Period of Examinations
Summer Semester 2015

HOCHSCHULE RHEIN-WAAL
Rhine-Waal University of Applied Sciences

Study Course: ______________________________

Module Title: **Measurement Engineering and Controls**

Examination Part: **Measurement Engineering and Controls**

Points: 100

Duration: 120 Minutes

Please write legibly!

Date: ____________________

Family Name: ____________________ ______________________________

First Name: ____________________ Signature (Student)

Student No.: ____________________

FOR INTERNAL USE ONLY:

Question Number	Tick Questions Attempted	Points	Question Number	Tick Questions attempted	Transfer Points
1		/ 9	13		
2		/ 15	14		
3		/ 14	15		
4		/ 11	16		
5		/ 12	17		
6		/ 11	18		
7		/ 18	19		
8		/ 10	20		
9			21		
10			22		
11			23		
12			24		
	SUM			TOTAL	/ 100

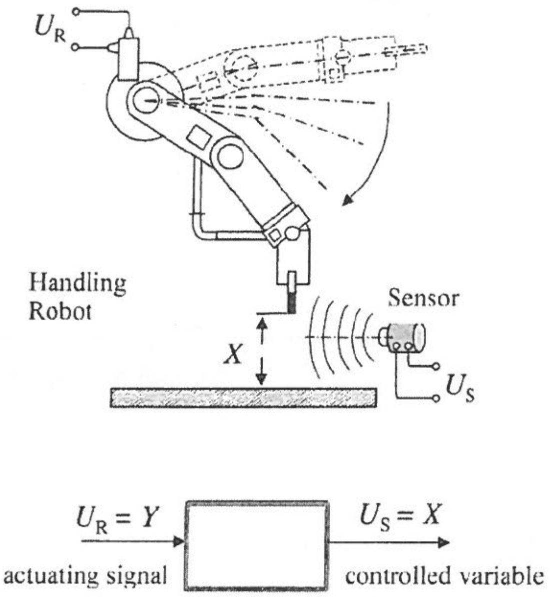
Graded by		Checked by

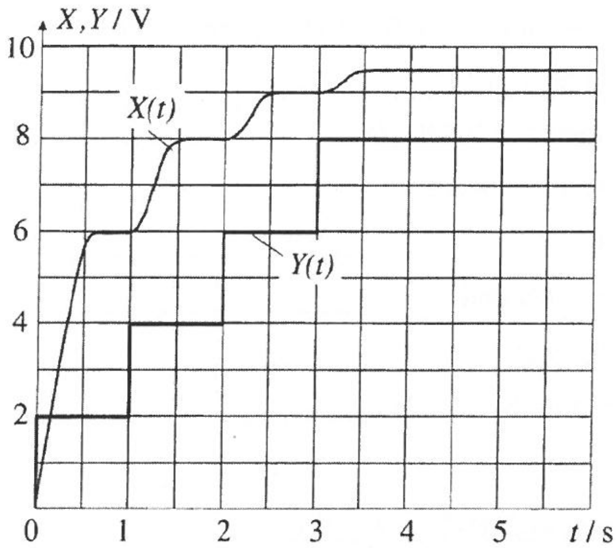
Final Grade

Regular grading key.	
Adjusted grading key. (Please add the adjusted grading key to the exam-results)	

Student No.: ____________________

Question 1: **Points: 9**





The actuating signal $Y(t) = U_R(t)$ of a controlled robot system is gradually increased in regular intervals. The response of the controlled variable (distance) $X(t) = U_S(t)$ is given in the time plot on the right.

a) Sketch the static steady state characteristic of the controlled system, by drawing $X_{stat} = f(Y_{stat})$ and sketch an interpolated curve between the points. Label the axes accordingly.

b) Is the given system linear? Please explain.

c) Linearize the steady state characteristic curve graphically about the operating point $Y_0 = 4\,V$ by a tangent line and determine the equation of the tangent $X_{lin} = m\,Y + b$.

Question 2: **15**

A system is represented through a differential equation

$$\ddot{y}(t) + 5\dot{y}(t) + 6y(t) = \dot{u}(t)$$

where $y(t)$ is the output and $u(t)$ is the input of the system.

a) Determine the transfer function of the system $G(s) = \frac{Y(s)}{U(s)}$.

L: $Y(s)\,(s^2+5s+6) = U(s)\,s$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{s}{s^2+5s+6}$$

b) What is the system type / system characteristic of the given system?

DT2 - system

Student No.: ____________________

c) Is the system stable? Please explain.

$$G(s) = \frac{s}{(s+2)(s+3)} \Longrightarrow \text{Poles: } P_1 = -2,\ P_2 = -3$$

since all the poles is Located at the LHP in the s-plane, then the system is stable.

d) Solve the output $y(t)$ for a ramp function input, thus $u(t) = t$, if all initial conditions are zero.

for a ramp input $u(t) = t \Rightarrow U(s) = \frac{1}{s^2}$

$$Y(s) = \frac{\cancel{s} \cdot 1}{s^{\cancel{2}}(s+2)(s+3)} = \frac{1}{s(s+2)(s+3)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+3}$$

$$\Rightarrow 1 = A(s+2)(s+3) + Bs(s+3) + Cs(s+2)$$

at $s=0 \Rightarrow \boxed{A = \frac{1}{6}}$

at $s=-2 \Rightarrow \boxed{B = -\frac{1}{2}}$ $\qquad \therefore Y(s) = \frac{1}{6s} + \frac{1}{3(s+3)} - \frac{1}{2(s+2)}$

at $s=-3 \Rightarrow \boxed{C = \frac{1}{3}}$

$$\mathcal{L}^{-1}: \boxed{y(t) = \frac{1}{6}(t) + \frac{1}{3}e^{-3t} - \frac{1}{2}e^{-2t}}$$

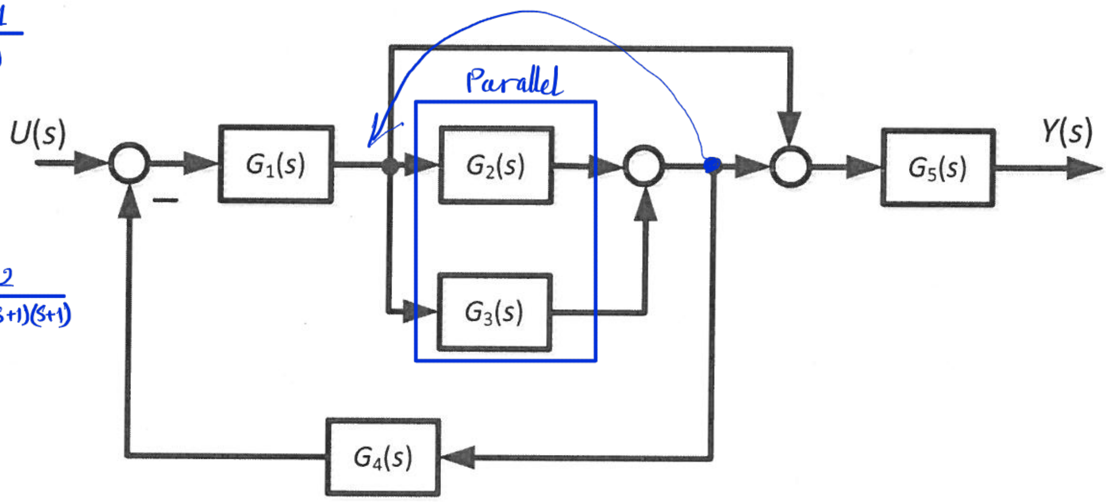
Student No.: ____________________

Question 3: **14**

Find the equivalent transfer function, $G(s) = \frac{Y(s)}{U(s)}$, for the system shown in the figure below, with

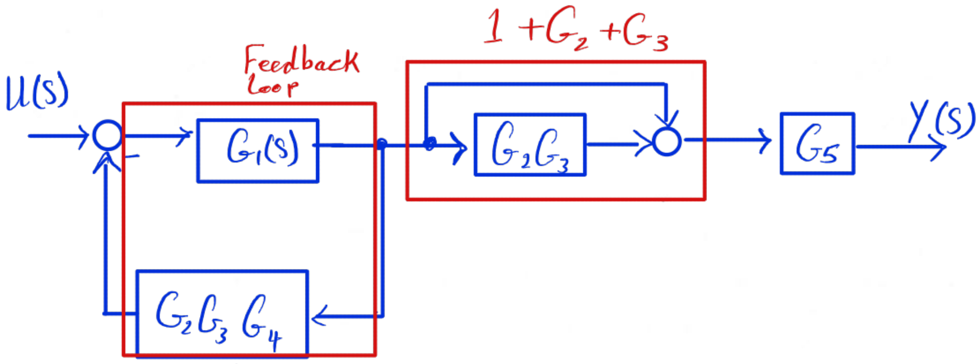
$$G_1(s) = \frac{s+1}{3s+1}, G_2(s) = \frac{1}{s}, G_3(s) = \frac{2}{s+1}, G_4(s) = \frac{1}{s+1}, G_5(s) = \frac{3s+1}{s+1}.$$

$G_2 + G_3 = \frac{3s+1}{s(s+1)}$

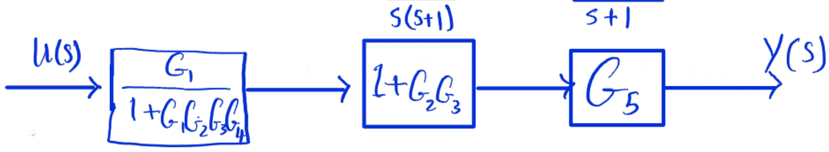


$G_1G_2G_3G_4$

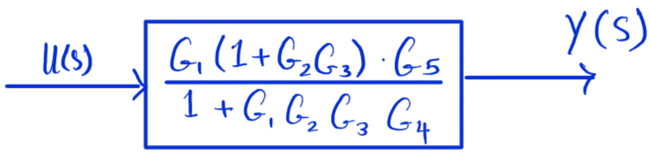
$= \frac{2\cancel{(s+1)}}{s(3s+1)(s+1)^{\cancel{2}}} = \frac{2}{s(3s+1)(s+1)}$



$\frac{G_1}{1+G_1G_2G_3G_4}$



$\frac{G_1}{1+G_1G_2G_3G_4} = \frac{\frac{(s+1)}{(3s+1)}}{1+\frac{2}{s(3s+1)(s+1)}} = \frac{s(s+1)^2}{s(3s+1)(s+1)+2}$

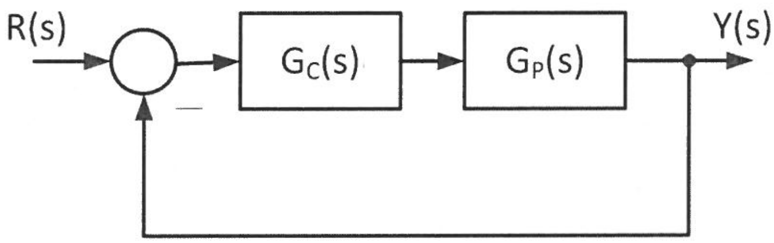


Student No.: ____________________

$$G(s) = \frac{Y(s)}{U(s)} = \frac{G_1(1+G_2G_3)\cdot G_5}{1+G_1G_2G_3G_4} = \frac{(s^2+4s+1)(3s+1)}{s(3s+1)(s+1)+2}$$

Question 4: **11**

A plant with the transfer function $G_P(s) = \frac{1}{s}$ is controlled by a PI-controller, characterized through the transfer function $G_C(s) = K_P + \frac{K_I}{s}$, according to the figure below.



a) Calculate the closed-loop transfer function $G(s) = \frac{Y(s)}{R(s)}$.

$$G_{ol} = G_c \cdot G_p = \frac{K_P}{s} + \frac{K_I}{s^2} = \frac{sK_P + K_I}{s^2}$$

$$G_{cl} = \frac{G_c \cdot G_p}{1+G_cG_p} = \frac{\frac{sK_P+K_I}{s^2}}{1+\frac{sK_P+K_I}{s^2}} = \frac{sK_P + K_I}{s^2+K_Ps+K_I}$$

Student No.: ____________________

b) Determine a reasonable damping ration D for the closed-loop transfer function, so that the maximum overshoot is 5 %.

$$S^2 + \overset{\to 2D\omega_0}{K_p} S + \overset{\omega_0^2}{K_I}$$

$$\omega_0 = \sqrt{K_I} \quad , \quad 2D\sqrt{K_I} = K_p$$

$$D = \frac{K_p}{2\sqrt{K_I}}$$

$$5\% = \text{overshoot} = e^{\frac{-D\pi}{\sqrt{1-D^2}}}$$

$$\therefore \frac{-D\pi}{\sqrt{1-D^2}} = \ln(0.05) \Rightarrow D^2\pi^2 = \ln(0.05)^2 - \ln(0.05)^2 D^2$$

$$D = \frac{-\ln(0.05)}{\sqrt{\pi^2 + \ln(0.05)^2}} \approx 0.7$$

c) Calculate the controller parameters K_P and K_I, so that the closed-loop system behaves with the given damping ratio chosen in b) and with an eigenfrequency of $\omega_0 = 4\,\frac{rad}{s}$.

$$\omega_0 = \sqrt{K_I} = 4\,rad/s \Longrightarrow K_I = 16$$

$$D = \frac{K_p}{2\sqrt{K_I}} \Longrightarrow K_p = 2(0.7)(4\,rad/s) =$$

Question 5: **12**

The open-loop system $G_S(s)$ is described by the transfer function

$$G_S(s) = \frac{s+5}{s(s+30)} = \frac{\cancel{5}\left(\frac{s}{5}+1\right)}{\cancel{30}_{6}\,s\left(\frac{s}{30}+1\right)} = \frac{1}{6}\cdot\frac{1}{\frac{s}{30}+1}\cdot\left(\frac{s}{5}+1\right)$$

Sketch the asymptotes of the Bode-diagram of the open-loop system including its magnitude and phase shift.

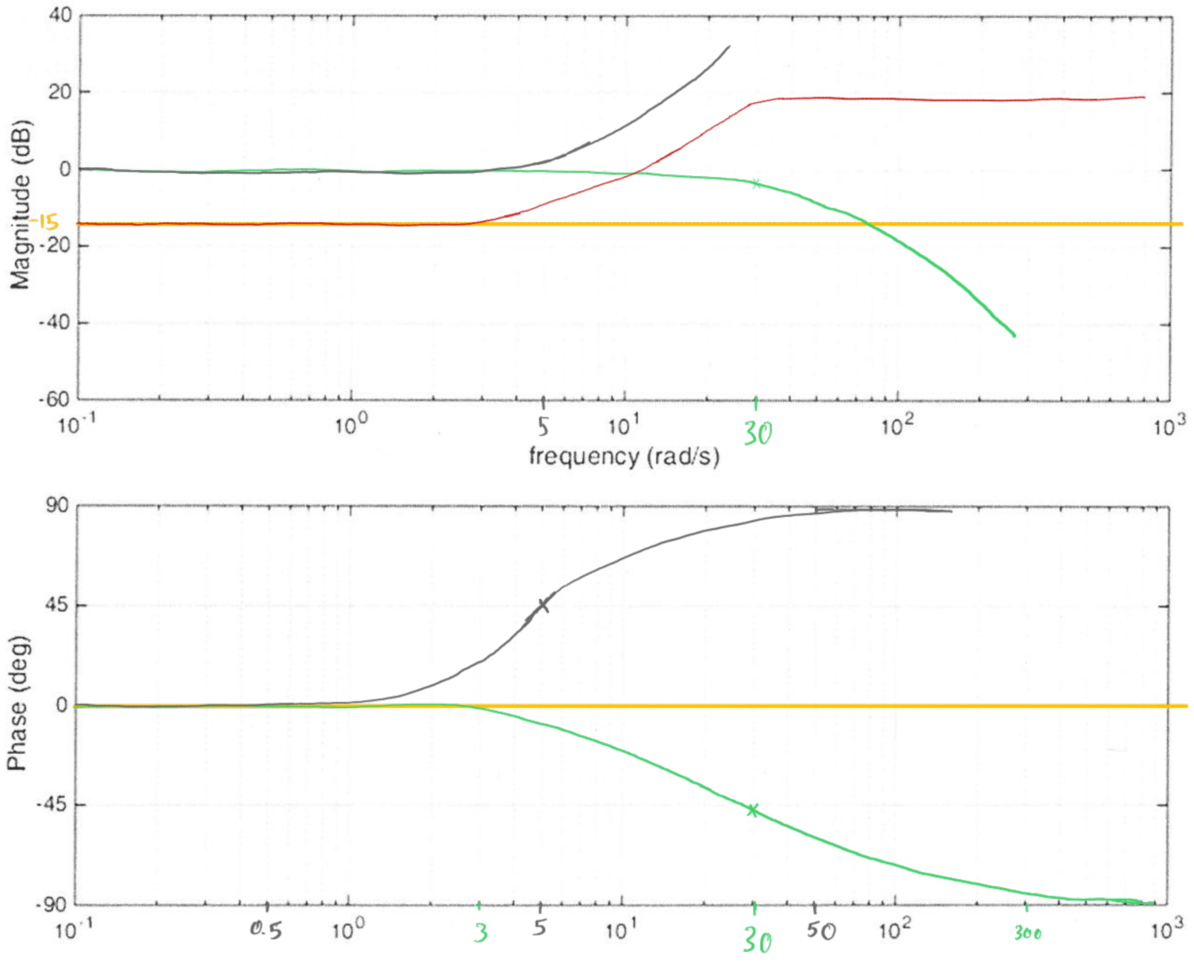
Student No.: ______________________

For $G_1(s)=\frac{1}{6} \Rightarrow G_{dB}=20\log(\frac{1}{6})=-15.56\,dB \Rightarrow \phi=0^\circ$ Yellow

for $G_2(s)=\frac{1}{\frac{s}{w_0}+1}=\frac{1}{\frac{s}{30}+1} \Rightarrow w_0=30\,rad/s,\ G_{dB}=-20\log\left(\sqrt{1+\frac{w^2}{30^2}}\right),\ \phi=\tan^{-1}\left(-\frac{w}{30}\right)$ Green

for $G_3(s)=\frac{s}{5}+1=\frac{s}{w_0}+1 \Rightarrow w_0=5\,rad/s,\ G_{dB}=20\log\left(\sqrt{1+\frac{w^2}{25}}\right),\ \phi=\tan^{-1}\left(\frac{w}{5}\right)$ gray

The $G_s(s)$ Bode diagram is the sum of all of 3 plots red color



Student No.: ____________________

Question 6:

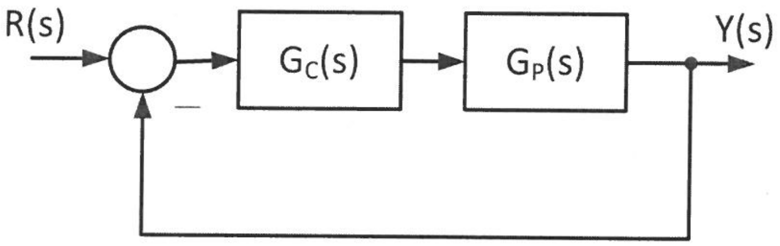
11

For the feedback system shown in the figure, with

$$G_C(s) = \frac{2s + K}{s}$$

and

$$G_P(s) = \frac{2}{s^3 + 7s^2 + s + 1}$$



Find the range of K for which the system is stable.

$$G_{CL} = \frac{G_c G_p}{1 + G_c G_p} = \frac{\frac{2(2S+K)}{S^4+7S^3+S^2+S}}{1 + \frac{2(2S+K)}{S^4+7S^3+S^2+S}} = \frac{2(2S+K)}{S^4+7S^3+S^2+5S+2K}$$

Routh-Hurwitz method:

S^4	$a_4=1$	$a_2=1$	$a_0=2K$
S^3	$a_3=7$	$a_1=5$	0
S^2	$b_1 = \frac{-\begin{vmatrix}1 & 1\\ 7 & 5\end{vmatrix}}{7} = \frac{2}{7}$	$b_2=2K$	0
S^1	$C_1=5-49K$	$C_2=0$	
S^0	$d_1=2K$		

$2K > 0 \Rightarrow K > 0$

and $5 - 49K > 0$

$K < \frac{5}{49}$

$\therefore$ To have a stable system

$$\boxed{0 < K < \frac{5}{49}}$$

Student No.: ____________________

Question 7: 18

An open-loop transfer function is given as,

$$G(s) = \frac{1}{s(s+2)}$$

a) Find the analytical expressions for the magnitude-frequency response $|G(j\omega)|$ and the phase response $\phi(\omega)$.

$$G(s=jw) = \frac{1}{(jw)^2 + 2jw} = \frac{1}{2jw - w^2} * \frac{2jw + w^2}{2jw + w^2} = \frac{2jw + w^2}{-4w^2 - w^4}$$

$$G(jw) = -\frac{2jw + w^2}{w^2(w^2+4)} = -\frac{1}{(w^2+4)} - \frac{2}{w(w^2+4)} j$$

$$Re = -\frac{1}{w^2+4} \qquad Im = -\frac{2}{w(w^2+4)} j$$

$$|G(jw)| = \sqrt{Re^2 + Im^2} = \sqrt{\left(\frac{1}{w^2+4}\right)^2 + \left(\frac{2}{w(w^2+4)}\right)^2} = \sqrt{\frac{w^2+4}{w^2(w^2+4)^2}}$$

$$\phi(w) = \tan^{-1}\left(\frac{Im}{Re}\right) = \tan^{-1}\left(\frac{2}{w}\right)$$

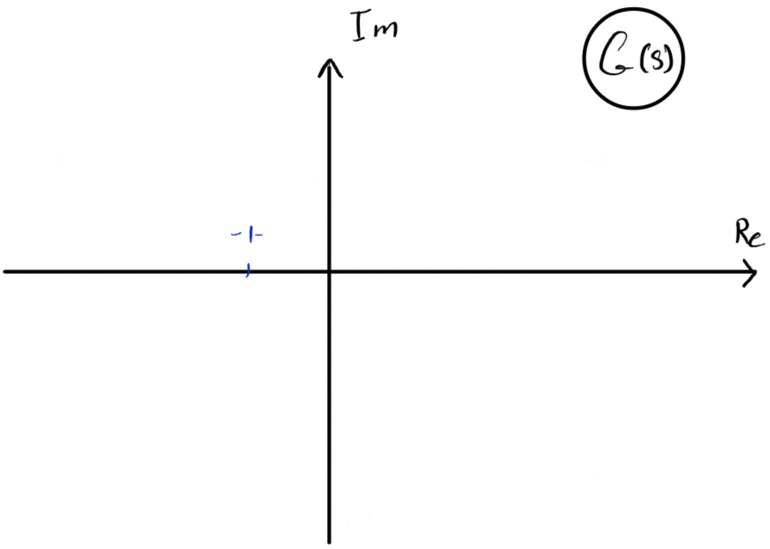
because both Im & Re are negative,
The plote is in the third quadrint

Student No.: ____________________

b) Draw the Nyquist-plot using the data points in the table below. Label the axes of the Nyquist-plot accordingly.

ω	$\lvert G(j\omega)\rvert$	$\phi(\omega)$
0	0.5	90°
0.1	4.884	87.14°
0.5	0.916	76°
1	0.447	63.435°
10	0.073	11.31°
∞	0	0°

$+180^\circ$



c) Is the closed-loop system stable? Explain by means of the Nyquist-plot obtained in b).

since the critical point $(-1, 0j)$ is located to the left of the nyquist plot at the Re axis,
Thus the open-loop is stable $\Rightarrow$ as well as the closed-loop.

Question 8: **10**

No answer expected. Here you will get the points from the laboratory.

Good luck!