TU1

Gauss elimination:

$$2x + 3y - z + 8r = 7$$

$$4x + 2y + 3z + r = 1$$

$$3x + z + 2r = 4$$

$$1x + 2y + 9z + r = 0$$

$$A \cdot \vec{x} = \vec{b}$$

$$\begin{array}{c} \begin{array}{cccc} x & y & z & r \end{array} \\ \left[\begin{array}{cccc|c} 2 & 3 & -1 & 8 & 7 \\ 4 & 2 & 3 & 1 & 1 \\ 3 & 0 & 1 & 2 & 4 \\ 1 & 2 & 9 & 1 & 0 \end{array}\right] \end{array}$$

$$\begin{array}{c} \begin{array}{cccc} y & x & z & r \end{array} \\ \left[\begin{array}{cccc|c} 3 & 2 & -1 & 8 & 7 \\ 0 & 3 & 1 & 2 & 4 \\ 2 & 4 & 3 & 1 & 1 \\ 2 & 1 & 9 & 1 & 0 \end{array}\right] \end{array} \begin{array}{l} \\ \\ III - \frac{2}{3} I \\ IV - \frac{2}{3} I \end{array} \Longrightarrow \left[\begin{array}{cccc|c} 3 & 2 & -1 & 8 & 7 \\ 0 & 3 & 1 & 2 & 4 \\ 0 & \frac{8}{3} & \frac{11}{3} & -\frac{13}{3} & -\frac{11}{3} \\ 0 & -\frac{1}{3} & \frac{29}{3} & -\frac{13}{3} & -\frac{14}{3} \end{array}\right] \begin{array}{l} \\ \\ III - \frac{8}{9} II \\ IV + \frac{1}{9} II \end{array}$$

$$\left[\begin{array}{cccc|c} 3 & 2 & -1 & 8 & 7 \\ 0 & 3 & 1 & 2 & 4 \\ 0 & 0 & \frac{25}{9} & -\frac{55}{9} & -\frac{65}{9} \\ 0 & 0 & \frac{88}{9} & -\frac{37}{9} & -\frac{38}{9} \end{array}\right] \begin{array}{l} \\ \\ \\ IV - \frac{88}{25} III \end{array}$$

$$\begin{array}{c} \begin{array}{cccc} y & x & z & r \end{array} \\ \left[\begin{array}{cccc|c} 3 & 2 & -1 & 8 & 7 \\ 0 & 3 & 1 & 2 & 4 \\ 0 & 0 & \frac{25}{9} & -\frac{55}{9} & -\frac{65}{9} \\ 0 & 0 & 0 & \frac{87}{5} & \frac{106}{5} \end{array}\right] \end{array}$$

Backward substitution:

$$r = \frac{5}{87} \cdot \frac{106}{5} = \frac{106}{87} \approx 1.2184$$

$$z = \frac{\frac{55}{9} r - \frac{65}{9}}{25/9} = \frac{7}{87} \approx 0.08046$$

$$x = \frac{-z - 2r + 4}{3} = \frac{43}{87} \approx 0.494252874$$

$$y = \frac{-2x + z - 8r + 7}{3}$$

$$y = -\frac{106}{87} \approx -1.2184$$

$2x + 3y - z = 7$

$4x + 2y + 3z = 1$

$3x - 2y + z = 4$

Gauss:

$$\begin{array}{ccc|c} x & y & z & \\ \end{array}$$

$$\left[\begin{array}{ccc|c} 2 & 3 & -1 & 7 \\ 4 & 2 & 3 & 1 \\ 3 & -2 & 1 & 4 \end{array}\right] \begin{array}{l} \\ II - 2I \\ III - \frac{3}{2}I \end{array}$$

$$\left[\begin{array}{ccc|c} 2 & 3 & -1 & 7 \\ 0 & -4 & 5 & -13 \\ 0 & -\frac{13}{2} & \frac{5}{2} & -\frac{13}{2} \end{array}\right] III - \frac{13}{8} II \qquad \left[\begin{array}{ccc|c} 2 & 3 & -1 & 7 \\ 0 & -4 & 5 & -13 \\ 0 & 0 & -\frac{45}{8} & \frac{117}{8} \end{array}\right]$$

backward substitution:

$z = \left(-\frac{8}{45}\right)\left(\frac{117}{8}\right) = \frac{-13}{5} = -2.6$

$y = \frac{-5z - 13}{-4} = 0$

$x = \frac{-3y + z + 7}{2} = \frac{11}{5} = 2.2$

If we are going to use LU-decom.

we already have $U = \begin{bmatrix} 2 & 3 & -1 \\ 0 & -4 & 5 \\ 0 & 0 & -\frac{45}{8} \end{bmatrix}$

$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ \frac{3}{2} & \frac{13}{8} & 1 \end{bmatrix}$

Forward sub.

$L \cdot \vec{c} = \vec{b} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ \frac{3}{2} & \frac{13}{2} & 1 \end{bmatrix} \cdot \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 1 \\ 4 \end{bmatrix}$

(if $\vec{b}$ changes, we just start from here again!)

$c_1 = 7$

$c_2 = 1 - 2c_1 = -13$

$c_3 = -\frac{3}{2}c_1 - \frac{13}{8}c_2 + 4 = \frac{117}{8} = 14.625$

same

Backward sub. $U \cdot \vec{x} = \vec{c}$

1- Solve the following system of linear equations using the Jacobi method:

$4x + y - z = 3 \longrightarrow x^{k+1} = \frac{3 - y^k + z^k}{4}$

$-2x + 5y + 2z = 9 \longrightarrow y^{k+1} = \frac{9 + 2x^k - 2z^k}{5}$

$x - y + 7z = -6 \longrightarrow z^{k+1} = \frac{-6 - x + y}{7}$

Use an initial guess of x0 = [0 0 0], and iterate until the solution converges to within a tolerance of 0.01.

$$\vec{x} = \begin{bmatrix} x^0 \\ y^0 \\ z^0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

acceptable error = 0.01

K	x^k	y^k	z^k
0	0	0	0
1	$\frac{3}{4}$	$\frac{9}{5}$	$-\frac{6}{7}$
2	$\frac{3}{35} \approx 0.086$	$\frac{171}{70} \approx 2.443$	$-\frac{99}{140} \approx -0.707$
3	$-\frac{3}{80} \approx -0.0375$	$\frac{741}{350} \approx 2.1171$	$-\frac{51}{98} \approx -0.5204$
4	$\frac{111}{1225} \approx 0.09$	1.9931632 65	-0.5493367
5	0.114375	2.05598	-0.58535
6	0.0896675		

Convergence check

$\mathcal{E}(q_1) = 3 - 4x - y + z$

$\mathcal{E}(q_2) = 9 + 2x - 5y - 2z$

$\mathcal{E}(q_3) = -6 - x + y - 7z$

$x = \frac{15}{167} \approx 0.089$

$y = \frac{345}{167} \approx 2.06587$

$z = -\frac{96}{167} \approx -0.575$