Period of Examinations
Winter Semester 2019/20

HOCHSCHULE RHEIN-WAAL
Rhine-Waal University of Applied Sciences

Course of study: Robotics / Module: SE 5 2910

Examination …

Points: 70

Duration of examination: 120 Minutes

Please write legibly!

Date: ____________________

Name: ____________________

Register No.:____________________

Study Course: ____________________

Hints:

Make sure that you enter your matriculation number in the header of each examination sheet.

Basically, multiple answers to each task or question can be correct. Please mark the box or boxes (☒) with the correct answers or enter the solutions into the appropriate field (___). Wrong answers to multiple choice tasks only lead to a reduction of points within the same task.

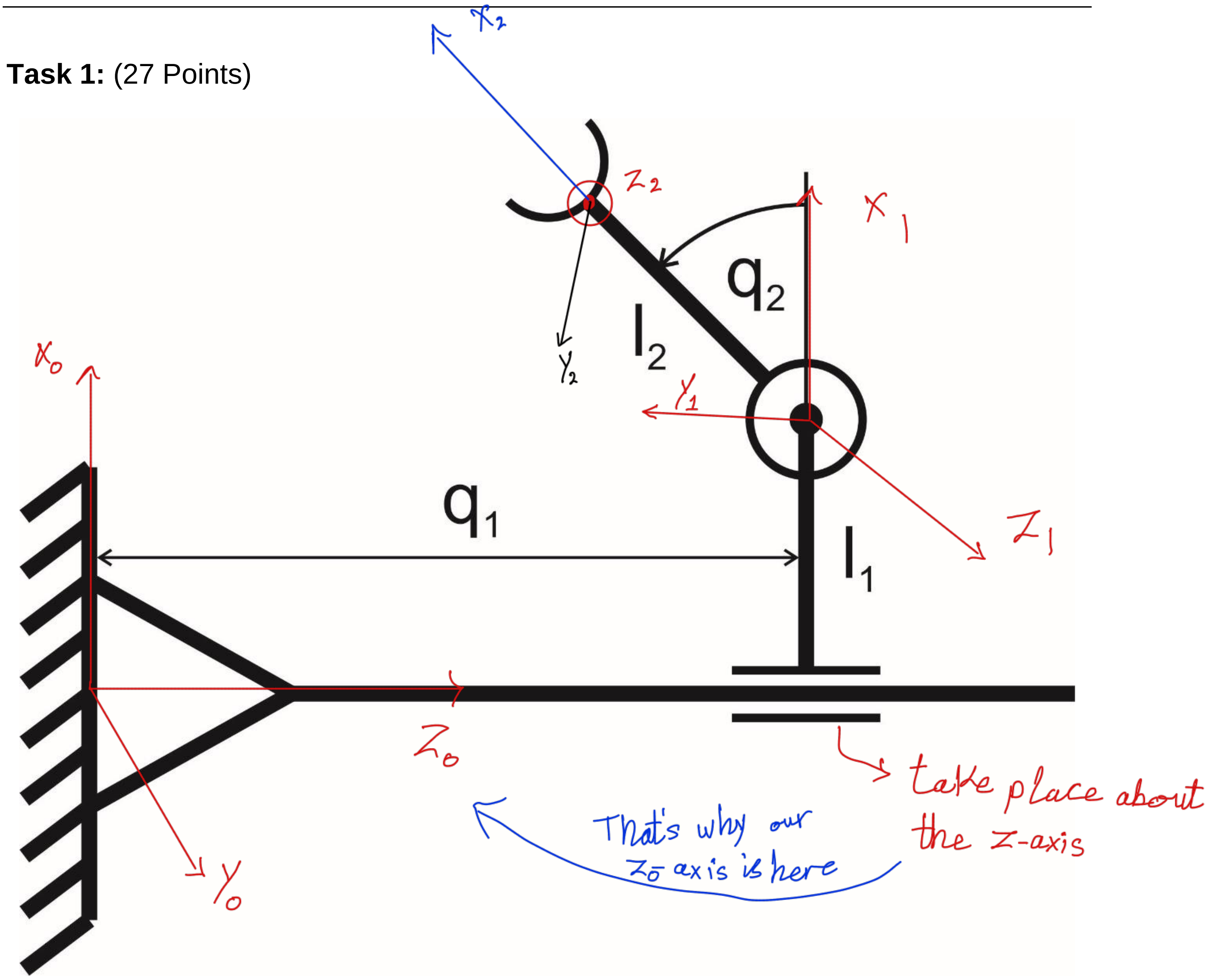
Example:

(2 points)

☐ wrong	☐ wrong	☒ wrong	☒ wrong
☒ right	☐ right	☒ right	☒ right
☐ wrong	☐ wrong	☐ wrong	☒ wrong
☒ right	☒ right	☐ right	☐ right
_______	_______	_______	_______
2 points	1 point	0 points	0 points

Matriculation number.: ____________________

Task 1: (27 Points)



A planar robot with two joints is considered. The first joint is a prismatic one, while the second is revolute. Furthermore $q_1 > 0$ and $-\frac{\pi}{2} \leq q_2 \leq \frac{\pi}{2}$ holds.

a) (3 Points)

Draw the necessary local coordinate systems due to the Denavit-Hartenberg algorithm in the illustration of the robot under consideration.

b) (4 Points)

Compute the parameters according to the Denavit-Hartenberg algorithm of the robot under consideration.

In order to see if there is an offset angel, we put $q_2 = 0$, and see if x_0 & x_1 are parallel or not.

θ	0	q_2
d	q_1	0
a	L_1	L_2
α	$-\frac{\pi}{2}$	0

Matriculation number.: ____________________

Appendix:

Denavit-Hartenberg algorithm

Definition of the coordinate systems:

1. Definition of the initial coordinate system at the base of the robot. The z_0-axis lies within the axis of movement of the first joint in direction of the kinematic chain. Define the x_0- and y_0-axes in order to generate an orthogonal right-handed system.
2. For i=1,…,n-1 do the following steps:
3. The z_i-axis is to arrange in direction of the axis of movement of joint i+1 (rotational or translational joint)
4. The origin of coordinate system lies within the intersection point of the z_i- and z_{i-1}-axes or within the intersection point of the z_i-axis with the collective perpendicular of the z_i- and z_{i-1}- axes
5. In case of an intersection of z_{i-1}- and z_i-axis, the x_i-axis is orthogonal to both z_{i-1}- and z_i-axis. Otherwise the x_i-axis lies in direction of the perpendicular between z_i- and z_{i-1}-axis.
6. Choose the y_i-axis in order to generate an orthogonal right-handed system $(x,y,z)_i$.
7. Definition of the TCP coordinate system: The z_n-axis lies in direction of z_{n-1}-axis. The x_n-axis is orthogonal to both z_n- and z_{n-1}-axis.

Denavit-Hartenberg parameters:

θ_i Angle between x_{i-1}-axis and x_i-axis around the z_{i-1}-axis-

d_i Distance from origin of coordinate system $(x,y,z)_{i-1}$ to the intersection point of z_{i-1}-axis and x_i-axis, measured along the z_{i-1}-axis.

a_i Distance from intersection point of z_{i-1}-axis and x_i-axis to the origin of coordinate system $(x,y,z)_i$, measured along the x_i-axis (or the shortest distance between z_{i-1}-axis and z_i-axis).

α_i Angle between the z_{i-1}-axis and z_i-axis around the x_i-axis.

Transformation matrix

$$\mathbf{A}_i = \mathrm{Rot}_{z,\theta_i}\mathrm{Trans}_{z,d_i}\mathrm{Trans}_{x,a_i}\mathrm{Rot}_{x,\alpha_i} =$$
$$\begin{bmatrix} \cos(\theta_i) & -\sin(\theta_i)\cos(\alpha_i) & \sin(\theta_i)\sin(\alpha_i) & a_i\cos(\theta_i) \\ \sin(\theta_i) & \cos(\theta_i)\cos(\alpha_i) & -\cos(\theta_i)\sin(\alpha_i) & a_i\sin(\theta_i) \\ 0 & \sin(\alpha_i) & \cos(\alpha_i) & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Matriculation number.: ______________________

---

c) (4 points)

Compute the local transformation matrices $\mathbf{A}_i$.

$$\mathbf{A}_i = \mathrm{Rot}_{z,\theta_i}\mathrm{Trans}_{z,d_i}\mathrm{Trans}_{x,a_i}\mathrm{Rot}_{x,\alpha_i} = \begin{bmatrix} \cos(\theta_i) & -\sin(\theta_i)\cos(\alpha_i) & \sin(\theta_i)\sin(\alpha_i) & a_i\cos(\theta_i) \\ \sin(\theta_i) & \cos(\theta_i)\cos(\alpha_i) & -\cos(\theta_i)\sin(\alpha_i) & a_i\sin(\theta_i) \\ 0 & \sin(\alpha_i) & \cos(\alpha_i) & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{A}_1 = \begin{bmatrix} \cos(0)=1 & -\sin(0)\cos(0)=0 & 0 & L_1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & q_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\cos(q_2)$

$\sin(q_2)$

$$\mathbf{A}_2 = \begin{bmatrix} C_2 & -S_2 & 0 & L_2C_2 \\ S_2 & C_2 & 0 & L_2S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

d) (4 points)

Compute the forward kinematics of the robot under consideration. Moving 0-coordinates to 2-coordinates

we could also get them from here but, we have to make sure that q_1 & q_2 are independant from each-other.

$x_o = L_1 + L_2 \cdot C_2 \longrightarrow q_2 = \cos^{-1}\left(\frac{x_o - L_1}{L_2}\right)$

$y_o = 0$

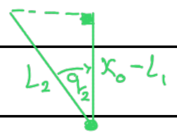
$z_o = q_1 - L_2 S_2$

e) (4 points)

Compute the inverse kinematics of the robot under consideration.

$q_1 = \pm\sqrt{L_2^2 - (x_o - L_1)^2} + z_o$

$q_2 = \cos^{-1}\left(\frac{x_o - L_1}{L_2}\right)$



f) (4 points)

Compute the Jacobian of the robot under consideration.

Matriculation number.: ____________________

$$\underline{\underline{J}} = \begin{bmatrix} \frac{\partial x}{\partial q_1} & \frac{\partial x}{\partial q_2} \\ \frac{\partial z}{q_1} & \frac{\partial z}{q_2} \end{bmatrix}$$

$$\mathbf{J} = \begin{bmatrix} 0 & -L_2 S_2 \\ 1 & -L_2 C_2 \end{bmatrix}$$

g) (4 points)

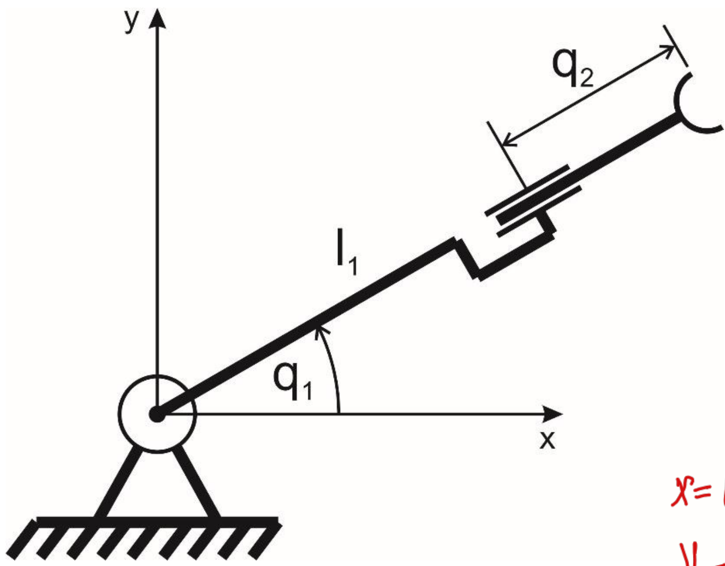
Calculate possible singular configurations of the robot under consideration.

$$\det(\underline{\underline{J}}) = 0 - (-L_2 S_2) = L_2 S_2$$

singular at $q_2 = 0, \pi, \ldots$

Matriculation number.: ____________________

Task 2: (8 points)



$x = (L_1 + q_2)\cdot\cos(q_1)$

$y = (L_1 + q_2)\cdot\sin(q_1)$

A planar robot with a rotational and a prismatic joint is considered.

a) (4 points)

Compute the Jacobian matrix of the system under consideration.

$$\mathbf{J} = \begin{array}{c} \\ f_x \\ f_y \end{array}\begin{array}{c} \begin{array}{cc} q_1 & q_2 \end{array} \\ \begin{bmatrix} -(L_1+q_2)\sin(q_1) & \cos(q_1) \\ (L_1+q_2)\cos(q_1) & \sin(q_1) \end{bmatrix} \end{array}$$

b) (4 points)

For a current configuration $\boldsymbol{q} = \begin{bmatrix} \frac{\pi}{4} & \frac{1}{4} \end{bmatrix}'$, $\dot{\boldsymbol{x}} = \begin{bmatrix} 1 & \frac{1}{2} \end{bmatrix}'$ and $l_1 = \cancel{\frac{3}{4}}\ \frac{1}{2}$ holds. Compute the appropriate joint velocities.

$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \underline{\underline{J}} \cdot \dot{\underline{q}} \Rightarrow \dot{\underline{q}} = \underline{\underline{J}}^{-1} \cdot \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix}$

$\underline{\underline{J}} = \begin{bmatrix} -\frac{3\sqrt{2}}{8} & \frac{\sqrt{2}}{2} \\ \frac{3\sqrt{2}}{8} & \frac{\sqrt{2}}{2} \end{bmatrix}$

$\det(\underline{\underline{J}}) = -\frac{3}{4}$

$\underline{\underline{J}}^{-1} = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{3\sqrt{2}}{8} & -\frac{3\sqrt{2}}{8} \end{bmatrix} * \left(-\frac{4}{3}\right)$

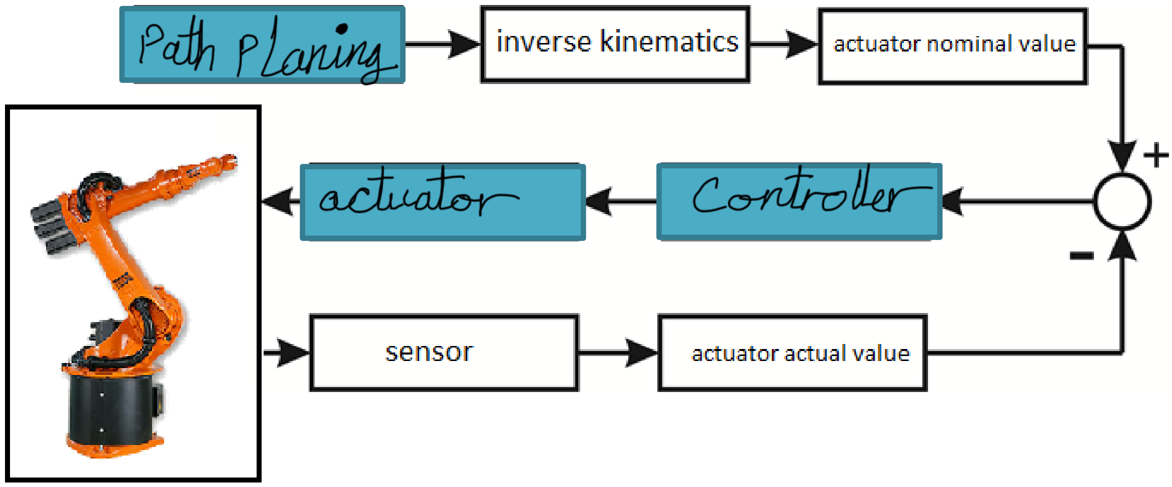
$\dot{q}_1 = -\frac{4\sqrt{2}}{6}\dot{x} + \frac{4\sqrt{2}}{6}\dot{y} = \frac{2\sqrt{2}}{3}(\dot{y} - \dot{x}) = -\frac{\sqrt{2}}{3}$

$\dot{q}_2 = -\frac{\sqrt{2}}{2}\dot{x} + \frac{\sqrt{2}}{2}\dot{y} = \frac{\sqrt{2}}{2}(\dot{y} - \dot{x}) = -\frac{\sqrt{2}}{4}$

$\underline{\underline{J}}^{-1} = \begin{bmatrix} -\frac{4\sqrt{2}}{6} & \frac{4\sqrt{2}}{6} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$

Matriculation number.: ____________________

Task 3: (3 points) Label all blocks in the diagram



Task 4: (2 points)

Is the given matrix a rotation matrix (give an explanatory statement)?

$$R = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 1 \\ 0.5 & 0 & 0 \end{bmatrix}$$

no, row & coulmn should be perpendicular & orthogonal

$R^T = \begin{bmatrix} 0.5 & 0 & 0.5 \\ 0 & 0.5 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

$\det(R) = 0.5 \begin{vmatrix} 0 & 0 \\ 0.5 & 1 \end{vmatrix} = 0$

No, because a determinant of a rotation matrix is always = 1

Task 5: (30 points)

1.) (2 points)

How many degrees of freedom does the end effector of a spatial robot have at maximum?

- ☐ 3
- ☐ 2
- ☒ 6
- ☐ 8

2.) (2 points)

Which of the statements concerning a rotation matrix **R** are true?

- ☐ $\mathbf{R}^T=\mathbf{R}$
- ☒ $\mathbf{R}^T=\mathbf{R}^{-1}$
- ☐ $\mathbf{R}^T=-\mathbf{R}$
- ☐ $\mathbf{R}=-\mathbf{R}$

3.) (2 points)

What are the properties of a rotation matrix **R**?

- ☒ **R** is square.
- ☒ **R** is orthogonal.
- ☐ **R** is diagonal.
- ☐ **R** is singular.

4.) (2 points)

How many solutions does the forward kinematics of a spatial serial manipulator usually have?

- ☐ 3
- ☐ 6
- ☒ 1
- ☐ none

5.) (2 points)

The mapping of the joint velocities onto the end effector velocities by use of a Jacobian is

- ☒ linear.
- ☐ non-linear. → Position
- ☐ singular.
- ☐ proportional.

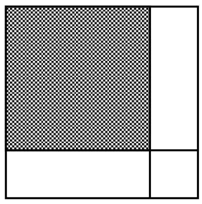
6.) (2 points)

How many independent parameters does a rotation matrix have at maximum to describe the orientation of a coordinate system with respect to another coordinate system?

- ☐ 9
- ☐ 1
- ☒ 3
- ☐ 0

7.) (2 points)

The upper left 3x3 sub-matrix of a homogenous transformation matrix has the meaning of a



- ☐ translation.
- ☒ rotation.
- ☐ bias.
- ☐ scaling.

8.) (2 points)

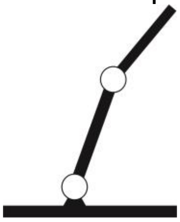
The norm of a row or a column of a rotation matrix is equal to

☒ 1.
☐ π.
☐ $\frac{\pi}{2}$.
☐ -1.

9.) (2 points)

The workspace of a plain robot with two rotational joints can have the shape of

☐ a circle.
☒ a circular ring.
☐ a square.
☐ an ellipse.



if both length are the same → a circle

10.) (2 points)

In which cases is the pseudo-inverse of a Jacobian typically used?

☐ The Jacobian is square and diagonal.
☐ The TCP is outside the workspace.
☒ The Jacobian is of non-square type.
☒ The number of joints is larger than the amount of degrees of freedom of the end effector.

11.) (2 points)

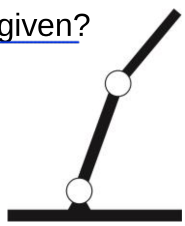
Singularities appear typically if

☒ the Jacobian matrix is singular.
☒ the robot is in a stretched position.
☐ the Jacobian matrix is square.
☐ the end effector leaves the planned path.

12.) (2 points)

How many possible solutions does the inverse kinematics of the pictured robot have, assuming only the position of the tool center point is given?

☐ one solution
☐ no solution
☒ two solutions
☐ infinite amount of solutions



13.)(2 points)

What is the dimension of the Jacobian of a robot with 7 actuators and 6 end effector degrees of freedom?

☐ 7x6
☐ 6x7x6
☒ 6x7
☐ 6x6

14.) (2 points)

How many variables of a transformation matrix (as used in robotics for the description of the kinematical relationship of two links coupled by a joint) depend on time?

- ☐ 4
- ☐ 3
- ☐ 2
- ☒ 1

15.) (2 points)

How many actuators does a robot need at least, assuming three independent degrees of freedom for the end effector are required?

- ☐ 1 actuator
- ☐ 6 actuators
- ☒ 3 actuators
- ☐ 5 actuators

Appendix:

Denavit-Hartenberg algorithm

Definition of the coordinate systems:

1. Definition of the initial coordinate system at the base of the robot. The z_0-axis lies within the axis of movement of the first joint in direction of the kinematic chain. Define the x_0- and y_0-axes in order to generate an orthogonal right-handed system.
2. For i=1,…,n-1 do the following steps:
3. The z_i-axis is to arrange in direction of the axis of movement of joint i+1 (rotational or translational joint)
4. The origin of coordinate system lies within the intersection point of the z_i- and z_{i-1}-axes or within the intersection point of the z_i-axis with the collective perpendicular of the z_i- and z_{i-1}- axes
5. In case of an intersection of z_{i-1}- and z_i-axis, the x_i-axis is orthogonal to both z_{i-1}- and z_i-axis. Otherwise the x_i-axis lies in direction of the perpendicular between z_i- and z_{i-1}-axis.
6. Choose the y_i-axis in order to generate an orthogonal right-handed system $(x,y,z)_i$.
7. Definition of the TCP coordinate system: The z_n-axis lies in direction of z_{n-1}-axis. The x_n-axis is orthogonal to both z_n- and z_{n-1}-axis.

Denavit-Hartenberg parameters:

θ_i Angle between x_{i-1}-axis and x_i-axis around the z_{i-1}-axis-

d_i Distance from origin of coordinate system $(x,y,z)_{i-1}$ to the intersection point of z_{i-1}-axis and x_i-axis, measured along the z_{i-1}-axis.

a_i Distance from intersection point of z_{i-1}-axis and x_i-axis to the origin of coordinate system $(x,y,z)_i$, measured along the x_i-axis (or the shortest distance between z_{i-1}-axis and z_i-axis).

α_i Angle between the z_{i-1}-axis and z_i-axis around the x_i-axis.

Transformation matrix

$$\mathbf{A}_i = \mathrm{Rot}_{z,\theta_i}\mathrm{Trans}_{z,d_i}\mathrm{Trans}_{x,a_i}\mathrm{Rot}_{x,\alpha_i} =$$
$$\begin{bmatrix} \cos(\theta_i) & -\sin(\theta_i)\cos(\alpha_i) & \sin(\theta_i)\sin(\alpha_i) & a_i\cos(\theta_i) \\ \sin(\theta_i) & \cos(\theta_i)\cos(\alpha_i) & -\cos(\theta_i)\sin(\alpha_i) & a_i\sin(\theta_i) \\ 0 & \sin(\alpha_i) & \cos(\alpha_i) & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$