When To Use L'Hopital's rule?

if $\lim\limits_{x \to a} \frac{f(x)}{g(x)}$

$\Rightarrow$ where $\frac{f(a)}{g(a)} = \frac{0}{0}$ or $\frac{\pm\infty}{\mp\infty}$

L'Hopital's rule: $\boxed{\lim\limits_{x \to a} \frac{f(x)}{g(x)} = \lim\limits_{x \to a} \frac{f'(x)}{g'(x)}}$

<u>Conditions to use the rule:</u>

(i) f, g are differentiable on interval L that contains a, & $g'(x) \neq 0$ for all $x \in L$.

(ii) $\lim\limits_{x \to a} f(x) = 0$ & $\lim\limits_{x \to a} g(x) = 0$ type "$\frac{0}{0}$"

or

$\lim\limits_{x \to a} f(x) = \pm\infty$ & $\lim\limits_{x \to a} g(x) = \pm\infty$ type "$\frac{\pm\infty}{\infty}$"

(iii) $\lim \frac{f'(x)}{g'(x)}$ exists or is $\pm\infty$

*If conditions didn't apply, & you can't use l'hopital's rule, it doesn't mean necessarily that there is no limit.

Examples:

8. $\lim\limits_{x \to 3} \dfrac{x-3}{x^2-9} \Rightarrow \lim\limits_{x \to 3} \dfrac{f(x)}{g(x)}$

(i) $\left.\begin{array}{l} f'(x) = 1 \\ g'(x) = 2x \end{array}\right\}$ exists

(ii) type "$\frac{0}{0}$" or "$\frac{\infty}{\infty}$"

$\lim\limits_{x \to 3} (f(x)) = 0$, $\lim (g(x)) = 0$

$\therefore$ "$\frac{0}{0}$" type

(iii) $\lim\limits_{x \to 3} \dfrac{f'(x)}{g'(x)} = \lim\limits_{x \to 3} \dfrac{1}{2x} = \dfrac{1}{6}$ Exists

$\therefore \lim\limits_{x \to 3} \dfrac{x-3}{x^2-9} = \dfrac{1}{6}$

9. $\lim\limits_{x \to 4} \dfrac{x^2-2x-8}{x-4} = \lim\limits_{x \to 4} \dfrac{\cancel{(x-4)}(x+2)}{\cancel{x-4}} = 6$

10. $\lim\limits_{x \to -2} \dfrac{x^3+8}{x+2} = \lim\limits_{x \to -2} \dfrac{f(x)}{g(x)}$

* $\lim\limits_{x \to -2} (f(x)) = 0$, $\lim\limits_{x \to -2} (g(x)) = 0$, "$\frac{0}{0}$" type

* $f'(x) = 3x^2$, $g'(x) = 1$

* $\lim\limits_{x \to -2} \dfrac{f'(x)}{g'(x)} = \lim\limits_{x \to -2} \dfrac{3x^2}{1} = 12$

L'hopital's rule $\lim\limits_{x \to -2} \dfrac{f(x)}{g(x)} = 12$ as well

11. $\lim\limits_{x \to 1} \dfrac{x^7-1}{x^3-1} = \lim\limits_{x \to 1} \dfrac{f(x)}{g(x)}$

(i) $\left.\begin{array}{l} \lim\limits_{x \to 1} f(x) = 0 \\ \lim\limits_{x \to 1} g(x) = 0 \end{array}\right\}$ "$\frac{0}{0}$" type

(ii) $f'(x) = 7x^6$, $g'(x) = 3x^2$

both exists

(iii) $\lim\limits_{x \to 1} \dfrac{f'(x)}{g'(x)} = \lim\limits_{x \to 1} \dfrac{7x^6}{3x^2} = \dfrac{7(1)^6}{3(1)^2} = \dfrac{7}{3}$

according to L'hopitals rule:

$\lim\limits_{x \to 1} \dfrac{f(x)}{g(x)} = \lim\limits_{x \to 1} \dfrac{f'(x)}{g'(x)} = \dfrac{7}{3}$

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12. $\lim\limits_{x \to 4} \dfrac{\sqrt{x}-2}{x-4} * \dfrac{\sqrt{x}+2}{\sqrt{x}+2}$

$\Rightarrow \lim\limits_{x \to 4} \dfrac{(\sqrt{x})^2-(2)^2}{(x-4)(\sqrt{x}+2)} = \lim\limits_{x \to 4} \dfrac{\cancel{x-4}}{\cancel{(x-4)}(\sqrt{x}+2)} = \dfrac{1}{(\sqrt{4}+2)} = \dfrac{1}{4}$

13. $\lim\limits_{x \to \pi/4} \dfrac{\sin x - \cos x}{\tan(x)-1} * \dfrac{\sin x + \cos x}{\sin x + \cos x} = \lim\limits_{x \to \pi/4} \dfrac{\sin^2 x - \cos^2 x}{\left(\frac{\sin x}{\cos x} - \frac{\cos x}{\cos x}\right)\left(\frac{\sin x + \cos x}{1}\right)}$

$\Rightarrow \lim\limits_{x \to \pi/4} \dfrac{\cancel{\sin^2 x - \cos^2 x}}{\frac{\cancel{\sin^2 x - \cos^2 x}}{\cos x}} = \lim\limits_{x \to \pi/4} \cos(x) = \cos(\frac{\pi}{4}) = \dfrac{\sqrt{2}}{2}$

14. $\lim\limits_{x \to 0} \dfrac{\tan 3x}{\sin 2x} = \lim\limits_{x \to 0} \dfrac{f(x)}{g(x)}$

(i) $\left.\begin{array}{l} \lim\limits_{x \to 0} f(x) = 0 \\ \lim\limits_{x \to 0} g(x) = 0 \end{array}\right\}$ type "$\frac{0}{0}$"

(ii) $f'(x) = 3\sec(3x)\tan(3x)$

$g'(x) = 2\cos(2x)$

(iii) $\lim\limits_{x \to 0} \dfrac{f'(x)}{g'(x)} = \dfrac{3}{2} \lim\limits_{x \to 0} \dfrac{\sec(3x)\tan(3x)}{\cos(2x)} = \dfrac{3}{2} \cdot \dfrac{1 \cdot (0)}{1} = 0$

according to L'hopital's rule:

$\lim\limits_{x \to 0} \dfrac{f(x)}{g(x)} = \lim\limits_{x \to 0} \dfrac{f'(x)}{g'(x)} = 0$

15. $\lim\limits_{t \to 0} \dfrac{e^{2t}-1}{\sin(t)} = \lim\limits_{t \to 0} \dfrac{f(t)}{g(t)}$

(i) $\left.\begin{array}{l} \lim\limits_{t \to 0} f(t) = 1-1 = 0 \\ \lim\limits_{t \to 0} g(t) = \sin(0) = 0 \end{array}\right\}$ "$\frac{0}{0}$" type

(ii) $f'(t) = 2e^{2t}$, $g'(t) = \cos(t)$

(iii) $\lim\limits_{t \to 0} \dfrac{f'(t)}{g'(t)} = 2\lim\limits_{t \to 0} \dfrac{e^{2t}}{\cos(t)} = 2(\frac{1}{1}) = 2$

according to L'hopital's rule:

$\lim\limits_{t \to 0} \dfrac{f(t)}{g(t)} = \lim\limits_{t \to 0} \dfrac{f'(t)}{g'(t)} = 2$