Period of Examinations
Summer Semester 2016

HOCHSCHULE RHEIN-WAAL
Rhine-Waal University of Applied Sciences

Study Course: ______________________

Module Title: **Measurement Engineering and Controls**

Examination Part: **Measurement Engineering and Controls**

Points: 100

Duration: 120 Minutes

Please write legibly!

Date: ______________

Family Name: ______________

First Name: ______________

Student No.: ______________

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Signature (Student)

FOR INTERNAL USE ONLY:

Question Number	Tick Questions Attempted	Points	Question Number	Tick Questions attempted	Transfer Points
1		/ 17	13		
2		/ 12	14		
3		/ 12	15		
4		/ 11	16		
5		/ 16	17		
6		/ 10	18		
7		/ 12	19		
8		/ 10	20		
9			21		
10			22		
11			23		
12			24		
	SUM			TOTAL	/ 100

Graded by		Checked by

Final Grade

Regular grading key.	
Adjusted grading key. (Please add the adjusted grading key to the exam-results)	

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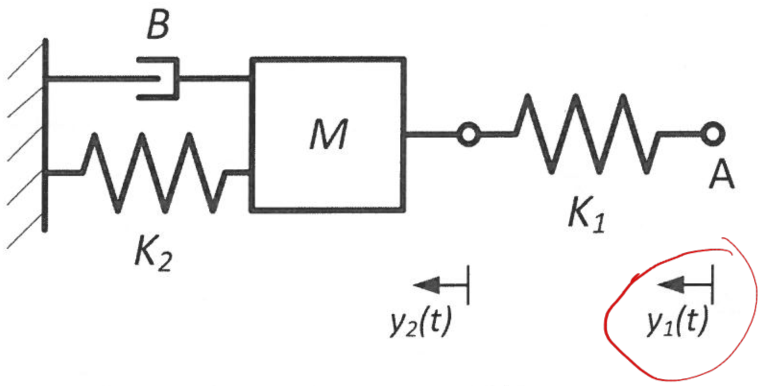
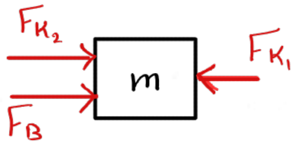
Question 1: **Points: 17**

A mechanical system is given according to the figure below. A mass M is connected to a spring (spring constant K_2) and a damper (damper constant B) on the left to a rigid wall, and connected to another spring on the right (spring constant K_1). $y_1(t)$ represents the displacement of the spring node and is considered as the input variable, whereas $y_2(t)$ represents the displacement of the mass and is considered as the output of the system.



a) Formulate the transfer function $G(s) = \frac{Y_2(s)}{Y_1(s)}$ of the system.



$$F_{K_1} = K_1\left(y_1(t) - y_2(t)\right)$$

$$F_{K_2} = K_2\, y_2(t)$$

$$F_B = B\, \dot{y}_2(t)$$

$$\sum F = m\ddot{y}_2(t) = F_{K_1} - F_{K_2} - F_B$$

$$m\ddot{y}_2 = K_1 y_1 - K_1 y_2 - K_2 y_2 - B\dot{y}_2$$

$$\boxed{K_1 y_1 = m\ddot{y}_2 + B\dot{y}_2 + (K_2 + K_1) y_2}$$

$$\mathcal{L} = \quad K_1 Y_1(s) = Y_2(s)\left(m s^2 + Bs + (K_2 + K_1)\right)$$

$$\boxed{G(s) = \frac{Y_2(s)}{Y_1(s)} = \frac{K_1/m}{s^2 + \frac{B}{m}s + \frac{K_2 + K_1}{m}}}$$

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b) What is the standard system type?

PT2 - System

Given are the parameters $M = 2\ kg$, $K_1 = 12\ N/m$, and $K_2 = 6\ N/m$.

c) Determine the eigenfrequency ω_0 of the system.

$$S^2 + \underbrace{\frac{B}{m}}_{2D\omega_0} S + \frac{K_1 + K_2}{m}\ \omega_0^2$$

$$\omega_0 = \sqrt{\frac{K_1 + K_2}{m}} = \sqrt{\frac{(12+6)\,N/m}{2\,Kg}} = 3\ rad/s$$

d) Calculate the quantity of the damping B, so that the system has a damping ratio of $D = 0.5$.

$$2D\omega_0 = \frac{B}{m} \Longrightarrow B = 2(0.5)(3\,rad/s)(2\,Kg) = 6\ Kg/s$$

In the following a system is characterized through the non-linear differential equation $\ddot{y}_2(t) + 3\dot{y}_2(t) + \sqrt{y_2(t)} = 4\, y_1^2(t)$.

e) Linearize the system about the operating point $y_{1,0} = 1$.

$$\ddot{y}_2(t) + 3\dot{y}_2(t) + \underbrace{\sqrt{y_2(t)}} - 4y_1^2(t) = 0$$

$$\Rightarrow \ddot{y}_2(t) + 3\dot{y}_2(t) + \frac{\partial y_2}{\partial t}\Big|_{op} \cdot y_2(t) - \frac{\partial y_1}{\partial t}\Big|_{op} \cdot y_1(t) = 0$$

$$\ddot{y}_2(t) + 3\dot{y}_2(t) + \left(\frac{1}{2\sqrt{y_2}}\right)\Bigg|_{y_2=1} \cdot y_2(t) - (8y_1)\Big|_{y_1=1} \cdot y_1(t) = 0$$

$$\ddot{y}_2(t) + 3\dot{y}_2(t) + \frac{1}{2} y_2(t) - 8 y_1(t) = 0$$

$$\boxed{8y_1(t) = \ddot{y}_2(t) + 3\dot{y}_2(t) + \frac{1}{2} y_2(t)}$$

Question 2: **12**

A system is described through the differential equation

$\dot{y}(t) + 3y(t) = 6u(t)$

where $y(t)$ represents the output variable, the initial conditions are $y(0) = 1$ and the input approaches a step, thus $u(t) = 1(t)$.

Calculate the dynamic response of the system in the time domain by using the method of the Laplace-transform.

$$\mathcal{L}: \quad sY(s) - y(0) + 3Y(s) = 6\,\cancel{U(s)}\ \tfrac{1}{s}$$ step input

$$Y(s)(s+3) - 1 = \frac{6}{s} \Rightarrow Y(s)(s+3) = \frac{s+6}{s}$$

$$Y(s) = \frac{s+6}{s(s+3)} = \frac{A}{s} + \frac{B}{s+3}$$

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$$S+6 = A(S+3) + BS$$

at $S=0 \Rightarrow \boxed{A = \frac{6}{3} = 2}$ at $S=-3 \Rightarrow \boxed{B = -1}$

$$\therefore Y(S) = \frac{S+6}{S(S+3)} = \frac{2}{S} - \frac{1}{S+3}$$

$\mathcal{L}^{-1}$: $\boxed{y(t) = 2(t) - e^{-3t}}$ The system's respones.

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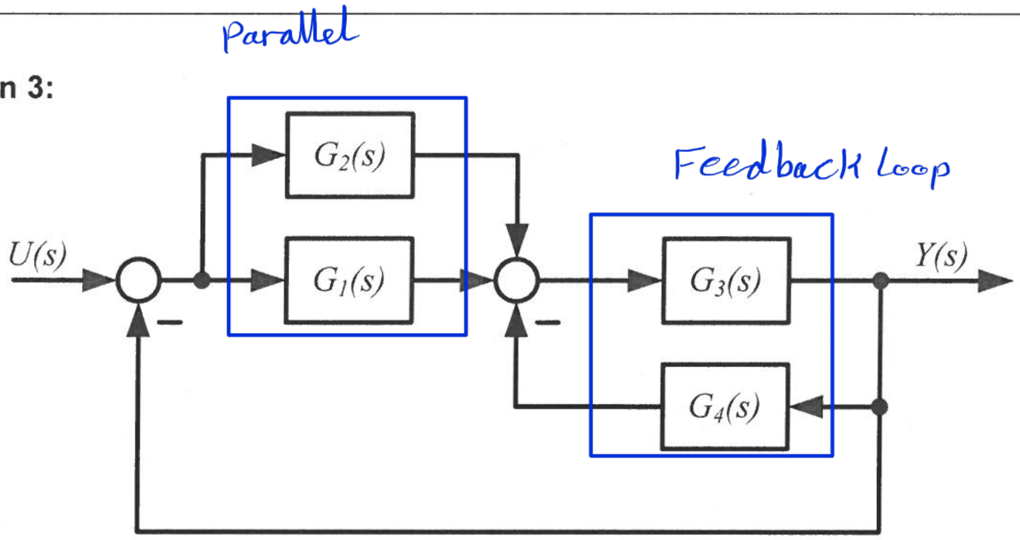
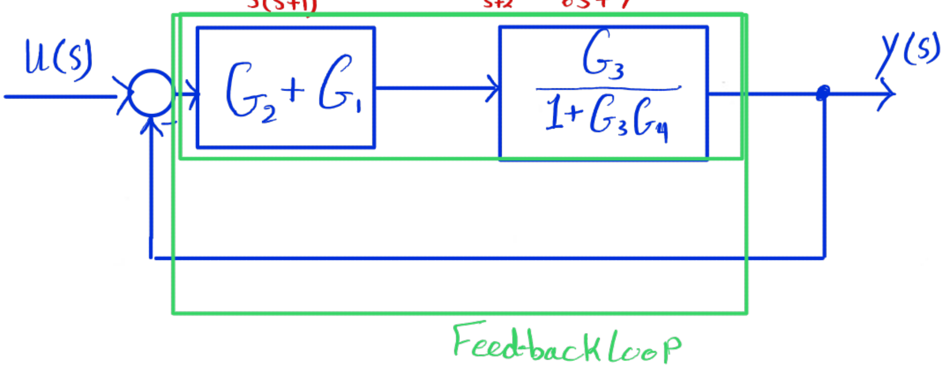
Question 3: 12

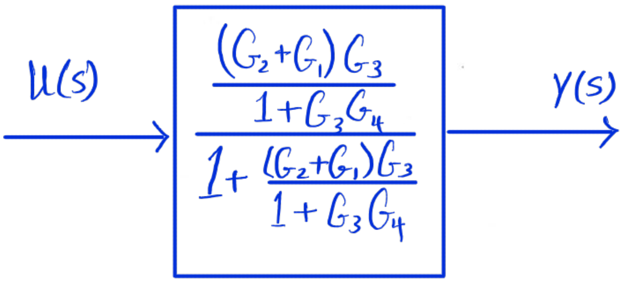


a) Find the equivalent transfer function, $G(s) = \frac{Y(s)}{U(s)}$ for the system shown in the figure, with

$$G_1(s) = \frac{2}{s+1}, G_2(s) = \frac{1}{s}, G_3(s) = \frac{s+1}{s+2}, G_4(s) = 5.$$

$$\frac{3s+1}{s(s+1)} \qquad \frac{\frac{s+1}{s+2}}{1+\frac{5s+5}{s+2}} = \frac{s+1}{6s+7} \qquad \text{series}$$





$$\frac{Y(s)}{U(s)} = \frac{\frac{(3s+1)(s+1)}{s(s+1)(6s+7)}}{1+\frac{(3s+1)(s+1)}{s(s+1)(6s+7)}} = \frac{(3s+1)\cancel{(s+1)}}{\cancel{(s+1)}(s(6s+7)+(3s+1))} = \frac{3s+1}{6s^2+10s+1}$$

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b) What is the standard system type?

PDT_2

c) Where are the zeros and poles of $G(s)$ located?

Zeros $\Rightarrow$ $3s+1=0 \Rightarrow \boxed{z_1 = -\frac{1}{3}}$

Poles: $6s^2+10s+1=0 \Rightarrow P_{1,2} = \frac{-5 \pm \sqrt{19}}{6}$

$\Rightarrow P_1 \approx -0.107$, $P_2 \approx -1.56$

d) Is the system $G(s)$ stable? Please explain.

Yes, because all the system poles are located at the LHP of the s-plane.

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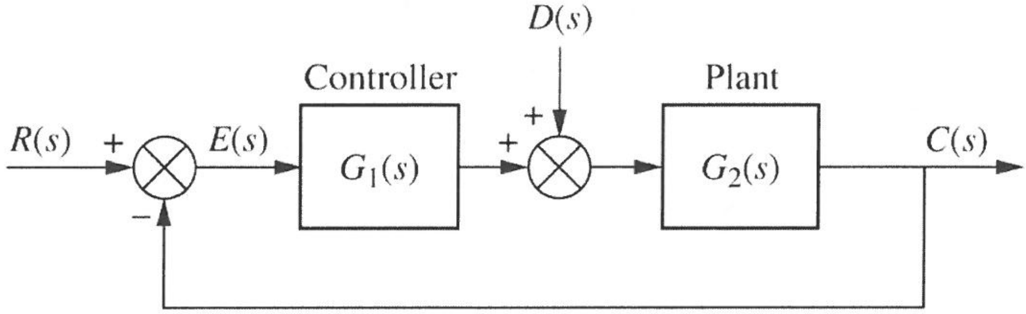
Question 4: **11**



A plant with DT1 behaviour ($K_D = 1$ and $T_1 = 1$) is controlled by an ideal I-controller, with the reference $R(s)$, the controlled variable $C(s)$ and the disturbance $D(s)$.

a) Calculate the reference transfer function $G_R(s) = \frac{C(s)}{R(s)}$.

Reference TF $\Rightarrow$ $D(s)=0$

$$G_p(s) = \frac{K_D S}{T_1 S + 1} = \frac{S}{S+1}$$

$$G_c(s) = \frac{K_I}{S}$$

$$G_R(S) = \frac{C(S)}{R(S)} = \frac{G_1 G_2}{1+G_1 G_2} = \frac{G_c(s)G_p(s)}{1+G_c(s)G_p(s)}$$

$$G_R(S) = \frac{\frac{K_I S}{S(S+1)}}{1+\frac{K_I S}{S(S+1)}} = \frac{K_I S}{S(S+K_I+1)}$$

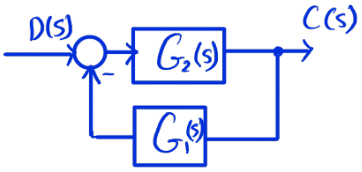
b) Calculate the gain K_I of the I-controller, so that the time constant of the closed-loop system is $T_{cl} = 0.5\ [sec]$.

c) Calculate the disturbance transfer function $G_D(s) = \frac{C(s)}{D(s)}$.

$G_p(s) = \frac{S}{S+1}$

$G_c(s) = \frac{K_I}{S}$

Disturbance TF $\Longrightarrow R(s) = 0$



$$G_D(s) = \frac{C(s)}{D(s)} = \frac{G_2}{1+G_2G_1} = \frac{G_P}{1+G_PG_C}$$

$$G_D(s) = \frac{\frac{S}{S+1}}{1+\frac{K_I S}{S(S+1)}} = \frac{S^2}{S(S+K_I+1)}$$

d) Draw the time response characteristic of the controlled variable $c(t)$ for $r(t) = 0$ and $d(t) = 1(t)$.

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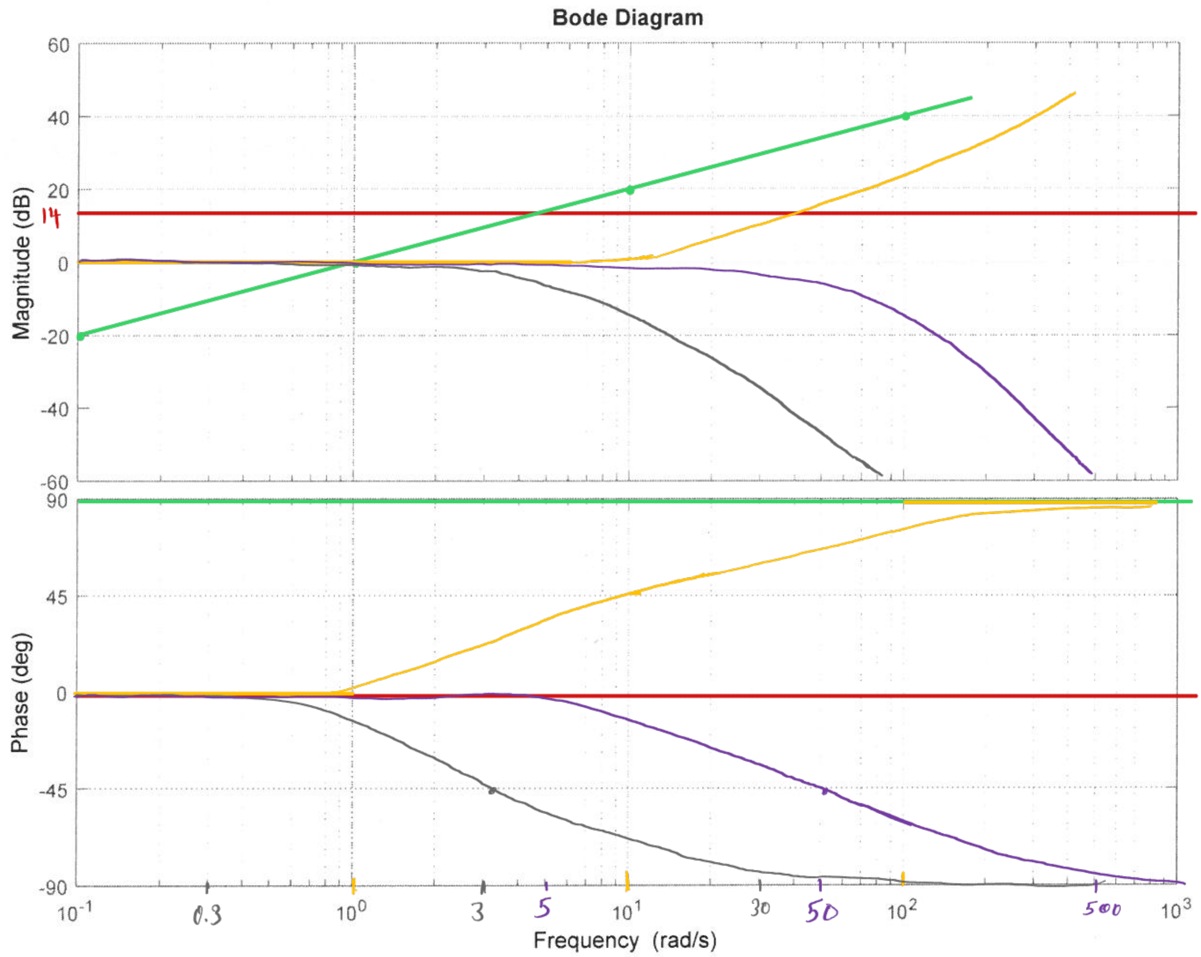
Question 5: **16**

The open-loop system $G_S(s)$ is described by the transfer function

$$G_S(s) = \frac{5s\left(\frac{1}{10}s+1\right)}{\left(\frac{1}{3}s+1\right)\left(\frac{1}{50}s+1\right)} = 5 \cdot S \cdot \left(\frac{s}{10}+1\right) \cdot \frac{1}{\frac{s}{3}+1} \cdot \frac{1}{\frac{s}{50}+1}$$

Sketch the asymptotes of the Bode-diagram of the open-loop system including its magnitude and phase shift.



TF	G_{dB} & ϕ	$W=0.1$	$W=1$	$W=10$
5	$G_{dB} = 20\log(5)$ $\phi = 0°$	$G_{dB} = 14dB$ $\phi = 0°$	//	//
S	$G_{dB} = 20\log(w)$ $\phi = +90°$	$G_{dB} = -20dB$ $\phi = 90°$	$G_{dB} = 0$ //	$G_{dB} = 20$ //
$\left(\frac{S}{10}+1\right)$	$G_{dB} = 20\log\left(\sqrt{1+\frac{w^2}{100}}\right)$ $\phi = \tan^{-1}\left(\frac{w}{10}\right)$	0	$\simeq 0$	$G_{dB} = 3dB$ $\phi = 45°$
$\left(\frac{1}{\frac{S}{3}+1}\right)$	$G_{dB} = -20\log\left(\sqrt{1+\frac{w^2}{9}}\right)$ $\phi = \tan^{-1}\left(-\frac{w}{3}\right)$	0	$G_{dB} = 0$ $\phi = -18°$	$G_{dB} = -11$ $\phi = -73°$
$\left(\frac{1}{\frac{S}{50}+1}\right)$	$G_{dB} = -20\log\left(\sqrt{1+\frac{w^2}{50^2}}\right)$ $\phi = \tan^{-1}\left(-\frac{w}{50}\right)$	0	$\simeq 0$	$G_{dB} =$ $\phi = -11°$

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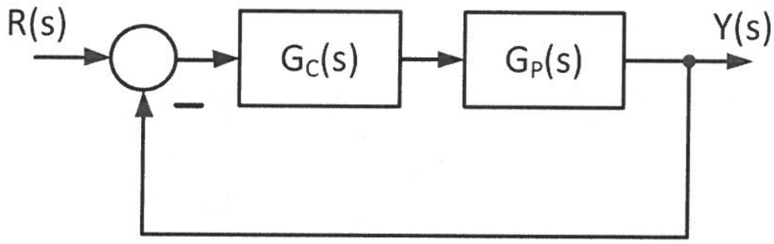
Question 6:

10

The plant with the transfer function

$$G_P(s) = \frac{4}{s^2 + 0.5s + 4}$$

is controlled by a PI-controller (see figure), where the integrator gain is set to $K_I = 1$.



Find the range of the proportional gain K_P for which the closed-loop system is stable.

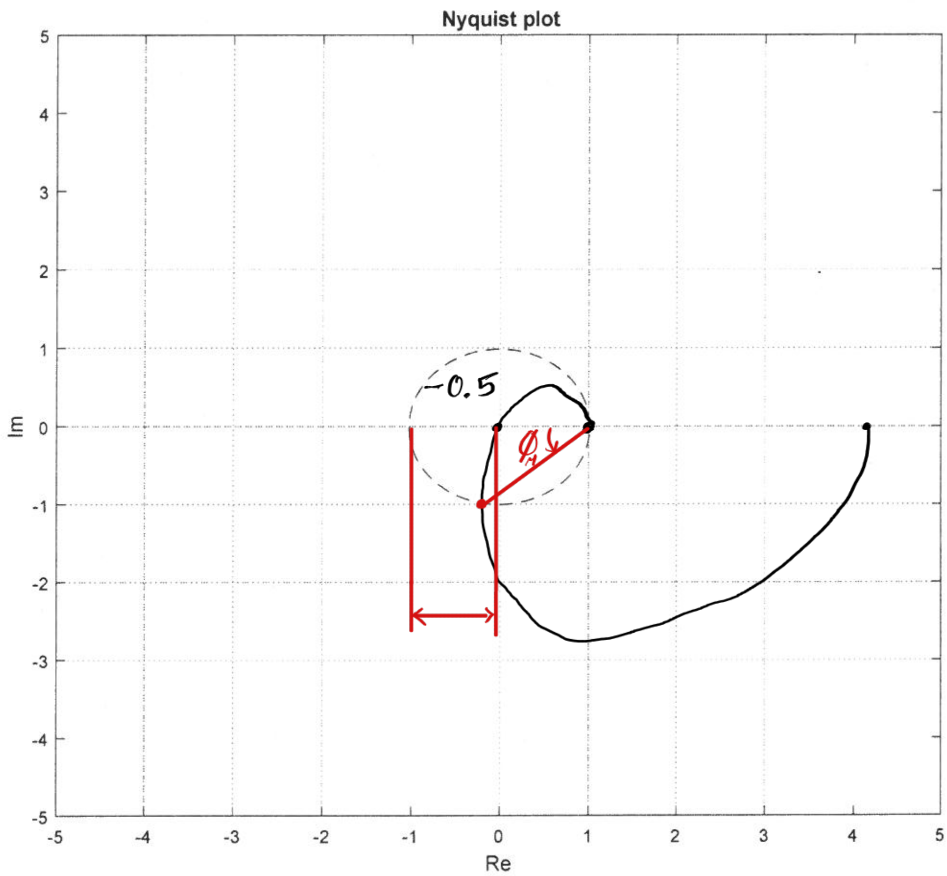
Question 7: **12**

An open-loop transfer function is given as,

$$G(s) = \frac{140}{(s+2)(s+3)(s+5)}$$

a) For the above system, the magnitude and the phase have been calculated and are given in the table below. Draw the Nyquist-plot using the data points from the table below.

ω [rad/s]	$\lvert G(j\omega)\rvert$	$\phi(\omega)$
0	4.7	0 °
0.1	4.7	-6 °
1	3.9	-56 °
$\sqrt{31}$	$\boxed{0.5}$	-180 °
10	0.1	-215 °
100	1.4 10^{-4}	-264 °



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b) Is the closed-loop system stable? Explain by means of the Nyquist-plot obtained in a).

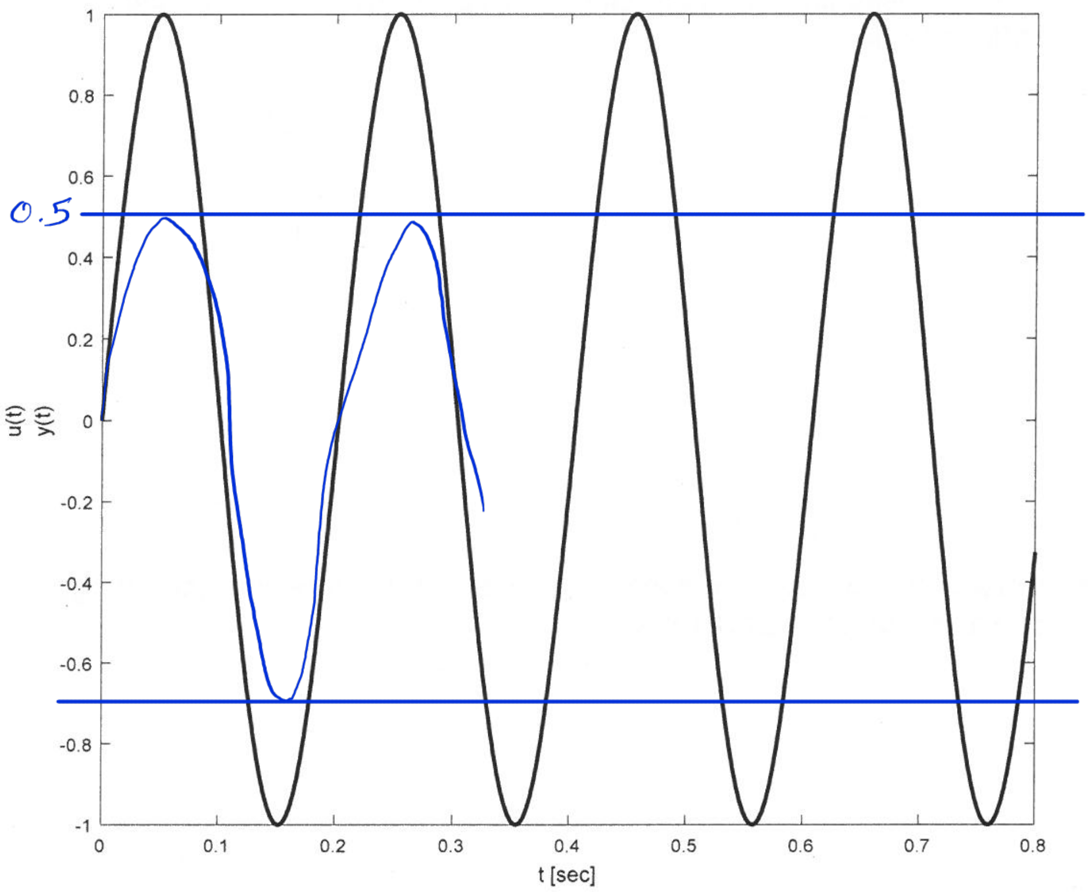
c) In case the closed-loop system is stable, determine the gain margin G_M and phase margin ϕ_M, graphically.

$$a = \frac{1}{0.5} = 2$$

$$G_M = 20 \log(2) = 6\,dB$$

$$\phi_M = 45^\circ$$

d) The input $u(t)$ at the open-loop system $G(s)$, is a sinusoidal-signal with an amplitude $\hat{u} = 1$ and a frequency of $\omega = \sqrt{31}\,\frac{rad}{s}$, see figure below. Use the Nyquist-plot and draw the expected output $y(t)$ of the open-loop system $G(s)$ into the diagram.



Question 8: **10**

No answer expected. Here you will get the points from the laboratory from summer term 2016.

Good luck!