Perspective Correct Interpolation of Attributes

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Abstract

In this document we will take a look at how we can interpolate attributes across a triangle in a perspective correct manner. We will use the result derived in the previous lecture of how we interpolate the depth value across a triangle and extend this to interpolation of any quantity/attribute in a perspective correct manner. This is an essential part of a rasterisation pipeline as we will generally do computations on each vertext and then interpolate out this quantity across the surface of the triangle.

Lets say that we have an attribute c that we want to interpolate out across the triangle. The attribute can be a scalar a vector, in the latter case we will just do the interpolation element-wise. Now we want this attribute to change across the triangle in the same way as the depth z changes. This means that we have the following relationship,

$$\frac{z - z_0}{z_1 - z_0} = \frac{c - c_0}{c_1 - c_0}. \tag{1}$$

From our previous derivation we know how we can write z as a function of a linear interpolation in screen space as,

$$z = \frac{1}{\frac{1}{z_0}(1-q) + \frac{1}{z_1}q},$$

where q is the scalar specifying the convex combination. Now if we use this result together with the relationship in Eq. **??** we can rewrite the left-hand side.

$$\begin{align}
\frac{z - z_0}{z_1 - z_0} &= \frac{\frac{1}{\frac{1}{z_0}(1-q)+\frac{1}{z_1}q} - z_0}{z_1 - z_0} \tag{2}\\
&= \frac{\frac{z_1 z_0}{z_1(1-q)+z_0 q} - z_0}{z_1 - z_0} \tag{3}\\
&= \frac{\frac{z_1 z_0 - z_1 z_0(1-q)+z_0^2 q}{z_1(1-q)+z_0 q}}{z_1 - z_0} \tag{4}\\
&= \frac{\frac{z_0 q(z_1 - z_0)}{z_1(1-q)+z_0 q}}{z_1 - z_0} \tag{5}\\
&= \frac{z_0 q}{z_1(1-q)+z_0 q} = \frac{z_0 q}{q(z_0 - z_1) + z_1} \tag{6}
\end{align}$$

Now we have got the relationship written in terms of the value q which is the scalar specifying the convex combination, i.e. the value that we interpolate over, the extreme values and the unknown c. What we want is an expression for c as a function of q. So we rewrite Eq.**??** and write c as a function of q,

$$\frac{z_0 q}{q(z_0 - z_1) + z_1} = \frac{c - c_0}{c_1 - c_0} \tag{7}$$

$$c = \frac{z_0 q(c_1 - c_0)}{z_1(1-q)z_0 q} + c_0 \tag{8}$$

$$= \frac{z_0 q(c_1 - c_0) + c_0(z_1(1-q) + z_0 q)}{z_1(1-q) + z_0 q} \tag{9}$$

$$= \frac{z_0 q c_1 + c_0 z_1 (1-q)}{z_1(1-q) + z_0 q}. \tag{10}$$

The above equation allows us to interpolate out any quantity c across the triangle in a perspective correct manner. However, what you would usually find is that you often first interpolate out the depth for each pixel before you interpolate out any quantity. This means that usually we know the depth value at each pixel at the stage where we want to interpolate out c. We can use this fact and simplify Eq.**??** to use z.

$$c = \frac{z_0 q c_1 + c_0 z_1 (1-q)}{z_1(1-q) + z_0 q} \tag{11}$$

$$= \frac{\frac{1}{z_1 z_0}(c_0 z_1(1-q) + c_1 z_0 q)}{\frac{1}{z_1 z_0}(z_1(1-q) + z_0 q)} \tag{12}$$

$$= \frac{\frac{c_0}{z_0}(1-q) + \frac{c_1}{z_1} q}{\underbrace{\frac{1}{z_0}(1-q) + \frac{1}{z_1} q}_{=\frac{1}{z}}} \tag{13}$$

$$= z\left(\frac{c_0}{z_0}(1-q) + \frac{c_1}{z_1} q\right) \tag{14}$$

So now we have our final result, both for the case where we do not know z and for when we have already calculated the depth.

$$c = \tfrac{z_0 q c_1 + c_0 z_1 (1-q)}{z_1(1-q) + z_0 q} \qquad c = z\left(\tfrac{c_0}{z_0}(1-q) + \tfrac{c_1}{z_1} q\right)$$