Period of Examinations
Summer Semester 2021 – 21th July
Exam

Module:Modelling and Simulation, Prof. Brandt

Examination

Points: 60

Duration of examination: 120 Minutes (including time for printing / scanning / upload etc.)

Please write legibly!

Date: 19.6.2022

Name: Timon te Kempel

Register No.: 26797

Course of Study: MSE

Before you turn in your solution please sign the declaration in lieu of oath:

I, ______________________________ [full name, matriculation number], hereby confirm in lieu of an oath that I am the person who was admitted to this examination. Further, I confirm that the submitted work is my own and was prepared without the use of any unauthorised aid or materials.

____

Signature

You can print the declaration and then sign and scan it. Alternatively, you can also sign it digitally or transcribe it by hand and then sign and scan it.

Please make sure that all documents that you upload contain your name and matriculation number.

Good luck!

Problem	Possible Points	Result
1	4+2+6+2=14	
2	6	
3	8	
4	2	
5	2	
Sum	**32**	

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1.) The following equations

$$\dot{x}_1 = 3x_1(2 - x_1 - 2x_2)$$

$$\dot{x}_2 = x_2(1 - 2x_1 - 5x_2)$$

describe a sample dynamic system.

a.) Indicate which of the following points are equilibria of the system. **(4 points)**

☒ $\boldsymbol{x}_{10} = \begin{bmatrix} 0.0 \\ 0.2 \end{bmatrix}$

☒ $\boldsymbol{x}_{20} = \begin{bmatrix} 2.0 \\ 0.0 \end{bmatrix}$

○ $\boldsymbol{x}_{30} = \begin{bmatrix} 0.0 \\ 2.0 \end{bmatrix}$

○ $\boldsymbol{x}_{40} = \begin{bmatrix} 0.2 \\ 0.0 \end{bmatrix}$

Plug in: $\vec{x}_{10}$: $\dot{x}_1 = 3 \cdot 0(2 - 0 - 2 \cdot 0.2) = 0$

$\dot{x}_2 = 0.2(1 - 2 \cdot 0 - 5 \cdot 0.2) = 0$

$\rightarrow$ Equilibrium : no change of states

$\vec{x}_{20}$: $\dot{x}_1 = 3 \cdot 2(2 - 2 - 0) = 0$

$\dot{x}_2 = 0(\ldots) = 0$

$\rightarrow$ Equilibrium : no change of states

$\vec{x}_{30}$: $\dot{x}_1 = 3 \cdot 0(\ldots) = 0$

$\dot{x}_2 = 2(1 - 2 \cdot 0 - 5 \cdot 2) = -22$

$\rightarrow$ no Equilibrium

$\vec{x}_{40}$: $\dot{x}_1 = 3 \cdot 0.2(2 - 0.2 - 2 \cdot 0) = 0.6 \cdot 1.8$

$\dot{x}_2 = 0(\ldots) = 0$

$\rightarrow$ no Equilibrium

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b.) Find a further equilibrium x_{50}. **(2 points)**

$\dot{x}_1 = f(x_1, x_2) = 3x_1(2 - x_1 - 2x_2)$

$\dot{x}_2 = g(x_1, x_2) = x_2(1 - 2x_1 - 5x_2)$

$$\vec{x}_{30} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

<u>Check:</u> $\dot{x}_1 = 3 \cdot 0 (\ldots) = 0$

$\dot{x}_2 = 0 \cdot (\ldots) = 0$

$\rightarrow$ no change in state variables

c.) Linearize the system about $x_{op} = \begin{bmatrix} 1.0 \\ 0.0 \end{bmatrix}$. **(6 points)**

linearize f and g, in order to work with the system easier!

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} \frac{\partial f}{\partial x_1} & \frac{\partial f}{\partial x_2} \\ \frac{\partial g}{\partial x_1} & \frac{\partial g}{\partial x_2} \end{pmatrix} \cdot (\vec{x} - \vec{x}_{op})$$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 6 - 6x_{1op} - 6x_{2op} & -6x_{1op} \\ -2x_{2op} & 1 - 2x_{1op} - 10x_{2op} \end{pmatrix} \cdot \begin{pmatrix} x_1 - 1 \\ x_2 \end{pmatrix}$$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 6 - 6 \cdot 1 - 0 & -6 \cdot 1 \\ -2 \cdot 0 & 1 - 2 \cdot 1 - 10 \cdot 0 \end{pmatrix} \begin{pmatrix} x_1 - 1 \\ x_2 \end{pmatrix}$$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & -6 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x_1 - 1 \\ x_2 \end{pmatrix}$$

c) linearize the system about $x_{op} = \begin{bmatrix} 1.0 \\ 0.0 \end{bmatrix}$

$$\dot{x}_1 = 3x_1(2 - x_1 - 2x_2)$$

$$\dot{x}_2 = x_2(1 - 2x_1 - 5x_2)$$

$$x_{op} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\underline{f}(x) \simeq \underline{f}(x_{op}) + \left.\frac{\partial f}{\partial x}\right|_{\underline{x}=\underline{x}_{op}} \cdot \Delta \underline{x}$$

$$\frac{\partial f}{\partial x} = \begin{bmatrix} 6 - 6x_1 - 6x_2 & -6x_1 \\ -2x_2 & 1 - 2x_1 - 10x_2 \end{bmatrix}$$

$$\left.\frac{\partial f}{\partial x}\right|_{\underline{x}=\underline{x}_{op}} = \begin{bmatrix} 0 & -6 \\ 0 & -1 \end{bmatrix}$$

$$\underline{f}(x) \simeq \begin{bmatrix} 3 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & -6 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} x_1 - 1 \\ x_2 - 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & -6 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & -6 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \end{bmatrix}}$$

$$= \begin{bmatrix} 3 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & -6 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

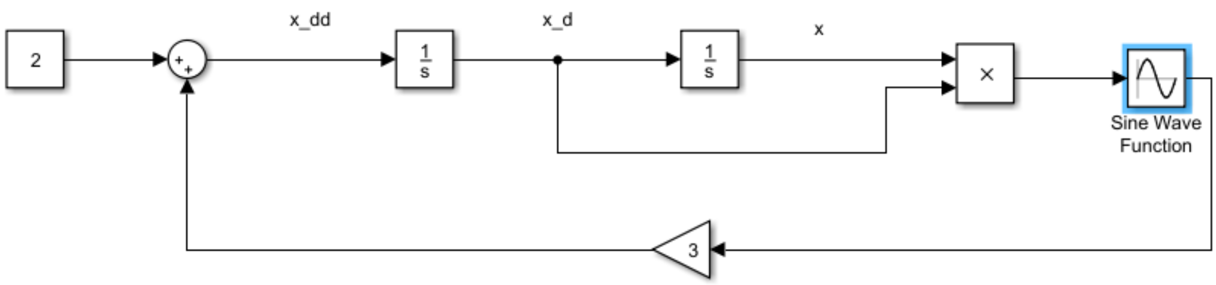
$$= \underbrace{\begin{bmatrix} 0 & -6 \\ 0 & -1 \end{bmatrix}}_{\underline{\underline{A}}} \cdot \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\underline{x}} + \underbrace{\begin{bmatrix} 3 \\ 0 \end{bmatrix}}_{\underline{\underline{B}}\,\underline{u}}$$

$$\dot{\underline{x}} = \underline{\underline{A}}\,\underline{x} + \underline{\underline{B}} \cdot \underline{u}$$

d.) Write the linearized system in the form $\dot{x} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u}$. **(2 points)**

$$\underbrace{\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\end{pmatrix}}_{\dot{\underline{x}}} = \underbrace{\begin{pmatrix}0 & -6\\ 0 & -1\end{pmatrix}}_{\underline{\underline{A}}} \underbrace{\begin{pmatrix}x_1-1\\ x_2\end{pmatrix}}_{\underline{x}}$$

2.) Give the ODE, which is modelled by the following Simulink model. Then, transform it into a system of first order differential equations (ODEs). **(6 points)**

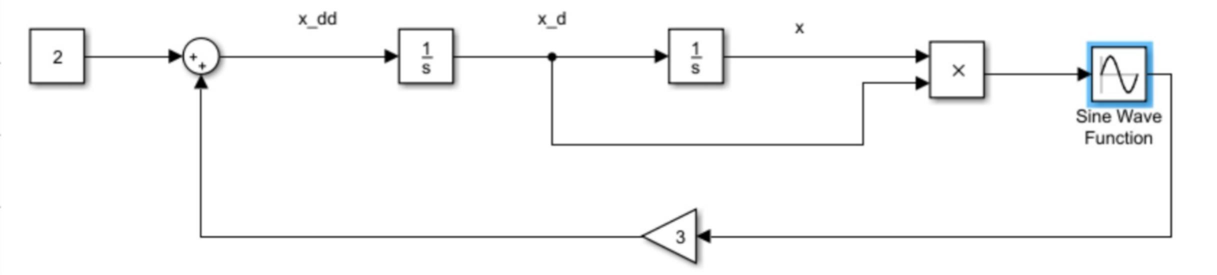


$\ddot{x} = 2 + 3(\sin(\dot{x} \cdot x))$ $\quad\quad$ $\ddot{x} = \dot{v}$

Define: $\quad x_1 = \dot{x}_2$

$x_2 = x$

$$\boxed{\begin{aligned}\dot{x}_1 &= 2 + 3(\sin(\dot{x}_2 \cdot x_2))\\ \dot{x}_2 &= x_1\end{aligned}}$$



$$\ddot{x} = 2 + 3\sin(x \cdot \dot{x})$$

Then transfer it into a system of 1st-order DE

$$x_1 = x$$

$$x_2 = \dot{x} = \dot{x}_1$$

$$\dot{x}_1 = x_2$$ → don't forget this equ.!

$$\dot{x}_2 = \ddot{x} = 2 + 3\sin(x_1 \cdot x_2)$$

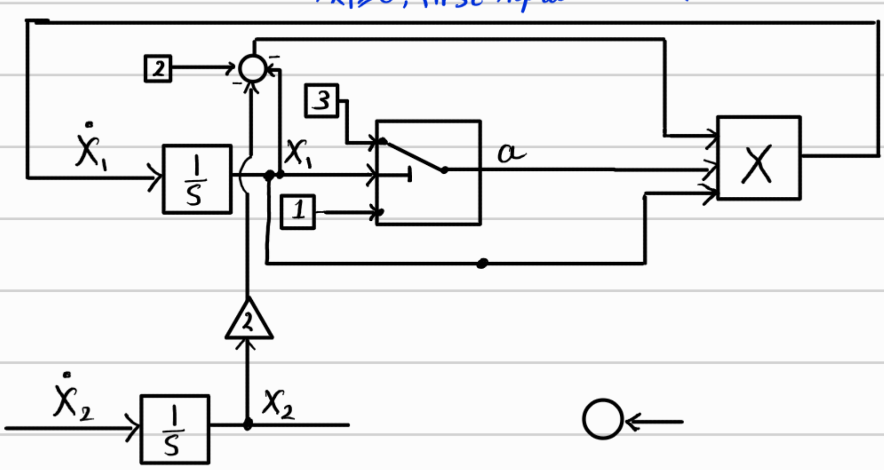
Q3

$$\dot{x}_1 = a \cdot x_1(2 - x_1 - 2x_2)$$

$$\dot{x}_2 = x_2(1 - 2x_1 - 5x_2)$$

$$a = \begin{cases} 3 & if\ x_1 \geq 0 \\ 1 & if\ x_1 < 0 \end{cases}$$ Triger that we will see the switch block

$*x_1 \geq 0$, first input → output



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3.) The following modified equations

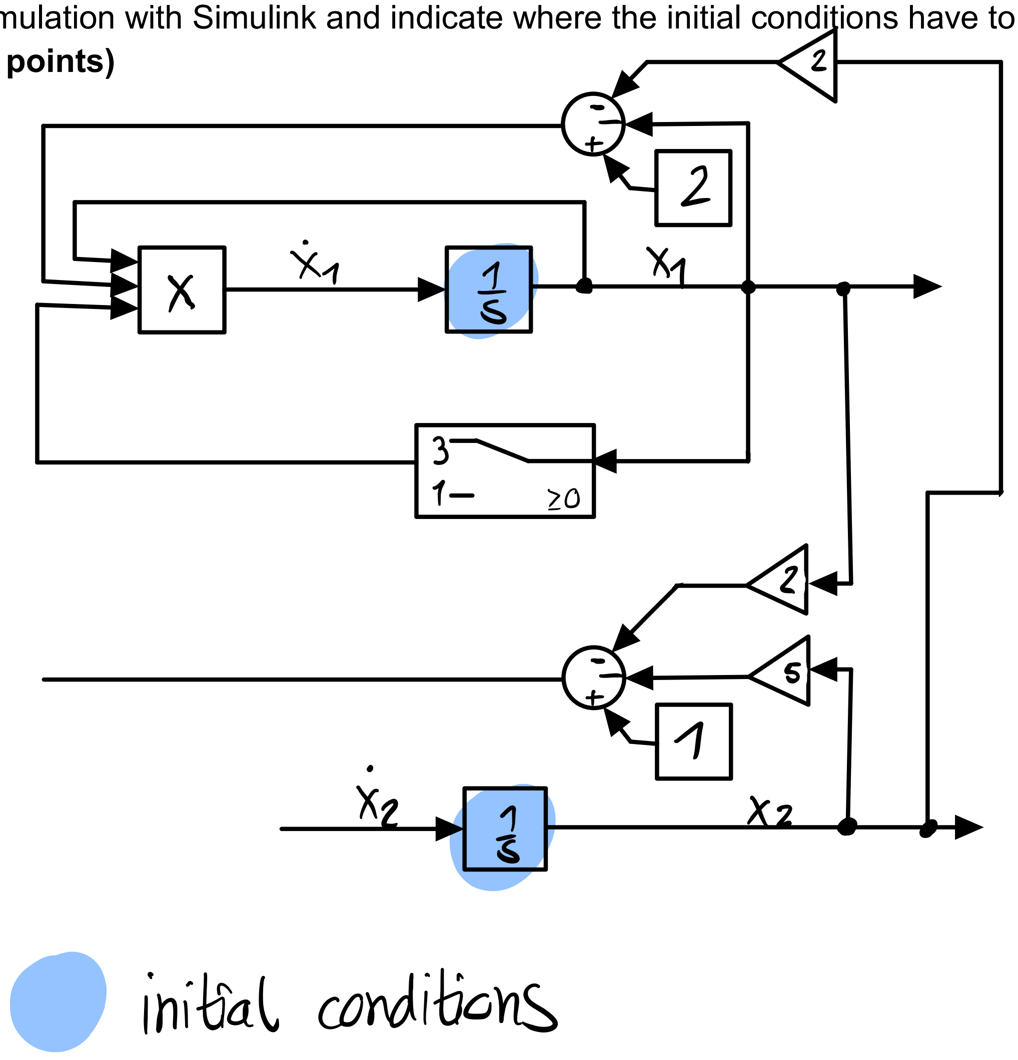
Must have

$$\dot{x}_1 = a \cdot x_1(2 - x_1 - 2x_2)$$

$$\dot{x}_2 = x_2(1 - 2x_1 - 5x_2)$$

$$a = \begin{cases} 3 & if \;\; x_1 \geq 0 \\ 1 & if \;\; x_1 < 0 \end{cases}$$

describe a systems with a discontinuity for the parameter *a*, which changes depending on the value of the state variable x_1. Draw the block-diagram for a simulation with Simulink and indicate where the initial conditions have to be set.
(8 points)



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4.) Indicate for all ODEs whether they are linear or nonlinear and give the order of all ODEs! **(2 points)**

$\dddot{y} = \ddot{y} + 12^2 + y^0 - \sin 3\pi$	3rd order linear ODE
$\sin(y) + y^1 = \dot{y}^2$	nonlinear 1st order
$3(\dot{y} + 4\ddot{y}) = 9a^2 * \dot{y}$; with $a = \begin{cases} y \; if \; \ddot{y} > 0 \\ \dot{y} \; else \end{cases}$	nonlinear 2nd order
$y * \dot{y} = \ddot{y}$	nonlinear 2nd order

5.) How can we react when a simulation becomes numerically unstable? **(2 points)**

I) Change the type of solver (e.g. from an explicit to implicit method), in order to increase the region of convergence.

II) Change/fix the step size and thereby force the solver to use the chosen stepsize
→ no variable change (→ more calculations), but solver will not "miss" numerically critical regions/points

Q5

2 ways:

1- Changing step size

2- change the solver