

Period of Examinations Winter semester 2020/21

Your course of study: _____ (Use the abbreviation ME or IE or SE respectively)

Module Title: **ME 3 2708 / SE 3 2708 / IE 5 2708 Thermodynamics**

Family Name: _____

First Name: _____

Student No.: _____

I hereby confirm in lieu of an oath that I am the person who was admitted to this examination. Further, I confirm that the submitted work is my own and was prepared without the use of any unauthorised aides or materials. I am fully aware of the legal consequences of making a false declaration.*

Date: 09.02.2021 Signature: _____

Examination form: Written exam

Points: 75

Duration: 120 Minutes

*Authorised aides and materials are:

- Non-programmable pocket calculator
- Geometrical tools (no coloured pencils!)
- All lectures slides and exercises available in the Moodle course “Thermodynamics B.Sc. WS20/21”
- Your personal handwritten notes
- Moran/Shapiro Steam Tables A2 – A6 (either printed or on the screen)

Use your own scratch paper for the solution. Each page must contain your register number, your name and your signature.

Please write legibly. Thank you very much and good luck.

Register No.: _____

FOR INTERNAL USE ONLY:

	Q1	Q2	Q3	Q4	Q5	Written exam	Stirling motor	Practical Training	Total
Max. Points	15	16	14	10	20	75	13	12	100
Achieved									
						≥ 38 Yes No		Grade	

Graded by	Checked by

Final Grade

Regular grading key.	
Adjusted grading key. (Please add the adjusted grading key to the exam-results)	X

Question 1 ... Work and polytropic change of state**Points: 15 (7+5+3)****Q1.1**

Oxygen gas expands in a flexible container following a relationship of

$$p \cdot v^{1.15} = \text{constant.} \quad n=1.15$$

The mass of the oxygen is 750 g, and initially the pressure and temperature of the oxygen are 1 MPa and 65°C, respectively. The expansion continues until the volume is double the original volume. Determine the final pressure and temperature of the oxygen, and determine the work done by the oxygen during the expansion. A $V_2 = 2V_1$

Oxygen behaves as an ideal gas, i.e. $p \cdot v = R \cdot T$

Given: Specific gas constant of oxygen $R = 0.260 \frac{\text{kJ}}{\text{kgK}}$

Solution:

$$T_2 = 304\text{K} \quad p_2 = 451 \text{ kPa} \quad W_{12} = 43.1 \text{ kJ}$$

Q1.2

Air expands in an isothermal process from a volume of 0.5 m³ and a pressure of 850 kPa, to a volume of 1.2 m³. The temperature of the air is 25°C. Determine the work done by the air in this expansion process.

Air behaves as an ideal gas, i.e. $p \cdot v = R \cdot T$

Given: Specific gas constant of air $R = 0.287 \frac{\text{kJ}}{\text{kgK}}$

Solution:

$$W_{12} = 372 \text{ kJ}$$

Q1.3

1.5 kg of water vapor at 500 kPa fills a balloon. The specific volume of the water vapor is 0.3749 m³/kg. The water condenses at constant pressure until a liquid-vapor mixture with a specific volume of 0.0938 m³/kg is present. Determine the work done in the process.

Solution:

$$W_{12} = 211 \text{ kJ}$$

Closed system

Q1.1

$$m = 0.75 \text{ kg}$$

$$P_1 = 1 \text{ MPa}$$

$$T_1 = 65^\circ C = 338 \text{ K}$$

$$P \cdot V^{1.15} = \text{cons.}$$

$$n = 1.15$$

$$T_2 = ?$$

$$P_2 = ?$$

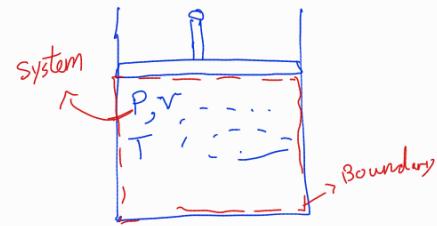
$$W_{12} = ?$$

$$P_1 \cdot V_1^n = P_2 \cdot V_2^n \quad (2V_1)^n \Rightarrow P_2 = \frac{P_1 V_1}{2^n} = \frac{1 \times 10^3 \text{ kPa}}{2^{1.15}}$$

$$\Rightarrow P_2 = 450.6 \text{ kPa}$$

From lecture polytropic processes (slide 8)

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} \Rightarrow T_2 = (338 \text{ K}) \left(\frac{450.6 \text{ kPa}}{1000 \text{ kPa}} \right)^{\frac{1.15-1}{1.15}} \Rightarrow T_2 = 304.62 \text{ K}$$



$$V_1 = \frac{m R T_1}{P_1} \Rightarrow V_2 = \frac{m R T_2}{P_2} \quad , \quad P = \frac{\text{cons.}}{V^n}$$

$$W_{12} = \int_1^2 P dV = \int_1^2 \frac{\text{cons.}}{V^n} dV = \frac{P_2 V_2 - P_1 V_1}{1-n} = \frac{m R (T_2 - T_1)}{1-n}$$

$$W_{12} = \frac{m R (T_2 - T_1)}{1-n} = \frac{0.75 \text{ kg} (0.26 \text{ kJ/kg}\cdot\text{K}) (304.62 - 338) \text{ K}}{1-1.15} \Rightarrow W_{12} = 43.4 \text{ kJ}$$

Q1.2

$$V_1 = 0.5 \text{ m}^3$$

$$P_1 = 850 \text{ kPa}$$

$$V_2 = 1.2 \text{ m}^3$$

$$T = 25^\circ C = 298 \text{ K}$$

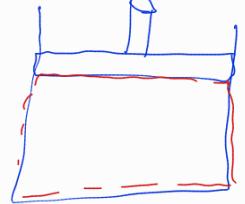
$$W_{12} = ?$$

$$\begin{aligned} P_1 V_1 &= \underbrace{m R T}_\text{constant} \\ P_2 V_2 &\leq m R T \end{aligned} \Rightarrow \frac{P_1 V_1}{P_2} \leq \frac{P_1 V_1}{V_2}$$

$$W_{12} = \int_1^2 V dP = \int_1^2 \frac{\text{cons.}}{P} dP \Rightarrow W_{12} = P_1 \cdot V_1 \cdot \ln\left(\frac{P_2}{P_1}\right) = P \cdot V_1 \cdot \ln\left(\frac{P_1 V_1}{P_2 V_2}\right) = P_1 V_1 \cdot \ln\left(\frac{V_1}{V_2}\right)$$

$$W_{12} = 850 \text{ kPa} (0.5 \text{ m}^3) \cdot \ln\left(\frac{0.5 \text{ m}^3}{1.2 \text{ m}^3}\right) = \boxed{372.07 \text{ kJ}}$$

Closed system
i.e. m = cons.



Q1.3

$$m = 1.5 \text{ kg}$$

Closed system
 $\rightarrow m = \text{cons.}$

$$P_1 = 500 \text{ kPa}$$

$$V_1 = 0.3749 \text{ m}^3/\text{kg}$$

$$P_2 = P_1$$

$$V_2 = 0.0938 \text{ m}^3/\text{kg}$$

$$W_{12} = \int_1^2 P dV = -m \cdot P (V_2 - V_1) \Rightarrow W_{12} = -(1.5 \text{ kg}) (500 \text{ kPa}) (0.0938 - 0.3749) \text{ m}^3/\text{kg}$$

$$W_{12} = 210.825 \text{ kJ}$$

Question 2 Thermodynamic properties

Points: 16 (2+3+6+5)

Use Moran/Shapiro Steam Tables for your solution!

Q2.1

0.5 kg of water at 120°C occupies a volume of 0.12 m³.

Find the quality x of the saturated mixture.

$$V_f = 1.0603 \times 10^{-3} \text{ m}^3/\text{kg}, V_g = 0.8919$$

$$V = \frac{V}{m} = \frac{0.12 \text{ m}^3}{0.5 \text{ kg}} = 0.24 \text{ m}^3/\text{kg}$$

$$x = \frac{V - V_f}{V_g - V_f} = 0.268 = 26.8\%$$

Q2.2

Use the steam table to determine the phase of water (i.e. compressed liquid, saturated liquid, saturated mixture, saturated vapour, or superheated vapour) with a pressure of 1 MPa and a temperature of

- a) 30°C Compressed liquid
- b) 179.9°C Saturated liquid/vapour ($0 \leq x \leq 1$)
- c) 300°C Superheated vapour

Q2.3

Determine the phase of water if

- a) T = 200°C and u = 1,500 kJ/kg
- b) p = 500 kPa and h = 3,000 kJ/kg

Q2.4

At 400 kPa, water has $v_f = 0.0010836 \text{ m}^3/\text{kg}$ and $v_g = 0.4625 \text{ m}^3/\text{kg}$. The mass of water is 0.5 kg. For each of the following volumes, determine whether the water is a compressed liquid, saturated liquid, saturated mixture, saturated vapour, or superheated vapour, and determine the quality x of the water:

- a) $V = 0.00045 \text{ m}^3 \rightarrow V < v_f \rightarrow$ Compressed liquid $\rightarrow x = \text{undefined}$
- b) $V = 0.000542 \text{ m}^3 \rightarrow V \approx v_f \rightarrow$ Saturated liquid
- c) $V = 0.025 \text{ m}^3 \rightarrow V > v_g \rightarrow$ Saturated mixture $\rightarrow x = \frac{V - v_f}{v_g - v_f} = 0.106 = 10.6\%$
- d) $V = 0.20 \text{ m}^3 \rightarrow V > v_g \rightarrow$ Saturated mixture $\rightarrow x = \frac{V - v_f}{v_g - v_f} = 0.866 = 86.6\%$
- e) $V = 0.4 \text{ m}^3 \rightarrow V > v_g \rightarrow$ Superheated vapour $\rightarrow x = \text{undefined}$

Question 3 1st rule of thermodynamics

Points: 14 (6+8)

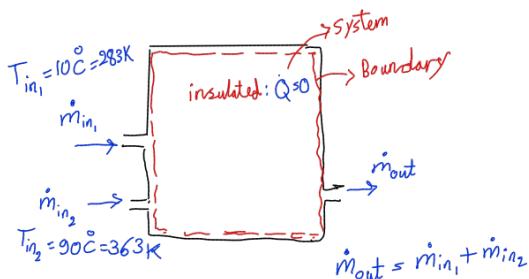
Q3.1

$$\dot{Q} = 0$$

Liquid water at 10°C enters an insulated mixing chamber at a rate of 180 kg/s . In the chamber, the water is mixed with a second stream of water, which enters at 90°C . Determine the exit temperature of the combined stream for mass flow rates of the second stream of

- a) 20 kg/s
- b) 100 kg/s
- c) 300 kg/s

State 1: Cool liquid inlet
 State 2: Warm liquid inlet
 State 3: Outlet



- a) $m_{out} = 200 \text{ kg/s}$
- b) $m_{out} = 280 \text{ kg/s}$
- c) $m_{out} = 480 \text{ kg/s}$

Assumptions:

- Adiabatic system
- Kinetic energy and potential energy negligible
- Steady-state mixing process
- Constant specific heats

Given:

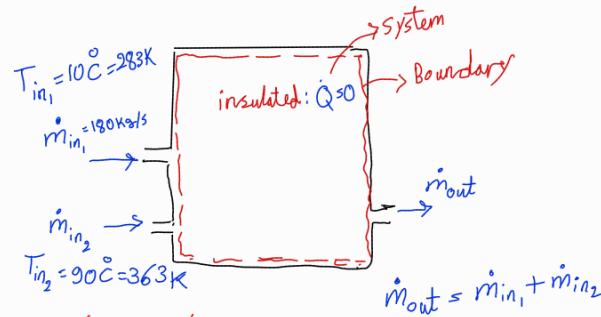
Specific heat capacity $\bar{c}_p = 4.18 \frac{\text{kJ}}{\text{kgK}} = \text{constant}$

Solution:

a) $T_3 = 18^\circ\text{C}$

- a) $\dot{m}_{in_2} = 20 \text{ kg/s}$
 b) $\dot{m}_{in_2} = 100 \text{ kg/s}$
 c) $\dot{m}_{in_2} = 300 \text{ kg/s}$

Open system



1st Law:

$$\dot{m}_{out} \cdot h_{out} - \dot{m}_{in_1} h_1 - \dot{m}_{in_2} h_2 = \cancel{\sum Q} + \cancel{\sum P} + \cancel{O}$$

$$\dot{m}_{in_1} h_{out} + \dot{m}_{in_2} h_{out} - \dot{m}_{in_1} h_1 - \dot{m}_{in_2} h_2 = 0 \Rightarrow \dot{m}_{in_1} (h_{out} - h_1) + \dot{m}_{in_2} (h_{out} - h_2) = 0$$

$$dh = \bar{C}_p dT \Rightarrow \dot{m}_{in_1} \cdot \bar{C}_p (T_{out} - T_{in_1}) = -\dot{m}_{in_2} \cdot \bar{C}_p (T_{out} - T_{in_2})$$

$$\dot{m}_{in_1} \cdot \bar{C}_p T_{out} + \dot{m}_{in_2} \bar{C}_p T_{out} = \dot{m}_{in_1} \bar{C}_p T_{in_1} + \dot{m}_{in_2} \bar{C}_p T_{in_2}$$

$$T_{out} = \frac{\bar{C}_p (\dot{m}_{in_1} T_{in_1} + \dot{m}_{in_2} T_{in_2})}{\bar{C}_p (\dot{m}_{in_1} + \dot{m}_{in_2})}$$

solution

a) $\dot{m}_{in_2} = 20 \text{ kg/s} \rightarrow T_{out} = \frac{(180 \cdot 283 + 20 \cdot 363) \text{ K}}{(180 + 20) \text{ kg/s}} = 291 \text{ K} = 18^\circ \text{C}$

b) $\dot{m}_{in_2} = 100 \text{ kg/s} \rightarrow T_{out} = 311.6 \text{ K} = 38.6^\circ \text{C}$

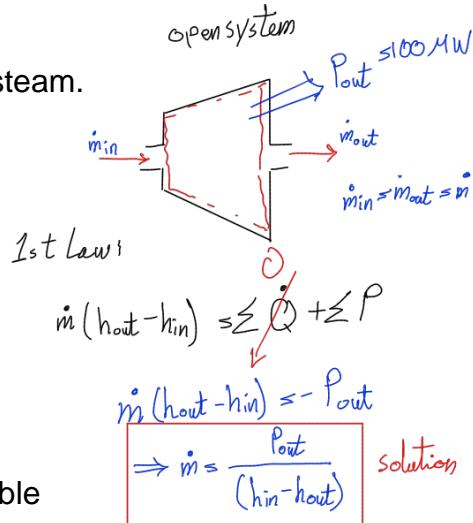
c) $\dot{m}_{in_2} = 300 \text{ kg/s} \rightarrow T_{out} = 333 \text{ K} = 60^\circ \text{C}$

Q3.2

An **insulated** steam turbine is to produce **100 MW** of power. Superheated steam enters the turbine at **10 MPa** and **600°C**, and saturated water exists the turbine at a pressure of **100 kPa** and a quality of **0.93**.

Determine the necessary mass flow rate of the steam.

- State 1: Steam inlet
State 2: Steam outlet



Assumptions:

- Steady-state conditions
- Adiabatic system
- Kinetic energy and potential energy negligible

Solution:

$$\dot{m} = 90 \frac{\text{kg}}{\text{s}}$$

$$\text{from Moran/Shapiro table A-4} \rightarrow h_{\text{in}} = 3625.3 \text{ kJ/kg}$$

$$\text{from A-3} \rightarrow h_{g_2} = 2675.5 \text{ kJ/kg}, h_{f_2} = 417.46 \text{ kJ/kg}$$

$$h_{\text{out}} = x_2 h_{g_2} + (1-x_2) h_{f_2} = 0.93(2675.5 \text{ kJ/kg}) + (1-0.93)(417.46 \text{ kJ/kg})$$

$$\Rightarrow h_{\text{out}} = 2517.43 \text{ kJ/kg}$$

$$\dot{m} = \frac{100 \times 10^3 \text{ kJ/s}}{(3625.3 - 2517.43) \text{ kJ/kg}} = 90.26 \text{ kg/s}$$

Question 4 2nd rule of thermodynamics**Points: 10 (5+5)****Q4.1**

A **reversible heat engine** operates between a hot thermal reservoir at 677°C and a cold reservoir at 27°C. The heat engine delivers **525 kW** of power.

 P_{net}

Determine the rate at which heat is received from the hot reservoir.

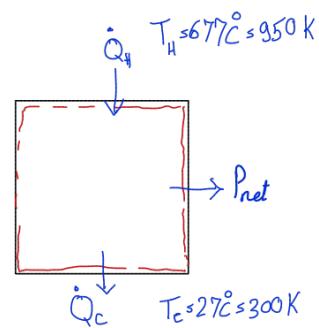
Solution:

$$\dot{Q}_H = 767 \text{ kW}$$

$$\text{reversible} \rightarrow \text{ideal} \rightarrow \eta_{\text{Carnot}} = \eta_{\text{th}}$$

$$1 - \frac{T_c}{T_H} = \frac{P_{\text{net}}}{\dot{Q}_H} \Rightarrow \dot{Q}_H = \frac{P_{\text{net}}}{1 - \frac{T_c}{T_H}}$$

$$\dot{Q}_H = \frac{525 \text{ kW}}{1 - \frac{300 \text{ K}}{950 \text{ K}}} = 767.31 \text{ kW}$$

**Q4.2**

An engineer proposes to develop a power plant that will operate at a thermal efficiency of **65%**. The steam flowing through the power plant reaches a peak temperature of **660 K**, and rejects heat to the environment at a temperature of **290 K**.

Is this proposed power plant feasible, or does it violate the second rule?

To answer "yes" or "no" gives no points. You have to provide an adequate explanation.

It does violate the second rule of thermodynamics.

Since the ideal case for a power plant to have an efficiency = $\eta_{\text{Carnot}} = 1 - \frac{T_c}{T_H}$

which in our case $\eta_{\text{Carnot}} = 1 - \frac{290 \text{ K}}{660 \text{ K}} = 56\% < \eta_{\text{th}} = 65\%$
can't be.

Question 5 Rankine cycle

Points: 20 (4+10+4+2)

Steam is the working fluid in an ideal Rankine cycle (i.e. no irreversibilities).

$$\text{Your aim is to realize a thermal efficiency of } 35\%. \eta_{th} = 0.35 = \frac{P_1 - P_2}{Q_B} = \frac{h_1 - h_2}{h_1 - h_4}$$

The condenser should operate at atmospheric pressure, i.e.

$$p_2 = p_3 = 1.0 \text{ bar}$$

$$0.35h_1 - 0.35h_4 = h_1 - h_2$$

$$h_1 - 0.35h_1 = h_2 - 0.35h_4$$

$$h_1 = \frac{h_2 - 0.35h_4}{1 - 0.35} = 3891.37 \text{ kJ/kg}$$

Assume that the vapour at the exit of the turbine is saturated, and that saturated liquid exits the condenser.

- a) Sketch the cycle on T-s coordinates and specify the nature of each process (adiabatic or isothermal or polytropic or isentropic or isobaric, expansion or compression and so on). ✓
- b) Determine pressure p_1 and temperature T_1 which are necessary to achieve the desired efficiency. ✓

Assume that the turbine operates under isentropic conditions and that the enthalpy at state 3 is approximately equal to the enthalpy at state 4.

- d) Determine the isentropic efficiency of the turbine for the case that irreversibilities at an amount of $\sigma = 0.3 \text{ kJ/kg}\cdot\text{K}$ are present. ✓
- e) Taking the isentropic efficiency into account, sketch the real expansion process in the turbine on T-s coordinates. Use the same template as in Point a). ✓

Assumptions:

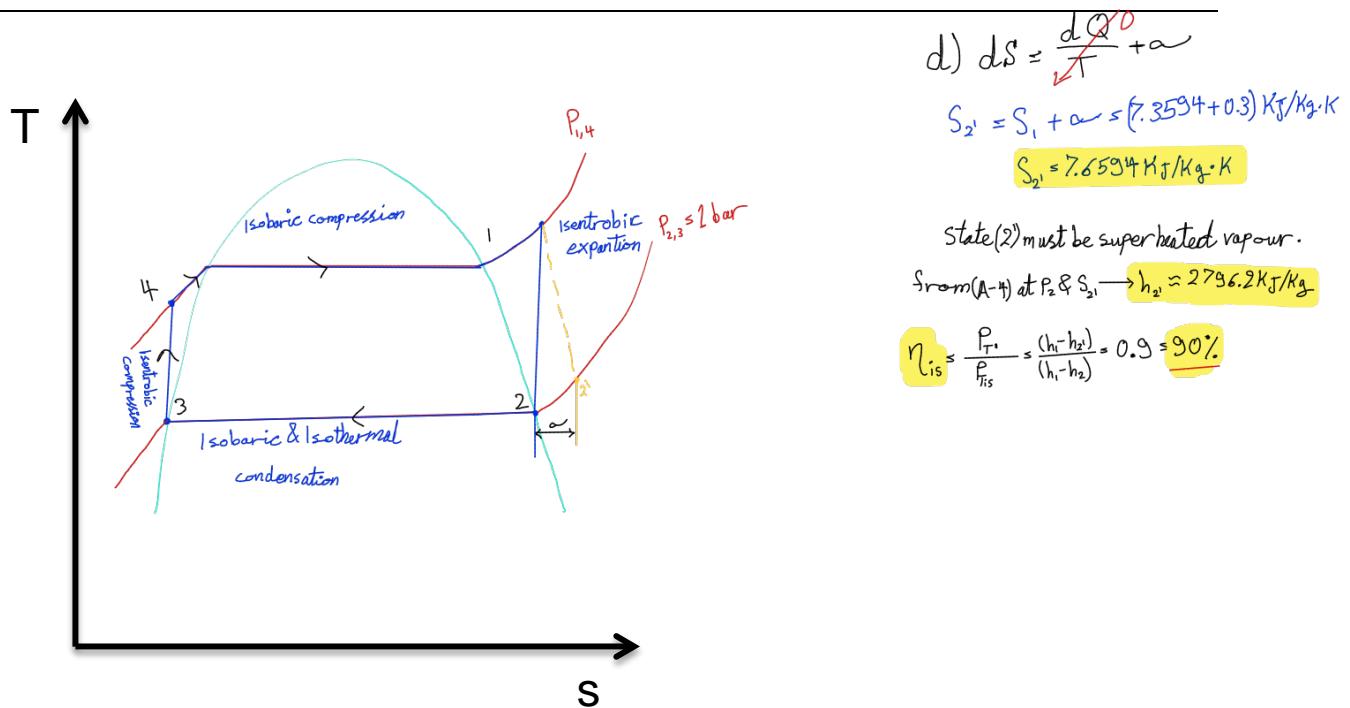
- Label the states as follows
 - 1 ... Turbine in
 - 2 ... Turbine out
 - 3 ... Condenser out
 - 4 ... Boiler in
- Neglect kinetic and potential energy effects.
- Turbine and pump operate adiabatically.
- Interpolation not required. An educated guess is sufficient.

b)

In A-4: at $P_i = 60 \text{ bar}$

S_i is in between 700°C , 864°C
an educated guess would be:

$$T_i = 670^\circ\text{C}$$



	$p [\text{bar}]$	$T [{}^\circ\text{C}]$	$h [\text{kJ}/\text{kg}]$	$s [\text{kJ}/\text{kgK}]$	$x [-]$
1	60	670	3891.37	7.3594	---
2	1	99.63	$h_{g_2} \approx 2675.5$	$S_{g_2} \approx 7.3594$	1
3	1	99.63	$h_{f_3} \approx 417.46$	$S_{f_3} \approx 1.3026$	0
4	60	---	$h_4 \approx h_3$	1.3026	---

End of exam