Integration:

Newton-Coates: $I = \int_a^b f(x)\,dx \approx \tilde{I} = \sum_{i=0}^{n} f(x_i)\cdot W_i$

$$W_i = \int_a^b L_i(x)\,dx = \int_a^b \prod_{\substack{j=0\\ j\neq i}}^{n} \frac{x-x_j}{x_i-x_j}\,dx$$

$$\tilde{I} = \sum_{i=0}^{n} f(x_i)\cdot \int_a^b \prod_{\substack{j=0\\ j\neq i}}^{n} \frac{x-x_j}{x_i-x_j}\,dx$$

$$x = hs + a$$

$$\boxed{h = \frac{\overset{x_n}{b} - \overset{x_0}{a}}{n}} \qquad = \sum_{i=0}^{n} f(x_i)\cdot h\cdot \int_0^n \prod_{\substack{j=0\\ j\neq i}}^{n} \frac{s-j}{i-j}\,ds$$

n	Closed Newton-Cotes Formula	Simplified Newton-Cotes Formula
1	$h\cdot\frac{f(x_0)+f(x_1)}{2}$	$(b-a)\cdot\frac{f(x_0)+f(x_1)}{2}$
2	$\frac{1}{3}h\left[f(x_0)+4f(x_1)+f(x_2)\right]$	$(b-a)\cdot\frac{f(x_0)+4f(x_1)+f(x_2)}{6}$
3	$\frac{3}{8}h\left[f(x_0)+3f(x_1)+3f(x_2)+f(x_3)\right]$	$(b-a)\cdot\frac{f(x_0)+3f(x_1)+3f(x_2)+f(x_3)}{8}$
4	$\frac{2}{45}h\left[7f(x_0)+32f(x_1)+12f(x_2)+32f(x_3)+7f(x_4)\right]$	$(b-a)\cdot\frac{7f(x_0)+32f(x_1)+12f(x_2)+32f(x_3)+7f(x_4)}{90}$

at $n=1 \longrightarrow$ Trapezoidal formula

at $n=2 \longrightarrow$ Simpson's $\frac{1}{3}$ formula

at $n=3 \longrightarrow$ Simpson's $\frac{3}{8}$ formula

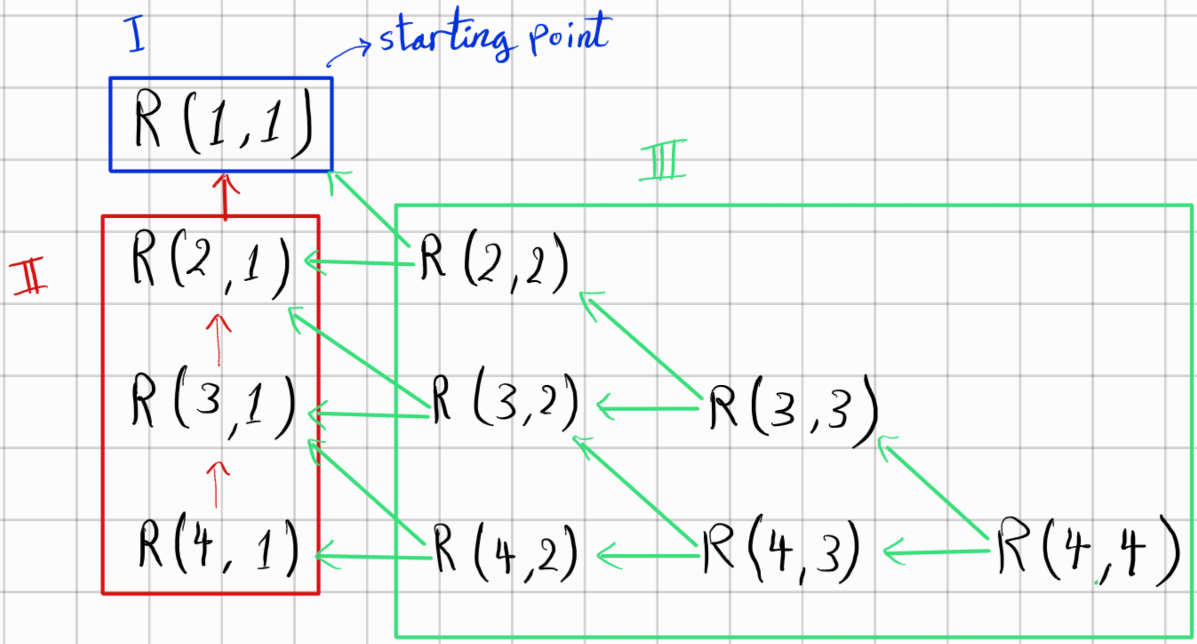
Universal Trapezoidal:

$$\boxed{I = \frac{h}{2}\left[f(a) + f(b) + 2\sum_{i=1}^{n-1} f(a+ih)\right]}$$

Universal simpson:

$$\boxed{\hat{I}(x) = \frac{h}{3}\left[f(x_0) + 4f(x_{n=odd}) + 2f(x_{n=even}) + f(x_n)\right]}$$

Romberg:



$$\boxed{R(1,1) = \frac{h}{2}\left[f(a)+f(b)\right]} \qquad h = b-a$$

$$\boxed{R(i,1) = \frac{h}{2}\left[f(a) + f(b) + 2\sum_{i=1}^{n-1} f(a+ih)\right]} \qquad n = 2^{i-1} \rightarrow h = \frac{b-a}{n}$$

$$\boxed{R(N,m) = \frac{4^{m-1} \cdot R(N,m-1) - R(N-1,m-1)}{4^{m-1} - 1}} \Leftrightarrow R(n,m) = R(n,m-1) + \frac{R(n,m-1) - R(n-1,m-1)}{4^{m-1} - 1}$$

$$R(2,2) = \frac{4\ R(2,1) - R(1,1)}{4-1} = \frac{4R(2,1) - R(1,1)}{3}$$