University of BRISTOL

COMS 30115

Raytracing

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Introduction

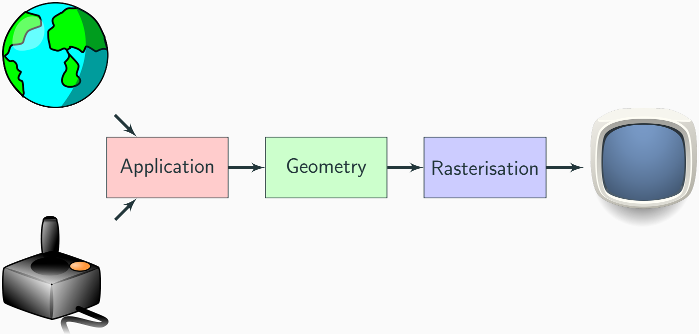
Today

- First rendering method
  - Raytracing/Raycasting
- Cameras
- Intersection calculations
- After today you should be able to start with Lab 1

- Introduction to Ray Tracing: a Simple Method for Creating 3D Images
- An Overview of the Ray-Tracing Rendering Technique

Raytracing

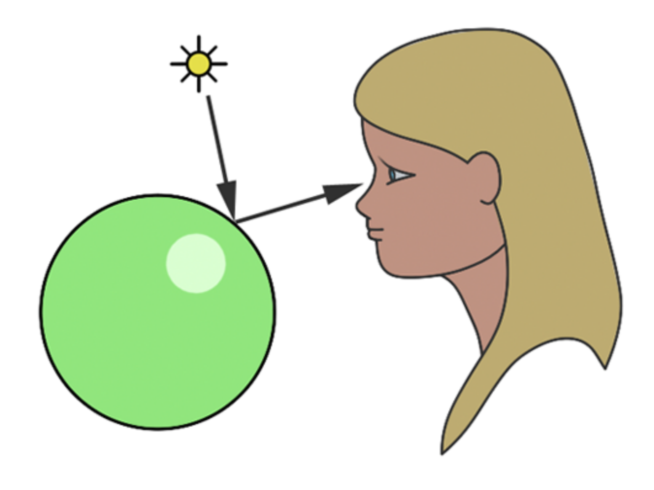
Pipeline



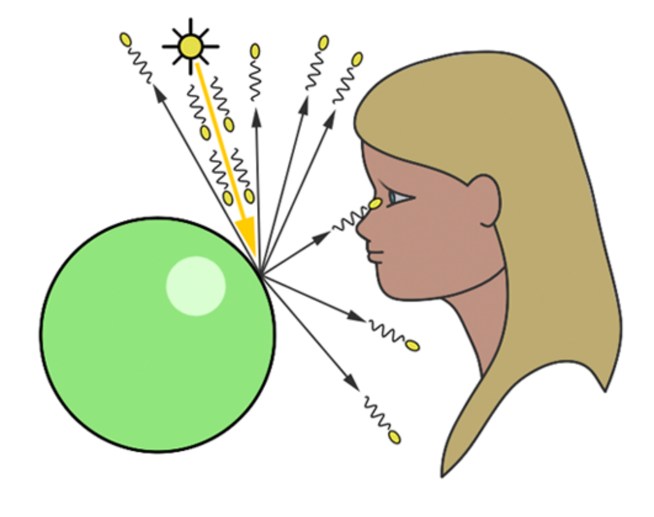
Pipeline


Geometry
View Transformation
Lighting
Projection
Clipping
Screen Mapping
Primitives

Raytracing



Raytracing



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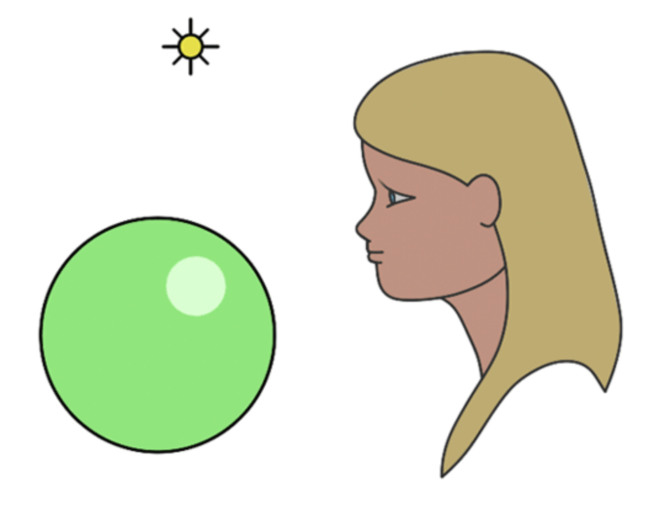

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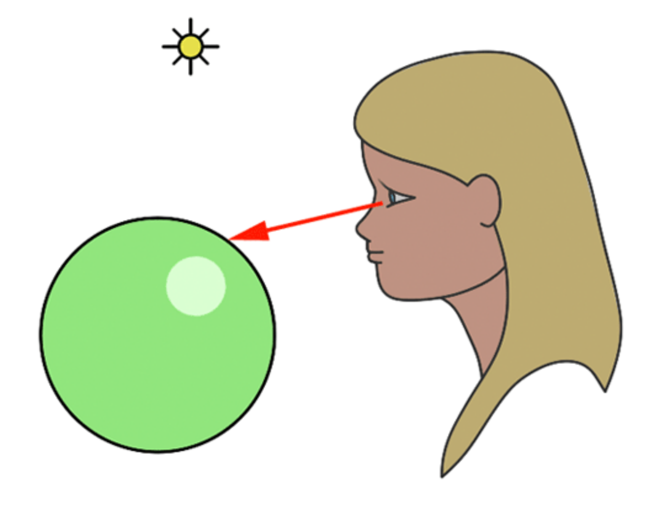
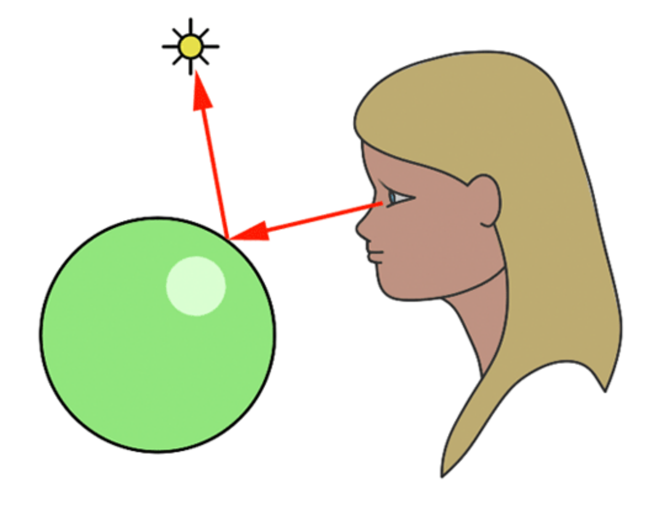


Raytracing



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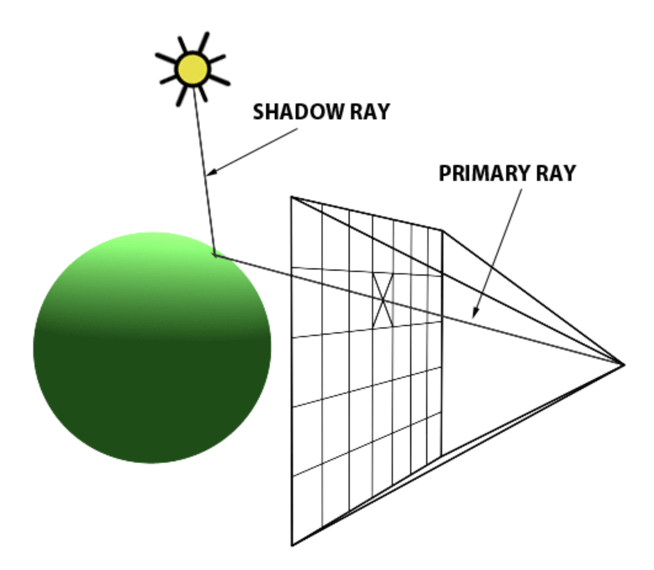

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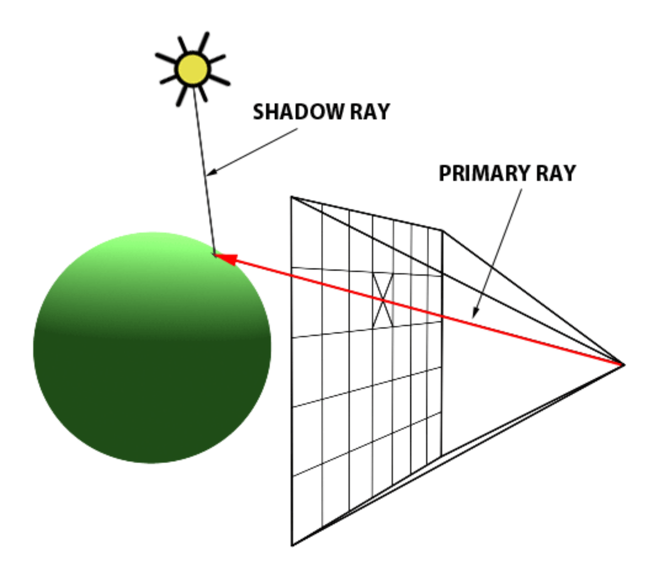

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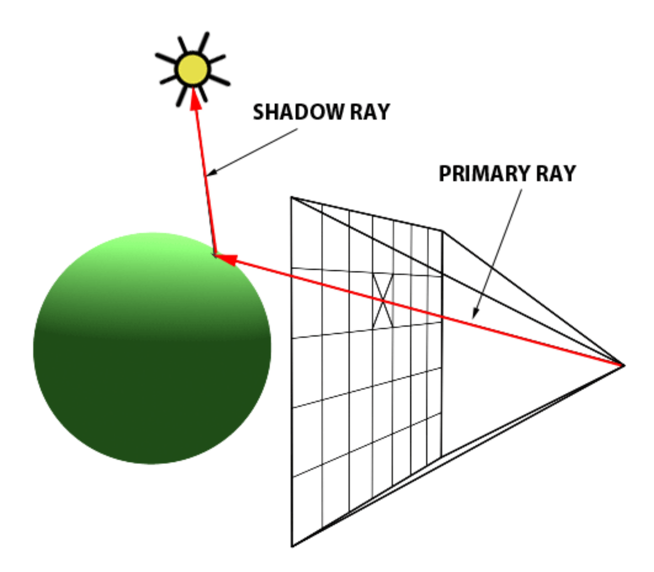

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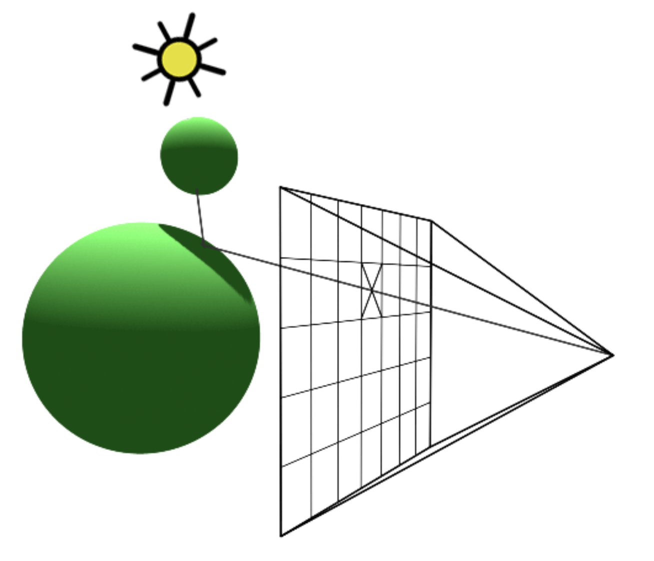

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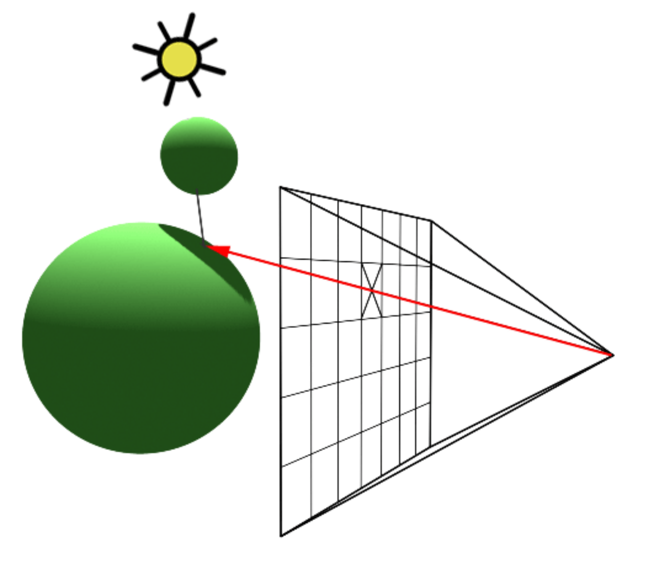

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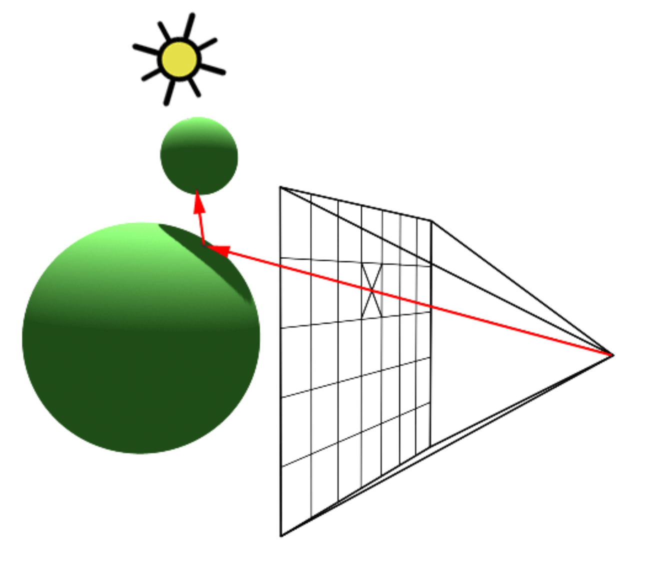
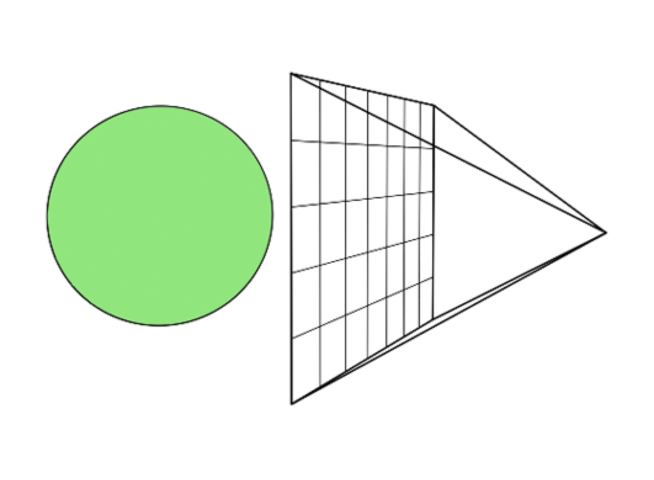


Raycasting Anim



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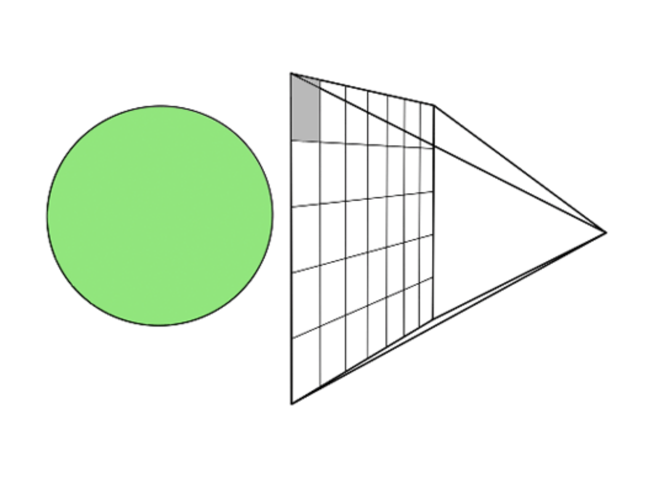

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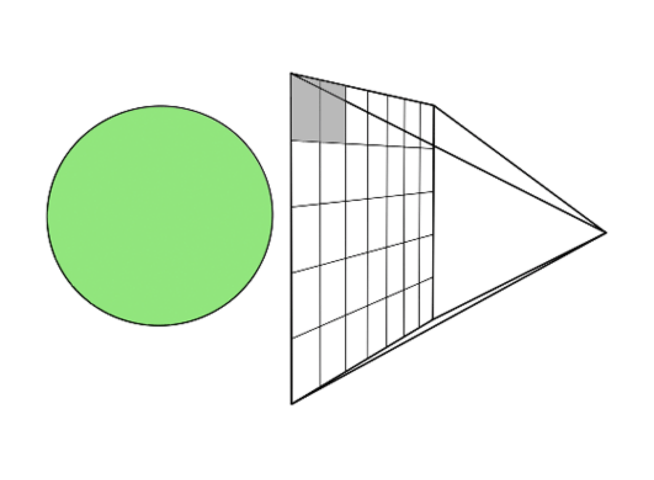

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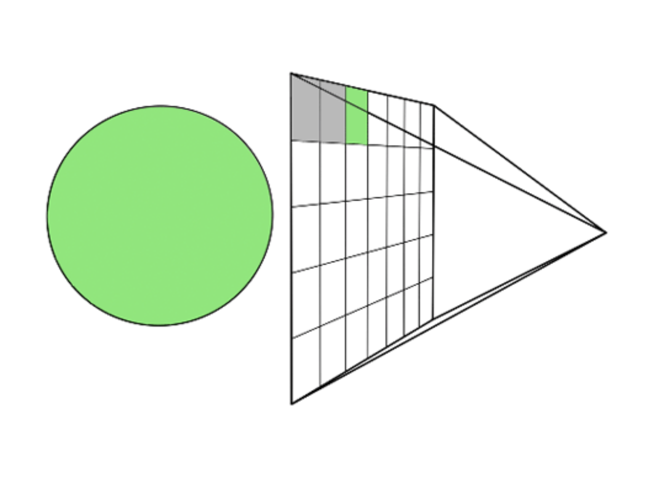

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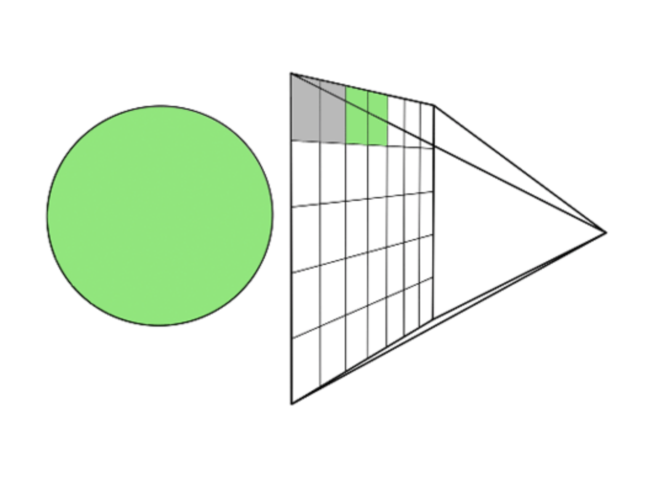

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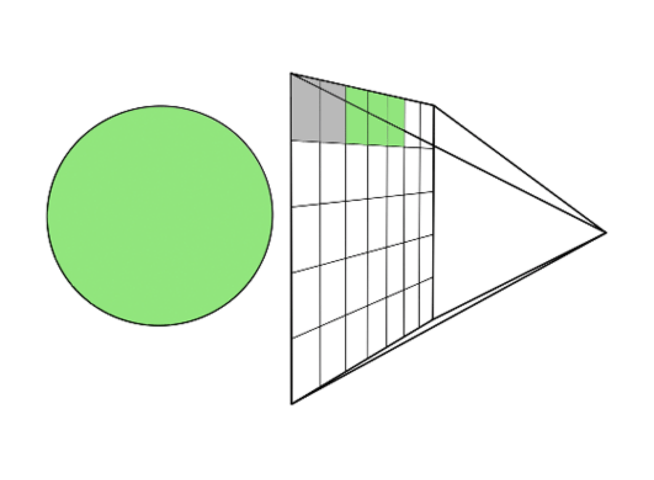

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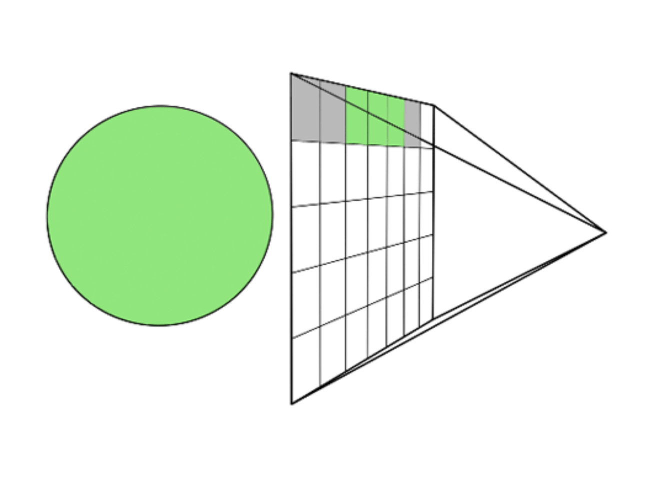

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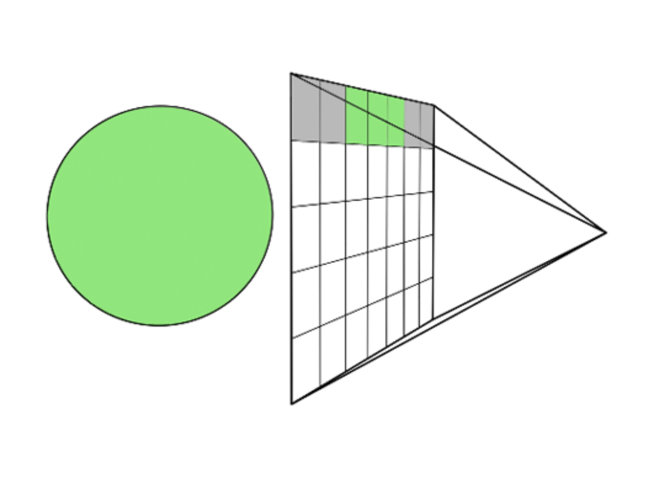

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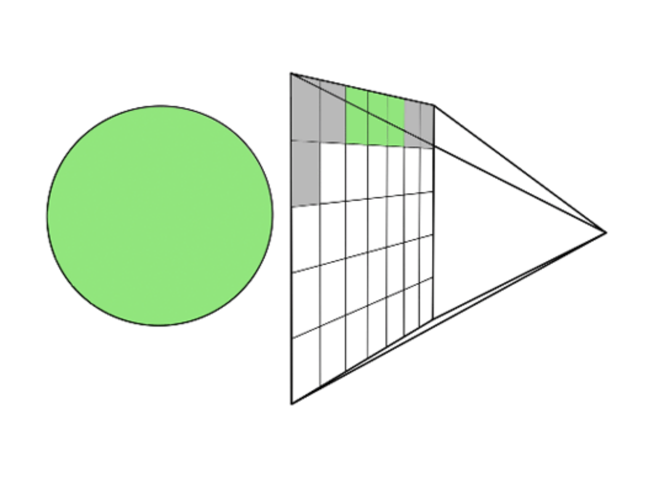

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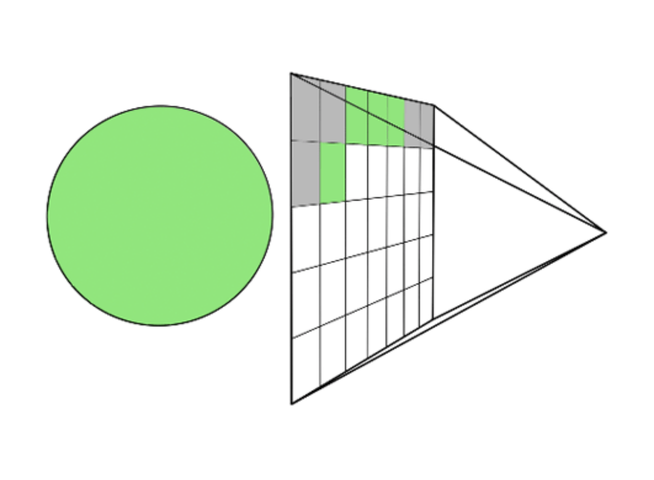

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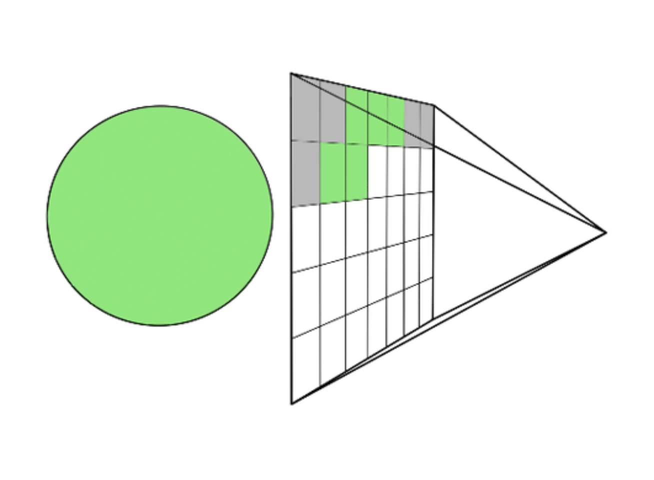

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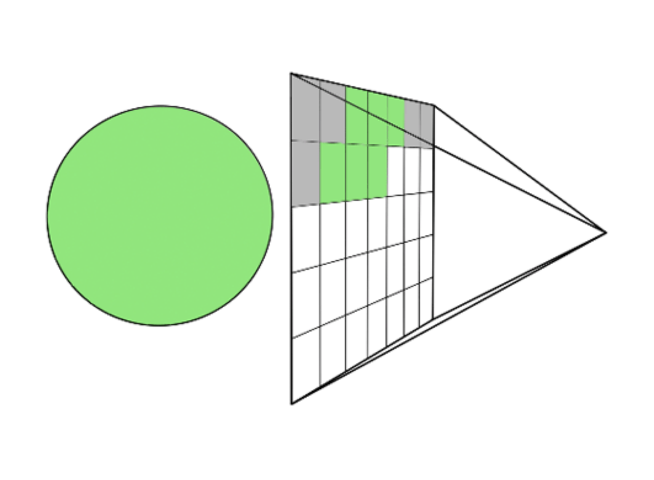

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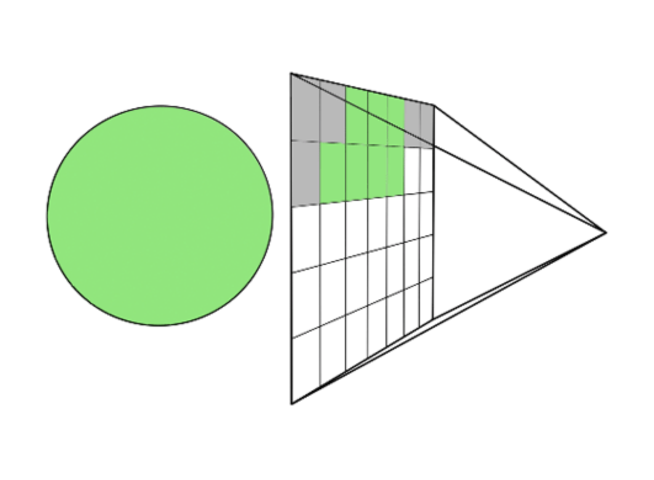

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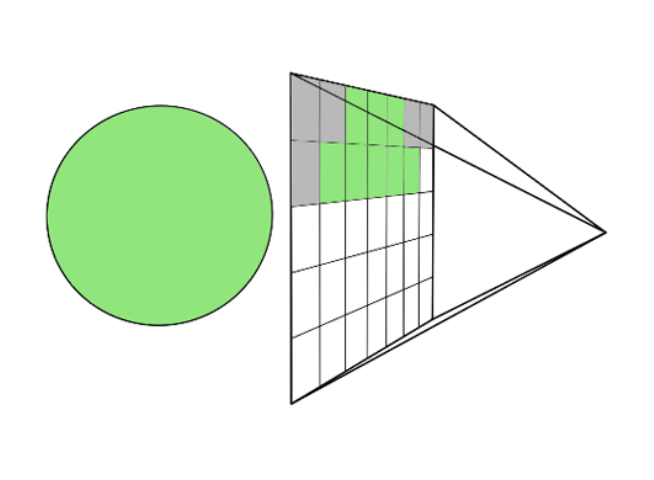

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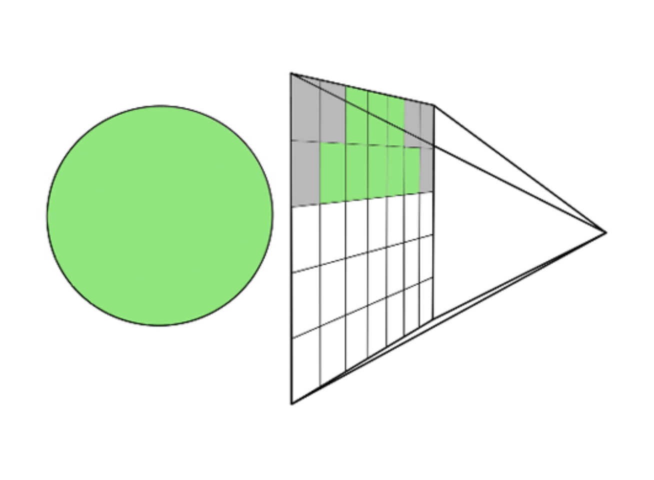

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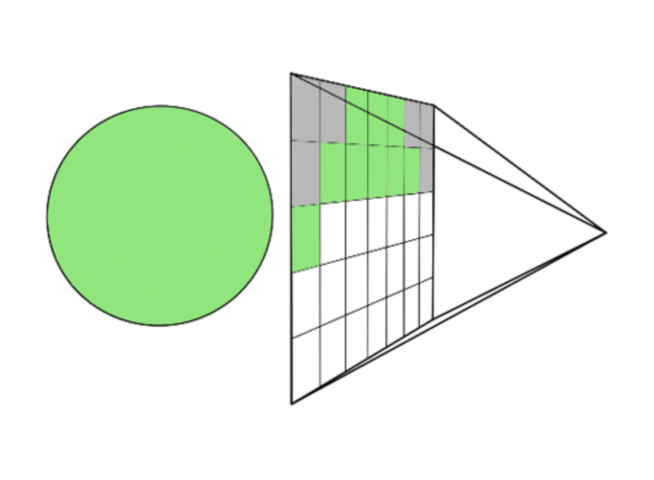

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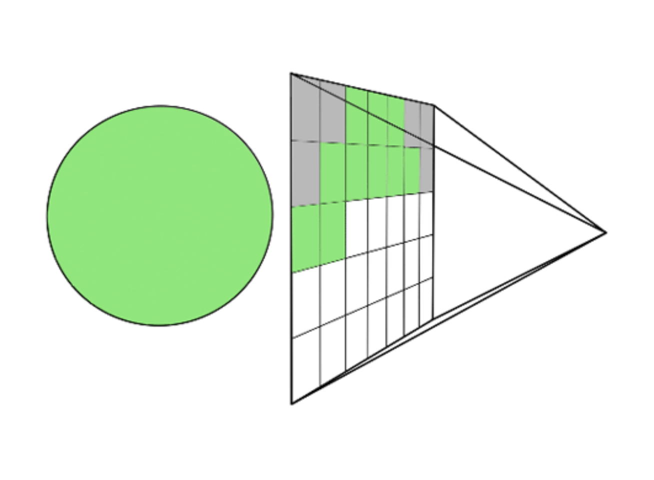

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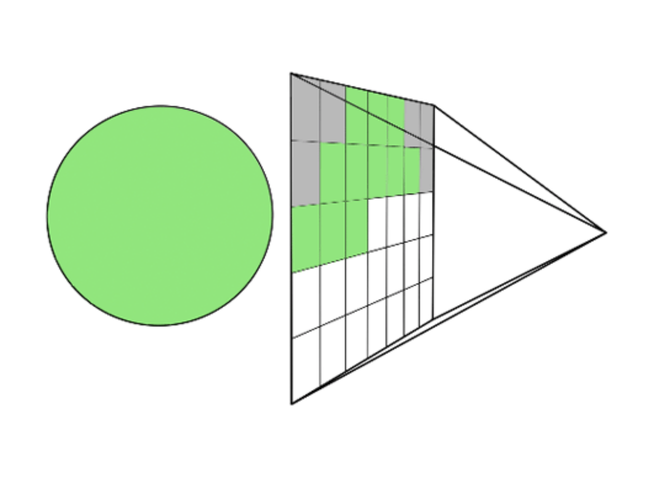

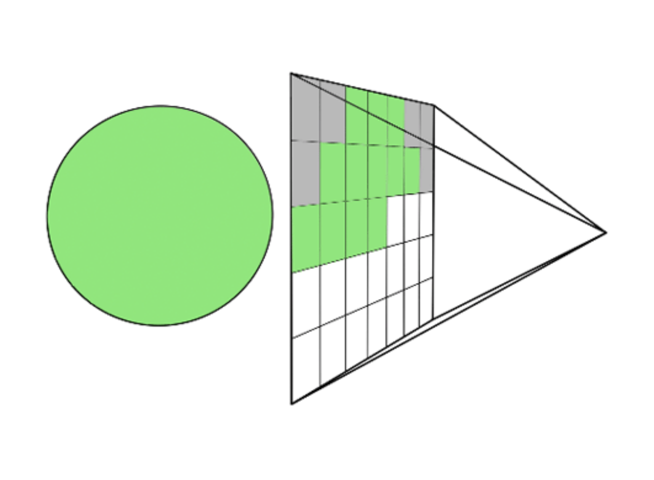

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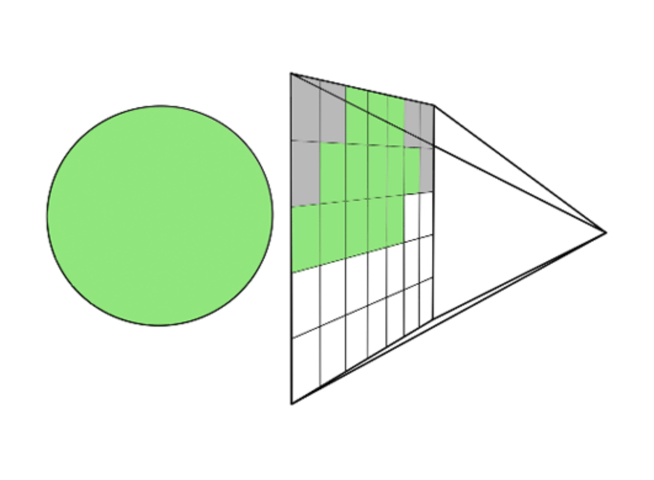

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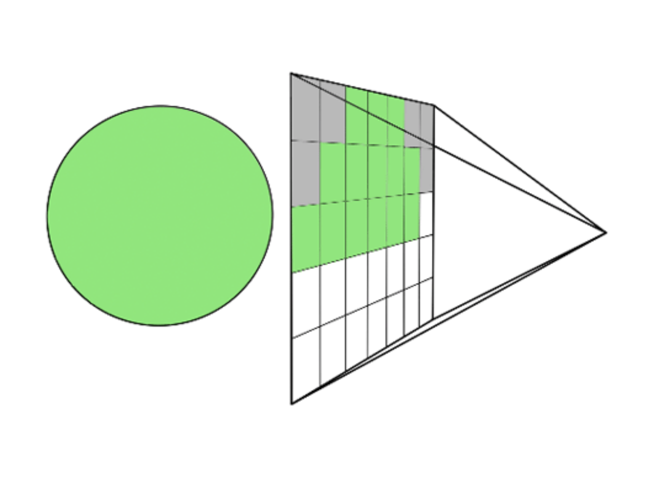

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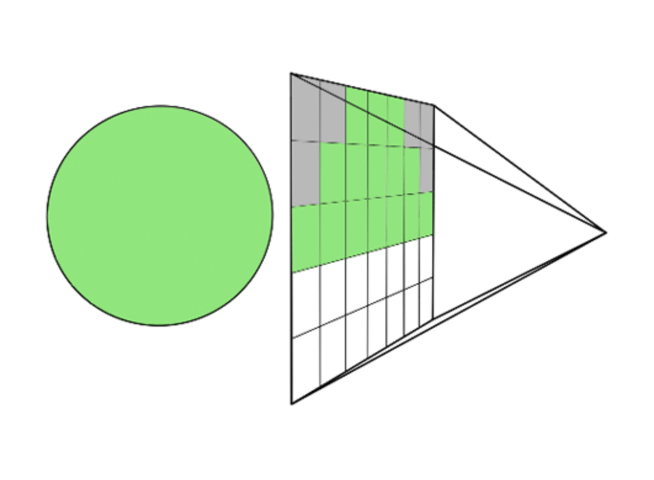

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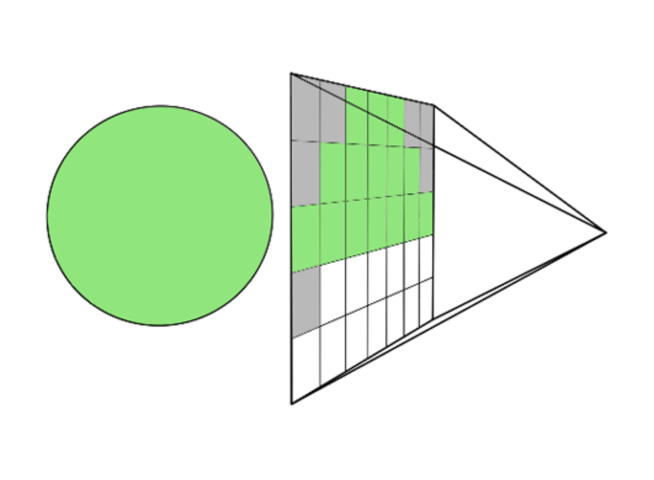

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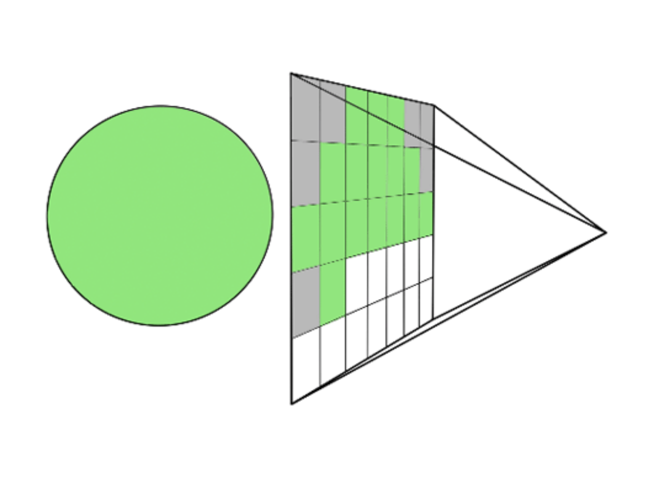

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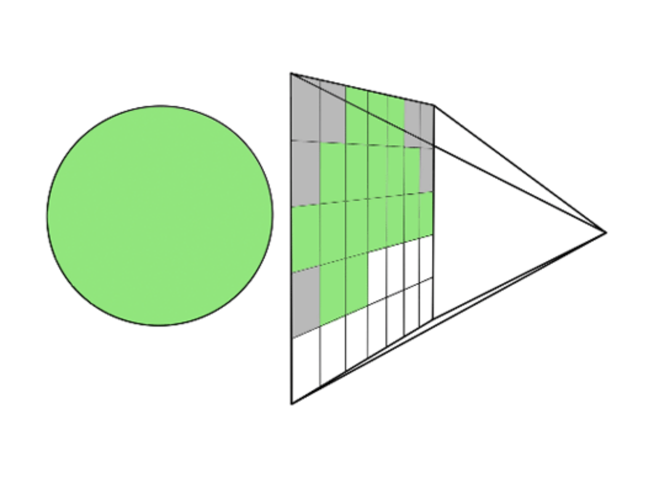

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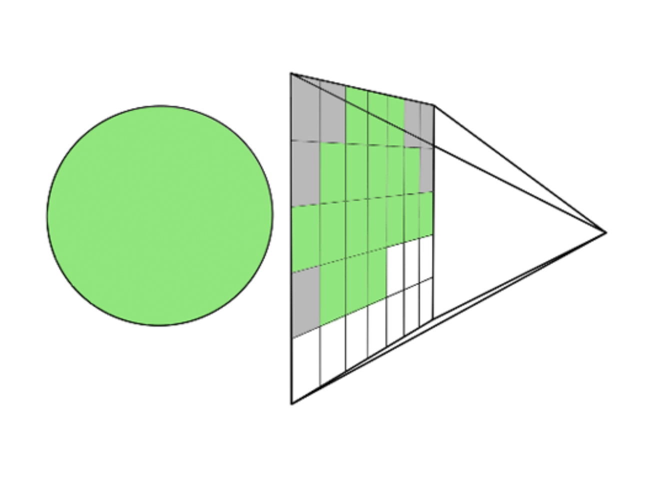

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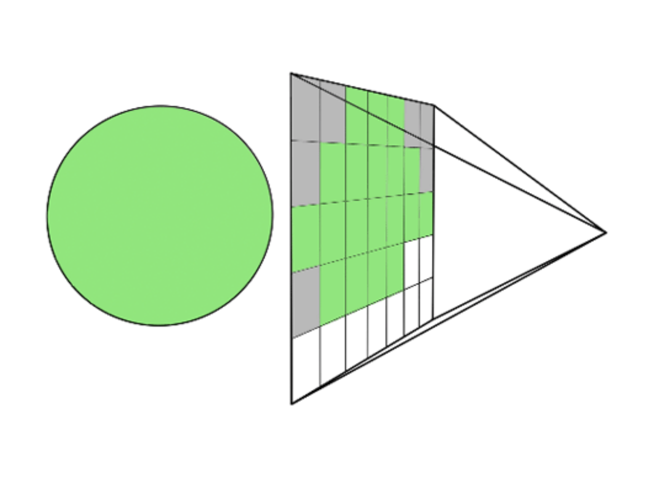

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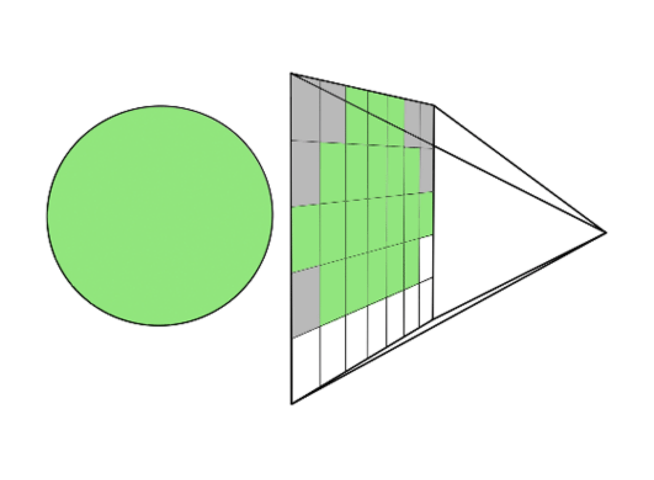

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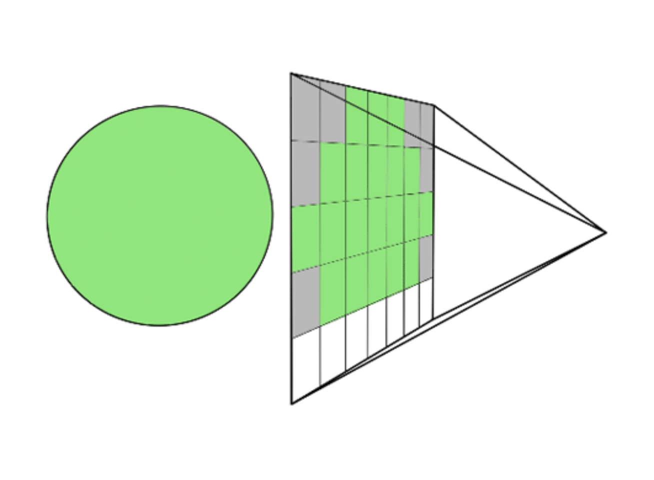

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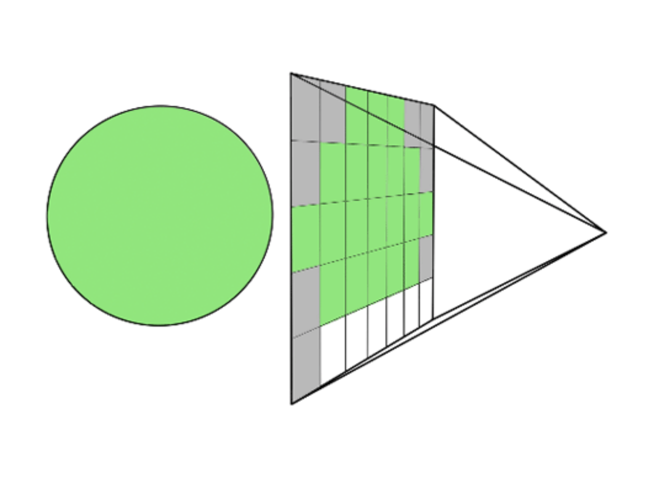

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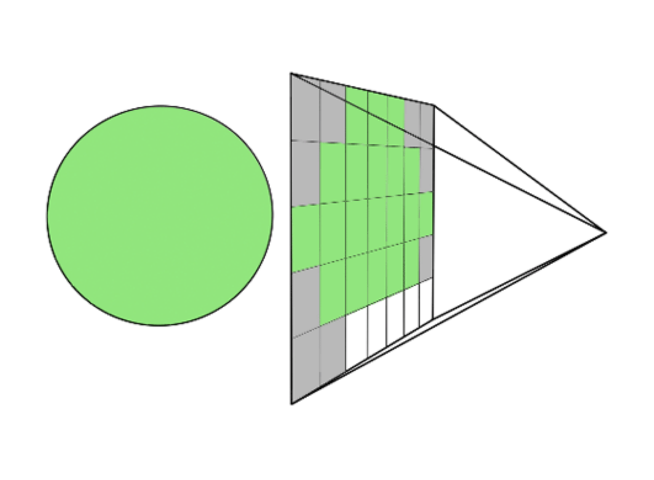

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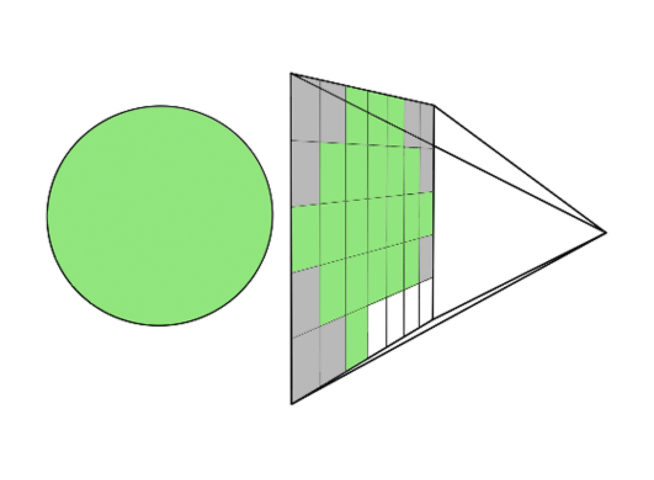

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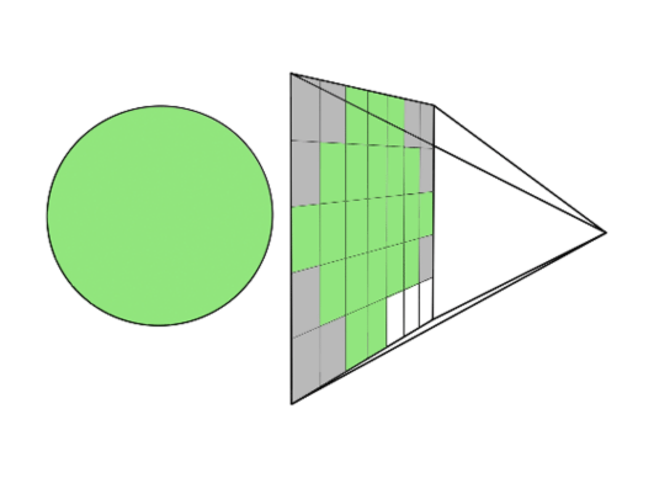

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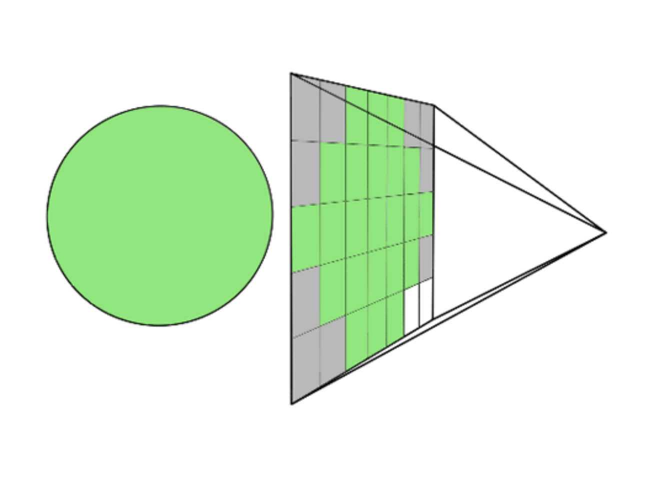

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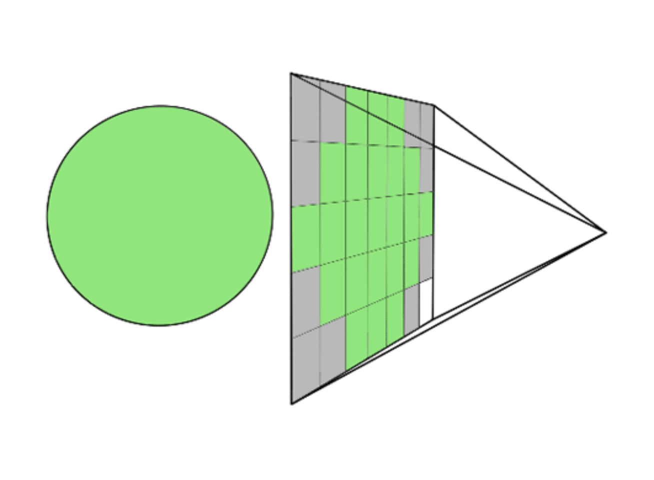

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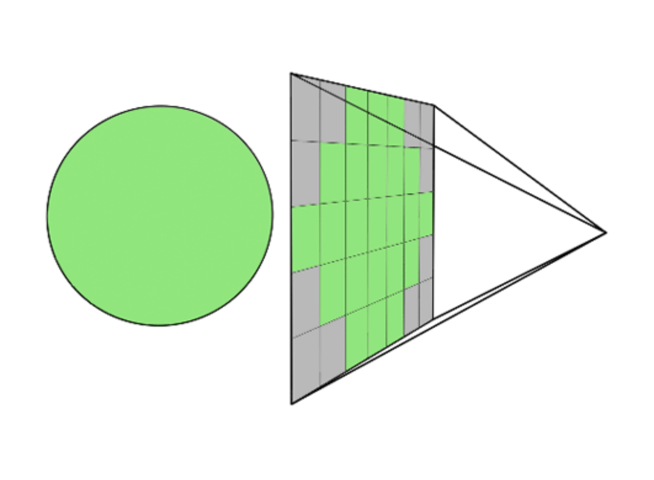


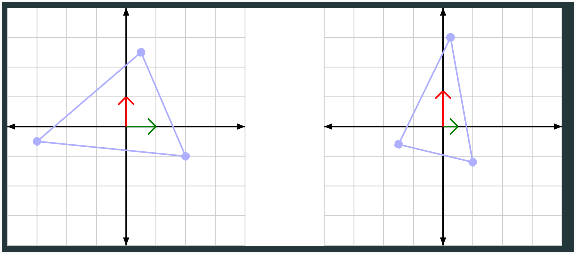
Linear Algebra

Vector Spaces

Point in vector space does not make sense without basis

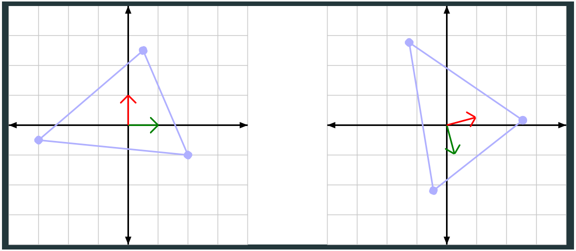
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = [\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3] \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Scaling



$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = [v_x \mathbf{e}_1, v_y \mathbf{e}_2, v_z \mathbf{e}_3] \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} v_x & 0 & 0 \\ 0 & v_y & 0 \\ 0 & 0 & v_z \end{bmatrix} [\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3] \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Rotation



$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \mathbf{R}\,[\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3] \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Rotation

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Matrix multiplication is not commutative
    - $\mathbf{AB} \neq \mathbf{BA}$
    - Need to determine order convention
- Pick and order and stick with it

Rotation

$$R_{xyz} = \begin{bmatrix} \cos\theta_y \cos\theta_z & -\cos\theta_x \sin\theta_z + \sin\theta_x \sin\theta_y \cos\theta_z & \sin\theta_x \sin\theta_z + \cos\theta_x \sin\theta_y \cos\theta_z \\ \cos\theta_y \sin\theta_z & \cos\theta_x \cos\theta_z + \sin\theta_x \sin\theta_y \sin\theta_z & -\sin\theta_x \cos\theta_z + \cos\theta_x \sin\theta_y \sin\theta_z \\ -\sin\theta_y & \sin\theta_x \cos\theta_y & \cos\theta_x \cos\theta_y \end{bmatrix}$$

- Matrix multiplication is not commutative
    - $\mathbf{AB} \neq \mathbf{BA}$
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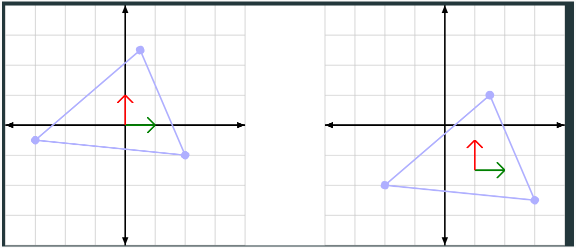
Translation



$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = [\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3] \begin{bmatrix} x + dx \\ y + dy \\ z + dz \end{bmatrix}$$

Homogenous Coordinates[1]

- We do not want to deal with translation differently compared to other transformations
- We can work in *projective* or *homogenous* coordinate representations instead
- We will append the space with one dimension and specify an equivivalence class of all projections

[1] `https://en.wikipedia.org/wiki/Homogeneous_coordinates`

Homogenous Coordinates

- Perspective Projection

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \frac{x}{z} \\ \frac{y}{z} \end{bmatrix}$$

- Specify equivivalence class between all projected points

$$\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \sim \begin{bmatrix} 4 \\ 6 \\ 8 \end{bmatrix}$$

- Homogenous coordinates

$$\begin{bmatrix} \frac{2}{4} \\ \frac{3}{4} \\ \frac{4}{4} \end{bmatrix} = \begin{bmatrix} \frac{4}{8} \\ \frac{6}{8} \\ \frac{8}{8} \end{bmatrix} = \begin{bmatrix} \frac{2}{4} \\ \frac{3}{4} \\ 1 \end{bmatrix}$$

Homogenous Coordinates

- Cartesian 3D space

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

- Allows for representing points in infinity

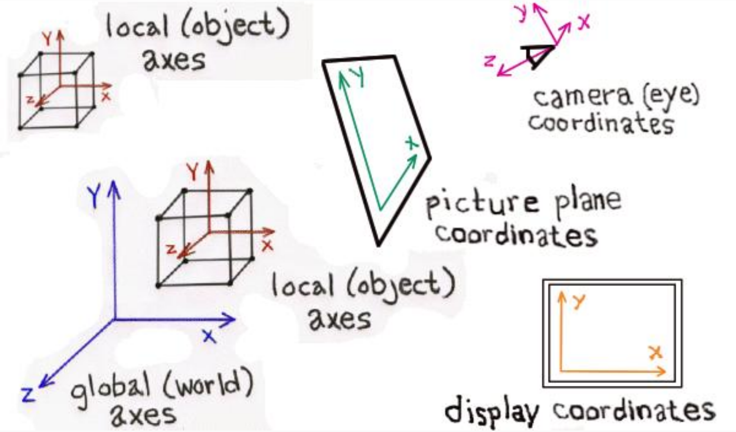
$$\lim_{d \to \infty} \begin{bmatrix} x + d \\ y + d \\ z + d \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ 0 \end{bmatrix}$$

- For example, under projection two parallell lines will meet

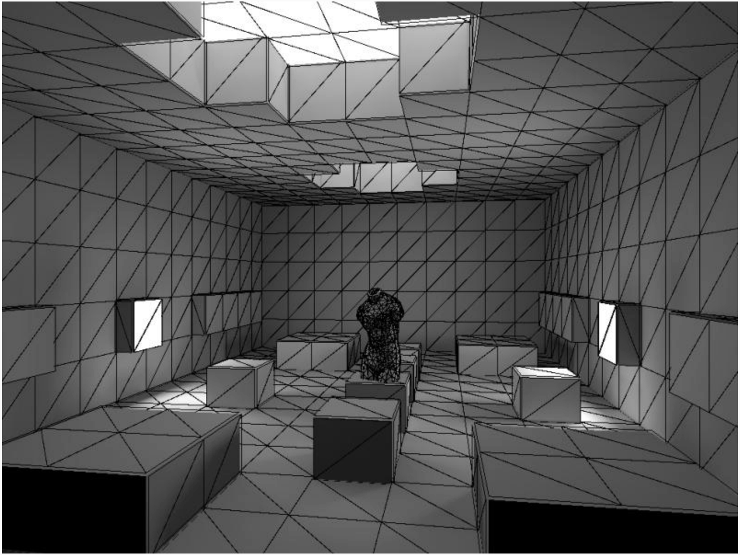
Homogenous Coordinates

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} v_x \cdot R_{11} & R_{12} & R_{13} & dx \\ R_{21} & v_y \cdot R_{22} & R_{23} & dy \\ R_{31} & R_{32} & v_z \cdot R_{33} & dz \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

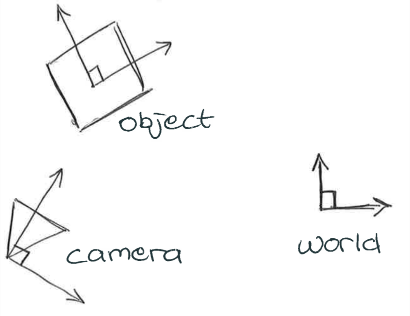
Cordinate Transforms



Scene



View Transformations



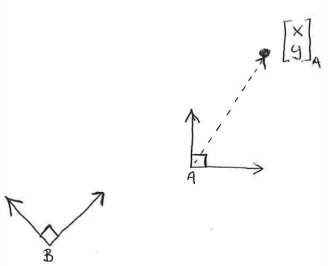
View Transformations

1. Translate

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}_A - \begin{bmatrix} x \\ y \end{bmatrix}_{O_B}$$

2. Rotate (negative)

$$\begin{bmatrix} x \\ y \end{bmatrix}_B = \begin{bmatrix} cos(-\theta) & -sin(-\theta) \\ sin(-\theta) & cos(-\theta) \end{bmatrix}_B \begin{bmatrix} x' \\ y' \end{bmatrix}$$



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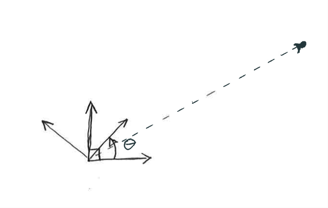

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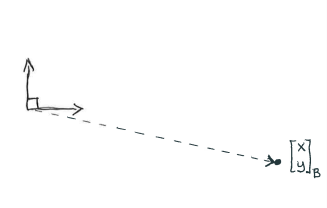

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View Transformations

Transformation

Always the same order for every transformation of coordinate systems

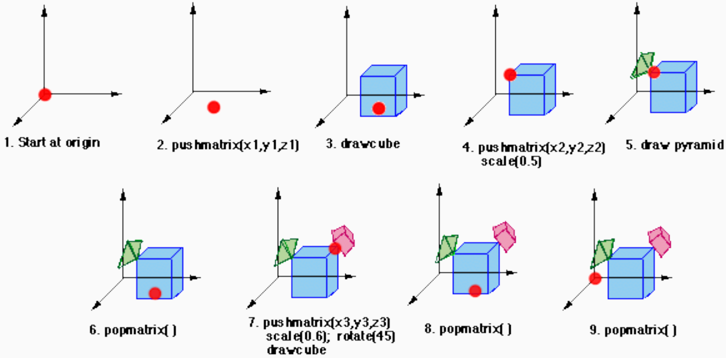
1. Translate
2. Rotate

Transformation Homogenous Coordinates

$$\begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} R_{11}(\theta) & R_{12}(\theta) & R_{13}(\theta) & a \\ R_{21}(\theta) & R_{22}(\theta) & R_{23}(\theta) & b \\ R_{31}(\theta) & R_{32}(\theta) & R_{33}(\theta) & c \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & -a \\ 0 & 1 & 0 & -b \\ 0 & 0 & 1 & -c \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Rotate θ degrees around x-axis at $[a, b, c]^{\mathrm{T}}$
- Remember that matrix multiplications are not cummutative

Transformation Stack

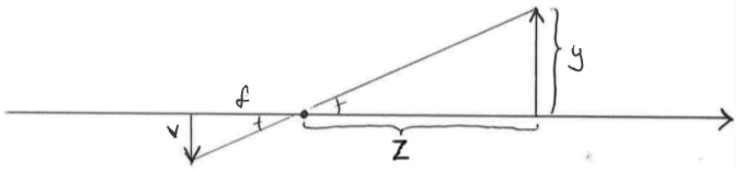


Raytracing

Camera

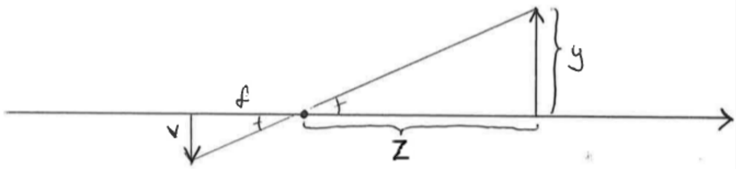
- Cameras create images
- Lens
  - captures rays
- Focal lenth
- Shutter opening

Pinhole camera



- Simplest form camera, where one ray of light lits up each pixel
- Defined by a *location*, *focal length*, *view-direction* and *up-direction*
- is this a realistic camera?

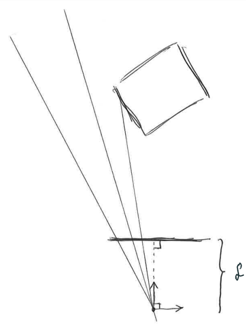
Pinhole camera



$$\frac{v}{f} = \frac{y}{z}$$
$$\Rightarrow v = \frac{f}{z} y$$

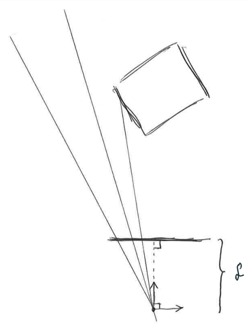
Raycasting

1. Camera at $[0, 0, 0]^{\mathrm{T}}$



Raycasting

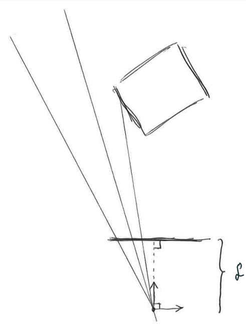
1. Camera at $[0, 0, 0]^{\mathrm{T}}$
2. View direction along z-axis $[0, 0, 1]^{\mathrm{T}}$



Raycasting

1. Camera at $[0, 0, 0]^{\mathrm{T}}$
2. View direction along z-axis $[0, 0, 1]^{\mathrm{T}}$
3. Image plane

$$\{[u, v, f]^{\mathrm{T}} | -\frac{W}{2} \leq u < \frac{W}{2}, -\frac{H}{2} < v < \frac{H}{2}, f\}$$



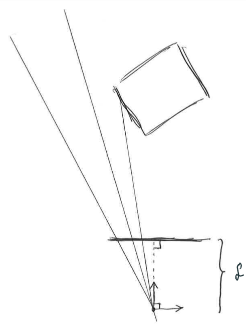
Raycasting

1. Camera at $[0, 0, 0]^{\mathrm{T}}$
2. View direction along z-axis $[0, 0, 1]^{\mathrm{T}}$
3. Image plane

$$\{[u, v, f]^{\mathrm{T}} | -\frac{W}{2} \leq u < \frac{W}{2}, -\frac{H}{2} < v < \frac{H}{2}, f\}$$

4. Compute normalised vector from origin to image-plane

$$\mathbf{d}_{ij} = \frac{1}{\sqrt{[u_i, v_j, f][u_i, v_j, f]^{\mathrm{T}}}} [u, v, f]^{\mathrm{T}}$$



Raycasting

1. Camera at $[0, 0, 0]^{\mathrm{T}}$
2. View direction along z-axis $[0, 0, 1]^{\mathrm{T}}$
3. Image plane

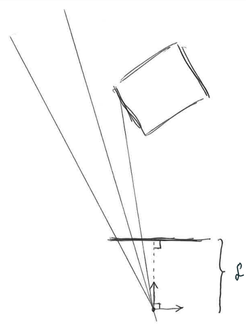
$$\{[u, v, f]^{\mathrm{T}} | -\frac{W}{2} \leq u < \frac{W}{2}, -\frac{H}{2} < v < \frac{H}{2}, f\}$$

4. Compute normalised vector from origin to image-plane

$$\mathbf{d}_{ij} = \frac{1}{\sqrt{[u_i, v_j, f][u_i, v_j, f]^{\mathrm{T}}}}[u, v, f]^{\mathrm{T}}$$

5. Parmatrise Ray

$$\mathbf{r}_{ij} = \mathbf{s} + t \cdot \mathbf{d}_{ij}$$



Raycasting

1. Camera at $[0, 0, 0]^{\mathrm{T}}$
2. View direction along z-axis $[0, 0, 1]^{\mathrm{T}}$
3. Image plane

$$\{[u, v, f]^{\mathrm{T}} | -\frac{W}{2} \leq u < \frac{W}{2}, -\frac{H}{2} < v < \frac{H}{2}, f\}$$

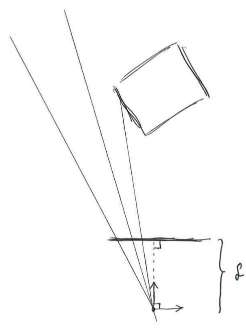
4. Compute normalised vector from origin to image-plane

$$\mathbf{d}_{ij} = \frac{1}{\sqrt{[u_i, v_j, f][u_i, v_j, f]^{\mathrm{T}}}} [u, v, f]^{\mathrm{T}}$$

5. Parmatrise Ray

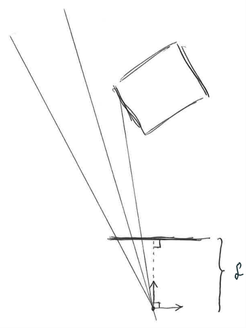
$$\mathbf{r}_{ij} = \mathbf{s} + t \cdot \mathbf{d}_{ij}$$

6. One degree-of-freedom

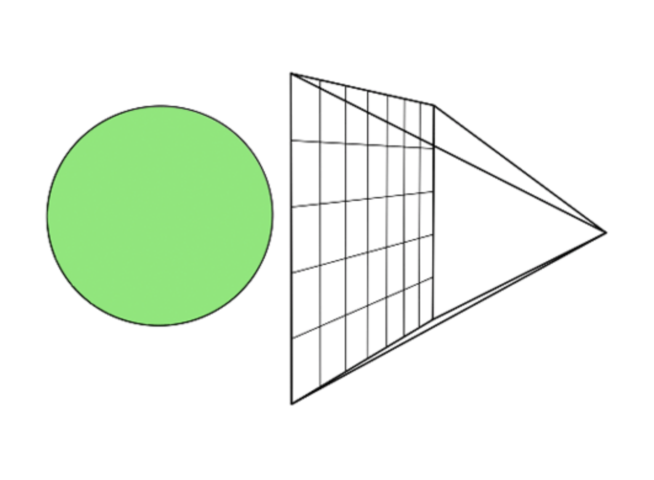


Raycasting

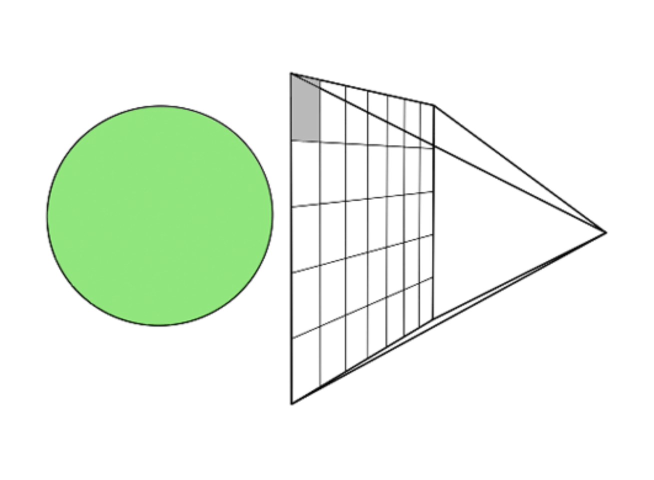
- Trace a ray for each pixel
- Compute intersection with all objects in world
- Plot colour of intersecting surface
- No "lighting" often called Raycaster



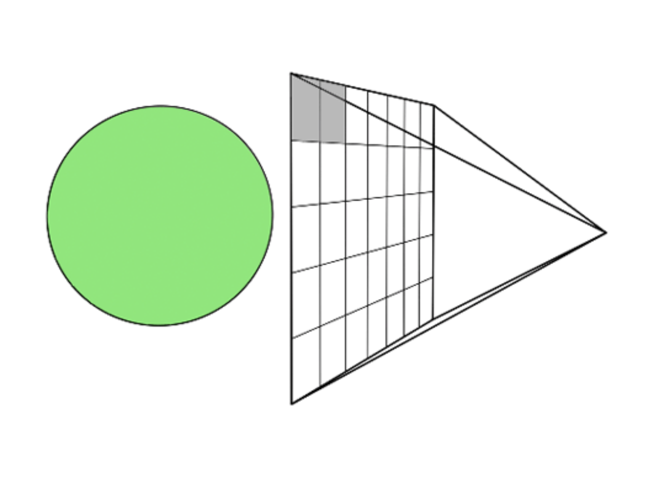
Raycasting Anim



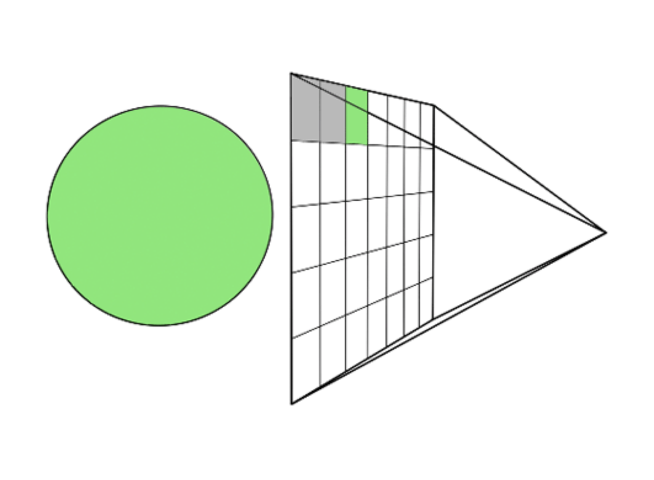
Raycasting Anim



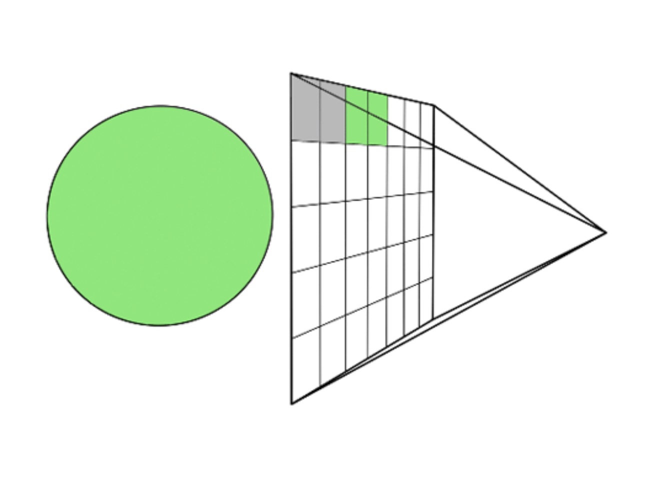
Raycasting Anim



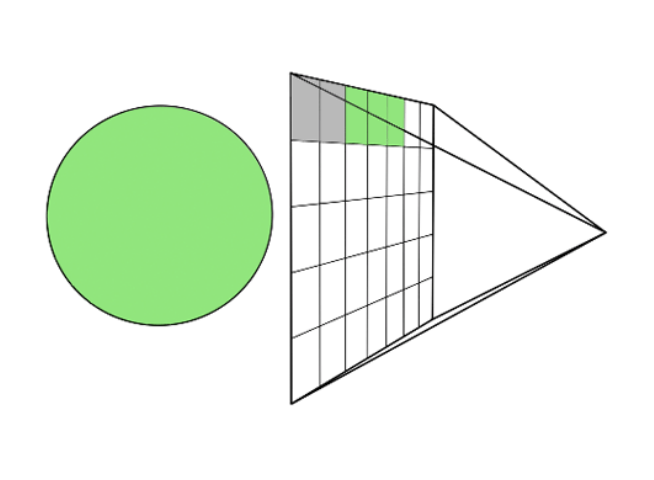
Raycasting Anim



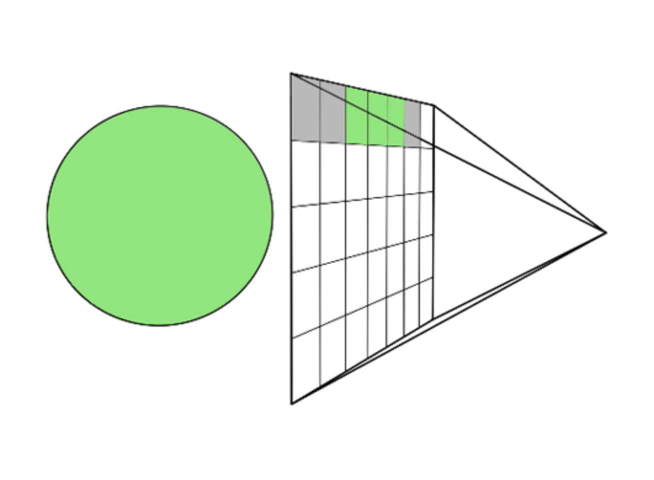
Raycasting Anim



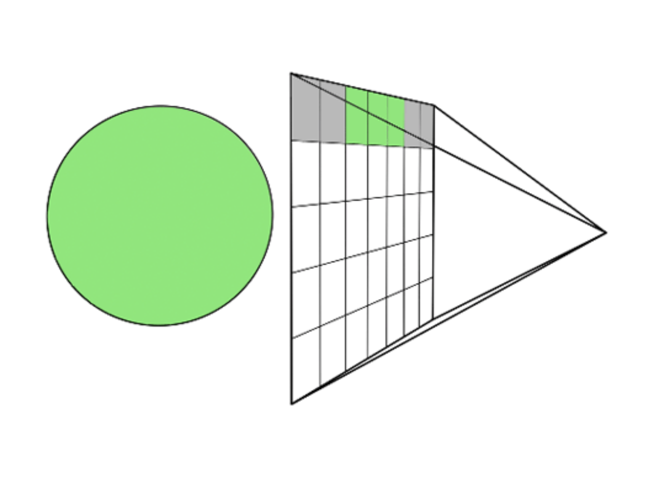
Raycasting Anim



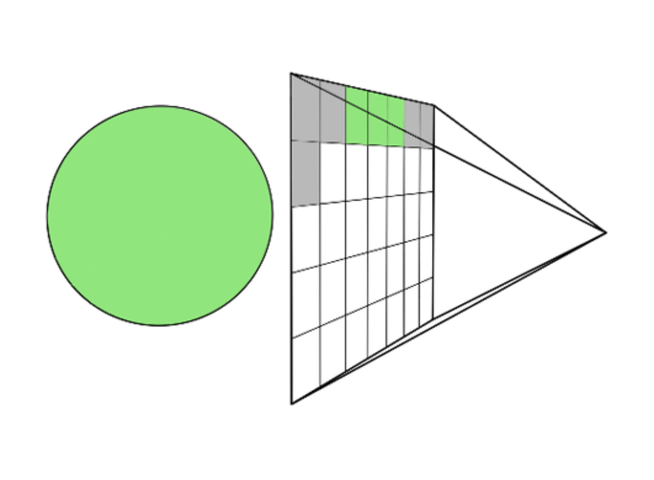
Raycasting Anim



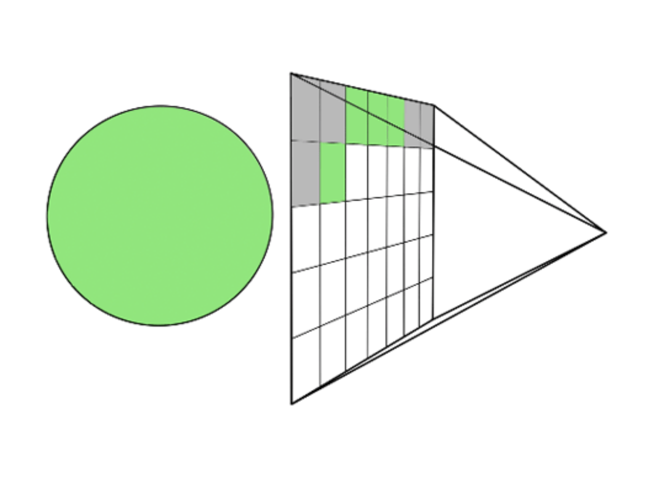
Raycasting Anim



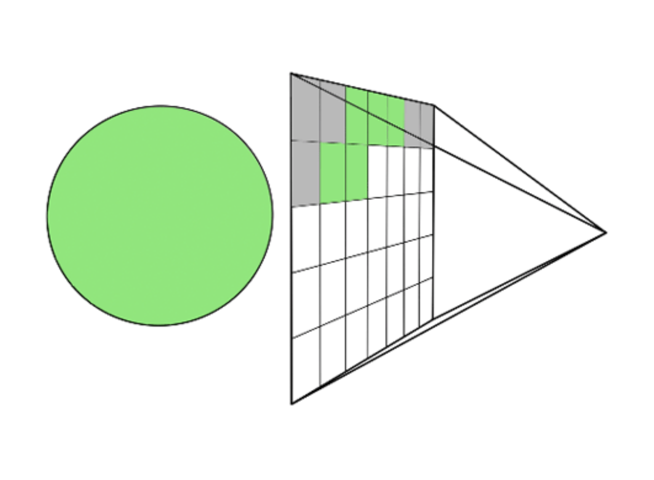
Raycasting Anim



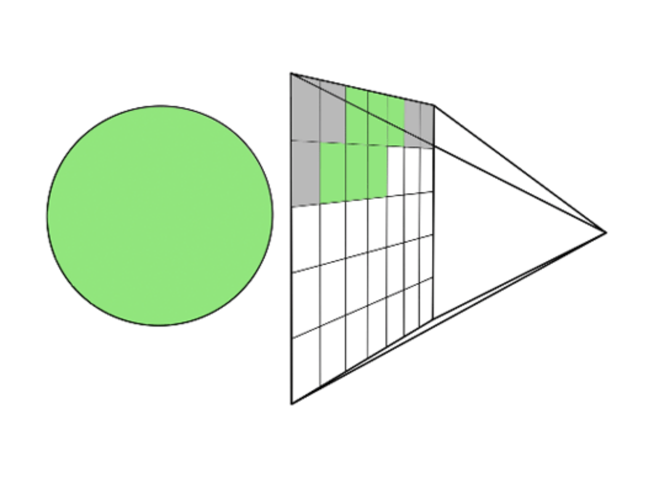
Raycasting Anim



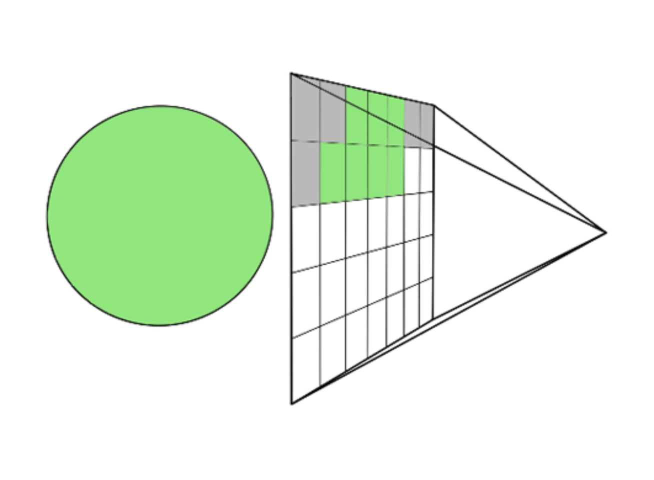
Raycasting Anim



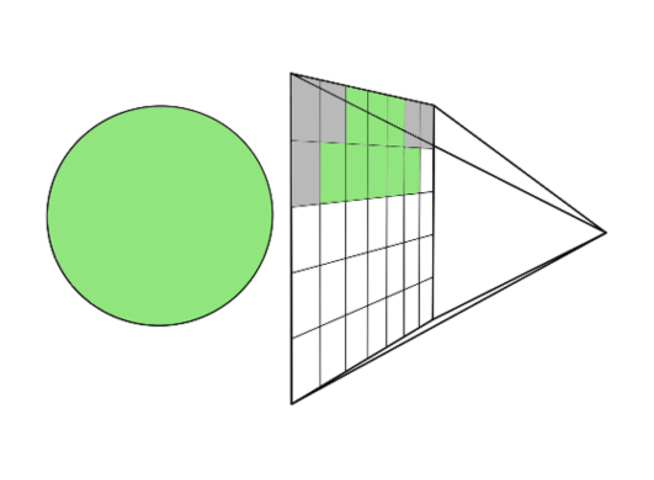
Raycasting Anim



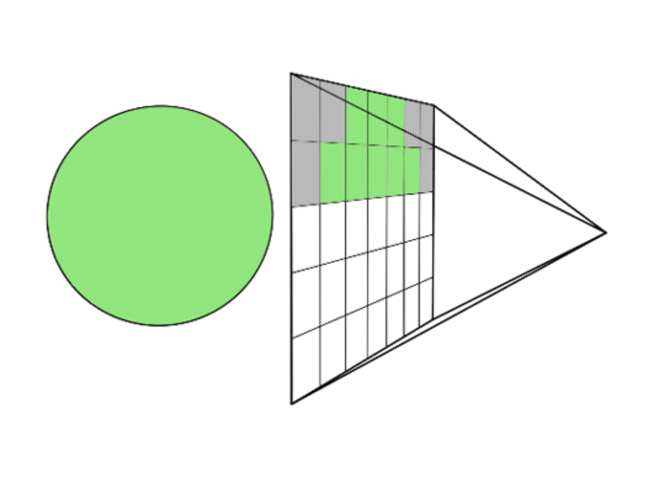
Raycasting Anim



Raycasting Anim



Raycasting Anim



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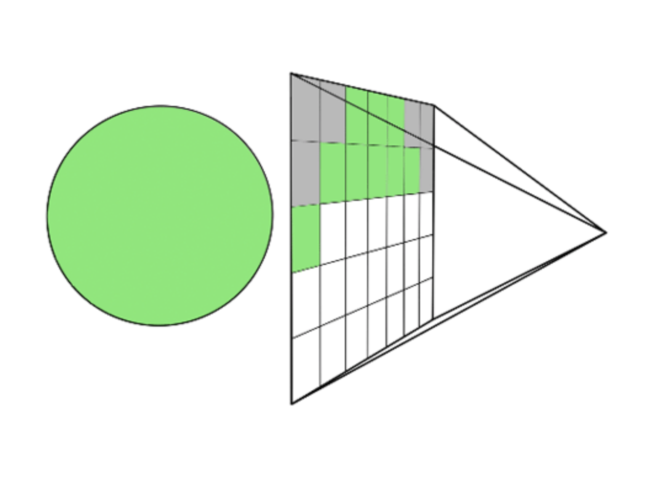

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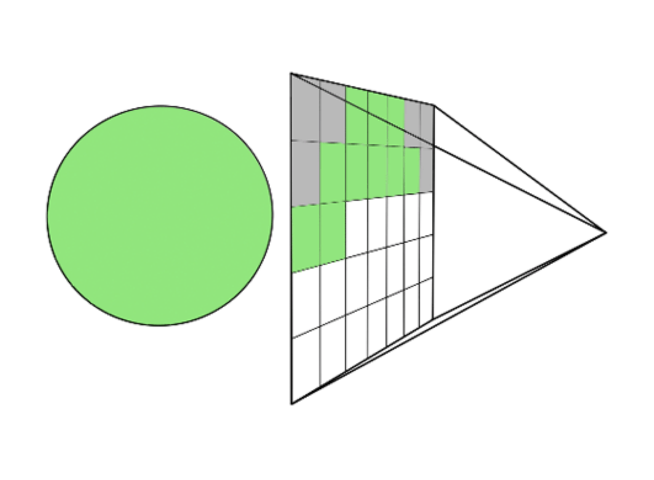

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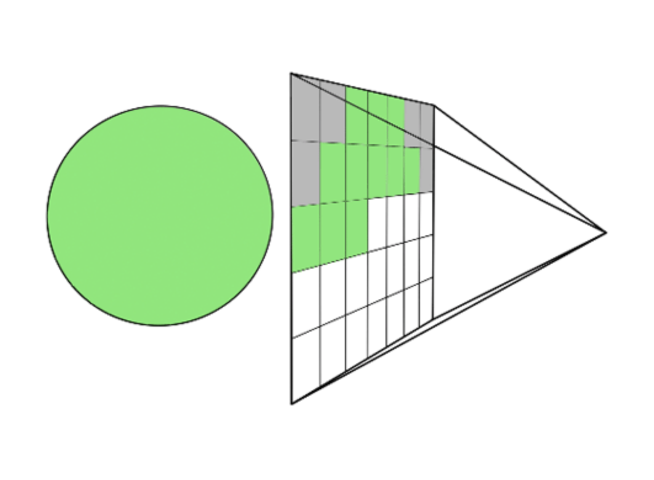

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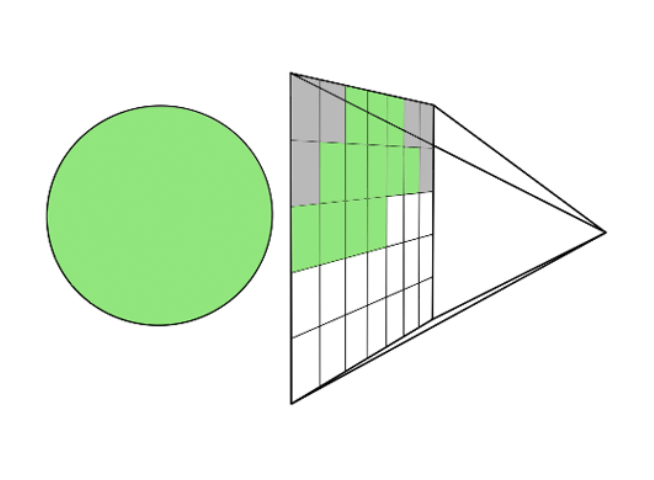

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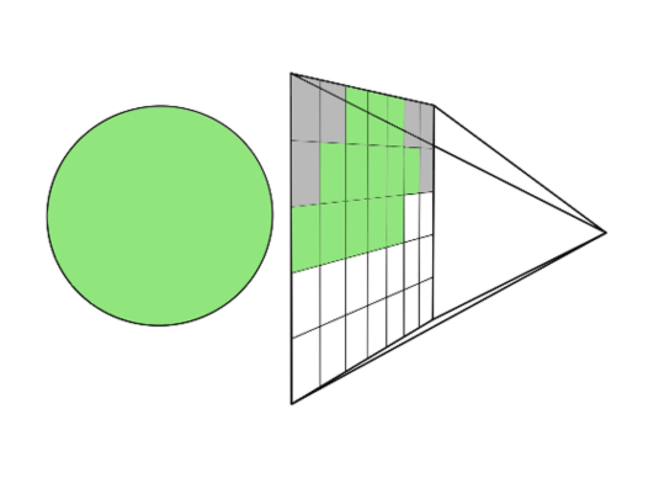

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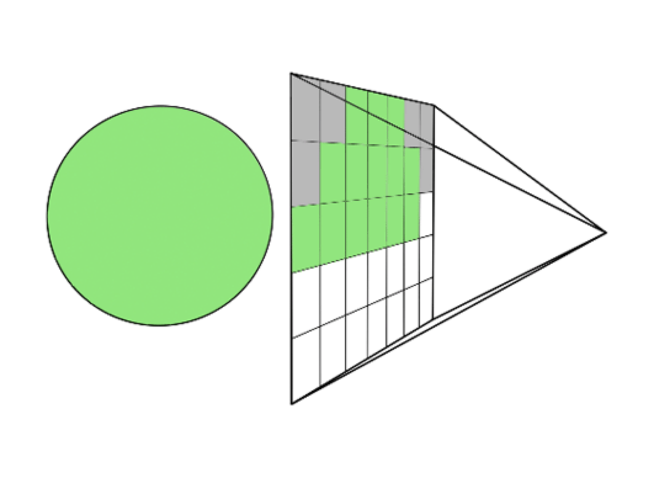
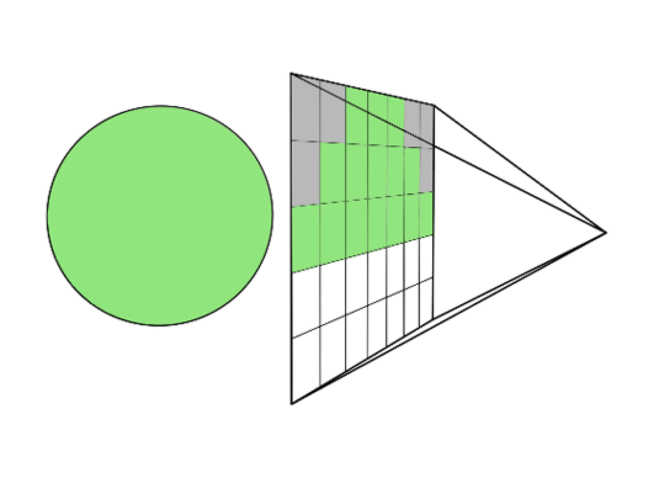


Raycasting Anim



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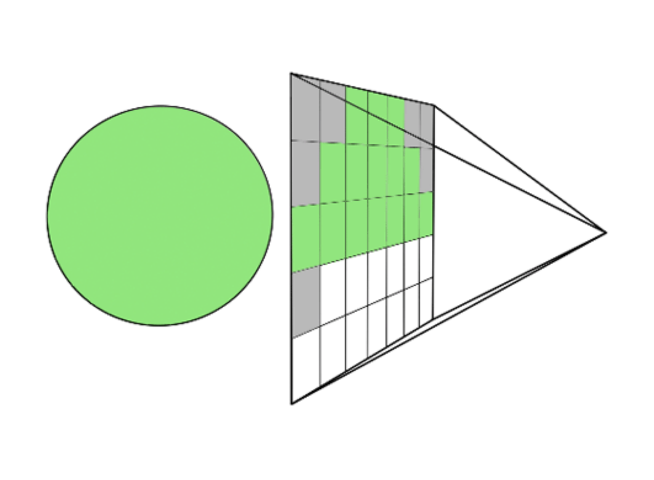

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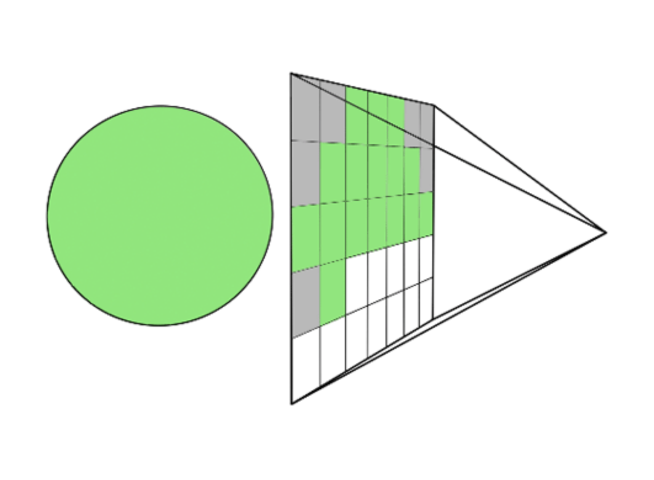
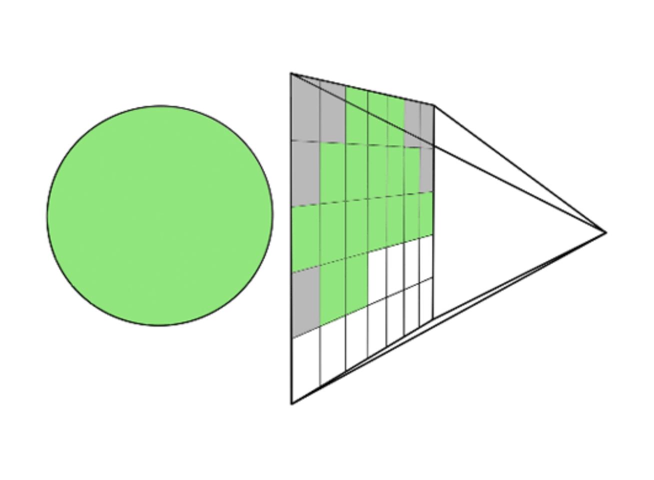
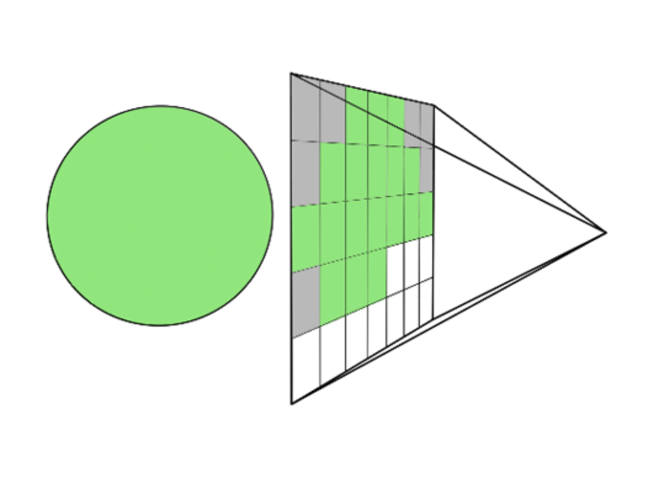


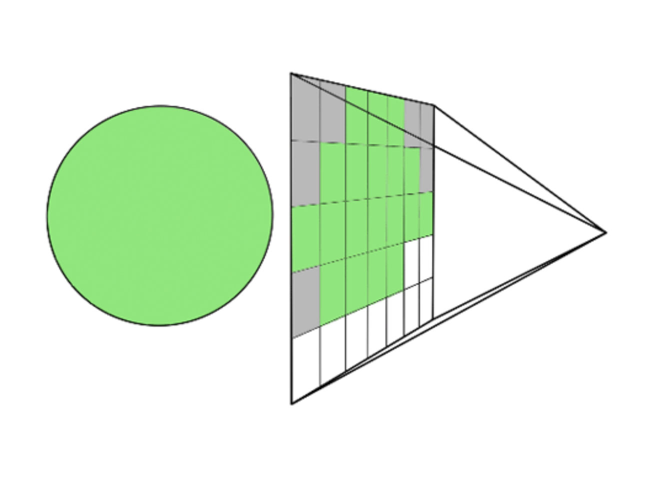
Raycasting Anim



Raycasting Anim



Raycasting Anim



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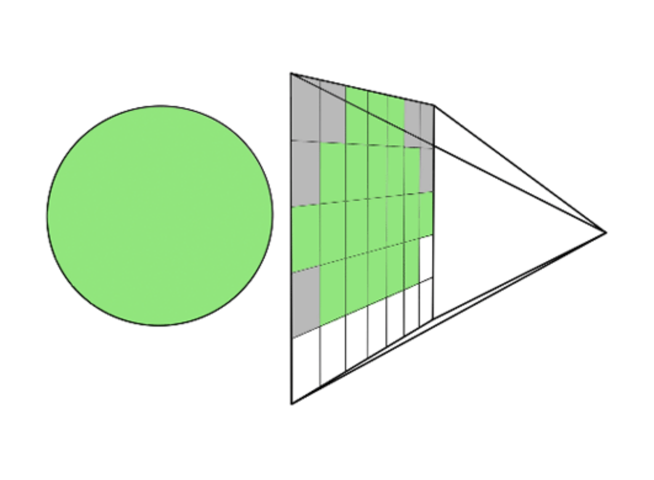

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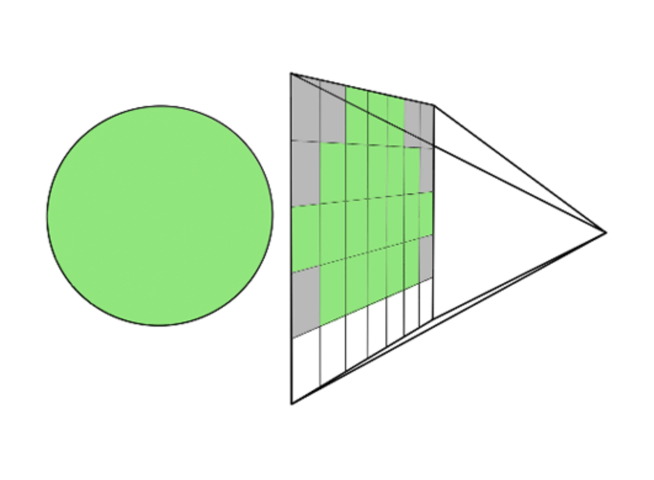

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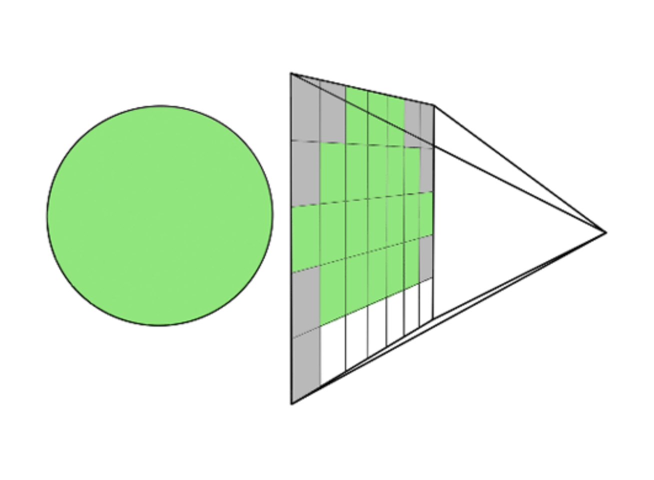

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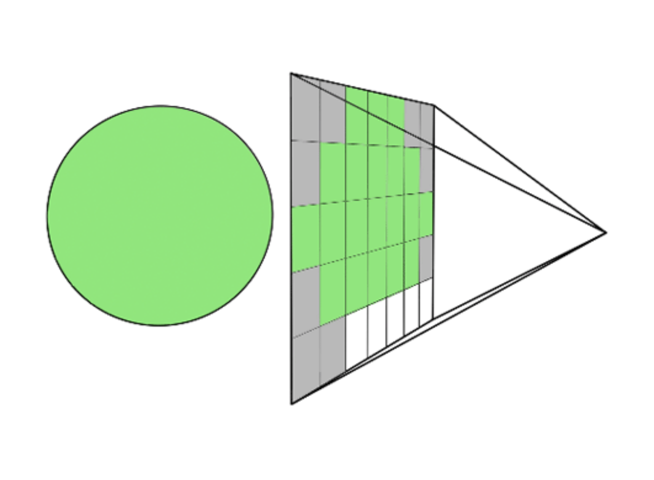

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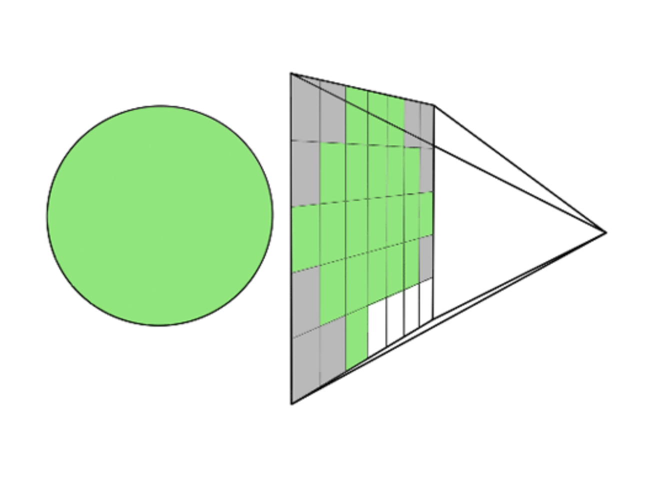

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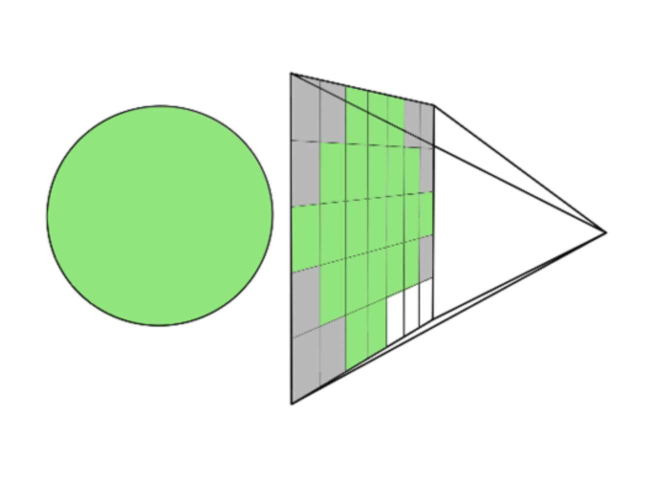

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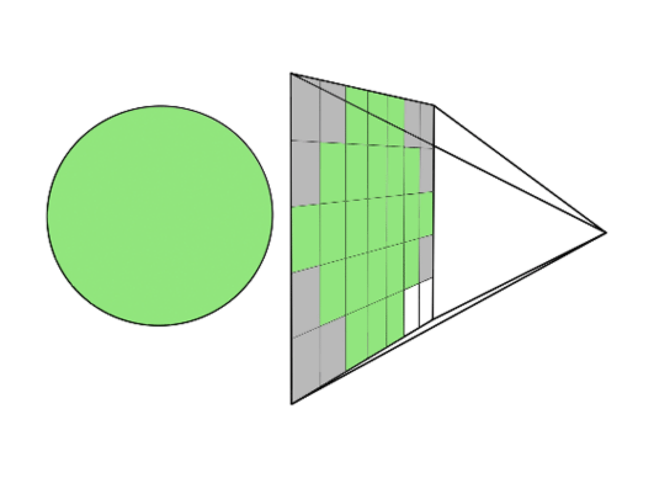
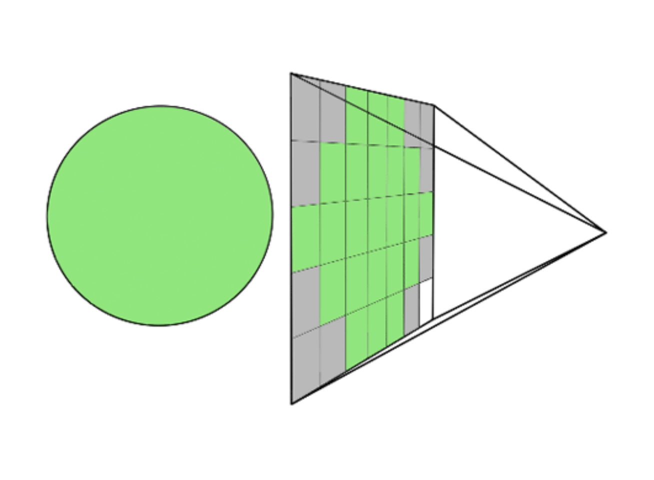
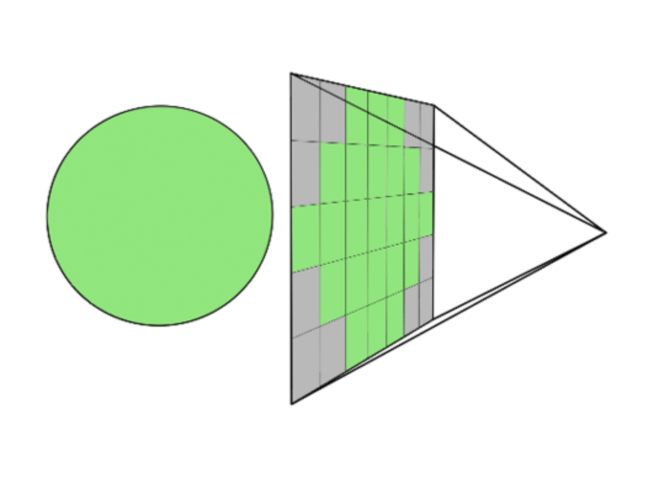


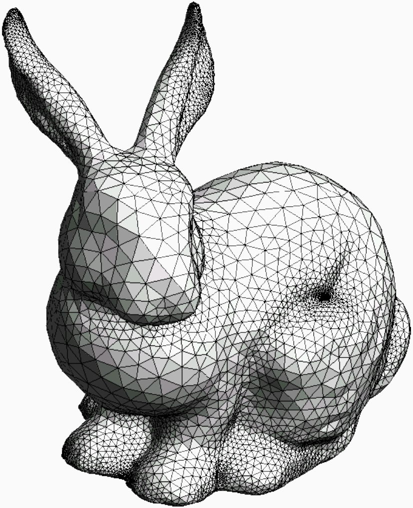
Raycasting Anim



Raycasting Anim



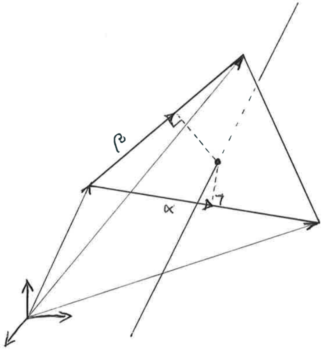
Intersections



Ray-Triangel Intersection

- Compute two vectors in plane

$$\mathbf{e}_1 = \mathbf{v}_1 - \mathbf{v}_0$$
$$\mathbf{e}_2 = \mathbf{v}_2 - \mathbf{v}_0$$



Ray-Triangel Intersection

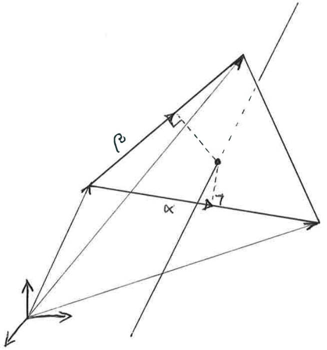
- Compute two vectors in plane

$$\mathbf{e}_1 = \mathbf{v}_1 - \mathbf{v}_0$$

$$\mathbf{e}_2 = \mathbf{v}_2 - \mathbf{v}_0$$

- All points on plane that triangle lies in

$$\mathbf{r} = \mathbf{v}_0 + u\mathbf{e}_0 + v\mathbf{e}_1$$



Ray-Triangel Intersection

- Compute two vectors in plane

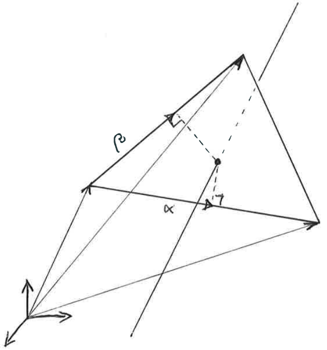
$$\mathbf{e}_1 = \mathbf{v}_1 - \mathbf{v}_0$$

$$\mathbf{e}_2 = \mathbf{v}_2 - \mathbf{v}_0$$

- All points on plane that triangle lies in

$$\mathbf{r} = \mathbf{v}_0 + u\mathbf{e}_0 + v\mathbf{e}_1$$

- Two degrees-of-freedom



Ray-Triangel Intersection

- Compute two vectors in plane

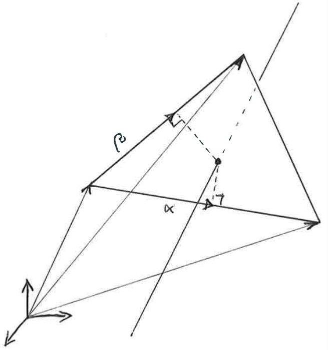
$$\mathbf{e}_1 = \mathbf{v}_1 - \mathbf{v}_0$$
$$\mathbf{e}_2 = \mathbf{v}_2 - \mathbf{v}_0$$

- All points on plane that triangle lies in

$$\mathbf{r} = \mathbf{v}_0 + u\mathbf{e}_0 + v\mathbf{e}_1$$

- Two degrees-of-freedom
- Inside triangle

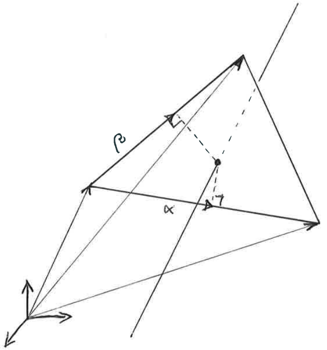
$$\{u, v | u \geq 0, v \geq 0, v+u \leq 1\}$$



Ray-Triangel Intersection

- Solve for intersection

$$\mathbf{r} = \mathbf{v}_0 + u\mathbf{e}_0 + v\mathbf{e}_1$$
$$\mathbf{r}_{ij} = \mathbf{s} + t \cdot \mathbf{d}_{ij}$$
$$\mathbf{s} + t \cdot \mathbf{d}_{ij} = \mathbf{v}_0 + u\mathbf{e}_0 + v\mathbf{e}_1$$



Ray-Triangel Intersection

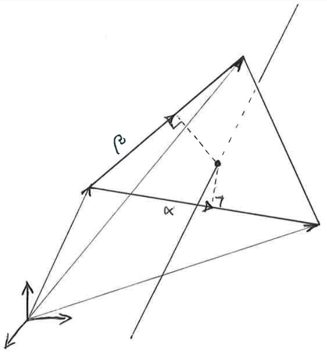
- Solve for intersection

$$\mathbf{r} = \mathbf{v}_0 + u\mathbf{e}_0 + v\mathbf{e}_1$$

$$\mathbf{r}_{ij} = \mathbf{s} + t \cdot \mathbf{d}_{ij}$$

$$\mathbf{s} + t \cdot \mathbf{d}_{ij} = \mathbf{v}_0 + u\mathbf{e}_0 + v\mathbf{e}_1$$

- Unknown: $[u, v, t]^{\mathrm{T}} \in \mathbb{R}^3$



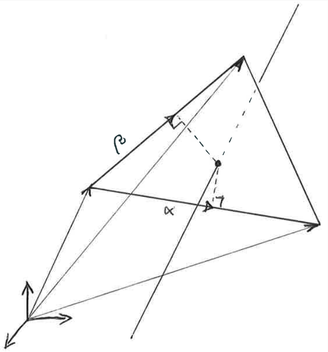
Ray-Triangel Intersection

- Solve for intersection

$$\mathbf{r} = \mathbf{v}_0 + u\mathbf{e}_0 + v\mathbf{e}_1$$

$$\mathbf{r}_{ij} = \mathbf{s} + t \cdot \mathbf{d}_{ij}$$

$$\mathbf{s} + t \cdot \mathbf{d}_{ij} = \mathbf{v}_0 + u\mathbf{e}_0 + v\mathbf{e}_1$$



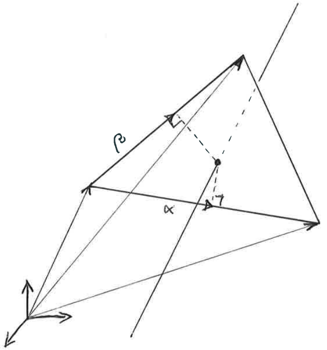
- Unknown: $[u, v, t]^{\mathrm{T}} \in \mathbb{R}^3$
- Three equations three unknowns!

Ray-Triangel Intersection

- Solve for intersection

$$\mathbf{r} = \mathbf{v}_0 + u\mathbf{e}_0 + v\mathbf{e}_1$$
$$\mathbf{r}_{ij} = \mathbf{s} + t \cdot \mathbf{d}_{ij}$$
$$\mathbf{s} + t \cdot \mathbf{d}_{ij} = \mathbf{v}_0 + u\mathbf{e}_0 + v\mathbf{e}_1$$

- Unknown: $[u, v, t]^{\mathrm{T}} \in \mathbb{R}^3$
- Three equations three unknowns!
- Intersection of plane



Ray-Triangel Intersection

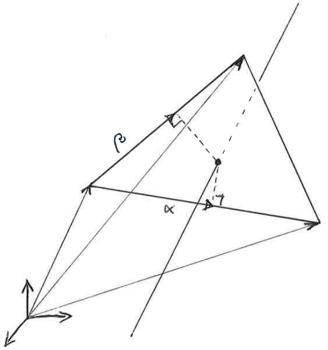
- Solve for intersection

$$\mathbf{r} = \mathbf{v}_0 + u\mathbf{e}_0 + v\mathbf{e}_1$$

$$\mathbf{r}_{ij} = \mathbf{s} + t \cdot \mathbf{d}_{ij}$$

$$\mathbf{s} + t \cdot \mathbf{d}_{ij} = \mathbf{v}_0 + u\mathbf{e}_0 + v\mathbf{e}_1$$

- Unknown: $[u, v, t]^{\mathrm{T}} \in \mathbb{R}^3$
- Three equations three unknowns!
- Intersection of plane
- Check boundary conditions of triangle



Ray-Triangle Intersection

- Ray in plane equation

$$\mathbf{s} + t \cdot \mathbf{d}_{ij} = \mathbf{v}_0 + u\mathbf{e}_0 + v\mathbf{e}_1$$

$$s - \mathbf{v}_0 = -t \cdot \mathbf{d}_{ij} + u\mathbf{e}_0 + v\mathbf{e}_1$$

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- Write on matrix form

$$\begin{bmatrix} -d_{ij}^x & e_0^x & e_1^x \\ -d_{ij}^y & e_0^y & e_1^y \\ -d_{ij}^z & e_0^z & e_1^z \end{bmatrix} \cdot \begin{bmatrix} t \\ u \\ v \end{bmatrix} = \begin{bmatrix} s^x - v_0^x \\ s^y - v_0^y \\ s^z - v_0^z \end{bmatrix}$$

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- This we know from basic math as

$$\mathbf{Ax} = \mathbf{b}$$

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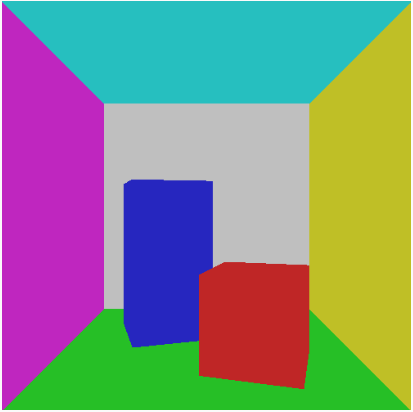
- This we know from basic math as

$$\mathbf{A}\mathbf{x} = \mathbf{b}$$

- *silliest solution*

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$$

Intersections



Ray-Sphere Intersection

- A point $\mathbf{p}$ lies the surface of a sphere with radius r and center $\mathbf{c}$ iff

$$\sqrt{(\mathbf{p}-\mathbf{c})^{\mathrm{T}}(\mathbf{p}-\mathbf{c})}-r=0$$

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- Insert ray-equation into sphere

$$\begin{aligned}\sqrt{(\mathbf{s}+t\mathbf{d}-\mathbf{c})^{\mathrm{T}}(\mathbf{s}+t\mathbf{d}-\mathbf{c})}-r&=0\\(\mathbf{s}+t\mathbf{d}-\mathbf{c})^{\mathrm{T}}(\mathbf{s}+t\mathbf{d}-\mathbf{c})&=r^2\\t^2+2t(\mathbf{d}^{\mathrm{T}}(\mathbf{s}-\mathbf{c}))+(\mathbf{s}-\mathbf{c})^{\mathrm{T}}(\mathbf{s}-\mathbf{c})-r^2&=0\end{aligned}$$

Ray-Sphere Intersection

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$$\begin{aligned}\sqrt{(\mathbf{s}+t\mathbf{d}-\mathbf{c})^{\mathrm{T}}(\mathbf{s}+t\mathbf{d}-\mathbf{c})} - r &= 0\\ (\mathbf{s}+t\mathbf{d}-\mathbf{c})^{\mathrm{T}}(\mathbf{s}+t\mathbf{d}-\mathbf{c}) &= r^2\\ t^2 + 2t(\mathbf{d}^{\mathrm{T}}(\mathbf{s}-\mathbf{c})) + (\mathbf{s}-\mathbf{c})^{\mathrm{T}}(\mathbf{s}-\mathbf{c}) - r^2 &= 0\end{aligned}$$

- Quadratic expression i t

$$t = -\mathbf{d}^{\mathrm{T}}(\mathbf{s}-\mathbf{c}) \pm \sqrt{(\mathbf{d}^{\mathrm{T}}(\mathbf{s}-\mathbf{c}))^2 - (\mathbf{s}-\mathbf{c})^{\mathrm{T}}(\mathbf{s}-\mathbf{c}) - r^2}$$

Ray Intersections

- A Raytracer spends most its time doing intersection computations (profile your code)
- Rules of Thumb
  - Can we trivially reject something?
  - Can we re-use computations?
- If we need to do several tests do them in order of computational cost
- Realtime

Ray Intersections

- A Raytracer spends most its time doing intersection computations (profile your code)
- Rules of Thumb
    - Can we trivially reject something?
    - Can we re-use computations?
- If we need to do several tests do them in order of computational cost
- Realtime
- *Its really fun optimisations as the code is often very small and the solutions are really cute*

Ray-Sphere Intersection

- Sphere intersection

$$t = -\mathbf{d}^{\mathrm{T}}(\mathbf{s} - \mathbf{c}) \pm \sqrt{\left(\mathbf{d}^{\mathrm{T}}(\mathbf{s} - \mathbf{c})\right)^2 - (\mathbf{s} - \mathbf{c})^{\mathrm{T}}(\mathbf{s} - \mathbf{c}) - r^2}$$

- Look at square-root
  - $= 0$ Only one solution, no need to compute square-root
  - < 0 No real solution, no intersection
  - > 0 We have to compute square root (two intersections)

Ray-Sphere Intersection

- Sphere intersection

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- Look at square-root
  - $= 0$ Only one solution, no need to compute square-root
  - < 0 No real solution, no intersection
  - > 0 We have to compute square root (two intersections)
- *Which of the above cases do we have most of the time?*

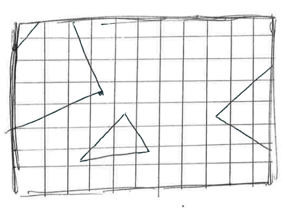
Raytracer Pipeline

Pipeline


Geometry
View Transformation
Lighting
Projection
Clipping
Screen Mapping
Primitives

Clipping and Screen Mapping

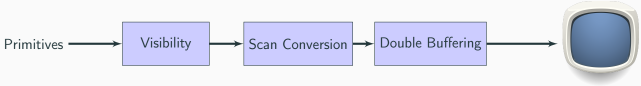
- Clip primitives to fit onto image
- Discretise visible primitives to pixels
- Our primitives are discrete pixels
- Trivial for a Raytracer

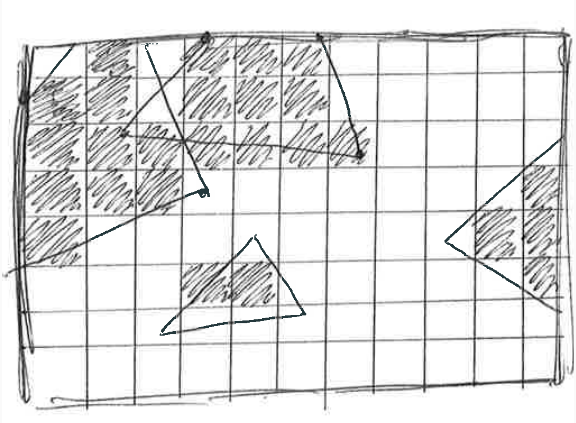


Pipeline


Geometry
View Transformation
Lighting
Projection
Clipping
Screen Mapping
Primitives

Rasterisation





Raycaster

```
for (int y=-h/2;y<h/2;y++)
  {
    for(int x=-w/2;x<w/2;x++)
      {
        /*compute primary ray*/
        for(int i=0;i<N_primitives;i++)
          {
            /*1. compute intersection
              2. check closest*/
          }
      }
  }
```

Next Time

Next Time

Lab Start with Lab 1

- Implement a camera structure and infrastructure to deal with transformations

Lecture Thursday 2nd of February

- 11-12 WMB 3.31
- Illumination: Light I
- How to decide colour of pixel

END

eof