Period of Examinations
Summer Semester 2020 - July
Test-Exam

Multibody Dynamics

Examination

Estimated duration of examination: 120 Minutes

Time to load exam up answers: 240 Minutes

Please write legibly!

Date: ______________________

Name: ______________________

Register No.: ______________________

Course of Study: ______________________

Please put your name and your matriculation number in the following declaration and sign it:

I, ____ ,
full name, matriculation number

hereby confirm in lieu of an oath that I am the person who was admitted to this examination. Further, I confirm that the submitted work is my own and was prepared without the use of any unauthorised aid or materials.

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Signature

1. **Preparing a Multibody System for Computer Simulation**

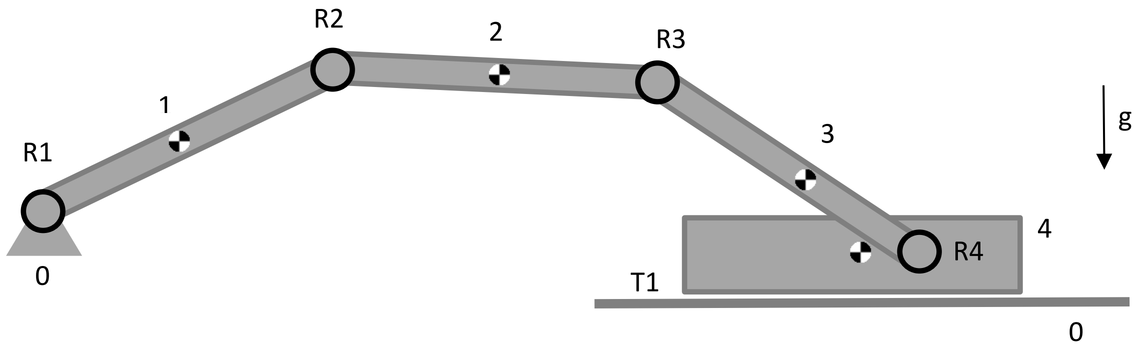

Figure 1: Sample Mechanism

In Figure 1 you find a sample mechanism that consists of four bodies (1-4), four revolute joints (R1-R4) and one translational joint (T1). Revolute joint R1 and the translational joint T1 connect the mechanism to the ground (0).

The following parameter are known:

Masses	m_1,m_2,m_3, and m_4
Moments of Inertia	J_1, J_2, J_3, J_4
gravitational acceleration	g

To prepare the mechanism for computer simulation based on a body-coordinate formulation

a) Define reference frames in terms of body-coordinates (use the enlarged figure of the mechanism on the extra sheet),
b) Define the necessary points and vectors to formulate the constraints of the system in body-coordinate formulation (use the enlarged figure of the mechanism on the extra sheet).

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2. Describing technical joints (5 points)

Formulate the constraints of joint R3 in Figure 1 on position level.

R_3: Geometrically:

$$\underline{r}_2 + \underline{s}_2^{R_3} - \underline{r}_3 - \underline{s}_3^{R_3} = \underline{0}$$

In global coordination:

$$\underline{r}_2 + \underline{A}_2 \cdot \underline{\$}_2^{R_3} - \underline{r}_3 - \underline{A}_3 \cdot \underline{\$}_3^{R_3} = \underline{0}$$

T_1: $\quad u_0^{\vee T} \cdot \underline{d} = 0$

$$\phi_4 - \phi_0 - \phi_c = 0$$

3. Degrees of freedom (10 points)

a) How many degrees of freedom (d.o.f.) has the mechanism in Figure 1? **(3 points)**

Dof $= 3*4 - 2*5 = 12 - 10 = 2$ degrees of freedom

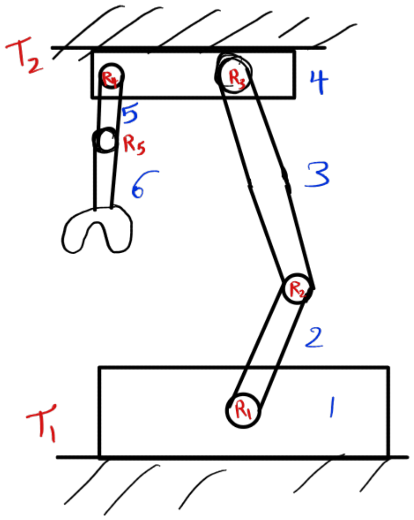
b) How many constraint equations have to be formulated for the mechanism in Figure 1 using the body-coordinate formulation? **(3 points)**

$2*(4+1) = 10$ Constraints

c) Draw a mechanism with four degrees of freedom including at least one revolute and one translational joint. **(4 points)**

$n_b = 6$

$n_j = 7$



num. of bodies: 6

num of Coordination $= 3*6 = 18$

num. of constraints $= \underbrace{2*5}_{\text{rev. joint}} + \underbrace{2*2}_{\text{Trans. joints}}$

4. Dynamics (16 points)

The equations of motion containing the constraint forces have the form

$$\mathbf{M}\ddot{\boldsymbol{c}} = \boldsymbol{h} + \boldsymbol{D}'\boldsymbol{\lambda}$$

For the mechanism shown in Figure 1,

a.) Define the mass matrices $\mathbf{M}_1$, $\mathbf{M}_2$, $\mathbf{M}_3$ and $\mathbf{M}_4$ **(2 points)**

$$M_1 = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_1 & 0 \\ 0 & 0 & J_1 \end{bmatrix}, \quad M_2 = \begin{bmatrix} m_2 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & J_2 \end{bmatrix}$$

b.) Define the mass matrix **M** for the complete mechanism. **(1 points)**

$$\underline{\underline{M}} = \begin{bmatrix} \underline{\underline{M}}_1 & 0 & 0 & 0 \\ 0 & \underline{\underline{M}}_2 & 0 & 0 \\ 0 & 0 & \underline{\underline{M}}_3 & 0 \\ 0 & 0 & 0 & \underline{\underline{M}}_4 \end{bmatrix}$$

c.) Define the array of forces $\mathbf{h}_1$, $\mathbf{h}_2$, $\mathbf{h}_3$ and $\mathbf{h}_3$. **(2 points)**

$$\underline{h}_1 = \begin{bmatrix} 0 \\ -m_1 g \\ 0 \end{bmatrix}, \quad \underline{h}_2 = \begin{bmatrix} 0 \\ -m_2 g \\ 0 \end{bmatrix}, \quad \underline{h}_3 = \begin{bmatrix} 0 \\ -m_3 g \\ 0 \end{bmatrix}, \quad \underline{h}_4 = \begin{bmatrix} 0 \\ -m_4 g \\ 0 \end{bmatrix}$$

d.) Define the force array **h** for the complete mechanism. **(1 points)**

$$\underline{h} = \begin{bmatrix} \underline{h}_1 \\ \underline{h}_2 \\ \underline{h}_3 \\ \underline{h}_4 \end{bmatrix}$$

e.) Define the vector of accelerations $\ddot{\boldsymbol{c}}$. **(2 points)**

$$\ddot{\underline{C}}_1 = \begin{bmatrix} \ddot{x}_1 \\ \ddot{y}_1 \\ \ddot{\theta}_1 \end{bmatrix}$$, same for all $\ddot{\underline{C}}_{2,3,4}$. $$\ddot{\underline{C}} = \begin{bmatrix} \ddot{\underline{C}}_1 \\ \ddot{\underline{C}}_2 \\ \ddot{\underline{C}}_3 \\ \ddot{\underline{C}}_4 \end{bmatrix}$$

a.) Mark the non-zero entries in each column of the the systems's Jacobian matrix for the mechanism shown in figure 1 by an "x".

	(1)	(2)	(3)	(4)
R1	X X			
R2	X X	X X		
R3		X X	X X	
R4			X X	X X
T1				X X

5.) Four-bar linkage

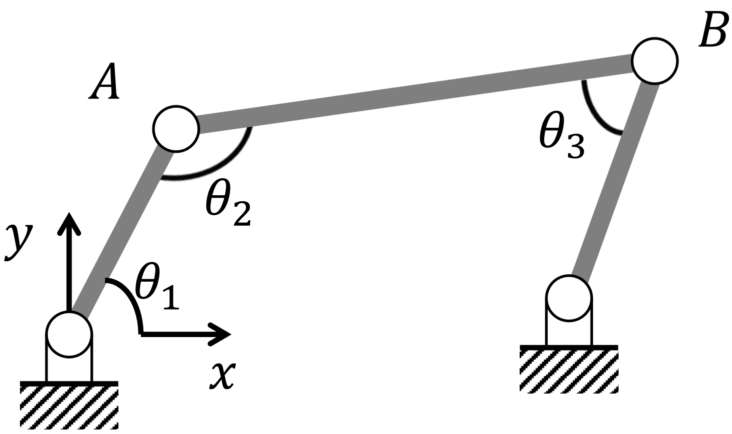


Figure 2: Four-bar mechanism

f.) How many degrees of freedom has the four-bar mechanism shown in Figure 2?

$Dof = 3 * n_b - 2 * n_j = 3(3) - 2(4) = 9 - 8 = 1$

g.) How many constraint equations are necessary in terms of body-coordinates?

4 jevolute joints $\Rightarrow 2 * 4 = 8$ Constraints

h.) How many constraints are necessary using the angles Θ_1, Θ_2 and Θ_3 shown in Figure 2 as coordinates for the four-bar mechanism?

2 constraints

i.) How many driver constraints are necessary for kinematic analysis of the four-bar mechanism in Figure 2?

Driver Constraints = Dof = 1

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6.) Rotation matrix (4 points)

Are matrices **A** and **B** both rotation matrices?

$\mathbf{A} = \begin{bmatrix} 0.6216 & -0.7833 \\ 0.7833 & 0.6216 \end{bmatrix}$ **B**= $\begin{bmatrix} 0.8253 & 0.5646 \\ 0.5646 & 0.8253 \end{bmatrix}$

Explain your answer!

A is rotation matrix since rotation matrix = $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$
(we can see that A has the same formate)

B is not a rotation matrix for the same reason, $\sin\theta \neq -\sin\theta$
where $\sin\theta \neq 0$

7.) Kinematic Analysis (12 points)

The following postion contraints are given:

$$1.0\cos\theta_1 + 3.0\cos\theta_2 - 2.2\cos\theta_3 - 2.0 = 0$$

$$1.0\sin\theta_1 + 3.0\sin\theta_2 - 2.2\sin\theta_3 - 0.5 = 0$$

a.) Calculate the velocity constraints.

$-1\sin\theta_1\dot{\theta}_1 - 3\sin\theta_2\dot{\theta}_2 - 2.2\sin\theta_3\cdot\dot{\theta}_3 = 0$

$\boxed{1\cos\theta_1\cdot\dot{\theta}_1 + 3\cos\theta_2\cdot\dot{\theta}_2 - 2.2\cos\theta_3\cdot\dot{\theta}_3 = 0}$

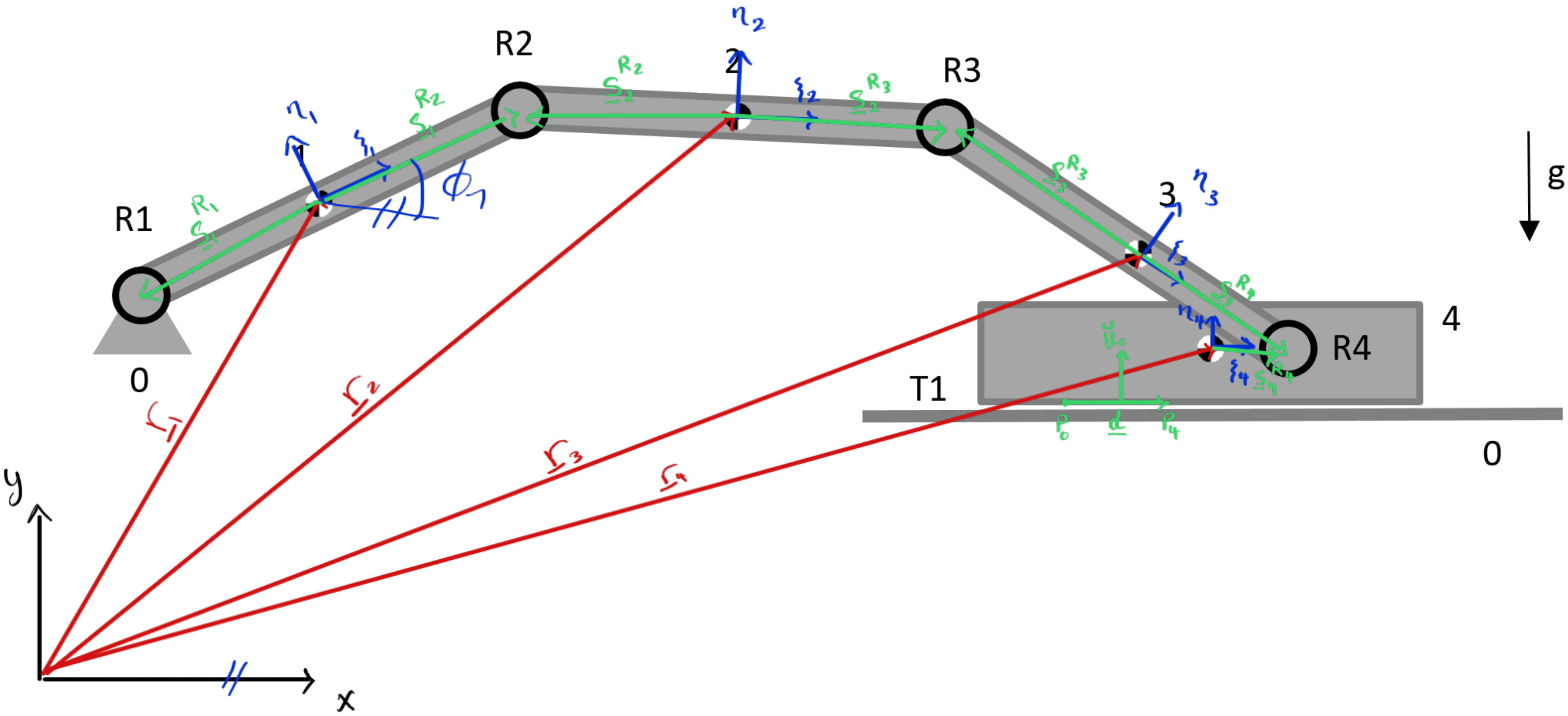
b.) Identify the Jacobian matrix of the system.

$$\underline{\underline{D}} = \begin{array}{c} \begin{array}{ccc} \dot{\theta}_1 & \dot{\theta}_2 & \dot{\theta}_3 \end{array} \\ \begin{bmatrix} -\sin(\theta_1) & -3\sin\theta_2 & -2.2\sin\theta_3 \\ \cos(\theta_1) & 3\cos\theta_2 & -2.2\cos\theta_3 \end{bmatrix} \end{array}$$

a.) Explain the computational difference between solving the constraints on position, velocity and acceleration level.

Position level: non-linear $\Rightarrow$ numeric solution

velocity/acceleration level: linear $\Rightarrow$ Analytic solution


R1
R2
2
R3
3
4
R4
T1
0
0
g
y
x