Period of Examinations
Summer Semester 2021

HOCHSCHULE RHEIN-WAAL
Rhine-Waal University of Applied Sciences

Study Course: ________________________________

Module Title: **System Theory and Controls**

Points: 100

Duration: 90 Minutes + 15 Minutes for scanning and upload = 105 Minutes

Please write legibly!

Date: ____________________

Family Name: ____________________

First Name: ____________________

Student No.: ____________________

I, ________________________ [full name, matriculation number], hereby confirm in lieu of an oath that I am the person who was admitted to this examination. Further, I confirm that the submitted work is my own and was prepared without the use of any unauthorised aid or materials.

With uploading the examination results, I confirm and agree the terms and conditions and that the examination is assessed and rated.

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Signature (Student)

If the exam / first page isn't signed, the examination is graded as failed.

Student No.: ____________________ / Student Name: ______________________

Points:

Question 1: 15

Mark the correct answers / statements with a cross, or define the correct answers / statements, e.g. mentioning a.1). For each correct cross / definition you will receive 1.5 points, each cross which is not correct will subtract 1.5 points from the total score. The total score for the entire question cannot be negative.

a) A system with PT2-characteristic has a damping ratio $D = 0.3$.
- ○ a.1) The system is critically damped.
- ○ a.2) The system is always stable.
- ○ a.3) The system has two zeros.
- ○ a.4) The imaginary part of the poles are nonzero.

b) A system is characterised through the differential equation
$y(t)\,\ddot{y}(t) + 2t^2 y(t) + y^3(t) = \ddot{u}(t)$.
- ○ b.1) The system is linear.
- ○ b.2) The system is causal.
- ○ b.3) The system is time-variant.
- ○ b.4) The system is of IT3 characteristic.

c) A system with DT2-characteristic
- ○ c.1) is characterised through two parameters.
- ○ c.2) has one zero at $z = 0$.
- ○ c.3) has a constant magnitude for the entire frequency range.
- ○ c.4) has a phase within the range of +90° - -90°.

d) A PID-controller
- ○ d.1) is characterized through three parameters.
- ○ d.2) is described by a P-, I-, and D-system in parallel.
- ○ d.3) is useless in practice.
- ○ d.4) guarantees always a stable closed-loop system.

e) A reference transfer function
- ○ e.1) is the inverse of a disturbance transfer function.
- ○ e.2) for a closed-loop control system should be $G_R(s) = 1$.
- ○ e.3) is always stable.
- ○ e.4) is calculated in that the disturbance is set to zero.

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Question 2: 20

Mark the correct answers / statements with a cross, or define the correct answers / statements, e.g. mentioning a.1). For each correct cross / definition you will receive 2.5 points, each cross which is not correct will subtract 2.5 points from the total score. The total score for the entire question cannot be negative.

a) A system is characterized through the differential equation
$2\,\ddot{y}(t) + 12\,\dot{y}(t) + 200\,y(t) = 400\,u(t)$.

- O a.1) The eigenfrequency of the system is $10\,\frac{rad}{s}$.
- O a.2) The damping ratio of the system is 0.3.
- O a.3) For a step input the steady state output is 0.5.
- O a.4) The system has a conjugated complex pole pair.

b) A system is characterized through the transfer function $G(s) = \frac{s+2}{s^2+s-20}$.

- O b.1) The system is of PDT2 characteristic.
- O b.2) The system is unstable.
- O b.3) The phase angle goes to -180 deg when the input frequency goes to infinity.
- O b.4) The slope of the magnitude is -20 dB/dec when the input frequency goes to infinity.

c) A plant with $G_P(s) = \frac{4}{s}$ is controlled by a D-controller in a standard control loop.

- O c.1) The closed-loop characteristic is according to a PT1-system.
- O c.2) Without any disturbance, the controller isn't capable to control the reference exactly.
- O c.3) With the controller gain $K_D = 1$, the time constant of the closed-loop system is 0.25 sec.
- O c.4) The plant is unstable.

d) A system is characterised through the transfer function $G(s) = \frac{5}{s^4+3s^3+2s^2+8s+4}$.

- O d.1) The system is unstable.
- O d.2) The system can be described through the differential equation
$2\,y^{(IV)}(t) + 6\,\dddot{y}(t) + 4\,\ddot{y}(t) + 16\,\dot{y}(t) + 8 = 5\,u(t)$.
- O d.3) The step response of the system converges to 0 when $t \to \infty$.
- O d.4) The system is not causal.

Student No.: ____________________ / Student Name: ______________________

Question 3: 14

A system is characterized through the differential equation

$$\dot{y}(t) + 2\, y(t) = \dot{u}(t)$$

where $u(t)$ is the input variable and $y(t)$ the output variable, respectively. Calculate the response $y(t)$, by use of the method of the Laplace transformation, when the system is excited by a ramp (thus $u(t) = t^2$) and the initial condition is $y(0) = 3$.

Student No.: ____________________ / Student Name: ______________________

Question 4: 13

A system is characterized through the poles (denoted X) and zeros (denoted O), according to the pole-zero map in figure 4.1.

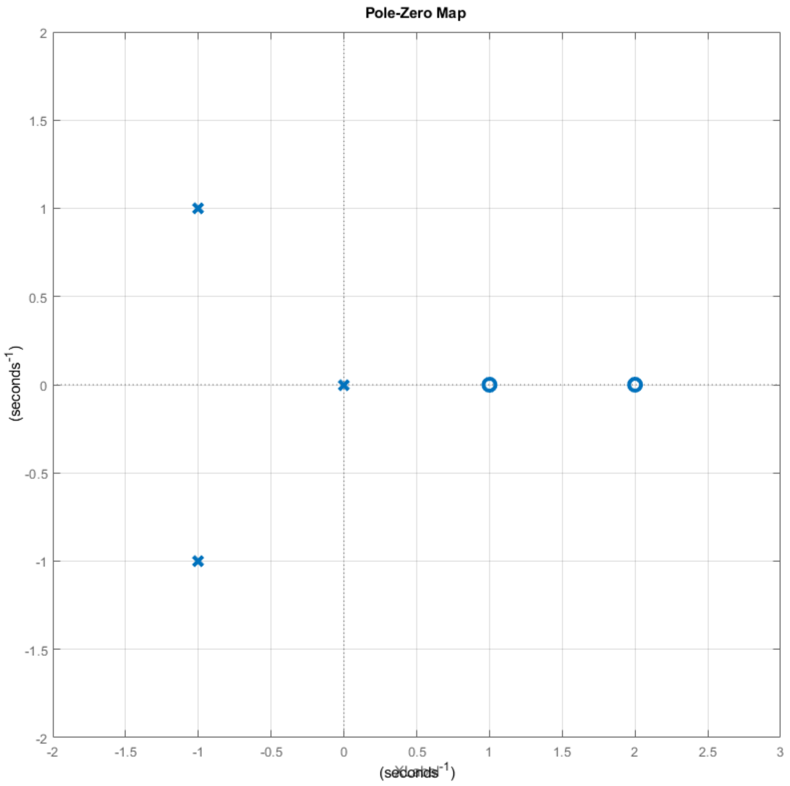


Figure 4.1: Pole-Zero-Map of a function

a) Formulate the transfer function of the system. (6 points)

$s(as^2+bs+c)=0$

$P_3=0$, $P_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = -1 \pm i$ $\Rightarrow$ $P_{1,2} = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$

let $a=1$, $-\frac{b}{2} = -1 \Rightarrow \boxed{b=2}$ $\frac{\sqrt{b^2-4c}}{2} = \sqrt{-1} \Rightarrow (2)^2-4c=-4$

$\Rightarrow c=2$

$$G(s) = \frac{(s-2)(s-1)}{s(s^2+2s+2)}$$

b) Is the system stable? Please explain. (2 points)

No, Boundary stable because we have one of the poles at 0

c) Sketch the step response of the system. (3 points)

d) The system is excited by a harmonic signal. Determine the phase of the response, when the input frequency is very high ($\omega \rightarrow \infty$). (2 points)

$$G(s) = \frac{(s-2)(s-1)}{s(s^2+2s+2)} = \frac{\cancel{2}\left(\frac{s}{2}-1\right)(s-1)}{s\cancel{2}\left(\frac{1}{2}s^2+s+1\right)}$$

$$\phi_{w \rightarrow \infty} = -180^\circ$$

Question 5: 28

A cruise (velocity) controller of an automobile has to be designed and analysed.

Therefore the longitudinal dynamics of the car according to figure 5.1 can be described through the simplified differential equation

$$\ddot{y}(t) + \dot{y}(t) = f(t)$$

where $y(t)$ is the position of the car, $\dot{y}(t) = v(t)$ the velocity of the car respectively, and $f(t)$ the propulsion force.

Figure 5.1: Longitudinal dynamics

a) Formulate the transfer function $G_2(s) = \frac{V(s)}{F(s)}$ of the car, where the input is the propulsion force and the output is the velocity of the vehicle.

b) Is the vehicle behaviour stable? Please explain.

In the following the vehicle (plant) is controlled by a PI-controller, according to the block diagram in figure 5.2, where $C(s)$ is the controlled variable, $R(s)$ the reference/target speed, and $D(s)$ the disturbance, e.g. road grade.

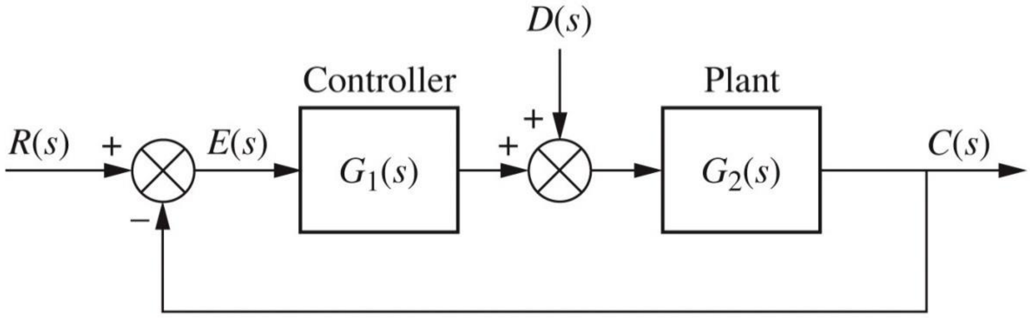


Figure 5.2: Block diagram of controlled system

c) Formulate the reference transfer function $G_R(s) = \frac{C(s)}{R(s)}$.

d) Is the controller able to control the speed exactly, when $D(s) = 0$? Please explain.

e) Formulate the disturbance transfer function $G_D(s) = \frac{C(s)}{D(s)}$.

f) Is the controller able to compensate disturbances? Please explain.

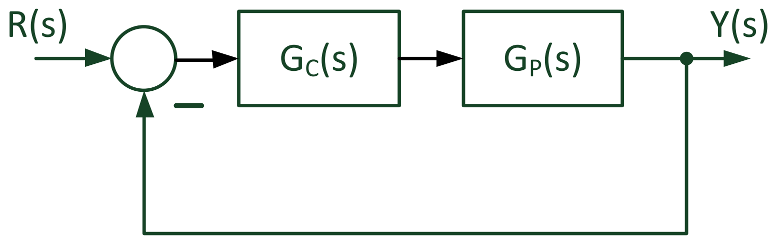
g) Calculate the controller gains K_P and K_I, so that the closed-loop control system has an eigenfrequency of $\omega_0 = 2\,\frac{rad}{s}$ and a damping ratio $D = 1$.

h) Sketch the response of the car, when the reference speed is constant and a disturbance is applied according to a step input.

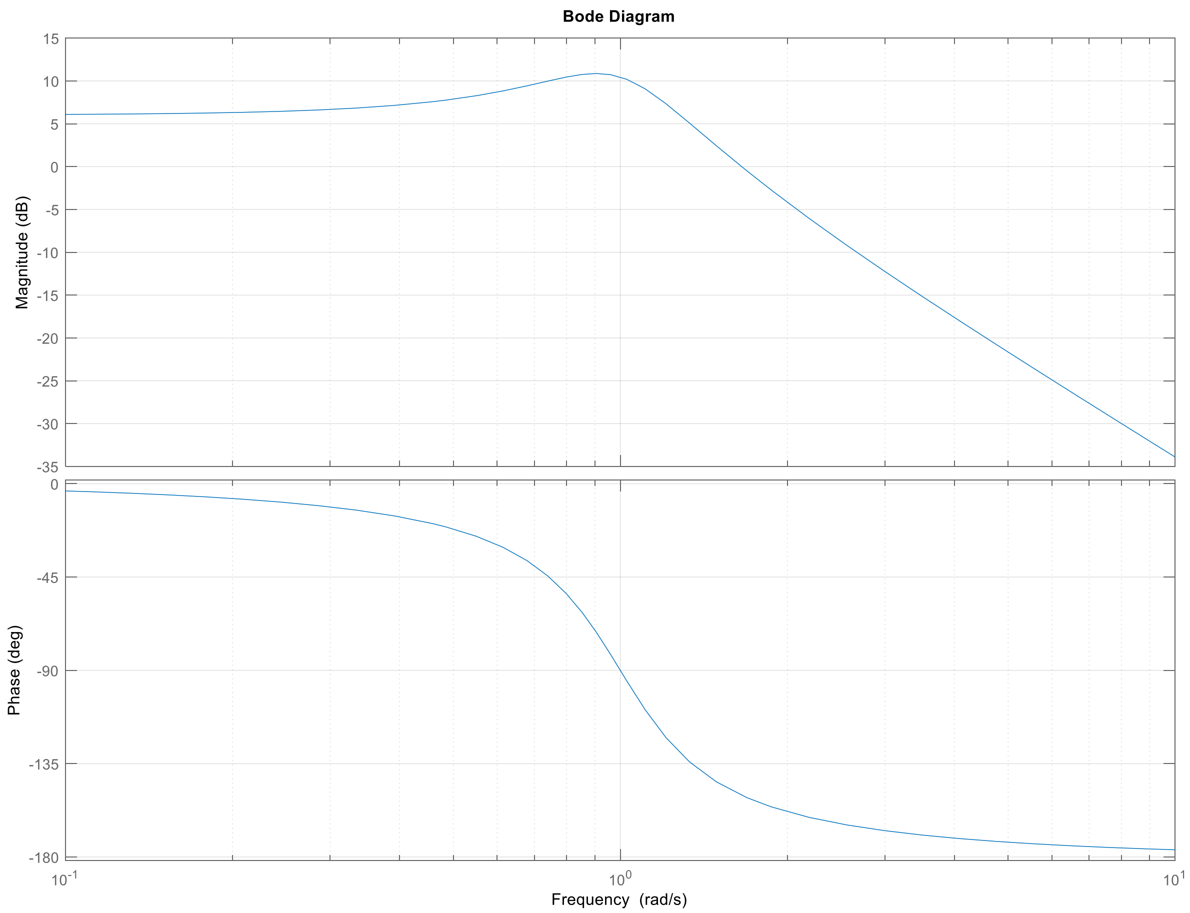
Student No.: ____________________ / Student Name: ______________________

Question 6: 10

A controlled system is arranged according to the block diagram below, where $G_C(s)$ is the controller, $G_P(s)$ the plant with a PT2-characteristic, $R(s)$ the reference, and $Y(s)$ the controlled variable.



The Bode-diagram of the plant has been determined according to the graph below.



a) Determine the output amplitude of the plant $\hat{y}$, when the plant is excited with a harmonic signal with the input amplitude $\hat{u} = 5$ and an excitation frequency of $\omega = 1\,\frac{rad}{s}$.

In the following the plant is controlled by a P-controller with $G_C(s) = K_P$.

b) Is the closed-loop system stable for any K_P? Please explain.

c) Determine the phase margin Φ_M for $K_P = 1$.

Good luck!