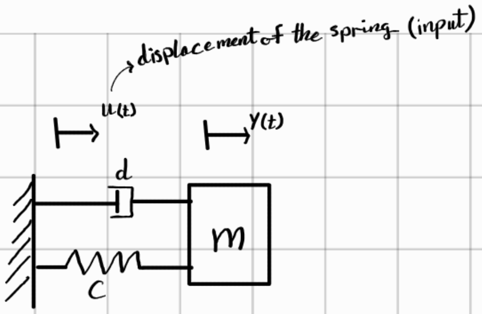
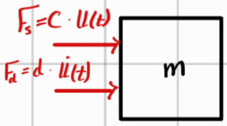
Summer 2014

Q_1





Equation of motion

$$\sum F_x = \boxed{m\ddot{y}(t) = d\,\dot{u}(t) + C\,u(t)}$$

$$\mathcal{L}:\; m s^2 Y(s) = U(s)(ds + C)$$

$$\boxed{G(s) = \frac{Y(s)}{U(s)} = \frac{ds + C}{m s^2}} \quad TF$$

Let's assume a system represented by the TF, $G(s) = \dfrac{Y(s)}{U(s)} = \dfrac{10s + 25}{s^2 + 9s + 20}$

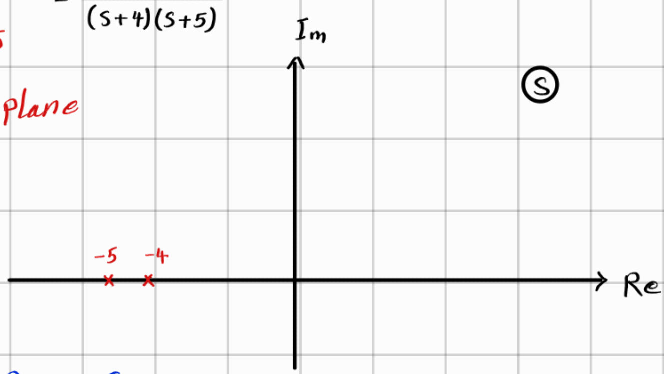
$$= \frac{10s + 25}{(s+4)(s+5)}$$

c) is stabe?

Since we have two negative poles $P_1 = -4$, $P_2 = -5$

$\Rightarrow$ The poles will be in the LHP of the s-plane

$\Rightarrow$ Yes, system will be stable!



d) solve for $y(t)$:

$$Y(s) = \underbrace{U(s)}_{U(s)=\frac{1}{s}} \cdot \frac{10s + 25}{(s+4)(s+5)} = \frac{10s + 25}{s(s+4)(s+5)} = \frac{A}{s} + \frac{B}{(s+4)} + \frac{C}{(s+5)}$$

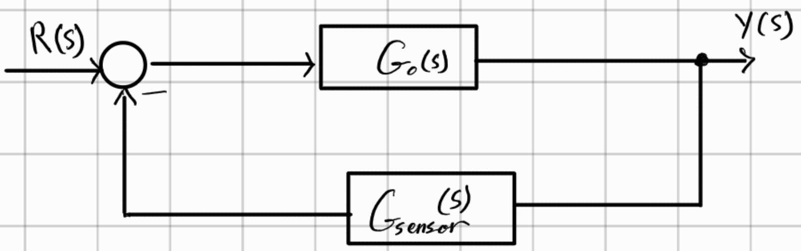
$$\Rightarrow 10s + 25 = A(s+4)(s+5) + B s(s+5) + C s(s+4)$$

at $s = 0$ $\boxed{A = \frac{25}{20} = \frac{5}{4}} = 1.25$, at $s = -4$ $\boxed{B = \frac{15}{4}}$, at $s = -5$ $\boxed{C = -5}$

$$Y(s) = \frac{5/4}{s} + \frac{15/4}{(s+4)} - \frac{5}{(s+5)}$$

$$\mathcal{L}^{-1}:\; \boxed{y(t) = \frac{5}{4}(t) + \frac{15}{4} e^{-4t} - 5 e^{-5t}}$$

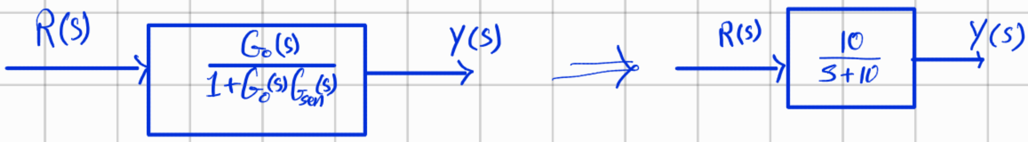
Q_2



$$G_o(s) = \frac{10}{s}$$

a) TF of the closed-loop, $G(s) = \dfrac{Y(s)}{R(s)} = \dfrac{G_o(s)}{1 + G_o(s) G_{sensor}(s)}$

assuming an ideal sensor (input = output) $\Rightarrow G_{sen}(s) = 1$



$$\boxed{G(S) = \frac{\frac{10}{S}}{1+\frac{10}{S}(1)} = \frac{10}{S+10}}$$

Type of the system? PT1

Let's assume a sensor with PT1-Charactaristics $G_{sen}(s) = \frac{1}{TS+1}$

Range of T for a stable system:-

$$G(S) = \frac{\frac{10}{S}}{1+\frac{10}{S(TS+1)}} = \frac{10(TS+1)}{TS^2+S+10} = \frac{10(TS+1)}{a_2S^2+a_1S+a_0}$$

S^2	$a_2 = T$	$a_0 = 10$
S^1	$a_1 = 1$	0
S^0	$b_1 = \frac{-\begin{vmatrix} a_2 & a_0 \\ a_1 & 0 \end{vmatrix}}{a_1} = \frac{-\begin{vmatrix} T & 10 \\ 1 & 0 \end{vmatrix}}{1}$	

$b_1 = 10$

$$\boxed{T > 0}$$

for a stable system.

Calculate T, for a damping ratio $D = 0.5$

$$S^2 + \underbrace{\frac{1}{T}}_{2D\omega_o} S + \frac{10}{T} \quad \omega_o^2$$

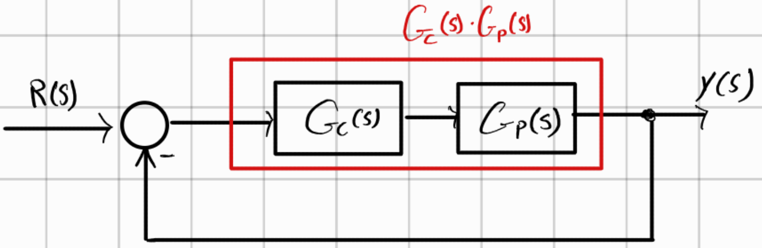
$$\omega_o = \sqrt{\frac{10}{T}}$$

$$2D\frac{\sqrt{10}}{\sqrt{T}} = \frac{1}{T} \Rightarrow T = \frac{1}{10(2D)^2} = \frac{1}{10(2(0.5))^2} = \frac{1}{10}$$

$$\boxed{T = 0.1}$$

Q3

$G_c(S) = K$ & $G_p = \frac{3}{S^4+6S^3+S^2+2S}$



$$G_{CL}(S) = \frac{G_c(S)\,G_p(S)}{1+G_c(S)\cdot G_p(S)} = \frac{\frac{3K}{S^4+6S^3+S^2+2S}}{1+\frac{3K}{S^4+6S^3+S^2+2S}} = \frac{3K}{S^4+6S^3+S^2+2S+3K}$$

$$G_{Closed}(s) = \frac{3K}{s^4+6s^3+s^2+2s+3K}$$

Routh Hurwitz methode:

s^4 $\quad a_4 = 1 \quad a_2 = 1 \quad a_0 = 3K$

s^3 $\quad a_3 = 6 \quad a_1 = 2 \quad 0$

s^2 $\quad b_1 = \frac{-\begin{vmatrix} 1 & 1 \\ 6 & 2 \end{vmatrix}}{6} = \frac{2}{3} \quad b_2 = \frac{18K}{6} = 3K \quad 0$

s^1 $\quad C_1 = \frac{\frac{4}{3} - 18K}{2/3} \quad C_2 = 0$

$C_1 = 2 - 27K$

s^0 $\quad d_1 = 3K$

For a stable system

$3K > 0 \quad \& \quad 2 - 27K > 0$

$K > 0 \quad \& \quad K < \frac{2}{27}$

$\therefore \boxed{0 < K < \frac{2}{27}}$

For a stabe system

Q4 $G_s(s) = \frac{s+50}{s(s+10)}$ TF of an open-loop system

Sketch the asymplotes of Bode diagram

$|G_{dB}| = 20\log(5) = 13.98 \approx 14$

$$G(s) = \frac{\overset{5}{\cancel{50}}\left(\frac{s}{50}+1\right)}{\cancel{10}s\left(\frac{s}{10}+1\right)} = 5 \cdot \left(\frac{s}{50}+1\right) \cdot \frac{1}{s} \cdot \frac{1}{\left(\frac{s}{10}+1\right)}$$

$$G(j\omega) = 5 \cdot \left(\frac{j\omega}{50}+1\right) \cdot \frac{1}{j\omega} * \frac{j}{j} \cdot \frac{1}{\left(\frac{j\omega}{10}+1\right)} * \frac{\frac{j\omega}{10}-1}{\frac{j\omega}{10}-1} = 5 \cdot \left(\frac{j\omega}{50}+1\right) \cdot \frac{-j}{\omega} \cdot \frac{\frac{j\omega}{10}-1}{1-\frac{\omega^2}{100}}$$

at $\omega = 10^{-2}$

$|G(j\omega)| \approx 1 \Rightarrow |G_{dB}| = 0$

$|G(j\omega)| = 100 \Rightarrow G_{dB} = 20\log(100) = 40$

$|G(j\omega)| \approx 1 \Rightarrow G_{dB} = 0$