conditions:

Power output: $P_T = 2.5\ \text{MW}$

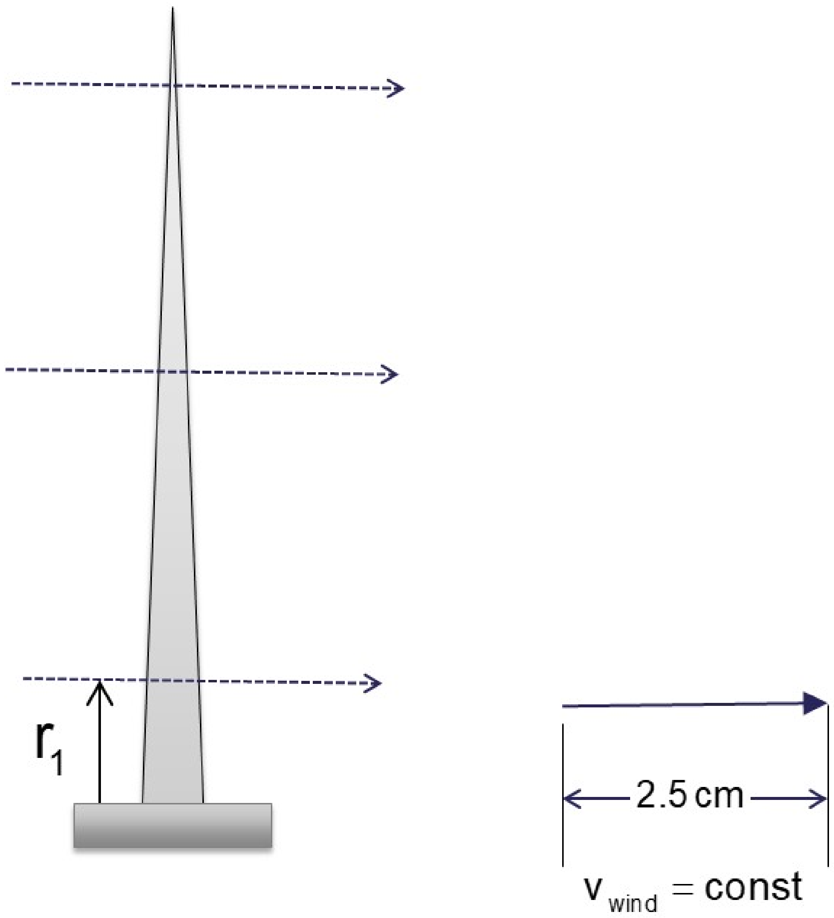
Wind velocity: $v = 30\ \dfrac{\text{km}}{\text{h}}$

Density of air: $\rho_{Air} = 1.2\ \dfrac{\text{kg}}{\text{m}^3}$

Betz coefficient: $c_P = 0.5$

<u>Solution:</u>

$D = 135\ \text{m}$



End of exam