Period of Examinations Summer Semester 2021 - July Exam

Module: Multibody Dynamics, Prof. Brandt

Examination

Points: 60

Duration of examination: 90 Minutes (+ 30 Minutes for technical issues)

Please write legibly!

Date: ______________________

Name: ______________________

Register No.: ______________________

Course of Study: ______________________

Before you turn in your solution in Moodle please sign the declaration in lieu of oath:

I, ____ [full name, matriculation number], hereby confirm in lieu of an oath that I am the person who was admitted to this examination. Further, I confirm that the submitted work is my own and was prepared without the use of any unauthorised aid or materials.

____

Signature

You can print the declaration and then sign and scan it. Alternatively, you can also sign it digitally or transcribe it by hand and then sign and scan it.

Please make sure that all documents that you upload contain your name and matriculation number.

Good luck!

Problem	Possible Points	Result
1	15	
2	5	
3	10	
4	14	
5	4	
6	4	
7	8	
Sum	**60**	

1. Preparing a Multibody System for Computer Simulation (15 points)

In Figure 1 you find a sample mechanism that consists of six bodies (1-6), five revolute joints (R1-R5) and two translational joints (T1-T2). The translational joint T1 connects the mechanism to the ground (0). Therefore, body (1) can only slide in horizontal direction and not move vertically. Revolute joint R5 connects body (6) to the ground.

Between body (3) and body (6) an actuator with actuator force ${}^{(a)}f$ is applied. The actuator is placed in a way that the line of action of the actuator force always passes through the centers of gravity of bodies (3) and (6).

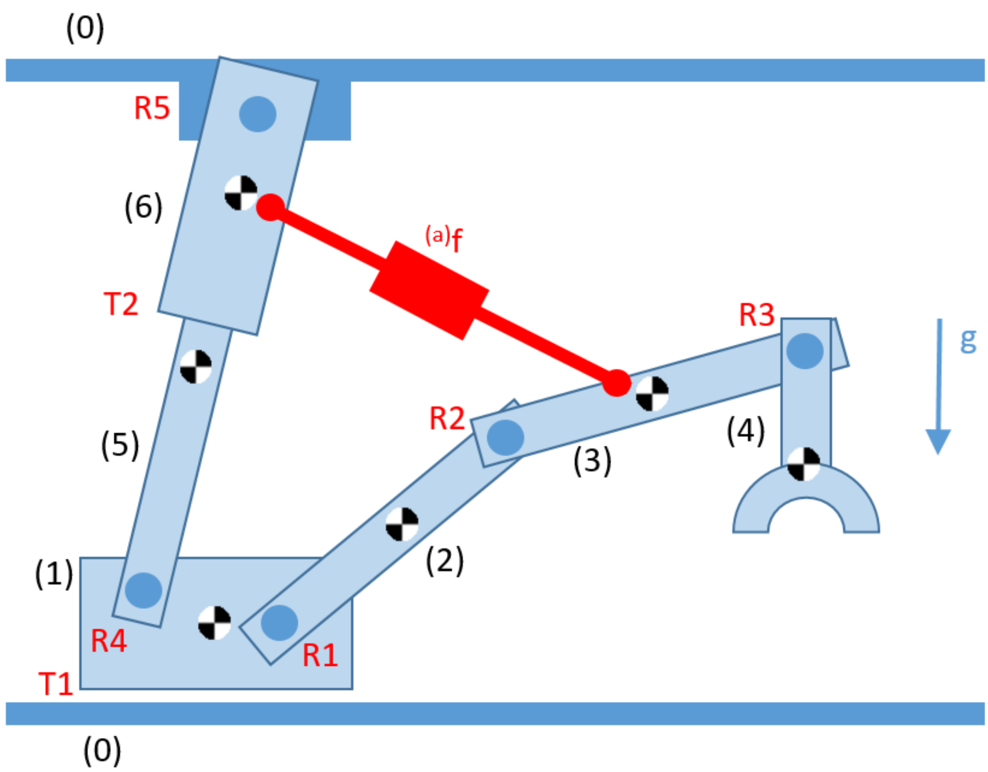


Figure 1: Multibody mechanism

The following parameter are known:

Masses	m_1,m_2,m_3, m_4, m_5, and m_6
Moments of Inertia	J_1, J_2, J_3, J_4, J_5, J_6, and J_6
Gravitational acceleration	g
Actuator force	${}^{(a)}f$

To prepare the mechanism for computer simulation based on a body-coordinate formulation

a) Define reference frames in terms of body-coordinates (use the enlarged figure of the mechanism on the extra sheet), **(5 points)**

b) Define the necessary points and vectors to formulate the constraints of the system in body-coordinate formulation and to integrate the actuator force into the equations of motion (use the enlarged figure of the mechanism on the extra sheet). **(10 points)**

2. Describing technical joints (5 points)

Depending on the last digit of your matriculation number, formulate the constraints of the following joints in Figure 1 on position level.

Last digit of matriculation number	Joints
0	R1, T2
1	R2, T2
2	R3, T2
3	R4, T2
4	R5, T2
5	R1, T1
6	R2, T1
7	R3, T1
8	R4, T1
9	R5, T1

R_1:

Geometrically:

$$\underline{r}_1 + \underline{s}_1^{R_1} - \underline{r}_2 - \underline{s}_2^{R_1} = \underline{0}$$

In global coordination:

$$\underline{r}_1 + \underline{\underline{A}}_1 \cdot \underline{\$}_1^{R_1} - \underline{r}_2 - \underline{\underline{A}}_2 \underline{\$}_2^{R_1}$$

T_2:

$$\underline{u}_6^{UT} \cdot \underline{d} = 0$$

$$\phi_5 - \phi_6 - \phi_c = 0$$

R_3: Geometrically:

$$\underline{r}_3 + \underline{s}_3^{R_3} - \underline{r}_4 - \underline{s}_4^{R_3} = \underline{0}$$

In global coordination

$$\underline{r}_3 + \underline{\underline{A}}_3 \underline{\$}_3^{R_3} - \underline{r}_4 - \underline{\underline{A}}_4 \underline{\$}_4^{R_3} = 0$$

T_1:

$$\underline{u}_0^{UT} \cdot \underline{d} = 0$$

$$\phi_1 - \phi_0 - \phi_c = 0$$

3. Degrees of freedom (10 points)

a) How many constraint equations have to be formulated for the mechanism in Figure 1 using the body-coordinate formulation? **(2 points)**

$2 * 7 = 14$

b) How many degrees of freedom (d.o.f.) has the mechanism in Figure 1? **(2 points)**

Dof = 3 * num. of bodies − 2 * num. of joints = $3*6 - 2*7 = 18 - 14 = 4$

c) How many driver constraints are necessary for kinematic analysis of the mechanism in Figure 1? **(2 points)**

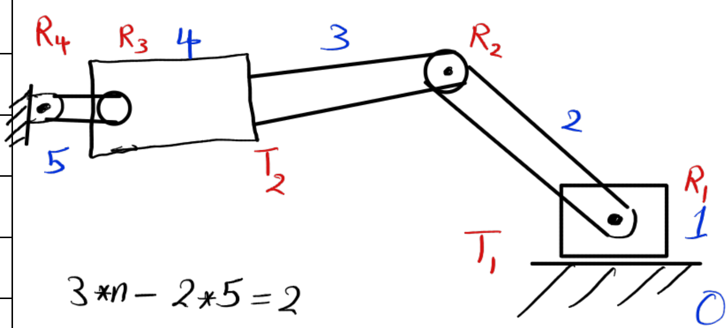
Diver constrains = Dof = 4

d) Draw a mechanism including at least one revolute and one translational joint. The mechanisam should have the following number of degrees of freedom according to your matriculation number. **(4 points)**

Last digit of matriculation number	Number of degrees of freedom (d.o.f.)
1	4
2	3
3	2
4	1
5	4
6	3
7	2
8	1
9	4
0	3

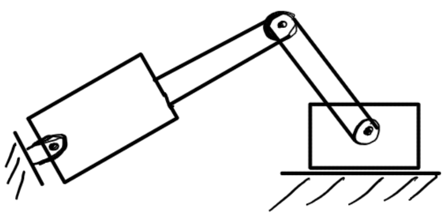
$3*n - 2*6 = 3$

$n = 5$



$3*n - 2*5 = 2$

$n = 4$



4. Dynamics (14 points)

The equations of motion containing the constraint forces have the form

$$\mathbf{M}\ddot{\boldsymbol{c}} = \boldsymbol{h} + \boldsymbol{D}'\boldsymbol{\lambda}$$

For the mechanism shown in Figure 1,

a.) Define the mass matrices **M**$_1$, **…, M**$_6$ **(2 points)**

$$\underline{M}_1 = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_1 & 0 \\ 0 & 0 & J_1 \end{bmatrix}, \quad \underline{M}_2 = \begin{bmatrix} m_2 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & J_2 \end{bmatrix}, \quad \underline{M}_3 = \begin{bmatrix} m_3 & 0 & 0 \\ 0 & m_3 & 0 \\ 0 & 0 & J_3 \end{bmatrix}$$

Some

b.) Define the mass matrix **M** for the complete mechanism. **(1 points)**

c.) Define the array of forces **h**$_1$**,…,h**$_{6a}$ **(2 points)**

$$\underline{h}_3 = \begin{bmatrix} f_x^a \\ f_y^a - m_3 g \\ 0 \end{bmatrix}, \quad \underline{h}_6 = \begin{bmatrix} f_x^a \\ f_y^a - m_6 g \\ 0 \end{bmatrix}$$

d.) Define the force array **h** for the complete mechanism. **(1 points)**

e.) Define the vector of accelerations $\ddot{\boldsymbol{c}}$**. (2 points)**

$$\ddot{C}_1 = \begin{bmatrix} \ddot{x}_1 \\ \ddot{y}_1 \\ \ddot{\phi}_1 \end{bmatrix}$$

f.) Mark the non-zero entries in each column of the systems's Jacobian matrix for the mechanism shown in Figure 1 by an "x". **(6 points)**

	(1)	(2)	(3)	(4)	(5)	(6)
R1	X X	X X				
R2		X X	X X			
R3			X X	X X		
R4	X X				X X	
R5						X X
T1	X X					
T2					X X	X X

5.) Four-bar linkage (4 points)

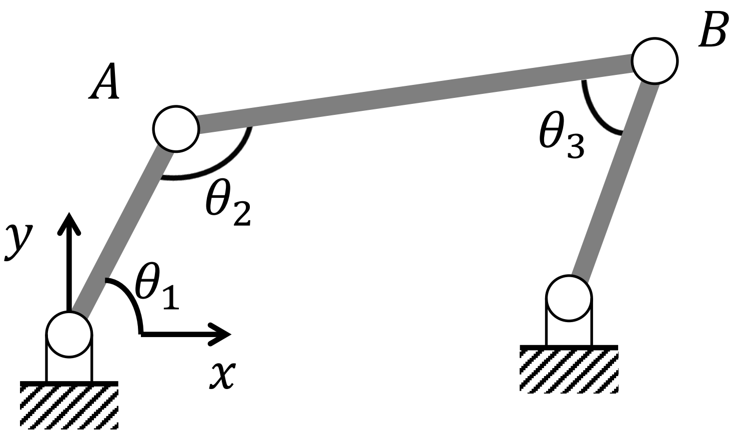


Figure 2: Four-bar mechanism

4 revolute joints ⇒ $4 \times 2 = 8$ constraints

a.) What dimension will the Jacobian matrix of the system have when body-coordinates are used? **(2 points)** $3 * 3 = 9$

8×9

b.) How many constraints are necessary using the angles Θ_1, Θ_2 and Θ_3 shown in Figure 2 as coordinates for the four-bar mechanism? **(2 points)**

2 constraints

Matrikel Nr.: ______________________

6.) Rotation matrix (4 points)

Are matrices **A** and **B** both rotation matrices?

$$\mathbf{A} = \begin{bmatrix} -0.99 & -0.1411 \\ 0.1411 & -0.99 \end{bmatrix} \qquad \mathbf{B} = \boldsymbol{A} * \boldsymbol{A}$$

$\overline{2 - \frac{2}{49}}$

Explain your answer!

7.) Kinematic Analysis (8 points)

The following position constraints are given:

$$2.0x_1 + 7.0x_2 - {x_3}^2 - 4.0 = 0$$

$2\dot{x}_1 + 7\dot{x}_2 - 2x_3\dot{x}_3 - 0 = 0$

$$2.0\sin x_1 + 7.0\sin x_2 - x_4 - 2 = 0$$

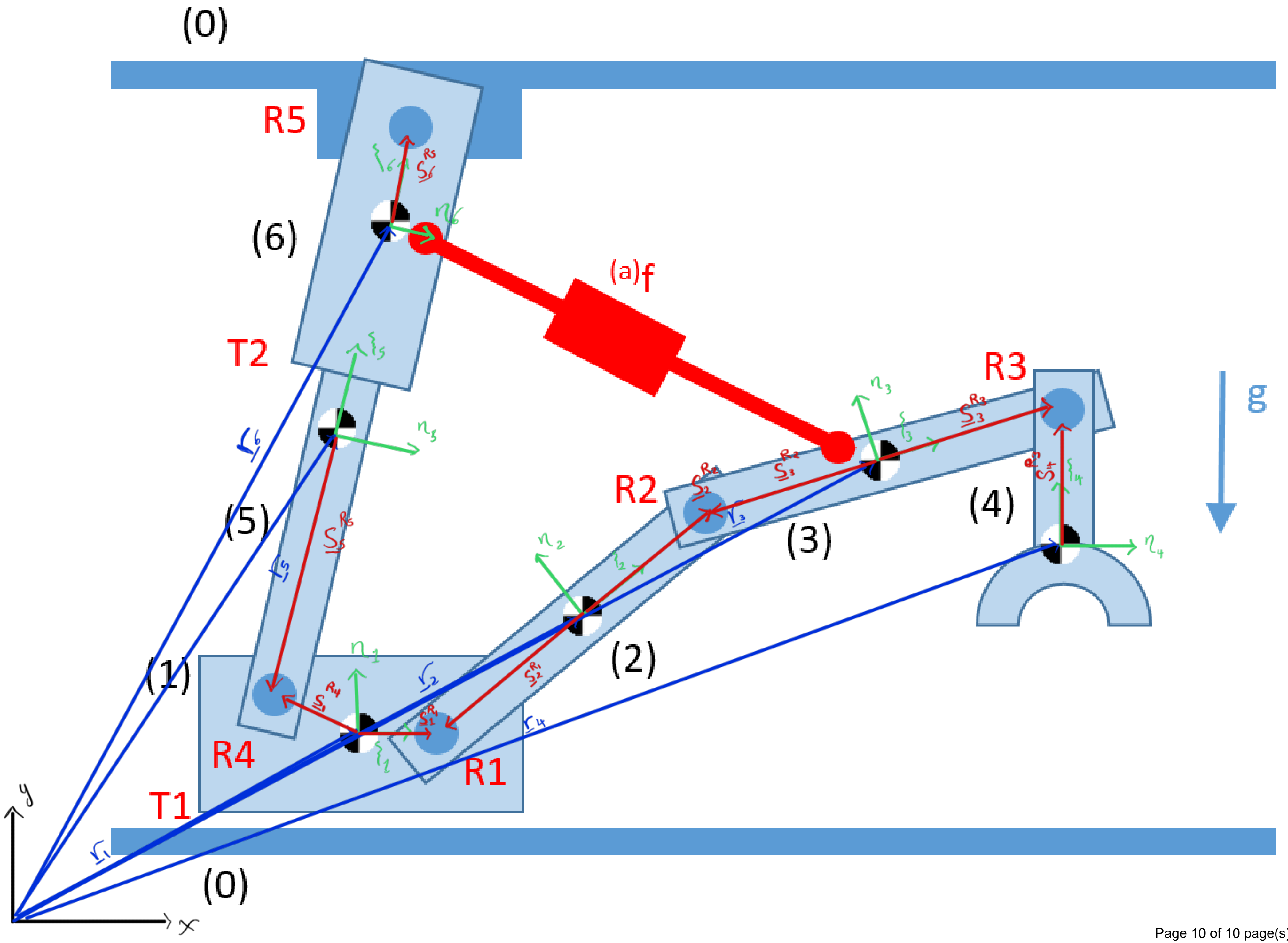
$2\cos x_1 \cdot \dot{x}_1 + 7\cos x_2 \cdot \dot{x}_2 - 1 \cdot \dot{x}_4 - 0$

$$2.0x_1 \cdot x_2 + 7.0x_2 \cdot x_3 \cdot x_4 = 0$$

$2x_2\dot{x}_1 + 2x_1\dot{x}_2 + 7x_3x_4\dot{x}_2 + 7x_3x_2\dot{x}_4 + 7x_2x_4\dot{x}_3 = 0$

a.) Calculate the velocity constraints. **(4 points)**

b.) Identify the Jacobian matrix of the system. **(4 points)**


(0)
R5
(6)
(a)f
T2
R3
g
R2
(5)
(4)
(3)
(2)
(1)
R4
R1
T1
(0)
x
y