HOCHSCHULE RHEIN-WAAL
Rhine-Waal University of Applied Sciences

Period of Examinations
Summer Semester 2020 – 15th July
Exam

Module:Modelling and Simulation, Prof. Brandt

Examination

Points: 60

Duration of examination: 120 Minutes (+ 120 Minutes for technical issues)

Please write legibly!

Date: ____________________

Name: ____________________

Register No.: ____________________

Course of Study: ____________________

Please consider the exam rules, which are summarized in the exam section in Moodle. There you find a summary containing your different options to upload your solutions or to withdraw from the exam.

Before you turn in your solution please sign the declaration in lieu of oath:

I, ____________________________________ [full name, matriculation number], hereby confirm in lieu of an oath that I am the person who was admitted to this examination. Further, I confirm that the submitted work is my own and was prepared without the use of any unauthorised aid or materials.

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Signature

You can print the declaration and then sign and scan it. Alternatively, you can also sign it digitally or transcribe it by hand and then sign and scan it.

Please make sure that all documents that you upload contain your name and matriculation number.

Good luck!

Problem	Possible Points	Result
1	8+8+2=18	
2	4	
3	2	
4	3	
5	2+6	
6	2	
7	3	
Sum	**40**	

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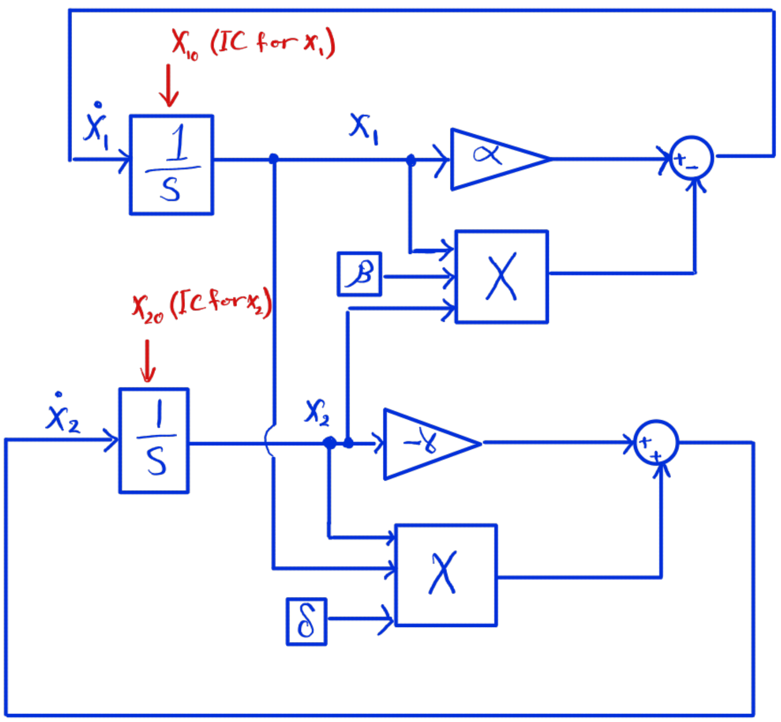
1.) In the following prey-predator system the amount of prey is modelled by x_1 and the amount of predator by x_2. The coefficients $\alpha, \beta, \gamma, \delta$ are known constants with positive values (>0). In the model it is assumed that only the two modelled species interact and that there is an infinite food resource for the prey. However, the only food resource for the predators is assumed to be the prey.

$$\dot{x}_1 = \alpha x_1 - \beta x_1 x_2$$

$$\dot{x}_2 = -\gamma x_2 + \delta x_1 x_2$$

a.) Draw the block-diagram for simulation with Simulink and indicate where the initial conditions have to be set.**(8 points)**



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b.) Linearize the prey-predator system above about $\mathbf{x}_0 = \begin{bmatrix} \frac{\gamma}{\delta} \\ \frac{\alpha}{\beta} \end{bmatrix}$. **(8 points)**

$$\underline{f}(\underline{x}) = \dot{\underline{x}} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \alpha x_1 - \beta x_1 x_2 \\ -\gamma x_2 + \delta x_1 x_2 \end{bmatrix}$$

$$\underline{f}(\underline{x}) \approx \underline{f}(\underline{x}_0) + \left.\frac{\partial f}{\partial x}\right|_{x \to x_0} \cdot \Delta \underline{x}$$

$$\underline{f}(\underline{x}_0) = \begin{bmatrix} \cancel{\frac{\alpha\gamma}{\delta} - \frac{\gamma\alpha}{\delta}} \\ \cancel{-\frac{\gamma\alpha}{\beta} + \frac{\gamma\alpha}{\beta}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \text{equilibrium}$$

$$\underline{\underline{J}} = \frac{\partial f}{\partial x} = \begin{bmatrix} \frac{\partial f(x_1)}{\partial x_1} & \frac{\partial f(x_1)}{\partial x_2} \\ \frac{\partial f(x_2)}{\partial x_1} & \frac{\partial f(x_2)}{\partial x_2} \end{bmatrix} = \begin{bmatrix} \alpha - \beta x_2 & -\beta x_1 \\ \delta x_2 & -\gamma + \delta x_1 \end{bmatrix}, \quad \left.\frac{\partial f}{\partial x}\right|_{x \to x_0} = \begin{bmatrix} \alpha - \cancel{\beta} \frac{\alpha}{\cancel{\beta}} & -\beta \frac{\gamma}{\delta} \\ \delta\left(\frac{\alpha}{\beta}\right) & -\gamma + \cancel{\delta} \frac{\gamma}{\cancel{\delta}} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \frac{-\beta\gamma}{\delta} \\ \frac{\delta\alpha}{\beta} & 0 \end{bmatrix}$$

$$\Delta \underline{x} = \underline{x} - \underline{x}_0 = \begin{bmatrix} x_1 - \frac{\gamma}{\delta} \\ x_2 - \frac{\alpha}{\beta} \end{bmatrix}$$

$$\underline{f}(\underline{x}) \approx \begin{bmatrix} 0 & \frac{-\beta\gamma}{\delta} \\ \frac{\delta\alpha}{\beta} & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 - \frac{\gamma}{\delta} \\ x_2 - \frac{\alpha}{\beta} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & \frac{-\beta\gamma}{\delta} \\ \frac{\delta\alpha}{\beta} & 0 \end{bmatrix}}_{\underline{\underline{A}}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\underline{x}} + \underbrace{\begin{bmatrix} \frac{\alpha\gamma}{\delta} \\ \frac{-\alpha\gamma}{\beta} \end{bmatrix}}_{\underline{\underline{B}} \cdot \underline{u}}$$

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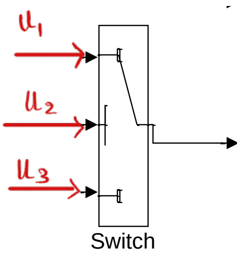
c.) Decide whether the operating point $\mathbf{x}_0 = \begin{bmatrix} \gamma \\ \delta \\ \alpha \\ \beta \end{bmatrix}$ is an equilibrium of the system or not. Justify your decision. **(2 points)**

yes it's equilibrium, because $\underline{f}(\underline{x}_0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

2.) A model that contains a discontinuity handles it by a switch. The switch has three inputs and one output. Explain how it works. **(4 points)**



u_1, and u_3 are signals which will be forworded to the output.

u_2 is the triger input, where u_2 is compared to the threshold, as a consequence the switch will know when to use u_1 or u_2 as an output.

It's like if statement, where we compare u_2 with the threshold value.

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3.) A vibration problem is described by the following equation:

$$\ddot{x} + d \cdot sign(\ddot{x}) \cdot \dot{x}^2 + {\omega_0}^2 \cdot x = \sin(\omega \cdot t)$$

The parameters ω, ${\omega_0}^2$and d are known. Is the problem linear or non-linear? How many integrators are necessary in Simulink in order to model the vibration problem? **(2 points)**

It's non-linear problem, since we have $\dot{x}^2$

4.) Transform the following ODE into a system of first order differential equations (ODEs). **(3 points)**

$$\ddot{x} + d \cdot \tan(x) \cdot \dot{x}^2 + {\omega_0}^2 \cdot x = \sin(\omega \cdot t)$$

$$\ddot{x} = -d \cdot \tan(x) \cdot \dot{x}^2 - w_0^2 \cdot x + \sin(w \cdot t)$$

$X_1 = X$

$X_2 = \dot{X} = \dot{X}_1$

$$\boxed{\begin{aligned} \dot{X}_1 &= X_2 \\ \dot{X}_2 &= -d \cdot \tan(X_1) \cdot X_2^2 - W_0^2 \cdot X_1 + \sin(Wt) \end{aligned}}$$

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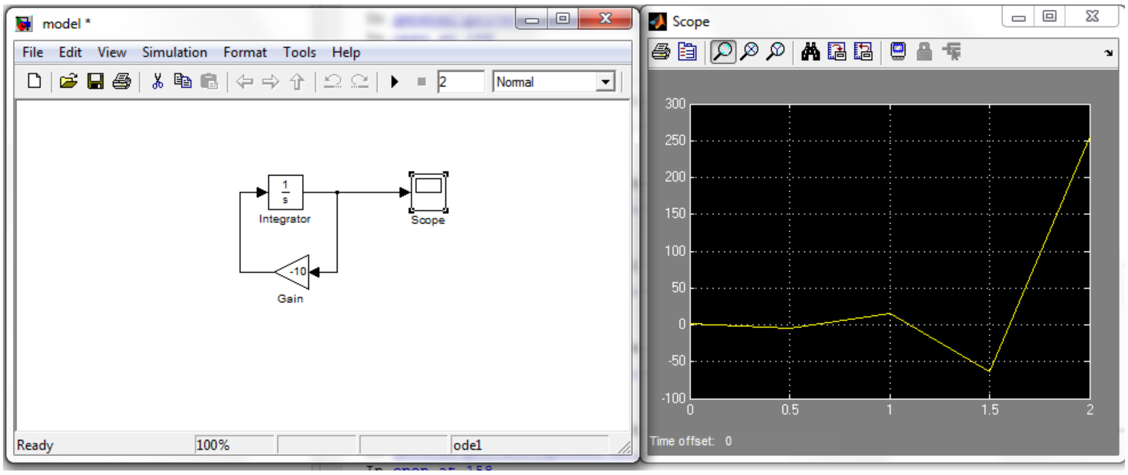
5.) The following simulation result is given.



a.) Why is the displayed signal oscillating? **(2 points)**

We have numerical instability. Bad combination between Ode-solver and step-size.

b.) Apply the explicit Euler Method (ode1) to the model. (Hint: 1st write down the *ode*, which is modelled in the above Simulink block diagram (gain=-10).) Calculate *five* steps of integration and compare the result to the signal displayed above. Explain your result. Depending on the last digit of your matriculation number use the following step-size *h*: **(6 points)**

$\alpha = 10,\ h = 0.8$

$x_0 = 0.1$

Last digit of matriculation number	*h*=
-0	1.0
-1	0.1
-2	0.2
-3	0.3
-4	0.4
-5	0.5
-6	0.6
-7	0.7
-8	0.8
-0	0.9

$x_1 = (1 - \alpha h)\, x_0$

$x_1 = (1-8)\, 0.1 = -0.7$

$x_2 = (1-8)\, x_1 = 4.9$

$x_3 = (1-8)\, x_2 = -34.3$

$x_4 = (1-8)\, x_3 = 240.1$

$x_5 = (1-8)\, x_4 = -1680.7$

still numerically unstable because the step size is too big like in the displayed signal above. The step size has to be $h < 0.1$ to be ustable

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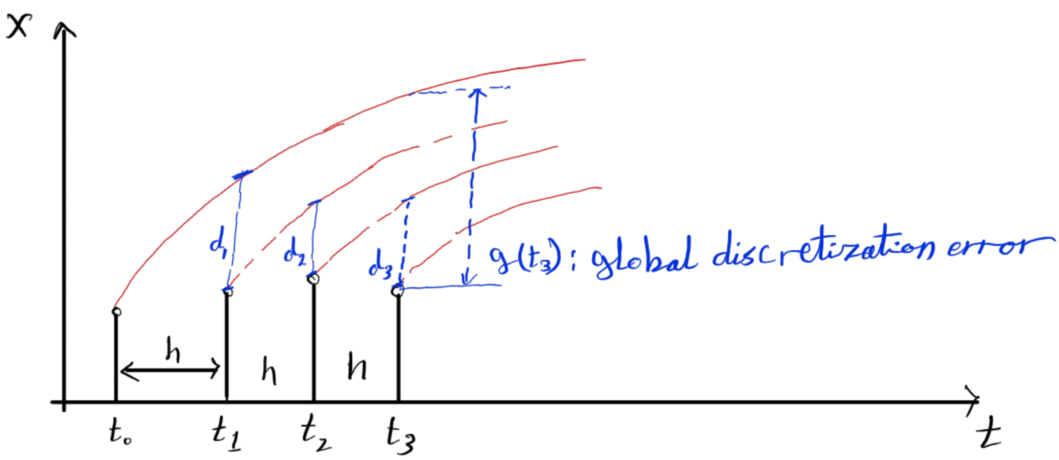
6.) Find all linear ODEs and give the order of all ODEs! **(2 points)**

$\dddot{y} = \ddot{y} + 3 + y^0$	Linear, 3rd order ODE
$\sin(y) + y^1 = \dot{y}$	non-linear, 1st order ODE
$3(\dot{y} + 4\ddot{y}) = 9a^2 * \dot{y}$; with $a = 100$	linear, 2nd order ODE
$y * y = \ddot{y}$	non-linear, 2nd order ODE

7.) Explain the term global discretization error. **(3 points)**



$d_{1,2,3}$ are local discretization errors (The error between each step)

where $g(t_3)$ is the global error between last step at $t = t_3$ and the true solution of $x(t_3)$