Period of Examinations
Summer Semester 2014

HOCHSCHULE RHEIN-WAAL
Rhine-Waal University of Applied Sciences

Study Course: **Mechanical Engineering / Mechatronic Systems Engineering / Electronics / Industrial Engineering**

Module Title: **Measurement Engineering and Controls**

Examination Part: **Measurement Engineering and Controls**

Points: 100

Duration: 120 Minutes

Please write legibly!

Date: ____________________

Family Name : ____________________ ____________________________

First Name: ____________________ Signature (Student)

Student No.: ____________________

FOR INTERNAL USE ONLY:

Question Number	Tick Questions Attempted	Points	Question Number	Tick Questions attempted	Transfer Points
1		/ 18	13		
2		/ 15	14		
3		/ 11	15		
4		/ 15	16		
5		/ 14	17		
6		/ 11	18		
7		/ 6	19		
8	Points from laboratory	/ 10	20		
9			21		
10			22		
11			23		
12			24		
	SUM			TOTAL	/ 100

Graded by		Checked by

Final Grade

Regular grading key.	
Adjusted grading key. (Please add the adjusted grading key to the exam-results)	

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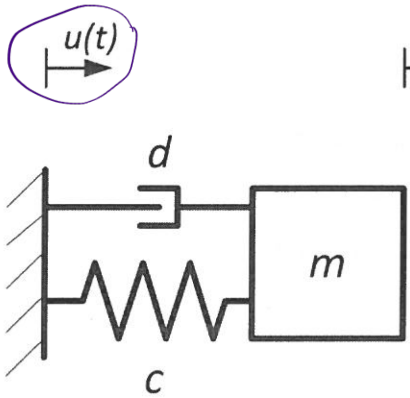
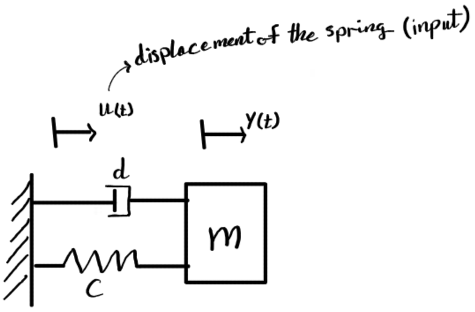
Question 1:

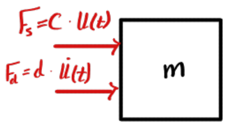
Points: **18**

A mechanical problem is given by the figure below. The model consists of a mass m and the position is described through the variable $y(t)$. The mass is attached through a parallel spring-damper arrangement (with spring constant c and damping constant d) to a rigid wall. Its displacement is described through the variable $u(t)$.



a) Determine the equation of motion of the system where $u(t)$ is the input variable and $y(t)$ represents the output variable of the system.





Equation of motion

$$\sum F_x = \boxed{m\,\ddot{y}(t) = d\,\dot{u}(t) + c\,u(t)}$$

b) Determine the transfer function of the system $G(s) = \frac{Y(s)}{U(s)}$.

$\mathcal{L}: \; m s^2 Y(s) = U(s)(ds + c)$

$$\boxed{G(s) = \frac{Y(s)}{U(s)} = \frac{ds + c}{m s^2}} \quad TF$$

Let´s assume a system is represented by the transfer function

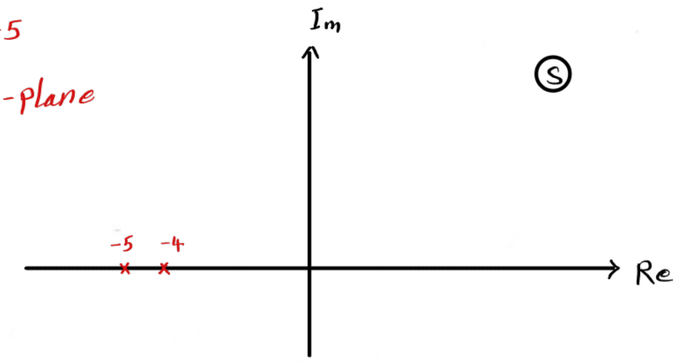
$$G(s) = \frac{Y(s)}{U(s)} = \frac{10s + 25}{s^2 + 9s + 20} = \frac{10s+25}{(s+4)(s+5)}$$

c) Is the system stable? Please explain.

Since we have two negative poles $P_1 = -4$, $P_2 = -5$

$\Rightarrow$ The poles will be in the LHP of the S-plane

$\Rightarrow$ Yes, system will be stable!



d) Solve the output $y(t)$ for an ideal step input, if all initial conditions are zero.

$$Y(s) = \underbrace{U(s)}_{U(s) = \frac{1}{s}} \cdot \frac{10s+25}{(s+4)(s+5)} = \frac{10s+25}{s(s+4)(s+5)} = \frac{A}{s} + \frac{B}{(s+4)} + \frac{C}{(s+5)}$$

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$$\Rightarrow 10s+25 = A(s+4)(s+5) + Bs(s+5) + Cs(s+4)$$

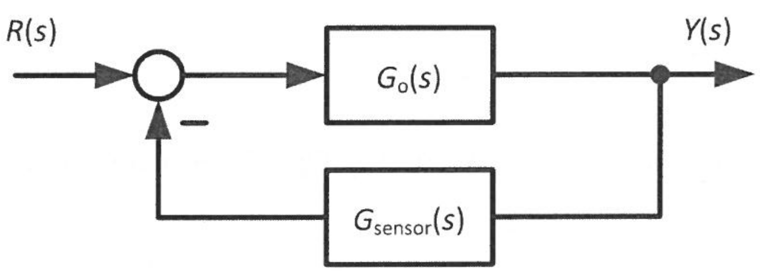
at $s=0$ $\boxed{A = \frac{25}{20} = \frac{5}{4}} = 1.25$, at $s=-4$ $\boxed{B = \frac{15}{4}}$, at $s=-5$ $\boxed{C=-5}$

$$Y(s) = \frac{5/4}{s} + \frac{15/4}{(s+4)} - \frac{5}{(s+5)}$$

$$\mathcal{L}^{-1}: \boxed{y(t) = \frac{5}{4}(t) + \frac{15}{4}e^{-4t} - 5e^{-5t}}$$

Question 2:

Points: **15**



A system $G_o(s)$ with the transfer function

$$G_o(s) = \frac{10}{s}$$

is controlled according to the block diagram above.

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a) Calculate the closed-loop transfer function $G(s) = \frac{Y(s)}{R(s)}$ of the system assuming an ideal sensor.

$$G(s) = \frac{Y(s)}{R(s)} = \frac{G_o(s)}{1+G_o(s)G_{sensor}(s)}$$

assuming an ideal sensor (input = output) $\Rightarrow G_{sen}(s) = 1$

$$R(s) \longrightarrow \boxed{\frac{G_o(s)}{1+G_o(s)G_{sen}(s)}} \xrightarrow{Y(s)} \quad \Rightarrow \quad \xrightarrow{R(s)} \boxed{\frac{10}{s+10}} \xrightarrow{Y(s)}$$

$$\boxed{G(s) = \frac{\frac{10}{s}}{1+\frac{10}{s}(1)} = \frac{10}{s+10}}$$

b) What is the system characteristic of the closed-loop system?

PT1

Let´s assume a sensor having a PT1-characteristic with

$$G_{sensor}(s) = \frac{1}{Ts+1}$$

where T is the time constant of the sensor.

c) Calculate the closed-loop transfer function $G(s) = \frac{Y(s)}{R(s)}$ of the system.

$$G(s) = \frac{\frac{10}{s}}{1+\frac{10}{s(Ts+1)}} = \frac{10(Ts+1)}{Ts^2+s+10} = \frac{10(Ts+1)}{a_2s^2+a_1s+a_0}$$

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d) What is the requirement for T so that the closed-loop system is stable?

$s^2 \quad a_2 = T \quad a_0 = 10$

$s^1 \quad a_1 = 1 \quad 0$

$s^0 \quad b_1 = \frac{-\begin{vmatrix} a_2 & a_0 \\ a_1 & 0 \end{vmatrix}}{a_1} = \frac{-\begin{vmatrix} T & 10 \\ 1 & 0 \end{vmatrix}}{1}$

$b_1 = 10$

$\boxed{T > 0}$

for a stable system.

e) Calculate the value of T, so that the damping ratio of the closed-loop system is $D = 0.5$.

$s^2 + \underbrace{\frac{1}{T}}_{2D\omega_0} s + \frac{10}{T} \quad \omega_0^2$

$\omega_0 = \sqrt{\frac{10}{T}}$

$2D \frac{\sqrt{10}}{\sqrt{T}} = \frac{1}{T} \Rightarrow T = \frac{1}{10(2D)^2} = \frac{1}{10(2(0.5))^2} = \frac{1}{10}$

$\boxed{T = 0.1}$

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Points:
11

Question 3:

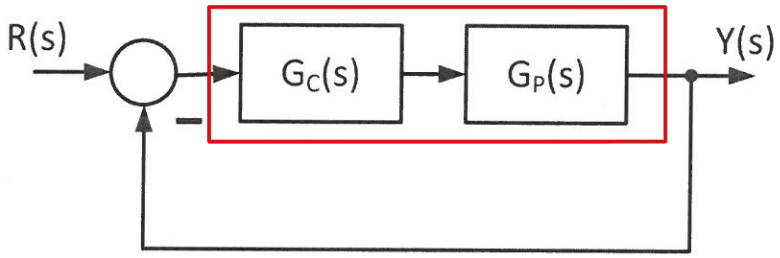
For the feedback system shown in the figure, with

$$G_C(s) = K$$

and

$$G_P(s) = \frac{3}{s^4 + 6s^3 + s^2 + 2s}$$

$G_c(s) \cdot G_p(s)$



Find the range of K for which the system is stable.

$$G_{cL}(s) = \frac{G_c(s)\,G_p(s)}{1 + G_c(s)\cdot G_p(s)} = \frac{\frac{3K}{s^4+6s^3+s^2+2s}}{1 + \frac{3K}{s^4+6s^3+s^2+2s}} = \frac{3K}{s^4+6s^3+s^2+2s+3K}$$

Routh Hurwitz methode:

s^4	$a_4 = 1$	$a_2 = 1$	$a_0 = 3K$
s^3	$a_3 = 6$	$a_1 = 2$	0
s^2	$b_1 = \frac{-\begin{vmatrix} 1 & 1 \\ 6 & 2 \end{vmatrix}}{6} = \frac{2}{3}$	$b_2 = \frac{18K}{6} = 3K$	0
s^1	$c_1 = \frac{\frac{4}{3} - 18K}{2/3}$, $c_1 = 2 - 27K$	$c_2 = 0$	
s^0	$d_1 = 3K$		

For a stable system

$3K > 0$ & $2 - 27K > 0$

$K > 0$ & $K < \frac{2}{27}$

$\therefore \boxed{0 < K < \frac{2}{27}}$

For a stabe system

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Question 4:

Points: **15**

The open-loop system $G_S(s)$ is described by the transfer function

$|G_{dB}| = 20Log(5) = 13.98 \approx 14$

$$G_S(s) = \frac{s+50}{s(s+10)} \qquad G(s) = \frac{\cancel{50}^{\,5}\left(\frac{s}{50}+1\right)}{\cancel{10}s\left(\frac{s}{10}+1\right)} = 5 \cdot \left(\frac{s}{50}+1\right) \cdot \frac{1}{s} \cdot \frac{1}{\left(\frac{s}{10}+1\right)}$$

a) Sketch the asymptotes of the Bode-diagram of the open-loop system including its magnitude and phase shift.

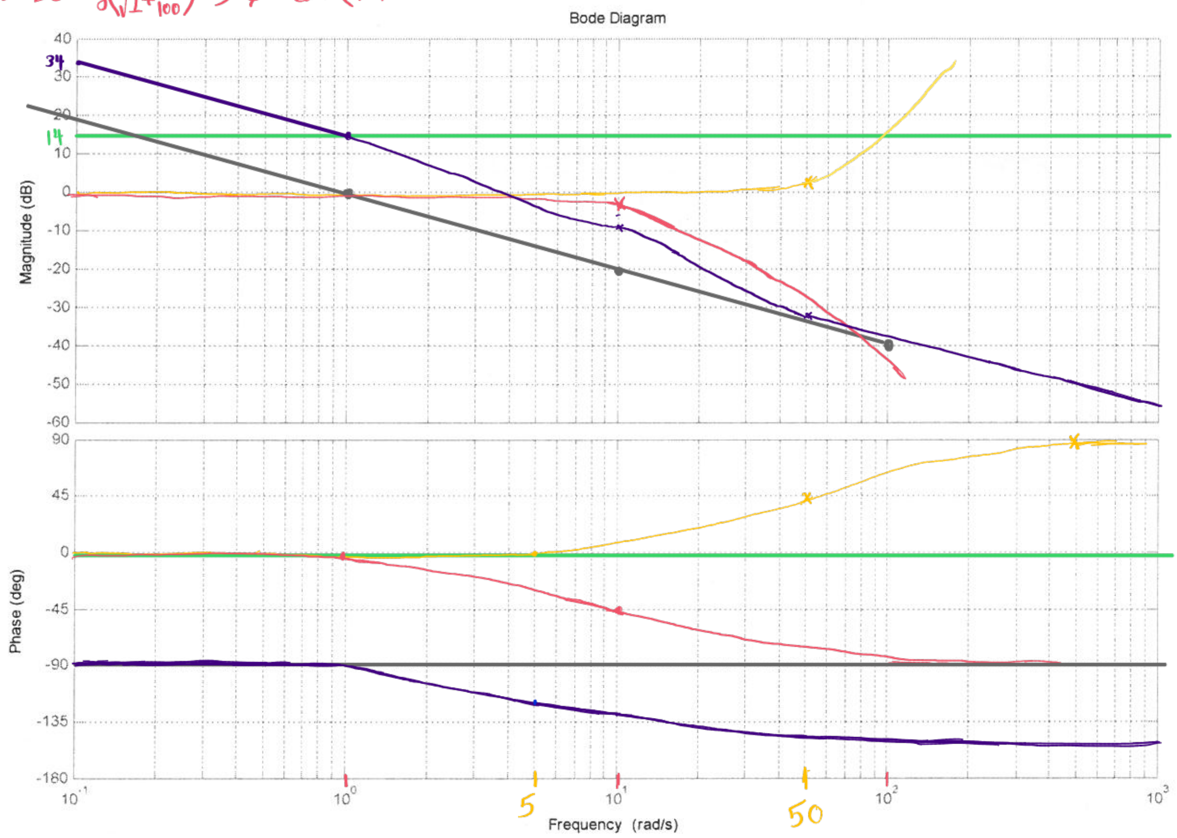
PT2

$G(s) = 5 \Longrightarrow G_{dB} = 20\log(5) \approx 14 \ , \ \phi = 0^\circ$

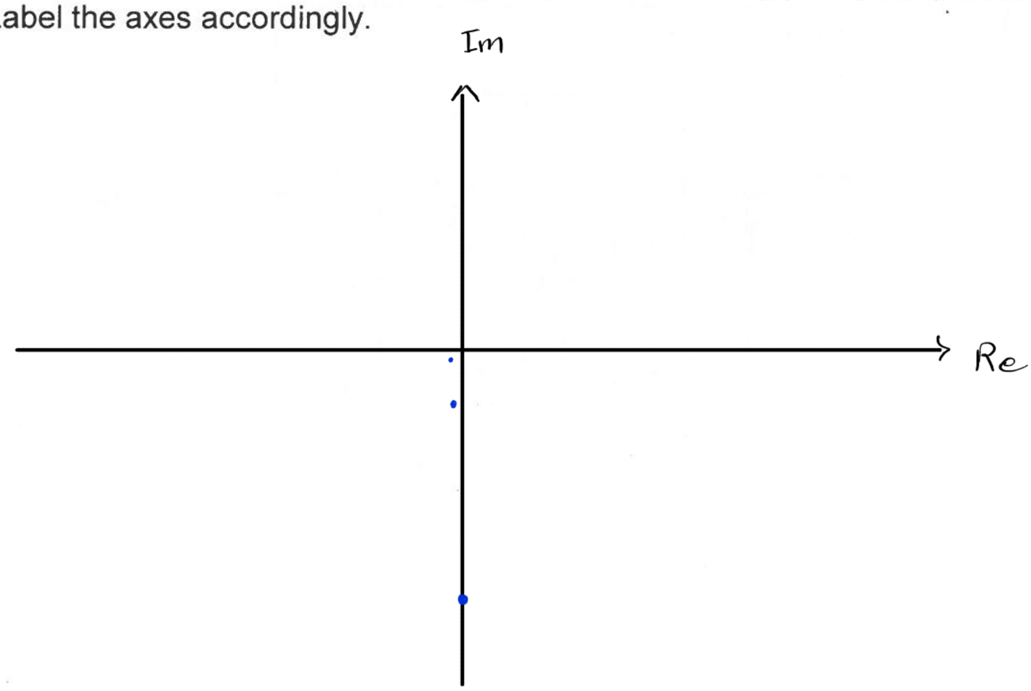
$G_{dB} = 20\log\left(\sqrt{\frac{w^2}{50^2}+1}\right) \ , \ \phi = \tan^{-1}\left(\frac{w}{50}\right)$

$G_{dB} = 20\log\left(\frac{1}{w}\right) \ , \ \phi = -90$

$G_{dB} = 20\log\left(\sqrt{1+\frac{w^2}{100}}\right) \ , \ \phi = \tan^{-1}\left(\frac{-w}{10}\right)$



b) Based on the sketched Bode-diagram, sketch the Nyquist plot qualitatively. Label the axes accordingly.



c) The open-loop system is controlled by a P-controller with the gain $K = 1$. Is the closed-loop system stable? Please explain by means of the Nyquist-criteria using the Bode-diagram above.

Question 5:

Points: **14**

Find the equivalent transfer function, $G(s) = \frac{Y(s)}{U(s)}$, for the system shown in the figure below, with

$G_1(s) = 5$, $G_2(s) = \frac{1}{s+2}$, $G_3(s) = \frac{1}{s}$, and $G_4(s) = \frac{2}{s+5}$.

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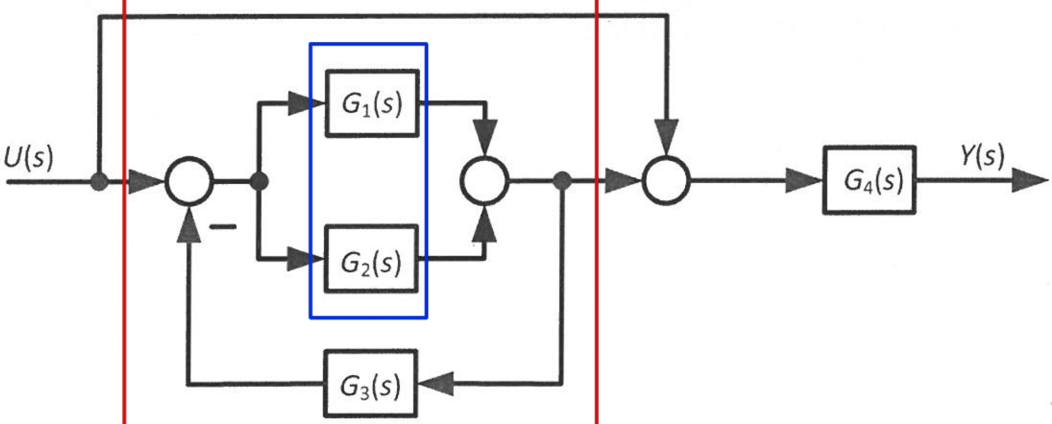
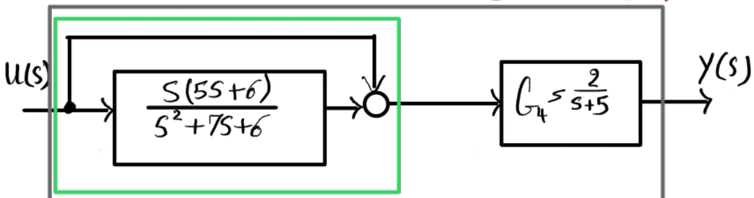
$$G_1+G_2=\frac{5s+6}{s+2}$$



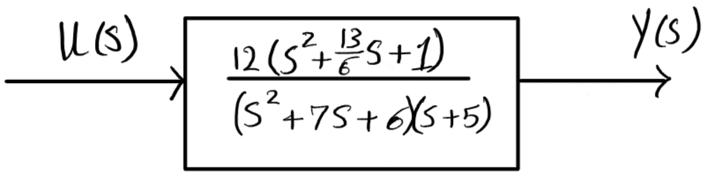
Feedback loop $\dfrac{G_1G_2}{1+G_1G_2G_3} = \dfrac{\frac{5s+6}{s+2}}{1+\frac{5s+6}{s(s+2)}} = \dfrac{s(5s+6)}{s^2+7s+6}$



$$\frac{S(5S+6)}{S^2+7S+6}+1$$

$$=\frac{6S^2+13S+6}{S^2+7S+6} \quad * \quad \frac{2}{S+5}$$

$$\frac{12\left(S^2+\frac{13}{6}S+1\right)}{(S^2+7S+6)(S+5)}$$



$$\boxed{\therefore G(S) = \frac{Y(S)}{U(S)} = \frac{12\left(S^2+\frac{13}{6}S+1\right)}{(S^2+7S+6)(S+5)}}$$

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Points: 11

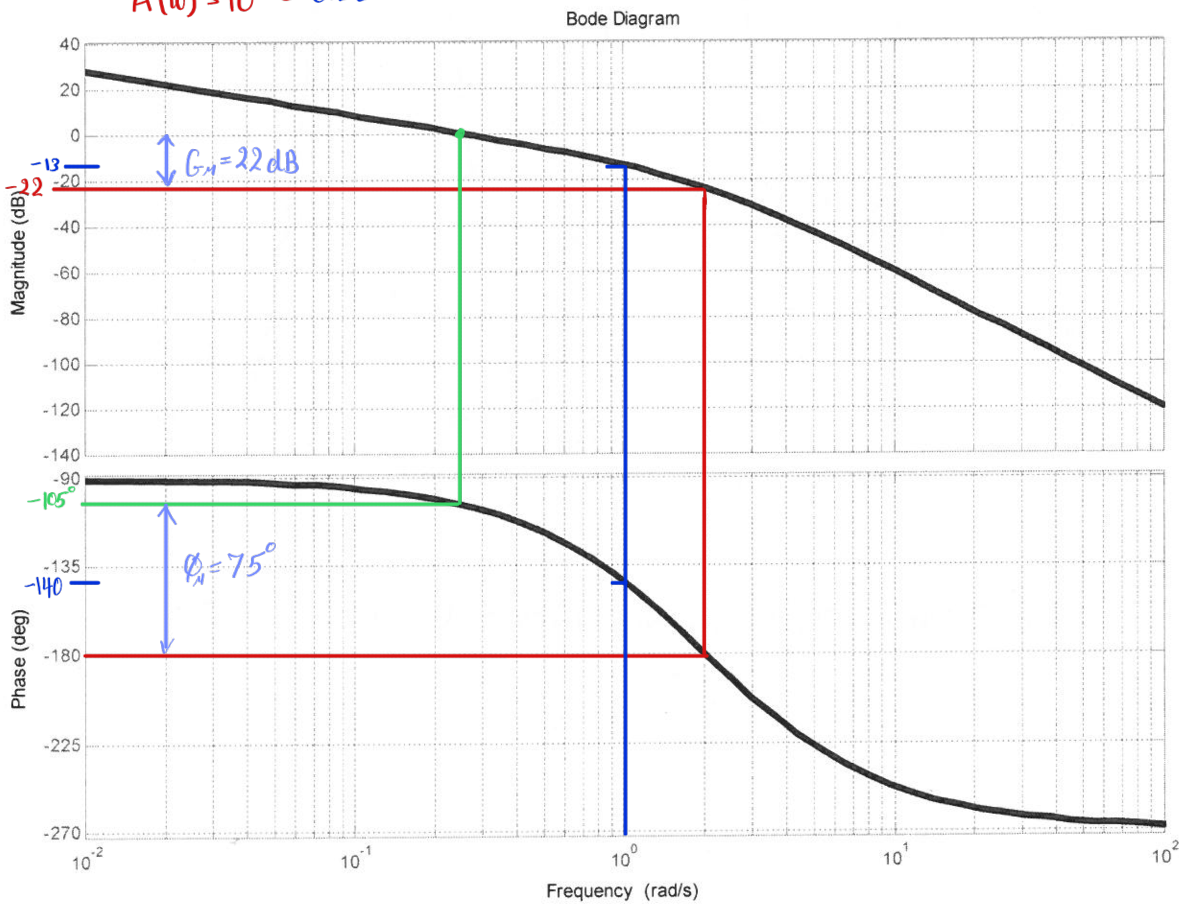
Question 6:

A unity negative feedback system has the open-loop transfer function,

$$G(s) = \frac{K}{s(s+2)^2}$$

The Bode-diagram for $K = 1$ is given in the figure below.

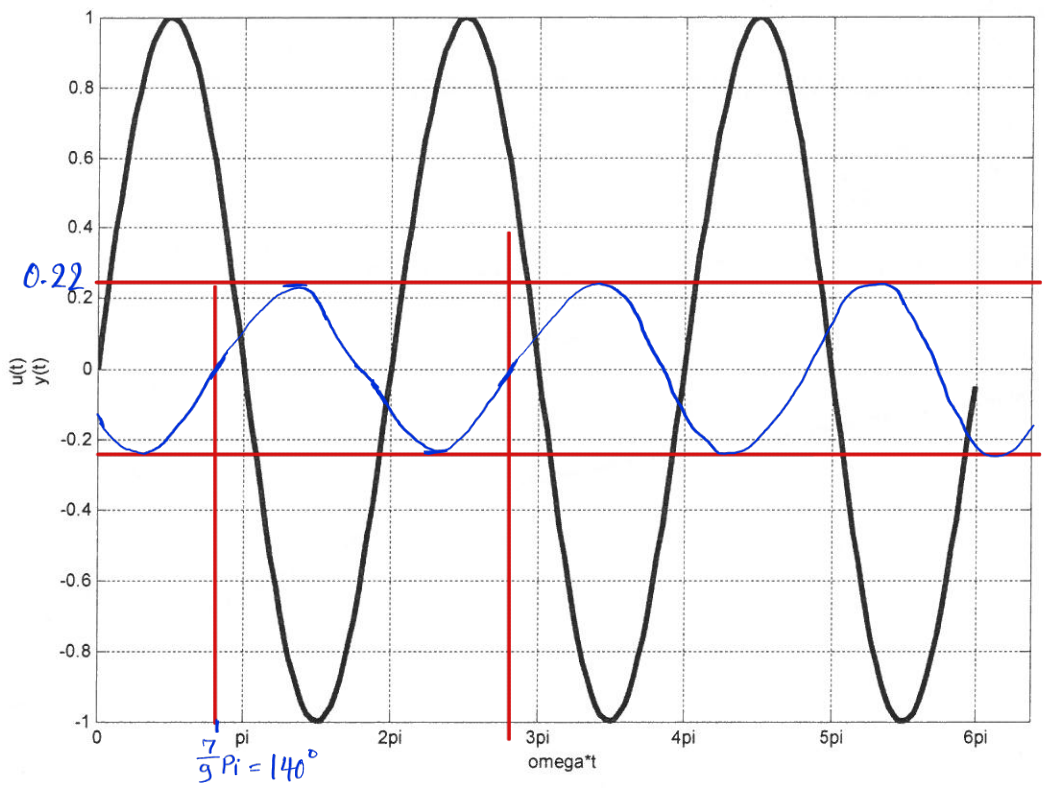
$A(\omega) \approx 10^{\frac{-13}{20}} \approx 0.22$



a) The input $u(t)$ at the open-loop system $G(s)$ with $K = 1$, is a sinusoidal-signal with an amplitude $\hat{u} = 1$ and a frequency of $\omega = 1\frac{rad}{s}$, see figure on next page. Use the Bode-diagram and draw the expected output $y(t)$ of the open-loop system $G(s)$ into the diagram on the next page.

Both input & output have same frequency but different amplitude & phase shift, thus we can use the Bode diagram above to investigate the output amplitude & phase shift at $\omega = 1$ rad/s

$A(\omega)_{out} = 10^{\frac{-13dB}{20}} \approx 0.22$ & $\phi_{out} = -140°$



b) Find the gain margin and phase margin for $K = 1$.

$G_M = 22\,dB$

$\phi_M = 75^{\circ}$

c) Find the gain margin for $K = 4$.

$A(\omega) = 0.126$ for $K = 1$

$\therefore A(\omega) = 4 * 0.126 \approx 0.5$ for $K = 4$

$a = \frac{1}{A(\omega)} \approx 2$

$G_M = 20\log(2) = 5.96\ dB$

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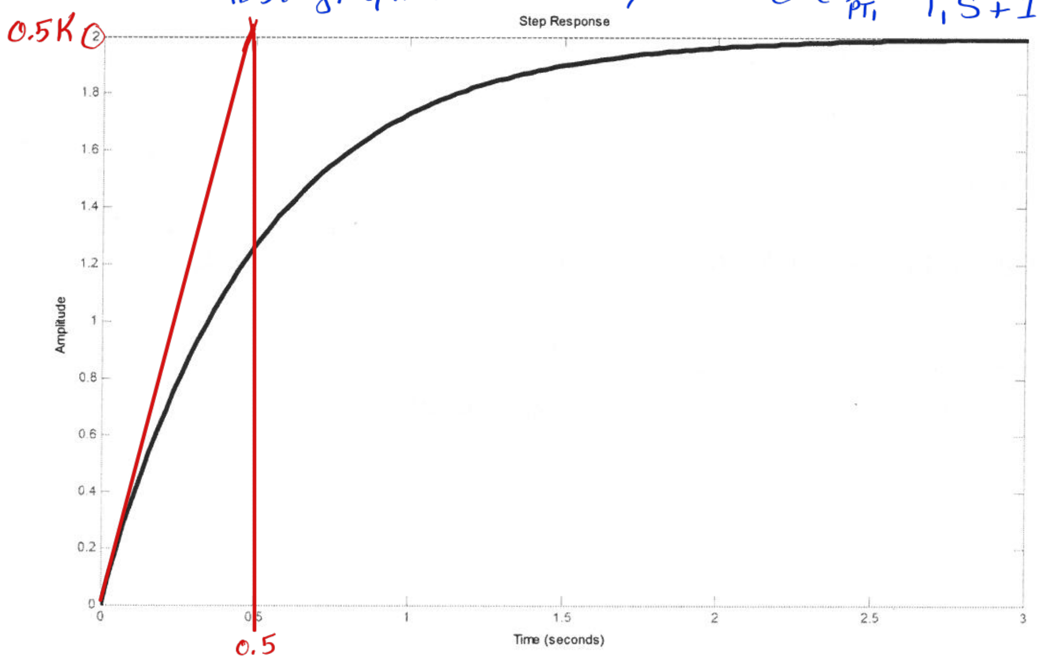
Question 7: Points: **6**

a) The response of a plant to a step with the amplitude of 0.5 is given by the measurement, shown in the graph below. Determine the transfer function of the plant $G_P(s)$ by analysing the measurement.

It's a graph of PT1-system $G(s)_{PT_1} = \frac{K}{T_1 S + 1}$

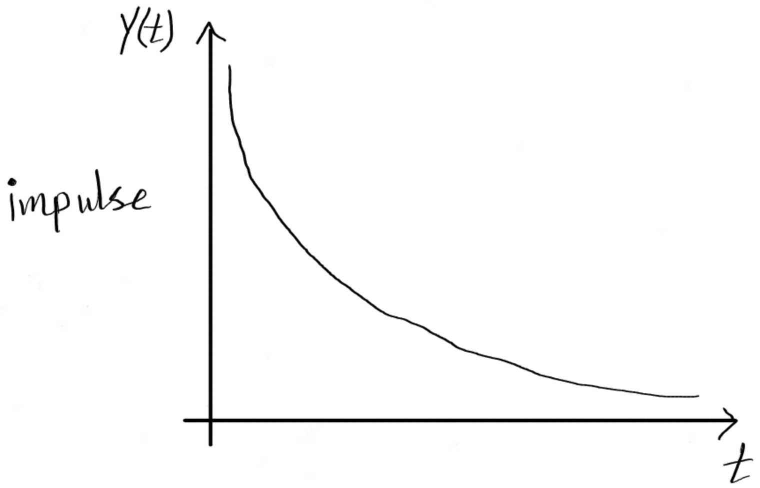


from graph

$$\text{amp.} = 0.5 \Rightarrow G(S=0) = \frac{2}{0.5} = 4 = K$$

$$\Rightarrow K = 4 \Rightarrow \quad \therefore \boxed{G(S) = \frac{4}{0.5S+1}}$$

b) Sketch the impulse and ramp response of the plant $G_P(s)$.



Question 8: Points: **10**

No answer expected. Here you will get the points from the laboratory.

Good luck!