

Thermodynamics

Important terms:

Isothermal $\rightarrow \Delta T = 0$, Temperature = cons.

Isobaric $\rightarrow \Delta P = 0$, Pressure = cons.

Isochoric $\rightarrow \Delta V = 0$, Volume = cons.

Rigid $\rightarrow V = \text{cons.}$

Adiabatic $\rightarrow \Delta Q = 0 \quad (\dot{Q} = 0)$

ISENTROPIC $\rightarrow \Delta S = \frac{\Delta Q}{T} = 0$ (Adiabatic + ideal)

Steady-State $\rightarrow \frac{dE}{dt} = 0$

Closed Sys. $\rightarrow \dot{m} = 0, m = \text{cons.}$

Open System $\rightarrow \dot{m} \neq 0$

Polytropic $\rightarrow P \cdot V^n = \text{constant}$

Ideal gas Law

$$P \cdot V = R \cdot T$$

$$\frac{P \cdot V}{n} = \frac{m \cdot R \cdot T}{n}$$

$$P \cdot \bar{V} = M \cdot R \cdot T$$

$$P \cdot \bar{V} = \bar{R} \cdot T$$

Universal gas cons. [J/mol·K]

$$\bar{R} = M \cdot R$$

$$\rightarrow R = \frac{\bar{R}}{M}$$

$$\dot{m} = \rho \frac{V}{\bar{V}} A \xrightarrow{\text{velocity}} \dot{m} = \rho \cdot \dot{V} \xrightarrow{\dot{m} = \rho V} \frac{m}{v} = \frac{\rho V}{v} = \frac{1}{\bar{V}}$$

1st law: (closed system)

$$dU = dQ + dW$$

$$(\text{ideal gas}): dU = C_v \cdot dT$$

$$W_v = P \cdot dV$$

1st law: (open system)

$$dh = dQ + dW$$

$$(\text{ideal gas}): dh = C_p \cdot dT$$

$$W_t = V \cdot dP$$

Entropy: $dS = \frac{dQ}{T}$ (ideal process)

(Real process): $dS = \frac{dQ}{T} + \alpha$

$\alpha > 0 \rightarrow$ irreversible

$\alpha = 0 \rightarrow$ reversible

$\alpha < 0 \rightarrow$ impossible

Interpolation:

T	S
T_1	S_1
T_2	S_2
T_3	S_3

$$\frac{T_1 - T_3}{S_1 - S_3} = \frac{T_2 - T_3}{S_2 - S_3}$$

Polytropic processes LC:

Ideal gas:

Slide 4

Closed system:

$$\Delta S_{2 \rightarrow 1} = C_v \cdot \ln\left(\frac{T_2}{T_1}\right) + R \cdot \ln\left(\frac{V_1}{V_2}\right)$$

Open System:

$$\Delta S_{2 \rightarrow 1} = C_p \cdot \ln\left(\frac{T_2}{T_1}\right) + R \cdot \ln\left(\frac{P_2}{P_1}\right)$$

(Isentropic): $C_v = \frac{R}{K-1}$, $C_p = \frac{K \cdot R}{K-1}$, $K = \frac{C_p}{C_v}$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{K-1}, \quad \frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{K-1}{K}}, \quad \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^K$$

Polytropic, open system, & Ideal gas: → Slide 8

Technical work: $W_{rev} = \frac{n}{n-1} \cdot (P_2 V_2 - P_1 V_1)$

$$P \cdot V = RT \rightarrow W_{rev} = \frac{n \cdot R}{n-1} \cdot (T_2 - T_1)$$

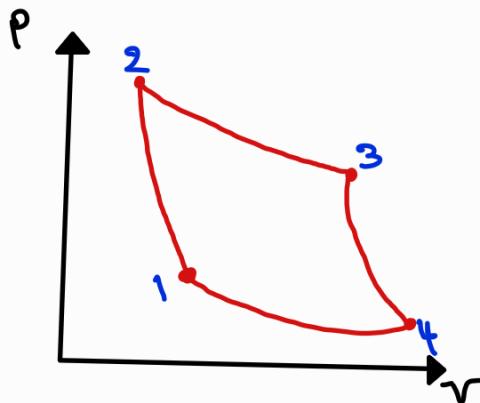
$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} \rightarrow W_{rev} = \frac{n}{n-1} \cdot R T_1 \cdot \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$$

special case ($n=1$) → $P \cdot V = \text{cons.} \rightarrow T = \text{cons.}$

$$W_{rev} = P_1 V_1 \cdot \ln\left(\frac{P_2}{P_1}\right)$$

$$W_{rev} = R \cdot T \cdot \ln\left(\frac{P_2}{P_1}\right)$$

Carnot cycle:

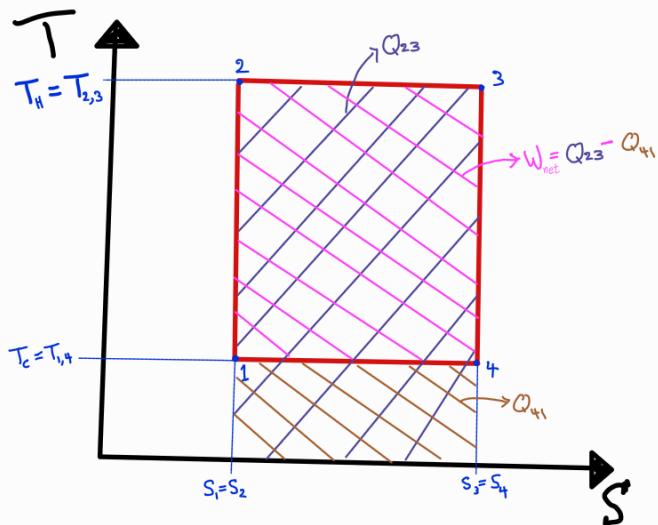


Process 1→2 (Adiabatic compression)

Process 2→3 (Isothermal expansion)

Process 3→4 (Adiabatic expansion)

Process 4→1 (Isothermal compression)

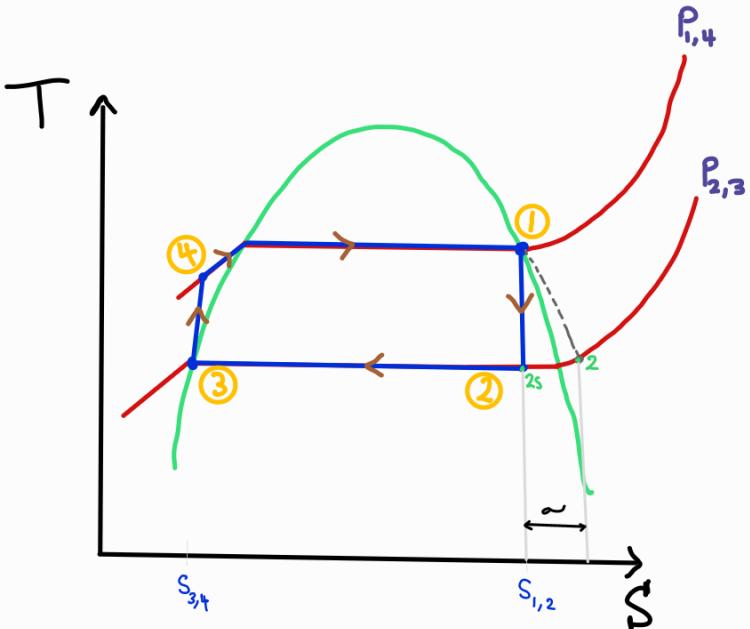
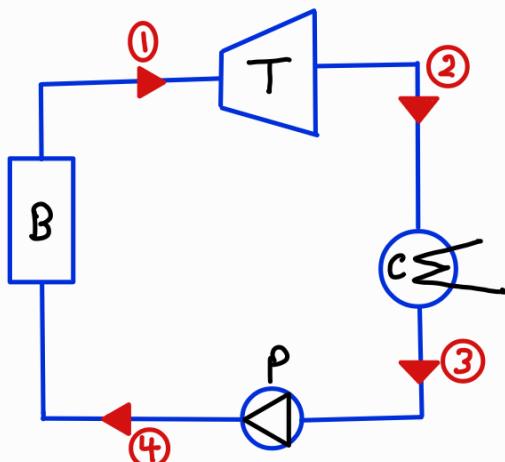


$$\eta_{\text{Carnot}} = 1 - \frac{T_c}{T_h}$$

$$\eta_{\text{th}} = \frac{W_{\text{net}}}{Q_{23}} = 1 - \frac{Q_{41}}{Q_{23}} = 1 - \frac{T_c (S_4 - S_1)}{T_h (S_3 - S_2)}$$

Rankine Cycle

Ideal cycle



$1 \rightarrow 2$ (Turbine) | Isentropic expansion (saturated vapour)

$$(\Delta S = 0 \rightarrow \dot{Q} = 0), \left(\frac{P_T}{\dot{m}} \right) = h_1 - h_2 \rightarrow x = 1, P_1 = P_4$$

$2 \rightarrow 3$ (condenser) | Sobaric & Isothermal condensation (saturated mixture)

$$(\Delta T = 0 \& \Delta P = 0) \rightarrow (P_2 = P_3, T_2 = T_3) \quad \text{Cooling water (cw): } (x=0)$$

$$\dot{Q}_c^{\text{out}} = h_2 - h_3, \quad 0 < x < 1$$

$$\dot{m}_{cw} = \frac{\dot{Q}_c}{h_{out} - h_{in}} \quad \text{or } dh = C_p \cdot dT, \quad C_{p,cw} = \text{Const.} = 4.2 \text{ kJ/kg}\cdot\text{K}$$

Shapiro table at T_{in} & T_{out}

$$\dot{m}_{cw} = \frac{\dot{Q}_c}{C_p(T_{out} - T_{in})}$$

$3 \rightarrow 4$ (Pump) | Isentropic compression (saturated Liquid) $\rightarrow x = 0$

$$(\Delta S = 0 \rightarrow \dot{Q} = 0), \quad \frac{P_p}{\dot{m}} = h_4 - h_3$$

$$(\text{assumptions: water is incompressible} \rightarrow v = \text{const.}) \rightarrow \frac{P_p}{\dot{m}} = w_t = v_3 \cdot (P_4 - P_3)$$

$4 \rightarrow 1$ (Boiler) | Isobaric compression (Compressed liquid)

$$(\Delta P = 0), \quad \dot{Q}_B = h_1 - h_4$$

$P_4 = P_1$

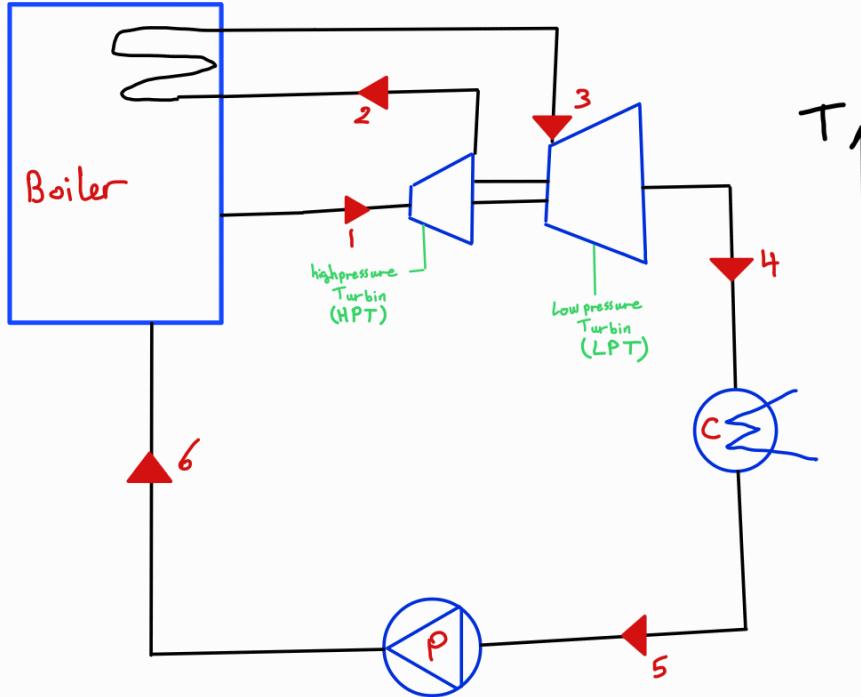
$$\eta_{th} = \frac{P_T - P_p}{Q_B} = \frac{(h_1 - h_2) - (h_4 - h_3)}{h_1 - h_4} = 1 - \frac{h_1 - h_2}{h_1 - h_4} \quad \left| \begin{array}{l} P_{net} = P_T - P_p = \dot{m}(h_1 - h_2) - \dot{m}(h_4 - h_3) \\ \dot{m} = \frac{P_{net}}{(h_1 - h_2) - (h_4 - h_3)} \end{array} \right.$$

$$(\text{Back-work ratio}) \eta_{bwr} = \frac{P_p}{P_T} = \frac{h_4 - h_3}{h_1 - h_2}$$

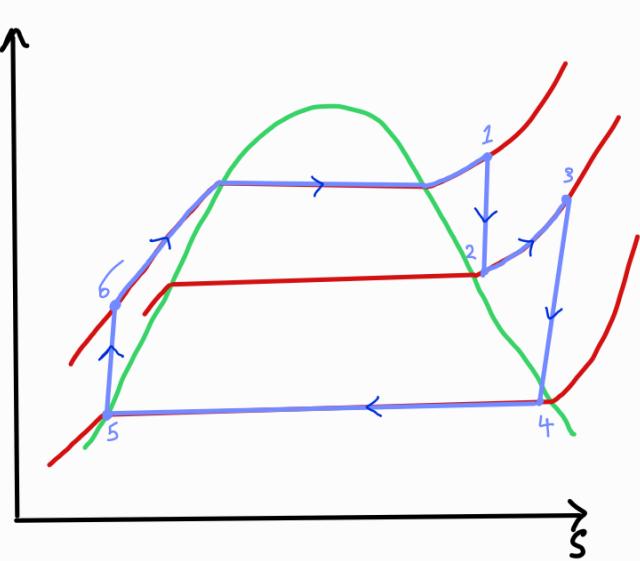
| Isentropic (ideal): $P_{Ts} = \dot{m}(h_1 - h_{2s})$

| Isentropic (real, } \alpha > 0\}: $P_T = \dot{m}(h_1 - h_2)$

$$|\text{Isentropic efficiency}| \eta_{is} = \frac{P_T}{P_{Ts}} \cdot \frac{h_1 - h_2}{h_1 - h_{2s}} < 1$$



• State(2) can be after or under the Dome



$$\eta_{th} = \frac{\text{HPT} + \text{LPT} + \text{Pump}}{\text{Boiler} + \text{Reheating}} , \dot{m} = \frac{W_{cycle/net}}{(h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5)} , \dot{Q}_c = \dot{m}(h_4 - h_5)$$

1st step: calculate x_2 (if under dome), & x_4 .

$$x_2 = \frac{s_2 - s_{f2}}{s_{22} - s_{f2}} \quad | \quad x_4 = \frac{s_4 - s_{f4}}{s_{24} - s_{f4}}$$

2nd step: calculate h_2 & h_4 .

3rd step: after bringing $h_5 = h_{fs}$ (because $x=0$), We need to find $h_6 = ?$

$$\begin{aligned} & \text{6} \quad \text{5} \quad \dot{m}_{in} \\ & \text{out} \quad \text{5} \\ & (\nu = \text{cons.}) \rightarrow w_t = \int_5^6 v dp = v_5(p_6 - p_5) \rightarrow h_6 = w_t + h_5 \end{aligned}$$

$$P_p = \dot{m}(h_6 - h_5) \rightarrow \frac{P_p}{\dot{m}} = w_t = (h_6 - h_5) \quad \text{↳ technical work}$$

4th step: calculate the η_{th} , and then the mass flow rate \dot{m} :

$$\eta_{th} = \frac{\text{HPT} + \text{LPT} + \text{Pump}}{\text{Boiler} + \text{Reheating}} , \dot{m} = \frac{P_{net}}{(h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5)}$$

$$\therefore P_{net} = P_{T_1} + P_{T_2} - P_p$$

$$\rightarrow \dot{m} = \frac{P_{net}}{(h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5)}$$