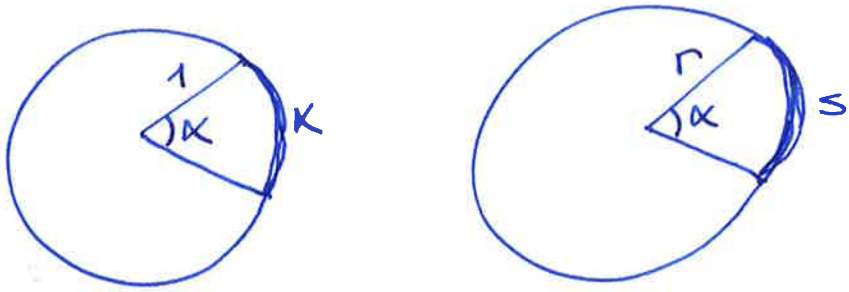
Solid Angles

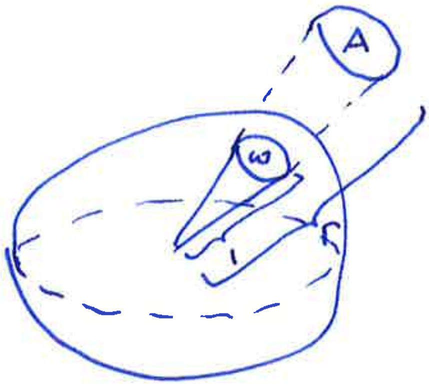


propotionality

$$\frac{\alpha}{s} = \frac{2\pi \cdot 1}{2\pi \cdot r}$$

$$\Rightarrow \alpha = \frac{s}{r}$$

Now lets do the same in 3D

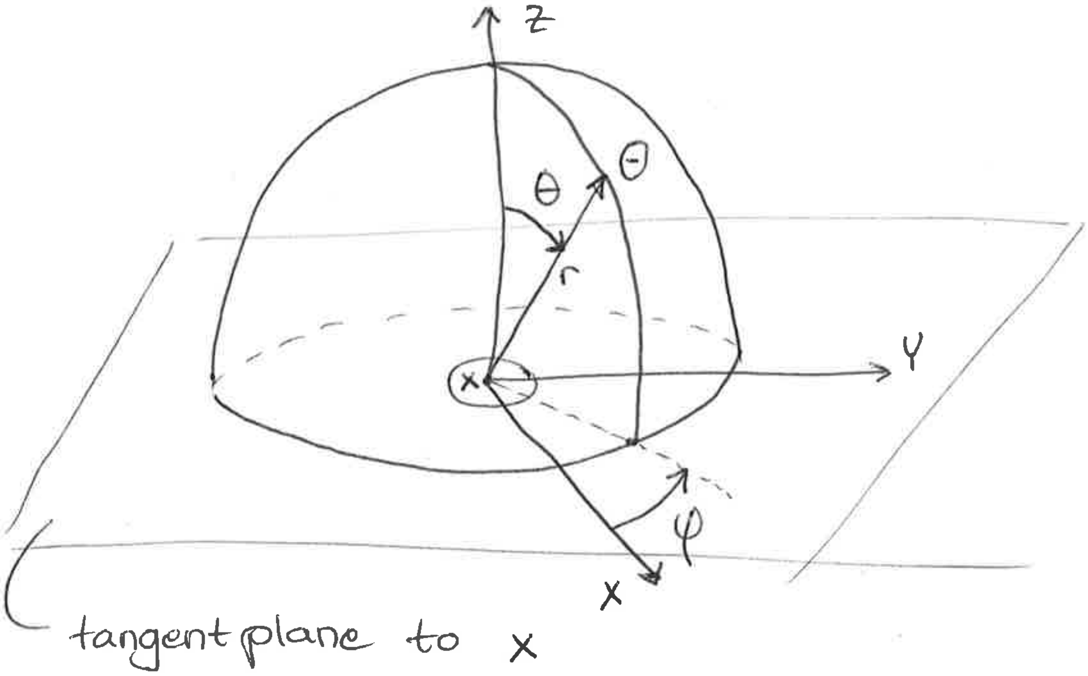


$$\frac{\omega}{A} = \frac{4\pi \cdot 1^2}{4\pi \cdot r^2}$$

$$\Rightarrow \omega = \frac{A}{r^2}$$

As a differential $d\omega = \frac{dA}{r^2}$

HEMISPHERICAL COORDINATES



Θ - direction on hemisphere

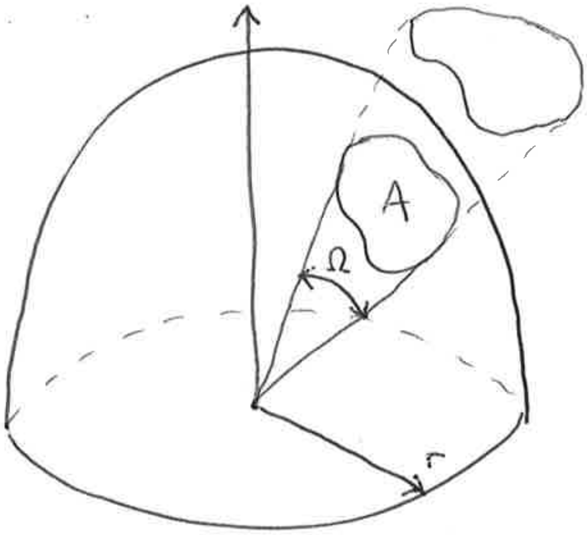
φ - azimuth angle "in" tangentplane to x $[0, 2\pi]$

θ - angle from normal $[0, \frac{\pi}{2})$

All points in half-plane in cartesian coordinates

$$\begin{cases} x = r \cdot \cos\varphi \cdot \sin\theta \\ y = r \cdot \sin\varphi \cdot \sin\theta \\ z = r \cdot \cos\theta \end{cases}$$

We will normally compute things going into or out of a point and use the sphere as an interface for these calculations therefore the radius does not make a difference so we can set it to 1



As a measure of quantity on the surface one commonly uses solid angles Ω

$$\Omega = \frac{A}{r^2}$$

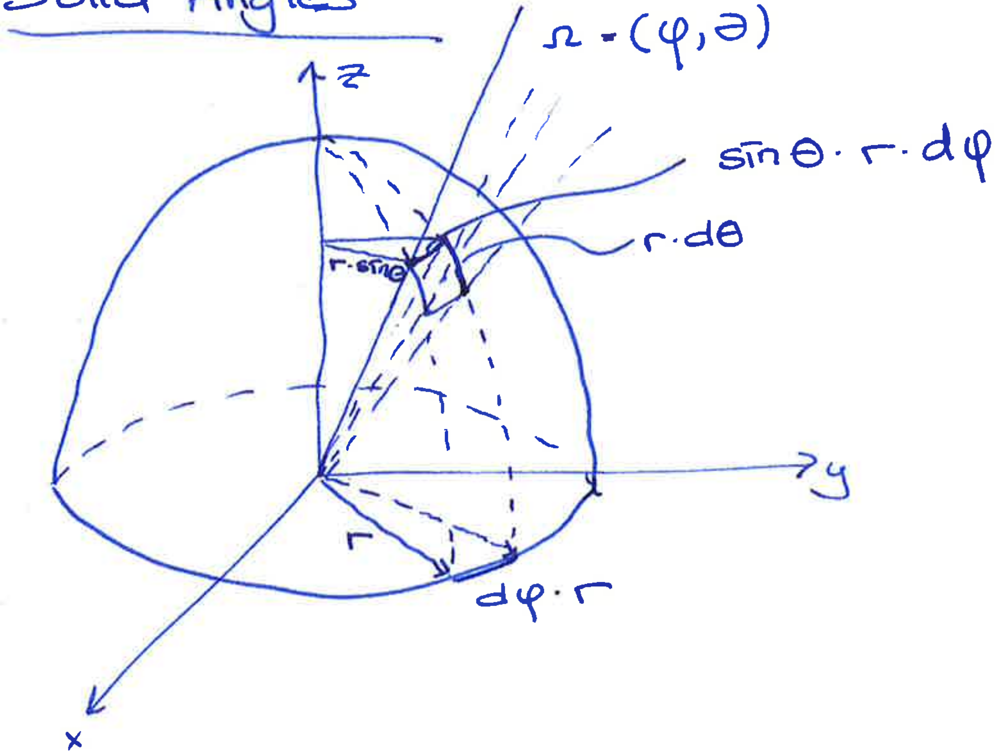
This means that it is the projected area onto the hemisphere.

Area of hemisphere surface = $2\pi r^2$

$$\Omega = \frac{A}{r^2} = \frac{2\pi r^2}{r^2} = 2\pi$$

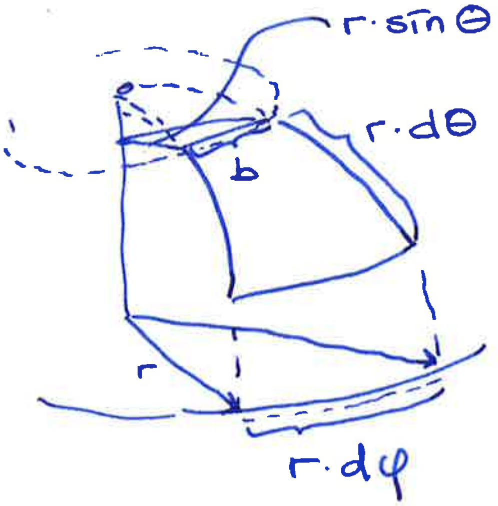
$\Rightarrow$ The solid angle that defines the whole sphere is 2π.

Solid Angles



$\Omega = (\varphi, \theta)$

now consider $\varphi + d\varphi$ and $\theta + d\theta$ which will now span a surface on the sphere



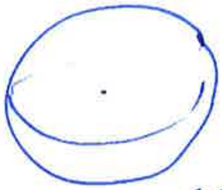
$$\frac{b}{\cancel{r} \cdot d\varphi} = \frac{r \cdot \sin\theta}{\cancel{r}}$$

$$\Rightarrow b = \sin\theta \cdot r \cdot d\varphi$$

$$\Rightarrow dA \approx (r \cdot \sin\theta \cdot d\varphi) \cdot (r \cdot d\theta) = r^2 \cdot \sin\theta \cdot d\varphi \cdot d\theta$$

$$d\omega = \frac{dA}{r^2} \approx \sin\theta \cdot d\varphi \cdot d\theta$$

Solid Angle



$$\Omega_{sphere} = \int_0^{2\pi}\int_0^{2\pi} \sin\theta \cdot d\theta \, d\varphi =$$

$$= \underbrace{\int_0^{2\pi} d\varphi}_{2\pi} \int_0^{\pi} \sin\theta \, d\theta = 2\pi \cdot \underbrace{\left[-\cos\theta\right]_0^{\pi}}_{} =$$

$$= 2\pi \cdot \underbrace{\left(-\left(-1-1\right)\right)}_{2} = 4\pi$$

$$\Omega_{hemisphere} = \int_0^{\pi/2}\int_0^{2\pi} \sin\theta \cdot d\theta \, d\varphi = 2\pi \cdot \left(-\left(0-1\right)\right) = 2\pi$$