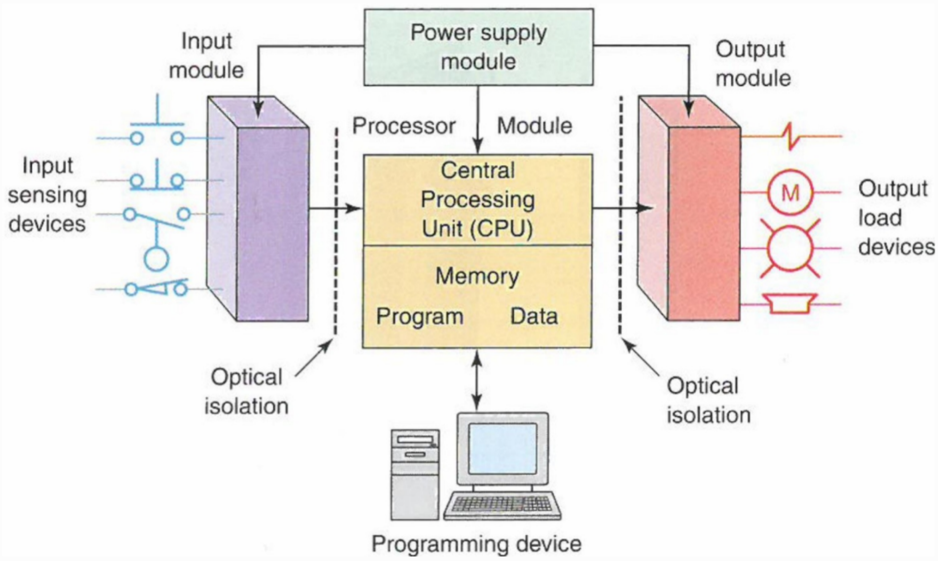
Disadvantages of relay control circuits:

- Cost
- Flexibility
- Reliability
- Communication capability
- Response time
- Troubleshooting

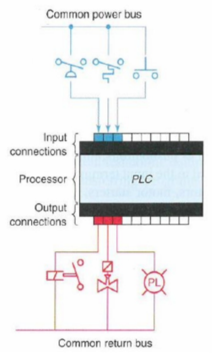
replaced by PLC

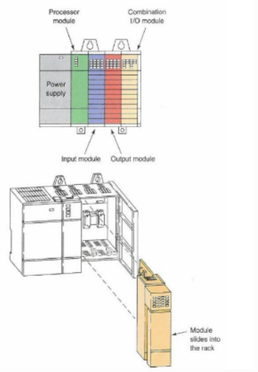
- Most widely used in industrial process control technology
- Programmable controller has eliminated much of the hardwiring
- Designed for multiple inputs and outputs
- Basically a digital computer designed for use in machine control (abbreviation PLC)
- Capable of performing switching, timing, counting, calculating, comparing, processing of analog signals

PLC Hardware: Relationship between input & output are determined by the user program.

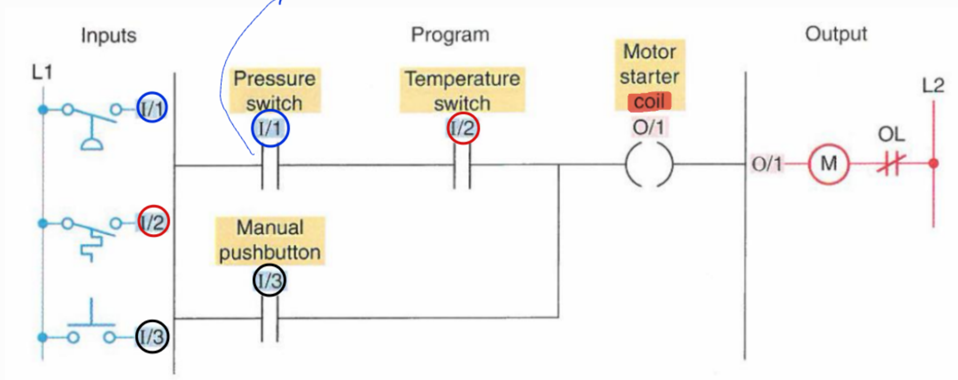


- Fixed I/O configuration
- Modular/scalable I/O configuration



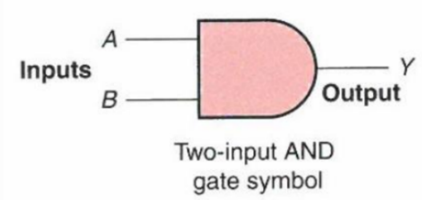


contactor



Logic gate:

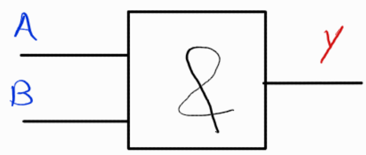
* AND Function: (Connected in series)



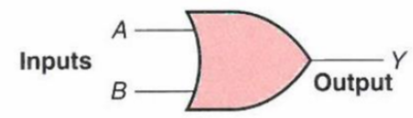
Two-input AND gate symbol

AND truth table

Inputs		Output
A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1



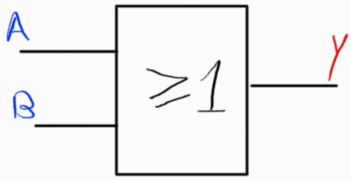
* OR Function: (connected in parallel)



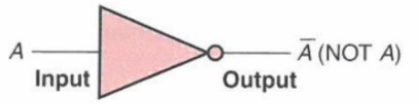
Two-input OR gate symbol

OR truth table

Inputs		Output
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

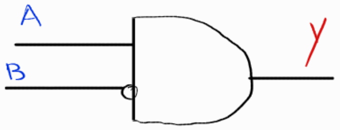


* NOT Function:

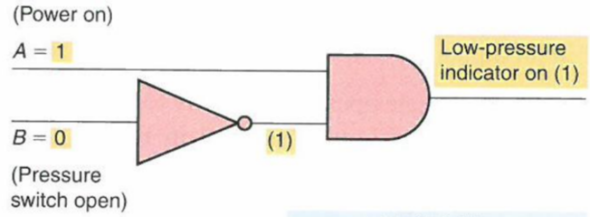


NOT truth table

A	NOT A
0	1
1	0

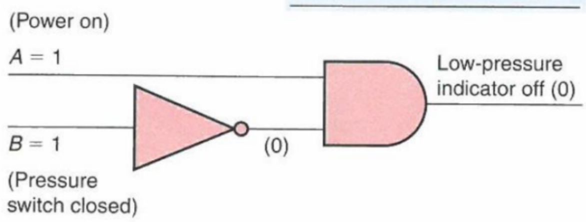


=

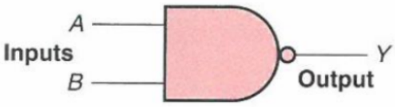


Truth table

Pressure switch	Power	Pressure indicator
0	1	1
1	1	0



NAND

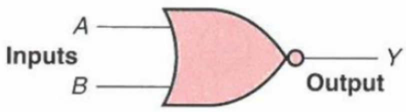


Two-input NAND gate

NAND truth table

Inputs		Output
A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

NOR

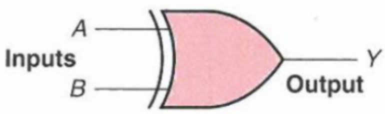


Two-input NOR gate

NOR truth table

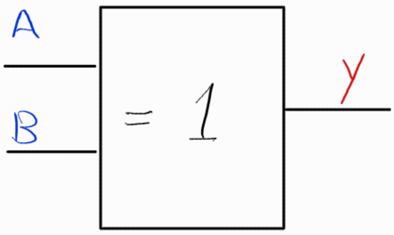
Inputs		Output
A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

* XOR Function: (Exclusive OR function)

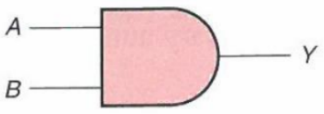
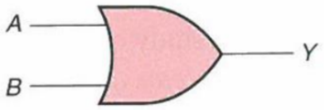
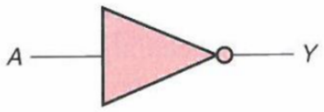


Truth table

Inputs		Output
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0



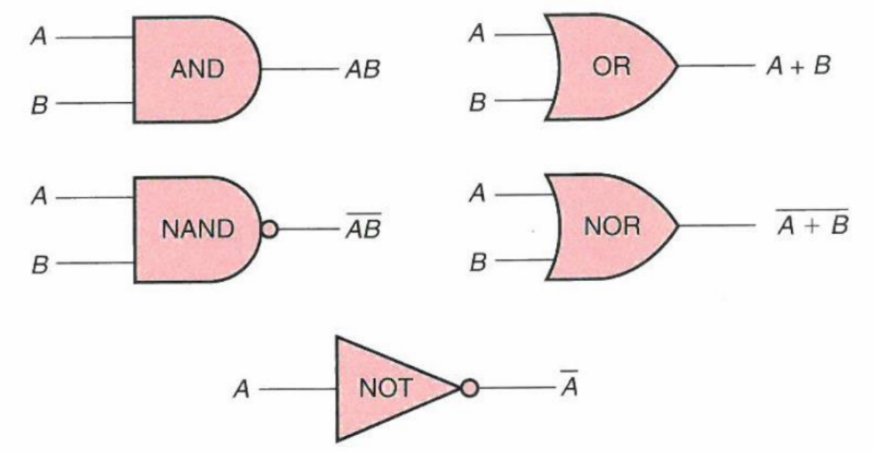
Boolean Algebra:

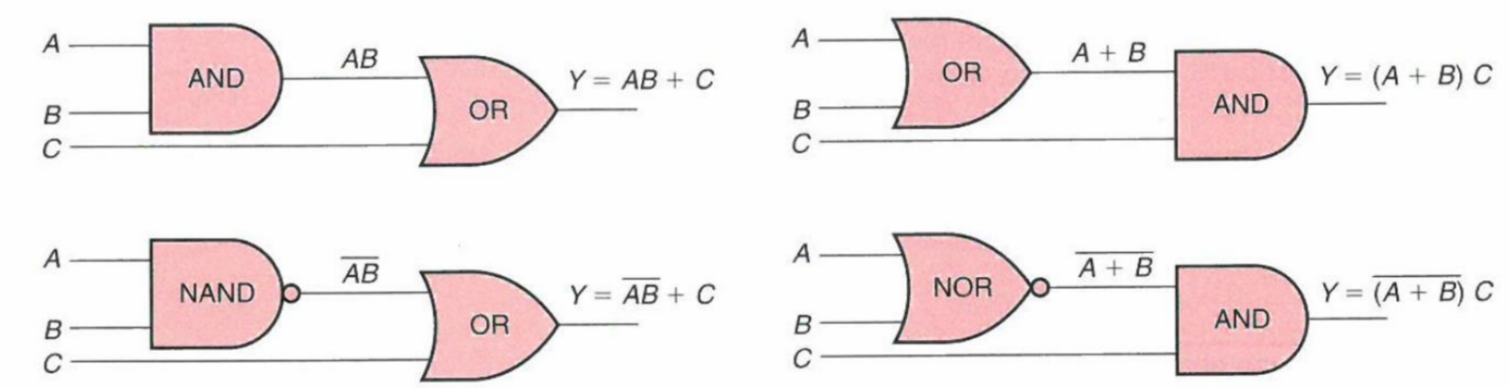
Logic symbol	Logic statement	Boolean equation	Boolean notations
	Y is 1 if A and B are 1	$Y = A \bullet B$ or $Y = AB$	Symbol / Meaning $\bullet$ and $+$ or $-$ not $\circ$ invert $=$ result in
	Y is 1 if A or B is 1	$Y = A + B$	
	Y is 1 if A is 0 Y is 0 if A is 1	$Y = \overline{A}$	

$\overline{AB} \neq \overline{A} \cdot \overline{B}$

XOR
↓
$\boxed{A \oplus B}$ Boolean exp.

Examples:





COMMUTATIVE LAW

$A + B = B + A$
$A \cdot B = B \cdot A$

ASSOCIATIVE LAW

$(A + B) + C = A + (B + C)$
$(A \cdot B) \cdot C = A \cdot (B \cdot C)$

DISTRIBUTIVE LAW

$A \cdot (B + C) = (A \cdot B) + (A \cdot C)$
$A + (B \cdot C) = (A + B) \cdot (A + C)$ → only in Boolean Algebra

This law holds true only in Boolean algebra.

$\overline{A}\,\overline{B} \neq \overline{A \cdot B}$

$\overline{A} \cdot \overline{B} = \overline{A + B}$

$\overline{A \cdot B} = \overline{A} + \overline{B}$

$\overline{A} + \overline{B} \neq \overline{A + B}$

XOR Function:

Truth Table:

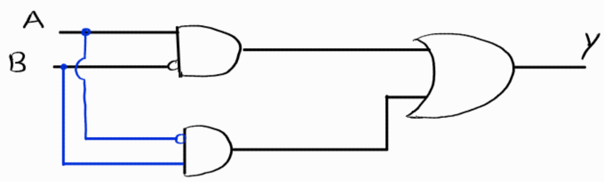
A	B	Y
0	0	0
1	0	1
0	1	1
1	1	0

Boolean expression:

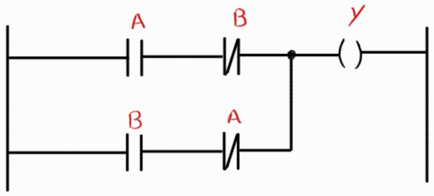
$$Y = A\bar{B} + \bar{A}B$$

$$Y = A \oplus B$$

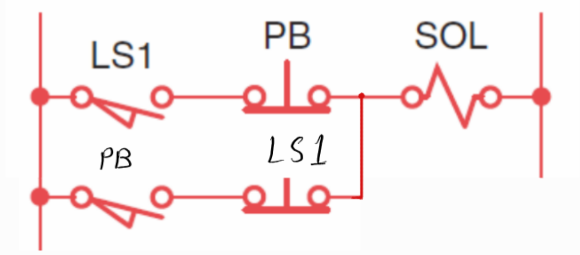
Gate logic:



Ladder logic program:



Relay schematic:



A motor control circuit contains two start/stop switches each. When either start switch (NO) is pressed, the motor runs. Either stop switch (NC) stops the motor when it is pressed

Normally open

Normally closed

- Relay schematic
- Ladder logic program
- Gate logic

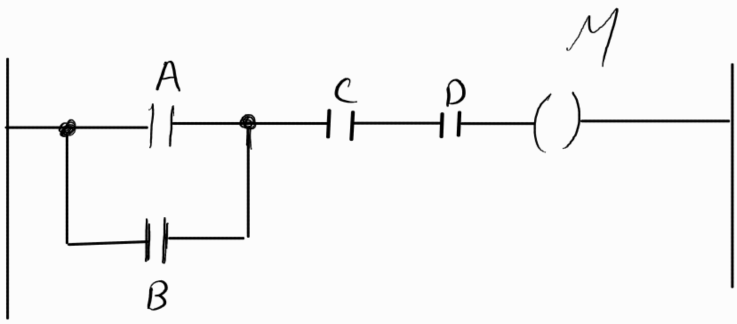
A: NO start switch 1

B: NO start switch 2

C: NC stop switch 1

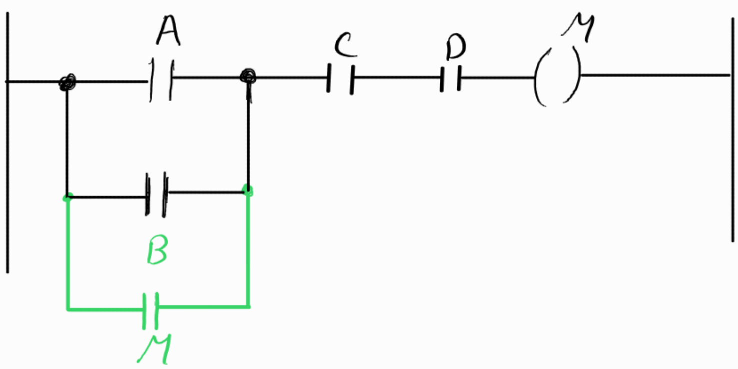
D: NC stop switch 2

M: output motor



Because it's switch, we don't need the seal in construct

When they're buttons not switches

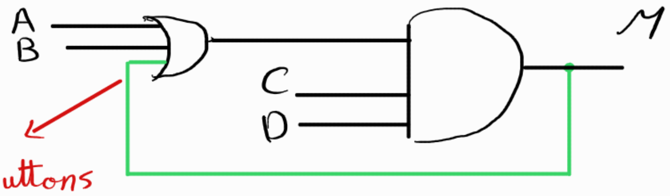


Boolean Expression:

(with switches) → $M = (A+B) \cdot CD$

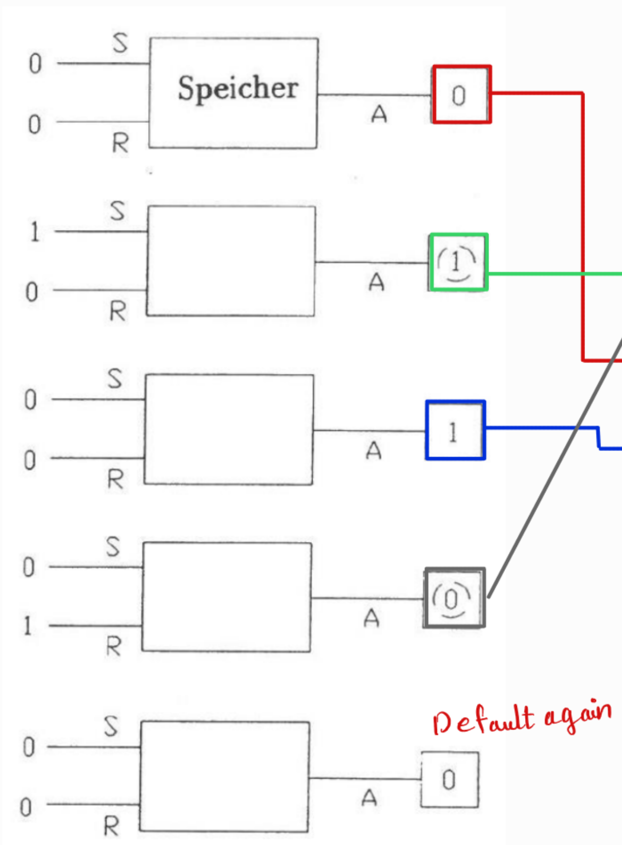
(with Buttons) → $M = (A+B+M) \cdot CD$

Gate Logic:



In case of Buttons

Example of a memory function



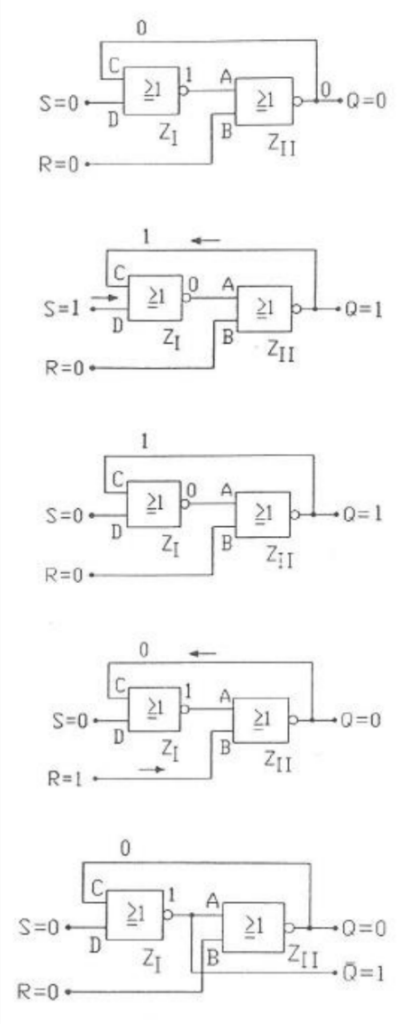
Find the correct sequence

- Reset memory
- Setting memory
- Default setting
- Save/maintain state

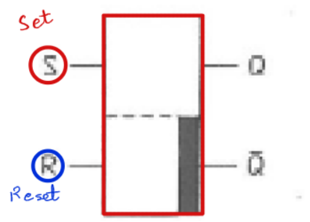
Flip-Flop elements

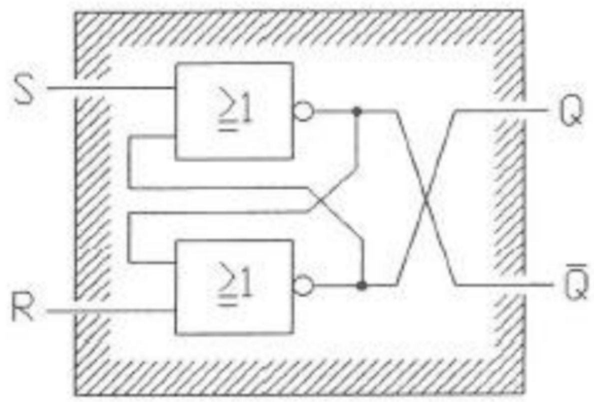
- Bistable
- means Can, on command, flip into one stable state of flop back again to the other
- Basis RS-Flip-Flop using NOR-Gates

1. Default setting
2. Setting memory
3. Save/maintain memory
4. Reset memory
5. Default setting

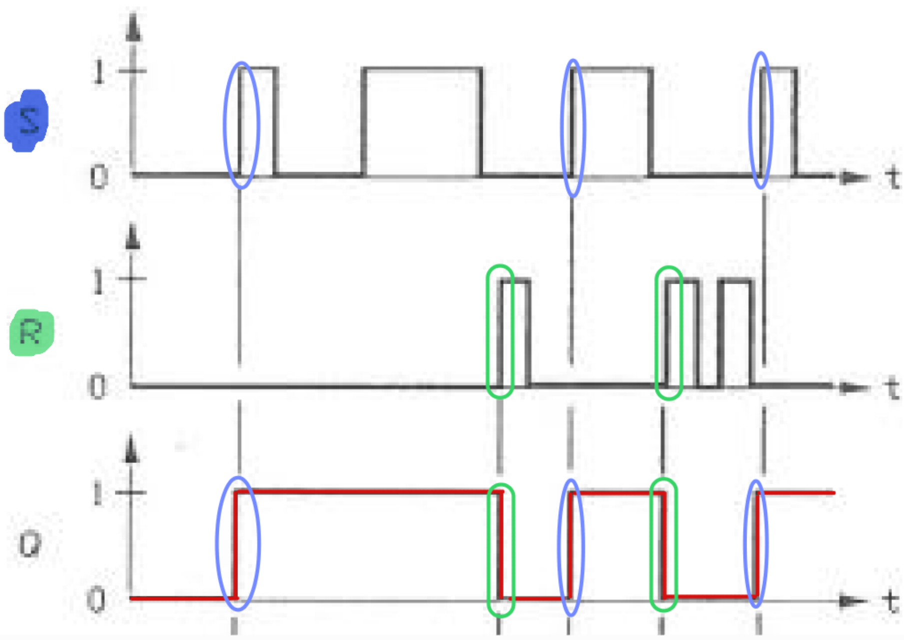


Symbol:

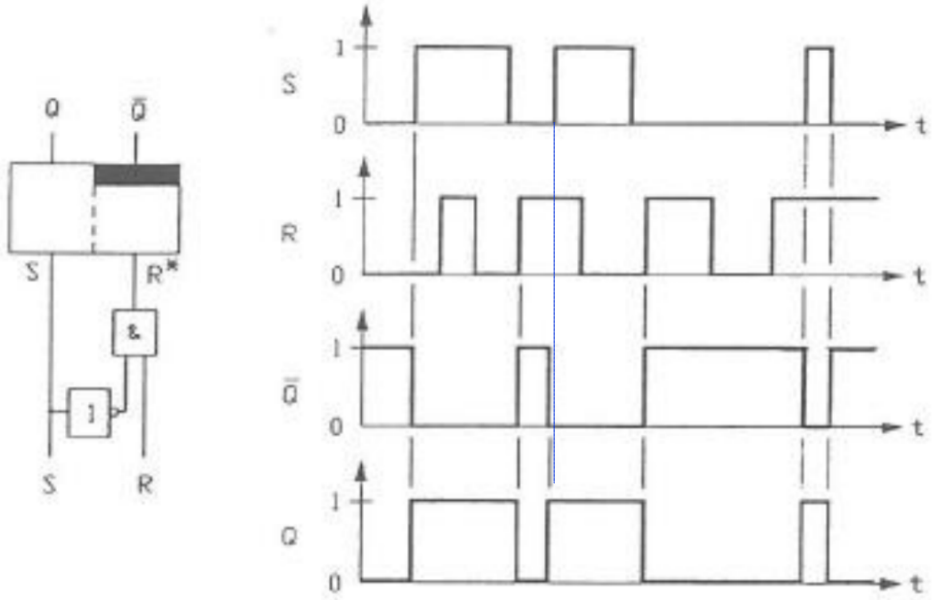




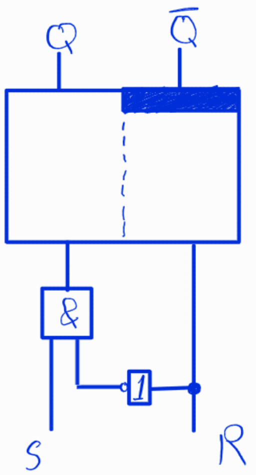
- Combination R = S = 1 not allowed
- Pre-logic defines priority of either R or S



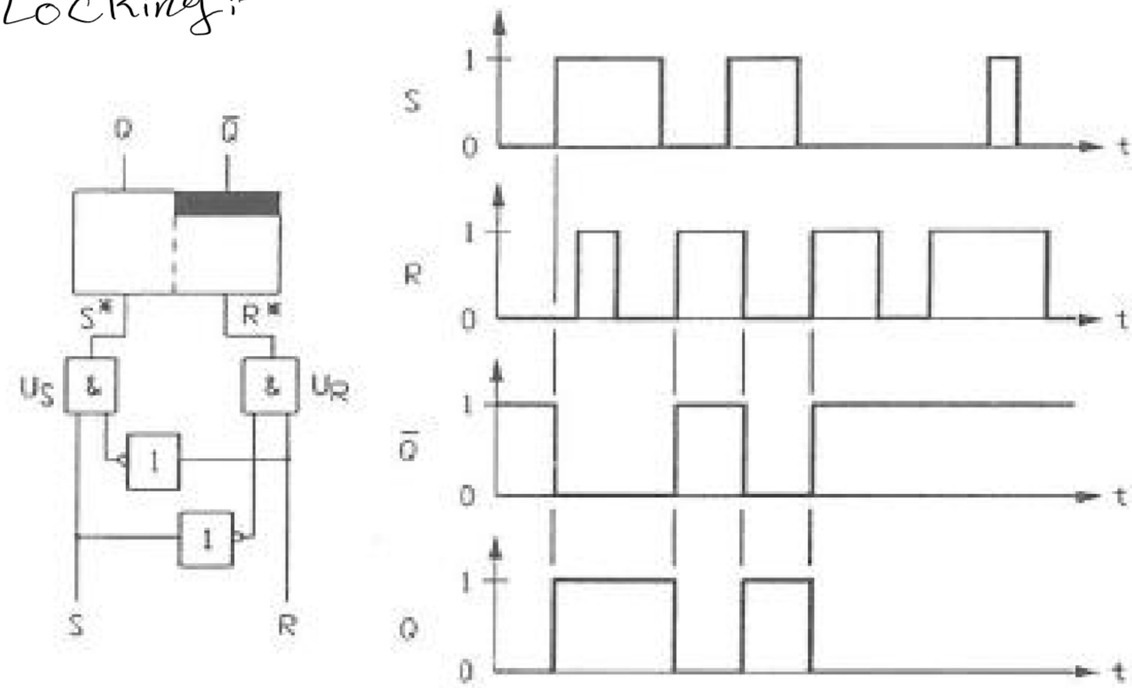
The pre-Logic in this case priorities S over R

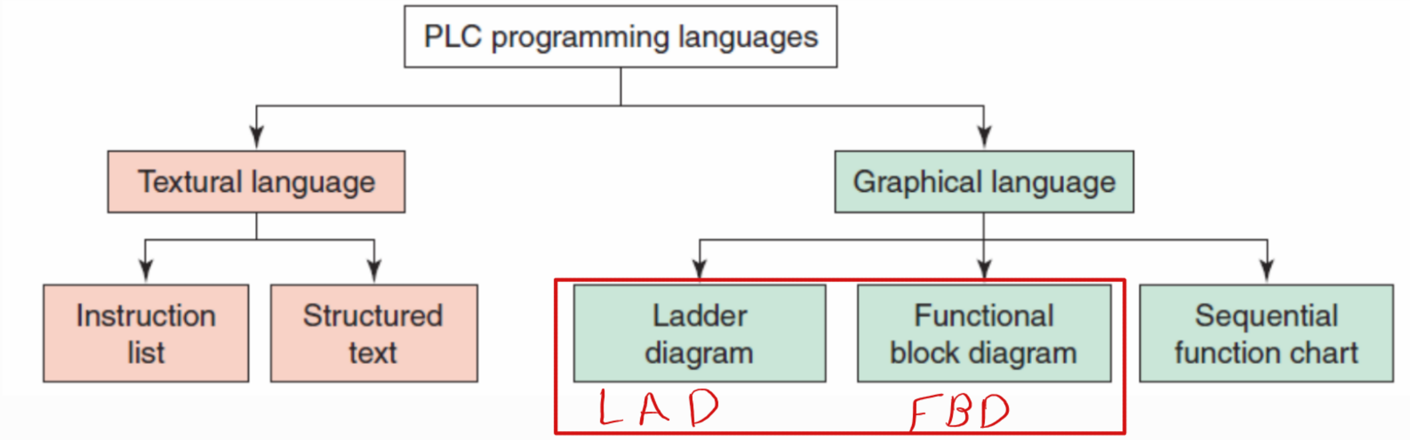


R has Priority

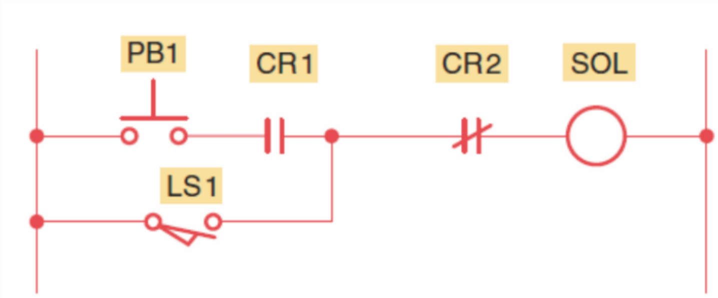


Locking:-





Instruction list: programming language consists of basic AND, OR, and NOT logic gate functions.



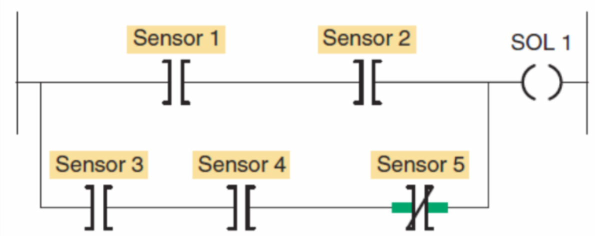
Hardwired relayed control circuit

⟹

START	PB1
AND	CR1
OR	LS1
AND NOT	CR2
OUT	SOL

Equivalent instruction list program

Structured text: Is a high level language primarily used to implement more complex procedures.



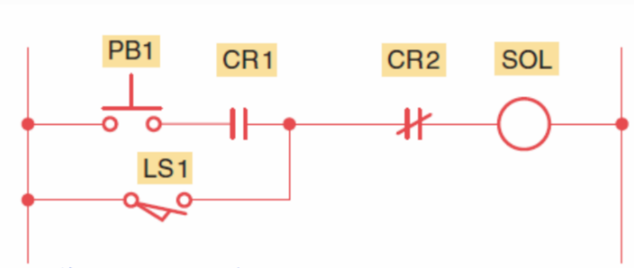
Lader Diagram

⟹

```
IF Sensor_1 AND Sensor_2 THEN
        SOL_1 := 1;
ELSEIF Sensor_3 AND Sensor_4 AND NOT Sensor_5 THEN
        SOL_1 := 1;
END_IF;
```

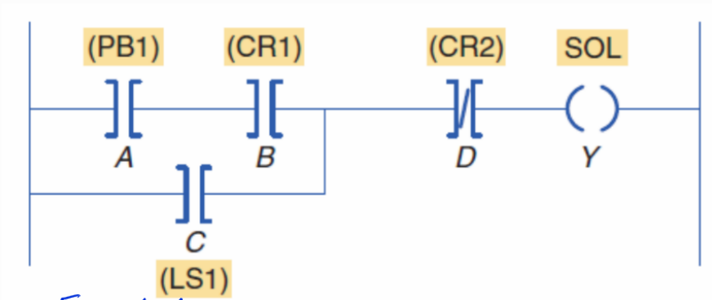
Equivalent structured text program

Ladder diagram: Is a common used PLC language and designed to mimic hardware relay logic.



Hardware relay control circuit

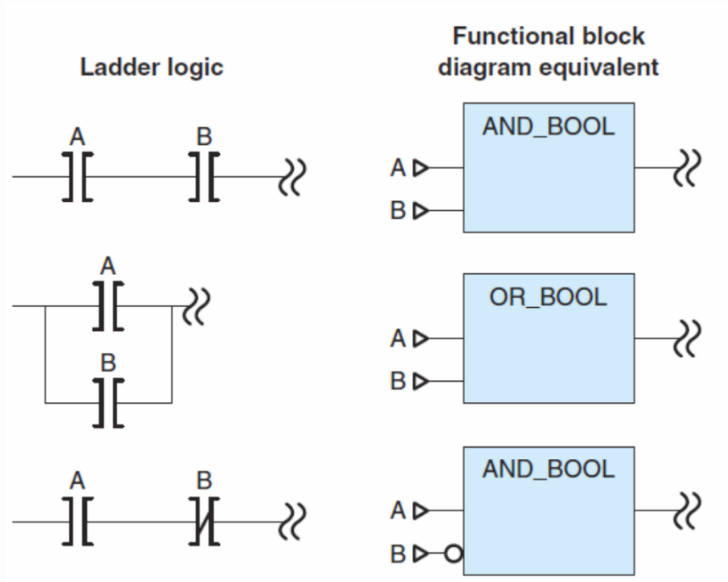
⟹

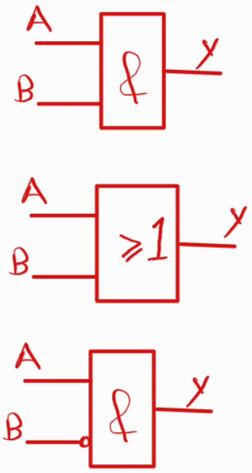


Equivalent Ladder diagram program

Functional block diagram:

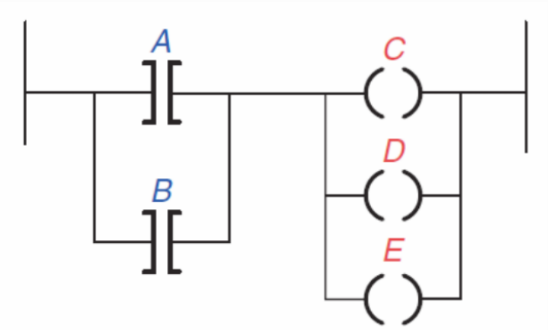
programming uses instructions that are programmed as blocks wired together to accomplish certain functions.



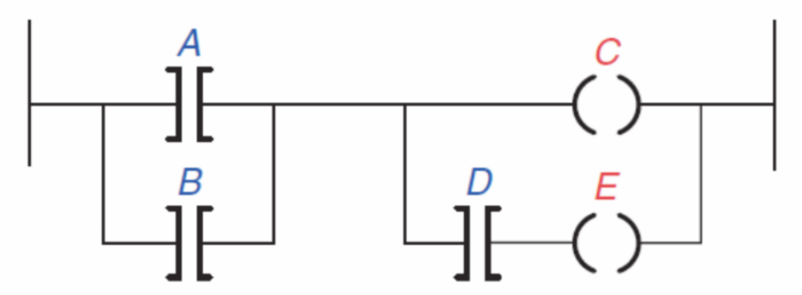


exam's expressions

Output branching allows a true logic path to control multiple outputs. However, a branch can only have one output. Thus, multiple outputs must be arranged in parallel.

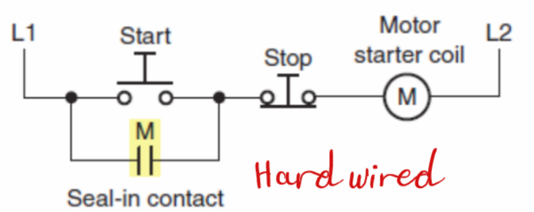


Either A or B provides a true logical path to all three output instructions: C, D, and E.



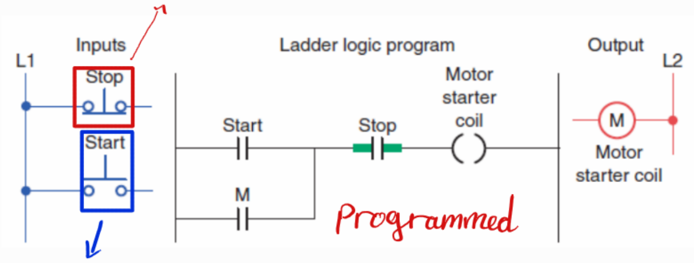
Additional input instructions can be programmed in the output branches.

A Seal-in Circuit is a method of maintaining current flow after a momentary switch has been pressed and released.



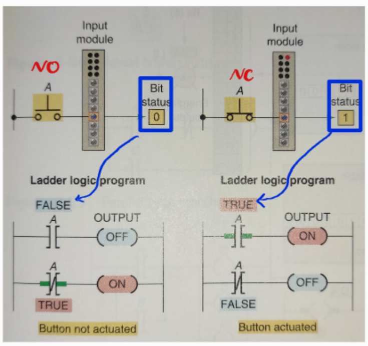
Hard wired

Hard wired → Normally closed (NC)



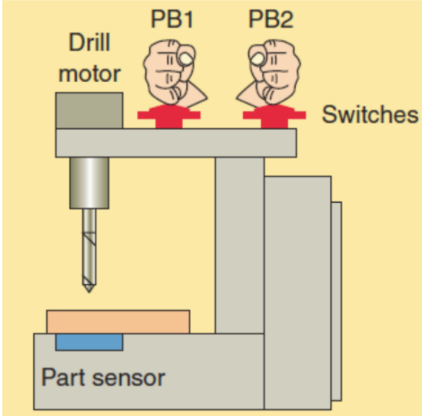
Programmed

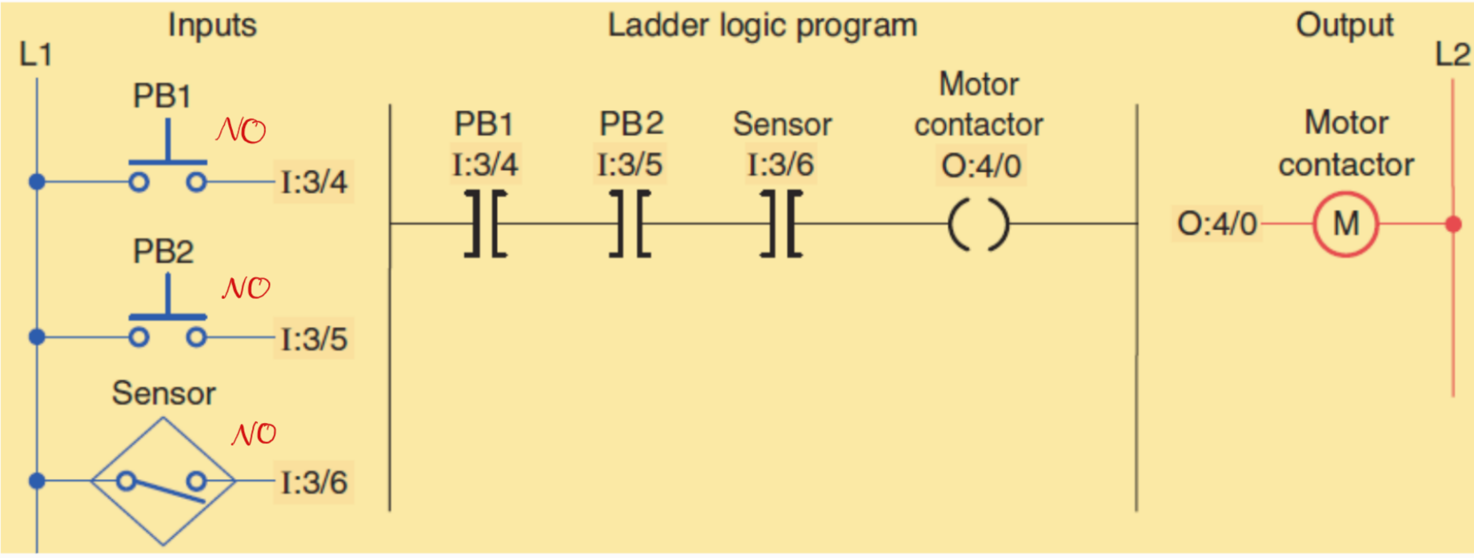
HW → Normally open (NO)



Writing a Ladder Logic Diagram from a narrative description

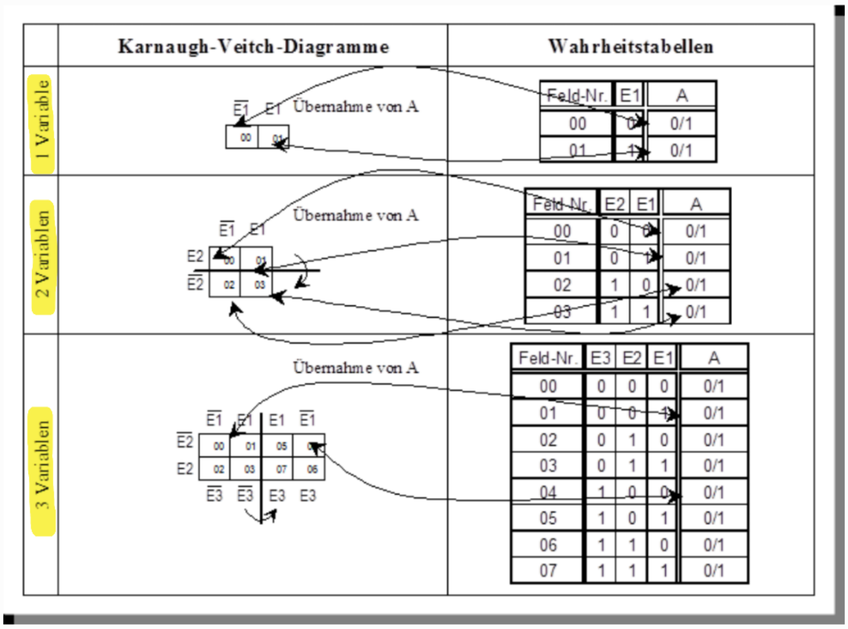
Drilling process that requires the drill press to turn on only if there is a part present and the operator has one hand on each of the start button.





Karnaugh-Veitch (KV) - Diagram:

- Tool to find optimized logical function.
- Truth table in a 2D-array
- Using symmetries and fields can be summarized. $(S1 + \overline{S1} = 1)$



4 Variables

	E3		$\overline{E3}$		
E4	1	0	0	1	$\overline{E2}$
	1	1	0	0	E2
$\overline{E4}$	1	1	1	1	
	0	1	1	1	$\overline{E2}$
	$\overline{E1}$	E1		$\overline{E1}$	

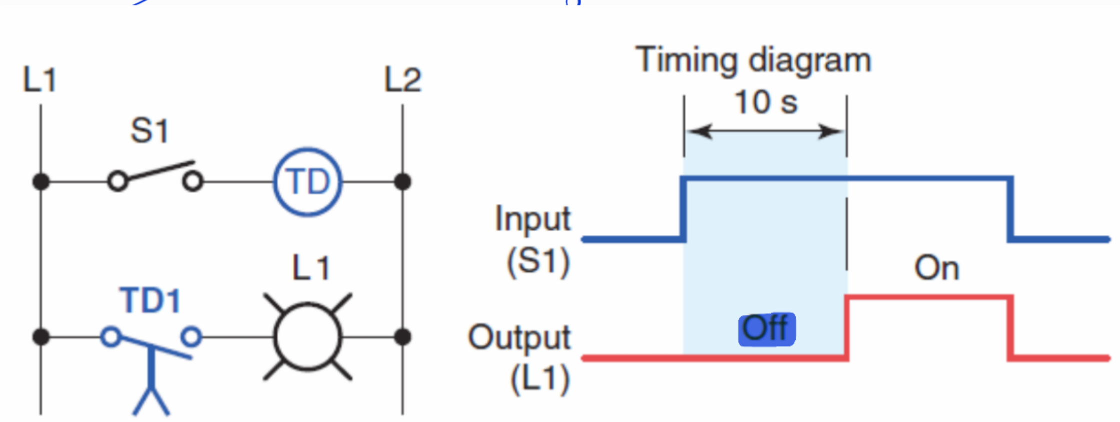
$$A = E2 \cdot E3 + E1 \cdot \overline{E4} + \overline{E3} \cdot \overline{E4} + \overline{E1} \cdot \overline{E2} \cdot \overline{E3}$$

Industrial Timing Tasks:

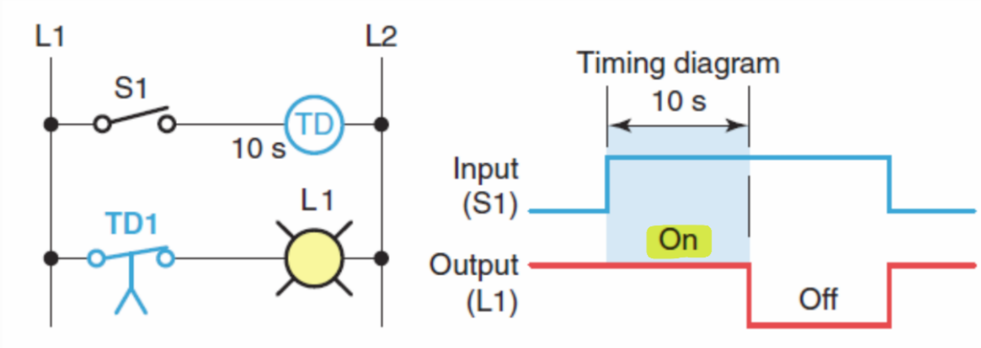
- Control of traffic lighs
- Time shift motor activation.
- Warning sequence

* On-delay timer (DOE):

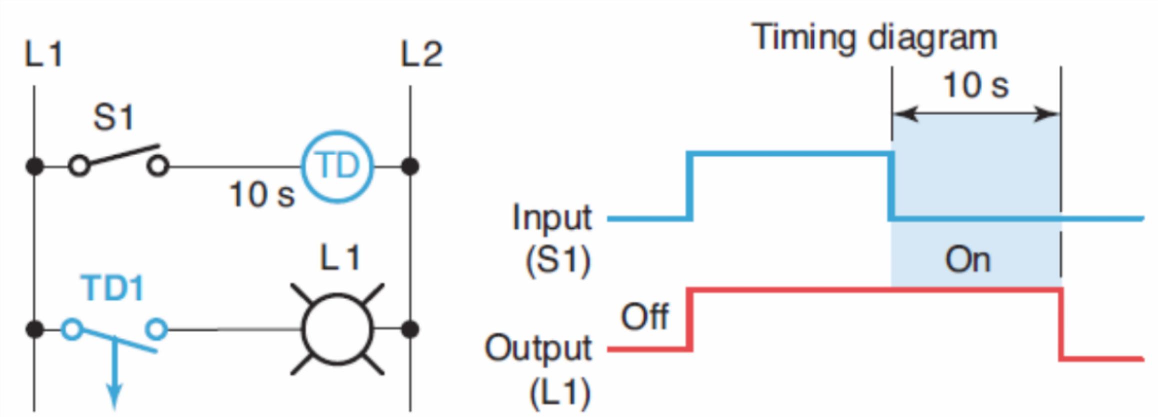
- It delays the ON time a given num. of seconds.



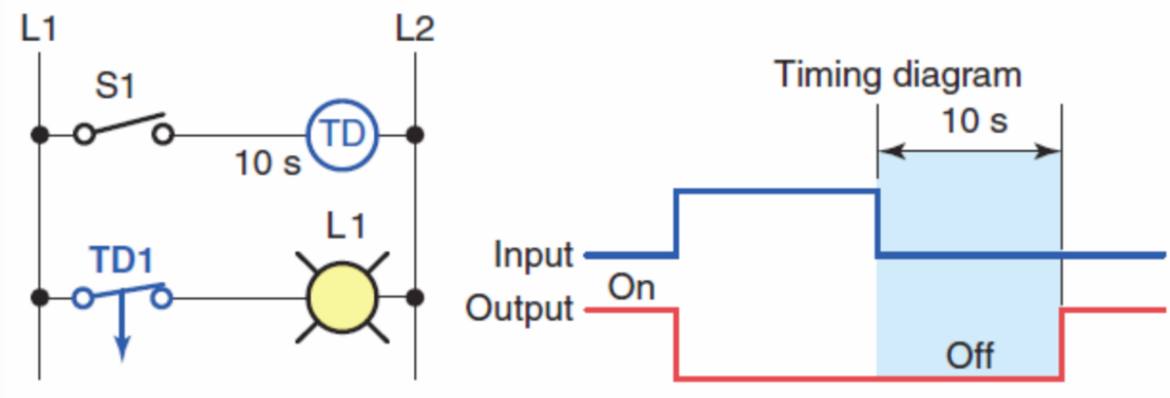
On-delay timer circuit that uses a **normally closed**, timed open (NCTO) contact.

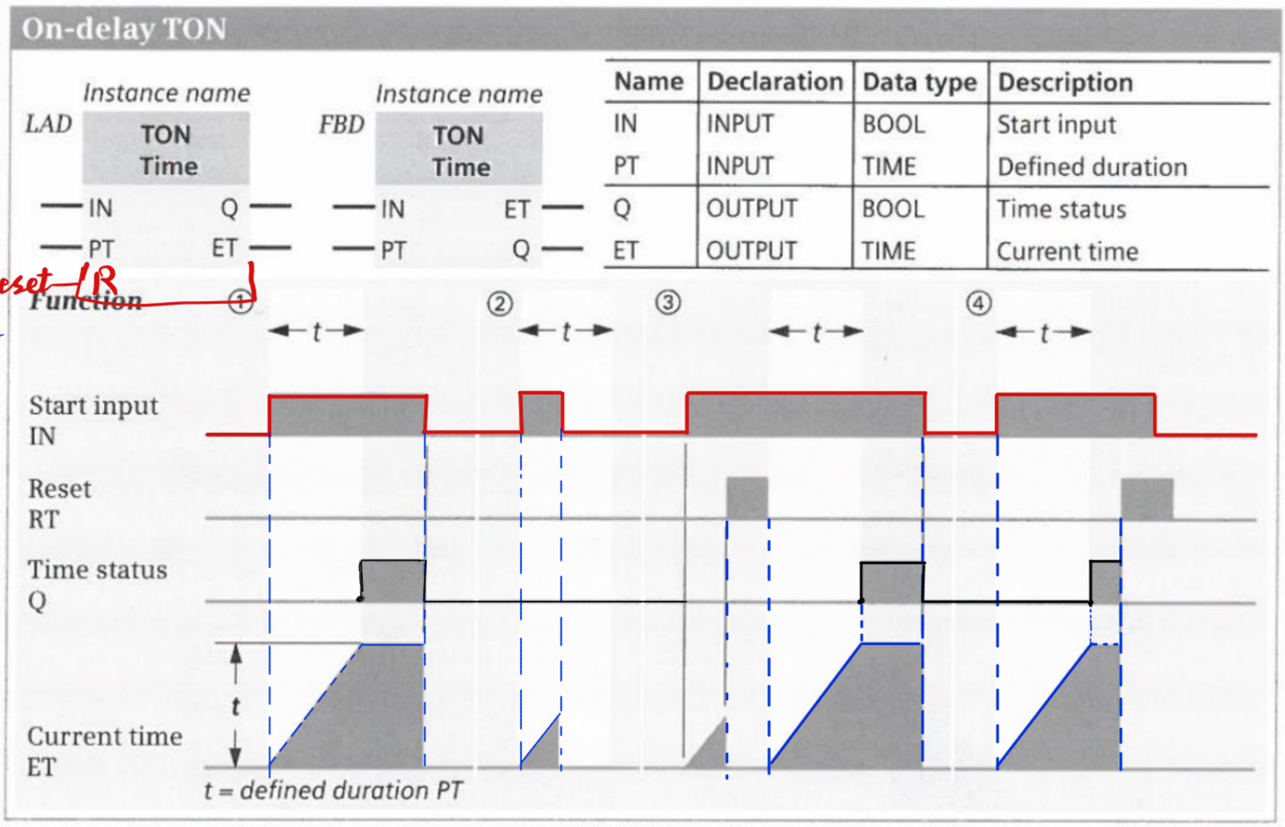


* Off-delay timer (DOD)



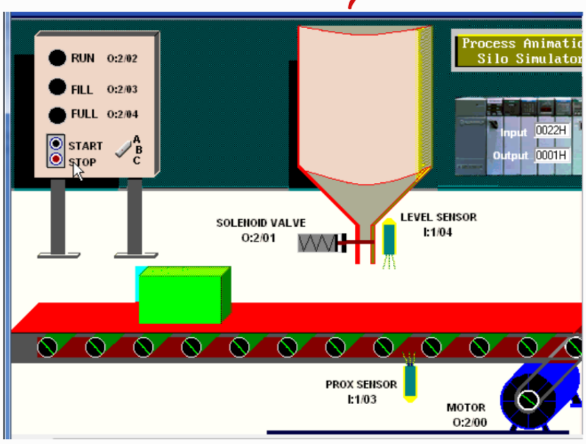
Off-delay timer circuit that uses a **normally closed**, timed closed (**NCTC**) contact.



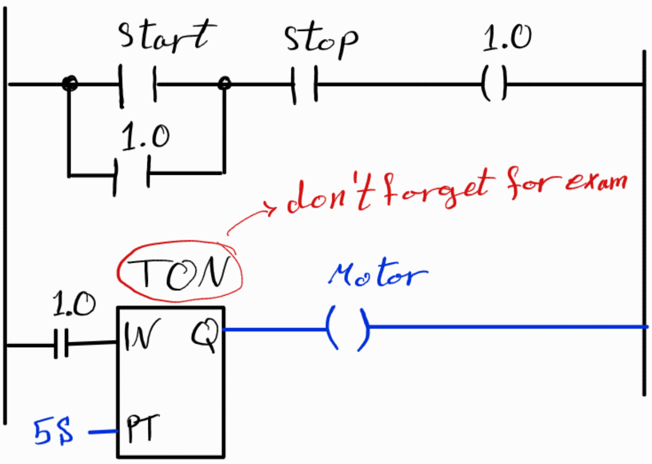


Name	Declaration	Data type	Description
IN	INPUT	BOOL	Start input
PT	INPUT	TIME	Defined duration
Q	OUTPUT	BOOL	Time status
ET	OUTPUT	TIME	Current time

Reset: either by using RT to reset, or give no input (0)

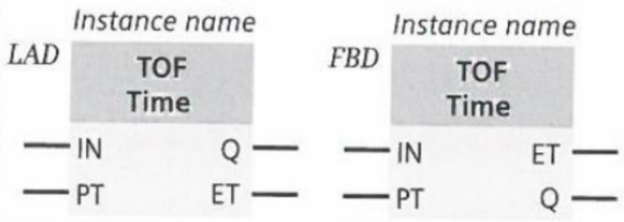


A conveyer belt starts 5sec after start button is activated.

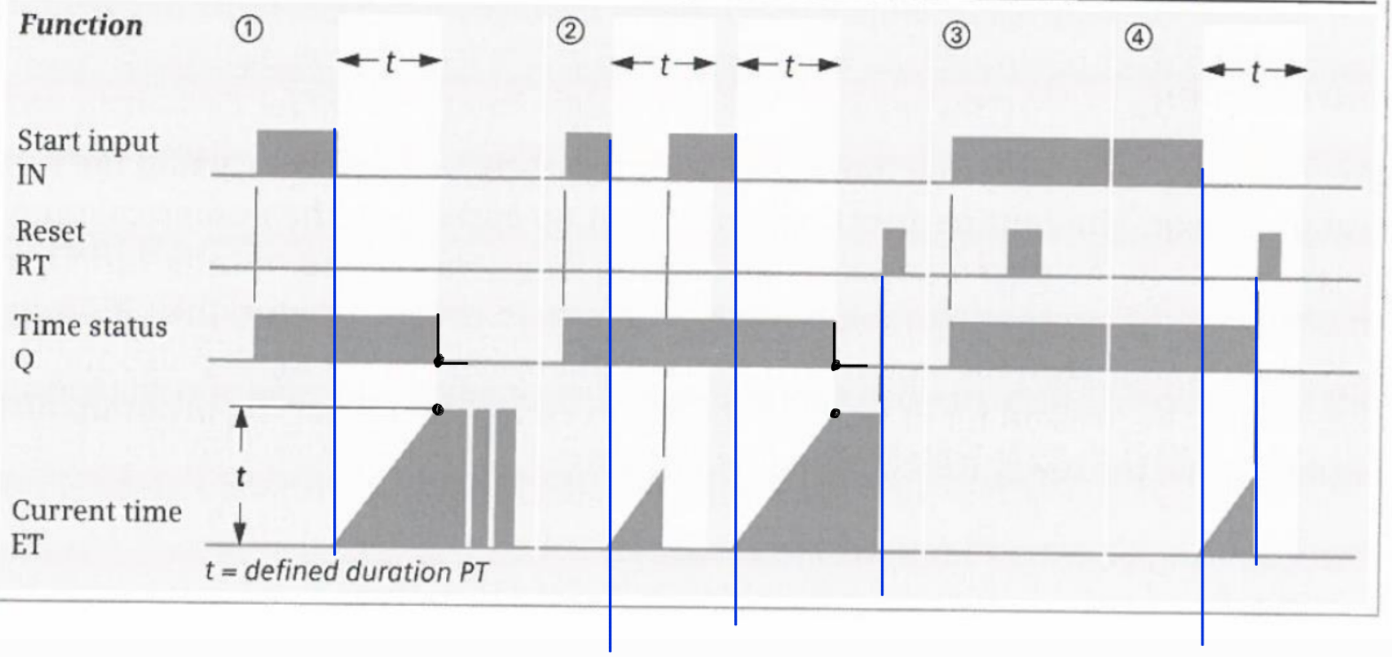


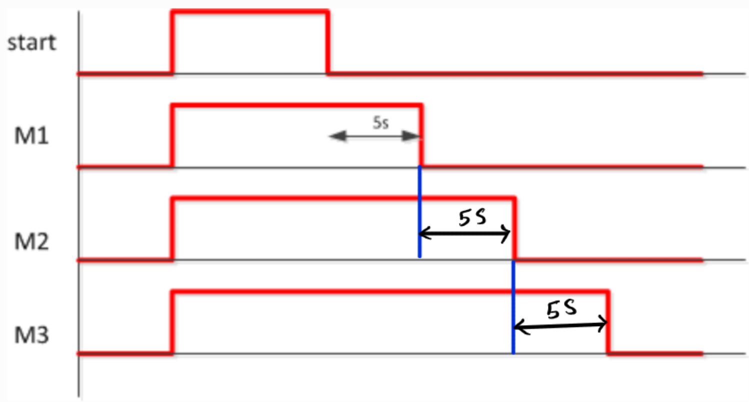
Off-delay TOF

طالما مفيش input داخل من الأول يبقى $Q = 0$

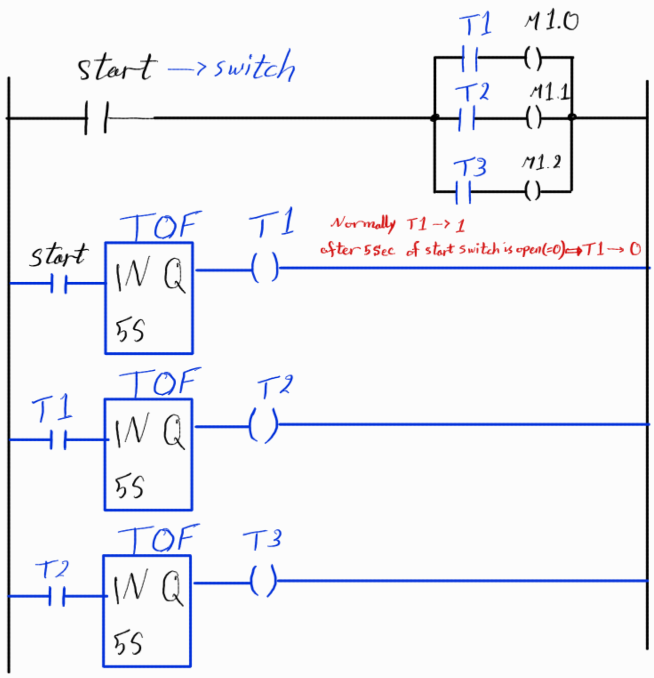


Name	Declaration	Data type	Description
IN	INPUT	BOOL	Start input
PT	INPUT	TIME	Defined duration
QO	UTPUT	BOOL	Time status
ET	OUTPUT	TIME	Current duration



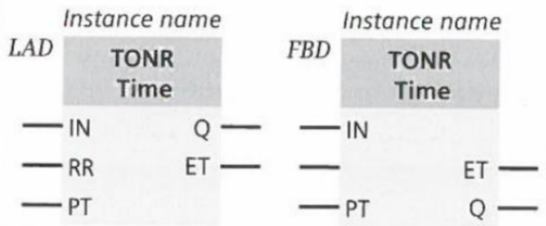


Usage of a off-delay timer instruction to switch motor off sequentially at 5s intervals.

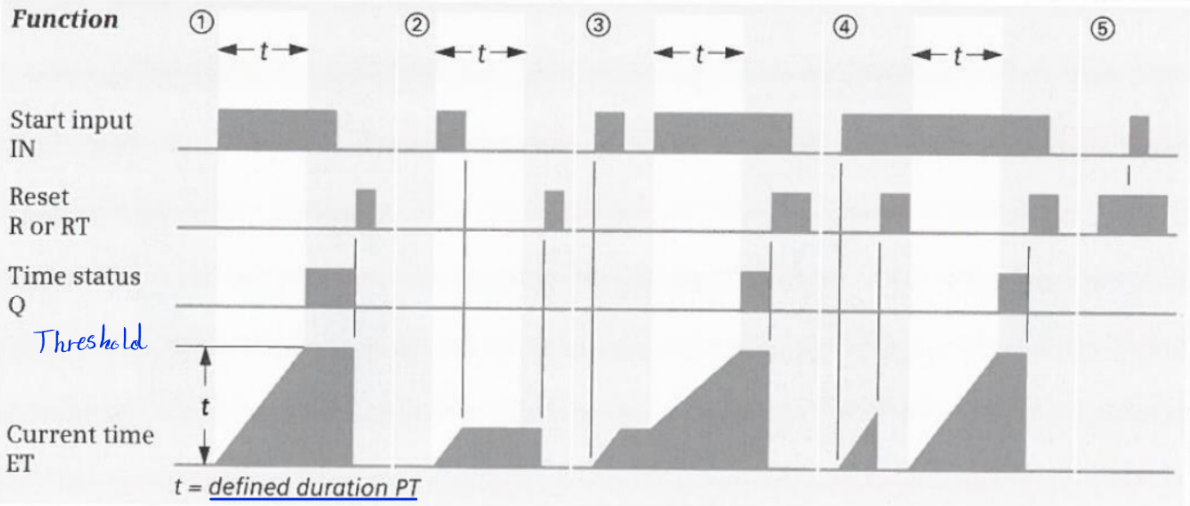


Retentive on-delay TONR / RTO, representation and function

Retentive on-delay TONR

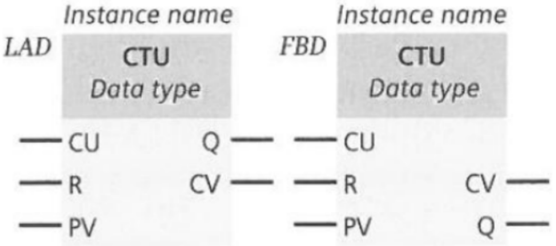


Name	Declaration	Data type	Description
IN	INPUT	BOOL	Start input
R	INPUT	BOOL	Reset input
PT	INPUT	TIME	Defined duration
Q	OUTPUT	BOOL	Time status
ET	OUTPUT	TIME	Current duration



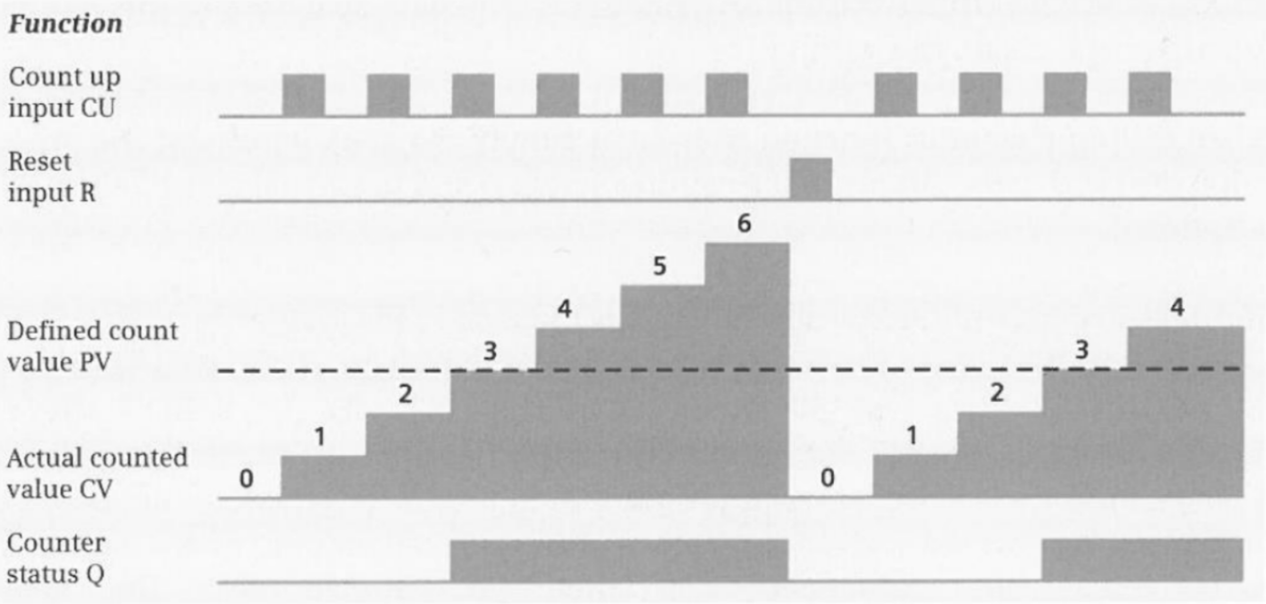
Counters:

Up counter CTU

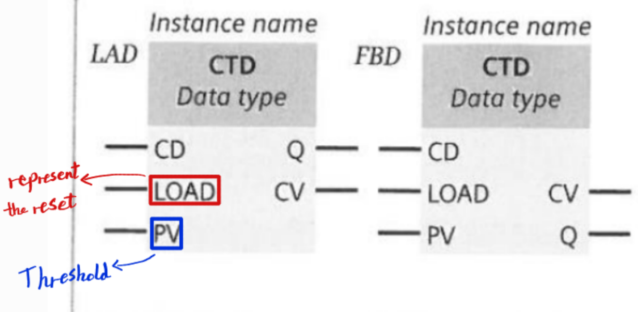


Name	Declaration	Data type	Description
CU	INPUT	BOOL	Count up input
R	INPUT	BOOL	Reset input
PV	INPUT	*Data type* *)	Defined counted value
Q	OUTPUT	BOOL	Counter status
CV	OUTPUT	*Data type* *)	Actual count value

*) USINT, UINT, UDINT, SINT, INT, DINT

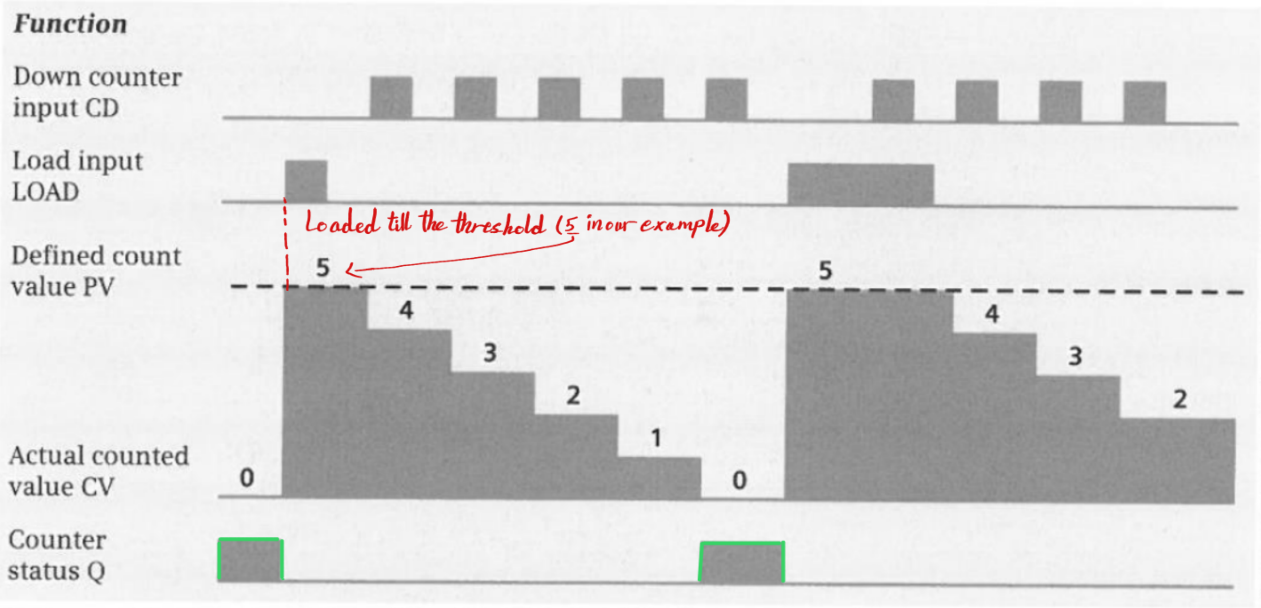


Down counter CTD

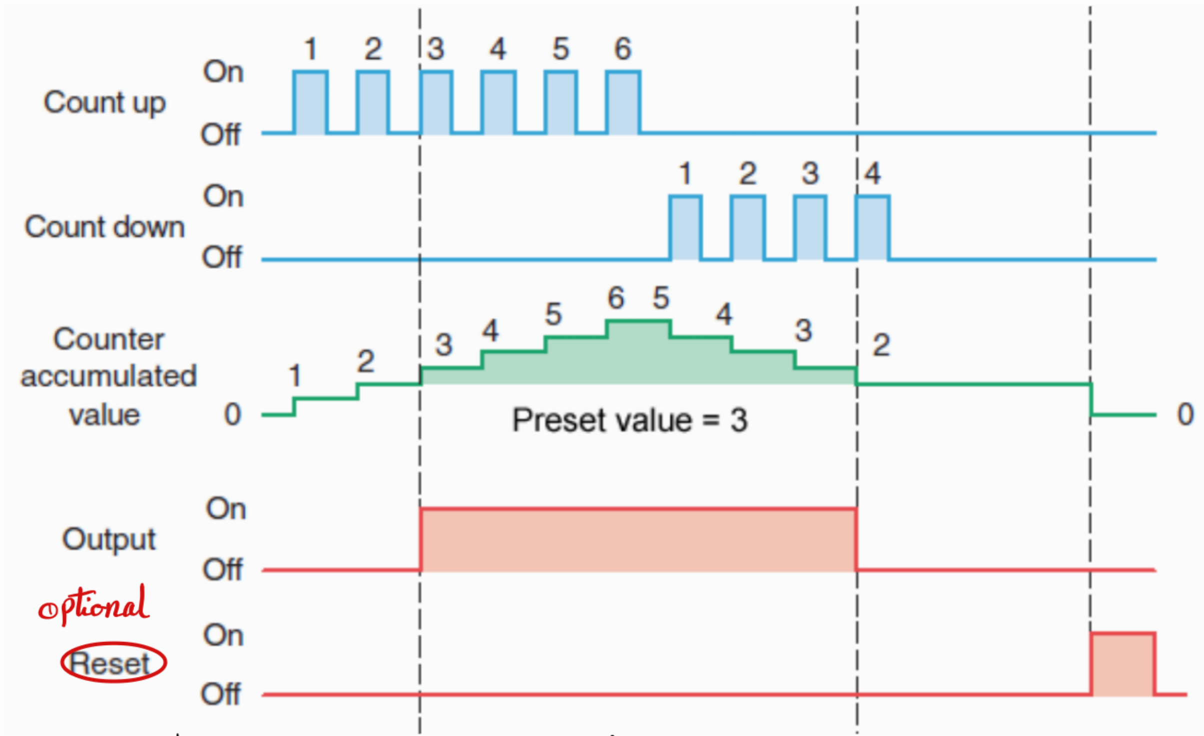


Name	Declaration	Data type	Description
CD	INPUT	BOOL	Count down input
LOAD	INPUT	BOOL	Load input
PV	INPUT	*Data type* *)	Defined count value
Q	OUTPUT	BOOL	Counter status
CV	OUTPUT	*Data type* *)	Actual count value

*) USINT, UINT, UDINT, SINT, INT, DINT



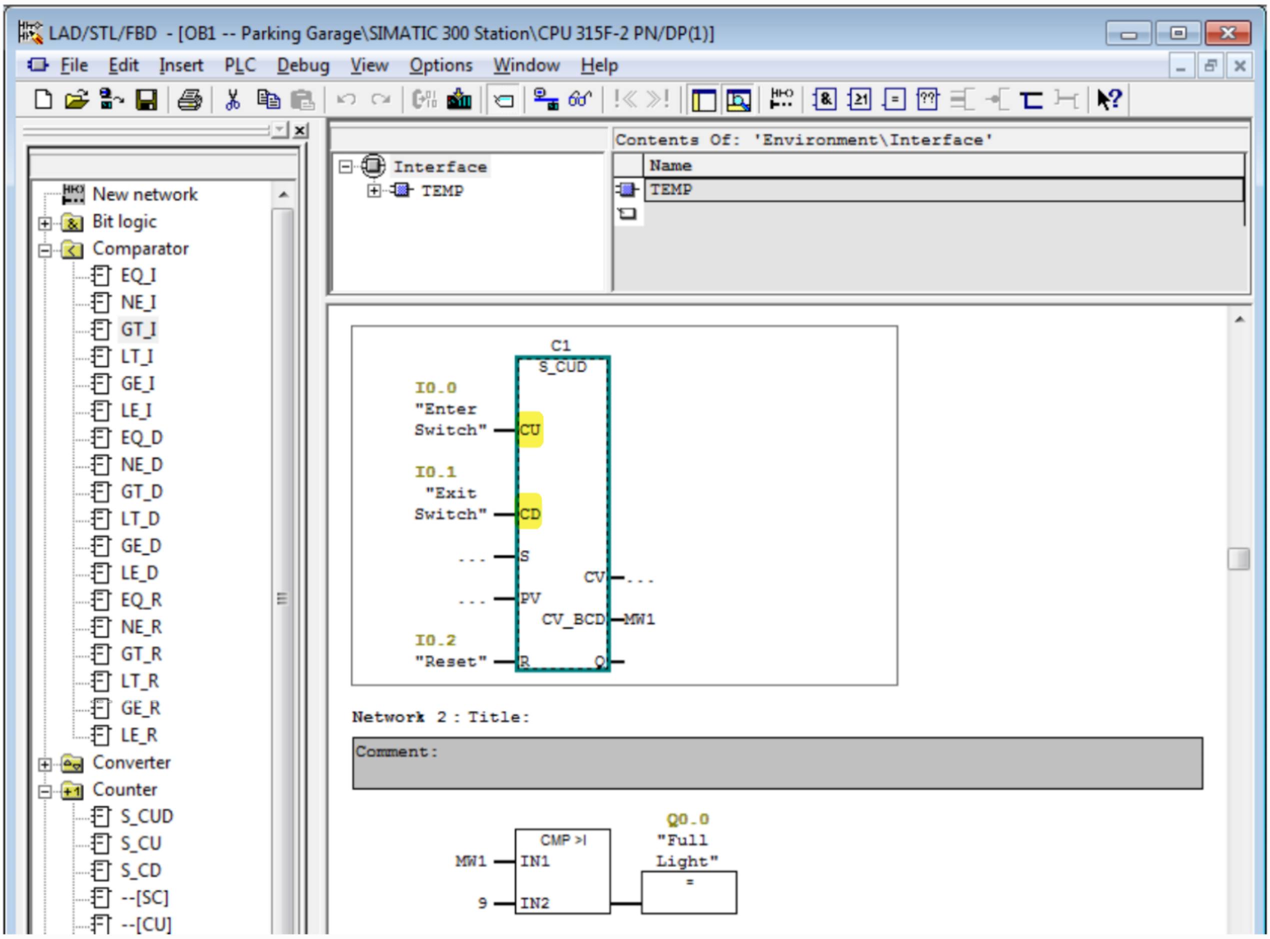
Combined Up/Down Counter : (reset is optional, unlike in single up/down it a must)



Application: parking Garage!

One application for an up/down-counter is to keep count of the cars that enter and leave a parking garage. The operation of a program can be summarized as follows:

- A car enters, the enter switch triggers the up-counter output instruction and increments the accumulated count by 1.
- As a car leaves, the exit switch triggers the down-counter output instruction and decrements the accumulated count by 1.
- Whenever the accumulated value of 150 equals the preset value of 150, a lot full light is activated.
- A reset button has been provided to reset the accumulated count.



Points:
Question 4: **16**

A packaging line is automated by use of a PLC. Figure 4.1 shows the hardware arrangement and consists of a NO start button, a NC stop button, a motor 1 to activate the main belt, and a proximity switch detecting the cans going into the packaging station. The specific sequence is as follows, when the start button is pushed.

1. The main belt is activated (by motor M1) and remains active, even when the start button is no longer pressed.
2. A proximity switch detects cans going into the packaging station.
3. Within 20 seconds (after activation according to task 1), 10 cans have to be in the packaging station, detected by the proximity switch. If this is not the case, the belt is shut off and will not be able to start again for another 30 seconds.
4. When 10 cans have passed the proximity switch, the 20 second timer for the next 10 cans is reset and the process continues.
5. The process continues and remains activated as long as the stop button is pressed. In that case the process shall stop immediately.

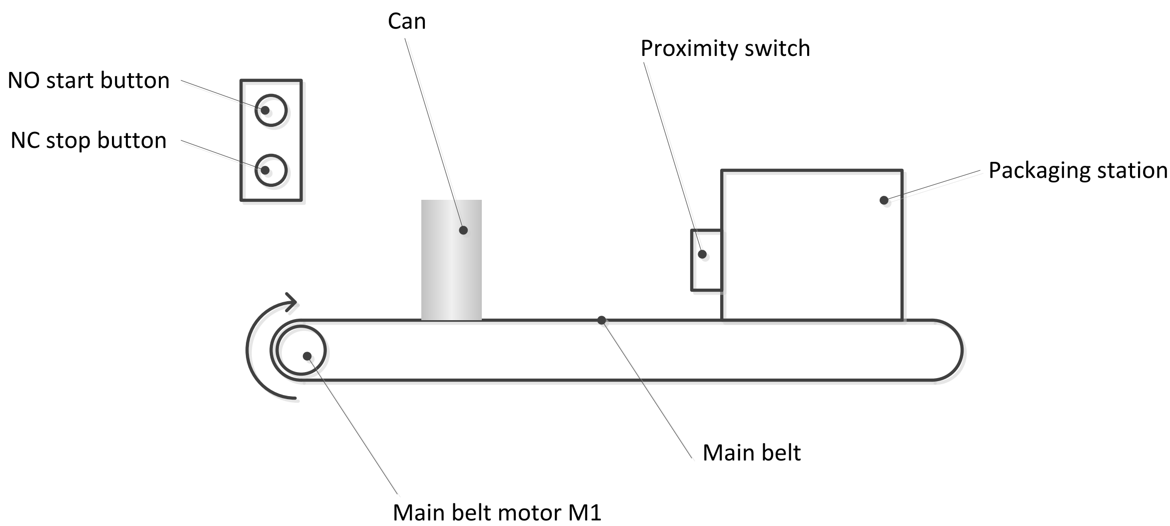
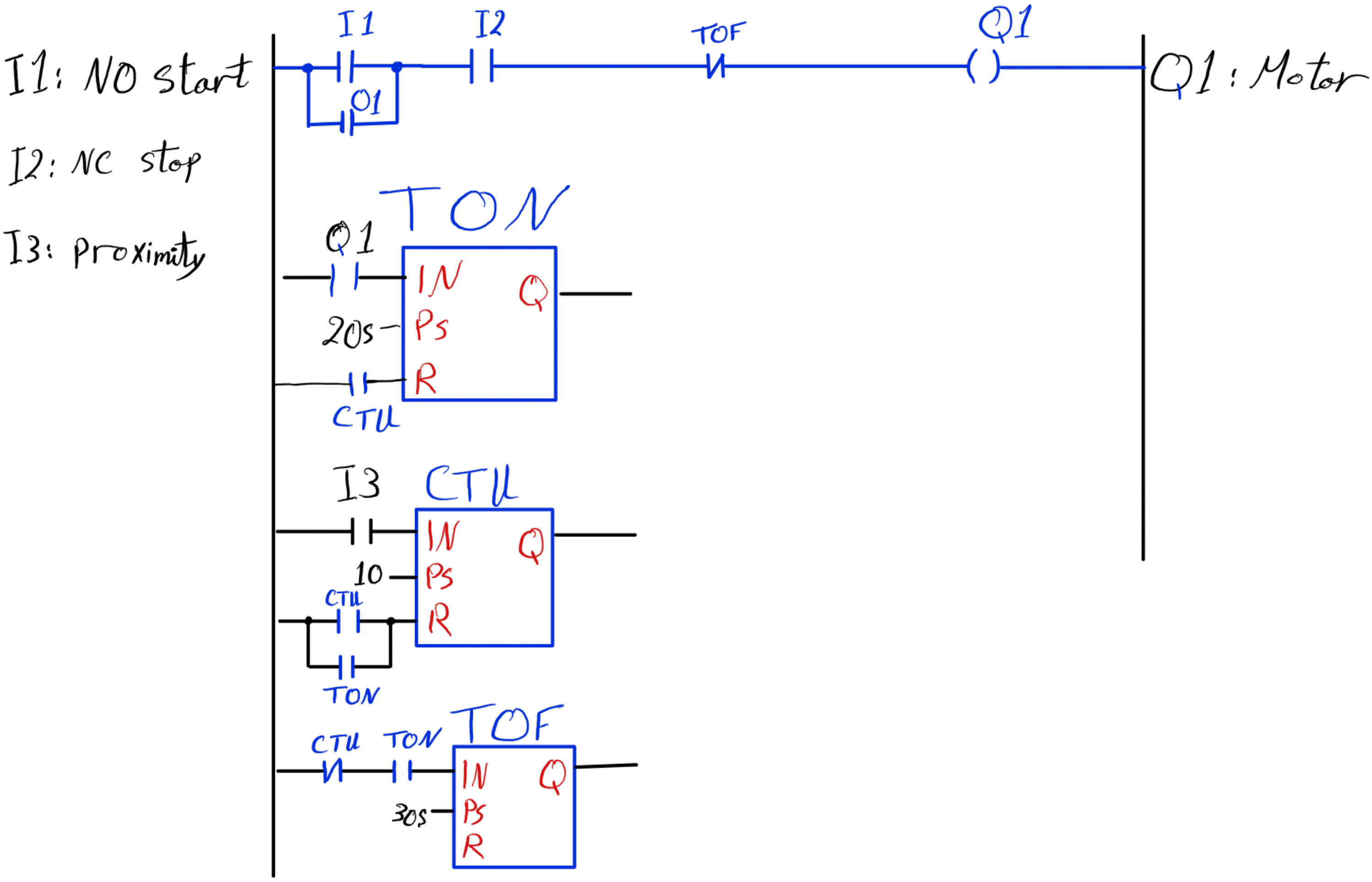


Figure 4.1: Packaging line

Develop and create the logic using a Ladder Diagram (LAD).

Student No.: ____________________


I1: NO start
I2: NC stop
I3: proximity
I1
I2
TOF
Q1
Q1
Q1: Motor
TON
Q1
IN
Q
20s
Ps
R
CTU
I3
CTU
IN
Q
10
Ps
CTU
R
TON
CTU
TON
TOF
IN
Q
30s
Ps
R

State-Space model :-

minimum number of state variables = order of differential equation

Newton´s Law yields

$$M\,\ddot{y}(t) + b\,\dot{y}(t) + k\,y(t) = u(t)$$

$x_1(t) = y(t) \xrightarrow{d/dt} \dot{x}_1(t) = \dot{y}(t) \Rightarrow \boxed{\dot{x}_1(t) = x_2(t)}$

$x_2(t) = \dot{y}(t) \xrightarrow{d/dt} \dot{x}_2(t) = \ddot{y}(t) \Rightarrow \boxed{\dot{x}_2(t) = -\frac{b}{M}x_2(t) - \frac{c}{M}x_1(t) + \frac{1}{M}u(t)}$

$M\ddot{y}(t) + b\dot{y}(t) + c\,y(t) = u(t)$

$M\dot{x}_2(t) + b\,x_2(t) + c\,x_1(t) = u(t) \Rightarrow \underline{\dot{x}_2(t) = \frac{1}{M}u(t) - \frac{b}{M}x_2(t) - \frac{c}{M}x_1(t)}$

output equation $y = x_1$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} \cdot u$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{c}{M} & -\frac{b}{M} \end{bmatrix}\underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\substack{\text{state vector} \\ \underline{x}}} + \begin{bmatrix} 0 \\ \frac{1}{M} \end{bmatrix} \cdot \underbrace{u(t)}_{\text{Input/control vector}}$$

$$\begin{aligned} \dot{x}_1 &= a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n + b_{11}u_1 + \cdots + b_{1m}u_m \\ \dot{x}_2 &= a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n + b_{21}u_1 + \cdots + b_{2m}u_m \\ &\vdots \\ \dot{x}_n &= a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n + b_{n1}u_1 + \cdots + b_{nm}u_m \end{aligned}$$

state vector $\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$

Input/control vector $\underline{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix}$

State differential equation

$$\boxed{\dot{x} = Ax + Bu}$$

Output equation

$$\boxed{y = Cx + Du}$$

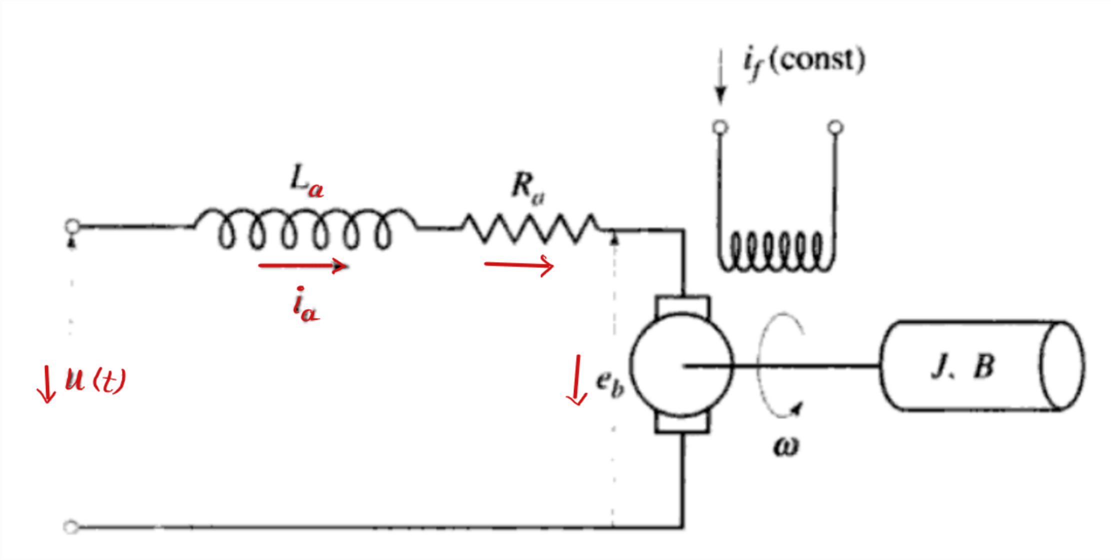
x := State vector
A := System matrix
u := Control vector
B := Input matrix
y := Output vector
C := Output matrix
D := Feedforward matrix

$\dot{\underline{x}} = \underline{\underline{A}}\,\underline{x} + \underline{\underline{B}}\,\underline{u}$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix} = n\underbrace{\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}}_{n}\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + n\underbrace{\begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1m} \\ b_{21} & b_{22} & \cdots & b_{2m} \\ \vdots & \vdots & & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nm} \end{bmatrix}}_{m}\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix}$$

$y = \underline{\underline{C}}\,\underline{x} + \underline{\underline{D}}\,\underline{u}$

$$y = 1\underbrace{\begin{bmatrix} \quad \end{bmatrix}}_{n}\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} + 1\underbrace{\begin{bmatrix} \quad \end{bmatrix}}_{m}\begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix}$$



$$u(t) = R_a \cdot i_a(t) + L_a \frac{di_a}{dt} + e_b(t) \qquad (1)$$

↳ back emf voltage

$$T_M = J \cdot \frac{dw}{dt} + B\,w(t) \qquad (2)$$

T_M: Torque from motor; J: moment of ineria; $\frac{dw}{dt}$: angular acceleration; B: viscos friction

$$T_M = K_T \cdot i_a(t) \qquad (3)$$

↳ motor torque const.

$$e_b(t) = K_b \cdot w(t) \qquad (4)$$

↳ back emf const.

$X_1 = w$, $X_2 = i_a$

from (2) and (3)

$$\boxed{\frac{dw}{dt} = -\frac{B}{J} w(t) + \frac{K_T}{J} i_a(t)}$$

from (1) and (4)

$$\boxed{\frac{di_a}{dt} = -\frac{K_b}{L_a} w(t) - \frac{R_a}{L_a} i_a(t) + u(t)}$$

$$\dot{X}_1 = -\frac{B}{J} X_1 + \frac{K_T}{J} X_2 \qquad \dot{X}_2 = -\frac{K_b}{L_a} X_1 - \frac{R_a}{L_a} X_2 + u(t)$$

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} -\frac{B}{J} & \frac{K_T}{J} \\ -\frac{K_b}{L_a} & -\frac{R_a}{L_a} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \cdot u$$

$$w = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} \cdot u$$

output equation

Phase variable canonical form:

$$\frac{Y(s)}{U(s)} = G(s) = \frac{b_3 s^3 + b_2 s^2 + b_1 s + b_0}{s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0} \cdot \frac{Z(s)}{Z(s)}$$

$$Y(s) = \left[b_3 S^3 + b_2 S^2 + b_1 S + b_0\right] \cdot Z(s)$$

$$U(s) = \left[S^4 + a_3 S^3 + a_2 S^2 + a_1 S + a_0\right] \cdot Z(s)$$

$$\mathcal{L}^{-1} \longrightarrow \quad y(t) = b_3 \dddot{Z}(t) + b_2 \ddot{Z}(t) + b_1 \dot{Z}(t) + b_0 Z(t)$$

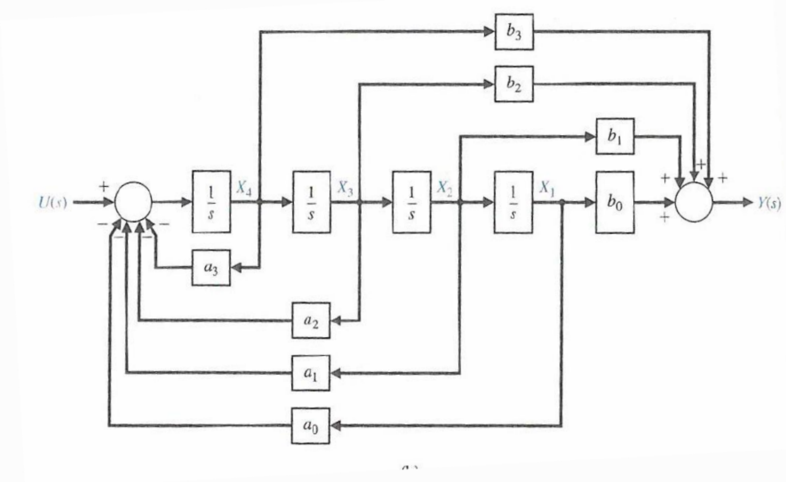
$$U(t) = \ddddot{Z}(t) + a_3 \dddot{Z}(t) + a_2 \ddot{Z}(t) + a_1 \dot{Z}(t) + a_0 Z(t)$$

$$\left.\begin{aligned} x_1 &= Z \\ x_2 &= \dot{Z} \\ x_3 &= \ddot{Z} \\ x_4 &= \dddot{Z} \end{aligned}\right\} \quad \boxed{\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= -a_0 x_1 - a_1 x_2 - a_2 x_3 - a_3 x_4 + U(t) \end{aligned}}$$

output equation: $\boxed{y = b_0 x_1 + b_1 x_2 + b_2 x_3 + b_3 x_4}$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 & -a_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \cdot u$$

$$y(t) = \begin{bmatrix} b_0 & b_1 & b_2 & b_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + [\,0\,] \cdot u$$



Converting from State Space to a TF

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}u$$
$$y = \boldsymbol{C}\boldsymbol{x} + \boldsymbol{D}u$$

where y is the single output and u is the single input

Let´s apply the Laplace transform, we get

$$s\boldsymbol{X}(s) = \boldsymbol{A}\boldsymbol{X}(s) + \boldsymbol{B}U(s) \quad (1)$$
$$Y(s) = \boldsymbol{C}\boldsymbol{X}(s) + \boldsymbol{D}U(s) \quad (2)$$

$G(s) = \frac{Y(s)}{U(s)}$

$$\underline{X}(s)\left[s\underline{\underline{I}} - \underline{\underline{A}}\right] = \underline{B}\, U(s) \qquad \text{with } \left[s\underline{\underline{I}} - \underline{\underline{A}}\right]^{-1} = \underline{\underline{\Phi}}(s)$$

$$\underline{X}(s) = \left[s\underline{\underline{I}} - \underline{\underline{A}}\right]^{-1} \underline{B}\, U(s)$$

$$\left[s\underline{\underline{I}} - \underline{\underline{A}}\right]^{-1} = \frac{\operatorname{adj}(s\underline{\underline{I}} - \underline{\underline{A}})}{\det(s\underline{\underline{I}} - \underline{\underline{A}})}$$

$$\underline{X}(s) = \underline{\underline{\Phi}}(s)\, \underline{B}\, U(s)$$

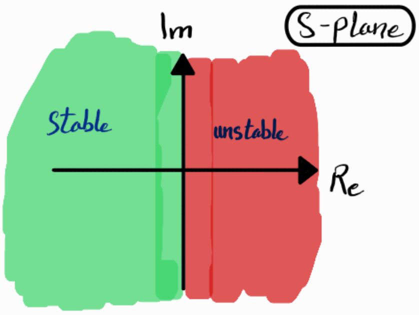
Into (2)

$$Y(s) = \underline{C}\, \underline{\underline{\Phi}}(s)\, \underline{B}\, U(s) + \underline{D}\, U(s)$$

$$\boxed{G(s) = \frac{Y(s)}{U(s)} = \underline{C}\, \underline{\underline{\Phi}}(s)\, \underline{B} + \underline{D}}$$

Stability: depends on the system's poles

denominator $= 0 \implies \boxed{\det(s\underline{\underline{I}} - \underline{\underline{A}}) = 0}$



$$\det(A) = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}\begin{matrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{matrix}$$

add the red product,
substract the green product.

Adjoint of 2x2 Matrix

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ — Interchange, Change signs

$$\operatorname{adj} A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Example

Given the system defined by the equation

$$\dot{\boldsymbol{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \boldsymbol{x}$$

find the TF $G(s) = \frac{Y(s)}{U(s)}$, where $U(s)$ is the input and $Y(s)$ the output.

$$[s\underline{\underline{I}} - A] = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} = \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 1 & 2 & s+3 \end{bmatrix}$$

$$\underline{\underline{M}}^{-1} = \frac{\operatorname{adj} \underline{\underline{M}}}{\det \underline{\underline{M}}}, \quad \underline{\underline{M}}^{-1} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}^{-1} = \frac{1}{\det(M)} \begin{bmatrix} ei-fh & ch-bi & bf-ce \\ fg-di & ai-cg & cd-af \\ dh-eg & bg-ah & ae-bd \end{bmatrix}$$

$$[s\underline{\underline{I}} - A]^{-1} = \frac{\begin{bmatrix} s(s+3)+2 & s+3 & 1 \\ -1 & s(s+3) & s \\ -s & -(1+2s) & s^2 \end{bmatrix}}{\underbrace{s(s^2+3s+2)+1(+1)}_{s^3+3s^2+2s+1 = \nabla}} = \underline{\underline{\Phi}}(s)$$

$$\underline{C} \cdot \underline{\underline{\Phi}} = [1 \quad 0 \quad 0] \cdot \underline{\underline{\Phi}}(s) = \frac{1}{\nabla} \cdot [s(s+3)+2 \quad (s+3) \quad 1]$$

$$G(s) = \underline{C} \cdot \underline{\underline{\Phi}} \cdot \underline{B} + \cancel{\underline{D}}^{\,0} = \frac{1}{\nabla} \cdot [s(s+3)+2 \quad (s+3) \quad 1] \cdot \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix} = \frac{10}{\nabla}(s^2+3s+2)$$

$$\boxed{G(s) = \frac{10(s^2+3s+2)}{s^3+3s^2+2s+1}}$$

stability: $\det(s\underline{\underline{I}} - \underline{A}) = 0 \Rightarrow s^3+3s^2+2s+1 = 0$

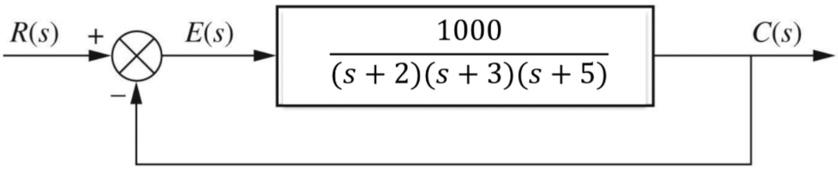
s^3	1	2
s^2	3	1
s^1	$b_1 = \frac{-\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix}}{3} = \frac{5}{3}$	$b_2 = \frac{-\begin{vmatrix} 1 & 0 \\ 3 & 0 \end{vmatrix}}{3} = 0$
s^0	$c_1 = \frac{-\begin{vmatrix} 3 & 1 \\ \frac{5}{3} & 0 \end{vmatrix}}{\frac{5}{3}} = 1$	$c_2 = \frac{-\begin{vmatrix} 3 & 0 \\ \frac{5}{3} & 0 \end{vmatrix}}{\frac{5}{3}} = 0$

No sign change in the 1st column

→ The system is stable

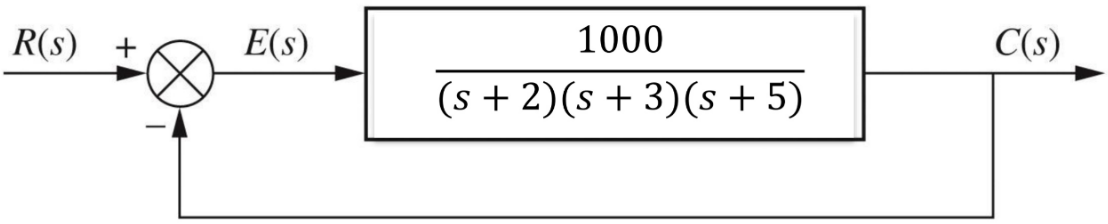
- System is stable if there are no sign changes in the first column of the Routh table

Example



s^3	a_3	a_1
s^2	a_2	a_0
s^1	$b_1 = \dfrac{-\begin{vmatrix} a_3 & a_1 \\ a_2 & a_0 \end{vmatrix}}{a_2}$	$b_2 = \dfrac{-\begin{vmatrix} a_3 & 0 \\ a_2 & 0 \end{vmatrix}}{a_2} = 0$
s^0	$c_1 = \dfrac{-\begin{vmatrix} a_2 & a_0 \\ b_1 & 0 \end{vmatrix}}{b_1}$	$c_2 = \dfrac{-\begin{vmatrix} a_2 & 0 \\ b_1 & 0 \end{vmatrix}}{b_1} = 0$

$$G(s) = \frac{\dfrac{1000}{(s+2)(s+3)(s+5)}}{1 + \dfrac{1000}{(s+2)(s+3)(s+5)}} = \frac{1000}{s^3+10s^2+31s+1030}$$



s^3	$a_3 = 1$	$a_1 = 31$
s^2	$a_2 = 10$	$a_0 = 1030$
s^1	$b_1 = \dfrac{-\begin{vmatrix} 1 & 31 \\ 10 & 1030 \end{vmatrix}}{10} = -72$	$b_2 = 0$
s^0	$c_1 = \dfrac{-\begin{vmatrix} 10 & 1030 \\ -72 & 0 \end{vmatrix}}{-72} = 1030$	$c_2 = 0$

Sign Change

$$G(s) = \frac{1000}{s^3+10s^2+31s+1030}$$

$$G(s) = \frac{1000}{a_3 s^3 + a_2 s^2 + a_1 s^1 + a_0}$$

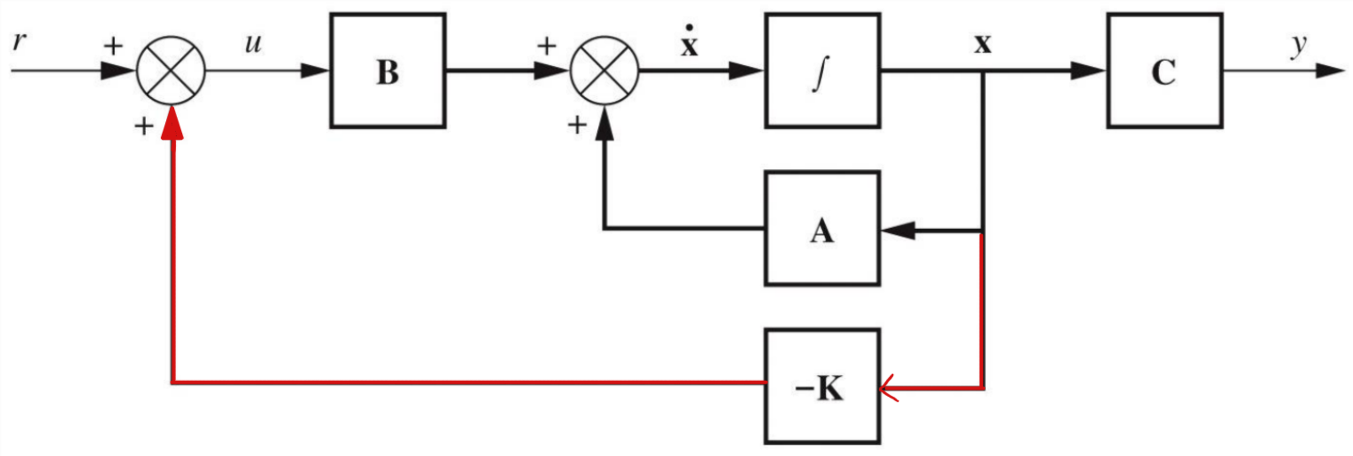
$\Rightarrow$ 2 sign change

$\Rightarrow$ 2 unstable poles.

Quadratic formula:

$$P_{1,2} = \frac{-b}{2} \pm \sqrt{\frac{b^2}{4} - c}$$

State equation for the closed-loop system:



$\dot{x} = B u + A x = B(r + (-K x)) + A x = (A - BK)x + Br$

$y = Cx$

Comparizon:

Open-loop: $\dot{x} = Ax + Bu \longrightarrow$ system matrix is A

Eigenvalues of system matrix = poles

Closed-loop: $\dot{x} = (A-BK)x + Br \longrightarrow$ system matrix is $(A-BK)$

Example: An unstable system, characterized through the state space model:

$\dot{x} = \underbrace{\begin{bmatrix} 4 & 8 \\ 1 & -5 \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{B} u$ is controlled by a feedback vector $K = [1 \quad k_2]$

Calculate k_2 so that the controlled system is stable

$BK = \begin{bmatrix} 1 & K_2 \\ 0 & 0 \end{bmatrix}$, $[A-BK] = \begin{bmatrix} 3 & 8-K_2 \\ 1 & -5 \end{bmatrix}$

$\det(sI - [A-BK]) = \begin{vmatrix} s-3 & K_2-8 \\ -1 & s+5 \end{vmatrix} = (s-3)(s+5) + (K_2-8) = s^2 + 2s + (K_2 - 23)$

Routh table

s^2	1	K_2-23
s^1	2	0
s^0	$\dfrac{-\begin{vmatrix} 1 & K_2-23 \\ 2 & 0 \end{vmatrix}}{2} = K_2-23$	

$K_2 - 23 > 0$

$K_2 > 23$

For a stable system with this feedback vector

Pole placement for plants in phase-variable form

(Where the system is in Canonical form and the Poles are given, we need to determine the feedback vector K)

Step 1) Represent the plant in phase-variable (canonical form).

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -a_0 & -a_1 & -a_2 & \cdots & -a_{n-1} \end{bmatrix}; B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$$

$$C = [c_1 \quad c_2 \quad \cdots \quad c_n]$$

$$G(s) = \frac{b_3s^3 + b_2s^2 + b_1s + b_0}{s^4 + a_3s^3 + a_2s^2 + a_1s + a_0}$$

The characteristic equation is thus

$$s^n + a_{n-1}s^{n-1} + \cdots + a_1s + a_0 = 0$$

Step 2) $K = [K_1 \quad K_2 \quad \cdots\cdots \quad K_n]$

Step 3) characteristic equ. for closed-loop system

$$\underline{A} - \underline{B}\,\underline{K} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -(a_0+K_1) & -(a_1+K_2) & -(a_2+K_3) & \cdots & -(a_{n-1}+K_n) \end{bmatrix}$$

$$\det(s\boldsymbol{I} - (\boldsymbol{A} - \boldsymbol{BK})) = s^n + (a_{n-1} + k_n)s^{n-1} + (a_{n-2} + k_{n-1})s^{n-2} + \cdots + (a_1 + k_2)s + (a_0 + k_1) = 0$$

Notice the relationship to the open-loop system → open-loop characteristic equation by adding the appropriate k_i to each coefficient.

Step 4) Use the poles to get the polynomial equ. of the deminator.

Now assume that the desired characteristic equation for proper pole placement is;

$$(S-P_1)(S-P_2)(S-P_n) = s^n + d_{n-1}s^{n-1} + d_{n-2}s^{n-2} + \cdots + d_2s^2 + d_1s + d_0$$

Step 5) Equating Coefficients (K_i)

$$d_i = a_i + K_{i+1} \; ; \quad i = 0, 1, 2, \ldots, n-1$$

from which $\Rightarrow \boxed{K_{i+1} = d_i - a_i}$

Controllability:

For SISO-Systems, the Controllability matrix is

$$C_M = [B \quad AB \quad A^2B \quad --- \quad A^{n-1}B] \quad , \text{rank}(C_M) = n$$

$\longrightarrow$ $n \times n$ matrix $\longrightarrow$ The system is Controllable if $\det(C_M) \neq 0$

Example: let's consider the system:

$$\dot{\underline{x}} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 \end{bmatrix}}_{A} \underline{x} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{B} u$$

Is the system Controllable?

$$C_M = [B \quad AB \quad A^2B]$$

$$AB = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -a_2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 \\ a_0a_2 & a_1a_2 & a_2^2 \end{bmatrix}$$

$$A^2B = \begin{bmatrix} 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 \\ a_0a_2 & a_1a_2 & a_2^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -a_2 \\ a_2^2 \end{bmatrix}$$

$$C_M = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -a_2 \\ 1 & -a_2 & a_2^2 \end{bmatrix}$$

$$\det(C_M) = 1 \cdot \begin{vmatrix} 0 & 1 \\ 1 & -a_2 \end{vmatrix} = -1 \neq 0$$

$\therefore$ The system is Controllable

Consider the system

$$G(s) = \frac{1}{s^2}$$

and check if the system is controllable. If so, determine the feedback gain to place the closed-loop poles at $p_{1,2} = -1 \pm j$.

$$\dot{\underline{x}} = \begin{bmatrix} 0 & 1 \\ \underbrace{0}_{-a_0} & \underbrace{0}_{-a_1} \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \cdot u$$

$$y = [\underbrace{1}_{b_0} \quad \underbrace{0}_{b_1}] \underline{x} + [0] \cdot u$$

$$\underline{C_M} = [B \quad AB] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad , \det(C_M) = -1 \neq$$

$\therefore$ The system is Controllable.

step 2) $\underline{K} = [K_1 \quad K_2]$

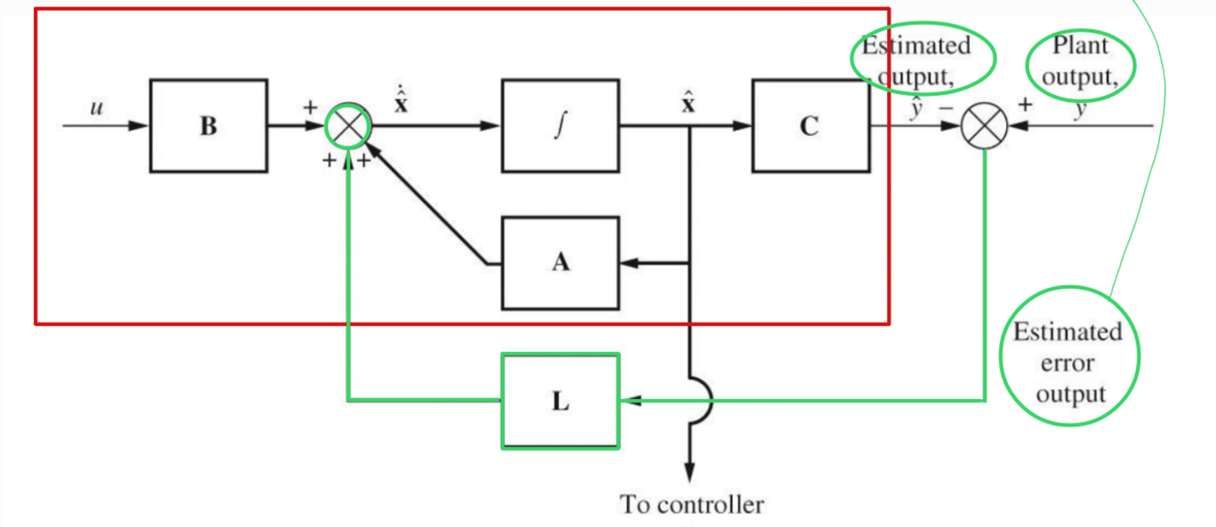
step 3) $\underline{\underline{A}} - \underline{B}\,\underline{K} = \begin{bmatrix} 0 & 1 \\ -K_1 & -K_2 \end{bmatrix}$

step 4) $(s+1-j)(s+1+j) = s^2+2s-\cancel{sj}+\cancel{sj}-\cancel{j}+\cancel{j}+1-(-1) = s^2 \underbrace{+2}_{d_1} s \underbrace{+2}_{d_0}$

step 5) $d_1 = -K_2 \Rightarrow \boxed{K_2 = -2}$, $d_0 = -K_1 \Rightarrow \boxed{K_1 = -2}$

$\therefore \underline{K} = [-2 \;\; -2]$ is the feedback gain to the closed-loop poles at $p_{1,2} = -1 \pm j$

In red is corrected with what is in green.



Digital Computer systems:

Application:

* Industry has grown over the past three decades.
* Transistor density
* Powerful
* Mobile capability
* Improved measurement sensitivity.
* Flexibility
* Reconfigure control algorithm in software

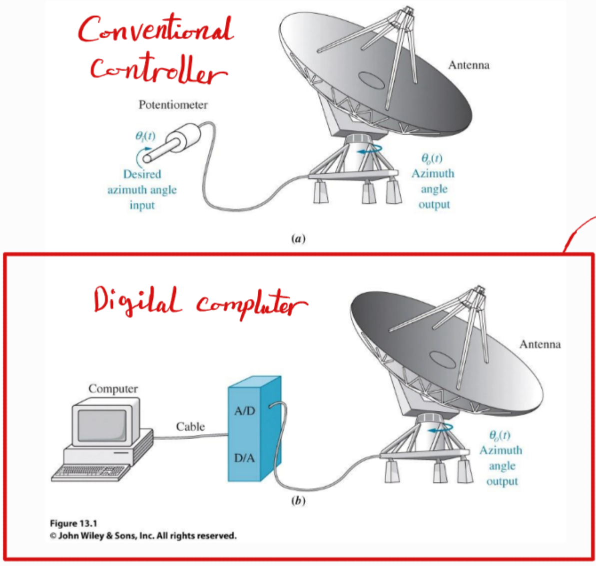
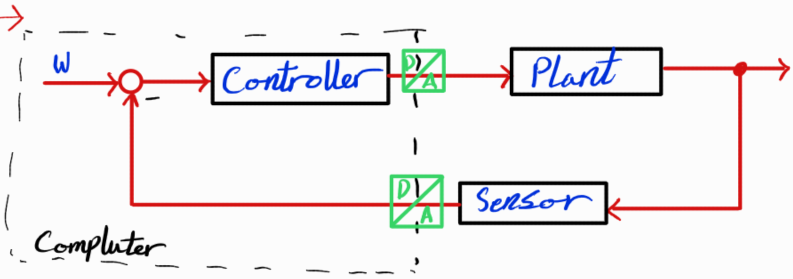
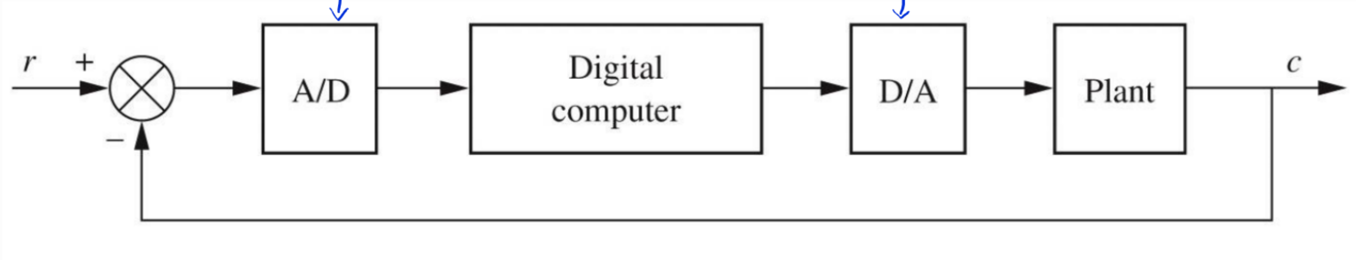


Figure 13.1



at input of digital comp.

at output



(b)

Figure 13.2

- Analog signal (a)
- Analog signal sampled at periodic intervals and held over the sampling interval (b)
- Device so called zero-order sample-and-hold (z.o.h.)
- After sampling and holding, the A/D-converter converts to a digital number (c)
- Quantization error
- Stability and transient response are now dependent on sampling rate

→ taking conversion into account during modeling

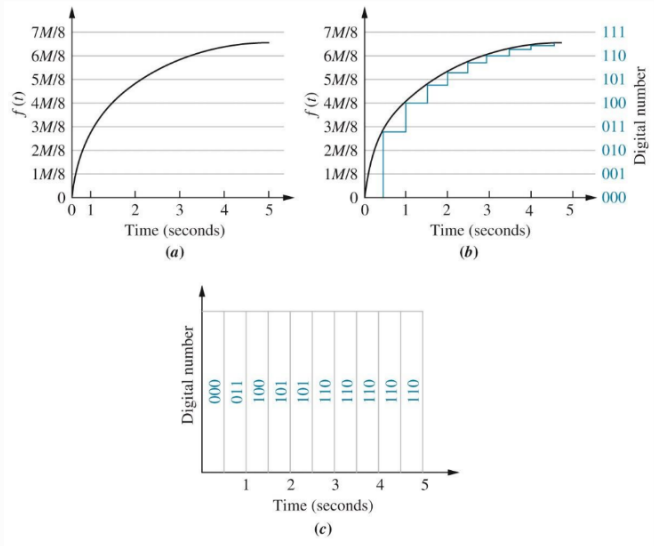
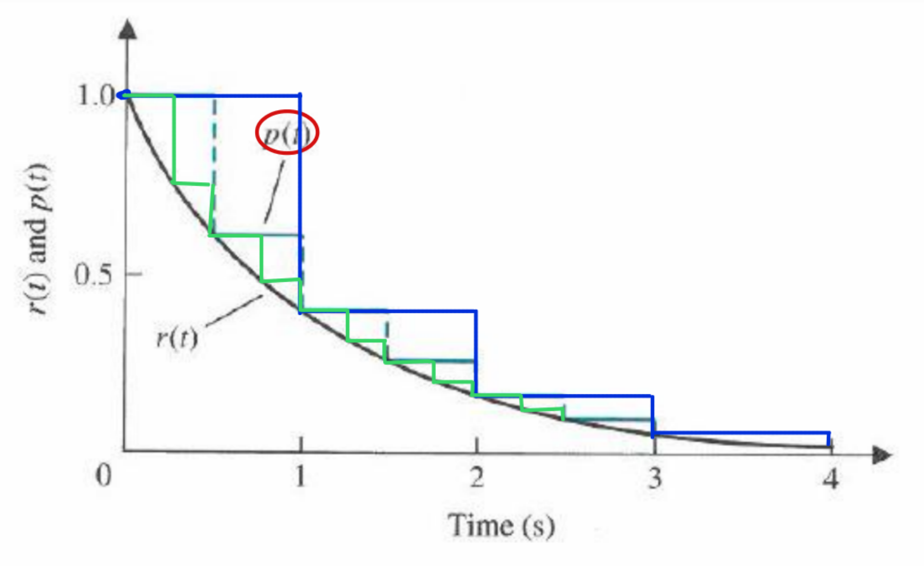


Figure 13.4



(a) $T = 0.5$ s

$T = 1s$

$T = 0.25$

Sampling time (T) is important for many factors, which stability is one of them.

$T \uparrow \Rightarrow$ stability $\downarrow$

$$Z.O.h(s) = \frac{1 - e^{-sT}}{s} \Rightarrow Z.O.h.(z) = (1 - z^{-1}) \cdot \mathcal{Z}\left\{\frac{1}{s}\right\}$$

$$\text{or} = \frac{z-1}{z}$$

➔ Difference equation

depends on $\alpha_n \cdots \alpha_0 + T$

$$y(k) + a_1 y(k-1) + a_2 y(k-2) + \cdots + a_m y(k-m) = b_0 u(k) + \cdots + b_m u(k-m)$$

$$G(z) = \frac{y(z)}{u(z)} = \frac{b_0 + b_1 z^{-1} + \cdots + b_m z^{-m}}{1 + a_1 z^{-1} + \cdots + a_m z^{-m}} = \frac{B(z)}{A(z)}$$

Matching coeffients with difference equation

TABLE 13.1 Partial table of z- and s-transforms

	$f(t)$	$F(s)$	$F(z)$	$f(kT)$
1.	$u(t)$	$\frac{1}{s}$	$\frac{z}{z-1}$	$u(kT)$
2.	t	$\frac{1}{s^2}$	$\frac{Tz}{(z-1)^2}$	kT
3.	t^n	$\frac{n!}{s^{n+1}}$	$\lim_{a\to 0}(-1)^n \frac{d^n}{da^n}\left[\frac{z}{z-e^{-aT}}\right]$	$(kT)^n$
4.	e^{-at}	$\frac{1}{s+a}$	$\frac{z}{z-e^{-aT}}$	e^{-akT}
5.	$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$	$(-1)^n \frac{d^n}{da^n}\left[\frac{z}{z-e^{-aT}}\right]$	$(kT)^n e^{-akT}$
6.	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$	$\frac{z \sin \omega T}{z^2 - 2z\cos \omega T + 1}$	$\sin \omega kT$
7.	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$	$\frac{z(z-\cos\omega T)}{z^2 - 2z\cos\omega T + 1}$	$\cos \omega kT$
8.	$e^{-at}\sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$	$\frac{ze^{-aT}\sin\omega T}{z^2 - 2ze^{-aT}\cos\omega T + e^{-2aT}}$	$e^{-akT}\sin \omega kT$
9.	$e^{-at}\cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$	$\frac{z^2 - ze^{-aT}\cos\omega T}{z^2 - 2ze^{-aT}\cos\omega T + e^{-2aT}}$	$e^{-akT}\cos \omega kT$

Table 13.1

TABLE 13.2 z-transform theorems

	Theorem	Name
1.	$z\{af(t)\} = aF(z)$	Linearity theorem
2.	$z\{f_1(t) + f_2(t)\} = F_1(z) + F_2(z)$	Linearity theorem
3.	$z\{e^{-aT}f(t)\} = F(e^{aT}z)$	Complex differentiation
4.	$z\{f(t-nT)\} = z^{-n}F(z)$	Real translation
5.	$z\{tf(t)\} = -Tz\frac{dF(z)}{dz}$	Complex differentiation
6.	$f(0) = \lim_{z\to\infty} F(z)$	Initial value theorem
7.	$f(\infty) = \lim_{z\to 1}(1-z^{-1})F(z)$	Final value theorem

Table 13.2

With $G_1(s) = \dfrac{1}{s+1}$ & $G_2(s) = \dfrac{1}{s+2}$

$$G_1(s)\cdot G_2(s) = \frac{1}{(s+1)(s+2)} = \frac{A}{(s+1)} + \frac{B}{s+2}$$

Partial fraction because we don't have a formula for this form

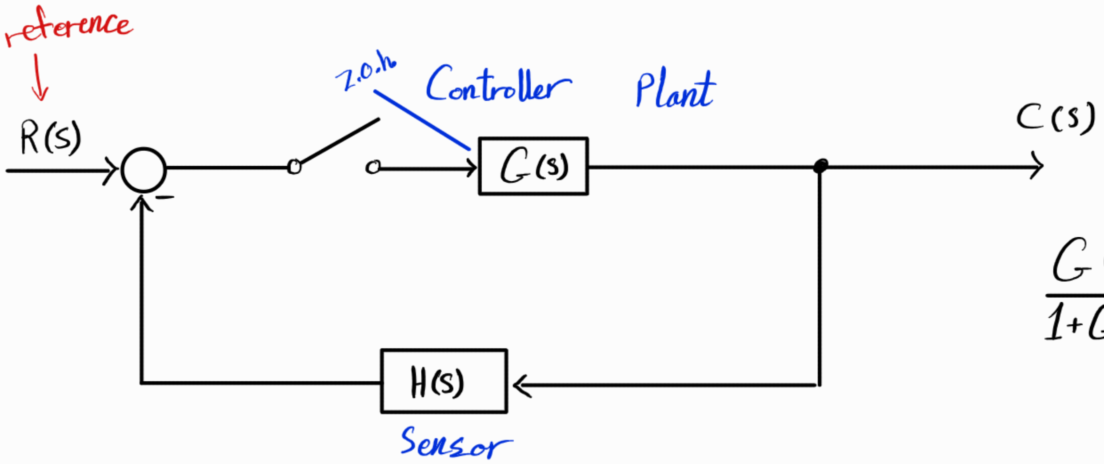
$$G_1(s)\cdot G_2(s) = \frac{1}{(s+1)(s+2)} = \frac{1}{s+1} - \frac{1}{s+2}$$

$$\mathcal{Z}\left\{G_1(s)\cdot G_2(s)\right\} = \mathcal{Z}\left\{\frac{1}{s+1} - \frac{1}{s+2}\right\} = \frac{z}{z-e^{-T}} - \frac{z}{z-e^{-2T}}$$

$$= \frac{z(z-e^{-2T}) - z(z-e^{-T})}{(z-e^{-T})(z-e^{-2T})} = \frac{z((\cancel{z}-e^{-2T}) - (\cancel{z}-e^{-T}))}{(z-e^{-T})(z-e^{-2T})} = \overset{G_1G_2(z)}{\boxed{\frac{z(e^{-T}-e^{-2T})}{(z-e^{-T})(z-e^{-2T})}}}$$

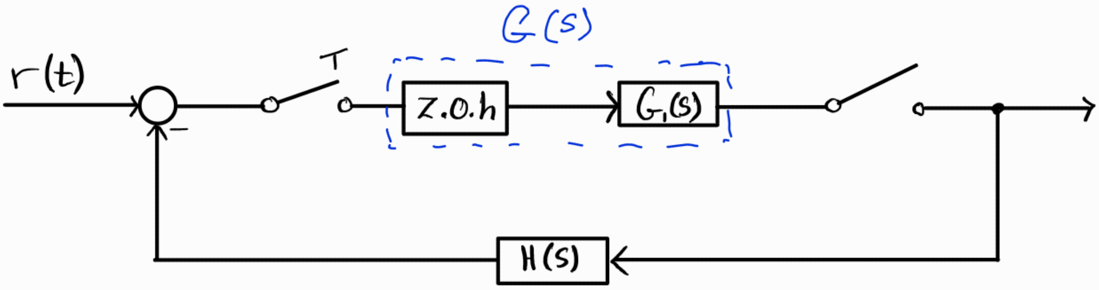
$$\mathcal{Z}\left\{G_1(s)\right\}\cdot\mathcal{Z}\left\{G_2(s)\right\} = \mathcal{Z}\left\{\frac{1}{s+1}\right\}\cdot\mathcal{Z}\left\{\frac{1}{s+2}\right\} = \frac{z}{z-e^{-T}}\cdot\frac{z}{z-e^{-2T}} = \overset{G_1(z)\cdot G_2(z)}{\frac{z^2}{(z-e^{-T})(z-e^{-2T})}}$$

$$\therefore \boxed{G_1G_2(z) \neq G_1(z)\cdot G_2(z)}$$



$$\frac{G(z)}{1+GH(z)}$$

Exam question:



with $G_1(s) = \frac{5}{s}$

$H(s) = \frac{10}{s+10}$

$G_{cL}(z) = \frac{C(z)}{R(z)}$

$$Z.O.h = \frac{1 - e^{-sT}}{s} \Rightarrow Z.O.h.(z) = (1 - z^{-1})$$

$$G_{cL}(z) = \frac{G(z)}{\underbrace{1 + G(z) \cdot H(z)}_{\text{because of the sampler in between}}}$$

$$G(z) = \mathcal{Z}\left\{\frac{1-e^{-sT}}{s} \cdot \frac{5}{s}\right\} = (1 - z^{-1})\,\mathcal{Z}\left\{\frac{5}{s^2}\right\} = \frac{\cancel{z-1}}{\cancel{z}} \cdot \frac{5T\cancel{z}}{(z-1)^{\cancel{2}}}$$

$$\boxed{G(z) = \frac{5T}{(z-1)}}$$

$$H(s) = \frac{10}{s+10} \Longrightarrow \boxed{H(z) = \frac{10z}{z - e^{-10T}}}$$

$$G_{cl}(z) = \frac{G(z)}{1 + G(z) \cdot H(z)} = \frac{5T/(z-1)}{1 + \frac{50Tz}{(z-1)(z-e^{-10T})}} = \frac{5T\,(\cancel{z-1})(z-e^{-10T})}{(50Tz + (z-1)(z-e^{-10T}))(\cancel{z-1})}$$

$$= \boxed{\frac{5T(z - e^{-10T})}{z^2 + z(50T - 1 - e^{-10T}) + e^{-10T}}}$$

Student No.: ____________________

Points:

Question 8: 17

A closed-loop sampled system with the reference input $r(t)$ and the actual output $c(t)$, as shown in block diagram fig. 8.1, consists of a system with the transfer function $G_1(s)$ in the feedforward path, a zero-order-hold in the feedforward path, two samplers which are synchronized with the sampling time T, and a system with the transfer function $H(s)$ in the feedback path.

The following transfer functions are given:

$G_1(s) = \frac{5}{s}$ and $H(s) = \frac{10}{s+10}$

$\tilde{G}(z)\cdot H(z) \neq \tilde{G}H(z)$

$\tilde{G}(s) = \frac{1-e^{-sT}}{s}\cdot\frac{5}{s}$

If this is not there

$G_{CL}(z) = \frac{\tilde{G}(z)}{1+\tilde{G}H(z)}$

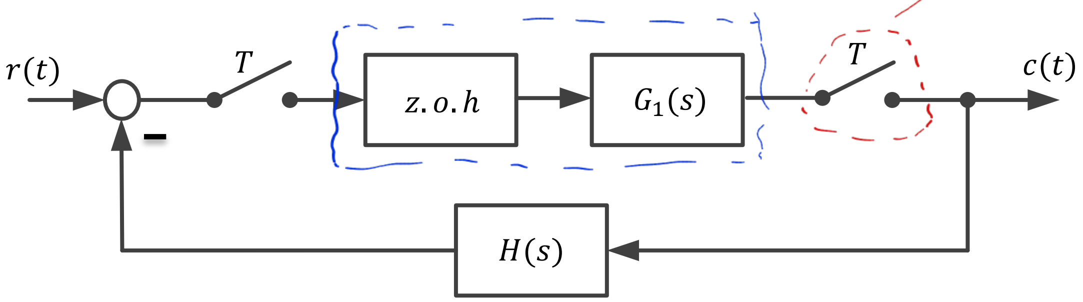


Figure 8.1: Block diagram of closed-loop system

a) Find the sampled-data transfer function $G(z) = \frac{C(z)}{R(z)}$ of the closed-loop system as a function of the sampling time T. *Hint: Check the sampler arrangement!*

b) Is the sampled closed-loop system stable for $T = 10\ [ms]$? Please explain.

$$\tilde{G}(z) = \mathcal{Z}\{1-e^{-sT}\}\cdot\mathcal{Z}\left\{\frac{5}{s^2}\right\} = \frac{\cancel{z-1}}{\cancel{z}}\cdot\frac{5T\cancel{z}}{(z-1)^{\cancel{2}}}$$

$$\boxed{\tilde{G}(z) = \frac{5T}{z-1}}$$

$$H(z) = \mathcal{Z}\left\{\frac{10}{s+10}\right\} = \frac{10z}{z-e^{-10T}}$$

$$G_{CL}(z) = \frac{C(z)}{R(z)} = \frac{\tilde{G}(z)}{1+H(z)\cdot\tilde{G}(z)} = \frac{\frac{5T}{z-1}}{1+\frac{50Tz}{(z-1)(z-e^{-10T})}} = \frac{\frac{5T}{\cancel{z-1}}}{\frac{z^2-z-e^{-10T}z+50Tz+e^{-10T}}{\cancel{(z-1)}(z-e^{-10T})}}$$

$$\boxed{G_{CL}(z) = \frac{5T(z-e^{-10T})}{z^2+z(50T-e^{-10T}-1)+e^{-10T}}}$$

3.2 Exercise

$\tilde{G}(s) \rightarrow \tilde{G}(z$

A sampled-data system is given in Figure 3.1.

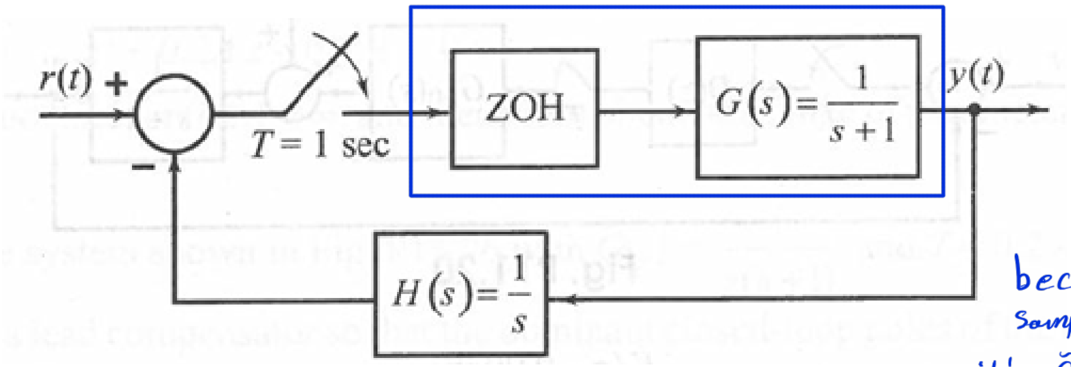


Figure 3.1. sampled-data system

because there is no sampler in between it's $\tilde{G}H(z)$

if there is a sampler $G_{cl}(z) = \frac{\tilde{G}(z)}{1+\tilde{G}(z)\cdot H(z}$

3.2.1 Task

Find out if the closed-loop system $G_{Cl}(z) = \frac{Y(z)}{R(z)}$ is stable or unstable and explain your result.

3.2.2 Solution

Please refer to ??.

$$G_{Cl}(z) = \frac{\tilde{G}(z)}{1+\tilde{G}H(z)}$$

$$\tilde{G}(s) = \frac{1-e^{-sT}}{s}\cdot\frac{1}{s+1}$$

$$\tilde{G}(z) = \mathcal{Z}\left\{\frac{1-e^{-sT}}{s}\cdot\frac{1}{s+1}\right\} = (1-z^{-1})\,\mathcal{Z}\left\{\frac{1}{s}-\frac{1}{s+1}\right\} = \frac{1-e^{-T}}{z-e^{-T}}$$

$$\tilde{G}(z) = \frac{0.63}{z-0.37}$$

$$\tilde{G}H(z) = \mathcal{Z}\left\{\frac{1-e^{-sT}}{s}\cdot\frac{1}{s+1}\cdot\frac{1}{s}\right\} = (1-z^{-1})\,\mathcal{Z}\left\{\frac{1}{s^2}-\frac{1}{s}+\frac{1}{s+1}\right\}$$

$$= \frac{z-1}{z}\left(\frac{Tz}{(z-1)^2}-\frac{z}{z-1}+\frac{z}{z-e^{-T}}\right) = \left(\frac{T}{z-1}-1+\frac{z-1}{z-e^{-T}}\right)$$

$$\tilde{G}H(z) = \frac{0.37z+0.26}{z^2-1.37z+0.37}$$

$$G_{Cl}(z) = \frac{0.63(z-1)}{z^2-z+0.63}$$

stable

3.3 Exercise

A sampled-data system is given in Figure 3.2. $G(s)$

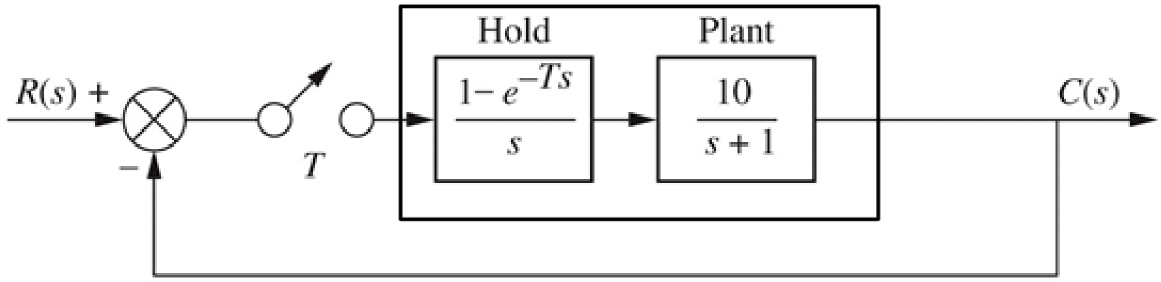


Figure 3.2. Diagram system

3.3.1 Task

Determine the range of sampling interval, T , which makes the system shown in Figure 3.2 stabel, and furthermore give the range that will lead to an unstable system.

3.3.2 Solution

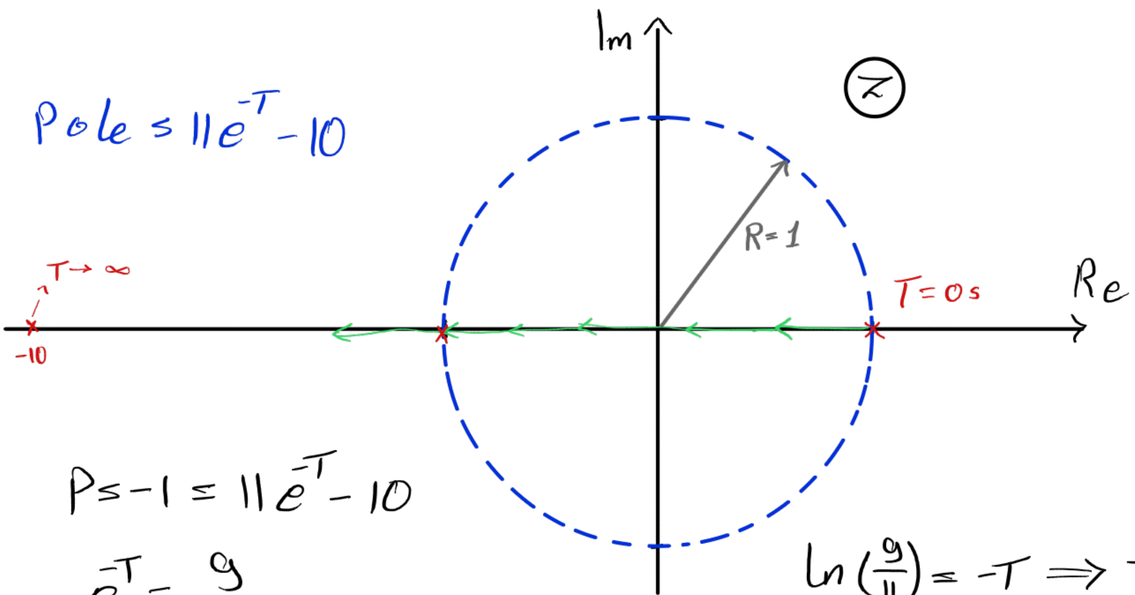
Please refer to ??.

$$G(z) = \mathcal{Z}\left\{\frac{1-e^{-Ts}}{s}\cdot\frac{10}{s+1}\right\} = \left(\frac{z-1}{z}\right)\mathcal{Z}\left\{\frac{1}{10s}-\frac{1}{10(s+1)}\right\}$$

$$G(z) = 10\cdot\frac{1-e^{-T}}{z-e^{-T}}$$

$$G_{cl}(z) = \frac{10(1-e^{-T})}{z-e^{-T}+10-10e^{-T}} = \frac{num}{z-11e^{-T}+10}$$

$$\text{Pole} = 11e^{-T}-10$$



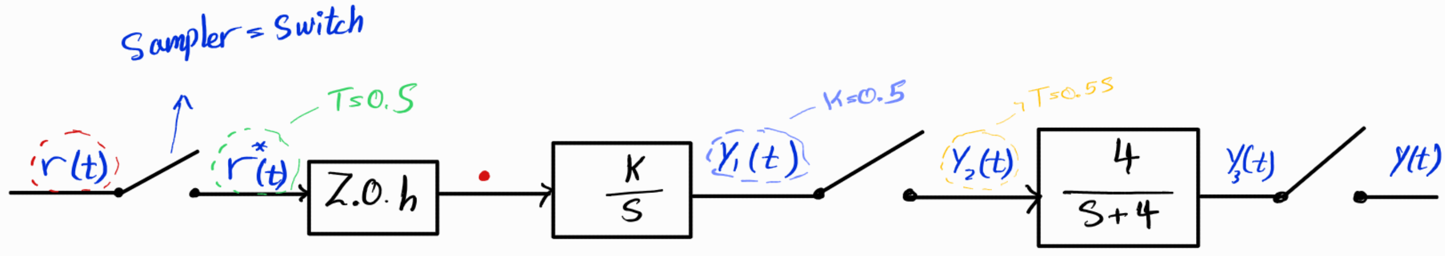
$$P = -1 = 11e^{-T}-10$$

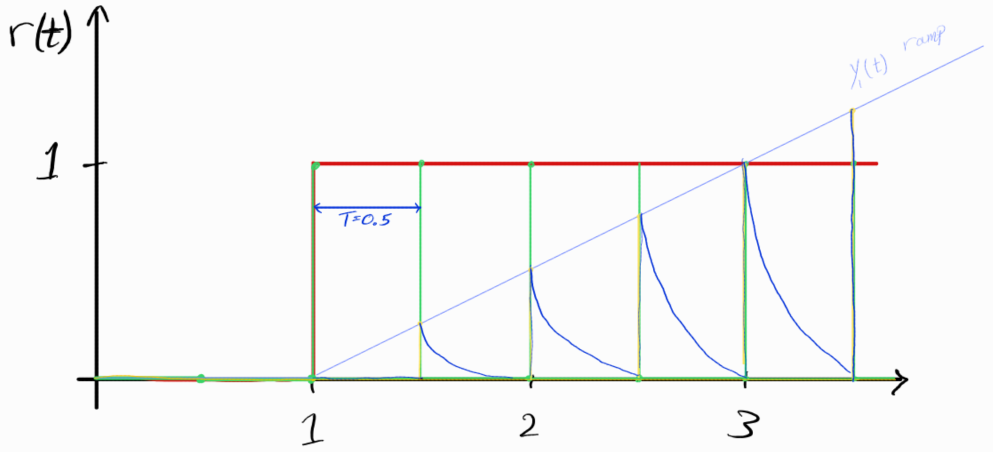
$$e^{-T} = \frac{9}{11}$$

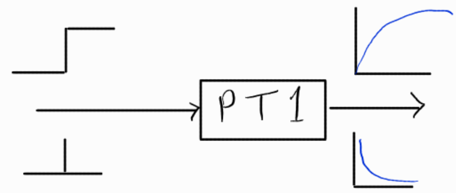
$$\ln\left(\frac{9}{11}\right) = -T \Rightarrow T = -\ln\left(\frac{9}{11}\right) = 0.2\,s$$

$$\therefore \boxed{0 < T < 0.2}$$

EXAM QUESTION:







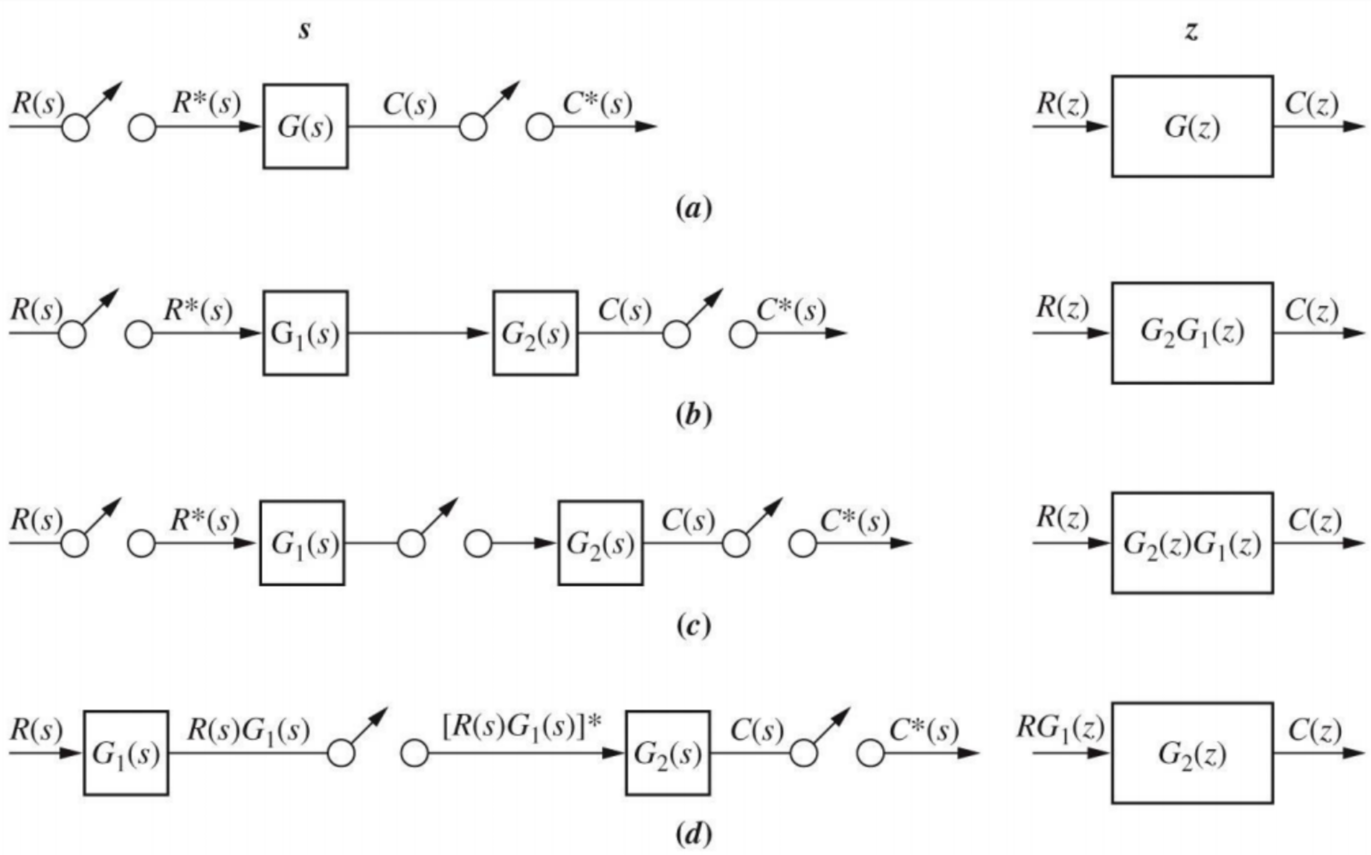


Figure 13.9

طالما في sampler مابينهم يبقى هيضربوا في z-domain

لو مافيش يبقى يضربوا في s-domin و بعدين يتحولوا الـ z