Controls

Exercise

Winter Semester 2021/2022

Revision 1

Table of Contents

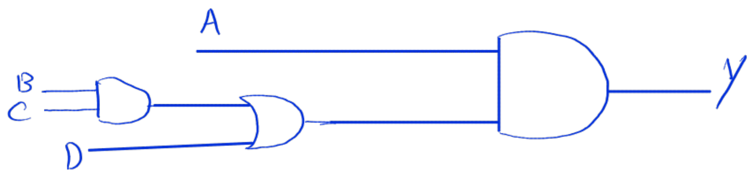
1 Programmable Logic Controllers

1.1 Exercise

Develop a logic gate circuit from the Boolean expression:

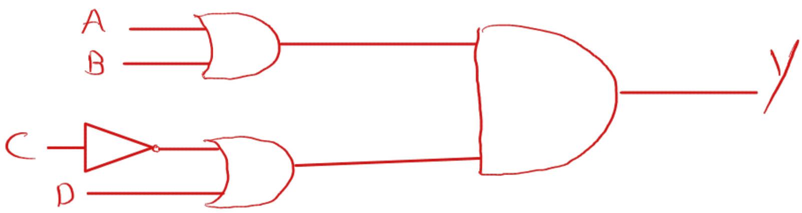
1.1.1 Task

$Y = A(BC + D)$



1.1.2 Task

$Y = (A + B)(\bar{C} + D)$



$$\bar{A}\,\bar{B} \neq \overline{A\cdot B}$$

$$\bar{A}\cdot\bar{B} = \overline{A+B}$$

$$\overline{A\cdot B} = \bar{A} + \bar{B}$$

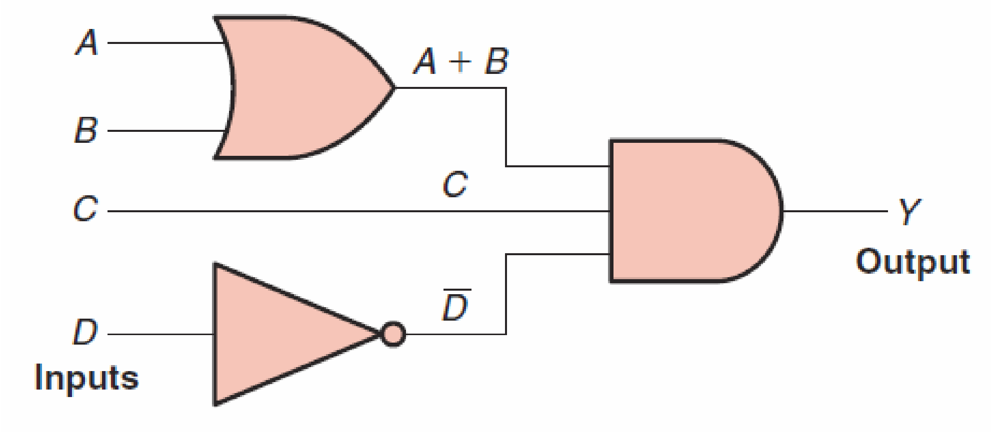
$$\bar{A}+\bar{B} \neq \overline{A+B}$$

1.2 Exercise

Develop the Boolean equation for a given logic gate circuit:

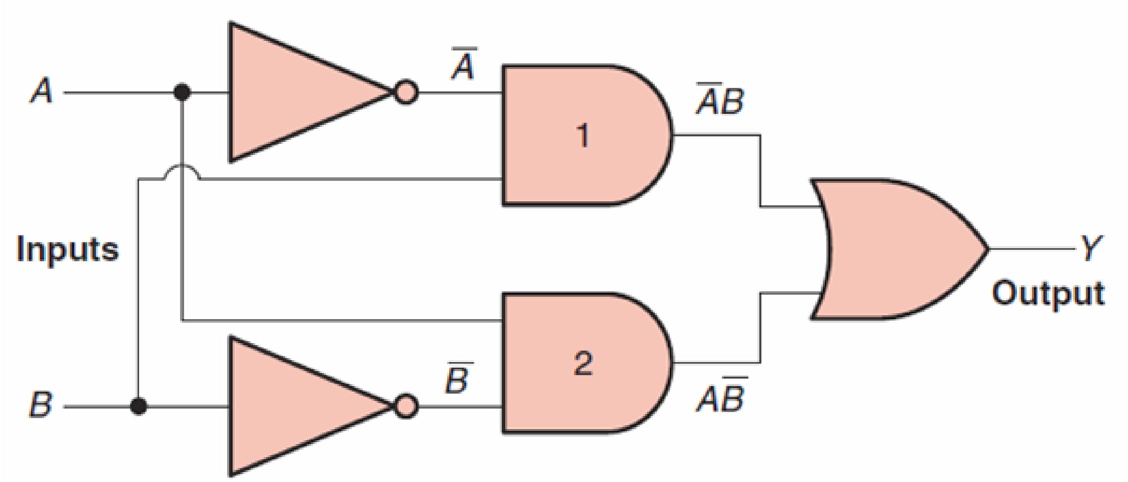
1.2.1 Task

$(A+B) \cdot C \cdot \overline{D} = Y$

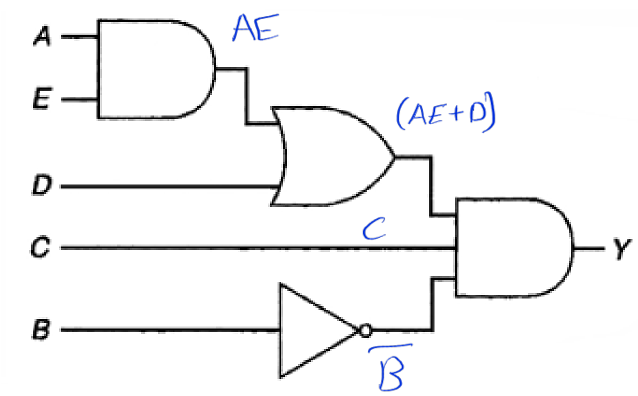


1.2.2 Task

$(\overline{A} \cdot B) + (A \cdot \overline{B}) = Y$



1.2.3 Task



1.3 Exercise

Draw a PLC ladder diagram program for the gate logic array shown in Figure 1.1.

1.3.1 Task

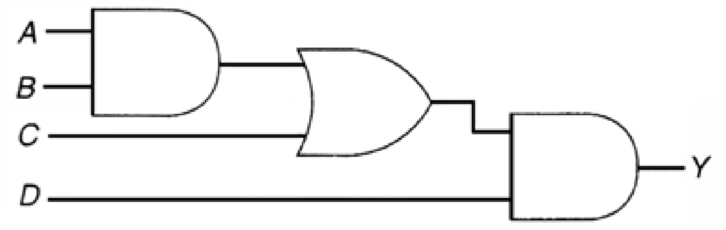
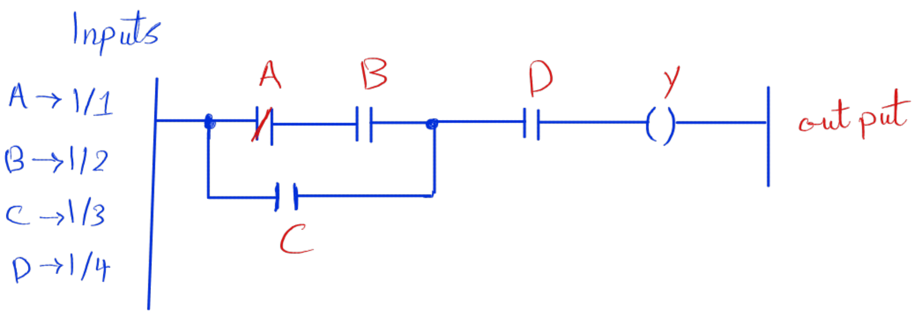


Figure 1.1. Gate logic array



$$(\bar{A}B)+C = \bar{A}C + BC$$

$$\overline{(\bar{A}B+C)} = \overline{\bar{A}B}\cdot\bar{C} = (A+\bar{B})\cdot\bar{C}$$

$$\overline{\bar{A}C+BC} = \underbrace{\overline{\bar{A}C}}_{(\bar{\bar{A}}+\bar{C})}\cdot\underbrace{\overline{BC}}_{(\bar{B}+\bar{C})}$$

$$\Rightarrow (A+\bar{C})\cdot(\bar{B}+\bar{C})$$

$$= \bar{C} + (A\bar{B})$$

1.4 Exercise

Draw the equivalent gate logic array for the PLC ladder diagram shown in Figure 1.2.

1.4.1 Task

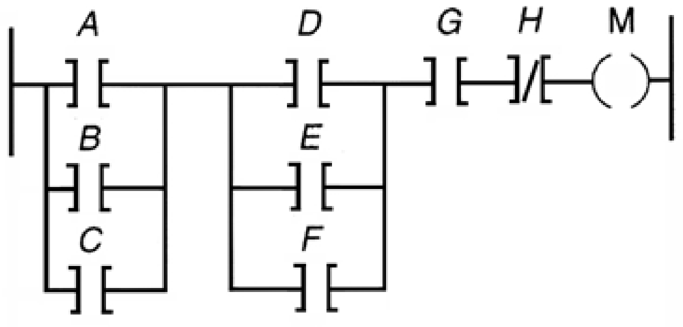
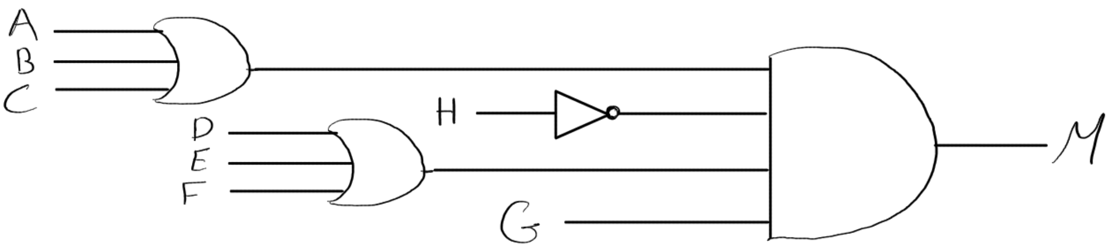


Figure 1.2. PLC ladder diagram



1.5 Exercise

Find the optimal function logic using a KV-diagram:

1.5.1 Task

independant of A

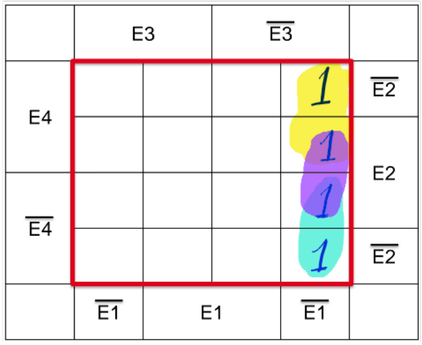
$Y = \boxed{\bar{A}\bar{B}\bar{C}} + \boxed{AB\bar{C} + \bar{A}B\bar{C}} + \boxed{\bar{A}\bar{B}C}$

Independant of C

1.5.2 Task

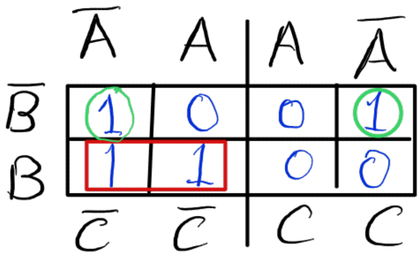
$Y = \boxed{\bar{A}\bar{B}\bar{C}\bar{D}} + \boxed{\bar{A}B\bar{C}\bar{D}} + \boxed{\bar{A}\bar{B}\bar{C}D} + \boxed{\bar{A}B\bar{C}D}$

$\bar{A}\bar{C}\bar{D} + \bar{A}\bar{C}D + \bar{A}B\bar{C}$



$\bar{A}\bar{C}D + \bar{A}B\bar{C} + \bar{A}\bar{C}\bar{D}$

$\boxed{1.5.1}$



$\Rightarrow Y = B\bar{C} + \bar{A}\bar{B}$

1.6 Exercise

A motorized overhead garage door, see Figure 1.3, is to be operated automatically to preset open and closed positions.

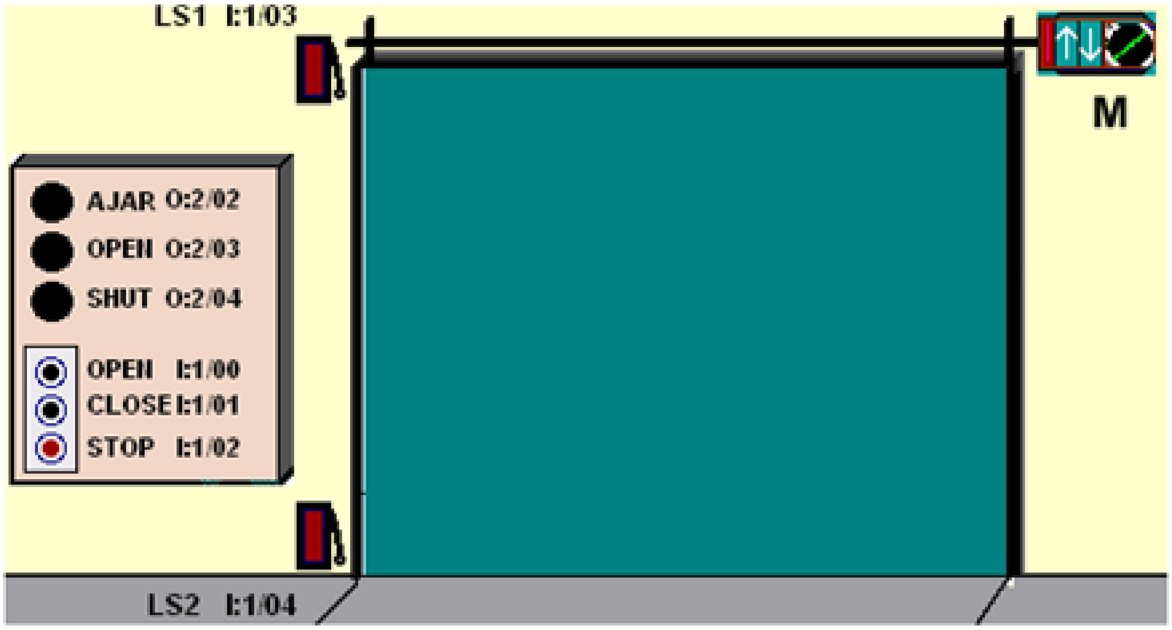


Figure 1.3. Garage door

The field devices include one of each of the following:

Information about the HW

- Reversing motor contactor for the up and down directions.
- Normally closed down limit switch to sense when the door is fully closed. 1
- Normally closed up limit switch to sense when the door is fully opened. 2
- Normally open door up button for the up direction. 3
- Normally open door down button for the down direction. 4
- Normally closed door button for stopping the door. 5
- Red door ajar light to signal when the door is partially open.
- Green door open light to signal when the door is fully open.
- Yellow door closed light to signal when the door is fully closed.

1.6.1 Task

Work out a LAD PLC program.

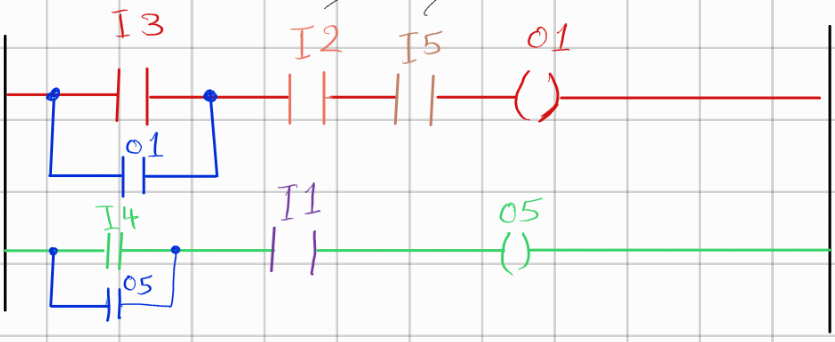
The sequence of operation requires that:

- When the up button is pushed, the up motor contactor energizes and the door travels upwards until the up limit switch is actuated.
- When the down button is pushed, the down motor contactor energizes and the door travels down until the down limit switch is actuated.
- When the stop button is pushed, the motor stops. Work out a LAD PLC program.

Why open contactor? check the info in the question about HW

Inputs:

- I1 : down limit sw
- I2 : up // //
- I3 : open button
- I4 : down //
- I5 : Stop button



outputs:

- 1: Motor up
- 2: red
- 3: yellow
- 4: green
- 5: Motor down

1.7 Exercise

Two conveyer belts (conveyer belt 1 and 2) transport material into a box, see Figure 1.4. Conveyer belt 1 is controlled through a motor M1 and conveyer belt 2 is controlled through a motor M2. The conveyer belt 1 is activated by a N/O push button 1 and the second belt is activated by a N/O push button 2. Two N/C push buttons (Stop1 and Stop2) guarantee that both belts stop, as soon as at least one stop button is pressed.

Due to safety reasons it has to be guaranteed, that only one conveyer belt can be activated. That means, if belt 1 is already active, belt 2 cannot be started and if belt 2 is already active, belt 1 cannot be started.

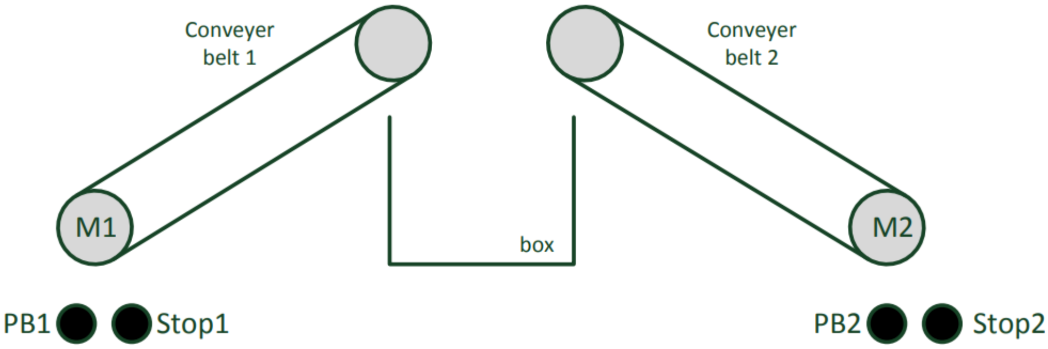


Figure 1.4. Conveyer belt arrangement

1.7.1 Task

The control of the conveyer belt arrangement should be done via PLC. Develop and create the logic using a Ladder Diagram (LAD).

(Exam question; 12 points == 12 min)

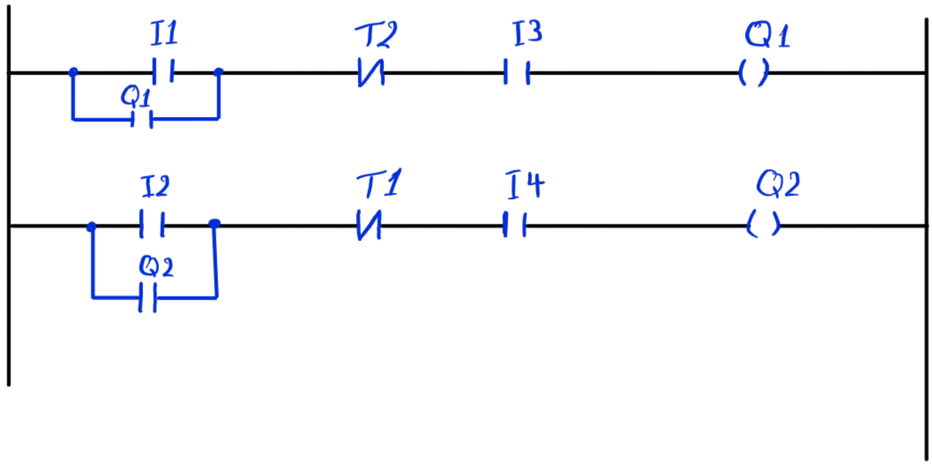
Inputs:

I1: N/O push button 1

I2: " " " 2

I3: N/C stop 1

I4: " " 2



outputs:

Q1: Motor 1

Q2: Motor 2

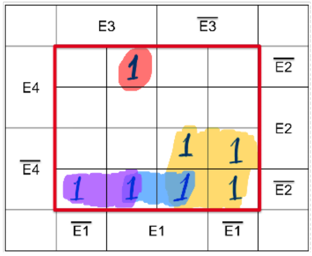
1.8 Exercise

The output setting of a logical system is realized by four inputs A, B, C, D. The logic can be described through the Boolean equation:
$Y = A\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}C\bar{D} + AB\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}\bar{D} + A\bar{B}C\bar{D} + \bar{A}B\bar{C}\bar{D} + A\bar{B}CD$

1.8.1 Task

$A\bar{B}CD + \bar{C}\bar{D} + A\bar{B}\bar{D} + \bar{B}C\bar{D}$

Find the optimized/minimum Boolean equation for the Boolean equation given above.



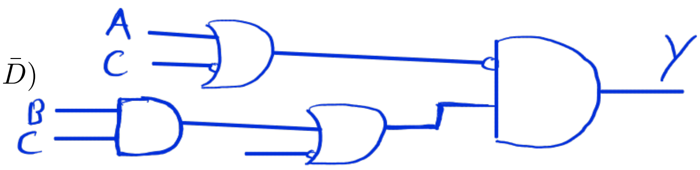
1.8.2 Task

Sketch the corresponding ladder diagram for the Boolean equation given above.

1.8.3 Task

Develop a logic gate circuit from the Boolean expression:

$Y = \overline{(A + \bar{C})}(BC + \bar{D})$



(Exam question from WiSe 2014/15; 14 points == 14 min)

1.9 Exercise

An automated packaging system, given in Figure 1.5, shall be programmed to work in the following way:

- A proximity switch detects cans coming off an assembly line for final packaging.
- Each package must contain 10 parts.
- When 10 cans are in the packaging station, a motor runs for 5 seconds.
- The process continues as long the start switch is pressed.

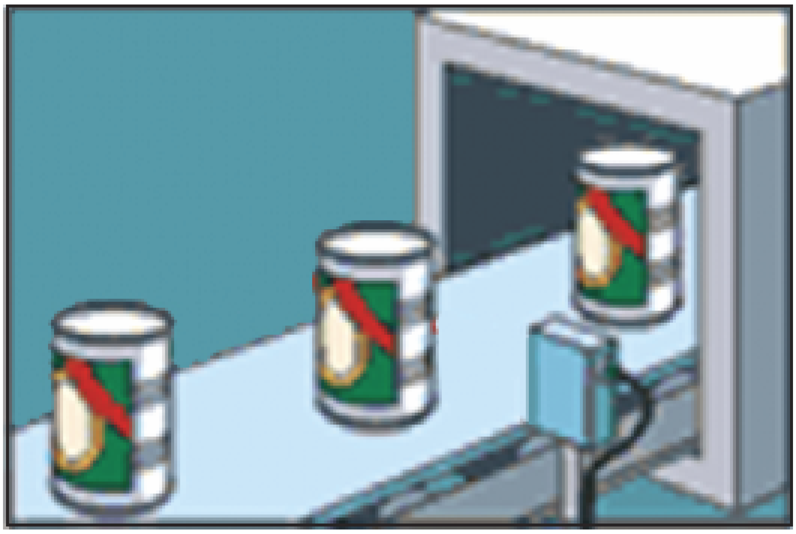
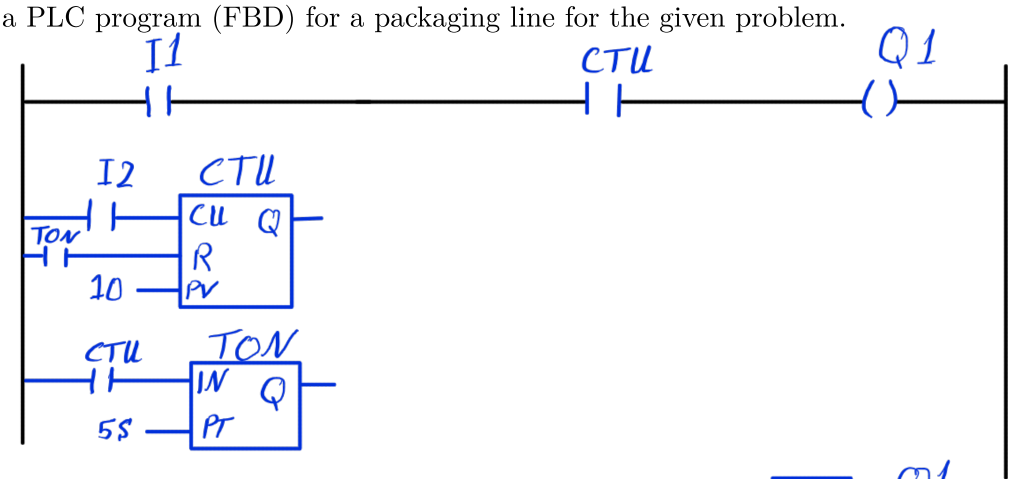


Figure 1.5. Packaging station

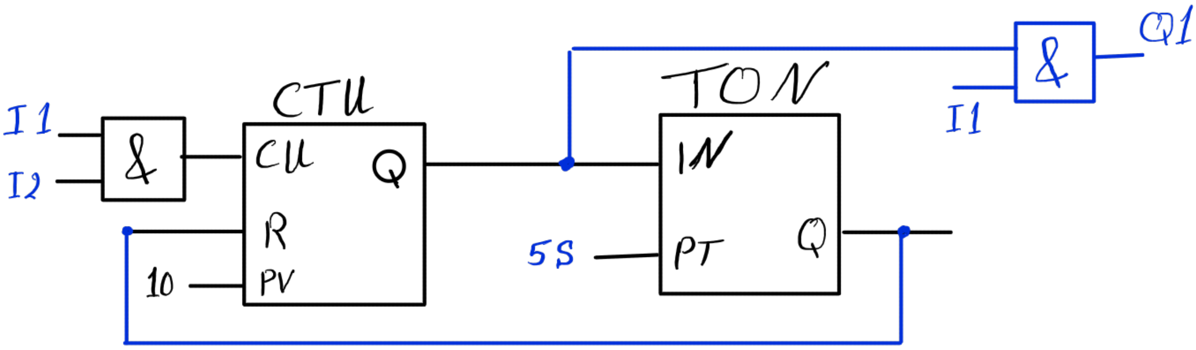
1.9.1 Task

Develop a PLC program (FBD) for a packaging line for the given problem.

Inputs:
I1: N/O start switch
I2: proximity switch



outputs:
Q1: Motor



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- A proximity switch detects cans coming off an assembly line for final packaging.
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- When 10 cans are in the packaging station, a motor runs for 5 seconds.
- The process continues as long the start switch is pressed.

Input proximity switch

10 parts/package

if 10 → motor 5s

as long as Start pressed

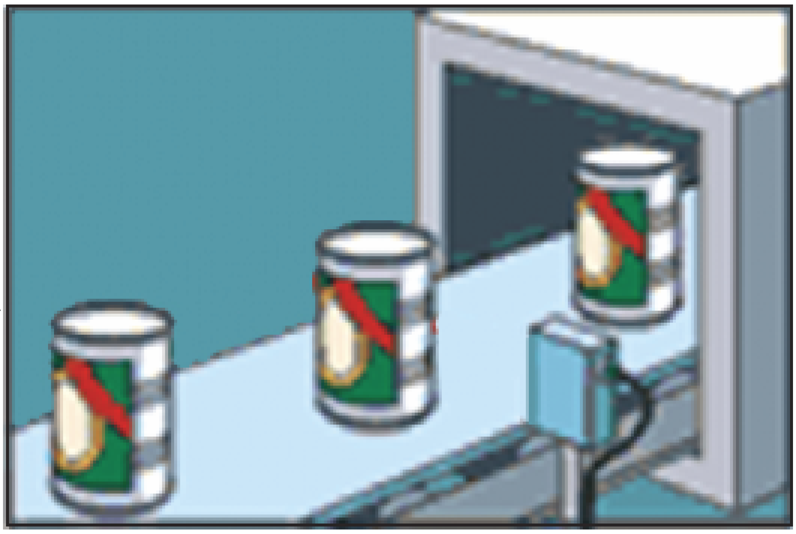

Figure 1.5. Packaging station

1.9.1 Task

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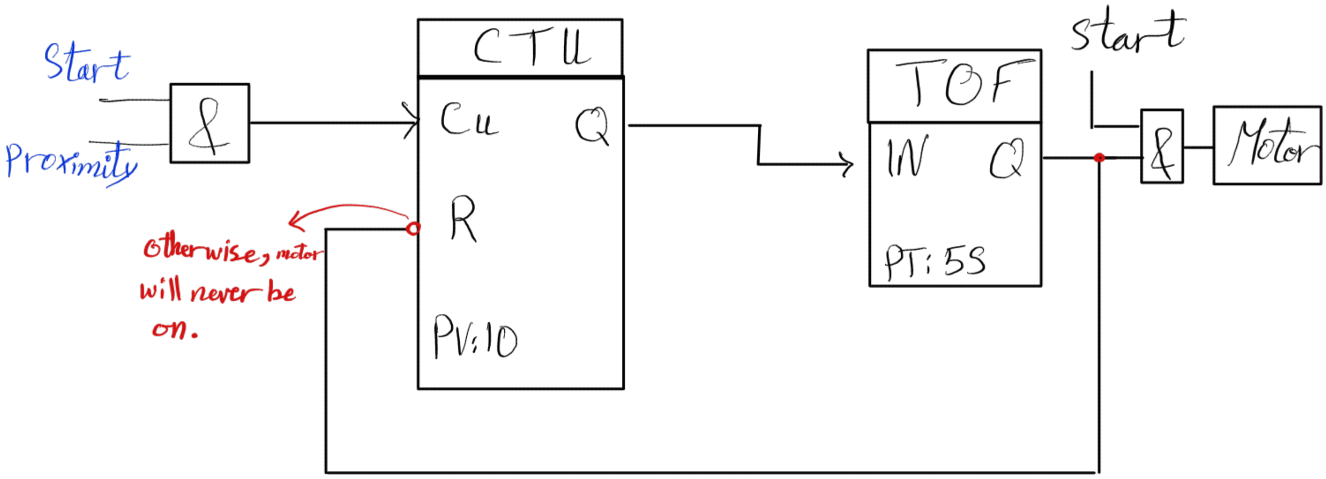


1.10 Exercise

A pump system, given in Figure 1.6, shall be programmed.

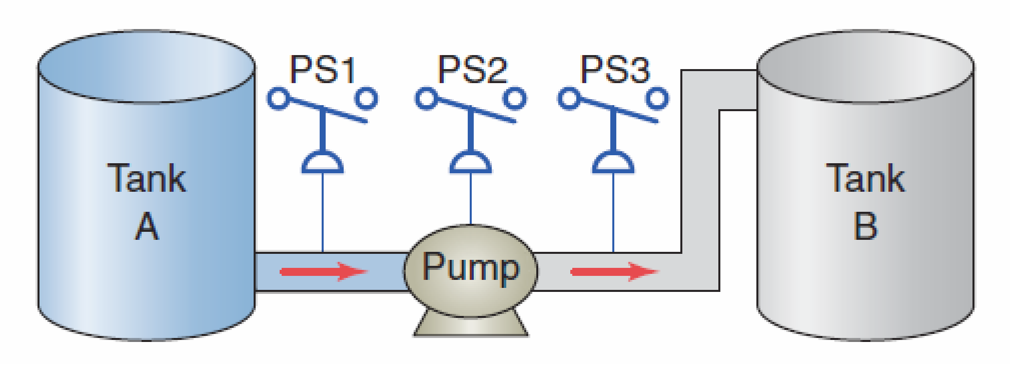


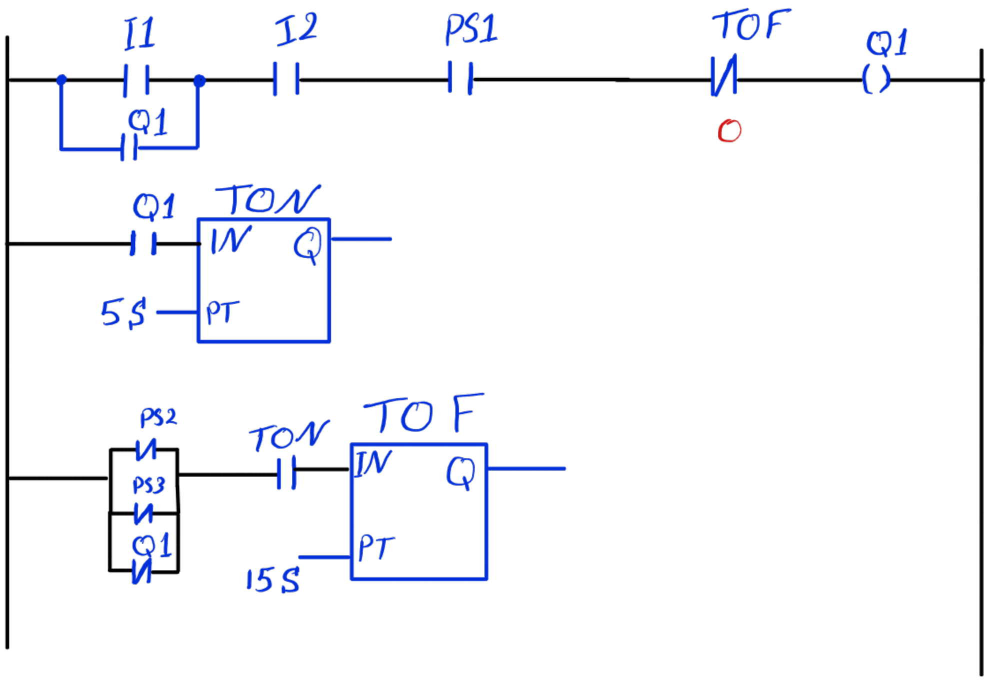
Figure 1.6. Pump setup

1.10.1 Task

Develop a PLC program for a pumping process for the following problem:

- The process involves pumping fluid from tank A to tank B.
- Before starting, PS1 must be closed.
- When the start button is pushed, the pump starts.
- PS2 and PS3 must be closed within 5s after the pump starts. If either PS2 or PS3 opens, the pump will shut off and will not be able to start again for another 15s.
- An emergency NC stop button should immediately stop the pump.

Inputs:
I1: N/O start button
I2: N/C stop button
PS1: N/O switch
PS2: " "
PS3: " "



output:
Q1: pump

1.10 Exercise

A pump system, given in Figure 1.6, shall be programmed.

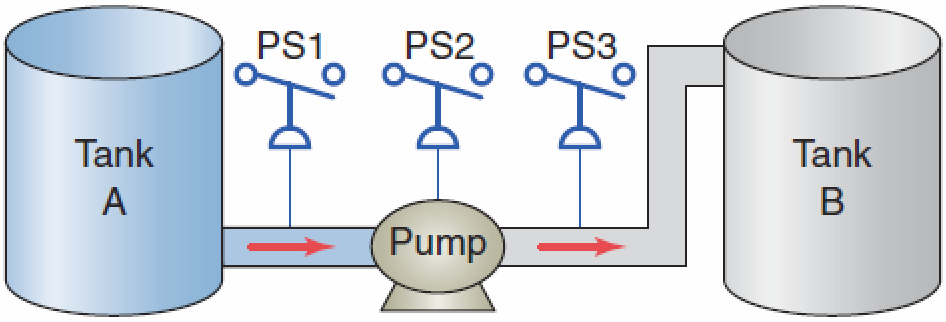


Figure 1.6. Pump setup

1.10.1 Task

Develop a PLC program for a pumping process for the following problem:

- The process involves pumping fluid from tank A to tank B.
- Before starting, PS1 must be closed.
- When the start button is pushed, the pump starts.
- PS2 and PS3 must be closed within 5s after the pump starts. If either PS2 or PS3 opens, the pump will shut off and will not be able to start again for another 15s.
- An emergency NC stop button should immediately stop the pump.

IN

Start

I0.0 emergancy NC

PS1
PS2
PS3 } switchs

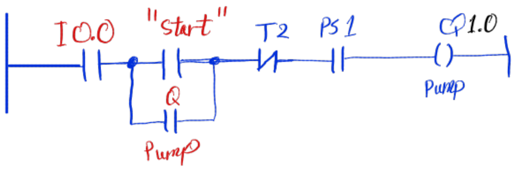
1x TON

1x TOF

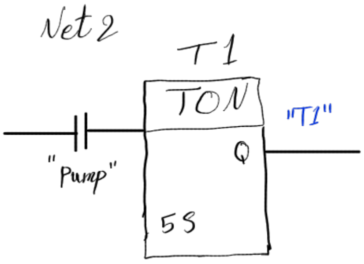
Net 1

Out

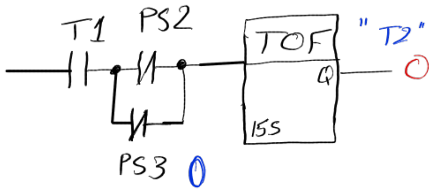
Pump



Net 2



Net 3



PS2
P3 } closed by hand

2 State-Space-Control

2.1 Exercise

An electrial network is given in Figure 2.1.

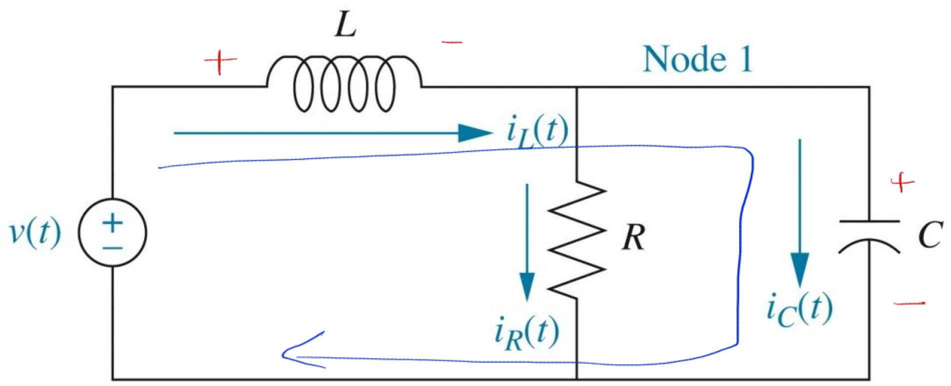


Figure 2.1. Electrical network for representation in state space

2.1.1 Task

Find a state-space representation if the output is the current through the resistor.

2.1.2 Solution

Please refer to ??.

1. State variable by use use of derivative equation for energy storage.

Capacitor: $C\frac{dv_c}{dt} = i_c$ (1)

inductor: $L\frac{di_L}{dt} = V_L$ (2)

2. Karchoff "Nod 1"

$i_c = -i_R + i_L$

$= -\frac{1}{R}V_c + i_L$ (3)

outer Loop:

$V(t) = V_L + V_c$

$V_L = -V_c + v(t)$ (4)

Substitude (3) (4) in (1) (2)

$C\frac{dV_c}{dt} = -\frac{1}{R}V_c + i_L$

$L\frac{di_L}{dt} = -V_c + v(t)$

$\frac{dV_c}{dt} = -\frac{1}{RC}V_c + i_L$

$\frac{di_c}{dt} = -\frac{1}{L}V_c + \frac{1}{L}v(t)$

$\boxed{2.1}$

4. Output function:

$$i_R = \frac{1}{R} V_c$$

$$\begin{bmatrix} \dot{V}_c \\ \dot{i}_L \end{bmatrix} = \begin{bmatrix} -\frac{1}{RC} & 1 \\ -\frac{1}{L} & 0 \end{bmatrix} \begin{bmatrix} V_c \\ i_L \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} V(t)$$

$$i_R = \begin{bmatrix} \frac{1}{R} & 0 \end{bmatrix} \begin{bmatrix} V_c \\ i_L \end{bmatrix}$$

2.2 Exercise

A mechanical system is given in Figure 2.2.

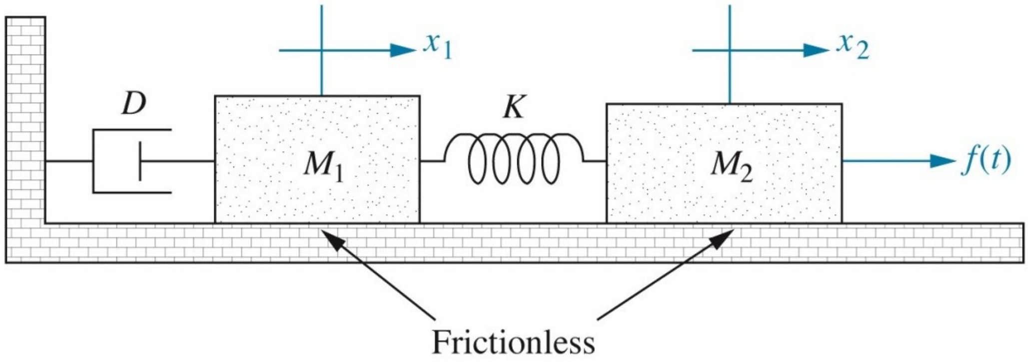


Figure 3.7

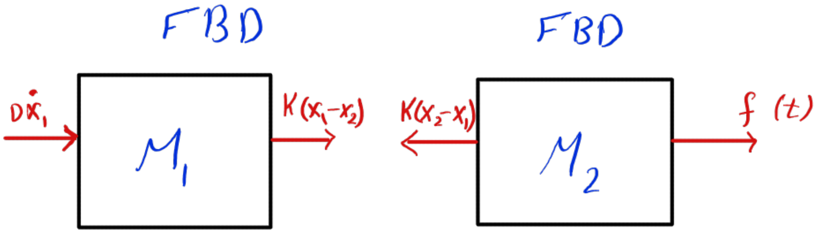
Figure 2.2. Translational mechanical System

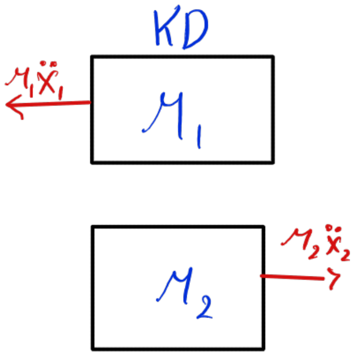
2.2.1 Task

Find the state equations for the translational mechanical system. The output is considered as the position of the mass M_1.

2.2.2 Solution

Please refer to ??.





$$M_1\ddot{x}_1 = -D\dot{x}_1 - K(x_1 - x_2)$$

$$M_2\ddot{x}_2 = f(t) - K(x_2 - x_1)$$

$$\dot{v}_1 = \ddot{x}_1 \qquad \dot{v}_2 = \ddot{x}_2 \quad \Bigg| \quad \left.\begin{matrix} x_1 \\ v_1 \\ x_2 \\ v_2 \end{matrix}\right\} \text{4 state variables} ,$$

$$\dot{x}_1 = v_1$$

$$\dot{v}_1 = -\frac{D}{M_1} v_1 - \frac{K}{M_1} x_1 + \frac{K}{M_1} x_2$$

$$\dot{x}_2 = v_2$$

$$\dot{v}_2 = \frac{K}{M_2} x_1 - \frac{K}{M_2} x_2 + \frac{1}{M_2} f(t)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{v}_1 \\ \dot{x}_2 \\ \dot{v}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{K}{M_1} & -\frac{D}{M_1} & \frac{K}{M_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{K}{M_2} & 0 & -\frac{K}{M_2} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ v_1 \\ x_2 \\ v_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{M_2} \end{bmatrix} \cdot f(t)$$

2.3 Exercise

A control system is defined by the transfer function:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{\overset{b_2}{2}s^2 + \overset{b_1}{8}s + \overset{b_0}{6}}{s^3 + \underset{a_2}{8}s^2 + \underset{a_1}{16}s + \underset{a_0}{6}} \tag{2.1}$$

2.3.1 Task

Convert the system model from the transfer function to a state space model and derive the block diagram.

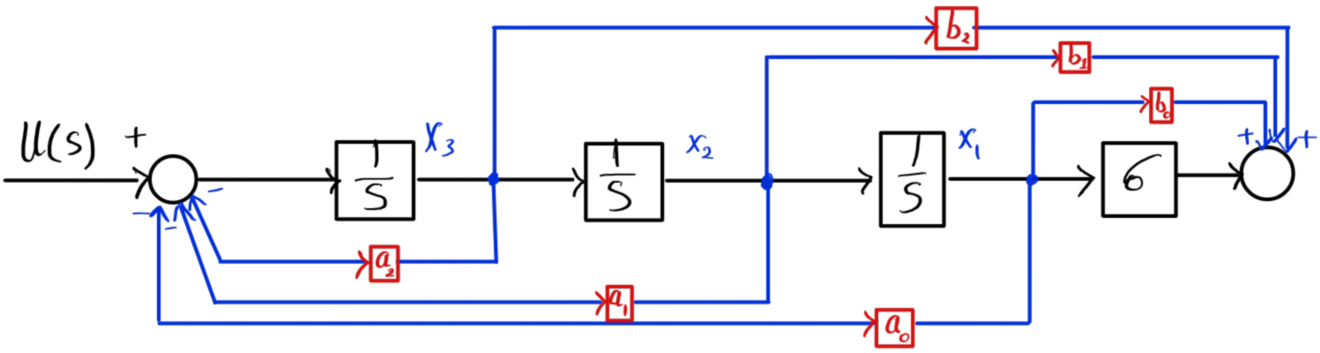
2.3.2 Solution

Please refer to ??.

in canonical form:

$$\underline{\dot{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -16 & -8 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \cdot u$$

$$y(t) = \begin{bmatrix} 6 & 8 & 2 \end{bmatrix} \cdot \underline{x} + \begin{bmatrix} 0 \end{bmatrix} \cdot u$$



2.4 Exercise

A State- and Output-Equation is given as follows::

$$\dot{x} = \underbrace{\begin{bmatrix} -4 & -1.5 \\ 4 & 0 \end{bmatrix}}_{\underline{\underline{A}}} x + \underbrace{\begin{bmatrix} 2 \\ 0 \end{bmatrix}}_{\underline{B}} u(t) \tag{2.2}$$

$$y = \underbrace{\begin{bmatrix} 1.5 & 0.625 \end{bmatrix}}_{\underline{C}} x + \underbrace{0}_{\underline{D}} \cdot u \tag{2.3}$$

2.4.1 Task

Convert the state and output equation to a transfer function.

2.4.2 Solution

Please refer to ??.

$$\boxed{G(s) = \frac{Y(s)}{U(s)} = \underline{C}\,\underline{\underline{\Phi}}(s)\cdot\underline{B} + \underline{D}}$$

$$\underline{\underline{\Phi}}(s) = \left[s\underline{\underline{I}} - \underline{\underline{A}}\right]^{-1} = \frac{\operatorname{adj}(s\underline{\underline{I}}-A)}{\det(s\underline{\underline{I}}-A)}$$

$$\operatorname{adj}(s\underline{\underline{I}}-A) = \begin{bmatrix} s & -1.5 \\ 4 & s+4 \end{bmatrix}$$

$$\det(s\underline{\underline{I}}-\underline{\underline{A}}) = \det\begin{pmatrix} s+4 & 1.5 \\ -4 & s \end{pmatrix} = s(s+4)+6 = s^2+4s+6$$

$$\underline{\underline{\Phi}}(s) = \frac{1}{s^2+4s+6}\begin{bmatrix} s & -1.5 \\ 4 & s+4 \end{bmatrix}$$

$$\underline{C}\,\underline{\underline{\Phi}}(s) = \frac{1}{s^2+4s+6}\begin{bmatrix} \frac{3}{2}s+\frac{5}{2} & -\cancel{\frac{9}{4}}+\frac{5}{8}s+\cancel{\frac{5}{2}}\ {+\frac{1}{4}} \end{bmatrix}$$

$$\underline{C}\,\underline{\underline{\Phi}}(s)\cdot\underline{B} = \frac{1}{\nabla}\begin{bmatrix} \frac{3}{2}s+\frac{5}{2} & \frac{5}{8}s+\frac{1}{4} \end{bmatrix}\cdot\begin{bmatrix} 2 \\ 0 \end{bmatrix} = \frac{1}{\nabla}\begin{bmatrix} 3s+5 \end{bmatrix}$$

$$G(s) = \underline{C}\,\underline{\underline{\Phi}}(s)\cdot\underline{B}+\underline{D} = \boxed{\frac{3s+5}{s^2+4s+6}}$$

2.5 Exercise

We find a matrix given in Equation 2.4.

$$\underline{\underline{A}} = \begin{bmatrix} 0 & 6 \\ -1 & -5 \end{bmatrix} \tag{2.4}$$

2.5.1 Task

Determine the State-Transition-Matrix $\underline{\underline{\Phi}}(s)$.

2.5.2 Solution

Please refer to **??**.

2.6 Exercise

A system in State-Space form is given in Equation 2.5.

$$\underline{\underline{A}} = \begin{bmatrix} 0 & 1 \\ -12 & 7 \end{bmatrix}, \underline{\underline{B}} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \underline{\underline{C}} = \begin{bmatrix} -10 & 4 \end{bmatrix} \tag{2.5}$$

2.6.1 Task

Calculate the poles of the system and dertermine whether the system is stable or unstable.

2.6.2 Solution

Please refer to ??.

we only need to check the dominator

$$\det(s\underline{\underline{I}} - A) = \det\begin{pmatrix} s & -1 \\ 12 & s-7 \end{pmatrix} = s(s-7) + 12$$

$$= s^2 - 7s + 12 = (s-3)(s-4)$$

Poles at $s=3$ & $s=4$

Poles are located at the RHP of s-domain

Thus the system is not stable

2.7 Exercise

A sytem in State-Space form is given in Equation 2.6.

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 4 \\ a & -2 \end{bmatrix}, \underline{\underline{B}} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \underline{\underline{C}} = \begin{bmatrix} 1 & 0 \end{bmatrix} \tag{2.6}$$

2.7.1 Task

Determine the condition for a , so that the given system is stable.

$$(s\underline{\underline{I}} - \underline{\underline{A}}) = \begin{bmatrix} s-1 & -4 \\ -a & s+2 \end{bmatrix}$$

2.7.2 Solution

Please refer to ??.

$$\det(s\underline{\underline{I}} - \underline{\underline{A}}) = (s-1)(s+2) - 4a$$

$$= s^2 + s - 2 - 4a$$

$$= s^2 + s - (2+4a)$$

$$a < -\frac{9}{16}$$

s^2 1 $-2-4a$

s^1 1 0

s^0 $b_1 = \dfrac{-\begin{vmatrix} 1 & -2-4a \\ 1 & 0 \end{vmatrix}}{1}$

$= -2-4a$

$$-2-4a > 0$$

$$-4a > 2$$

$$\boxed{a < -\frac{1}{2}}$$

$$p_{1,2} = -\frac{1}{2} \pm \sqrt{\frac{1}{4} + (2+4a)} = -\frac{1}{2} \pm \sqrt{\frac{9}{4} + 4a}$$

$$\sqrt{\frac{9}{4} + 4a} < \frac{1}{2}$$

2.8 Exercise

The state equation for a system is given in Equation 2.7.

$$\dot{x} = Ax + Bu = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} u \tag{2.7}$$

2.8.1 Task

Determine whether the system is controlable or not.

2.8.2 Solution

Please refer to ??.

$$C_M = [B \quad AB \quad A^2B]$$

$$A^2 = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$AB = \underbrace{\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}}_{3\times3} \underbrace{\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}}_{3\times1} = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}$$

$$A^2B = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 4 \end{bmatrix}$$

$$C_M = \begin{bmatrix} 0 & 1 & -2 \\ 1 & -1 & 1 \\ 1 & -2 & 4 \end{bmatrix}$$

$$\det(C_M) = -1 \cdot \begin{vmatrix} 1 & -2 \\ -2 & 4 \end{vmatrix} + 1 \cdot \begin{vmatrix} 1 & -2 \\ -1 & 1 \end{vmatrix}$$

$$= -(\overset{0}{\cancel{4-4}}) + (\overset{-1}{\cancel{1-2}})$$

$$= -1 \neq 0$$

Thus, the system is controllable!

2.9 Exercise

A system is represented by Equation 2.8 and Equation 2.9.

$$\dot{x_1} = -2x_1 + u \tag{2.8}$$

$$\dot{x_2} = -3x_2 + dx_1 \tag{2.9}$$

2.9.1 Task

Determine the condition ($d =$?) for controllability.

2.9.2 Solution

Please refer to ??.

$$\dot{\underline{x}} = \begin{bmatrix} -2 & 0 \\ d & -3 \end{bmatrix} \underline{x} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot u$$

$$C_M = \begin{bmatrix} B & AB \end{bmatrix}$$

$$AB = \begin{bmatrix} -2 & 0 \\ d & -3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ d \end{bmatrix}$$

$$C_M = \begin{bmatrix} 1 & -2 \\ 0 & d \end{bmatrix} \qquad \det(C_M) = d$$

The system is controllable when $d \neq 0$

2.10 Exercise

We find an unstable system, characterized through Equation 2.10.

$$\underline{\dot{x}} = \begin{bmatrix} 4 & 8 \\ 1 & -5 \end{bmatrix} \underline{x} \begin{bmatrix} 1 \\ 0 \end{bmatrix} u \tag{2.10}$$

The system is controlled by a feedback vector given in Equation 2.11.

$$\underline{\underline{K}} = \begin{bmatrix} 1 & k_2 \end{bmatrix} \tag{2.11}$$

2.10.1 Task

Calculate k_2 so that the controlled system is stable.

2.10.2 Solution

Please refer to **??**.

$$\det(s\underline{\underline{I}} - (\underline{\underline{A}} - \underline{\underline{B}}K))$$

2.11 Exercise

A third order system is given in Equation 2.12.

$$\dddot{y} + 5\ddot{y} + 3\dot{y} + 2y = u \tag{2.12}$$

$$Y(s)\left[s^3+5s^2+3x+2\right] = U(s)$$

2.11.1 Task

Design a state feedback controller so that the closed-loop system poles are located at $p_{1,2} = -4 \pm 3j$ and $p_3 = -4$.

2.11.2 Solution

Please refer to ??

$$G(s) = \frac{Y(s)}{U(s)} = \frac{1}{s^3+5s^2+3s+2}$$

Cononical form

step 1)

$$\underline{\dot{x}} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -3 & -5 \end{bmatrix}}_{\underline{\underline{A}}} \cdot \underline{x} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{\underline{B}} \cdot u$$

$$y = \underbrace{\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}}_{\underline{C}} \cdot \underline{x} + [0] \cdot u$$

step 2) $\underline{K} = [K_1 \quad K_2 \quad K_3]$

step 3) $[\underline{\underline{A}} - \underline{B}\,\underline{K}] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -(K_1+2) & -(K_2+3) & -(K_3+5) \end{bmatrix}$

step 4) $(s+4)(s+4-3j)(s+4+3j) = (s+4)(s^2+8s+25)$

$$= s^3 + \underbrace{12}_{d_2} s^2 + \underbrace{57}_{d_1} s + \underbrace{100}_{d_0}$$

step 5) $d_i = a_i + K_{i+1}$

$$K_{i+1} = d_i - a_i \Rightarrow K_1 = 100 - 2 = 98$$

$$K_2 = 57 - 3 = 54$$

$$K_3 = 12 - 5 = 7$$

$\therefore K = [98 \quad 54 \quad 7]$

3 Time-Discrete Systems

3.1 Exercise

We find a z.o.h in cascade with $G_1(s) = \frac{s+2}{s+1}$ or

$$G(s) = \frac{1 - e^{-Ts}}{s} \frac{(s + 2)}{(s + 1)} \tag{3.1}$$

3.1.1 Task

Find the sampled-data transfer function, $G(z)$, if the sampling time T is 0.5 second.

3.1.2 Solution

Please refer to ??.

$$G(z) = (1 - z^{-1}) \cdot \mathcal{Z}\left\{\frac{2}{s} - \frac{1}{s+1}\right\} = \left(\frac{z-1}{z}\right)\left(2 \cdot \frac{z}{z-1} - \frac{z}{z-e^{-T}}\right)$$

$$G(z) = \left(2 - \frac{z-1}{z-e^{-T}}\right) = \frac{2(z-e^{-T}) - z - 1}{z - e^{-T}} = \frac{z - e^{-T} - 1}{z - e^{-T}}$$

at $T = 0.5\,s \Rightarrow \boxed{G(z) = \frac{z - 1.6065}{z - 0.6065}}$

3.2 Exercise

$\tilde{G}(s) \rightarrow \tilde{G}(z$

A sampled-data system is given in Figure 3.1.

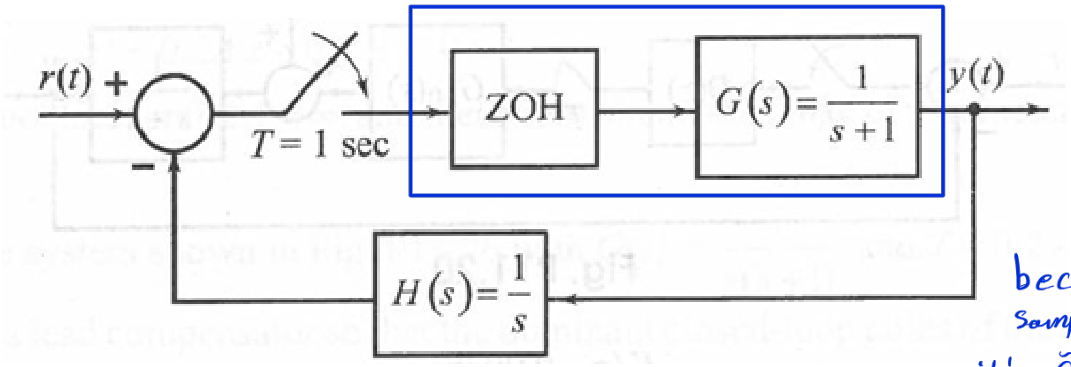


Figure 3.1. sampled-data system

because there is no sampler in between it's $\tilde{G}H(z)$

if there's a sampler $G_{cl}(z) = \frac{\tilde{G}(z)}{1+\tilde{G}(z)\cdot H(z}$

3.2.1 Task

Find out if the closed-loop system $G_{Cl}(z) = \frac{Y(z)}{R(z)}$ is stable or unstable and explain your result.

3.2.2 Solution

Please refer to ??.

$$G_{cl}(z) = \frac{\tilde{G}(z)}{1+\tilde{G}H(z)}$$

$$\tilde{G}(s) = \frac{1-e^{sT}}{s} \cdot \frac{1}{s+1}$$

$$\tilde{G}(z) = \mathcal{Z}\left\{\frac{1-e^{sT}}{s} \cdot \frac{1}{s+1}\right\} = (1-z^{-1})\,\mathcal{Z}\left\{\frac{1}{s} - \frac{1}{s+1}\right\} = \frac{1-e^{-T}}{z-e^{-T}}$$

$$\tilde{G}(z) = \frac{0.63}{z-0.37}$$

$$\tilde{G}H(z) = \mathcal{Z}\left\{\frac{1-s^{sT}}{s} \cdot \frac{1}{s+1} \cdot \frac{1}{s}\right\} = (1-z^{-1})\,\mathcal{Z}\left\{\frac{1}{s^2} - \frac{1}{s} + \frac{1}{s+1}\right\}$$

$$= \frac{z-1}{z}\left(\frac{Tz}{(z-1)^2} - \frac{z}{z-1} + \frac{z}{z-e^{-T}}\right) = \left(\frac{T}{z-1} - 1 + \frac{z-1}{z-e^{-T}}\right)$$

$$\tilde{G}H(z) = \frac{0.37z+0.26}{z^2-1.37z+0.37}$$

$$G_{cl}(z) = \frac{0.63(z-1)}{z^2-z+0.63}$$

stable

3.3 Exercise

A sampled-data system is given in Figure 3.2. $G(s)$

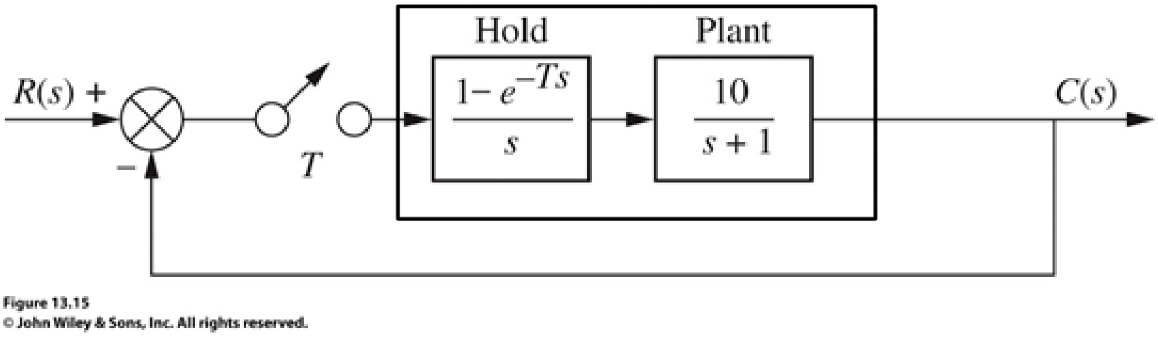


Figure 3.2. Diagram system

3.3.1 Task

Determine the range of sampling interval, T , which makes the system shown in Figure 3.2 stabel, and furthermore give the range that will lead to an unstable system.

3.3.2 Solution

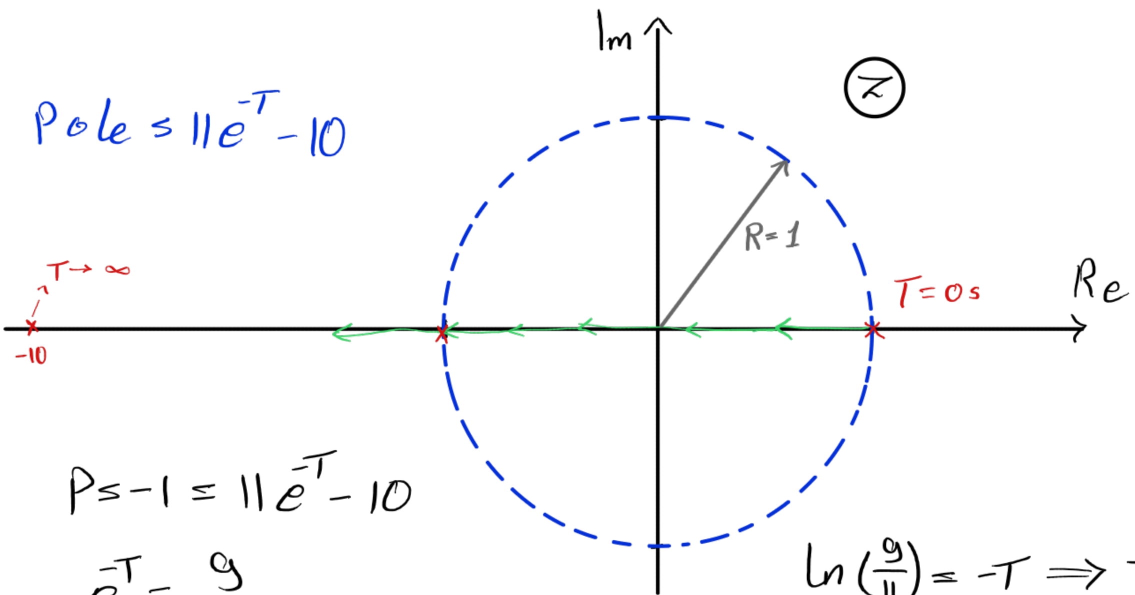
Please refer to ??.

$$G(z) = \mathcal{Z}\left\{\frac{1-e^{-Ts}}{s}\cdot\frac{10}{s+1}\right\} = \left(\frac{z-1}{z}\right)\mathcal{Z}\left\{\frac{1}{10s}-\frac{1}{10(s+1)}\right\}$$

$$G(z) = 10\cdot\frac{1-e^{-T}}{z-e^{-T}}$$

$$G_{cl}(z) = \frac{10(1-e^{-T})}{z-e^{-T}+10-10e^{-T}} = \frac{num}{z-11e^{-T}+10}$$

$$Pole = 11e^{-T}-10$$



$$P = -1 = 11e^{-T}-10$$

$$e^{-T} = \frac{9}{11}$$

$$\ln\left(\frac{9}{11}\right) = -T \Rightarrow T = -\ln\left(\frac{9}{11}\right) = 0.2\,s$$

$$\therefore \boxed{0 < T < 0.2}$$