

Newton's Law of viscosity

$$\tau = \eta \cdot \frac{dv}{dx}$$

Viscosity = cons.
when Temp = cons.

Dimensions

$$\Delta p = M L^{-1} T^{-2}, \eta = M L^{-1} T^{-1}, \rho = M L^{-3}, v = L T^{-1}$$

* For gases viscosity increases with temp. increase, but for liquids vice versa.

Pascal's law: $\frac{dp}{dz} = -\rho g$

Continuity eq.: $\dot{m}_{in} = \dot{m}_{out} \Leftrightarrow \rho_1 v_1 A_1 = \rho_2 v_2 A_2 \Rightarrow$ incompressible fluid $\rightarrow \rho = \text{cons.}$

Momentum eq.: $F = \dot{M}_2 - \dot{M}_1 = \dot{m}_2 v_2 - \dot{m}_1 v_1 = \rho_2 A_2 v_2^2 - \rho_1 A_1 v_1^2$ Solving using integral form.

Bernoulli equation assumption: * Steady-state. * Along streamline. * No viscous forces. * $\rho = \text{cons.}$

$$\Rightarrow p_1 + \frac{\rho}{2} v_1^2 + \rho g z_1 = p_2 + \frac{\rho}{2} v_2^2 + \rho g z_2$$

Similitude: $Eu_M = Eu_R = \frac{\Delta p}{\rho v^2}$ $Re_M = Re_R = \frac{\rho v d}{\eta}$ $Fr_M = Fr_R = \frac{v^2}{g d}$

Internal flow

Extended Bernoulli: $\dot{V}_1 (p_1 + \frac{\rho}{2} v_1^2 + \rho g h_1) + \dot{P} = \dot{V}_2 A_2 (p_2 + \frac{\rho}{2} v_2^2 + \rho g h_2 + \Delta p_{losses}) + \dot{P}$

$$\Delta p_{losses} = \Delta p_f + \Delta p_j$$

$$\Delta p_f = \lambda \cdot \frac{L}{d} \cdot \frac{\rho}{2} v^2$$

$$\Delta p_j = \sum \rho \frac{v^2}{2}$$

Tables

$Re \ll 2300$ (Laminar flow)

$Re \gg 2300$ (Turbulent)

$$\lambda = \frac{64}{Re}$$

Moody diagram

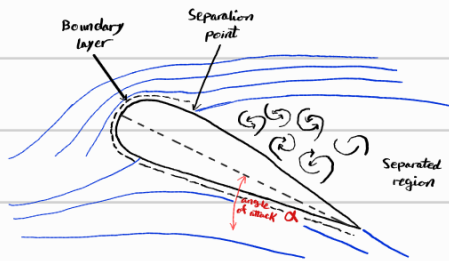
$$\text{reference Error } \Delta \xi = \frac{|V_{ref} - V|}{V} < 5\% - 10\%$$

* External Flow: Drag Force is acting in the direction of the flow: ($\vec{F}_D = \vec{F}_H$), $\vec{F}_{L,th}$ is normal to drag force \vec{F}_D .

$$C_D = \frac{2 F_D}{\rho \cdot v^2 \cdot A} = \underbrace{\tilde{C}_P}_{\text{Form}} + \underbrace{\tilde{C}_F}_{\text{Skin}} + \underbrace{\tilde{C}_{Ind}}_{\text{Induced}} = \frac{F_P + F_F}{\frac{1}{2} \rho v^2 A}, \quad C_L = \frac{F_L}{\frac{1}{2} \rho v^2 A} = \frac{P - P_{ref}}{\frac{1}{2} \rho v^2}, \quad \vec{F}_P = \oint p dA, \quad \vec{F}_F = \oint \tau dA$$

$Re < 1$
 $C_D = \frac{24}{Re}$

The graph



Stall is highly undesirable on aircraft at cruise conditions and leads to inefficiencies, when it occurs on turbine blades. However, it's used to provide the high drag needed when landing an aircraft.

Velocity low \rightarrow pressure high
 \rightarrow Aircraft will be "pushed down"

$F_B = \rho_f V g$
Buoyancy fluid

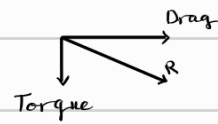
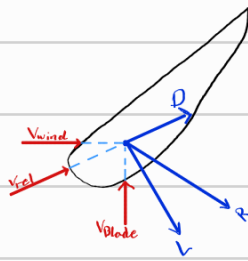
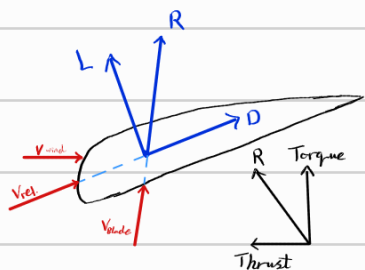
Sedimentation: With $A = \frac{\pi}{4} d^2$ & $V = \frac{\pi}{6} d^3 \Rightarrow v_{\infty} = \sqrt{\frac{2 g (\rho_s - \rho_f) \cdot 4 d^3}{6 C_D \rho_f}}$

If creeping flow ($Re < 1$) $\rightarrow C_D = \frac{24}{Re} \Rightarrow v_{\infty} = \sqrt{\frac{4 g (\rho_s - \rho_f) d \cdot Re}{3 \rho_f}} \Rightarrow v_{\infty} = \frac{2 (\rho_s - \rho_f) d^2}{18 \eta_f}$, If $1000 < Re < 2 \times 10^5 \Rightarrow C_D \approx 0.44 \Rightarrow v_{\infty} = \sqrt{\frac{2 g (\rho_s - \rho_f) \cdot 4 d^3}{6 (0.44) \rho_f}}$

Separating condition $\Rightarrow v_{\infty} \geq v_L = \frac{4 \dot{V}}{\pi D^2} \Rightarrow D \geq \sqrt{\frac{4 \dot{V}}{\pi v_{\infty}}} \rightarrow D$ is basin's diameter.

Propeller blade

Rotor blade



$$\frac{P_{turb}}{A} = C_p \cdot \frac{\rho}{2} \bar{V}_1^3, \quad C_p = \frac{P_{turb}/A}{P_{wind}/A_1} = \frac{1+x}{2} \cdot (1-x^2), \quad x = \frac{\bar{V}_2}{\bar{V}_1}, \quad \bar{V}_3 = \frac{1}{2} (\bar{V}_2 + \bar{V}_1)$$