Lagrange Polynomial Interponials: Creates intepolation function using Polynomials!

Lagrange Polynomials: $\boxed{\sum_{i=0}^{n}\left[y_i \prod_{\substack{j=0\\ j\neq i}}^{n} \frac{x-x_j}{x_i-x_j} \right]}$ where n = node (num. of datapoints) -1

$$\Rightarrow = y_0 \cdot \frac{x-x_1}{x_0-x_1} \cdot \frac{x-x_2}{x_0-x_2} \cdot \dots \cdot \frac{x-x_n}{x_0-x_n}$$

$$+ y_1 \cdot \frac{x-x_0}{x_1-x_0} \cdot \frac{x-x_2}{x_1-x_2} \cdot \dots \cdot \frac{x-x_n}{x_1-x_n}$$

$$+ y_2 \cdot \frac{x-x_0}{x_2-x_0} \cdot \frac{x-x_1}{x_2-x_1} \cdot \frac{x-x_3}{x_2-x_3} \cdot \dots \cdot \frac{x-x_n}{x_2-x_n} + \dots$$

$$+ y_n \cdot \frac{x-x_0}{x_n-x_0} \cdot \frac{x-x_1}{x_n-x_1} \cdot \dots \cdot \frac{x-x_{n-1}}{x_n-x_{n-1}}$$

First-Order Lagrange polynomial Example: Find the output of $x = 2.2$ for the following data set using Lagrange polynomial interpolation

$n = 2 - 1 = 1$

$\boxed{y = f(x) = 5x - 5}$

x	y
2	5
3	10

$$P(x) = \sum_{i=0}^{n=1} y_i \prod_{\substack{j=0\\ j\neq i}}^{1} \frac{x-x_j}{x_i-x_j} = y_0 \cdot \frac{x-x_1}{x_0-x_1} + y_1 \cdot \frac{x-x_0}{x_1-x_0}$$

$$P(x) = (5) \cdot \frac{x-3}{2-3} + (10) \cdot \frac{x-2}{3-2} = 10x - 20 - 5x + 15$$

$$\Rightarrow \boxed{P(x) = 5x - 5}$$

$$P(2.2) = 5(2.2) - 5 \Rightarrow \boxed{P(2.2) = 6}$$

2nd Example: Find the output of $x = 2.5$ for the following data set using Lagrange.

$n = 3 - 1 = 2$

x	y
1	4
3	8
7	10

$$P(x) = \sum_{i=0}^{n=2} y_i \prod_{\substack{j=0\\ j\neq i}}^{n=2} \frac{x-x_j}{x_i-x_j} = y_0 \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} + y_1 \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} + y_2 \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$$

$$P(x) = 4 \cdot \frac{(x-3)(x-7)}{(1-3)(1-7)} + 8 \cdot \frac{(x-1)(x-7)}{(3-1)(3-7)} + 10 \frac{(x-1)(x-3)}{(7-1)(7-3)}$$

$$= \frac{1}{3}(x^2 - 10x + 21) - (x^2 - 8x + 7) + \frac{5}{12}(x^2 - 4x + 3)$$

$$\boxed{P(x) = -\frac{1}{4}x^2 + 3x + \frac{5}{4}}$$

$$P(2.5) = 7.1875$$

Newton's Interpolation:

Interpolate the function f Knowing:

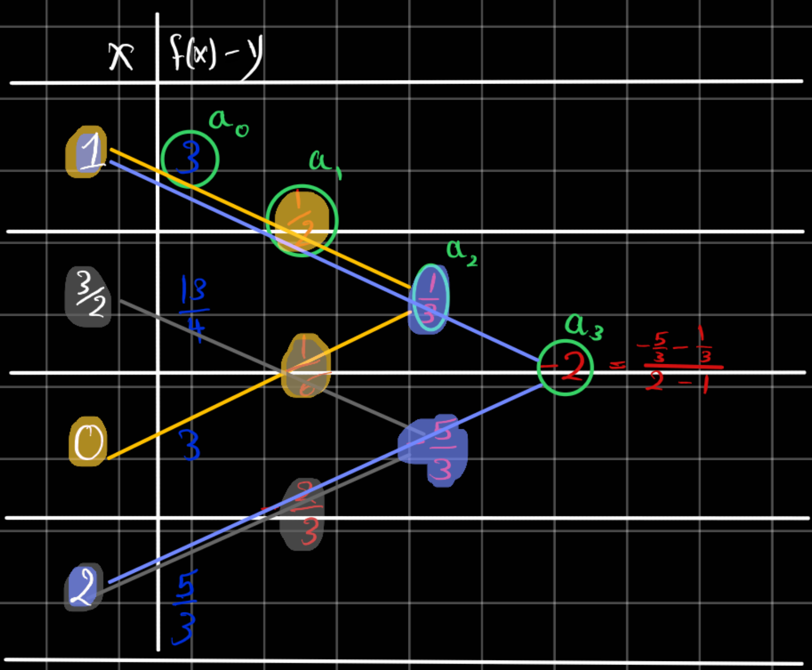
x	1	$\frac{3}{2}$	0	2
$f(x)$	3	$\frac{13}{4}$	3	$\frac{5}{3}$

$\Delta y_1 = \frac{13}{4} - 3 = \frac{1}{4}$

$\Delta x_1 = \frac{3}{2} - 1 = \frac{1}{2}$

$\frac{\Delta y_1}{\Delta x_1} = \frac{1/4}{1/2} = \frac{1}{2}$

solution



$\frac{\Delta y_3}{\Delta x_3} = \frac{\frac{5}{3} - 3}{2 - 0} = \frac{-\frac{4}{3}}{2}$

$= -\frac{2}{3}$

$\frac{\frac{1}{6} - \frac{1}{2}}{0 - 1} = \frac{2}{6} = \frac{1}{3}$

$\frac{-\frac{2}{3} - \frac{1}{6}}{2 - \frac{3}{2}} = \frac{-\frac{5}{6}}{\frac{1}{2}} = -\frac{5}{3}$

$$P(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) + a_3(x - x_0)(x - x_1)(x - x_2)$$

Horner's Algorithm

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p = a_n
  for  k = n-1 : -1 : 0
         p = a_k + x·p
  end
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n num. of order