COMS 30115

Global Illumination

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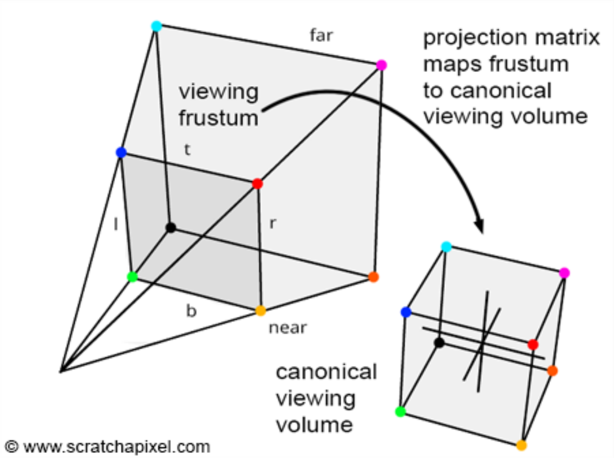
Clip Space


far
projection matrix
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Clip Space[1]

1. Map from world space to clip space

$$[x, y, z, 1]^{\mathrm{T}}] \rightarrow [x, y, z, \frac{z}{f}]^{\mathrm{T}}$$

[1] http://www.lighthouse3d.com/tutorials/view-frustum-culling/clip-space-approach-extracting-the-planes/

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2. Clip x and y plane of view frustrum

$$-w \cdot x_{\mathrm{max}} \leq x \leq w \cdot x_{\mathrm{max}}$$
$$-w \cdot y_{\mathrm{max}} \leq y \leq w \cdot y_{\mathrm{max}}$$

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3. Map homogenous coordinate to screen space by homgenising coordinate

$$[x_{clipped}, y_{clipped}, z, \frac{z}{f}]^{\mathrm{T}} \rightarrow [x_{clipped}\frac{f}{z}, y_{clipped}\frac{f}{z}, z\frac{f}{z}, \frac{z}{f}\frac{f}{z}]^{\mathrm{T}} = [u_{clipped}, v_{clipped}, f, 1]^{\mathrm{T}}$$

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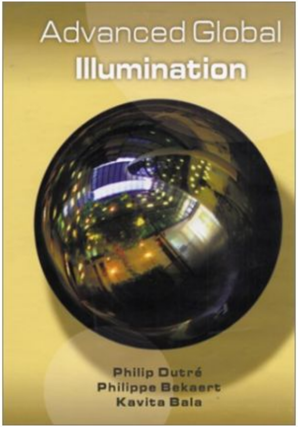
The Book

- Scratchapixel on Global Illumination
- Equation compendium that later turned into "the book"
- Monte-Carlo Methods in Global Illumination free textbook



Global Illumination

Rendering Equation

$$L(x \to \Theta) = L_e(x \to \Theta) + \int_{\Omega_x} f_r(x, \Psi \to \Theta) L(x \leftarrow \Psi) cos(\mathbf{n}_x, \Psi) \mathrm{d}\omega_\Psi$$

Overview

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  - what are the quantities that are involved in Global Illumination

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  - BRDF

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- Quantification of Light
    - what are the quantities that are involved in Global Illumination
- We want to have a simple way of summing incomming light to a specific location
    - hemisphere
- We want to be able to describe outgoing light as a function of incoming light
    - BRDF
- Formulate appearance as light transport
    - How to solve for equilibrium state

Quantification of Light

Radiant Power/Flux - the total amount of energy that flows per unit time

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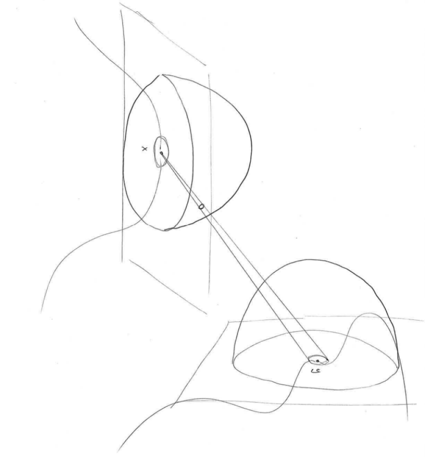
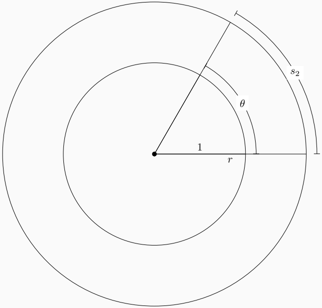
- L(x) - [$Watt/steradian \cdot m^2$]

Hemisphere

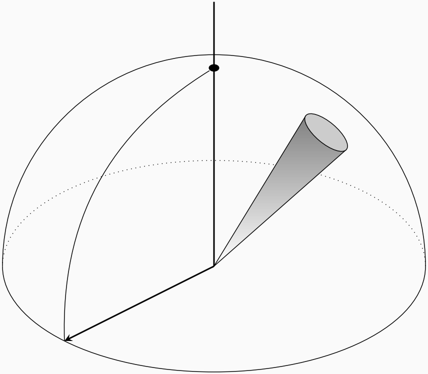


Solid Angles



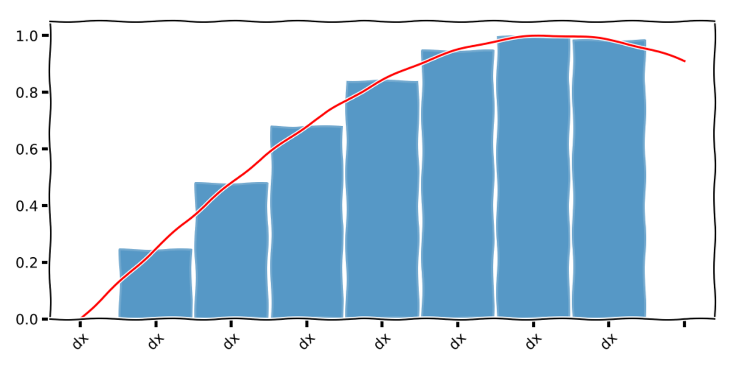
$$\frac{\theta}{2\pi \cdot 1} = \frac{s}{2\pi \cdot r} \quad \Rightarrow \quad \theta = \frac{s}{r}$$

Solid Angles



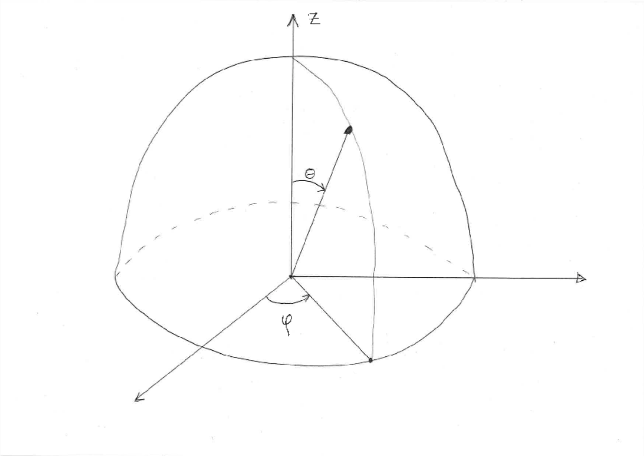
$$\frac{\omega}{4\pi \cdot 1^2} = \frac{A}{4\pi \cdot r^2} \quad \omega = \frac{A}{r^2}$$

Calculus



$$A = \int f(x)\mathrm{d}x$$

Solid Angles



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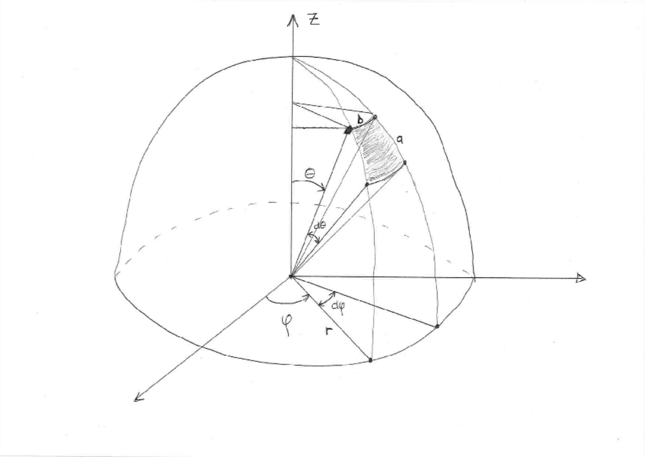
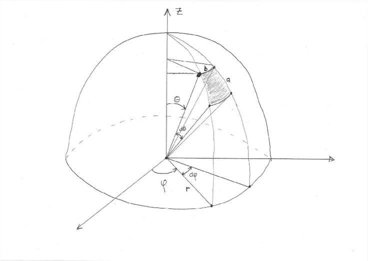


Solid Angles

$$
\begin{aligned}
dA = d\omega_\Theta &= \lim_{d\phi \to 0, d\theta \to 0} a \cdot b \\
b &= sin(\theta) r d\phi \\
a &= r d\theta \\
\Rightarrow d\omega_\Theta &= r^2 sin(\theta) d\phi d\theta
\end{aligned}
$$



We can parametrise a differiental surface which can be used as an interface for computing light transport through the geometry

Example: Surface of sphere

$$A_{sphere} = \int dA = \int d\omega$$

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$$= \underbrace{\int_{0}^{2\pi} \mathrm{d}\phi}_{2\pi} \int_{0}^{\pi} sin(\theta) \mathrm{d}\theta$$

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\begin{aligned}
A_{sphere} &= \int dA = \int d\omega = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} sin(\theta) \mathrm{d}\theta \mathrm{d}\phi \\
&= \underbrace{\int_0^{2\pi} \mathrm{d}\phi}_{2\pi} \int_0^{\pi} sin(\theta) \mathrm{d}\theta \\
&= 2\pi \cdot [-cos(\theta)]_0^{\pi} = 2\pi \underbrace{(-(-1-1))}_{2} = 4\pi
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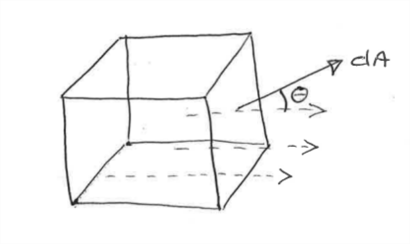
- Remember that $\mathrm{d}A = r^2 \cdot \mathrm{d}\omega$



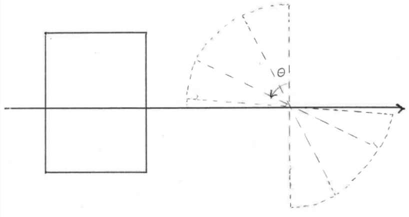
Radiance

the radiant power per unit projected area per unit solid angle

- L(x) - [$Watt/steradian \cdot m^2$]

$$L = \frac{d^2\Phi}{d\omega dA^{\perp}} = \frac{d^2\Phi}{d\omega \cos(\theta) dA}$$

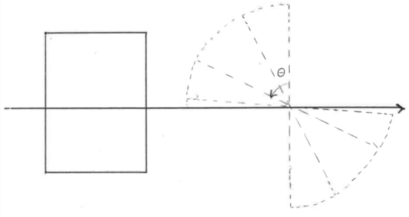
Quantification of Light



- Number of particles/photons that passes through surface $\mathrm{d}A$ in time $\mathrm{d}t$

$$N = p(x, \omega, \lambda)\mathrm{d}\omega\mathrm{d}\lambda \underbrace{c\mathrm{d}t\mathrm{d}A\cos(\theta)}_{\mathrm{d}V}$$

Quantification of Light



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- Flux is energy per unit time.

$$\Phi = E \cdot p(x, \omega, \lambda)\mathrm{d}\omega\mathrm{d}\lambda c\mathrm{d}t\mathrm{d}A\cos(\theta)$$

Energy of Light

Planck-Einstein relation

$$E = \frac{c}{\lambda} h = \frac{[m/s]}{[m]} [Joules \cdot s] = [Joules]$$

- h is Planck's constant

$$h \approx 6.626070040 \cdot 10^{-34} [Joules \cdot s]$$

Quantification of Light

- Radiance - *the radiant power per unit projected area per unit solid angle*

$$
\begin{aligned}
L(x, \omega, \lambda) &= \frac{\mathrm{d}^2}{\mathrm{d}\omega \mathrm{d}A \cos(\theta)} E \cdot p(x, \omega, \lambda) \mathrm{d}\omega \mathrm{d}\lambda c \mathrm{d}t \mathrm{d}A \cos(\theta) \\
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\end{aligned}
$$

- Flux, Irrandiance & Radiosity

$$
\begin{aligned}
\Phi(x) &= \int \int L(x \leftarrow \Theta) \cos(\theta) \mathrm{d}\omega_\Theta \mathrm{d}A_x \\
E(x) &= \frac{\mathrm{d}\Phi}{\mathrm{d}A_x} = \int L(x \leftarrow \Theta) \cos(\theta) \mathrm{d}\omega_\Theta \\
B(x) &= \frac{\mathrm{d}\Phi}{\mathrm{d}A_x} = \int L(x \rightarrow \Theta) \cos(\theta) \mathrm{d}\omega_\Theta
\end{aligned}
$$

Light Transport

$$B(x) = \int_{\Omega} L(x \rightarrow \Theta) cos(\theta) \mathrm{d}\omega_{\Theta}$$

- The total radiant power leaving area $\mathrm{d}A_x$ and arriving at area $\mathrm{d}A_y$ is,
$$(\mathrm{d}^2\Phi)_{xy} = L(x \rightarrow y) cos(\theta_x) \mathrm{d}\omega_{xy} \mathrm{d}A_x$$
- $\mathrm{d}\omega_{xy}$ solid angle under which $\mathrm{d}A_y$ is seen from x
- The total radiant power arriving at area $\mathrm{d}A_y$ and from area $\mathrm{d}A_x$ is,
$$(\mathrm{d}^2\Phi)_{yx} = L(y \leftarrow x) cos(\theta_y) \mathrm{d}\omega_{yx} \mathrm{d}A_y$$

Light Transport

If we assume that no energy is lost between the two surfaces i.e. the medium is vacum then,

$$(\mathrm{d}^2\Phi)_{xy} = (\mathrm{d}^2\Phi)_{yx}$$

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$$\left\{ \begin{array}{l} \mathrm{d}\omega_{xy} = \frac{\mathrm{d}A}{r^2} = \frac{cos(\theta_y)\mathrm{d}A_y}{r_{xy}^2} \\ \mathrm{d}\omega_{yx} = \frac{\mathrm{d}A}{r^2} = \frac{cos(\theta_x)\mathrm{d}A_x}{r_{xy}^2} \end{array} \right\}$$

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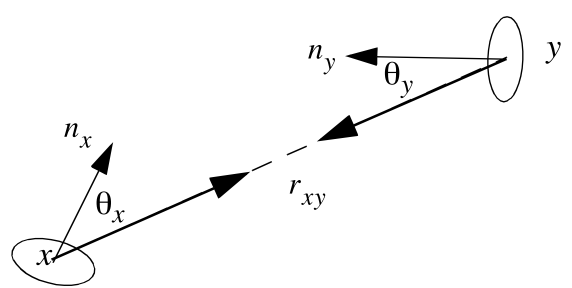
$$L(x \rightarrow y) cos(\theta_x) \frac{cos(\theta_y)\mathrm{d}A_y}{r_{xy}^2} \mathrm{d}A_x = L(y \leftarrow x) cos(\theta_y) \frac{cos(\theta_x)\mathrm{d}A_x}{r_{xy}^2} \mathrm{d}A_y$$

$$\Rightarrow L(x \rightarrow y) = L(y \leftarrow x)$$



Radiance

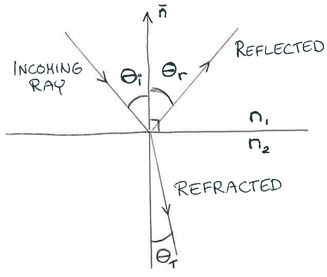
$$L(x \rightarrow y) = L(y \leftarrow x)$$

The radiance leaving point x directed towards point y is the same as the radiance arriving at point y leaving point x. Assuming that the light is travelling through vacuum.

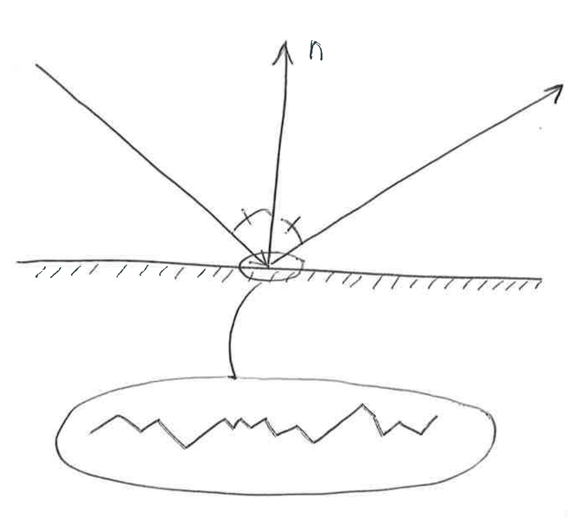
BRDFs

BRDFs

- Light still behaves the same
- Incoming light
- Outgoing
  - Relected
  - Refracted
- BRDF parametrises behaviour



BRDF



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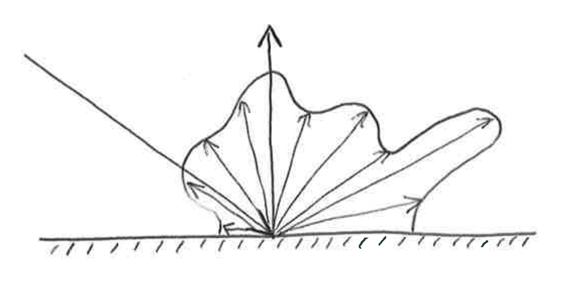

Bidirectional Reflectance Distribution Function

$$f_r(x, \Psi \to \Theta) = \frac{\mathrm{d}L(x \to \Theta)}{\mathrm{d}E(x \leftarrow \Psi)}$$

Definition (BRDF)

ratio of the differiental radiance reflected in an exitant direction Θ and the differiental irradiance incident through a differiental solid angle Ψ

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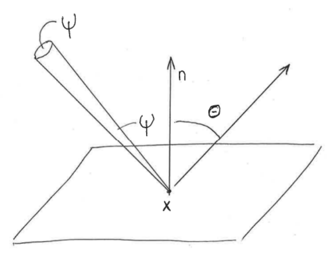
BRDF



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Properties of BRDF

- *Defined over whole sphere to represenent transparency*
- Dimension: *Four dimensional, input direction (2) and output direction (2)*
- Reciprocity: *Reversing the direction of light does not alter the BRDF*

$$f_r(x, \Psi \to \Theta) = f_r(x, \Theta \to \Psi)$$

Properties of BRDF

- Incident & reflected Radiance: *the BRDF is independent of irradiance from other incident angles. This means it is linear (i.e. addative) with respect to all incident directions*
- This means we can easily compute the total reflected radiance,

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$$\mathrm{d}L(x \to \Theta) = f_r(x, \Psi \to \Theta) L(x \leftarrow \Psi) cos(\mathbf{n}_x, \Psi) \mathrm{d}\omega_\Psi$$

- *Relating incoming to outgoing light!*

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$$L(x \rightarrow \Theta) = \int_{\Omega_x} f_r(x, \Psi \rightarrow \Theta) L(x \leftarrow \Psi) cos(\mathbf{n}_x, \Psi)\mathrm{d}\omega_\Psi$$

- *Relating incoming to outgoing light!*

Properties of BRDF

- Energy Conservation: *the amount of power reflected over all directions of a point must be the same or smaller than the total amount of energy incident to on the surface*

$$E(x) = \int_{\Omega_x} L(x \leftarrow \Psi) cos(\mathbf{n}_x, \Psi) \mathrm{d}\omega_\Psi$$
$$M(x) = \int_{\Omega_x} L(x \rightarrow \Theta) cos(\mathbf{n}_x, \Theta) \mathrm{d}\omega_\Theta$$

- Energy is conserved if $\frac{M(x)}{E(x)} \leq 1$

BRDF Energy Conservation

- Definition of BRDF allows us to write

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- We can re-write the reflected power $M(x)$

$$\int_{\Omega_x} \int_{\Omega_x} f_r(x, \Psi \to \Theta) L(x \leftarrow \Psi) cos(\mathbf{n}_x, \Psi) cos(\mathbf{n}_x, \theta) \mathrm{d}\omega_{\boldsymbol{\Psi}} \mathrm{d}\omega_{\theta}$$

BRDF Energy Conservation

- Definition of BRDF allows us to write

$$L(x \to \Theta) = \int_{\Omega_x} f_r(x, \Psi \to \Theta) L(x \leftarrow \Psi) cos(\mathbf{n}_x, \Psi) \mathrm{d}\omega_{\Psi}$$

- We can re-write the reflected power $M(x)$

$$\int_{\Omega_x} \int_{\Omega_x} f_r(x, \Psi \to \Theta) L(x \leftarrow \Psi) cos(\mathbf{n}_x, \Psi) cos(\mathbf{n}_x, \theta) \mathrm{d}\omega_{\Psi} \mathrm{d}\omega_{\theta}$$

- The relationship

$$\frac{\int_{\Omega_x} \int_{\Omega_x} f_r(x, \Psi \to \Theta) L(x \leftarrow \Psi) cos(\mathbf{n}_x, \Psi) cos(\mathbf{n}_x, \theta) \mathrm{d}\omega_{\Psi} \mathrm{d}\omega_{\theta}}{\int_{\Omega_x} L(x \leftarrow \Psi) cos(\mathbf{n}_x, \Psi) \mathrm{d}\omega_{\Psi}} \leq 1$$

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- The above has to be true for any incident radiance function $L(x \leftarrow \Psi)$

BRDF Energy Conservation

$$\frac{M(x)}{E(x)} = \frac{\int_{\Omega_x} \int_{\Omega_x} f_r(x, \Psi \rightarrow \Theta) L(x \leftarrow \Psi) cos(\mathbf{n}_x, \Psi) cos(\mathbf{n}_x, \theta) \mathrm{d}\omega_{\Psi} \mathrm{d}\omega_{\theta}}{\int_{\Omega_x} L(x \leftarrow \Psi) cos(\mathbf{n}_x, \Psi) \mathrm{d}\omega_{\Psi}}$$

BRDF Energy Conservation

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$$= \frac{\int_{\Omega_x} f_r(x, \Psi \to \Theta) L_{in} cos^2(\mathbf{n}_x, \theta) \mathrm{d}\omega_\theta}{L_{in} cos(\mathbf{n}_x, \theta)}$$

BRDF Energy Conservation

$$\frac{M(x)}{E(x)} = \frac{\int_{\Omega_x}\int_{\Omega_x} f_r(x, \Psi \to \Theta) L(x \leftarrow \Psi) cos(\mathbf{n}_x, \Psi) cos(\mathbf{n}_x, \theta) \mathrm{d}\omega_{\mathbf{\Psi}} \mathrm{d}\omega_\theta}{\int_{\Omega_x} L(x \leftarrow \Psi) cos(\mathbf{n}_x, \Psi) \mathrm{d}\omega_{\mathbf{\Psi}}}$$

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$$= \int_{\Omega_x} f_r(x, \Psi \to \Theta) cos(\mathbf{n}_x, \theta) \mathrm{d}\omega_\theta \leq 1$$

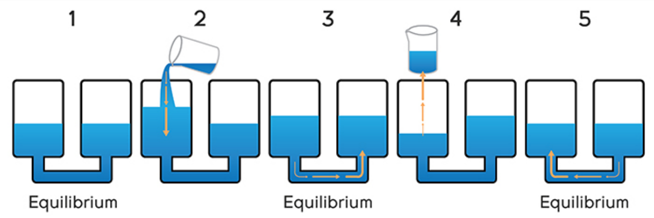
Rendering Equation

Rendering Equation[2]

- Now we have all the building blocks
    - Solid angles allows us to define computational interfaces
    - BRDFs allows us to parametrise interactions
    - Properties of BRDFs guarantees physical correctness

[2] http://blenderartists.org/forum/showthread.php?146041-Cornell-Box-with-BI

Rendering Equation



Light is very very fast, so we can assume that the equlibrium happens instantly (at least for world size scenes)

Rendering Equation

Radiance going out from a point x in a direction Θ

- Emitted radiance (light source):

$$L_e(x \to \Theta)$$

- Relfected radiance

$$L_r(x \to \Theta)$$

- Total outgoing radiance:

$$L(x \to \Theta) = L_e(x \to \Theta) + L_r(x \to \Theta)$$

Rendering Equation

- The BRDF tells us how to represent reflected radiance in terms of incoming radiance

$$L_r(x \to \Theta) = \int_{\Omega_x} f_r(x, \Psi \to \Theta) L(x \leftarrow \Psi) cos(\mathbf{n}_x, \Psi) \mathrm{d}\omega_{\Psi}$$

Rendering Equation

- The BRDF tells us how to represent reflected radiance in terms of incoming radiance

$$L_r(x \to \Theta) = \int_{\Omega_x} f_r(x, \Psi \to \Theta) L(x \leftarrow \Psi) cos(\mathbf{n}_x, \Psi) \mathrm{d}\omega_{\boldsymbol{\Psi}}$$

- Putting these together gives the rendering equation,

$$\begin{aligned} L(x \to \Theta) = L_e(x \to \Theta) \\ + \int_{\Omega_x} f_r(x, \Psi \to \Theta) L(x \leftarrow \Psi) cos(\mathbf{n}_x, \Psi) \mathrm{d}\omega_{\boldsymbol{\Psi}} \end{aligned}$$

Rendering Equation

- The BRDF tells us how to represent reflected radiance in terms of incoming radiance

$$L_r(x \to \Theta) = \int_{\Omega_x} f_r(x, \Psi \to \Theta) L(x \leftarrow \Psi) cos(\mathbf{n}_x, \Psi) \mathrm{d}\omega_{\Psi}$$

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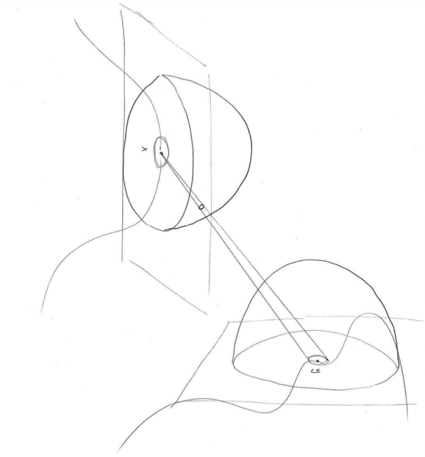
- We want to solve for the radiance $L(x \to \theta) \ \ \forall \{x, \theta\}$ for the whole scene, why is this complicated?

Rendering Equation



$$L(x \to \Theta) = L_e(x \to \Theta) + \int_{\Omega_x} f_r(x, \Psi \to \Theta) L(x \leftarrow \Psi) cos(\mathbf{n}_x, \Psi) \mathrm{d}\omega_\Psi$$

Summary

Summary

- We want to include indirect light in our rendering
- Hemispherical coordinates provides interface to do computations on
- Derivation of Rendering equation through BRDF
- Rendering as a transport problem

Next Time

Lecture Classical Radiosity

- First rendering technique for GI

Lab Finish up the 50% mark

- Think about extensions

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