

Period of Examinations Winter Semester 2020/2021

Study Course: _____

Module Title: **Measurement Engineering and Controls / System Theory and Controls**

Points: 100

Duration: 90 Minutes + 15 Minutes for scanning and upload = 105 Minutes

Please write legibly!

Date: _____

Family Name: _____

First Name: _____

Student No.: _____

I, _____ [full name, matriculation number], hereby confirm in lieu of an oath that I am the person who was admitted to this examination. Further, I confirm that the submitted work is my own and was prepared without the use of any unauthorised aid or materials.

With uploading the examination results, I confirm and agree the terms and conditions and that the examination is assessed and rated.

Signature (Student)

If the exam / first page isn't signed, the examination is graded as failed.

Question 1:**15**

Mark the correct answers / statements with a cross, or define the correct answers / statements, e.g. mentioning a.1). For each correct cross / definition you will receive 1.5 points, each cross which is not correct will subtract 1.5 points from the total score. The total score for the entire question cannot be negative.

a) A system with PT2-characteristic has a damping ratio $D = 0.3$.

☐ a.1) The system is critically damped.

☒ a.2) The system is always stable.

☐ a.3) The system has two zeros.

☒ a.4) The imaginary part of the poles are nonzero.

b) A system is characterised through the differential equation

$$y(t) \ddot{y}(t) + 2t^2 y(t) + y^3(t) = \ddot{u}(t).$$

☐ b.1) The system is linear.

☒ b.2) The system is causal.

☐ b.3) The system is time-invariant.

☐ b.4) The system is of IT3 characteristic.

c) A system with DT2-characteristic

☐ c.1) is characterised through two parameters.

☒ c.2) has one zero at $z = 0$.

☐ c.3) has a constant magnitude for the entire frequency range.

☐ c.4) has a phase within the range of $+90^\circ - -90^\circ$.

d) A PID-controller

☒ d.1) is characterized through three parameters.

☒ d.2) is described by a P-, I-, and D-system in parallel.

☐ d.3) is useless in practice.

☐ d.4) guarantees always a stable closed-loop system.

e) A reference transfer function

☐ e.1) is the inverse of a disturbance transfer function.

☒ e.2) for a closed-loop control system should be $G_R(s) = 1$.

☐ e.3) is always stable.

☒ e.4) is calculated in that the disturbance is set to zero.

Question 2:
20

Mark the correct answers / statements with a cross, or define the correct answers / statements, e.g. mentioning a.1). For each correct cross / definition you will receive 2.5 points, each cross which is not correct will subtract 2.5 points from the total score. The total score for the entire question cannot be negative.

- a) A system is characterized through the differential equation
 $2 \ddot{y}(t) + 12 \dot{y}(t) + 200 y(t) = 400 u(t)$.
 \checkmark a.1) The eigenfrequency of the system is $10 \frac{\text{rad}}{\text{s}}$.
☐ a.2) The damping ratio of the system is 0.3.
☐ a.3) For a step input the steady state output is 0.5.
☐ a.4) The system has a conjugated complex pole pair.
- b) A system is characterized through the transfer function $G(s) = \frac{s+2}{s^2+s-20}$.
 \checkmark b.1) The system is of PDT2 characteristic.
☐ b.2) The system is unstable.
☐ b.3) The phase angle goes to -180 deg when the input frequency goes to infinity.
 \checkmark b.4) The slope of the magnitude is -20 dB/dec when the input frequency goes to infinity.
- c) A plant with $G_p(s) = \frac{4}{s}$ is controlled by a D-controller in a standard control loop.
☐ c.1) The closed-loop characteristic is according to a PT1-system.
☐ c.2) Without any disturbance, the controller isn't capable to control the reference exactly.
☐ c.3) With the controller gain $K_D = 1$, the time constant of the closed-loop system is 0.25 sec.
☐ c.4) The plant is unstable.
- d) A system is characterised through the transfer function $G(s) = \frac{5}{s^4+3s^3+2s^2+8s+4}$.
 \checkmark d.1) The system is unstable.
☐ d.2) The system can be described through the differential equation
 $2 y^{(IV)}(t) + 6 \ddot{y}(t) + 4 \dot{y}(t) + 16 y(t) + 8 = 5 u(t)$.
 \checkmark d.3) The step response of the system converges to 0 when $t \rightarrow \infty$.
☐ d.4) The system is not causal.

Question 3:
27

A hydraulic system arrangement is shown in figure 3.1 and consists of the main components hydraulic cylinder and a valve, where $u(t)$ is the input voltage of the valve, $y(t)$ the output position (controlled variable) of the hydraulic cylinder, and $F(t)$ the load (disturbance) of the hydraulic cylinder.

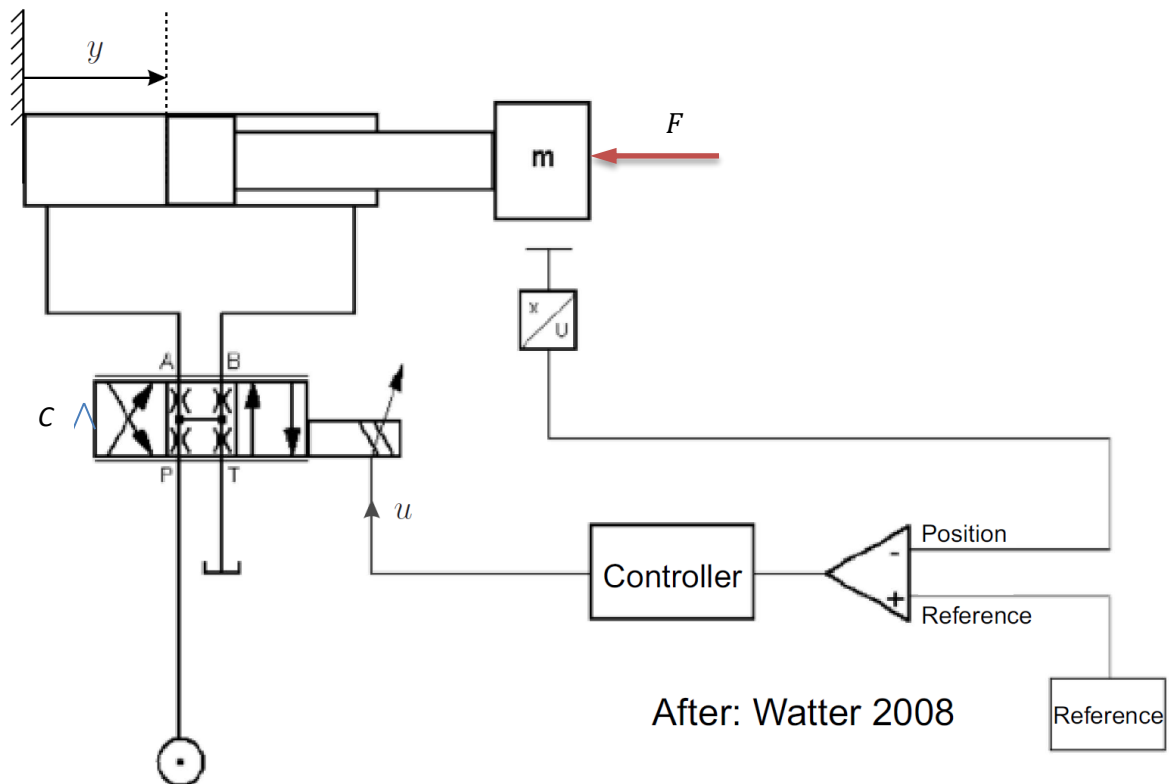


Figure 3.1: Hydraulic system arrangement

For the illustrated hydraulic cylinder, the equation of motion in positive y -direction can be considered as

$$m \ddot{y}(t) = A p(t) - B \dot{y}(t) - F(t)$$

where m is the mass of the cylinder, A is the cross section area of the cylinder piston, $p(t)$ the pressure in the corresponding chamber, and B the coefficient for viscous friction.

The pressure dynamic is dependent on fluid flow with

$$\dot{p}(t) = K_v [-A \dot{y}(t) + Q(t)] \Rightarrow \underset{L_i}{sP(s)} = K_v [EAS Y(s) + Q(s)] \Rightarrow SP(s) = K_v [-AS Y(s) + (K_1 U(s) + K_2 P(s))]$$

where K_v is a system constant and the flow $Q(t)$ can be approximated through the linearized function

$$Q(t) = K_1 u(t) + K_2 p(t) \Rightarrow L: Q(s) = K_1 U(s) + K_2 P(s)$$

with the valve constants K_1 and K_2 .

$$\Rightarrow SP(s) - K_v K_2 P$$

Student No.: _____ / Student Name: _____

a) Formulate the transfer function $G(s) = \frac{Y(s)}{U(s)}$

Student No.: _____ / Student Name: _____

b) What is the system characteristic (system type) of the system?

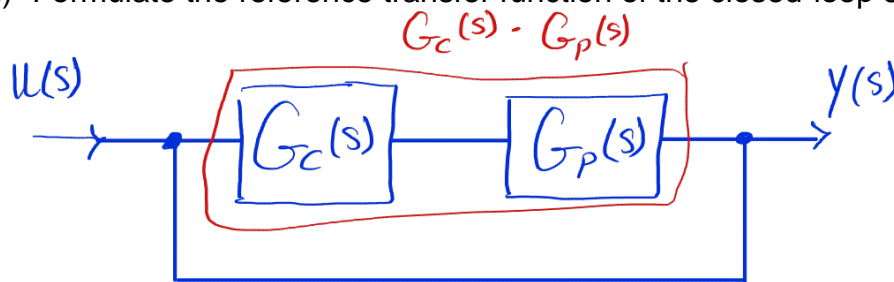
c) Is the hydraulic system stable? Please explain.

In the following the transfer function of the hydraulic system can be considered as

$$G_P(s) = \frac{Y(s)}{U(s)} = \frac{10}{s^3 + 4s^2 + 3s}$$

And is controlled by a P-controller with $G_C(s) = K_P$.

d) Formulate the reference transfer function of the closed-loop system.



$$G_R(s) = \frac{G_C \cdot G_P}{1 + G_C G_P} = \frac{10 K_P}{1 + 10 K_P}$$

e) Is the P-controller able to control the position of the cylinder exactly, thus steady-state error equal to zero, when the disturbance $F(t) = 0$? Please explain.

we set $s=0$
 $G_R(s) = 1$

Student No.: _____ / Student Name: _____

Question 4:**15**

Draw the ramp response of a DT1-plant (use the values of K_D and T_1 according to the table below, dependent on your student ID number) in a proper graph with proper axes scaling, with $5cm \cong 1 sec$ for the time axis and $2cm \cong 1$ for the output axis / y-axis. Clearly show in the graph, where K_D and T_1 can be found.

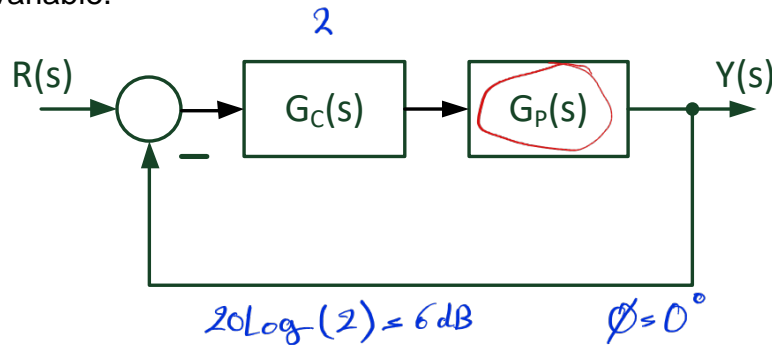
4 th digit of student ID	0	1	2	3	4	5	6	7	8	9
K_D	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5

5 th digit of student ID	0	1	2	3	4	5	6	7	8	9
T_1 [sec]	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1

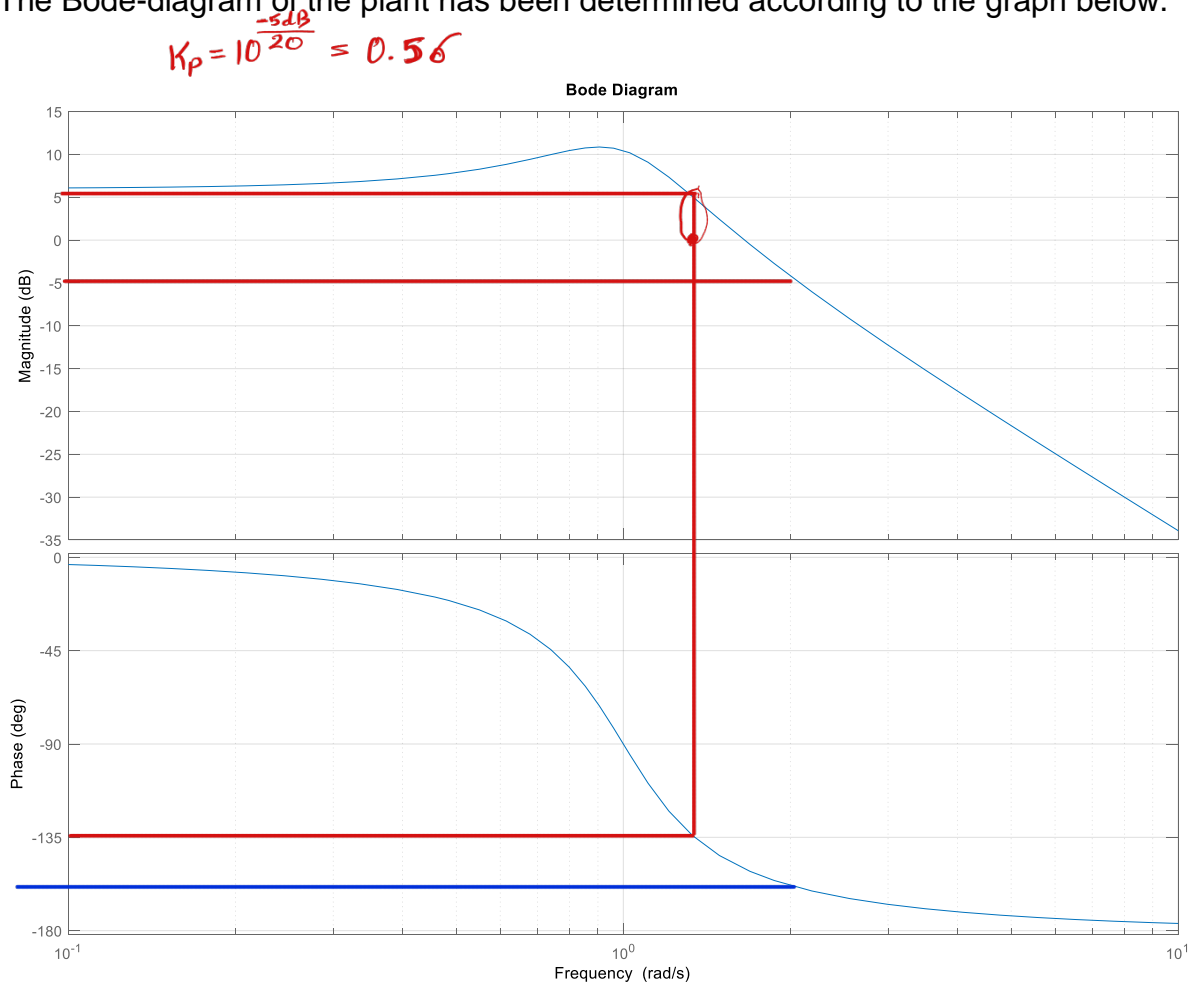
Question 5:

23

A controlled system is arranged according to the block diagram below, where $G_C(s)$ is the controller, $G_P(s)$ the plant with a PT2-characteristic, $R(s)$ the reference, and $Y(s)$ the controlled variable.



The Bode-diagram of the plant has been determined according to the graph below.



- a) Determine the output amplitude of the plant \hat{y} , when the plant is excited with a harmonic signal with the input amplitude $\hat{u} = 5$ and an excitation frequency of $\omega = 1 \frac{rad}{s}$.

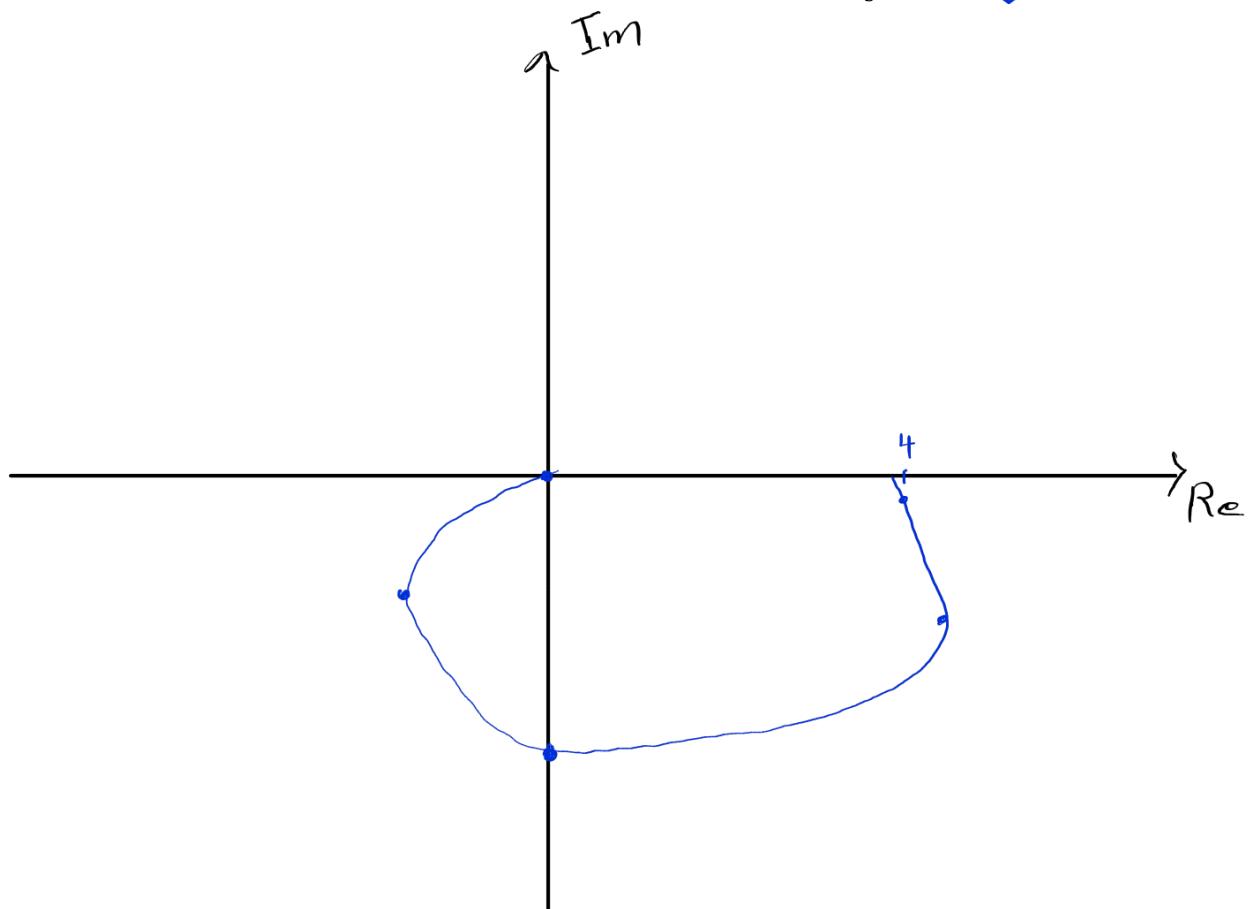
$$G_{dB} = 10 dB \Rightarrow A(\omega) = \frac{\hat{y}}{\hat{u}} = 10^{\frac{10dB}{20}} = 10$$

$$\hat{y} = 5 * 10^{\frac{10dB}{20}} = 15.8$$

ω	0.1	0.7	1	2	10
$ G $	$10^{\frac{12}{20}}$	$10^{\frac{15}{20}}$	$10^{\frac{16}{20}}$	$10^{\frac{2}{20}}$	$10^{\frac{-27}{20}}$
ϕ	-15°	-40°	-90°	-155°	$\approx 180^\circ$

In the following the plant is controlled by a P-controller with $G_C(s) = K_P$.

- b) Draw the Nyquist-plot of the open-loop control system with $K_P = 2$. (Hint: Use five appropriate data points with $\omega = 0.1/0.7/1/2/10 \frac{\text{rad}}{\text{s}}$)



c) Is the closed-loop system stable for any K_p ? Please explain.

Yes, —

d) Determine the phase margin Φ_M for $K_p = 1$.

$$\Phi_M \approx 40^\circ$$

e) Determine the controller gain K_p so that the phase margin is $\Phi_M = 45^\circ$.

$$\text{For } \Phi_M = 45^\circ$$

$$\text{we need } K_p \approx 0.56$$

Good luck!