



Mechatronics Engineering Project

Sorting Object System

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Introduction

The shift towards industrial automation is a cornerstone of modern industry, aiming to enhance competitiveness, increase operational efficiency, and ensure the highest product quality standards.

In this context, automated sorting systems are of paramount importance across various industrial sectors, particularly those requiring precision and speed in processing.

Manual and traditional methods of product sorting are characterized by low productivity and a high probability of human error, leading to increased waste and product damage, and consequently, significant economic losses for industrial facilities.

Therefore, the urgent need has arisen for the design and development of advanced automated systems capable of providing effective and reliable engineering solutions to address these challenges.

Summary of References

A-Design of Arduino Uno-Based Automatic Object Sorter Belt Conveyor System Using Ultrasonic Sensors:

1-Objective and Main Idea: The objective is to automate the sorting of packages and objects based on their height. This system was designed to overcome the issues of time, cost, and labor-intensive manual sorting and transport in small and medium industries.

2-Components and Methodology:

- **Key Components:** The system is based on the Arduino Uno as the microcontroller, an Ultrasonic sensor for measuring object height, Servo motors for sorting, and a Belt Conveyor.
- **Process:** The ultrasonic sensor detects the height (distance between the sensor and the object) as the object passes on the conveyor belt. Based on this reading, the Arduino determines the object's category. The servo motors then open the path to push the object into its designated area.

3- Key Results:

- **Sorting Success:** Objects were successfully sorted into Area A (for 12 cm height) and Area B (for 15 cm height), with unread objects running to Area C "Reject".
- **Minimum Continuous Distances:** To avoid errors during continuous sorting, the minimum distance between sequential boxes was determined to be:

For Box A (12 cm): Minimum distance of 30 cm.

For Box B (15 cm): Minimum distance of 70 cm.

E_ Object Sorting System Using Robotic Arm

This project, "Object Sorting System Using Robotic Arm," details the design and implementation of an automated industrial solution aimed at increasing manufacturing speed and accuracy by replacing manual labor in monotonous sorting tasks. The system utilizes an Arduino Uno microcontroller as its brain, coordinating a 6DOF Robotic Arm to perform "Pick and Place" operations and a Pixy CMUcam5 camera for intelligent color identification (color signature). The process begins with objects placed on a conveyor belt which stops when the camera detects an

object. The camera identifies the color, sends the data to the Arduino, which then executes the sorting algorithm and commands the robotic arm to move the object to its designated color-coded bin. The expected result is a fully built system that successfully sorts objects of different colors. Future work includes expanding the system's capabilities to sort by other features like shape and size, and developing a specialized mobile application to assist colorblind individuals.

B-Mathematical Modeling and Control of DC Motor:

This document focuses on developing the mathematical model of a DC motor to relate the applied armature voltage (V_a) to the motor's angular velocity (ω_a). This modeling is essential for simulation and control system design.

1. The Fundamental Mathematical Model:

- The model is derived from two primary balance equations: one for the electrical characteristics and one for the mechanical characteristics.

A. Electrical Characteristics:

Applying Kirchhoff's Voltage Law around the equivalent electrical circuit of the armature:

$$V_a - i_a R_a - L_a \frac{di_a}{dt} - K_v \omega_a = 0$$

Where: V_a is the Applied Voltage, i_a is the Armature Current, R_a is the Armature Resistance, $L_a \frac{di_a}{dt}$ is the voltage across the Inductance, and $K_v \omega_a$ is the Back EMF (V_c).

B. Mechanical Characteristics:

Performing a torque balance on the rotating system:

$$K_t i_a - J \frac{d\omega_a}{dt} - B\omega_a - T_L = 0$$

Where: $K_t i_a$ is the Electromagnetic Torque (T_e), $J \frac{d\omega_a}{dt}$ is the Acceleration Torque, $B\omega_a$ is the Friction Torque, and T_L is the Mechanical Load Torque.

2. Motor Control: Closed-Loop System:

The closed-loop system is designed to ensure Reference Tracking for both speed (ω_a) and position (angle).

PI Controller Implementation: A Proportional-Integral (PI) Controller is used to adjust the motor's dynamic response, ensuring the speed or position reaches the desired reference quickly.

Importance of Control: The controller is essential for Load Rejection, ensuring the motor maintains its set speed even when a sudden mechanical load is applied.

C-MODELING DC SERVOMOTORS CONTROL SYSTEMS TECH NOTE

This summary focuses on deriving the Transfer Function ($G_m(s)$) which links the input control voltage (V_a) to the motor's angular displacement (θ_m), representing the essential modeling tool for your object sorting line project.

1. The Complete Motor Transfer Function (Full Model)

This mathematical formula represents the motor system as a Third-Order system (by considering the armature inductance L_a):

- Transfer Function from Voltage to Angular Displacement:

$$G_m(s) = \frac{\theta_m(s)}{V_a(s)} = \frac{K_t}{L_a J_e S^3 + (R_a J_e + L_a D_e) S^2 + (R_a D_e + K_t K_b) S}$$

2. Fundamental Equations of Motion (Time Domain)

The mathematical modeling process relies on deriving three fundamental equations of motion in the Time Domain:

a. Motor Torque Equation: The resulting torque (T_m) is shown to be directly proportional to the armature current (I_a), where K_t is the torque constant.

$$T_m(t) = K_t \cdot I_a$$

b. Back EMF Equation: This force is directly proportional to the angular velocity of the motor shaft.

$$V_b(t) = K_b \cdot \frac{d\theta_m}{dt}$$

(Where K_b is the back EMF constant).

c. Mechanical Motion Equation (Newton's Law): This equation is written in the Laplace domain by summing the forces acting on the shaft, using the equivalent inertia and damping coefficients (J_e, D_e).

$$T_m = (J_e \cdot S^2 + D_e \cdot S) \cdot \theta_m$$

3. System Representation using the Feedback Model

Deriving the transfer function requires using Ohm's Law for impedances to write the armature current (I_a) equation in the Laplace domain:

$$I_a(s) = \frac{V_a(s) - V_b(s)}{R_a + s L_a}$$

The motor's block diagram shows that the back EMF (V_b) acts as a Negative Feedback signal to the applied voltage (V_a). This allows the transfer function to be simplified into the standard form:

$$G_m(s) = \frac{\theta_m(s)}{V_a(s)} = \frac{G}{1 + G \cdot H}$$

Important Note: The plus sign (+) in the denominator is evidence that the motor system uses Negative Feedback, which ensures the system's stability and controllability

D-DC motors: dynamic model and control techniques

The reference explains the dynamic modeling of DC motors and the relationships between magnetic field, voltage, and current.

It discusses the physical principles behind force and torque generation in the motor using Lorentz's law and the Back EMF effect.

It also presents the electrical and mechanical equations that describe motor performance, clarifying the relationship between voltage, current,

Key Equations:

The main relationships for an ideal motor are:

- **Stator (Field) Electric Circuit (governs flux generation):**

$$V_e = L_e * \frac{di_e}{dt} + R_e i_e$$

- **Rotor (Armature) Electric Circuit (incorporates back EMF):**

$$v_a(t) = L_a * \frac{di_a}{dt} + R_a i_a + e$$

Motor Torque(T_m) (proportional to field and armature current):

$$T_m = k * i_e * i_a$$

- **Back Electromotive Force (e) (proportional to field current and angular speed ω):**

$$e = k i_e \omega$$

- **Mechanical Equation (relates net torque to inertia and friction):**

$$T_m - T_L = J \frac{d\omega}{dt} + F\omega$$

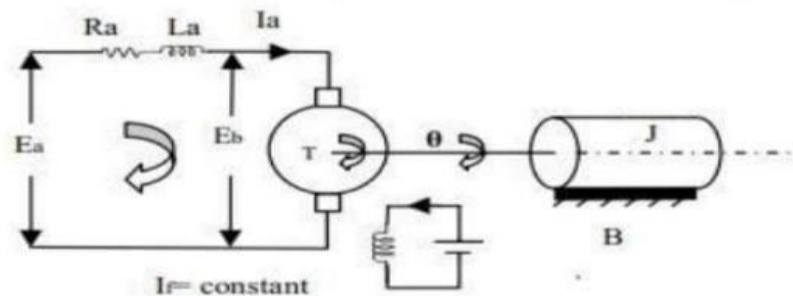
- v_e, i_e : Stator voltage and current.
- v_a, i_a : Armature voltage and current.
- L_e, R_e : Stator inductance and resistance.
- L_a, R_a : Armature inductance and resistance.
- T_L : Load torque.
- J : Rotor inertia, F : Viscous friction coefficient.
- K : A motor constant related to flux and geometry.

Results and Conclusions :

The study shows that a DC motor can be represented by either a linear or nonlinear dynamic model depending on the control method. It demonstrates that the Back **EMF** acts as a natural negative feedback that helps stabilize the system.

E-System Identification for a Mathematical Model of DC Motor System :

Mathematical modelling will be done in general with the help of the DC motor. As previously stated, the DC motor system is divided into two parts: electrical and mechanical. The DC motor circuit is depicted in the diagram below.



Kirchoff's voltage law (KVL) may be used to represent the electrical part based on Fig. 1, which comprises a voltage source, armature resistance, armature inductance, armature current, and the back emf voltage. The equation for the electrical portion can be derived using these parameters and the KVL as shown below.

$$E_a = R_a I_a + \frac{di_a}{dt} + E_b \quad (1)$$

I_a = armature current (A)

E_b = back emf voltage (V)

E_a = voltage source (V)

R_a = armature resistance (Ω)

L_a = armature inductance (H)

As a result, the back emf, E_b which is directly proportional to the shaft's angular velocity, θ by a constant factor, K_b may be stated as follows.

$$E_b = k_b \frac{d\theta}{dt} \quad (2)$$

The following may be deduced from the mechanical part of electric motor:

$$J_m \frac{d^2\theta}{dt^2} + B_m \frac{d\theta}{dt} = T_m \quad (3)$$

The rotor moment of inertia J_m the frictional coefficient B_m and the motor's torque T_m are all specified in the equation above.

$$T_m = k_t I_a \quad (4)$$

The torque of the motor is only proportional to the armature current, I_a by a constant ratio, K_t

The substitution approach will be applied to (1) and (2) generated from the preceding (3). By adding (2) into (1), we obtain a new equation, (5).

$$E_a = R_a I_a + \frac{di_a}{dt} + k_b \frac{d\theta}{dt} \quad (5)$$

Then, substitute (4) into (3) which will create (6).

$$J_m \frac{d^2\theta}{dt^2} + B_m \frac{d\theta}{dt} = k_t I_a \quad (6)$$

Both (5) and (6) may be expressed using the Laplace transform.

$$E_a(s) - K_b S \theta(s) = (R_a + SL_a) I_a(s) \quad (7)$$

$$J_m S^2 \theta(s) + B_m S \theta(s) = K_t I_a(s)$$

Next, substitute (8) into (7), then the transfer function from the voltage source, $E_a(s)$ to the output angle $\theta(s)$, directly follows:

G-Ultrasonic Distance Sensor With IO-Link Reference Design:

Key Inferences and Equations for the Project

Distance Measurement and Accuracy Constraint

Resolution ≈ 1 mm (at } 300 kHz

Resolution ≈ 1 cm (at } 58 kHz

- **Distance Range and Operational Trade-off**

The sensor's detection range dictates the physical layout of your sorting line:

Distance Range 10 cm to 30 cm at 300 kHz

OR 30 cm to 5 m (at } 58 KHZ

➤ **H-Application of ultrasonic sensor for measuring distances in robotics**

temperature's effect on the speed of sound to maintain high positional accuracy

- Governing Equation for Distance Measurement (ToF)

$$D = \frac{1}{2} C \Delta T$$

- Accuracy Constraint: Temperature Compensation

$$c \approx 331.3 + 0.606 T$$

- System Speed Constraint (Sampling Rate)

$$T_{\text{measurement}} \approx \frac{2D_m}{c}$$

K_ MODELLING OF IMPULSE FUNCTIONS OF ULTRASONIC SENSORS WHEN TILT

ANGLE OF REFLECTING SURFACE IS CHANGED

Goal: To model the impulse functions of ultrasonic sensors to identify the **tilt angle of a metallic reflecting surface** (from 0^0 to 30)

Methodology: The system modeled the sensor's impulse response using a Transfer Function ($G(s)$) and used the **Hilbert transform** to determine the signal's envelope

Key Finding: The main result was an extraordinarily strong non-linear correlation ($r = -0.9905$) found between two key parameters of the modeled function:

The steady-state value ($y(\infty)$) the complex integral of the impulse function)

The front time (the time parameter of the transient characteristic).

Conclusion: This strong non-linear dependence directly confirms the state reached when the slope vector of the reflecting surface is parallel to the sensor array, providing a critical metric for determining surface tilt and geometry.

M_ Development of Smart Sorting Machine Using Artificial Intelligence for Chili Fertigation Industries

This research developed a smart chili sorting machine using image processing and Artificial Neural Networks (ANN). Images of red and green chili were captured and processed in MATLAB to extract color features, which were used to train the classifier. Training with more samples improved accuracy from 80% to 85%. The system shows good potential for real-time agricultural use, with room for improvement using better cameras or advanced machine-learning methods.

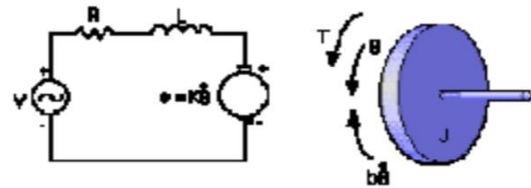
Mathematical Model

A_ DC Motor Model

Electrical System

$$V = iR + L \frac{di}{dt} + E_b$$

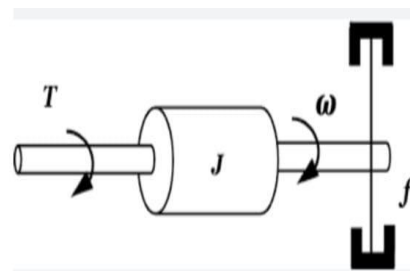
$$V_{(s)} = (R + Ls)I_{(s)} + E_{b(s)} \quad (1)$$



• Mechanical System:

$$T = J\dot{\omega} + D\omega$$

$$T_{(s)} = (Js + D)\omega_{(s)}$$



- **Transfer Function:**

$$\therefore E_{b(s)} \propto \omega(s) \quad \therefore E_{b(s)} = k_b \omega(s)$$

$$\therefore T(s) \propto I(s) \quad \therefore T(s) = k_t I(s) \quad \therefore I(s) = \frac{T(s)}{k_t} = \frac{(JS+D) \omega(s)}{k_t}$$

- **By Substituting into Equation (1)**

$$\therefore V(s) = \left(\frac{(JS + D) \omega(s)}{k_t} \right) (R + LS) + k_b \omega(s)$$

$$\therefore V(s) = \left(\frac{(JS + D)(R + LS)}{k_t} + k_b \right) \omega(s)$$

$$\therefore G(s) = \frac{\omega(s)}{v(s)} = \frac{1}{\left(\frac{(JS + D)(R + LS)}{k_t} \right) + K_b}$$

$$\therefore G(s) = \frac{\omega(s)}{v(s)} = \frac{k_t}{(JS + D)(R + LS) + (k_b k_t)}$$

State space to DC Motor

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

$$v = R_a * I_a + L \frac{dia}{dT} + E_b \quad E_b = k_b * w$$

$$v = R_a * I_a + L \frac{dia}{dT} + k_b * w$$

$$T_T = J\dot{w} + Dw \quad T_T = K_T * Ia$$

$$K_T Ia = J\dot{w} + Dw$$

$$u_T = v$$

$$x_1 = I_a \quad \dot{x}_1 = \frac{di_a}{dT} = \frac{-R_a * I_a - k_b * w}{L} = -\frac{R_a}{L} x_1 - \frac{k_b}{L} x_2 + \frac{1}{L} u_{(T)}$$

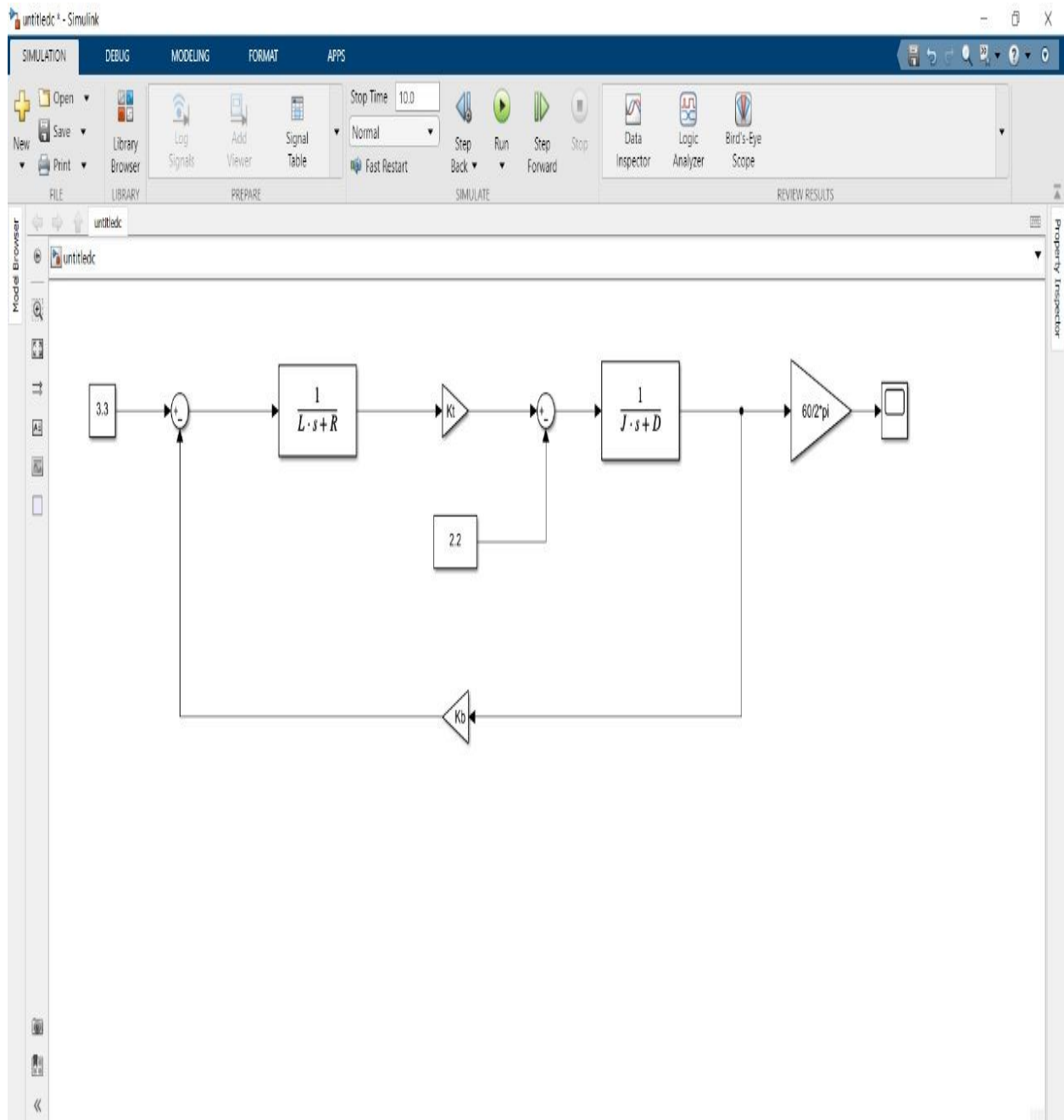
$$x_2 = w \quad \dot{x}_2 = \dot{w} = \frac{k_T * I_a - Dw}{J} = \frac{k_T}{J} x_1 - \frac{D}{J} x_2$$

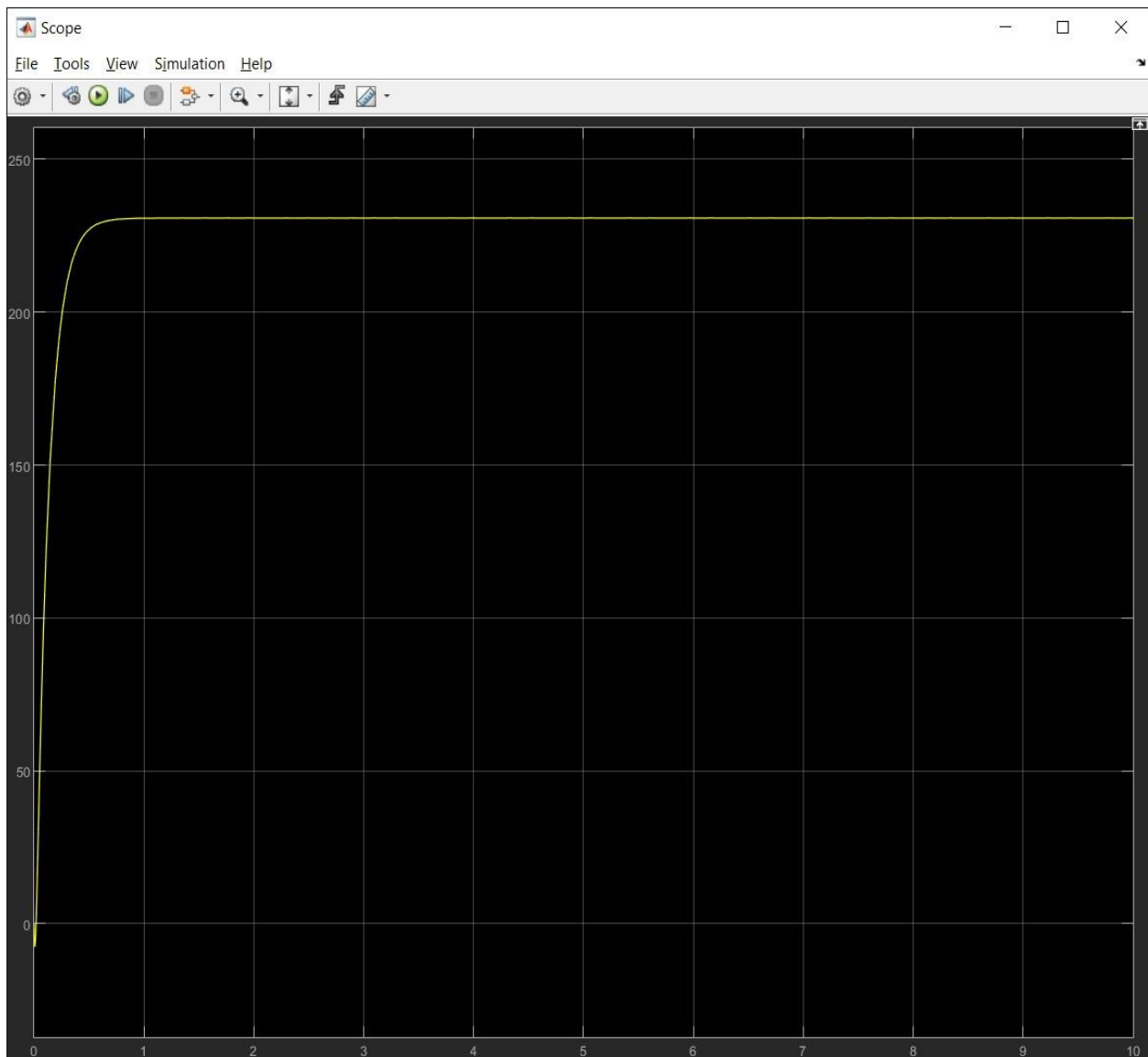
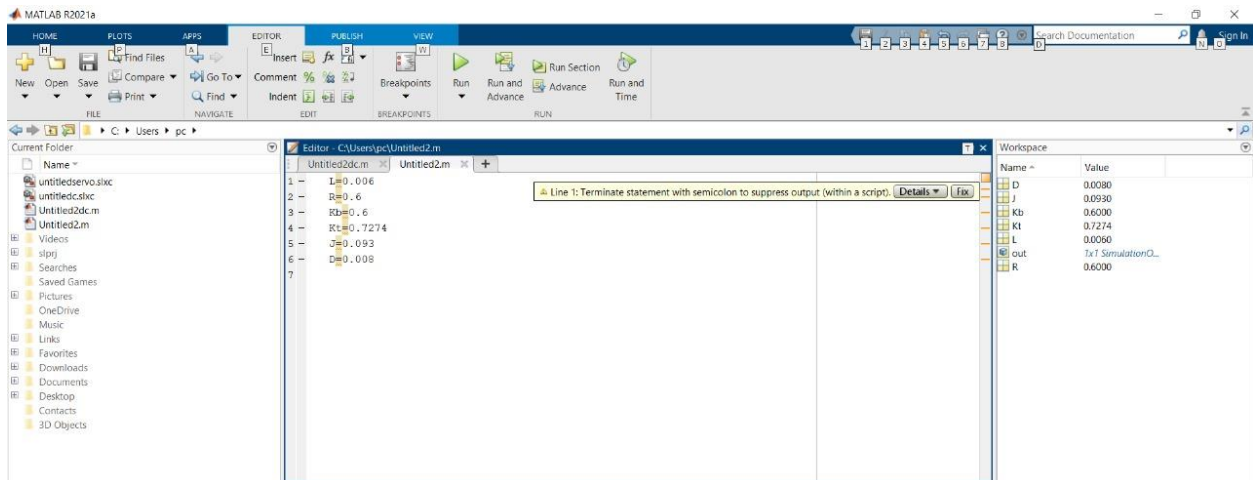
$$y = w = x_2$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \frac{-R_a}{L} & \frac{-k_b}{L} \\ \frac{k_T}{J} & \frac{-D}{J} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} u_{(T)}$$

$$y = [0 \ 1] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Simulink MATLAB





B_DC Servo Motor

Electrical System:

$$e_i(t) = ia(t) + Ra + la \frac{dia(t)}{dt} + eb(t)$$

Symbol	Description
$ei(t)$	<i>applied armature Voltage</i>
$ia(t)$	<i>armature Currnt</i>
Ra	<i>armature Resistance</i>
$La(t)$	<i>armature Inductance</i>
$eb(t)$	<i>Back EMF</i>

—

the back EMF is Proportion to the Motor Angular Velocity

$$eb(t) = ke w(t) = Ke \frac{d\theta}{dt}$$

Ke=Motor Voltage constant back **EMF** Constant

Mechanical System :

$$T(t) = J \frac{d^2\theta(t)}{dt^2} + B \frac{d\theta(t)}{dt}$$

Symbol	Description
$T(t)$	<i>applied Torque</i>
J	<i>Total Moment of Inertia</i>
B	<i>Viscosity friction Coefficient</i>
$\theta(t)$	<i>Angular Position</i>

- **Transfer Function:**

$$\because E_{b(s)} \propto \omega_{(s)} \quad \because \omega_{(s)} = S \theta_{(s)} \quad \therefore E_{b(s)} = k_b S \theta_{(s)}$$

$$\because T_{(s)} \propto I_{(s)} \quad \therefore T_{(s)} = k_t I_{(s)} \quad \therefore I_{(s)} = \frac{T_{(s)}}{k_t} = \frac{(JS^2 + DS)\theta_{(s)}}{k_t}$$

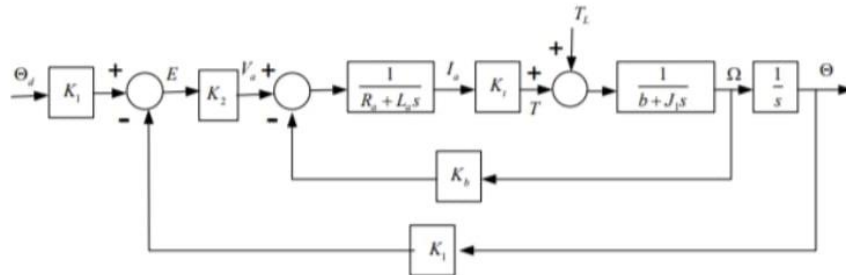
- **By Substituting into Equation (2)**

$$\therefore V_{(s)} = \left(\frac{(JS^2 + DS)\theta_{(s)}}{k_t} \right) (R + LS) + k_b S \theta_{(s)}$$

$$\therefore V_{(s)} = \left(\frac{(JS^2 + DS)(R + LS)}{k_t} + k_b S \right) \theta_{(s)}$$

$$\therefore G_{(s)} = \frac{\theta_{(s)}}{v_{(s)}} = \frac{1}{\left(\frac{(JS^2 + DS)(R + LS)}{k_t} \right) + K_b S}$$

$$\therefore G_{(s)} = \frac{\theta_{(s)}}{v_{(s)}} = \frac{k_t}{(JS^2 + DS)(R + LS) + (k_b k_t S)}$$



State Space to Servo Motor

$$v = RI + l \frac{di}{dt} + k_b \dot{\theta}$$

$$k_t I = J \ddot{\theta} + D \dot{\theta}$$

$$u(t) = v$$

$$x_1 = \theta \quad \dot{x}_1 = \dot{\theta} = x_2$$

$$x_2 = \dot{\theta} \quad \dot{x}_2 = \ddot{\theta} = \frac{k_t I - D \dot{\theta}}{J} = \frac{k_t}{J} x_3 - \frac{D}{J} x_2$$

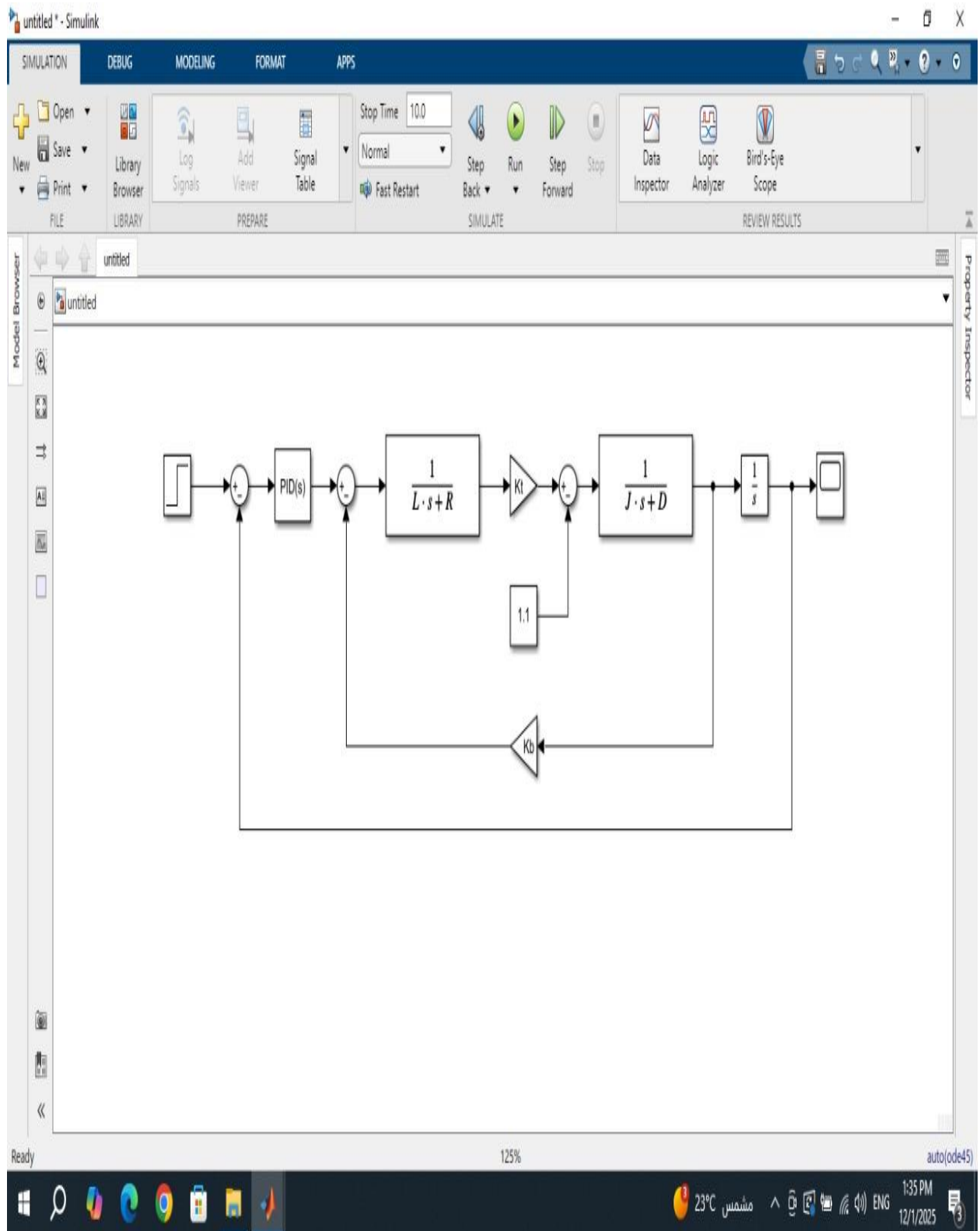
$$x_3 = I \quad \dot{x}_3 = \dot{I} = \frac{v - RI - Kb \dot{\theta}}{l} = \frac{1}{l} u(t) - \frac{R}{l} x_3 - \frac{Kb}{l} x_2$$

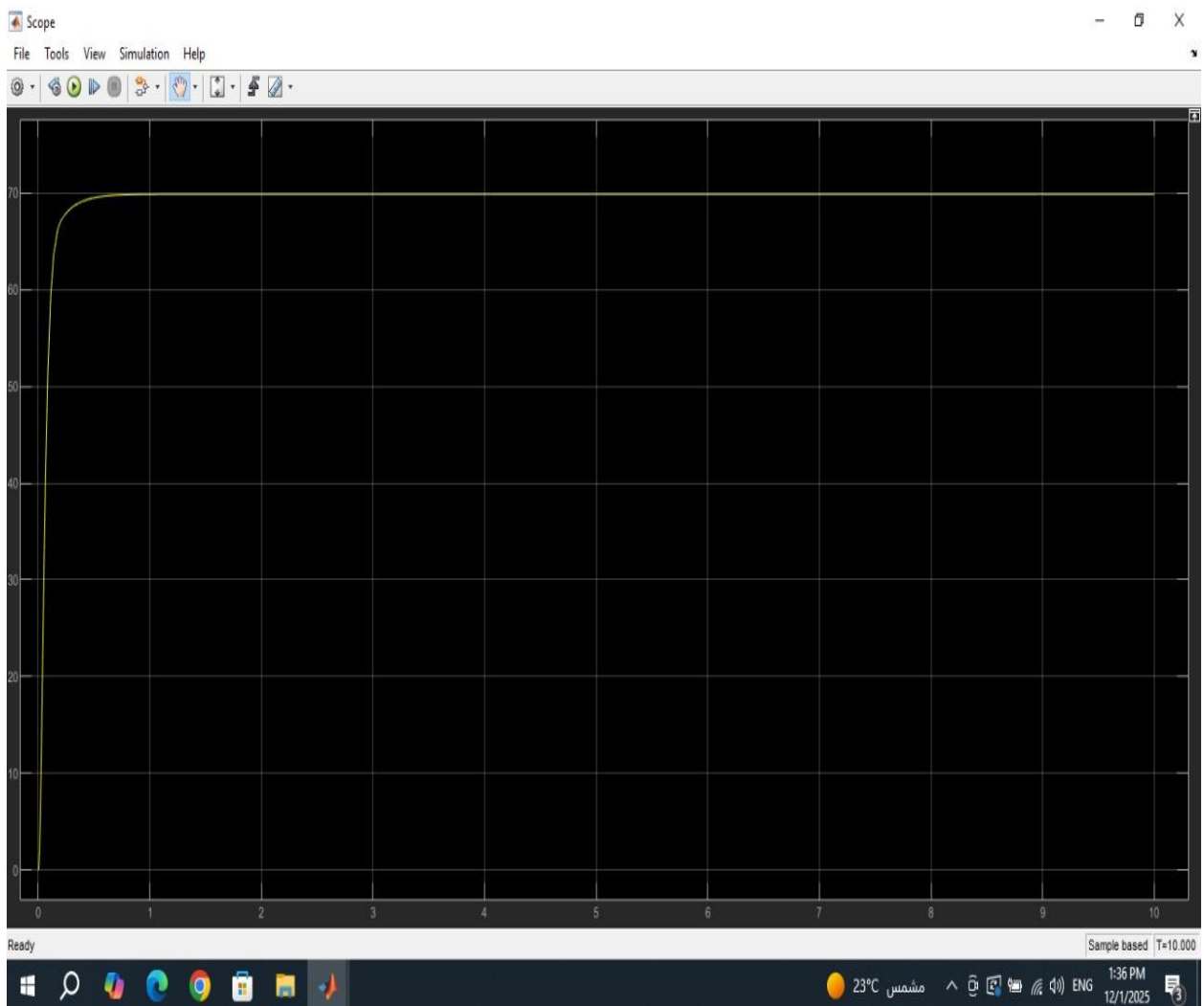
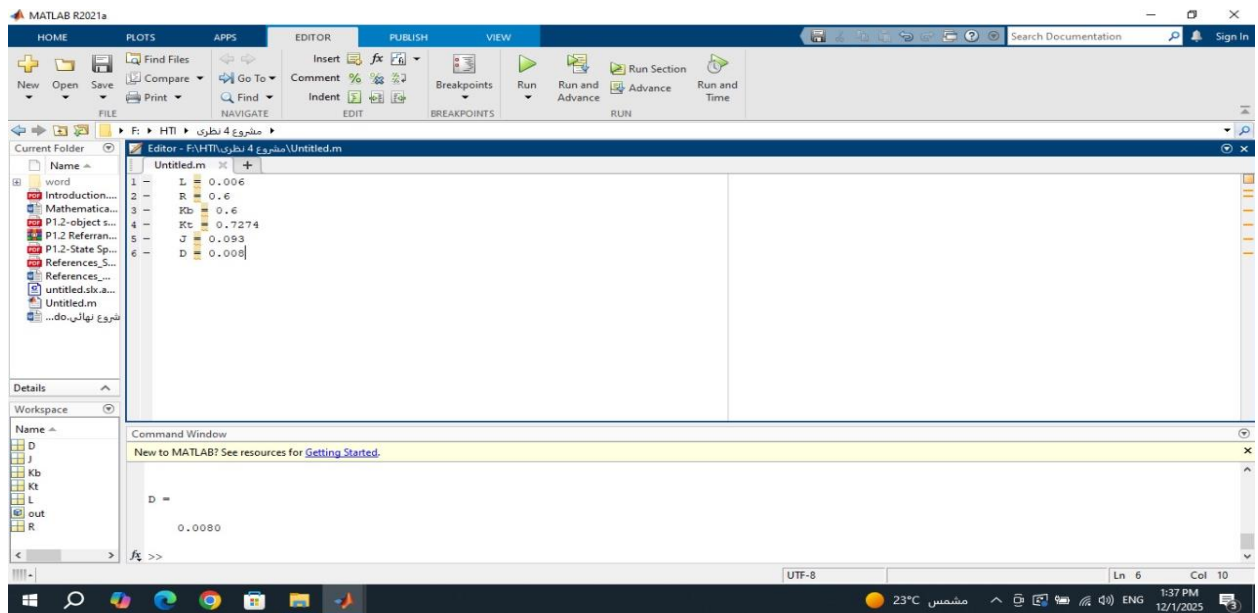
$$y(t) = \theta = x_1$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & \frac{-D}{J} & \frac{Kt}{J} \\ 0 & \frac{-Kb}{l} & \frac{-R}{l} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{l} \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Simulink MATLAB





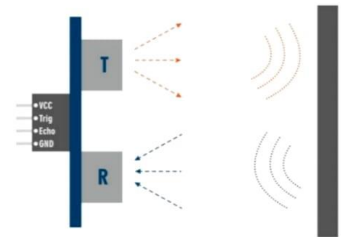
C- Ultrasonic sensor :

$$(D) = \frac{(C)*(t)}{2}$$

D : Distance between the body and the sensor.

C : Speed of sound in air (343 m/s).

t : Time of going and returning.



Mechanical operation on the Line

Total Load

$$F = m * g$$

Mass

M1 100g M 2 150g M 3 100

Total mass = 350g = 0.35 * 9.8 = 3.43N

Radius Pulley

r = 0.05m

Tension Stress

$$\sigma = F / A$$

Dimensions of Belt

Width = 15cm = 0.15m Thickness = 5mm = 0.005m

$$Area = width * thickniess$$

$$Area = 0.15 * 0.005 = 0.00075m^2$$

$$Tension stress = 3.45 / 0.00075 = 4573.3 \cong 4.6kpa$$

Require Torque

$$T = f * r$$

$$T = 3.45 * 0.05 = 0.1725 \cong 1.725 N.m$$

Calculation of Dynamic Load Torque (T_{load})

1. The Primary Load Torque Equation:

The load torque (T_{load}) is calculated as the required torque to overcome total inertia and viscous damping:

$$T_{load} = J_{eq} \cdot \alpha + D_{eq} \cdot \omega$$

Where:

- T_{load} : Total Dynamic Load Torque.
- J_{eq} : Total Equivalent Inertia.
- α : Angular Acceleration of the servo arm ($\frac{d^2\theta}{dt^2}$).
- D_{eq} : Total Equivalent Viscous Damping.
- ω : Angular Velocity of the servo arm ($\frac{d\theta}{dt}$).

2. Equivalent Inertia Component Breakdown:

The total equivalent inertia (J_{eq}) is the sum of all rotating mass resistances:

$$J_{eq} = J_{motor} + J_{arm} + J_{mass}$$

Where:

- J_{motor} : Inertia of the Servomotor rotor.
- J_{arm} : Inertia of the Arm structure itself.
- J_{mass} : Inertia of the Mass/Product being transported.

3. Equivalent Damping Component Breakdown:

The total equivalent damping (D_{eq}) is the sum of all frictional resistances in the system:

$$D_{eq} = D_{motor} + D_{bearings}$$

Where:

- D_{motor} : Viscous damping of the Motor (internal friction).
- $D_{bearings}$: Viscous damping of the external Bearings supporting the shaft.

4. Net Torque Equation (Simulink Application):

The net torque (T_{net}) is the torque that actually causes motion, calculated by subtracting the load from the motor output:

$$T_{net} = T_m - T_{load}$$

Where:

- T_{net} : Net Torque input to the mechanical motion model.
- T_m : Motor Torque generated by the electrical circuit.
- T_{load} : Dynamic Load Torque (calculated in Step 1).

Conclusion

Prototype: Highly effective, low-cost automatic sorting and counting solution.

Benefits: Escalated production, heightened consistency, and major reductions in labor costs and errors.

Target: Streamlines material handling for mass-producing industries.

References

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