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Numerical Mathematics

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9) Interpolation polynomials

Consider the function

$$f(x) = \ln(x) - \frac{2(x-1)}{x}$$

a) Make a sketch of the function.

b) Find the Newton interpolation polynomial for $x_0 = 1$, $x_1 = 2$, and $x_2 = 4$.

c) Find the Lagrange interpolation polynomial using the same nodes as in a).

d) Add another node at $x_3 = 8$. How does the Newton polynomial change ?

e) Calculate absolute and relative errors at $x = 2.9$ and $x = 5.25$ for all three polynomials.

10) Numerical differentiation

Consider the function

$$f(x) = x\,e^x + \cos(x^2)$$

a) Find an approximation depending only on the nodes for $f^{(2)}(x)$, using the formula derived in lecture 10.

b) Formulate $f^{(2)}(x)$ using $x_i = i\frac{\pi}{2}$, $i = 0, 1, \ldots, 2$.

c) Calculate absolute and relative errors at $x = 1$ and $x = 2$.

g) $f(x) = \ln(x) - \dfrac{2(x-1)}{x}$

a) * Find the zeros . . . –

* Find the domain: $D = \mathbb{R}^+$

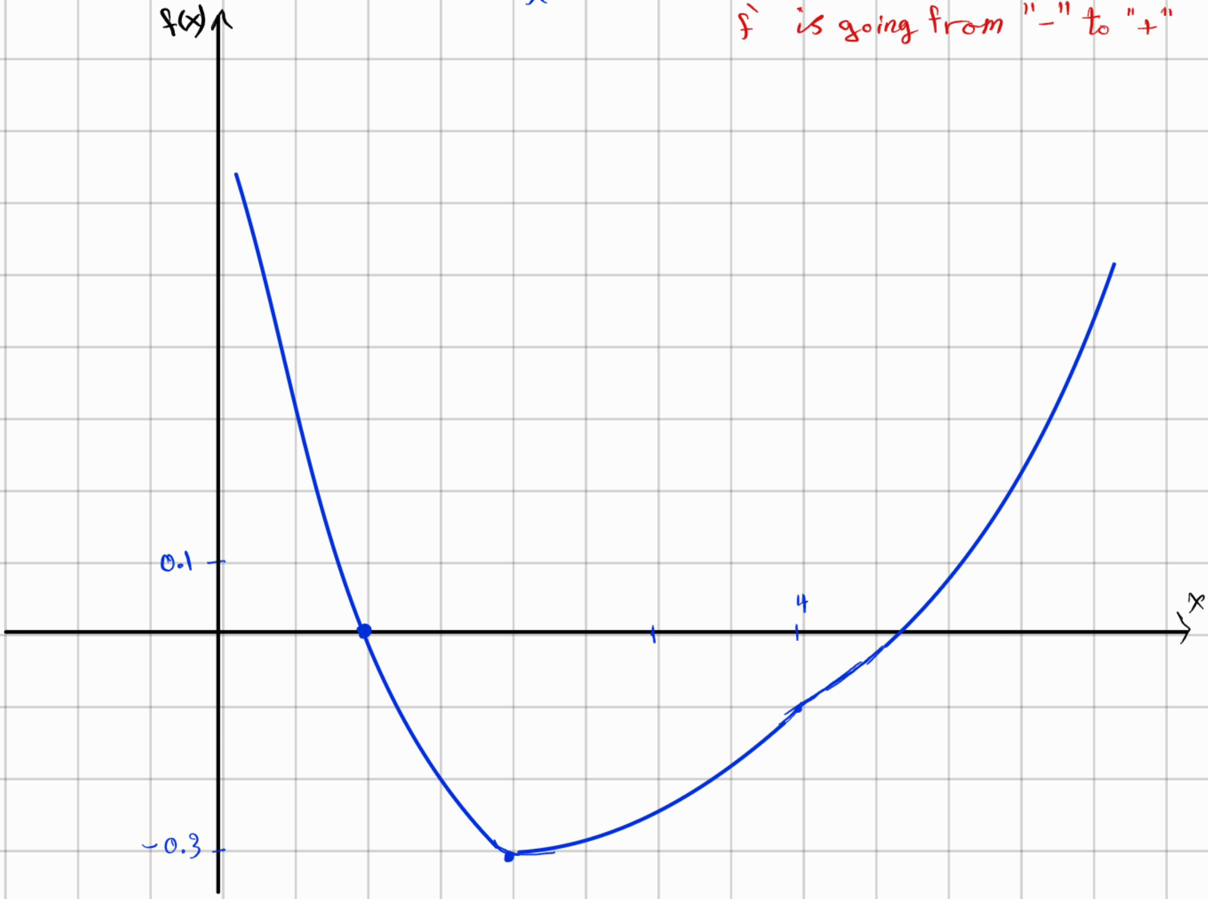
because x is going stronger to zero

* Asymptotes / limits: $\lim\limits_{x \to 0^+} \left(\ln(x) - 2 + \frac{2}{x}\right) = \lim\limits_{x \to 0^+} \dfrac{\overset{0}{\cancel{x\ln(x)}} - \overset{0}{\cancel{2x}} + 2}{x} = +\infty$

* Minimum / maximum: $f'(x) = \dfrac{1}{x} - \dfrac{2}{x^2} = \dfrac{x-2}{x^2} \Rightarrow f'(2) = 0$

f' is going from "–" to "+"

x	f(x)
1	0
2	-0.3069
4	-0.1137



b)

x	$f(x)=y$			
1	0 a_0			
		$\frac{\Delta y}{\Delta x} = -0.3069$ a_1		
2	-0.3069		0.1345 a_2	
		0.0966		-0.00189
4	-0.1137		---	

8	---			

$$P(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1) + a_3(x-x_0)(x-x_1)(x-x_2)$$

$$P(x) = 0 - 0.3069(x-1) + 0.1345(x-1)(x-2)$$

$$= -0.3069x + 0.3069 + 0.1345x^2 - 0.4035x + 0.4035$$

$$\boxed{P(x) = 0.1345x^2 - 0.7104x + 0.5759}$$

c) Lagrange:

$$P_{L_2}(x) = \sum_{i=0}^{2} y_i \underbrace{\prod_{\substack{j=0 \\ j\neq i}}^{2} \frac{x-x_j}{x_i-x_j}}_{L_i(x)} = \cancel{0 \cdot \frac{x-2}{1-2} \cdot \frac{x-4}{1-4}} \rightarrow 0 - 0.3069 \cdot \frac{x-1}{2-1} \cdot \frac{x-4}{2-4}$$

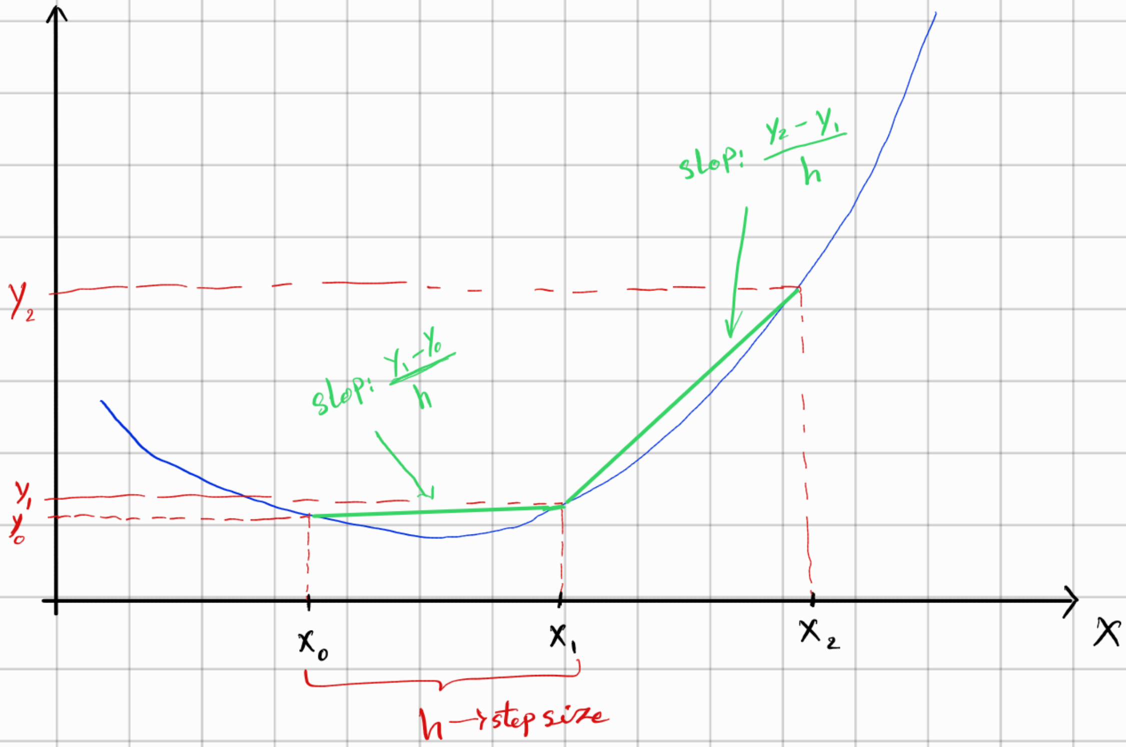
$$-0.1137 \cdot \frac{x-1}{4-1} \cdot \frac{x-2}{4-2}$$

$$= -0.3069 \frac{x^2-5x+4}{2} - 0.1137 \frac{x^2-3x+2}{6}$$

⑩ a) $f^{(2)}(x) \approx P_2^{(2)}(x) = \sum_{i=0}^{2} y_i \frac{(-1)^{2-i}}{h^2} \binom{2}{i}$ $\binom{n}{k} = \frac{n!}{(n-k)! \cdot k!}$

$= y_0 \cdot \frac{1}{h^2} \cdot 1 + y_1 \cdot \frac{(-1)}{h^2} \cdot 2 + y_2 \cdot \frac{1}{h^2} \cdot 1$

$= \frac{y_0 - 2y_1 + y_2}{h^2} \approx \frac{f'(y_2) - f'(y_1)}{h} \approx \frac{\frac{y_2 - y_1}{h} - \frac{y_1 - y_0}{h}}{h}$



Differentiate coefficients $\frac{f(x_0 + h) - f(x_0)}{(x_0 + h) - x_0}$

b) $f^{(2)}(x) \approx \underline{\underline{24}}$

$P_{2N}(x)$ ——— $+ 12 \cdot (x - \frac{\pi}{2}) \cdot (x - 0)$

x_i	$f[\cdot]$	$F[\cdot,\cdot]$	$F[\cdot,\cdot,\cdot]$
0	1		
		3.7	
$\frac{\pi}{2}$	6.8		12
		41.4	
π	71.8		