Period of Examinations
Summer Semester 2017

HOCHSCHULE RHEIN-WAAL
Rhine-Waal University of Applied Sciences

Study Course: **Mechanical Engineering / Mechatronic Systems Engineering / Electronics**

Module Title: **Controls**

Examination Part: **Controls**

Points: 100

Duration: 120 Minutes

Please write legibly!

Date: ____________________

Family Name : ____________________ ____________________________

First Name: ____________________ Signature (Student)

Student No.: ____________________

FOR INTERNAL USE ONLY:

Question Number	Tick Questions Attempted	Points	Question Number	Tick Questions attempted	Transfer Points
1		/ 12	13		
2		/ 12	14		
3		/ 11	15		
4		/ 13	16		
5		/ 12	17		
6		/ 11	18		
7		/ 10	19		
8		/ 13	20		
9		/ 6	21		
10	Bonus points from laboratory	/ 10	22		
11			23		
12			24		
	SUM			TOTAL	/ 100

Graded by		Checked by

Final Grade

Regular grading key.	
Adjusted grading key. (Please add the adjusted grading key to the exam-results)	

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Question 1: Points: **12**

The output setting of a logical system is realized by four inputs A, B, C, D. The logic can be described through the Boolean equation

$$Y = A\,B\,C\,D + \bar{A}\,\bar{B}\,\bar{C}\,\bar{D} + A\,\bar{C}\,D + A\,\bar{B}\,C\,D + \boxed{\bar{A}\,B\,\bar{C}\,\bar{D}}$$

a) Find the optimized/minimum Boolean equation.
b) Sketch the corresponding ladder diagram.
c) Develop a logic gate circuit from the Boolean expression $Y = (\overline{A\,B} + \bar{C})(\overline{D + \bar{B}})$.

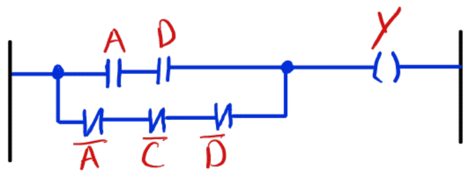
a) $Y = ACD + A\bar{C}D + \bar{A}\bar{C}\bar{D}$

$Y = AD + \bar{A}\bar{C}\bar{D}$

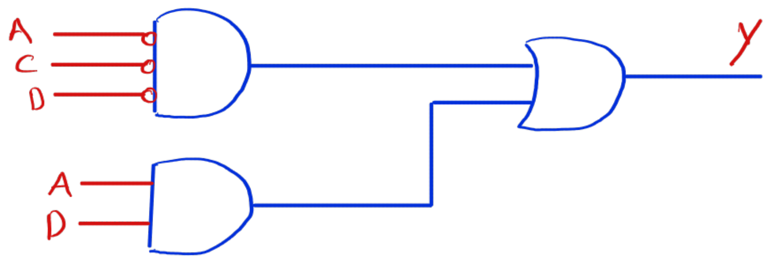
	E3		$\overline{E3}$		
E4		1	1		$\overline{E2}$
		1	1		E2
$\overline{E4}$				1	
				1	$\overline{E2}$
	$\overline{E1}$	E1		$\overline{E1}$	

$Y = AD + \bar{A}\bar{C}\bar{D}$

b)



c)



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Question 2:

Points:
12

Two conveyer belts (conveyer belt 1 and 2) transport material into a box, see figure 2.1. Conveyer belt 1 is controlled through a motor M1 and conveyer belt 2 is controlled through a motor M2. The conveyer belt 1 is activated by a N/O push button 1 and the second belt is activated by a N/O push button 2. Two N/C push buttons (Stop1 and Stop2) guarantee that both belts stop, as soon as at least one stop button is pressed.

Due to safety reasons it has to be guaranteed, that only one conveyer belt can be activated. That means, if belt 1 is already active, belt 2 cannot be started and if belt 2 is already active, belt 1 cannot be started.

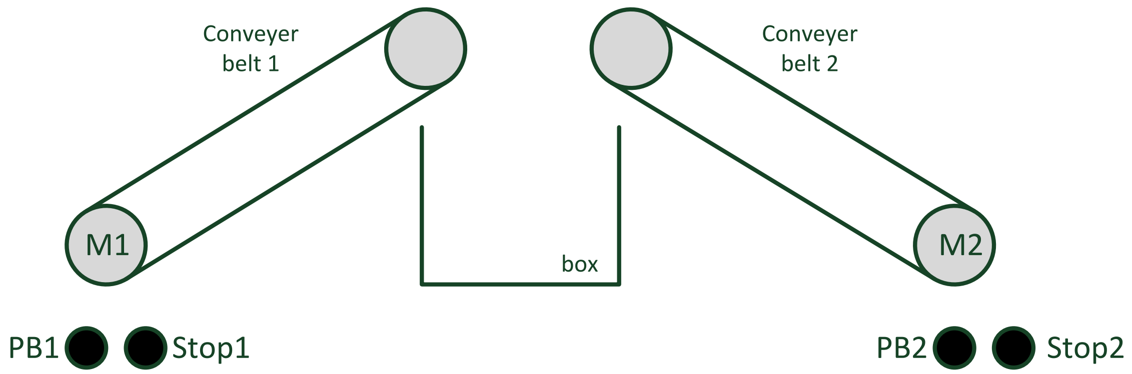
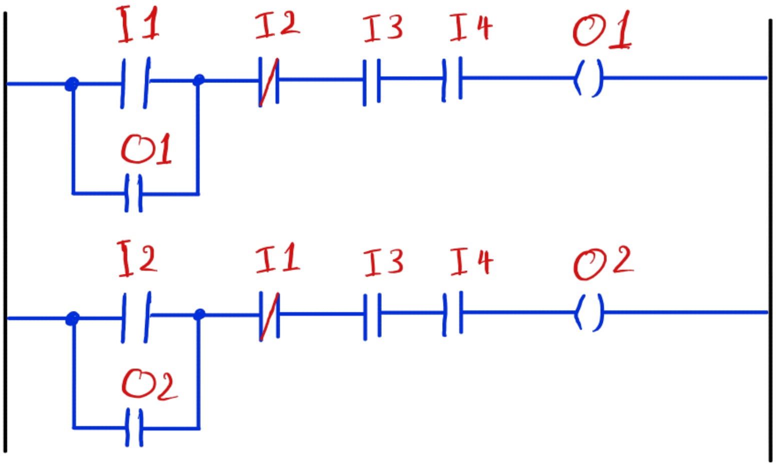


Figure 2.1: Conveyer belt arrangement

The control of the conveyer belt arrangement should be done via PLC. Develop and create the logic using a Ladder Diagram (LAD).

I1: N/O button1
I2: // // 2
I3: N/C Stop 1
I4: // // 2



O1: Motor 1
O2: // 2

Question 3:

Points: **11**

A process should be activated (and remains activated), when two N/O push buttons A and B are pressed for at least 5 seconds at the same time. When a N/C push button C is pressed, the process should stop immediately.

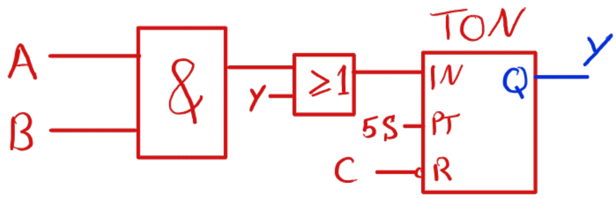
Develop and draw the logic using a Functional Block Diagram (FBD).

Input:-

A : N/O button

B: N/O button

C: N/C button



Output:-

Y: process

Question 4:

Points: **13**

A system is described through a set of differential equations

$$\dot{x}_1 = -4x_1 + a\,x_3 + u_1$$
$$\dot{x}_2 = 2x_1 + 2u_1$$
$$\dot{x}_3 = x_2 + u_2$$
$$y = 3x_1 + x_2 + u_1$$

where u_1 and u_2 are the inputs and y represents the output.

a) Derive a state-space model of the system and determine the matrices $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, and $\boldsymbol{D}$.

b) Determine the condition for the parameter a for which the system is stable.

a) $$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \underbrace{\begin{bmatrix} -4 & 0 & a \\ 2 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\begin{bmatrix} 1 & 0 \\ 2 & 0 \\ 1 & 0 \end{bmatrix}}_{B} \cdot \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$y = \underbrace{\begin{bmatrix} 3 & 1 & 0 \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{D} \cdot \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

b) $$G(s) = \frac{Y(s)}{U(s)} = \underline{C} \cdot \underline{\underline{\Phi}}(s) \cdot \underline{B} + \underline{D}$$

$$\underline{\underline{\Phi}}(s) = [s\underline{\underline{I}} - \underline{\underline{A}}]^{-1} \Longrightarrow \det(s\underline{\underline{I}} - \underline{\underline{A}}) = \det\begin{pmatrix} s+4 & 0 & -a \\ -2 & s & 0 \\ 0 & -1 & s \end{pmatrix} \begin{matrix} s+4 & 0 \\ -2 & s \\ 0 & -1 \end{matrix}$$

$$\det(s\underline{\underline{I}} - \underline{\underline{A}}) = s^2(s+4) - 2a \Rightarrow s^3 + 4s^2 - 2a$$

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Routh table:-

$$s^3 + 4s^2 - 2a$$

s^3	1	0
s^2	4	$-2a$
s^1	$= \frac{-\begin{vmatrix} 1 & 0 \\ 4 & -2a \end{vmatrix}}{4} = \frac{1}{2}a$	0
s^0	$= \frac{-\begin{vmatrix} 4 & -2a \\ -\frac{1}{2}a & 0 \end{vmatrix}}{-\frac{1}{2}a} = -2a$	0

$$\frac{1}{2}a > 0 \ \& \ -2a > 0$$

$$a > 0 \ \& \ a < 0$$

In this case, the system is only stable for $\boxed{a = 0}$

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Question 5:

Points: **12**

A plant is defined through the transfer function

$$G(s) = \frac{2}{s^2 + 4s + 3}$$

a) Convert the system model from the transfer function to a state space model.

In the following, the plant is controlled by use of a state-space controller with the feedback gain $\boldsymbol{K} = [17 \quad 5]$.

b) Calculate the poles of the closed-loop system.

The characteristic of the plant has now changed and is given through

$$G(s) = \frac{2s + 4}{s^2 + 4s + 3}$$

and is again controlled by a state-space controller.

c) Determine the feedback gain $\boldsymbol{K}$, so that the closed-loop poles are equal to those obtained in b).

a) Canonical form $G(s) = \frac{b_0}{s^2 + a_1 s + a_0}$

$a_1 = 4$, $a_0 = 3$, $b_0 = 2$

$$\dot{\underline{x}} = \underbrace{\begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix}}_{\underline{A}} \underline{x} + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{\underline{B}} \cdot u \qquad y = \underbrace{[2 \quad 0]}_{\underline{C}} \underline{x} + \underbrace{[0]}_{\underline{D}} \cdot u$$

b) $\dot{x} = [\underline{A} - \underline{B}\,\underline{K}] \cdot \underline{x} + \underline{B} r$

$$[\underline{A} - \underline{B}\underline{K}] = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 17 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -20 & -9 \end{bmatrix}$$

to get the poles: $\det(s\underline{I} - [\underline{A} - \underline{B}\underline{K}]) = \begin{vmatrix} s & -1 \\ 20 & s+9 \end{vmatrix} = s^2 + 9s + 20$

$= (s+4)(s+5)$

$\Longrightarrow \boxed{P_1 = -4}, \boxed{P_2 = -5}$

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c) $G(s) = \dfrac{2s+4}{s^2+4s+3}$

Step 1) $\underline{\underline{A}} = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix}$, $\underline{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C = [4 \;\; 2]$

Step 2) $K = [K_1 \;\; K_2]$

Step 3) $\underline{\underline{A}} - \underline{B}\,\underline{K} = \begin{bmatrix} 0 & 1 \\ -(K_1+3) & -(K_2+4) \end{bmatrix}$

$\det(s\underline{\underline{I}} - [\underline{\underline{A}} - \underline{B}\,\underline{K}]) = s^2 + (K_2+4)s + (K_1+3)$

Step 4) $(s+4)(s+5) = s^2 + \underbrace{9}_{d_1}s + \underbrace{20}_{d_0}$

Step 5) $d_i = a_i + K_{i+1} \Rightarrow 9 = K_2 + 4 \Rightarrow \boxed{K_2 = 5}$

$K_1 = d_0 - a_0 = 9 - 3 \Rightarrow \boxed{K_1 = 6}$

$\therefore K = [6 \;\; 5]$

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Question 6: Points: **11**

A control system is defined by a state space model

$$\dot{\boldsymbol{x}} = \begin{bmatrix} 0 & 1 & 4 \\ b & 0 & 0 \\ 1 & -8 & 1 \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \boldsymbol{x}$$

Determine the condition for b so that the system is [redacted].

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Question 7:

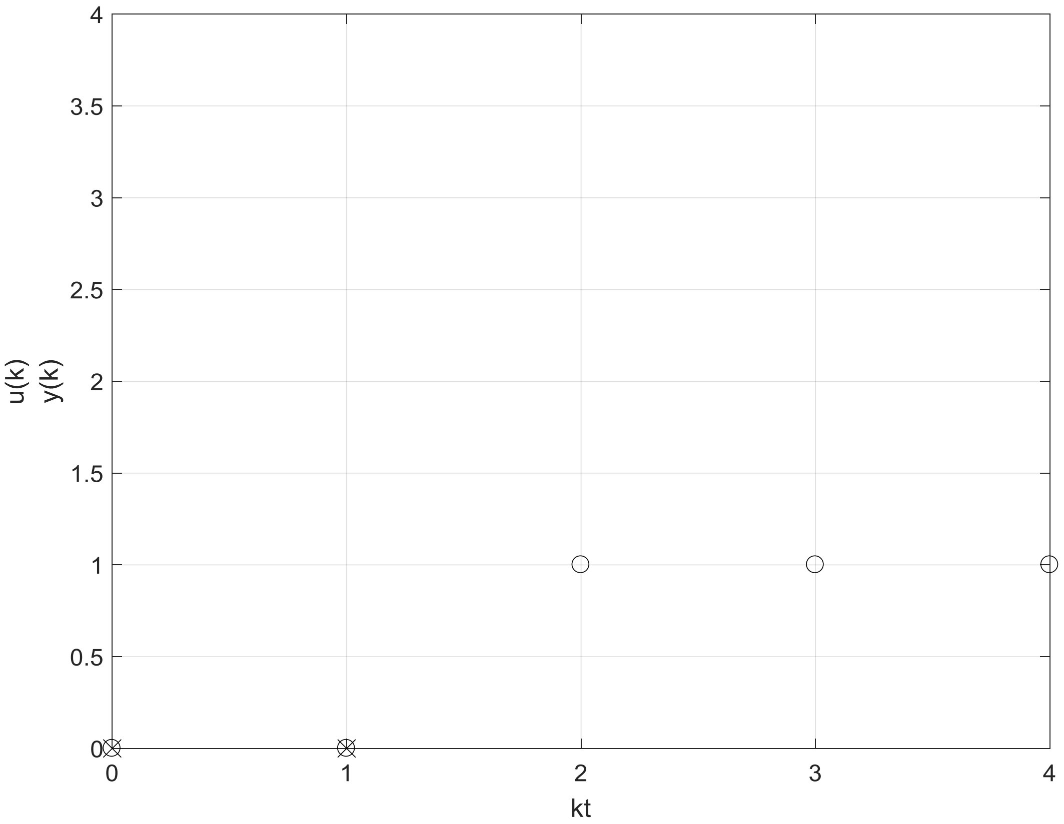
Points: **10**

A sampled data system consists of a time-discrete transfer function (in the z-domain)

$$G(z) = \frac{2}{1 - 0.5z^{-1} - 0.1z^{-2}}$$

a) Formulate the corresponding difference equation.

b) The sampled input signal $u(k)$ is given in the diagram below (marked by ○) as well as the first two sampled outputs $y(0)$ and $y(1)$ (marked by ×). Complete the diagram below by adding the sampled outputs $y(2)$, $y(3)$, and $y(4)$, considering the corresponding input $u(k)$.



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Question 8:

Points: **13**

A plant with the characteristic

$$G(s) = \frac{s}{(s+1)(s+2)}$$

is in series to a zero-order-hold and a sampler according to the block diagram in figure 8.1.

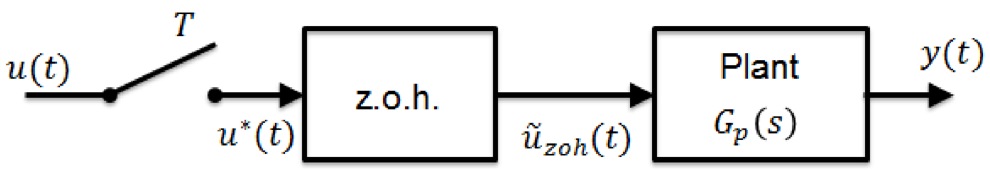

Figure 8.1: Block diagram of signal flow

a) Find the sampled-data transfer function $G(z)$ as a function of the sampling time T.
b) Determine the sampling time T_{crit}, for which the system is stable.

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Points:

Question 9: **6**

The step response of a system is given according to figure 9.1. As a digital computer is used for control, determine the maximum sampling period/time T, so that the theory of time continuous control can be applied.

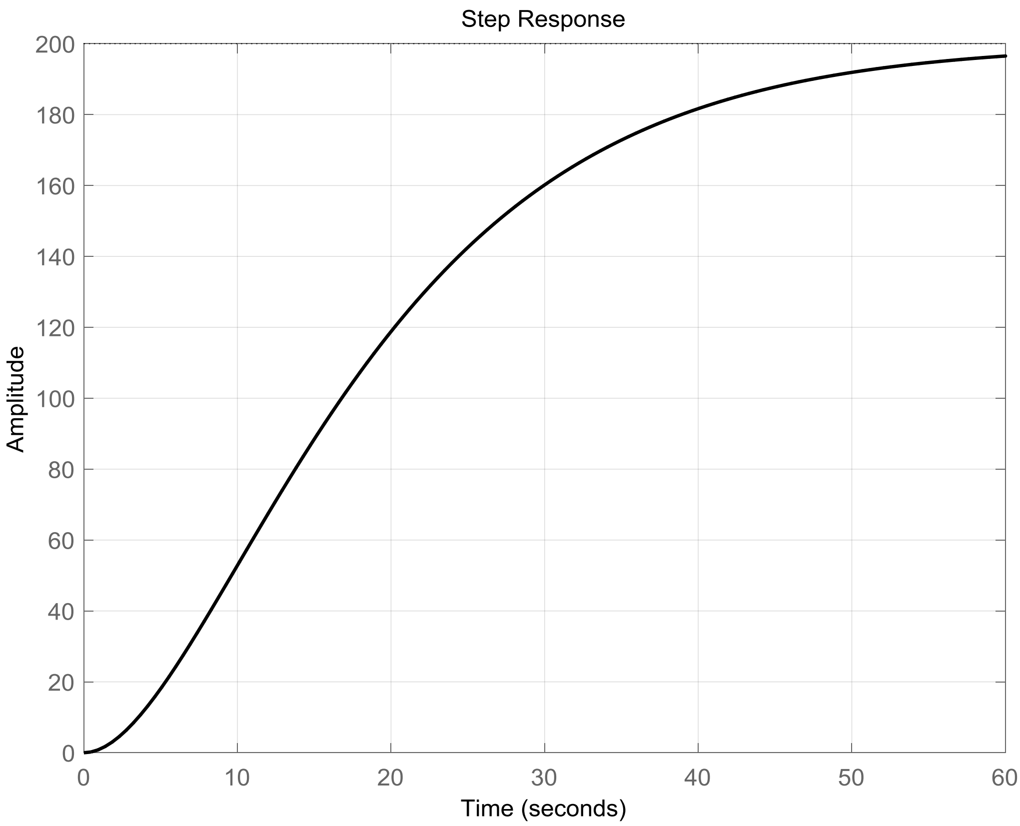

Figure 9.1: Step response

Good luck!