COMS 30115

Rasterisation

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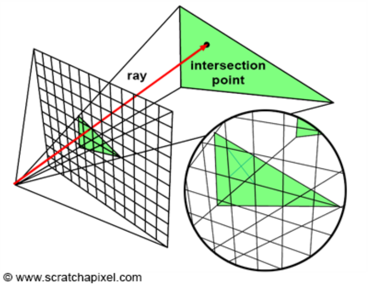
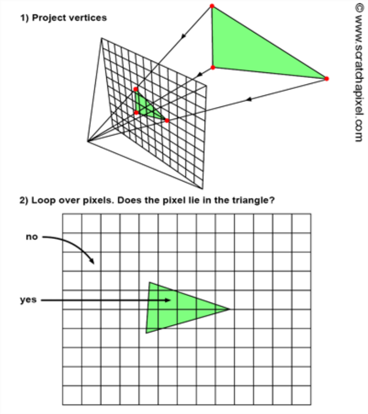
Rasterisation vs Raytracing





Line Drawing

Breshenham

- Create a decision function $f(x, y)$ (called implicit surface)
  - $f(x, y) > 0$ point below line
  - $f(x, y) < 0$ point above line
- Write decision function on integer form
  - $f(x, y) = \Delta y \cdot x - \Delta x \cdot y + \Delta x \cdot c$
- Reuse previous computations and make iterative update of decision

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  - $f(x, y) > 0$ point below line
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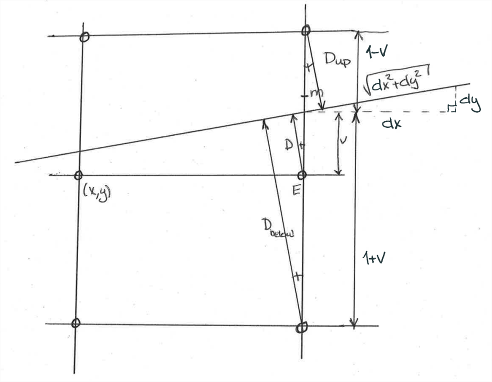
- Write decision function on integer form
  - $f(x, y) = \Delta y \cdot x - \Delta x \cdot y + \Delta x \cdot c$
- Reuse previous computations and make iterative update of decision
- Can extend to anti-aliasing using value of decision function

Gupta-Sproull



- Area that pixel covers important
- Weight pixels with perpendicular distance

Gupta-Sproull

Code

```
//compute constants A,B
//1. Run Breshenham and get d
if(d<0) //E pixel
  {
    D = A*(d+dx);
    Dup = B-D;;
    Dbelow = B+D;
    // look-up shading based on D
```

Today

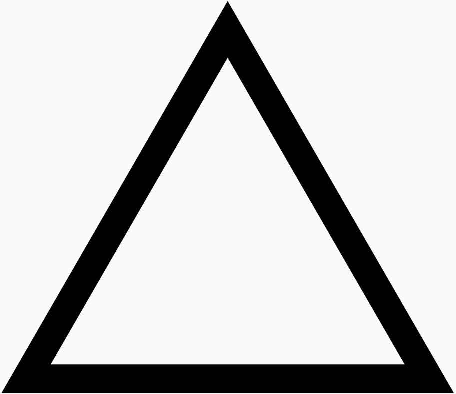
- Filled Triangles
- Prespective Correct Interpolation

The Book

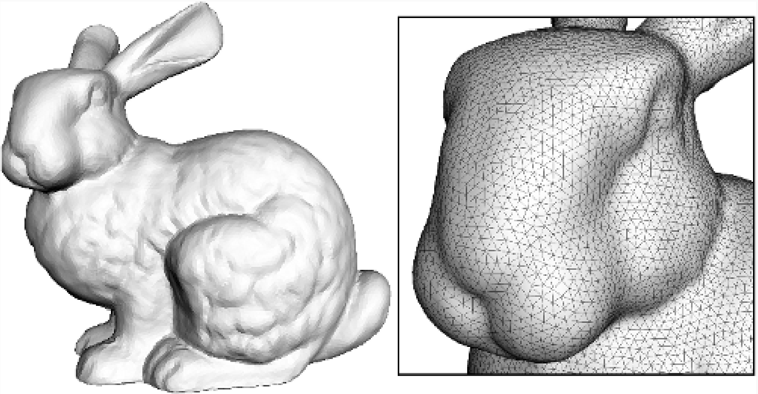
- Paper on line drawing algorithms URL
- Prespective Correct Interpolation URL
- Most of Part II is covered URL

Triangles

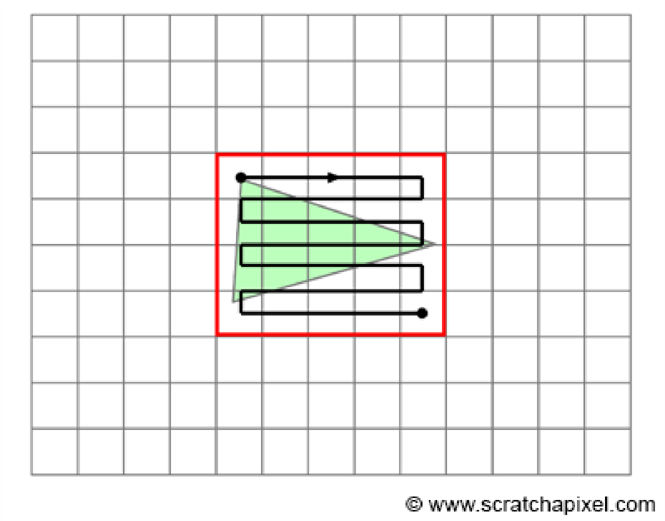
Triangles



Triangles

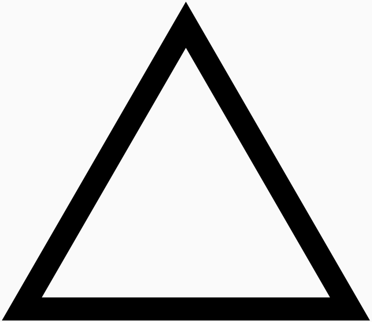


Triangles



Triangle Spantables

1. compute number of `ROWS`
2. compute left and right `x-values`
3. fill left->right



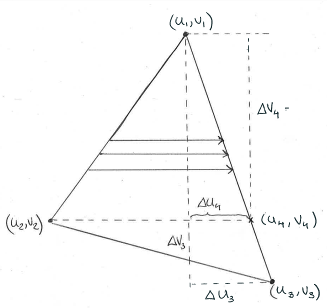
Triangle Spantables

Code

```
vector<ivec2> leftPixel(ROWS);
vector<ivec2> rightPixel(ROWS);
/*draw outer lines to fill*/
for(uint32_t i=0;i<ROWS;i++)
  {
    drawline(leftPixels[i],y0+i,
             rightPixel[i],y0+i);
  }
```

Triangle Spantables

- Drawing Flat triangle is easy
- Split triangle into two flat ones
- *special cases*
- Draw lines



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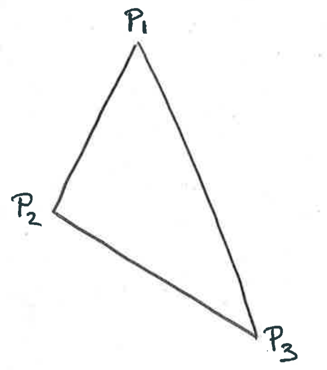

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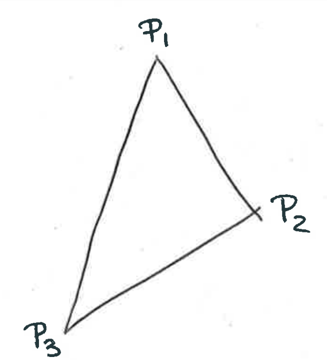

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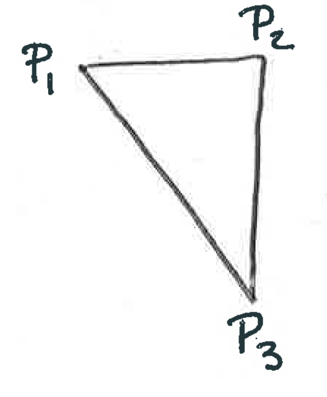

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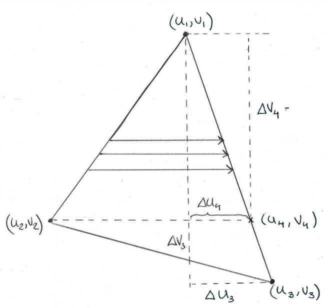
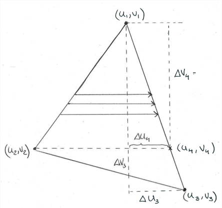


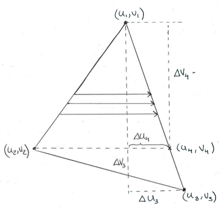
Triangle Spantables

$$\frac{\Delta v_3}{\Delta u_3} = \frac{\Delta v_4}{\Delta u_4}$$



Triangle Spantables

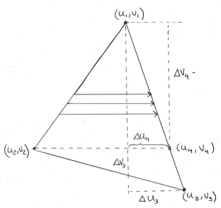
$$\frac{\Delta v_3}{\Delta u_3} = \frac{\Delta v_4}{\Delta u_4} \Rightarrow \Delta u_4 = \frac{\Delta v_4}{\Delta v_3} \Delta u_3$$



Triangle Spantables

$$\frac{\Delta v_3}{\Delta u_3} = \frac{\Delta v_4}{\Delta u_4} \Rightarrow \Delta u_4 = \frac{\Delta v_4}{\Delta v_3} \Delta u_3$$

$$= \{\Delta v_4 = v_4 - v_1, v_4 = v_2\}$$



Triangle Spantables

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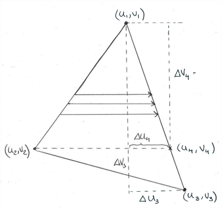
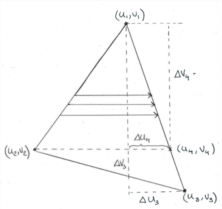
$$= \{\Delta v_4 = v_4 - v_1, v_4 = v_2\}$$
$$= \frac{v_2 - v_1}{v_3 - v_1}(u_3 - u_1)$$



Triangle Spantables

$$
\begin{aligned}
&\frac{\Delta v_3}{\Delta u_3} = \frac{\Delta v_4}{\Delta u_4} \Rightarrow \Delta u_4 = \frac{\Delta v_4}{\Delta v_3}\Delta u_3 \\
=&\{\Delta v_4 = v_4 - v_1, v_4 = v_2\} \\
=&\frac{v_2 - v_1}{v_3 - v_1}(u_3 - u_1) = \Delta u_4 = u_4 - u_1
\end{aligned}
$$



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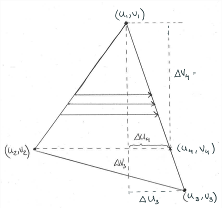
$$= \{\Delta v_4 = v_4 - v_1, v_4 = v_2\}$$

$$= \frac{v_2 - v_1}{v_3 - v_1}(u_3 - u_1) = \Delta u_4 = u_4 - u_1$$

Solve for: u_4



Triangle Spantables

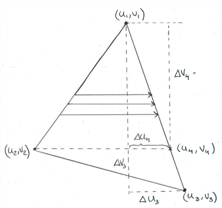
$$\frac{\Delta v_3}{\Delta u_3} = \frac{\Delta v_4}{\Delta u_4} \Rightarrow \Delta u_4 = \frac{\Delta v_4}{\Delta v_3}\Delta u_3$$

$$= \{\Delta v_4 = v_4 - v_1, v_4 = v_2\}$$

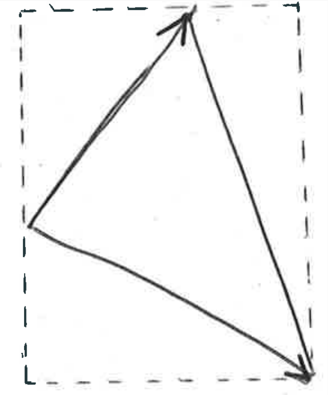
$$= \frac{v_2 - v_1}{v_3 - v_1}(u_3 - u_1) = \Delta u_4 = u_4 - u_1$$

Solve for: u_4

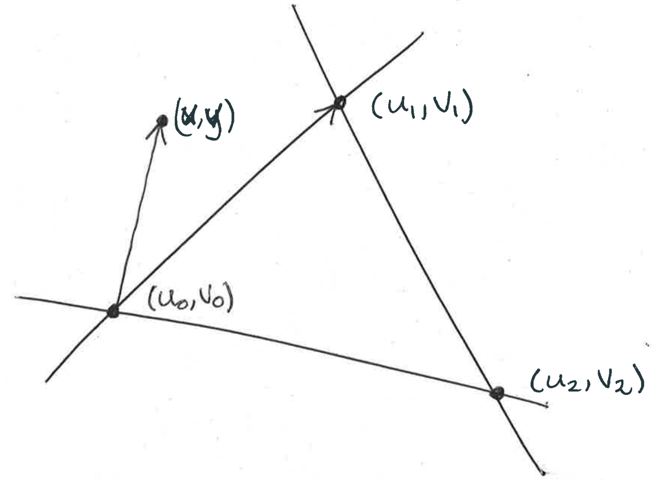
$$\Rightarrow u_4 = \frac{v_2 - v_1}{v_3 - v_1}(u_3 - u_1) + u_1$$



Triangle Halfplanes



Triangle Halfplanes



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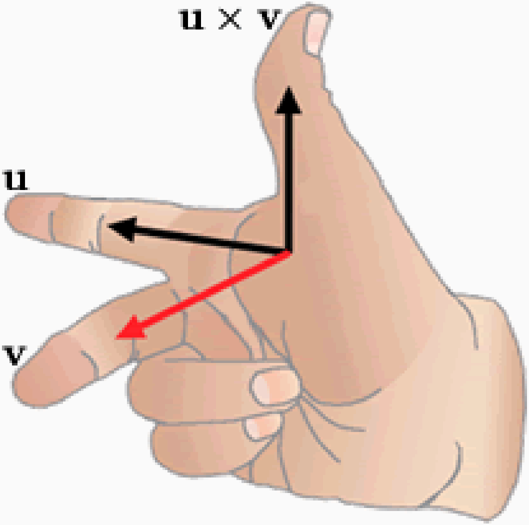

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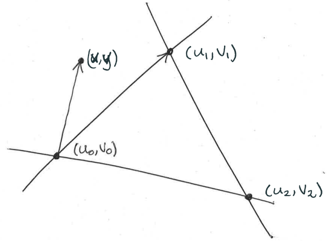
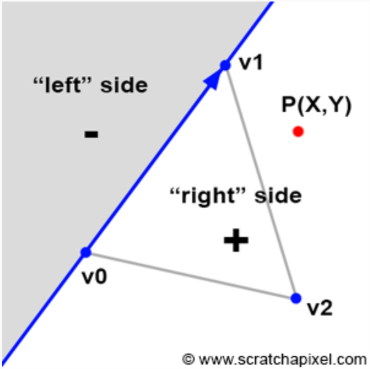
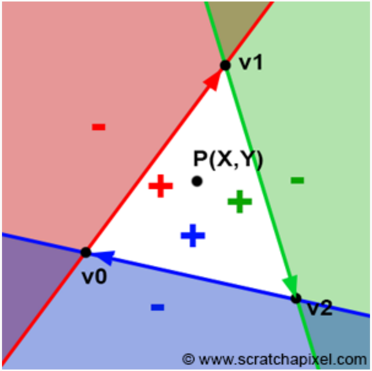


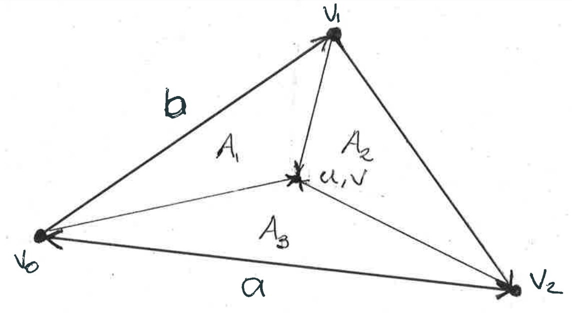
$$((u_1 - u_0, v_1 - v_0, 0 - 0) \times (u - u_0, v - v_0, 0 - 0)) =$$
$$((0, 0, (u - u_0)(v_1 - v_0) - (u_1 - u_0)(v - v_0))) = f_{01}(u, v)$$

Triangle Halfplanes



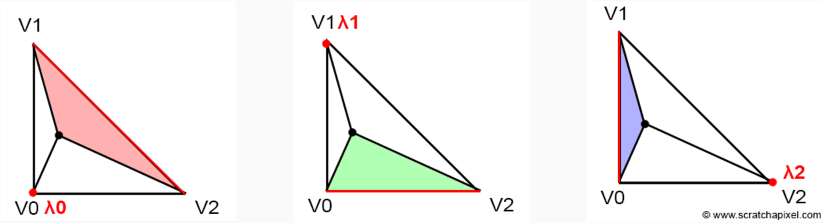


Barycentric coordinates

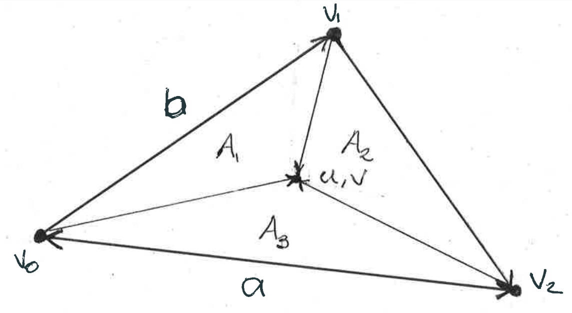


$$\mathbf{p} = \lambda_0 \mathbf{v}_0 + \lambda_1 \mathbf{v}_1 + \lambda_2 \mathbf{v}_2$$

Barycentric coordinates (Area)

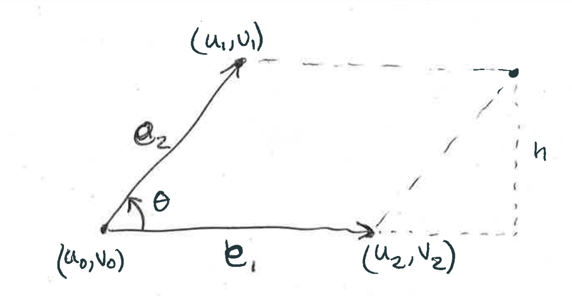


Barycentrinc coordinates



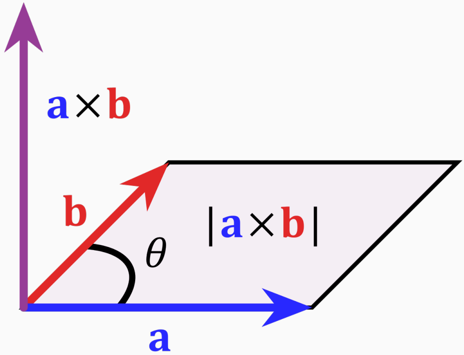
$$A = A_1 + A_2 + A_3$$

Paralellogram



$$A = ||\mathbf{e}_1|| \cdot ||\mathbf{e}_2|| \sin(\theta)$$

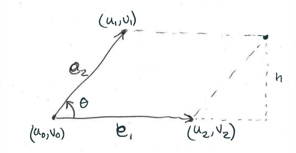
Cross-product

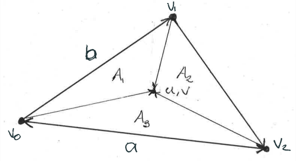


Definition (Cross-product)

$$\mathbf{a} \times \mathbf{b} = \underbrace{||\mathbf{a}|| \cdot ||\mathbf{b}|| \cdot \sin(\theta)}_{\text{length}} \cdot \mathbf{n}$$

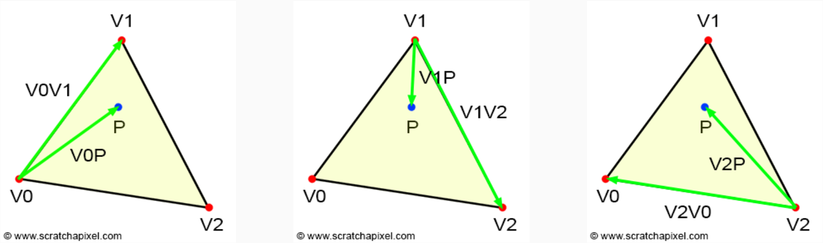
Paralellogram





$$A = A_1 + A_2 + A_3 = \frac{1}{2} f_{01}(u, v) + \frac{1}{2} f_{12}(u, v) + \frac{1}{2} f_{20}(u, v)$$

Barycentric Coordinates

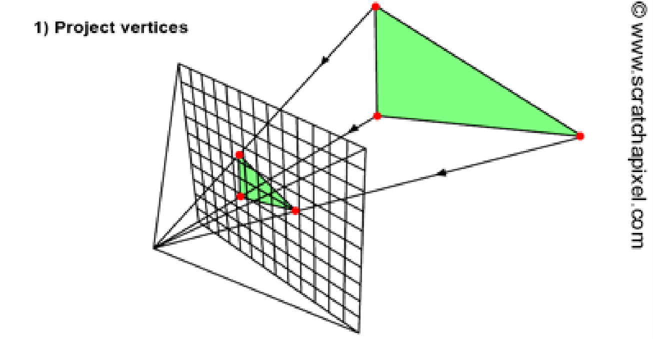


Triangle filling

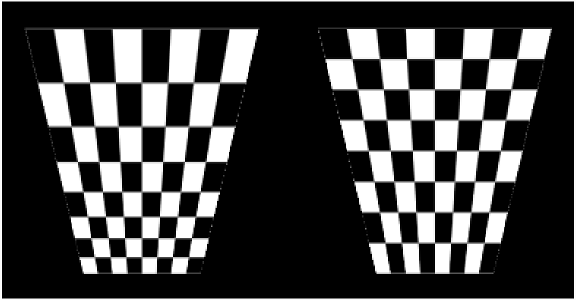
- The triangle filler in a rasteriser is like your closest intersection in the raytracer
    - make sure that it handles all the special cases
    - optimise
- There are many more methods
    - Möller-Trumbore

Perspective Correct

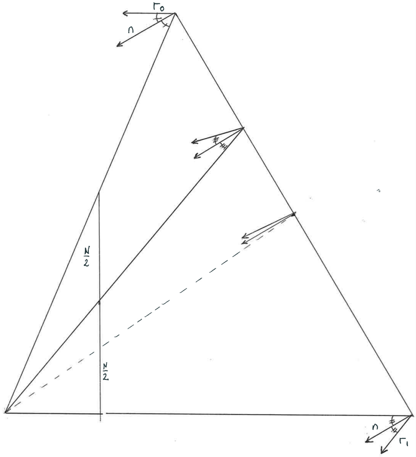
Rasterisation



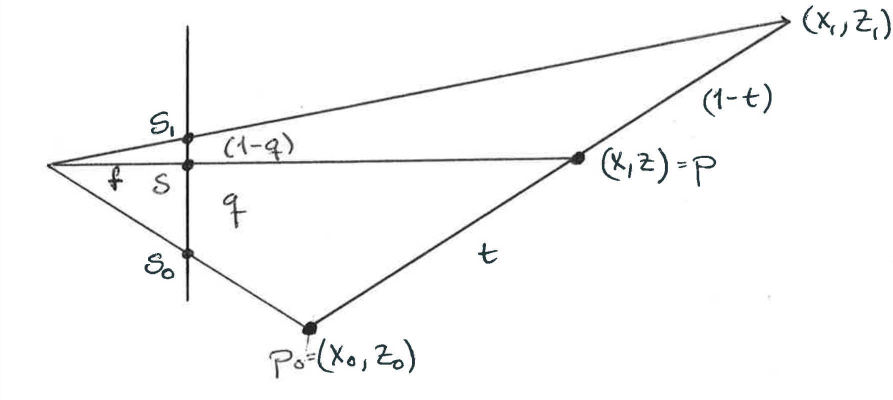
Perspective Error



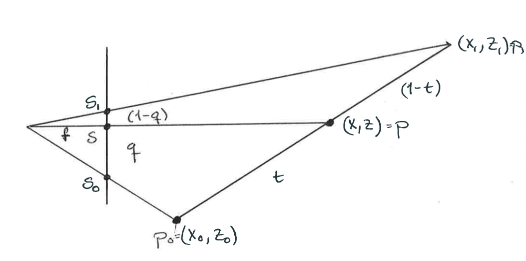
Perspective Correct



Perspective Correct



Perspective Correct Interpolation



$$[x, z]^{\mathrm{T}} = \begin{bmatrix} x_0 \\ z_0 \end{bmatrix} (1-t) + \begin{bmatrix} x_1 \\ z_1 \end{bmatrix} t = \begin{bmatrix} x_0 \\ z_0 \end{bmatrix} + t \left(\begin{bmatrix} x_1 - x_0 \\ z_1 - z_0 \end{bmatrix} \right)$$

$$s = s_0(1-q) + s_1 q = s_0 + q(s_1 - s_0)$$

can we write t as a function of q?

Perspective Correct Interpolation

- s is a projected point

$$\left[\; s = \frac{x}{z}, \quad s_0 = \frac{x_0}{z_0}, \quad s_1 = \frac{x_1}{z_1} \;\right]$$

- Write in terms of interpolated quantities

$$s = \frac{X}{Z}$$

Perspective Correct Interpolation

- s is a projected point

$$\left[\ s = \frac{x}{z}, \quad s_0 = \frac{x_0}{z_0}, \quad s_1 = \frac{x_1}{z_1}\ \right]$$

- Write in terms of interpolated quantities

$$s = \frac{x}{z} \Rightarrow z = \frac{x}{s}$$

Perspective Correct Interpolation

- s is a projected point

$$\left[\; s = \tfrac{x}{z}, \quad s_0 = \tfrac{x_0}{z_0}, \quad s_1 = \tfrac{x_1}{z_1} \;\right]$$

- Write in terms of interpolated quantities

$$s = \frac{x}{z} \Rightarrow z = \frac{x}{s} = \frac{x_0 + t(x_1 - x_0)}{s_0 + q(s_1 - s_0)}$$

Perspective Correct Interpolation

- s is a projected point

$$\left[\ s = \tfrac{x}{z}, \quad s_0 = \tfrac{x_0}{z_0}, \quad s_1 = \tfrac{x_1}{z_1}\ \right]$$

- Write in terms of interpolated quantities

$$s = \frac{x}{z} \Rightarrow z = \frac{x}{s} = \frac{x_0 + t(x_1 - x_0)}{s_0 + q(s_1 - s_0)}$$
$$= \{x = s \cdot z\}$$

Perspective Correct Interpolation

- s is a projected point

$$\left[\ s = \tfrac{x}{z},\quad s_0 = \tfrac{x_0}{z_0},\quad s_1 = \tfrac{x_1}{z_1}\ \right]$$

- Write in terms of interpolated quantities

$$\begin{aligned} s = \frac{x}{z} \Rightarrow z = \frac{x}{s} &= \frac{x_0 + t(x_1 - x_0)}{s_0 + q(s_1 - s_0)} \\ &= \{x = s \cdot z\} = \frac{s_0 z_0 + t(s_1 z_1 - s_0 z_0)}{s_0 + q(s_1 - s_0)} \end{aligned}$$

Perspective Correct Interpolation

- s is a projected point

$$\left[\ s = \tfrac{x}{z}, \quad s_0 = \tfrac{x_0}{z_0}, \quad s_1 = \tfrac{x_1}{z_1}\ \right]$$

- Write in terms of interpolated quantities

$$\begin{aligned} s = \frac{x}{z} \Rightarrow z = \frac{x}{s} &= \frac{x_0 + t(x_1 - x_0)}{s_0 + q(s_1 - s_0)} \\ &= \{x = s \cdot z\} = \frac{s_0 z_0 + t(s_1 z_1 - s_0 z_0)}{s_0 + q(s_1 - s_0)} \\ &= z_0 + t(z_1 - z_0) \end{aligned}$$

Perspective Correct Interpolation

$$z_0 + t(z_1 - z_0) = \frac{s_0 z_0 + t(s_1 z_1 - s_0 z_0)}{s_0 + q(s_1 - s_0)}$$

Perspective Correct Interpolation

$$z_0 + t(z_1 - z_0) = \frac{s_0 z_0 + t(s_1 z_1 - s_0 z_0)}{s_0 + q(s_1 - s_0)}$$

- Multiply denominator

$$z_0 s_0 + z_0 q(s_1 - s_0) + t s_0(z_1 - z_0) + tq(z_1 - z_0)(s_1 - s_0)$$
$$= s_0 z_0 + t(s_1 z_1 - s_0 z_0)$$

Perspective Correct Interpolation

$$z_0 + t(z_1 - z_0) = \frac{s_0 z_0 + t(s_1 z_1 - s_0 z_0)}{s_0 + q(s_1 - s_0)}$$

- Multiply denominator

$$z_0 s_0 + z_0 q(s_1 - s_0) + t s_0(z_1 - z_0) + tq(z_1 - z_0)(s_1 - s_0)$$
$$= s_0 z_0 + t(s_1 z_1 - s_0 z_0)$$

- Collect terms with the different "interpolants"

$$t\left(s_0(z_1 - z_0) + q(z_1 - z_0)(s_1 - s_0) - (s_1 z_1 - s_0 z_0)\right) = -q z_0(s_1 - s_0)$$

Perspective Correct Interpolation

$$z_0 + t(z_1 - z_0) = \frac{s_0 z_0 + t(s_1 z_1 - s_0 z_0)}{s_0 + q(s_1 - s_0)}$$

- Multiply denominator

$$z_0 s_0 + z_0 q(s_1 - s_0) + t s_0(z_1 - z_0) + tq(z_1 - z_0)(s_1 - s_0)$$
$$= s_0 z_0 + t(s_1 z_1 - s_0 z_0)$$

- Collect terms with the different "interpolants"

$$t\left(s_0(z_1 - z_0) + q(z_1 - z_0)(s_1 - s_0) - (s_1 z_1 - s_0 z_0)\right) = -qz_0(s_1 - s_0)$$

- Identify square on LHS

$$t(s_1 - s_0)(-z_1 + q(z_1 - z_0)) = -qz_0(s_1 - s_0)$$

Perspective Correct Interpolation

- Simplify

$$t(z_1 - q(z_1 - z_0)) = qz_0$$

Perspective Correct Interpolation

- Simplify

$$t(z_1 - q(z_1 - z_0)) = qz_0$$

- Result 1

$$\Rightarrow t = \frac{qz_0}{z_1 - q(z_1 - z_0)}$$

Perspective Correct Interpolation

- Simplify

$$t(z_1 - q(z_1 - z_0)) = qz_0$$

- Result 1

$$\Rightarrow t = \frac{qz_0}{z_1 - q(z_1 - z_0)}$$

- We have written t in terms of q, now replace and write z in terms of q

Perspective Correct Interpolation

- Write z in terms of screen space interpolant

$$z = z_0 + t(z_1 - z_0)$$

Perspective Correct Interpolation

- Write z in terms of screen space interpolant

$$z = z_0 + t(z_1 - z_0) = z_0 + \frac{qz_0}{z_1 - q(z_1 - z_0)}(z_1 - z_0)$$

Perspective Correct Interpolation

- Write z in terms of screen space interpolant

$$
\begin{aligned}
z = z_0 + t(z_1 - z_0) &= z_0 + \frac{qz_0}{z_1 - q(z_1 - z_0)}(z_1 - z_0) \\
&= z_0 + \frac{qz_1z_0 - qz_0^2}{qz_0 + z_1(1-q)}
\end{aligned}
$$

Perspective Correct Interpolation

- Write z in terms of screen space interpolant

$$
\begin{aligned}
z = z_0 + t(z_1 - z_0) &= z_0 + \frac{qz_0}{z_1 - q(z_1 - z_0)}(z_1 - z_0) \\
&= z_0 + \frac{qz_1 z_0 - qz_0^2}{qz_0 + z_1(1-q)} = \frac{qz_0^2 + z_0 z_1(1-q) + qz_1 z_0 - qz_0^2}{qz_0 + z_1(1-q)}
\end{aligned}
$$

Perspective Correct Interpolation

- Write z in terms of screen space interpolant

$$
\begin{aligned}
z &= z_0 + t(z_1 - z_0) = z_0 + \frac{qz_0}{z_1 - q(z_1 - z_0)}(z_1 - z_0) \\
&= z_0 + \frac{qz_1 z_0 - qz_0^2}{qz_0 + z_1(1-q)} = \frac{qz_0^2 + z_0 z_1(1-q) + qz_1 z_0 - qz_0^2}{qz_0 + z_1(1-q)} \\
&= \frac{z_0 z_1}{qz_0 + z_1(1-q)}
\end{aligned}
$$

Perspective Correct Interpolation

- Write z in terms of screen space interpolant

$$
\begin{aligned}
z &= z_0 + t(z_1 - z_0) = z_0 + \frac{qz_0}{z_1 - q(z_1 - z_0)}(z_1 - z_0) \\
&= z_0 + \frac{qz_1z_0 - qz_0^2}{qz_0 + z_1(1-q)} = \frac{qz_0^2 + z_0z_1(1-q) + qz_1z_0 - qz_0^2}{qz_0 + z_1(1-q)} \\
&= \frac{z_0z_1}{qz_0 + z_1(1-q)} = \frac{1}{\frac{qz_0}{z_0z_1} + \frac{z_1(1-q)}{z_0z_1}}
\end{aligned}
$$

Perspective Correct Interpolation

- Write z in terms of screen space interpolant

$$
\begin{aligned}
z &= z_0 + t(z_1 - z_0) = z_0 + \frac{qz_0}{z_1 - q(z_1 - z_0)}(z_1 - z_0) \\
&= z_0 + \frac{qz_1z_0 - qz_0^2}{qz_0 + z_1(1-q)} = \frac{qz_0^2 + z_0z_1(1-q) + qz_1z_0 - qz_0^2}{qz_0 + z_1(1-q)} \\
&= \frac{z_0z_1}{qz_0 + z_1(1-q)} = \frac{1}{\frac{qz_0}{z_0z_1} + \frac{z_1(1-q)}{z_0z_1}} \\
&= \frac{1}{\frac{q}{z_1} + \frac{1-q}{z_0}}
\end{aligned}
$$

Perspective Correct Interpolation

- Write z in terms of screen space interpolant

$$
\begin{aligned}
z &= z_0 + t(z_1 - z_0) = z_0 + \frac{qz_0}{z_1 - q(z_1 - z_0)}(z_1 - z_0) \\
&= z_0 + \frac{qz_1z_0 - qz_0^2}{qz_0 + z_1(1-q)} = \frac{qz_0^2 + z_0z_1(1-q) + qz_1z_0 - qz_0^2}{qz_0 + z_1(1-q)} \\
&= \frac{z_0z_1}{qz_0 + z_1(1-q)} = \frac{1}{\frac{qz_0}{z_0z_1} + \frac{z_1(1-q)}{z_0z_1}} \\
&= \frac{1}{\frac{q}{z_1} + \frac{1-q}{z_0}} = \frac{1}{\frac{1}{z_1} + q(\frac{1}{z_1} - \frac{1}{z_0})}
\end{aligned}
$$

Perspective Correct Interpolation

- Write z in terms of screen space interpolant

$$
\begin{aligned}
z &= z_0 + t(z_1 - z_0) = z_0 + \frac{qz_0}{z_1 - q(z_1 - z_0)}(z_1 - z_0) \\
&= z_0 + \frac{qz_1z_0 - qz_0^2}{qz_0 + z_1(1-q)} = \frac{qz_0^2 + z_0z_1(1-q) + qz_1z_0 - qz_0^2}{qz_0 + z_1(1-q)} \\
&= \frac{z_0z_1}{qz_0 + z_1(1-q)} = \frac{1}{\frac{qz_0}{z_0z_1} + \frac{z_1(1-q)}{z_0z_1}} \\
&= \frac{1}{\frac{q}{z_1} + \frac{1-q}{z_0}} = \frac{1}{\frac{1}{z_1} + q(\frac{1}{z_1} - \frac{1}{z_0})} = z
\end{aligned}
$$

Perspective Correct Interpolation

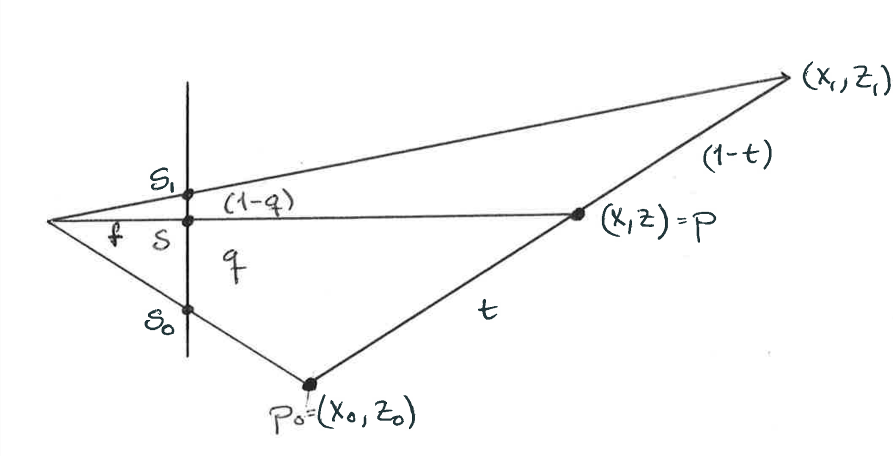
$$z = \frac{1}{\frac{1}{z_1} + q(\frac{1}{z_1} - \frac{1}{z_0})}$$

- This leads to the final result

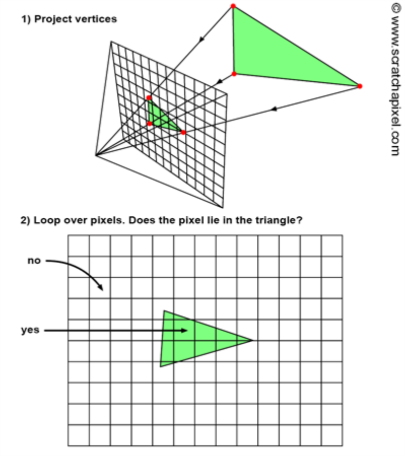
$$\frac{1}{z} = \frac{1}{z_0} + q\left(\frac{1}{z_1} - \frac{1}{z_0}\right)$$

- If you interpolate in screen space, you need to interpolate the inverse of the depth
- This is most likely the most important result in rasterisation

Perspective Correct



Perspective Correct

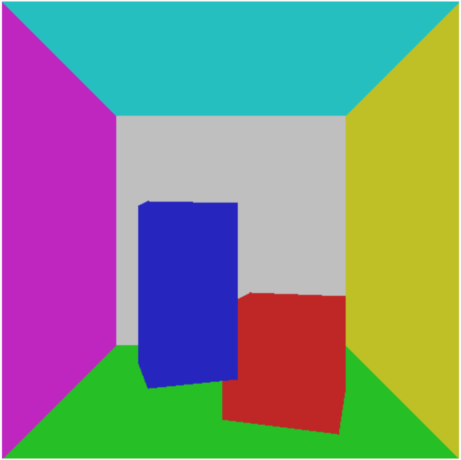


Summary

Summary

- Primitives
  - Lines
  - Triangles
- Perspective Correct
  - why is interpolation a bit challenging in image space
  - most likely the most important part of rasterisation

Lab



Next Time

Lecture interpolation of quantities

- Perspective correct interpolation
- Shading

Lab continue with Lab 2

- material is up to part 4

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