# Period of Examinations
# Winter Semester 2020/21 – 16th February Exam

HOCHSCHULE RHEIN-WAAL
Rhine-Waal University of Applied Sciences

**Module:Modelling and Simulation, Prof. Brandt**

**Examination**

Points: 40

Duration of examination: 120 Minutes (including 30 Minutes for technical issues)

Please write legibly!

Date: 16/02/2021

Name: Carine Silva Allen

Register No.: 24404

Course of Study: ME

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Please consider the exam rules, which are summarized in the exam section in Moodle. There you find a summary containing your different options to upload your.

Before you turn in your solution please sign the declaration in lieu of oath:

*I,* Carine Silva Allen, 24404 *[full name, matriculation number], hereby confirm in lieu of an oath that I am the person who was admitted to this examination. Further, I confirm that the submitted work is my own and was prepared without the use of any unauthorised aid or materials.*

Carine Silva Allen
*Signature*

You can print the declaration and then sign and scan it. Alternatively, you can also sign it digitally or transcribe it by hand and then sign and scan it.

Please make sure that all documents that you upload contain your name and matriculation number.

Good luck!

| Problem | Possible Points | Result |
|---|---|---|
| 1 | 10+8+2 | |
| 2 | 4 | |
| 3 | 6 | |
| 4 | 3+3 | |
| 5 | 2 | |
| 6 | 2 | |
| Sum | **40** | |

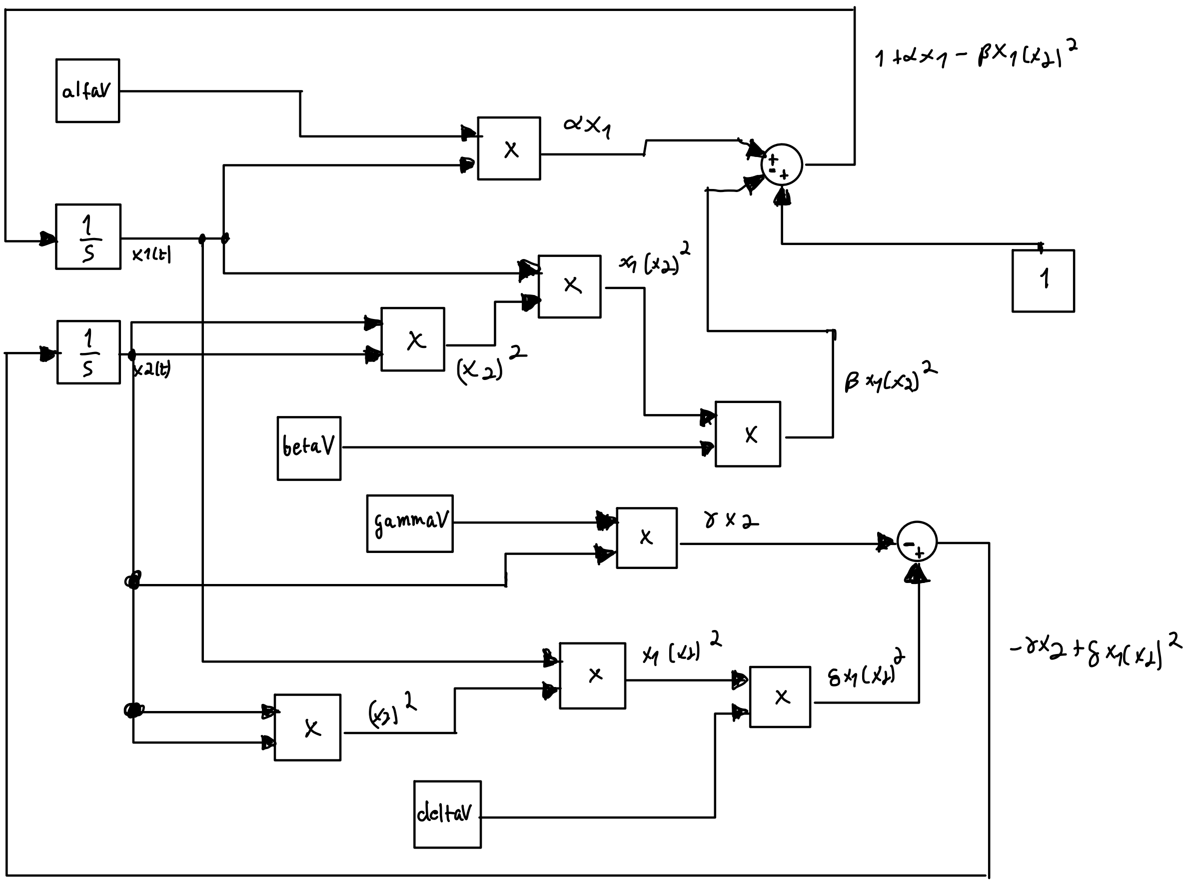
Register No.: 24404

1.) In the following modified prey-predator system the amount of prey is modelled by $x_1$ and the amount of predator by $x_2$. The coefficients $\alpha, \beta, \gamma, \delta$ are known constants with positive values (>0). In the model it is assumed that only the two modelled species interact and that there is an infinite food resource for the prey. However, the only food resource for the predators is assumed to be the prey.

$$\dot{x}_1 = 1 + \alpha x_1 - \beta x_1 (x_2)^2$$

$$\dot{x}_2 = -\gamma x_2 + \delta x_1 (x_2)^2$$

**a.)** Draw the block-diagram for a simulation with Simulink and indicate where the initial conditions have to be set. **(10 points)**



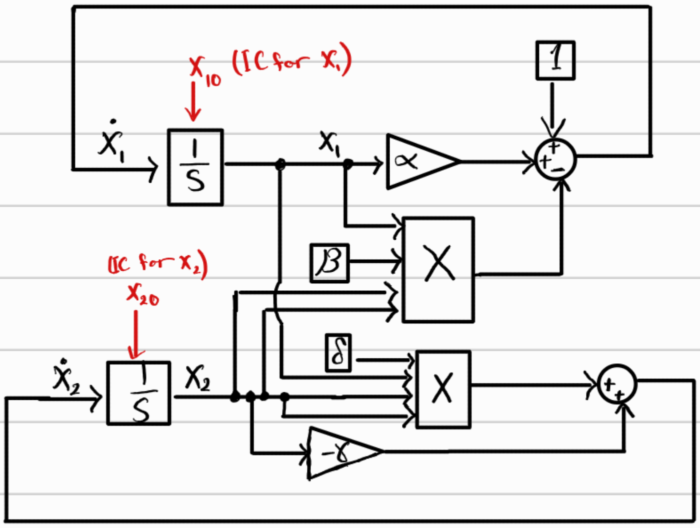

with alfaV = $\alpha$
betaV = $\beta$
gammaV = $\gamma$
detaV = $\delta$

Set $x_1(0)$ in the up integrator for $x_1(t)$

Set $x_2(0)$ in the bellow integrator for $x_2(t)$

$$\dot{x}_1 = 1 + \alpha x_1 - \beta x_1 (x_2)^2$$

$$\dot{x}_2 = -\gamma x_2 + \delta x_1 (x_2)^2$$




The initial conditions are set at the integerators of $x_1$ & $x_2$

b) linearize the prey-predator system about $x_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$\underline{\dot{x}} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underline{f}(\underline{x}) = \begin{bmatrix} 1 + \alpha x_1 - \beta x_1 (x_2)^2 \\ -\gamma x_2 + \delta x_1 (x_2)^2 \end{bmatrix}$$

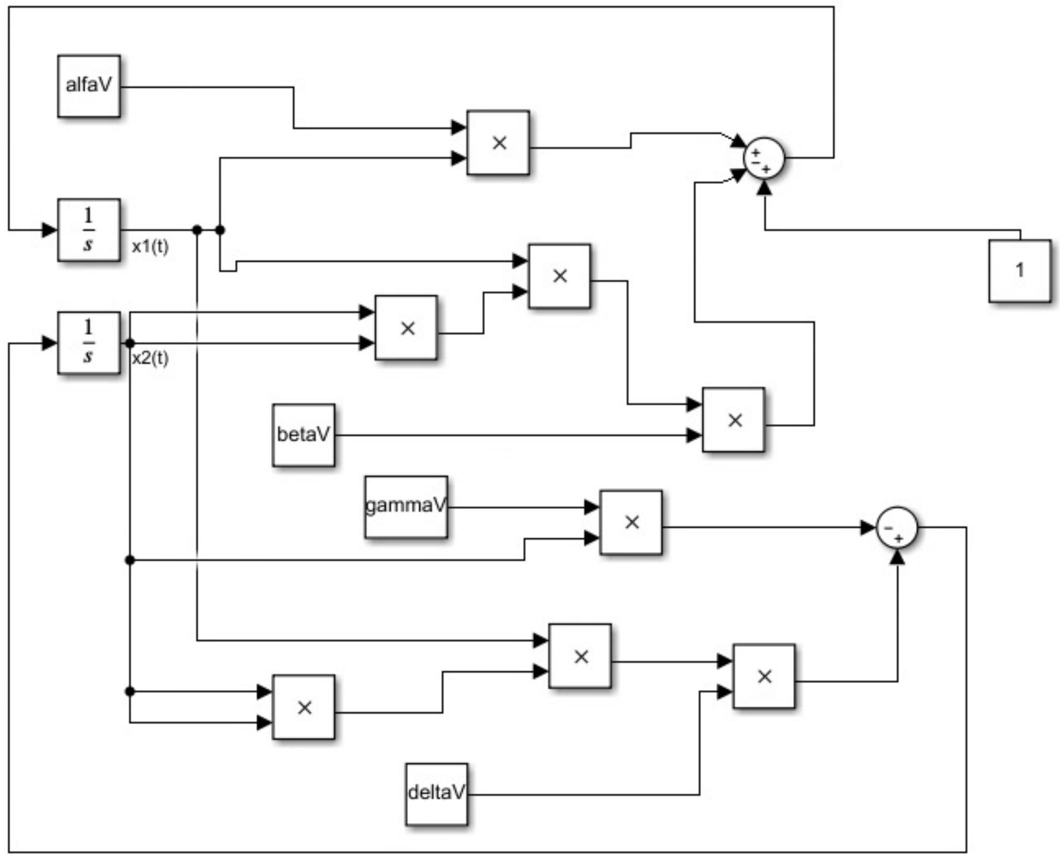
$$\underline{f}(\underline{x}) \simeq \underline{f}(\underline{x}_0) + \frac{\partial f}{\partial \underline{x}}\bigg|_{\underline{x} \to \underline{x}_0} \cdot \Delta \underline{x}$$

$$\underline{f}(x_0) = \begin{bmatrix} 1 + 0 - 0 \\ -0 + 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$ (There is no equilibrium!)

$$\underline{\underline{J}} = \begin{bmatrix} \frac{\partial f(x)}{\partial x_1} & \frac{\partial f(x_1)}{\partial x_2} \\ \frac{\partial f(x_2)}{\partial x_1} & \frac{\partial f(x_2)}{\partial x_2} \end{bmatrix} = \begin{bmatrix} \alpha - \beta x_2^2 & -2\beta x_1 x_2 \\ \delta x_2^2 & -\gamma + 2\delta x_1 x_2 \end{bmatrix}, \quad \underline{\underline{J}}_{x \to x_0} = \frac{\partial f}{\partial \underline{x}}\bigg|_{x \to x_0} = \begin{bmatrix} \alpha & 0 \\ 0 & -\gamma \end{bmatrix}$$

$$\Delta \underline{x} = \underline{x} - \underline{x}_0 = \begin{bmatrix} x_1 - 0 \\ x_2 - 0 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\underline{f}(\underline{x}) \simeq \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} \alpha & 0 \\ 0 & -\gamma \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \underbrace{\begin{bmatrix} \alpha & 0 \\ 0 & -\gamma \end{bmatrix}}_{\underline{\underline{A}}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\underline{x}} + \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{\underline{\underline{B}}\,\underline{u}}$$




Register No.: 24404

Register No.: 24404

b.) Linearize the prey-predator system above about $\mathbf{x} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. **(8 points)**

$$\underline{\underline{J}} = \begin{bmatrix} \alpha - \beta x_2^2 & -2\beta x_1 x_2 \\ \delta x_2^2 & -\gamma + 2\delta x_1 x_2 \end{bmatrix}$$

$$f \approx f(x) + \underline{\underline{J}}\big|_{x = x_0} (x - x_0)$$

$$f(x_0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$f \approx \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} \alpha & 0 \\ 0 & -\gamma \end{bmatrix} \begin{bmatrix} x_1 - 0 \\ x_2 - 0 \end{bmatrix}$$

$$f \approx \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} \alpha & 0 \\ 0 & -\gamma \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

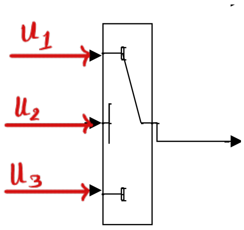
c.) Decide whether the operating point $\mathbf{x} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ is an equilibrium of the system or not. Justify your decision. **(2 points)**

The system is not in equilibrium, since the

$$f(x_0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \neq 0$$

Register No.: 24404

2.) A model that contains a discontinuity handles it by a switch. The switch has three inputs and one output. Explain how it works. **(4 points)**



Switch

$u_1$, $u_3$ signals that will be forwarded to the output.

$u_2$ : The triger input

The triger is compared to the threshold; as a consequence one of the signals $u_1$ or $u_3$ will be taken as an output.

When the second input satisfies the selected criteria ($U_2$ >= threashold or $U_2$ > threshold or $U_2$ ~= 0), the signal pass through input 1. Otherwise, the signal pass through input 3.

3) $\dddot{x} = \ddot{x} + \sin(x \cdot \dot{x})$

$x_1 = x$

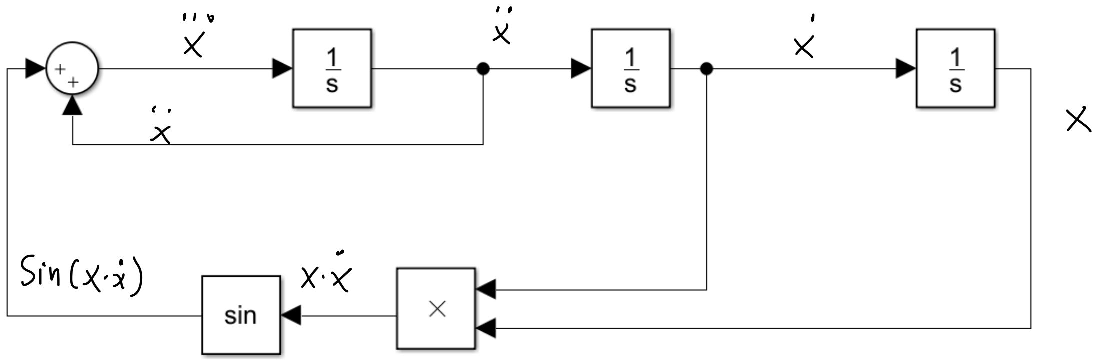
$x_2 = \dot{x}_1$

$x_3 = \dot{x}_2 = \ddot{x}$

$$\boxed{\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= x_3 + \sin(x_1 x_2) \end{aligned}}$$

Register No.: 24404

3.) Give the ODE, which is modelled by the following Simulink model. Then, transform it into a system of first order differential equations (ODEs). **(6 points)**




$$\dddot{x} = \sin(x \cdot \dot{x}) + \ddot{x}$$

$x = v_1$ $\qquad \dot{v}_1 = v_2$

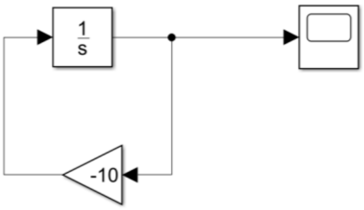
$\dot{x} = v_2$ $\qquad \dot{v}_2 = v_3$

$\ddot{x} = v_3$ $\qquad \dot{v}_3 = \sin(v_1 \cdot v_2) + v_3$

$$\begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \\ \dot{v}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \sin(v_1 \cdot v_2)$$

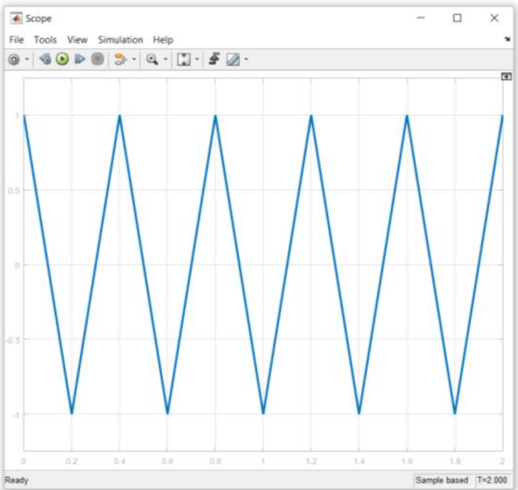
Register No.: 24404

**4.)** The following Simulink model describes the "test equation" $\dot{x}(t) = -\alpha x(t)$ with $\alpha = 10$.




Using the fixed step solver "ode1" in Simulink (explicit Euler method) with a step size of *h=0.2* and starting from an initial condition $x(0) = 1$ leads to the following result:

We have numerical instability.




Bad combination between ode-solver and step-size (in this case too big)

a.) Why is the displayed signal oscillating? **(3 points)**

it's a bad combination of the numerical method and the step size h

b.) How would you choose the step size for this model. Justify your choice. **(3 points)**

$(1-\alpha h) > 0 \Longrightarrow 1 > \alpha h \Longrightarrow h < \frac{1}{\alpha} \Longrightarrow h < 0.1$

we can get a usable solution when $h < \frac{1}{\alpha}$

So,

$h < \frac{1}{10}$, the $h < 0.1$

Register No.: 24404

5.) Are the following ODEs linear or nonlinear? Give the order of all ODEs! **(2 points)**

| | |
|---|---|
| $\ddot{y} = y + 3 + y^0 \cdot y^0$ | linear ; 2nd order |
| $\cos(y) + y^1 = \dot{y}$ | non-linear ; 1st order |
| $3(\dot{y} + 4\ddot{y}) = 9a^2 * \ddot{y}$ ; with $a = 100$ | linear ; 2nd order |
| $y * y = \dddot{y}$ | non-linear ; 3rd order |

6.) What is the benefit of a variable step-size solver compared to a fixed-step solver?

**(2 points)**

Using a fixed step size, the solver can be slow in case of a small step size or lead to numerical instabillity, in case of a big step size. Ode 45 adapts the step size according to the system.

Using a variable step size solver, looks at the rate of change of the problem and adapts the step size for the problem, instead of using the same step size.

The Fixed-step solver is not reacting to any change in the system response which could miss some information, and even if we made the fixed step very small for not missing any informations the simulation would be very slow. unlike in a variable step-size solver, we don't have to give up speed for getting all the information correctly.