Mapping Micro Economies in Digital Twins: A Case Study on Terra Luna's Economic Model

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Abstract

In the rapidly evolving landscape of digital currencies, the stark downfall of Terra Luna's market value from USD 22 billion to 3 billion within days stands as a stark reminder of the volatile nature of cryptocurrency economies. This paper delves into the intricate dynamics of Terra Luna's economic mechanisms by employing a digital twin simulation, aiming to bridge the gap between macroeconomic theories and the nuanced realities of microeconomic systems. Through this explorative study, we seek to unravel the factors that precipitated Terra Luna's dramatic crash and explore potential strategies for resilience and recovery within similar economic models.

The purpose of this research is to conduct an in-depth analysis of algorithmic stablecoins, centering particularly on the Terra Luna incident as a case study. This investigation seeks to dissect the dynamics and underlying factors contributing to the volatility of such cryptocurrencies. By examining the Terra Luna example, the study will identify key levers and strategies that could have mitigated or possibly prevented its dramatic crash. Through this analysis, the research aims to contribute to a better understanding of the operational challenges facing algorithmic stablecoins and propose viable solutions to enhance their stability and reliability in the digital currency market.

1. Introduction to Stablecoins

The stablecoin market has seen rapid expansion, notably led by Tether (USDT). Since 2017, stablecoin market capitalization and trading volumes have consistently risen. Stablecoin market capitalization now exceeds $128 billion, comprising nearly 9% of the total cryptocurrency market. Tether (USDT) dominates this market with an impressive $88 billion market value, representing around 70% of the total market capitalization (*DefiLlama*). Other significant players include USD Coin (USDC) with a market value of $24 billion and Dai with $5 billion (*Top Stablecoin Tokens by Market Capitalization*).

Stablecoins serve as a versatile payment method in the cryptocurrency ecosystem, facilitating various transactions from salary payments to purchasing goods and services. Their convenience eliminates traditional barriers such as approval processes from financial authorities, paperwork for remittances, and long processing times. This versatility and ease of use position stablecoins as essential tools in the digital economy, seamlessly integrating into diverse financial activities.

Stablecoins have emerged as a crucial component within the cryptocurrency ecosystem, offering a solution to the extreme volatility that often characterizes digital assets like Bitcoin and Ethereum. Unlike traditional cryptocurrencies, which can experience rapid price fluctuations, stablecoins are designed (*Stablecoins*, n.d.) to maintain a stable value, typically pegged to a fiat currency like the US dollar or a basket of assets.

1.1 Spectrum of Decentralization in Stablecoins

Stablecoins exist along a spectrum of decentralization, with different projects implementing various mechanisms to achieve stability while balancing decentralization and efficiency. Three prominent categories within this spectrum are overcollateralized stablecoins, fractional stablecoins, and algorithmic stablecoins.

1.1.1 Overcollateralized Stablecoins

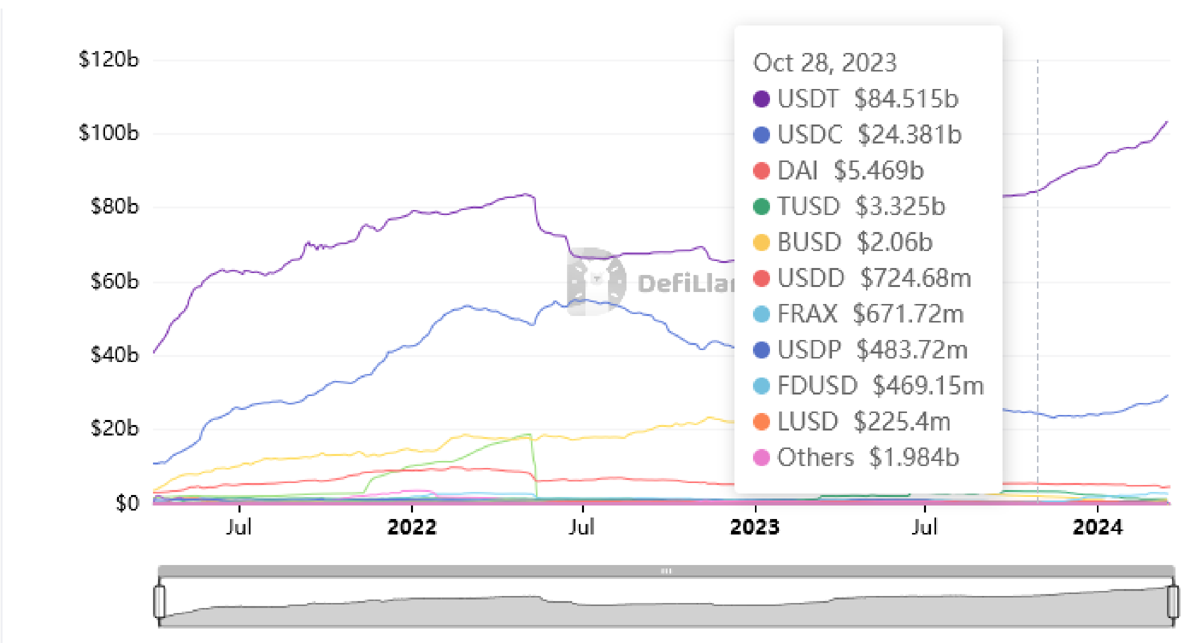
Overcollateralized stablecoins maintain stability by requiring users to collateralize their holdings with assets such as fiat or cryptocurrency, a value that exceeds the stablecoin's issuance. This ensures that the stablecoin is backed by assets of equal or greater value, providing confidence in its stability. Dai, a stablecoin built on the Ethereum blockchain, is a prominent example of an overcollateralized stablecoin. It is backed by a surplus of Ethereum-based assets, which are managed by a decentralized network of participants.

1.1.2 Fractional Stablecoins

Fractional stablecoins implement a fractional reserve model, where only a portion of the stablecoin's value is backed by collateral. This approach allows for greater efficiency and scalability compared to overcollateralized models. FRAX is an example of a fractional stablecoin that dynamically adjusts its collateral ratio based on market conditions. It uses a combination of collateralized assets (80% collateralized by crypto assets (*CoinDesk*, 2023)) and algorithmic mechanisms to maintain its stability, with the aim of achieving a stable value while optimizing collateral utilization.

1.1.3 Algorithmic Stablecoins

Algorithmic stablecoins rely on algorithmic mechanisms to regulate the stablecoin's supply and demand, aiming to achieve stability without the need for collateral backing. Terra is a notable example of an algorithmic stablecoin that utilizes a dual-token system consisting of Terra (LUNA) and Terra stablecoins (such as UST). LUNA serves as collateral and is used to stabilize the price of Terra stablecoins through an algorithmic mechanism that adjusts the token's supply based on market demand. This approach allows Terra to maintain stability while minimizing the need for external collateral.



2. Terra Ecosystem

The Terra-Luna ecosystem represents a public blockchain protocol with a primary emphasis on fostering a robust decentralized finance (DeFi) ecosystem, where algorithmic stablecoins play a central role. At its core, Terra comprises several key components. Firstly, the blockchain network is constructed on the Tendermint consensus mechanism and Cosmos SDK, providing a rapid and secure foundation for various DeFi applications to flourish. Secondly, the native token of the ecosystem, LUNA, serves as the driving force behind its operations, facilitating the algorithmic stablecoin mechanism and ensuring the system's vitality. Moreover, the algorithmic stablecoin, TerraUSD (UST), forms the cornerstone of the ecosystem, steadfastly pegged to the US dollar. This stability is maintained through a distinctive burning and minting process that involves LUNA, thus underpinning the resilience and reliability of the stablecoin within the broader Terra ecosystem. As of February 19, 2024, the Terra-Luna ecosystem has a market capitalization of around $1.48 billion and boasts over $25 billion in TVL.

2.1 Tokenomics of Terra-Luna

The foundation of the Terra-Luna ecosystem comprises Terra and Luna tokens, each serving distinct yet complementary functions. Terra stablecoins replicate and follow the value of fiat currencies, presenting a digital substitute for conventional money. The primary allure of Terra stablecoins rests in their promise of stability; for instance, Terra USD (UST) maintains parity with the US Dollar. Users engage with Terra akin to fiat currency, enjoying the added advantages of blockchain technology such as immutability, swift transactions, and minimal transaction fees. The process of generating new Terra stablecoins operates through a deflationary mechanism involving Luna, the native staking token of the Terra protocol. To create new Terra, users execute a burn transaction of Luna, injecting fresh Terra stablecoins into the system while concurrently diminishing the circulating supply of Luna. As demand for Terra rises, more Luna is burned to mint Terra, thereby reducing Luna's supply and increasing its value. This dynamic interaction establishes a reciprocal association wherein Luna's value aligns directly with Terra's utilization within the ecosystem.

Terra's operational framework represents a novel approach in the realm of stablecoins, distinguishing itself through a hybrid stability mechanism that intricately blends algorithmic strategies with a collateralized reserve, overseen by the Luna Foundation Guard. At the heart of Terra's stability and operational integrity is its innovative arbitration mechanism, designed to anchor Terra's value to the US Dollar.

This mechanism is pivotal for the protocol, enabling users to directly exchange 1 UST (Terra's stablecoin) for an equivalent value of $1 in LUNA (Terra's native cryptocurrency), thereby introducing a systematic arbitrage opportunity. This opportunity becomes particularly relevant and is actively pursued by market participants whenever there is a deviation from Terra's intended $1 peg.

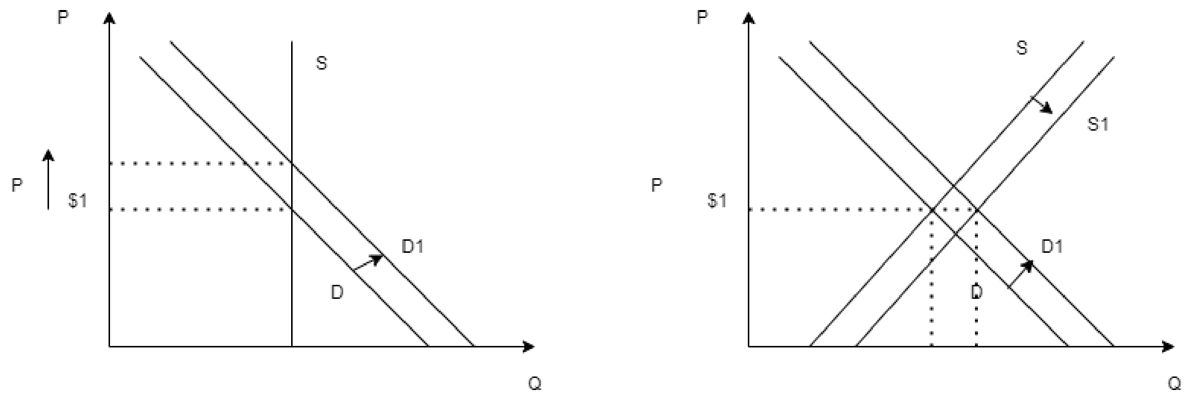


Figure 1. Demand and Supply Mechanism in Terra-Luna Market

The arbitration process is not just a theoretical construct but a practical tool that plays a critical role in the real-time adjustment of Terra's market value, ensuring its peg remains stable. A more detailed exploration of Terra's market module and its underlying mechanisms will be provided in subsequent sections.

Furthermore, Luna holders are incentivized for their contributions to network stability, receiving rewards and governance rights. The dynamic adjustment of metaphorical pools representing the circulating supplies of Terra and Luna plays a pivotal role in maintaining Terra's price peg. During expansion phases, users are encouraged to exchange Luna for Terra, aligning supply with demand until equilibrium is achieved at the pegged price. However, vulnerabilities in Terra's stability mechanism surfaced during periods of market stress, necessitating a deeper understanding of its operation and potential improvements to ensure resilience in the face of volatility.

2.2 Concept of Seigniorage

Seigniorage refers to the profit gained by the entity responsible for issuing and managing a currency, representing the difference between its face value and production cost. Historically associated with fiat currencies under central authority, seigniorage evolved from the medieval era to modern times, becoming a key component of monetary policy and government revenue.
In traditional fiat currencies, central banks generate seigniorage by issuing banknotes. With algorithmic stablecoins, seigniorage mechanisms operate through decentralized algorithms embedded in smart contracts, dynamically managing coin supply based on market conditions.

For instance, in the case of Terra and Luna, when demand for Terra rises, the system mints Terra, earning Luna in return. This process, known as seigniorage, involves minting currency at zero cost and burning Luna to create scarcity. The significance of seigniorage will be thoroughly examined within our study, particularly as we delve into the historical evolution of Terra's market-making module. This examination will elucidate the pivotal role seigniorage plays in maintaining the equilibrium of the ecosystem.

2.3 Terra-Luna Crash

In May 2022, the Terra-Luna ecosystem faced a significant crash, marking a catastrophic event that erased billions of dollars and reverberated across the cryptocurrency landscape. The crash unfolded as follows: A substantial portion of UST, the algorithmic stablecoin at the core of the ecosystem, lost its peg to the US dollar, sparking widespread panic selling that drove both UST and LUNA prices into a steep decline. The intricate stability mechanism tying UST to LUNA initiated a massive minting of LUNA tokens to burn UST and restore the peg, resulting in hyperinflation of LUNA and exacerbating its devaluation. Consequently, the price of LUNA plummeted from over $80 to virtually zero within a matter of days, inflicting staggering losses upon investors and participants within the ecosystem.

Anchor Protocol, renowned for offering a remarkable 20% annual percentage yield (APY) on deposits of UST, became a beacon for crypto investors seeking lucrative returns. This attractive proposition led to a significant influx of capital into the platform. However, the idyllic scenario quickly soured as the market plunged into panic reminiscent of a classic bank run. UST stablecoin holders, fearing a collapse, hurried to liquidate assets locked within Anchor Protocol in a frantic bid to salvage whatever value they could. This surge in withdrawals, particularly from protocols like Anchor, exacerbated the already precarious liquidity crisis, pushing UST further away from its peg. The situation reached a critical juncture with the startling withdrawal of 2 billion UST from Anchor, intensifying the panic gripping investors. As rumors of instability spread like wildfire, deposits in Anchor plummeted at an alarming rate, with funds being withdrawn at an astonishing pace of $10 million per minute. The ominous signs of a run on Anchor became glaringly evident on May 7, 2022, as two large addresses initiated withdrawals totaling 375 million UST, triggering a rapid decline in UST price and further exacerbating withdrawals from Anchor. Despite Terraform Labs' (TFL) attempts to stabilize the peg by purchasing UST, investor confidence continued to wane, leading to intensified withdrawals late on May 9, 2022. By May 13, Anchor found itself with less than 2 billion UST remaining, and the value of UST had plummeted to below $0.2, marking a dire turn of events for the ecosystem.

3. History and Evolution of Terra's Algorithmic Market Module

Before exploring the detailed aspects of our model, it's crucial to establish a foundational understanding of Terra's original system architecture, along with its evolutionary upgrades. This initial overview will encompass the core design principles, operational dynamics, and the motivations driving the system's enhancements over time. By doing so, we aim to provide a clear context for our subsequent analysis, ensuring that the rationale behind Terra's strategic updates is comprehensively understood. This approach is intended to facilitate a deeper appreciation of the model's relevance and accuracy in simulating Terra's economic mechanisms and stability strategies.

3.1 Columbus Updates

The Terra protocol, an ambitious project designed to combine the price stability and wide adoption of fiat currencies with the censorship resistance of Bitcoin, has undergone a series of significant upgrades known as the "Columbus" updates.

Throughout the progression of the Columbus upgrades, the Terra team has implemented numerous proposals aimed at fostering protocol growth and enhancing system stability. Given the extensive nature of these upgrades, this section intends to categorize them into coherent groups based on their implementation timelines

3.2 Pre-Columbus-3

Prior to the introduction of Columbus-3, the Terra protocol lacked a dedicated market module to facilitate the liquidity of Terra to Luna swaps. Transactions were executed directly through Terra Station, offering zero slippage and thereby maintaining a 1:1 dollar worth swap between Terra and Luna. This approach, while efficient in maintaining swap ratios, posed a risk of hyperinflation and oracle attacks. It had the potential to cause significant increases in Luna's supply within a short timeframe, or vice versa. Such dynamics could precipitate extreme volatility within the ecosystem.

To cater to this, the protocol had imposed a daily LUNA supply change cap which specified the max inflation or deflation that LUNA supply could experience in a given day, after which further swaps failed (*Market · Terra Documentation*). However, it was not meant to be a sustainable solution against the aforementioned attack vectors.

3.3 Columbus-3 & Terra Constant Product Swaps

The Columbus-3 upgrade marked a significant evolution for the Terra ecosystem with the introduction of a market module designed to streamline swaps between Terra's native coins. The Terra's Market Module is an algorithmic and adaptive response to both hyperinflation and oracle manipulation. This innovative module, while drawing conceptual parallels to Automated Market Maker (AMM) pools like those seen in Uniswap, distinguishes itself by utilizing virtual rather than actual reserves. In traditional AMM pools, tokens are physically added to or withdrawn from pools to facilitate swaps. In contrast, the Terra market module operates through a process of minting and burning tokens, employing a unique approach to maintaining liquidity and swap efficiency.

A key feature of this module is its adaptation of the constant product formula, which is slightly modified to prioritize the stability of the Luna pool's size in relation to Terra's pegged price, as opposed to directly correlating Terra's value to Luna's market price. Consequently, the swap rate between these two assets is determined not by Luna's fluctuating market price, but by the controlled size of the Luna pool (jp12, 2022).

$$CP = Pool_{Terra} \cdot Fiat_{Luna}$$

The module is underpinned by two virtual liquidity pools for Terra and Luna, governed by specific equations designed to manage the dynamics of swaps. Adjustments in the pool sizes are denoted by δ, with a mechanism in place to replenish this value, ensuring the pool reverts to its foundational size. This replenishment occurs at a rate defined by the "pool recovery period" parameter (Kwon, 2019).

$$Pool_{Terra} = Pool_{Base} + \delta$$

$$Pool_{Luna} = \frac{\left(Pool_{Base}\right)^2}{Pool_{Terra}}$$

A crucial concept introduced with this upgrade is the "Pool Base," which sets a threshold for the pool size beyond which swapping large quantities becomes economically infeasible due to slippage. This mechanism is designed to deter arbitrage that is not beneficial, though it still permits unrestricted Terra to Luna swaps within practical limits.

Before the Columbus-3 upgrade, the absence of a mechanism for managing reserves effectively meant there was no cap on the volume of off-chain liquidity that could be exchanged within Terra Station. This posed a risk of oracle manipulation attacks, whereby minor fluctuations in the off-chain price of LUNA/UST could be exploited to execute swaps at unrepresentative values within the virtual pool, potentially destabilizing the system.

The implementation of Terra's constant product swap mechanism significantly mitigates this risk. It ensures the cost of potential attacks exceeds their prospective gain by making on-chain liquidity a manageable parameter via governance[1], thus maintaining a strategic imbalance between on-chain and off-chain liquidity levels. Through the introduction of the base pool and adherence to the x * y = k curve, the system inherently increases the slippage—and consequently the cost—faced by potential attackers, bolstering the ecosystem's resilience against manipulation. A brief overview of the pool replenishment process is explained in the following section.

In the evolving landscape of Terra's economic model, the market module parameters had played a critical role in adapting the protocol to its growth and external market forces. Over time, a series of proposals had been passed, reflecting the community's responsiveness and the insights of leading

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[1] Virtual pool's recovery period will stall to quote the same prices provided by oracles until the pool is completely replenished.

market participants, such as Jump Crypto, a top market-maker with a deep understanding of the ecosystem (jp12, 2022). Here’s an integrated overview of the key parameter changes that had shaped Terra's market module:

1. **Proposal 10 (June 7, 2020):** This proposal aimed at increasing the Tobin tax from 0.25% to 0.35%. The primary objective was to mitigate the risk of arbitrage attacks, a measure indicating the community's proactive stance towards maintaining market stability.

2. **Proposal 11 (September 2, 2020)**: With a focus on enhancing liquidity and facilitating smoother swaps between Terra and Luna, this proposal reduced the minimum spread of Terra to Luna swaps from 2% to 0.5%. This change was crucial for improving the user experience and encouraging more transactions within the ecosystem.

3. **Proposal 12 (September 2, 2020):** This proposal significantly increased the basepool value from 250,000 SDR to 625,000 SDR. By enlarging the basepool, the protocol aimed to bolster its stability and resilience against market fluctuations, ensuring a more reliable mechanism for minting and burning operations.

4.**Proposal 18 (November 20, 2020):** Recognizing the need for a more dynamic adjustment mechanism, this proposal reduced the PoolRecoveryPeriod from a 24-hour cycle to an 8-hour cycle (4800 blocks). This adjustment allowed for a quicker response to market changes, enhancing the protocol's flexibility.

5. **Proposal 27 (January 25, 2021):**In response to growing demand and the need for increased minting capacity, this proposal further increased the basepool to 7M SDR and decreased the PoolRecoveryPeriod to 200 blocks, with an estimated minting capacity of $10M. These changes marked a significant step towards scaling the protocol's operations.

6. **Proposal 36 (February 10, 2021):** Continuing the trend of expansion, the basepool was increased to 13M SDR, and the PoolRecoveryPeriod was further decreased to 130 blocks. This adjustment set the estimated minting capacity at $20M, underscoring the protocol's commitment to growth and scalability.

7. **Proposal 90 (May 23, 2021):** With a dramatic increase in the base pool to 32.5M SDR and a decrease in the PoolRecoveryPeriod to 49 blocks, this proposal set an estimated minting capacity of $135M. This substantial enhancement reflected Terra's robust growth trajectory and the need for a larger liquidity pool to support its expanding ecosystem.

8. **Proposal 185 (February 1, 2022):** Marking the latest in a series of strategic adjustments, this proposal expanded the base pool to 50M SDR and reduced the PoolRecoveryPeriod to 36 blocks, with an estimated minting capacity of $293M. This move was indicative of the protocol's continued evolution and its efforts to maintain a balanced and thriving economic environment.

3.4 Columbus-5

This period saw continued refinement in preparation for the Columbus-5 upgrade. Discussions within the community were rife regarding the best approaches to sustain the peg of Terra amidst a growing ecosystem. The proposals considered included adjustments to the seigniorage distribution and the introduction of a burn mechanism for Luna to create a deflationary pressure and support Terra's value. Columbus-5 represented a pivotal change, shifting towards a full seigniorage burn model. This simplified the previous model by removing the budget allocation entirely, intending to create a clear and straightforward mechanism that could easily be audited and understood by users. The deflationary model introduced by burning all seigniorage was aimed at providing a long-term sustainable path for the currency's growth.

4. Replenishment System Overview

The system operates by continually adjusting the number of stable tokens in the virtual pool to align with a predefined baseline quantity, referred to as PoolBase. The adjustment process involves a decay function δ(t), which is intended to gradually bring the stable token quantity back to the baseline over a period defined by PoolRecoveryPeriod (PRP).

The initial design of the decay function guaranteed a decreasing replenishment rate δ(t) with each new block, offering a steady yet gradually diminishing rate of replenishment. This rate was directly linked to the blockchain's block production cycle, which occurred approximately every 6 seconds.

4.1 Two-Fold Purpose of the Replenishment System

Reserve Preservation

By ensuring that δ(n) never hits zero, the system avoids completely depleting its reserves. This is critical for maintaining trust in the stablecoin's value and the overall health of the ecosystem.

Limitation on Withdrawals

The replenishment rate also serves as a deterrent against large, rapid withdrawals that could destabilize the pool. By gradually offering worse rates over time, it disincentivizes large, quick sell-offs of the stablecoin, effectively pacing outflows to maintain stability.

4.2 Detailed Mechanism

The adjustment process for the value of delta, δ(t), which represents the deviation from the pool base, follows a specific decay function that ensures delta never reaches zero. This function is recalculated with every new block, approximately every six seconds. The formula for this adjustment can be shown as:

$$\delta(t) = \delta(t-1) \cdot \left(1 - \frac{1}{PRP}\right)$$

However, with this equation δ(t) would never go down to zero and could hinder the pool's ability to recover from stress scenarios rapidly. Therefore, we suggest an enhanced arbitrary version that allows δ to decrease to zero after a predetermined number of blocks, as defined by the PoolRecoveryPeriod.

4.3 Adaptive Replenishment System Modification

We propose a simple adaptive modification to the system to address the above issue. If the variation of δ exceeds a certain threshold i.e. 5% of the size of the base pool, the system responds by distributing the variation only across the first half of the recovery period or in simpler terms it divides the PoolRecoveryPeriod into half, effectively allowing the virtual pool's redemption capacity to double. This modification enables the system to recover more rapidly from a peg loss, adapting to highly volatile market scenarios. The function δ(t) then becomes:

$$\delta(t) = \delta(t-1) \cdot \left(1 - \frac{2}{PRP}\right)$$

In a crisis scenario where the price of UST deviates significantly from the peg, the adaptive replenishment system ensures that δ returns to zero in half the original time. This system adjustment allows the pool to dynamically manage liquidity shocks and maintain the stablecoin's peg, even under duress[2].

4.4 Balanced Approach

The implementation of an adaptive replenishment system enables the virtual pool to regain its functionality for arbitration processes following a depegging event. However, this approach introduces a vulnerability to manipulation by halving the Pool Recovery Period. This adjustment raises critical considerations regarding the optimal duration of the Pool Recovery Period to maximize the system's redemption capacity effectively.

To address this, it is essential to conduct comprehensive stress testing of the system's redemption module, taking into account various levels of volatility in the Luna market. Such analysis will provide valuable insights into establishing a robust market module. By simulating the stability module across a spectrum of volatilities and Pool Recovery Periods, we can identify a configuration that ensures optimal operational resilience and efficiency.

[2] In the simulations, we test the system under both normal and adaptive replenishment conditions to see if any difference is observed.

5. Model

Our methodology employs a comprehensive approach to construct a digital twin of Terra's economic model. This involves a detailed simulation of various components including swap operations, the establishment of both virtual and real liquidity pools, arbitration mechanics and user balances. These elements work together to emulate the dynamic behavior of Terra's actual system. The primary aim is to accurately simulate the stability mechanisms of the system, integrating specific scenarios that could potentially lead to system collapses.

In our simulation, each step represents a transaction, such as a swap, which alters the system's state by activating all conceivable interactions. These swaps are simulated using a random walk approach, a stochastic process selected for its ability to capture the inherent uncertainty of market behaviors. This stochastic model is pivotal in generating randomness among swaps, allowing for the modeling of a broad spectrum of user activities and how they might influence the stability of the stablecoin.

By adopting this strategy, we aim to delve into the intricacies of Terra's stability mechanisms, exploring how they respond under various conditions and identifying potential vulnerabilities that could lead to instability.

We consider three different cases to test the system’s resilience against peg deviations:

- Normal conditions representing the actual state of the Terra economy
- Adaptive Pool Replenishment case with different PoolRecoveryPeriods and base pool sizes.
- Reserve replenishment case where external protocol reserves are injected to restore the peg instead of any algorithmic adjustments.

5.1 Simulation Framework and Key Assumptions

5.1.1 Reference Benchmark

A critical component of our simulation model is the establishment of an absolute reference point for token prices. In this study, the US Dollar (USD) has been chosen as this reference, providing a stable benchmark against which the fluctuations of the algorithmic stablecoin’s price can be measured and analyzed. The choice of the USD as a reference point is instrumental in ensuring the clarity and relevance of the simulation outcomes, facilitating a direct comparison with real-world currency values.

5.1.2 Assumptions

The simulation's foundational assumptions are twofold and warrant explicit acknowledgment due to their significance in the modeling process. Firstly, the operation of the simulation in discrete time frames underscores the methodology, with each iteration representing a distinct transaction event. This discrete temporal structure is essential for capturing the dynamic nature of token exchanges and their effects on market stability. Secondly, the anchoring of the simulation’s pricing mechanism to the US

Dollar serves as the cornerstone of our model. This assumption provides a consistent and universally recognized value standard, enabling a coherent analysis of the algorithmic stablecoin's performance in simulated environments.

These elements combined—discrete time simulation, the importance of swap volumes, the observance of emergent behaviors, and the US Dollar as a price reference—form the foundation of our approach to modeling the complexities of algorithmic stablecoin stability. Through this framework, the research aims to unveil the intricate mechanics of stablecoin stabilization and the potential for systemic risks within the digital currency ecosystem.

5.1.3 Liquidity Pool Dynamics

Within our simulation model, three distinct liquidity pools are established to encapsulate the market interactions between different token pairings. The pools are as follows: one containing the pairing of Luna and USDC ($\Pi volatile$), another with Terra and USDC ($\Pi stable$).. These pools operate under the constant product market maker model, commonly utilized in decentralized exchanges like Uniswap.

In the stable and volatile pools 'xs' and 'xv' represent the base assets respectively. To establish an absolute reference point for the price of Terra, we use `USDC` `yu` as the quote asset in each of the pools. The state of these liquidity pools based on the constant product model (x .y = k) at each iteration `i` can then be defined as:

$$\Pi_{stable(i)} = x_{s(i)} \cdot y_{u(i)} = k(i)$$

$$\Pi_{volatile(i)} = x_{v(i)} \cdot y_{u(i)} = k(i)$$

5.1.4 Virtual Pool Dynamics

Next, to mimic the market module of Terra a virtual pool ($\Pi virtual$) is initialized containing both `xs` and `xv` i.e. Terra and Luna tokens along with a `Pool_Base` which is the amount of Terra SDR the pool starts with. The state of the virtual pool in terms of Terra and Luna can then be defined as:

$$x_s(i) = Pool_{Base} + \delta$$

$$x_v(i) = \frac{(Pool_{Base})^2}{x_s(i)}$$

Here δ represents the difference between the current quantity of Terra tokens at iteration (i) and the initial quantity `Pool_Base`.

5.1.5 User Wallet Balances

User wallets are initialized with balances `B` and are modeled using an exponential distribution with a parameter`(λ)`, indicating the rate of balance changes, a higher `λ` results in higher balance variations.

$$f(B;\lambda) = \begin{cases} \lambda e^{-\lambda B} & B \geq 0 \\ 0 & B < 0 \end{cases}$$

The choice to model balances with an exponential distribution assumes that changes in balances (due to transactions) happen independently and at a constant rate on average. It also implies an understanding that while most users may have relatively small balances, there's a long tail of the distribution representing a small number of users with significantly larger balances.

5.1.6 Swap Mechanics

The direction of the swap is modeled through a stochastic approach i.e. via a random walk. The system is initialized with a probability of `0.5` which represents an equilibrium condition.

For example in a liquidity pool, Π(a,b), the probability of purchasing token a is `p` and the probability of purchasing token b is `(1-p)`.

To implement the random walk, a 'Δ' variation is added to the probability at each iteration to incorporate market volatility. Delta follows a normal probability distribution with 'µ' initialized to 0 and standard deviation 'σ' initialized as a hyper parameter of the system representing market volatility. The new probability at each iteration (i) is then calculated as:

$$P_{new} = P_{old} + \Delta$$

Each swap conducted in the liquidity pool requires two inputs:

- The swap direction (explained in the previous section)
- The swap quantity `q`

The quantity of the tokens swapped is again a random variable with a truncated normal distribution, with a mean ('µ') of 0 and a standard deviation ('σ') calculated as:

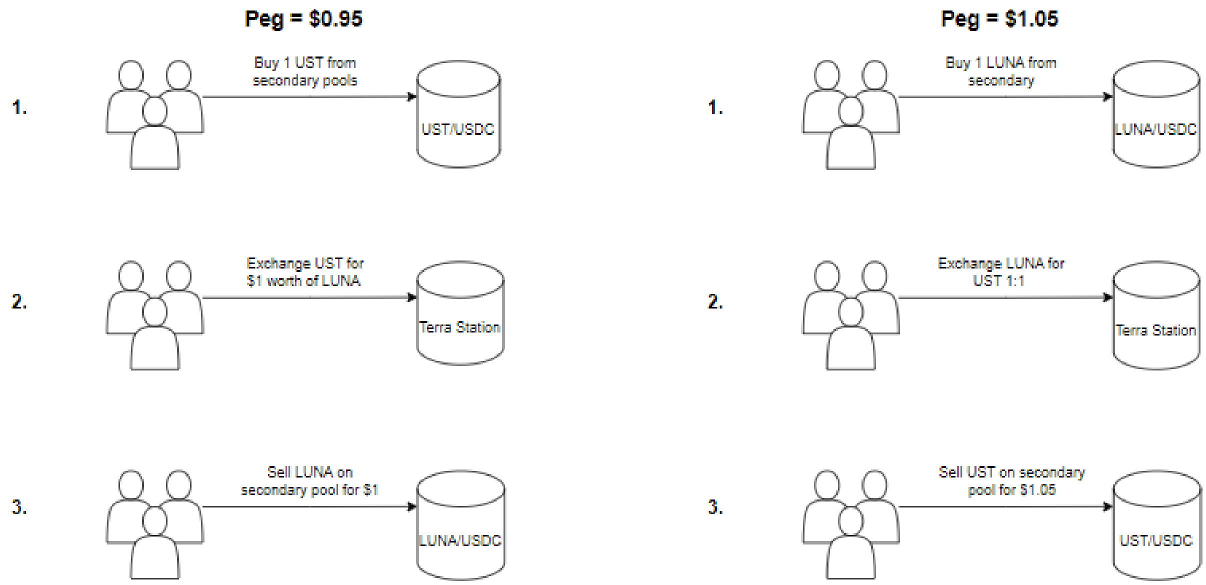
$$\sigma_q = B * (\sigma_\Delta * \phi)$$ [3]

The value of 'B' gives a range to the quantity of tokens swapped; it indicates that the upper cap is limited by the wallet balance. ' ϕ' is a constant that represents extreme market conditions where the price of UST deviates more than 5% from the peg.

5.1.7 Simulation Overview

The model comprises two distinct liquidity pools ($\Pi stable$, $\Pi volatile$) and a virtual pool ($\Pi virtual$). Each iteration of the model simulates a stochastic swap event within the stable and volatile pools, driven by the prevailing market price of the stable token, exemplified here by UST.

When the market price of UST deviates below its intended $1 peg, an arbitrage opportunity emerges. Participants can exploit this by purchasing UST at a discount from the $\Pi stable$. This UST can subsequently be traded for the volatile token (LUNA in this context) at Terra station, which guarantees a 1:1 exchange rate. The acquired LUNA can then be sold in the $\Pi volatile$, capturing a profit from the price difference between the pools. A similar arbitrage process applies when UST's price exceeds the $1 peg, but the order is reversed. Both the cases can be seen in the following figure:



After the swaps take place, the replenishment system of the virtual pool adjusts its reserves based on executed swaps, factoring in the Pool Recovery Period and the base pool's size.

[3]'σΔ' is a **hyper parameter** initialized at the beginning of the simulation and is increased after a certain number of iterations to imitate a scenario where volatility of the system keeps increasing with time.

6. Findings

In our study, we define "normal conditions" as the operational state of the Terra system before any collapse occurred. Through extensive analysis involving 30 simulations, each consisting of 100,000 iterations, we have identified a significant trend in the system's performance. Our findings indicate that under these conditions, the Terra economic model experiences a failure rate of approximately 80%, with the peg dropping below $0.5 in 24 out of 30 simulations. The average point of failure across these instances was identified at iteration number 78,284, highlighting a pronounced vulnerability within Terra's economic framework.

Following this, we applied an adaptive pool replenishment strategy in a series of stress tests on the system. The outcomes from these tests revealed an enhanced ability of the system to uphold the stablecoin's peg above $0.5 in 9 out of the 30 simulations. This improvement indicates a certain degree of resilience under specific market conditions. Interestingly, the average iteration number at which failures were observed during these simulations was 77,571, closely mirroring the results obtained under normal operational conditions. Therefore, while the adaptive replenishment strategy offers some improvements, there remains a fundamental instability that needs addressing.

To check if there is an inherent vulnerability in the algorithmic design of Terra stablecoins against stress tests, we take a reserve replenishment strategy where emergency reserves of the protocol are used to re-peg in case of extreme volatility. This approach involves deploying the protocol's emergency reserves to re-establish the stablecoin's peg whenever its price significantly deviates below the target. Essentially, the reserve pool serves as a financial safeguard, underpinning the stability of the system by facilitating the automatic repurchase of the stable token during critical downturns.

Our findings, based on 30 targeted stress test scenarios, indicate a high degree of robustness within this strategy. Specifically, the system successfully maintained its stability in 90% of the cases—that is, in 27 out of 30 scenarios—demonstrating its effectiveness in mitigating the impacts of severe market fluctuations. The average iteration at which failures occurred, when they did, was notably high, at 89,772, underscoring the strategy's capacity not only to prevent destabilization but also to prolong the duration before any potential depegging events might occur.

Despite the efficacy of using external collateral as a protective measure during instances of extreme market volatility, it's important to recognize that such collateral acts primarily as a safety net rather than a foundational stability mechanism for Terra stablecoins, which are algorithmic by design. The core stability mechanism for the protocol, both in principle and intended function, is predicated on the arbitrage dynamics facilitated by the Terra virtual pool.

The examination of the virtual pool's operational mechanics indicates notable constraints in its stabilization capabilities. Specifically, with a foundational pool of 50 million Special Drawing Rights (SDR) and an estimated capability for minting $293 million, combined with a recovery interval of 36 blocks, the framework was insufficiently provisioned to offset the daily net outflows originating from the Anchor protocol (credmark, 2022), which surpassed the virtual pool's minting potential. This inadequacy can be partially ascribed to the procedural delays inherent in governance-led parameter adjustments, as changes implemented through governance mechanisms require time to manifest their full impact. An algorithmic approach to parameter modification could potentially offer a more dynamic solution, enabling real-time adjustments to the arbitration mechanics. Such an approach could effectively address and mitigate the escalating sell-off pressures encountered during instances of de-pegging.

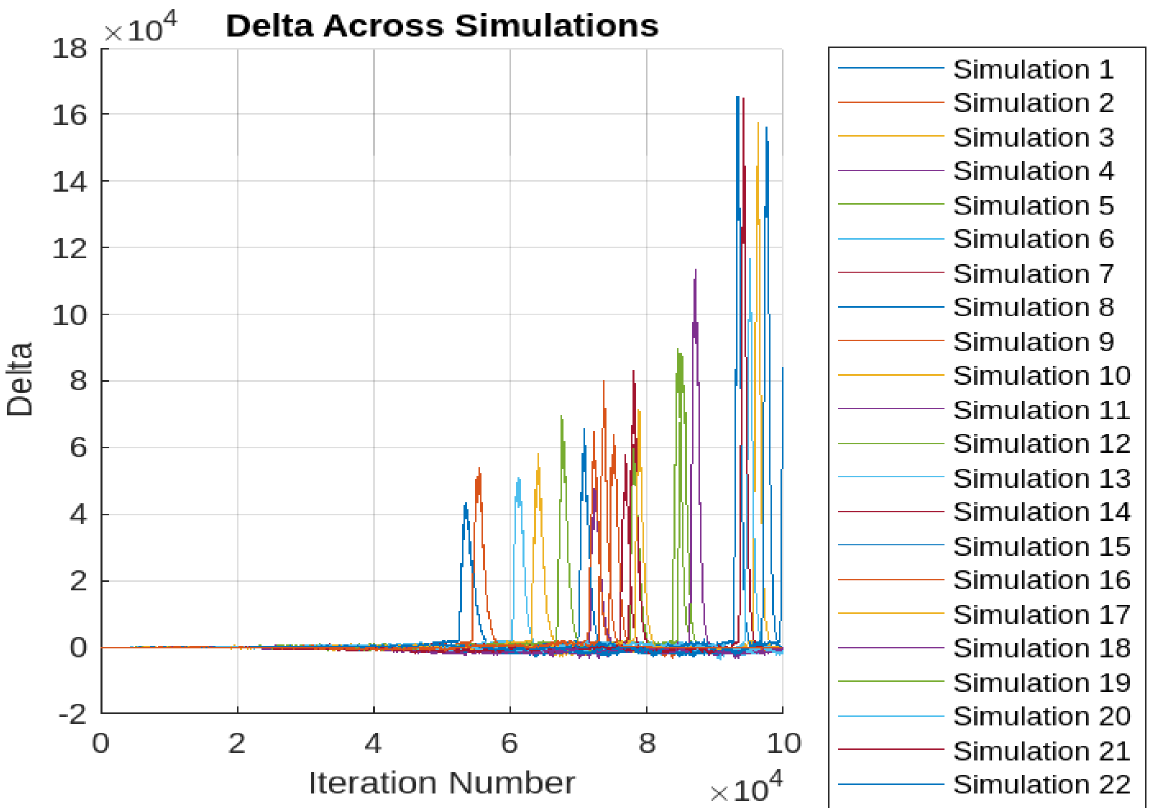
7. Appendix

Simulation	Original	Reserve Pool	Replenishing
01	53166	79270	77726
02	74665	Price never falls below $0.5	82688
03	Price never falls below $0.5	Price never falls below $0.5	71798
04	72076	Price never falls below $0.5	57702
05	84056	Price never falls below $0.5	Price never falls below $0.5
06	Price never falls below $0.5	Price never falls below $0.5	82679
07	76472	Price never falls below $0.5	96439
08	70361	91754	Price never falls below $0.5
09	Price never falls below $0.5	Price never falls below $0.5	78695
10	95739	Price never falls below $0.5	88716
11	Price never falls below $0.5	Price never falls below $0.5	61509
12	84693	Price never falls below $0.5	Price never falls below $0.5

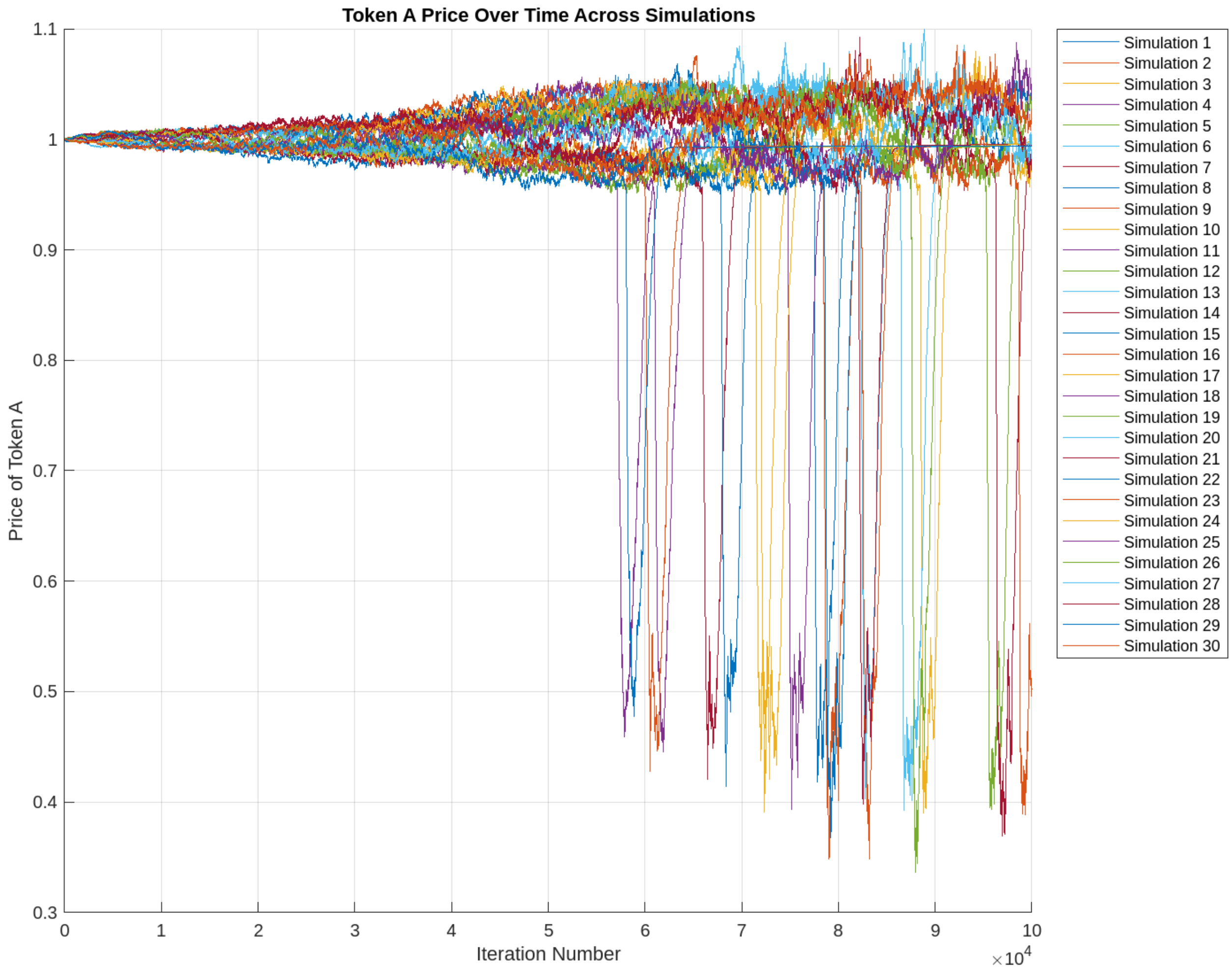
13	60700	Price never falls below $0.5	86691
14	77654	98292	823933
15	92929	Price never falls below $0.5	78864
16	54844	Price never falls below $0.5	60470
17	63561	Price never falls below $0.5	Price never falls below $0.5
18	86509	Price never falls below $0.5	75028
19	77761	Price never falls below $0.5	95542
20	94642	Price never falls below $0.5	Price never falls below $0.5
21	77604	Price never falls below $0.5	66331
22	97068	Price never falls below $0.5	68245
23	71823	Price never falls below $0.5	98761
24	78520	Price never falls below $0.5	72227
25	Price never falls below $0.5	Price never falls below $0.5	Price never falls below $0.5
26	67274	Price never falls below $0.5	87768
27	Price never falls below $0.5	Price never falls below $0.5	Price never falls below $0.5
28	93709	Price never falls below $0.5	Price never falls below $0.5
29	99769	Price never falls below $0.5	58721

30	73225	Price never falls below $0.5	Price never falls below $0.5
Total Collapses	**24**	**3**	**21**
Mean Collapse	**78284**	**89772**	**77571**

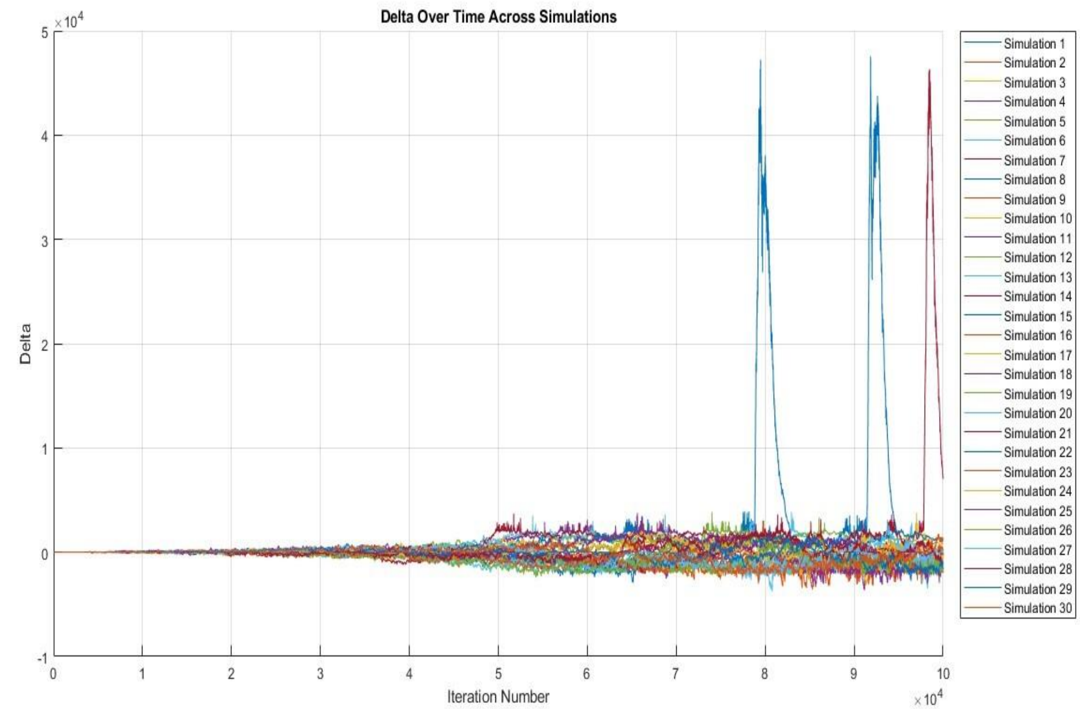
Table 1: Model Stress Test



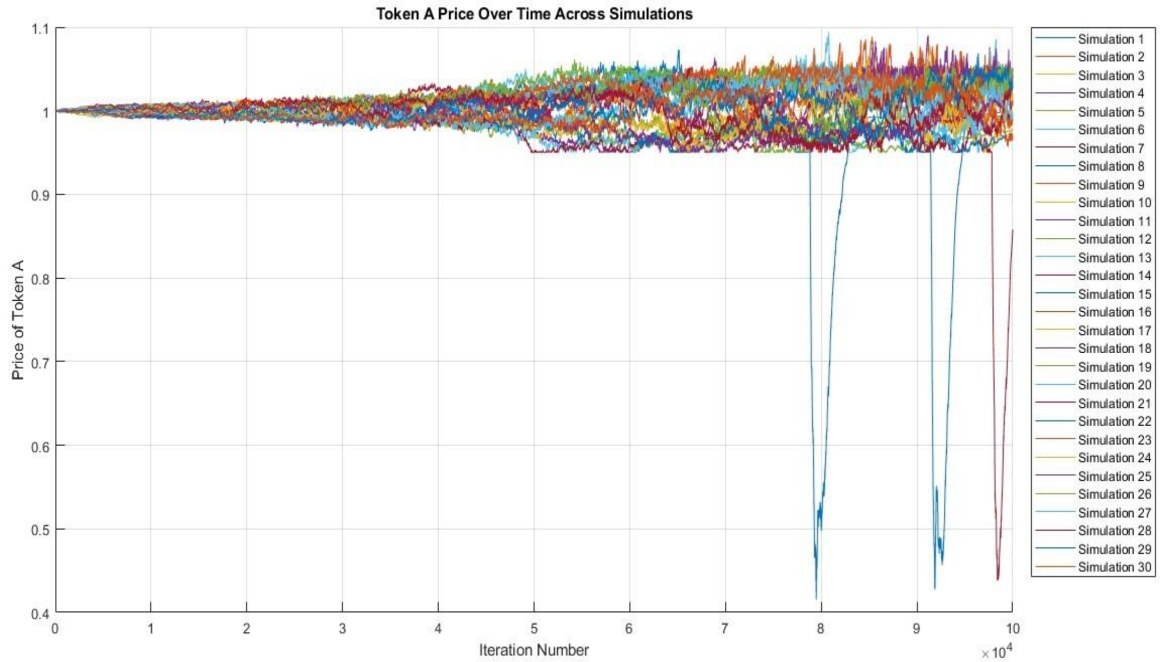
The graph is a visual representation of the dynamic adjustment of the delta parameter under normal conditions across multiple simulations, which is managed by a decay function recalculated at brief, regular intervals to prevent delta from reaching zero. The variability across simulations could imply sensitivity to initial conditions or the influence of random factors within the model.



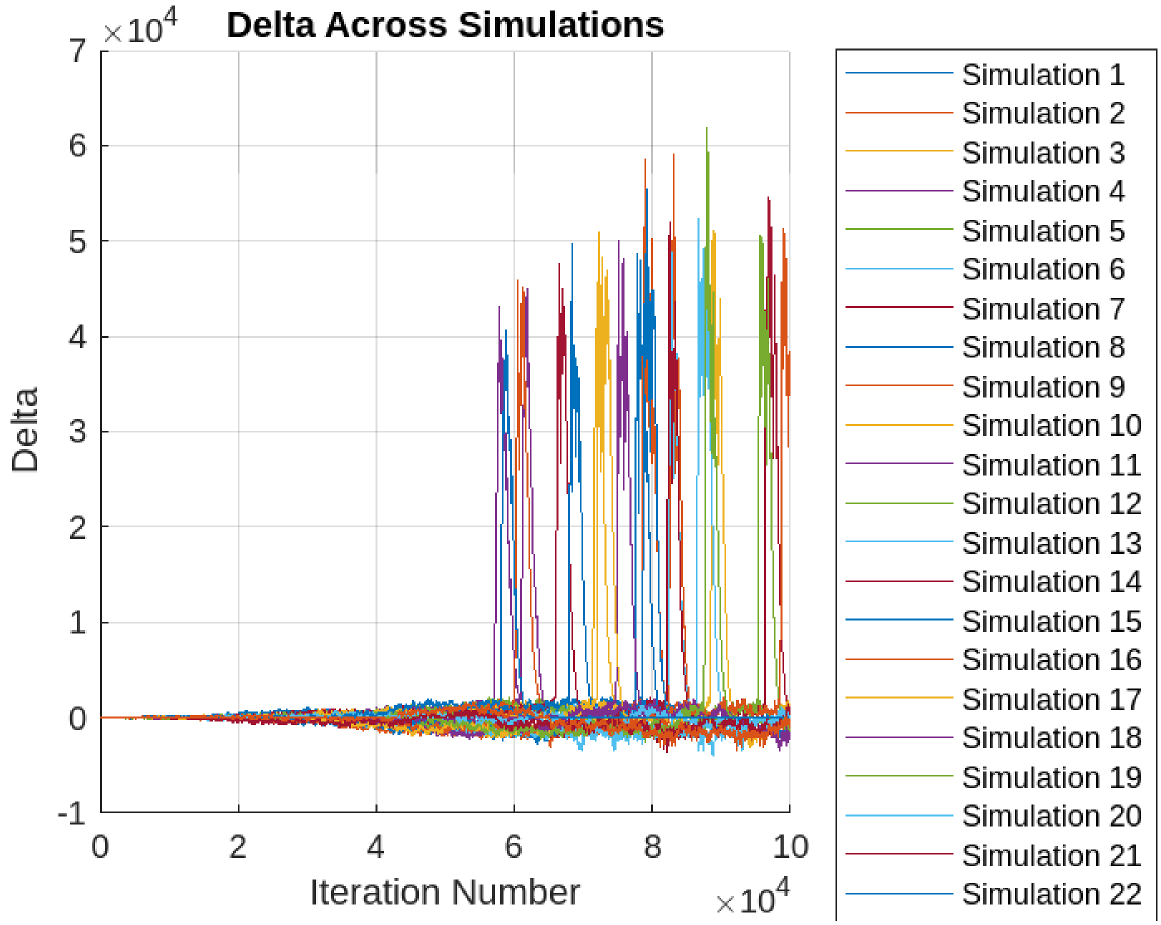
The graph titled "Token A Price Over Time Across Simulations" shows the price behavior of Token A across 30 simulations under normal conditions. Initially, the price is stable, hovering around 1. However, as iterations increase, all simulations exhibit sharp price declines, with some dropping as low as 0.2. These patterns suggest that Token A can experience significant volatility under certain conditions. The iteration count, reaching into tens of thousands, implies a comprehensive analysis of the token's price resilience and reaction to different simulated market scenarios.



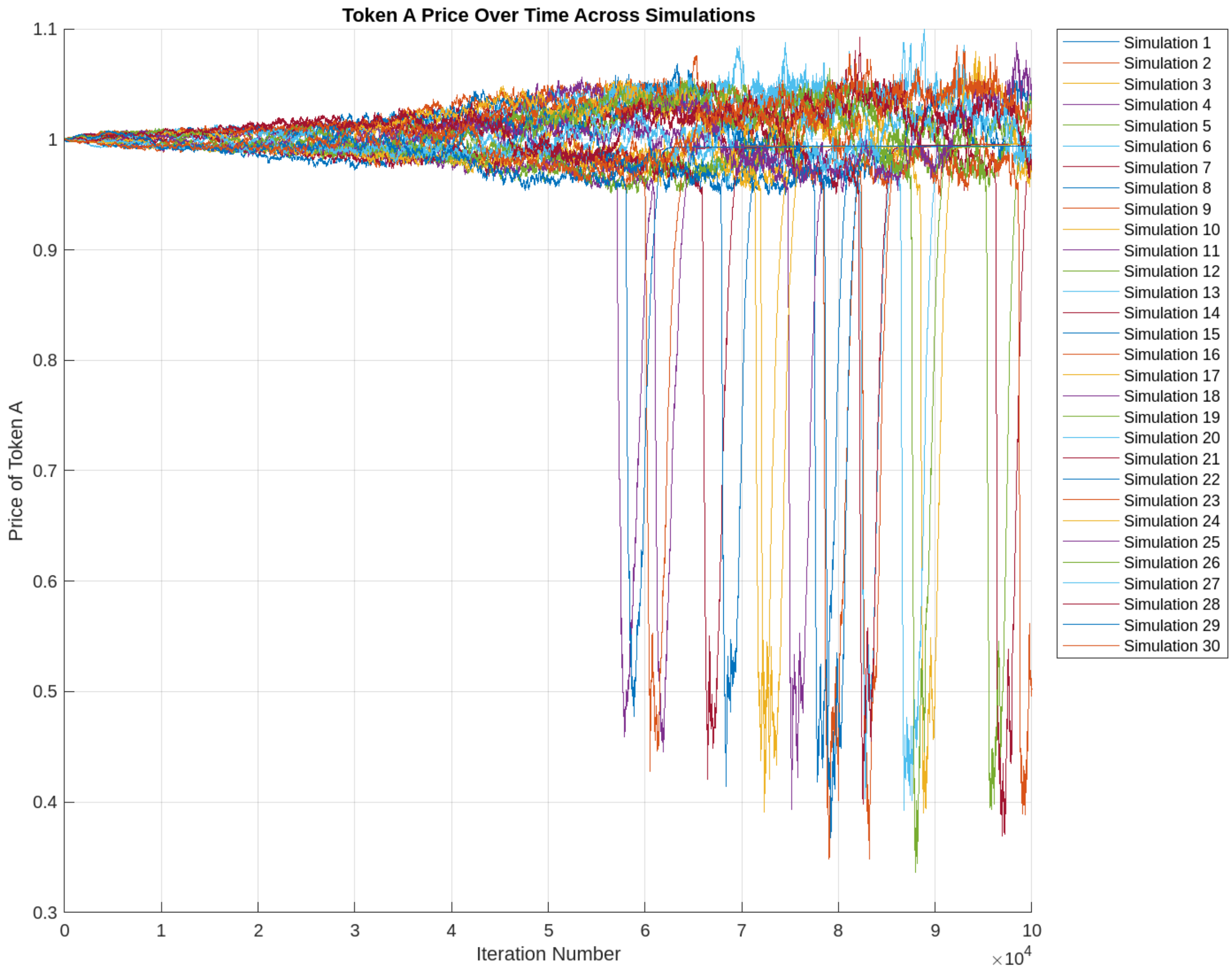
The graph "Delta Over Time Across Simulations" represents the delta parameter in the context of reserve interventions for maintaining a stablecoin's peg. Across 30 simulations, delta remains close to zero for most iterations, indicating minimal deviation from the target peg. However, occasional spikes in delta suggest that there are moments when the reserves must intervene significantly to maintain stability.



The graph "Token A Price Over Time Across Simulations" shows the price stability of a stablecoin and the effects of reserve interventions. The price remains close to 1 in most cases, indicating stability, but shows occasional sharp declines, suggesting market stress where reserves are used to maintain the peg. These interventions are infrequent but crucial to stabilizing the token's value across 30 different simulations.



The graph titled "Delta Across Simulations" shows the adjustment of delta values, representing the deviation from a baseline in a pool of assets across 30 simulations of the adaptive pool replenishment case. The delta values fluctuate minimally around zero for the majority of iterations, indicating stable conditions. Periodically, sharp spikes in delta are observed, suggesting corrective actions or rebalancing efforts within the system to maintain the pool's stability. These adjustments follow a decay function designed to prevent delta from reaching zero, signifying a mechanism that avoids complete depletion or convergence.



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