

**First period of examinations
Winter semester 2019/2020 January**

NEW CURRICULUM (PO 2017)

Study Course: _____

Module Title: **27101 Fluid Mechanics B.Sc. (Prof. Gebel)**

Written exam: 84 points

Duration: 120 min

Date: 28.01.2020

Family Name: _____

First Name: _____ Signature (Student)

Register No.: _____

The exam consists of 17 pages. First, check that your copy contains all pages.

All calculations and sketches should be done on the sheets provided for that purpose. If more than one solution is given, mark clearly which one should be rated.

Please write legibly! Good luck!

FOR INTERNAL USE ONLY:

	Q1	Q2	Q3	Q4	Q5	Q6	Written exam	Practical Training	Total
Max. Points	12	12	16	16	12	16	84	16	100
Achieved									
							≥ 42 Yes No	Grade	

Graded by	Checked by

Final Grade

Regular grading key.	
Adjusted grading key. (Please add the adjusted grading key to the exam-results)	X

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do not
use this
page.*

Question 1 ... Question/Answer Session

Points: 12

There is only one correct answer for each question.

1. What happens if the flow of water within a horizontal tube changes from laminar to turbulent?
 - The pressure losses remain constant.
 - The pressure losses will increase.
 - The pressure losses will decrease.

2. Consider laminar flow of water under steady-state conditions. What happens if the diameter of a horizontal tube smoothly increases?
 - The pressure will decrease and the velocity will increase.
 - Pressure and velocity remain constant.
 - Pressure and velocity will decrease.
 - The pressure will increase and the velocity will decrease.

3. Consider laminar flow of air under steady-state conditions through a horizontal tube. What happens if the temperature of the air decreases due to cooling of the tube?

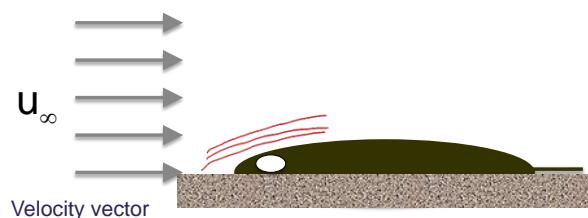
gas viscosity increases with tempreature increase, and viceversa with liquid

 - The pressure losses will increase.
 - The pressure losses will decrease.
 - The pressure remains constant.

since air Temp↓ $\Rightarrow \eta_{air} \downarrow$

$$\lambda_{Lam.} = \frac{64}{Re} = \frac{64 \eta}{V \rho d} \Rightarrow \lambda \downarrow \Rightarrow \Delta P_{losses} \downarrow$$

4. Plaice are bottom-living flatfish similar to flounder. At rest they constitute low, rounded humps on smooth, sandy bottoms. What happens if the current passing over the fish becomes stronger, i.e. higher flow velocity?



- The flatfish is pressed on the ground.
- The flatfish is lifted up into the current.
- The flatfish remains quiescent.

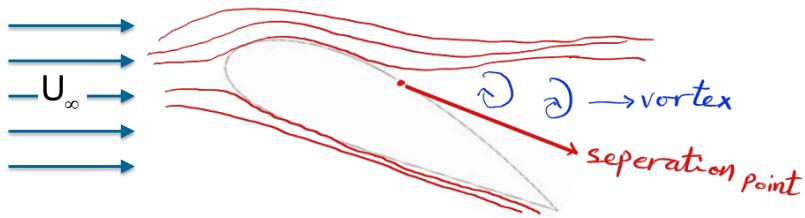
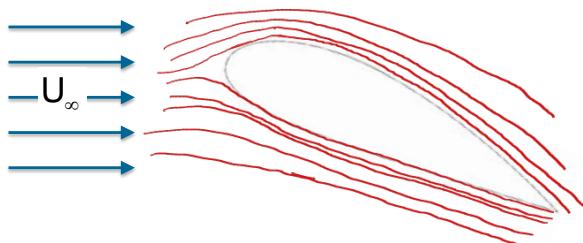
5. A streamline is a line in the flow possessing the following property:

- The velocity vector of each particle occupying a point on the streamline is normal to the streamline.
- The velocity vector of each particle occupying a point on the streamline is tangent to the streamline.
- The velocity vector of each particle occupying a point on the streamline is shifted by 45° with respect to the streamline.

6. Consider a Newtonian fluid contained between two large parallel plates of area A, which are everywhere separated by a very small distance Y. The system is at rest. Then the lower plate is set in motion at a constant velocity v. What velocity profile will appear at steady state, i.e. after acceleration?

- A parabolic velocity profile.
- A linear velocity profile.
- A hyperbolic velocity profile.

7. Sketch the streamlines over an airfoil at a large angle of attack for both attached flow and separated flow. (2 P)



Question 2 ... Equation of continuity**Points: 12**

Air at 16°C and 276 kPa (absolute) flows in a 10-cm-diameter pipe with a mass flow of 3 kg/s. The pipe undergoes a conversion to a 5 cm by 7.5 cm rectangular duct in which the temperature is 65°C and the pressure is 48 kPa (absolute).

Calculate the velocity in each section.

Given:

$$\text{Gas constant } R_{\text{Air}} = 287 \frac{\text{J}}{\text{kg K}}$$

$$pV = RT$$

$$\rho = \frac{P}{RT}$$

Answer sheet

Use this sheet for your answer only. Other notes will not be accepted.

$$\text{Equation of continuity: } w_2 = 5 \text{ cm}, t_2 = 7.5 \text{ cm}$$

$$\dot{m}_{\text{in}} = \dot{m}_{\text{out}}$$

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2 = 3 \text{ m/s}$$

$$V_2 = \frac{\dot{m}_{\text{out}}}{\rho_2 A_2} = \frac{\dot{m}_{\text{out}}}{\frac{P_2}{RT_2} \cdot w_2 t_2} = \frac{3 \text{ kg/s}}{\frac{48 \cdot 10^3 \text{ Pa}}{287 \frac{\text{J}}{\text{kg K}} (65+273) \text{ K}} \cdot (0.05 \text{ m}) (0.075 \text{ m})} \approx 1616.77 \text{ m/s}$$

$$V_1 = \frac{\dot{m}_{\text{in}}}{\rho_1 A_1} = \frac{\dot{m}_{\text{in}}}{\frac{P_1}{RT_1} \frac{\pi}{4} d_1^2} = \frac{3 \text{ kg/s}}{\frac{276 \times 10^3 \text{ Pa}}{287 \frac{\text{J}}{\text{kg K}} (16+273) \text{ K}} \cdot \frac{\pi}{4} (0.1 \text{ m})^2} \approx 114.79 \text{ m/s}$$

Question 3 ... Pressure drop

Points: 16

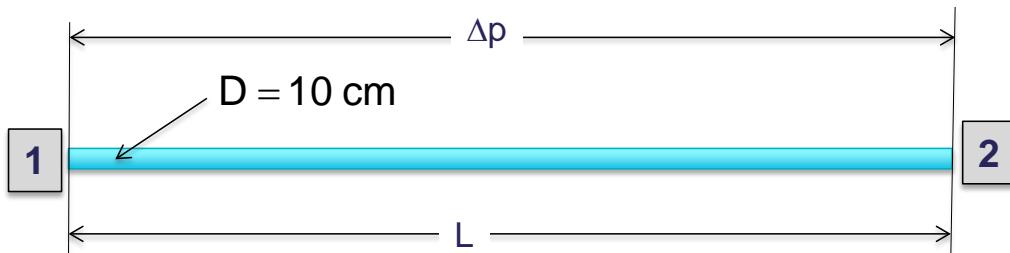
A pressure drop of 700 kPa is measured over a 300-m length of a horizontal, 10-cm diameter wrought iron pipe that transports oil.

- Under which conditions the so-called Bernoulli Equation may be applied.
- Apply Bernoulli Equation for a streamline between Point 1 and Point 2 taking pressure losses into account.
- Calculate the flow rate, in m³/s, within a relative error range of approx. 10%. Use the attached Moody diagram.

Given:

Density of oil $\rho_{oil} = 900 \frac{\text{kg}}{\text{m}^3}$

Kinematic viscosity $v_{oil} = 1 \times 10^{-5} \frac{\text{m}^2}{\text{s}} = \frac{\eta_{oil}}{\rho_{oil}}$



Data: $P_2 - P_1 = 700 \text{ kPa}$, $L = 300 \text{ m}$, $D = 10 \text{ cm}$, $\rho_{oil} = 900 \frac{\text{kg}}{\text{m}^3}$, $\frac{\eta_{oil}}{\rho_{oil}} = 10^{-5} \frac{\text{m}^2}{\text{s}}$

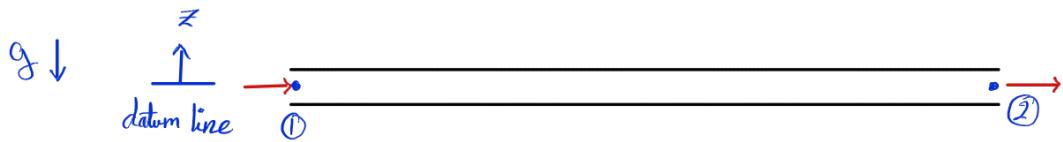
a) Bernoulli equation conditions :-

- * P = constant
- * along streamline
- * steady state
- * No viscous forces

Data: **Answer sheet**

Use this sheet for your answer only. Other notes will not be accepted.

$$P_1 - P_2 = 700 \text{ kPa}, L = 300 \text{ m}, D = 10 \text{ cm}, \rho_{oil} = 900 \frac{\text{kg}}{\text{m}^3} \Rightarrow \frac{m_{oil}}{P_{oil}} = 10^{-5} \frac{\text{m}^2}{\text{s}}$$



Extended Bernoulli equation :

$$P_1 + \frac{\rho}{2} V_1^2 + \rho g z_1 = P_2 + \frac{\rho}{2} V_2^2 + \rho g z_2 + \Delta p_{losses}$$

horizontal arrangement $z_1 = z_2$

Continuity equ. $\dot{m}_1 = \dot{m}_2$

$$\cancel{\rho} V_1 \cancel{\frac{\pi}{4} d^2} = \cancel{\rho} V_2 \cancel{\frac{\pi}{4} d^2}$$

$$d_1 = d_2 \Rightarrow V_1 = V_2$$

$$\Delta p_{losses} = P_1 - P_2 = 700 \text{ kPa}$$

$$\Delta p_{losses} \leq \lambda \cdot \frac{L}{d} \frac{\rho}{2} V^2$$

$$V = \sqrt{\frac{\Delta p_{losses} \cdot d}{\lambda \cdot L \cdot \frac{\rho}{2}}}$$

wrought iron pipe $\Rightarrow e = 0.046 \text{ mm}$

$$\frac{e}{d} = \frac{0.046 \text{ mm}}{100 \text{ mm}} = 4.6 \times 10^{-4}$$

at $\frac{e}{d} = 4.6 \times 10^{-4}$ and complete turbulence ($\lambda \neq f(Re)$) $\Rightarrow \lambda_1 = 0.016$

$$V_1 = \sqrt{\frac{700 \times 10^3 \text{ Pa}}{0.016} \cdot \frac{0.1 \text{ m}}{300 \text{ m}} \cdot \frac{2}{900 \text{ kg/m}^3}} \approx 5.7 \text{ m/s}$$

$$Re_1 = \frac{V_1 \rho d}{\eta} = \frac{V_1 d}{\eta_{oil}} = \frac{5.7 \text{ m/s} \cdot (0.1 \text{ m})}{10^{-5} \text{ m}^2/\text{s}} = 57000 > 2300 \quad (\text{Turbulent flow})$$

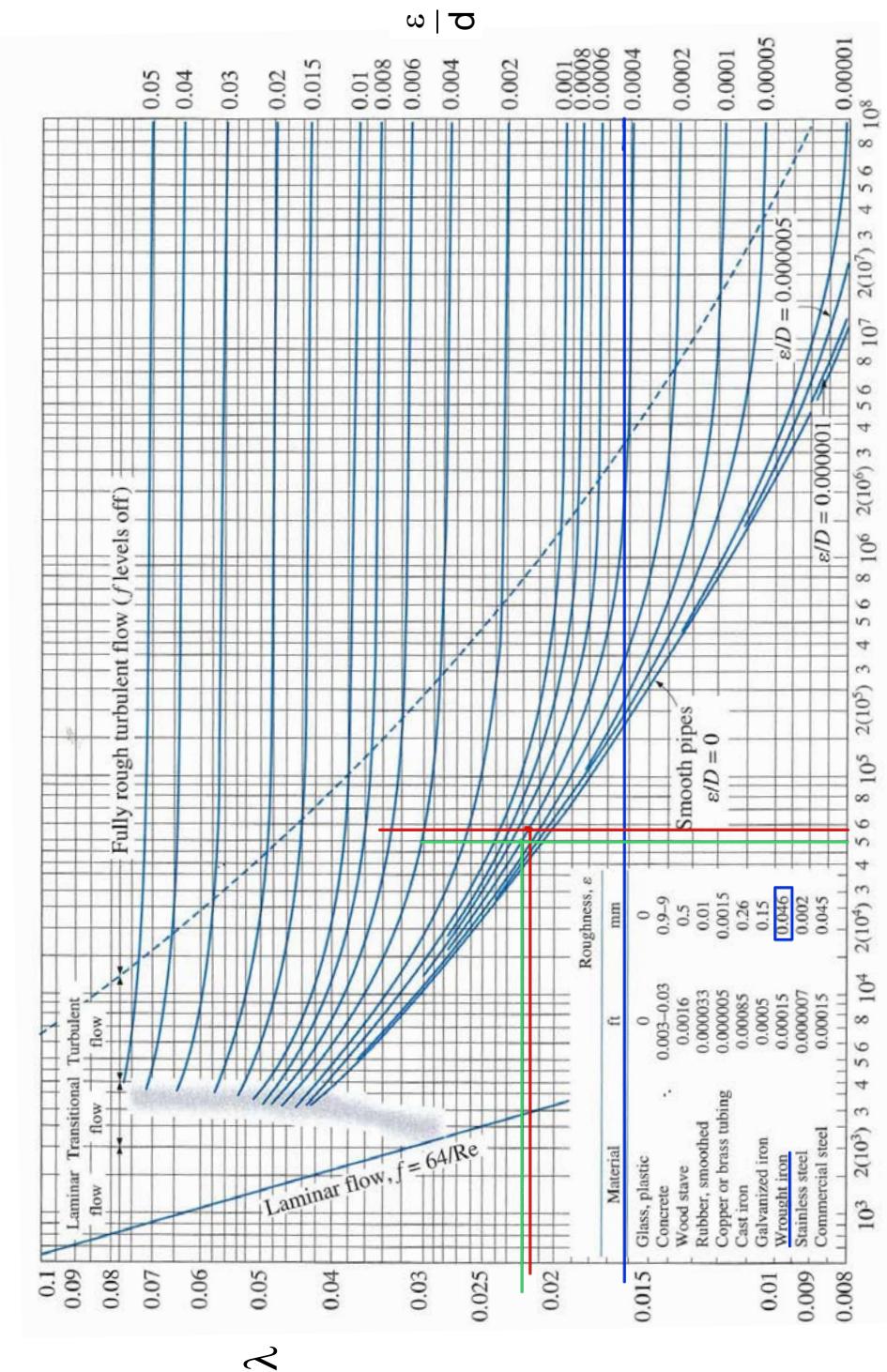
at $Re_1 = 5.7 \times 10^4 \Rightarrow \lambda_2 = 0.0217$

$$V_2 = 4.89 \text{ m/s} \Rightarrow Re_2 = \frac{V_2 \cdot d}{\eta_{oil}} = \frac{V_2 \cdot d}{\eta_{oil}} = 4.89 \times 10^4 > 2300 \quad (\text{Turbulent flow})$$

at $Re_2 \Rightarrow \lambda_3 = 0.022 \Rightarrow V_3 = 4.855 \text{ m/s}$

$$\text{ref. error} = \frac{|V_{i+1} - V_i|}{V_i} \times 100 = \frac{|4.855 - 4.89|}{4.89} \times 100 = 0.7\%$$

$$\dot{V} = V_3 A = 4.855 \text{ m/s} \cdot \frac{\pi}{4} (0.1 \text{ m})^2 = 0.0381 \text{ m}^3/\text{s}$$



$$Re = \frac{\rho v d}{\eta}$$

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Answer sheet

Use this sheet for your answer only. Other notes will not be accepted.

Question 4 ... Dimensional analysis and similitude**Points: 16**

Assume that the pressure losses of a flow through a centrifugal pump depend on the following variables:

$$\Delta p = f(v, \rho, \eta, d)$$

v velocity

ρ density

η dynamic viscosity

d diameter

	Δp	v	ρ	η	d	
M	1	0	1	1	0	repeating variables
L	-1	1	-3	-1	1	
T	-2	-1	0	-1	0	

$$\Pi_1 = \frac{\Delta p}{\rho \cdot v^2}$$

$$\Pi_2 = \frac{\eta}{d \cdot \rho \cdot v}$$

The task is to determine the dimensionless numbers (π -terms), which describe the dependency of the pressure losses from the given variables.

Buckingham theorem states that $(n-m)$ dimensionless groups are called π -terms, and can be represented as $\Pi_1 = f(\Pi_2, \Pi_3, \dots, \Pi_{n-m})$, where n is the number of variables

- a) Please describe in your own words what the Buckingham π -theorem states. *and m is the num. of basic dimensions*
- b) Identify appropriate repeating variables. $n=5, m=3 \Rightarrow n-m=2$
- c) Determine the π -terms by combining the repeating variables with each of the remaining variables. $\Pi_1 = \frac{\Delta p}{\rho v^2}, \Pi_2 = \frac{\rho v d}{\eta}$
- d) Which commonly known dimensionless numbers have you found? Please express these numbers as ratio of two forces. $\Pi_1 = \text{Euler number} = \frac{\text{pressure force}}{\text{inertia force}}$
 $\Pi_2 = \frac{\rho v d}{\eta} = \text{Reynolds number} = \frac{\text{Inertia force}}{\text{Viscous force}}$
- e) A test is to be performed on a proposed design for a prototype of a large pump that is to deliver $2.0 \text{ m}^3/\text{s}$ of water from a 50-cm -diameter impeller with a pressure of 500 kPa . A model with a 10-cm -diameter impeller is to be used.

$$Eu = f(Re)$$

$$Re_R = Re_M \quad \& \quad Eu_M = Eu_R$$

~~$$\frac{\rho v d R}{\eta} = \frac{\rho v_m d_m}{\eta}$$~~

water

 $\rho_{\text{cons.}}$ $T_{\text{cons.}} \Rightarrow \eta_{\text{cons.}}$

What flow rate, in m^3/s , should be used in the model?

$$\frac{\Delta P_M}{\rho V_M^2} = \frac{\Delta P_R}{\rho V_R^2} \Rightarrow \Delta P_M = \Delta P_R \left(\frac{V_M}{V_R} \right)^2 = 500 \text{ kPa} \left(\frac{5}{10} \right)^2 = 12500 \text{ kPa}$$

$$\frac{V_M}{V_R} = \frac{d_R}{d_M} = \frac{50 \text{ cm}}{10 \text{ cm}} = 5$$

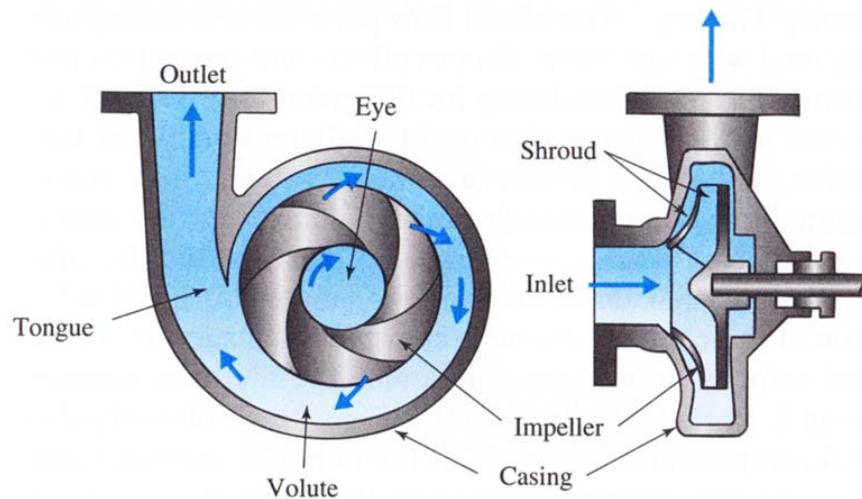
The model fluid is water at the same temperature as in the prototype.

$$\frac{\dot{V}_M}{\dot{V}_R} = \frac{V_M A_M}{V_R A_R} = \frac{V_M}{V_R} \cdot \frac{d_M^2}{d_R^2} = 5 \left(\frac{10 \text{ cm}}{50 \text{ cm}} \right)^2 = \frac{1}{5}$$

$$\dot{V}_M = \frac{\dot{V}_R}{5} = \frac{2 \text{ m}^3/\text{s}}{5} = 0.4 \text{ m}^3/\text{s}$$

Answer sheet

Use this sheet for your answer only. Other notes will not be accepted.



Question 5 ... Drag and lift

Points: 12

A small airplane has a total mass of 1,800 kg and a wing area of 42 m².

Determine the lift and drag coefficients of this airplane while cruising at an altitude of 4,000 m at a constant speed of 280 km/h and generating 190 kW of power.

Assumptions:

The lift and drag produced by parts of the plane other than the wings are not considered.

The fuel is used primarily to provide propulsive power.

Given:

Density of air at 4,000 m $\rho_{\text{Air}} = 0.82 \frac{\text{kg}}{\text{m}^3}$

Data: $m = 1800 \text{ kg}$, $A = 42 \text{ m}^2$, $h = 4000 \text{ m}$, $\bar{V} = 280 \text{ km/h} = \frac{700}{9} \text{ m/s}$, $P_{\text{motor}} = 190 \text{ kW}$ $[\text{KJ/s}] = [\text{N} \cdot \text{m/s}]$

$$C_D = \frac{2F_D}{\rho_{\text{air}} \cdot \bar{V}^2 \cdot A}$$

$$C_L = \frac{2F_L}{\rho_{\text{air}} \cdot \bar{V}^2 \cdot A}$$

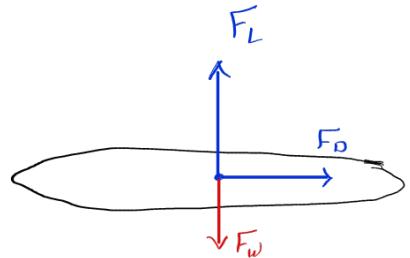
$$F_L = F_w = m g = 1800 \text{ kg} (9.81 \text{ m/s}^2)$$

$$F_L = 17658 \text{ N}$$

$$P_{\text{motor}} = F_{\text{thrust}} \cdot \bar{V} \Rightarrow F_{\text{thrust}} = F_p = \frac{P_{\text{motor}}}{\bar{V}}$$

$$C_D = \frac{2P_{\text{motor}}}{\rho_{\text{air}} \cdot \bar{V}^3 \cdot A} = \frac{2 \times 190 \times 10^3 \text{ W}}{0.82 \frac{\text{kg}}{\text{m}^3} \left(\frac{700}{9} \text{ m/s} \right)^3 (42 \text{ m}^2)} = 0.02345$$

$$C_L = \frac{2 \times 17658 \text{ N}}{0.82 \frac{\text{kg}}{\text{m}^3} \left(\frac{700}{9} \text{ m/s} \right)^2 (42 \text{ m}^2)} = 0.16951$$



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Answer sheet

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Question 6 ... Drag and terminal velocity**Points: 16**

A small drop of water in a rain cloud has a diameter $d = 30 \mu\text{m}$ (see figure below). The air temperature is 25°C, and its pressure is standard atmospheric pressure.

How fast does the air have to move vertically so that the drop will remain suspended in the air?

Hint: Start with a force balance for the falling sphere.

Apply, if possible, Stokes drag formula

$$c_D = \frac{24}{Re_d} \quad 1 \ll Re_d$$

Give rationale for this assumption.

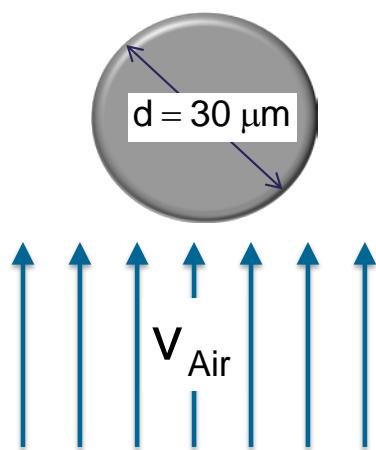
Given:

Density of water $\rho_{\text{Water}} = 1,000 \frac{\text{kg}}{\text{m}^3}$

Density of air $\rho_{\text{Air}} = 1.2 \frac{\text{kg}}{\text{m}^3}$

Dynamic viscosity of air $\eta_{\text{Air}} = 1.85 \times 10^{-5} \frac{\text{kg}}{\text{ms}}$

Volume of sphere $V_{\text{Sphere}} = \frac{\pi}{6} d^3$



Answer sheet

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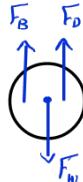
Data: $\rho_w = 10^3 \frac{\text{kg}}{\text{m}^3}$, $\rho_{\text{air}} = 1.2 \frac{\text{kg}}{\text{m}^3}$, $\eta_{\text{air}} = 1.85 \times 10^{-5} \frac{\text{kg}}{\text{m}\cdot\text{s}}$, $V_{\text{sphere}} = \frac{\pi}{6} d^3$, $T = 25^\circ\text{C}$, $d = 3 \times 10^{-5} \text{ m}$, $P = P_{\text{atm}}$

$$\sum F_y = 0$$

$$F_D = F_W - F_B$$

$$C_D \frac{\rho_{\text{air}}}{2} V^2 A = (\rho_w - \rho_{\text{air}}) V g$$

$$V_\infty = \sqrt{\frac{2(\rho_w - \rho_{\text{air}}) V g}{\rho_{\text{air}} C_D A}} = \sqrt{\frac{2g(\rho_w - \rho_{\text{air}}) \cdot 4d}{6 C_D \rho_{\text{air}}}}$$



$\downarrow g$

$$F_W = mg = \rho_w V g$$

$$F_B = \rho_{\text{air}} V g$$

$$F_D = C_D \frac{\rho_{\text{air}}}{2} V^2 A$$

$$\text{applying stoke's law (assuming } Re \ll 1) \Rightarrow C_D = \frac{24}{Re}$$

$$V_\infty = \sqrt{\frac{4g(\rho_w - \rho_{\text{air}}) d Re}{3 \rho_{\text{air}}}}^{\frac{1}{24}}$$

$$\text{with } Re = \frac{\rho_{\text{air}} V_\infty d}{\eta_{\text{air}}} \Rightarrow V_\infty = \boxed{\frac{g(\rho_w - \rho_{\text{air}}) d^2}{18 \eta_{\text{air}}}} = \frac{9.81 \text{ m/s}^2 (10^3 - 1.2) \frac{\text{kg}}{\text{m}^3} (3 \times 10^{-5})^2}{18 \times 1.85 \times 10^{-5} \frac{\text{kg}}{\text{m}\cdot\text{s}}} = 0.0265 \text{ m/s}$$

$$= 26.5 \text{ mm/s}$$

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Answer sheet

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Answer sheet

Use this sheet for your answer only. Other notes will not be accepted.

End of exam