

COMS 30115

Classic Radiosity

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Last Time

- Hemisphere
- BRDFs
- Rendering Equation

Quantification of Light

Radiant Power/Flux - the total amount of energy that flows per unit time

- Φ - [Watt = Joule/sec]

Irradiance the **incident** radiant power on a surface per unit surface area

- $E(x)$ - [Watt/m²]

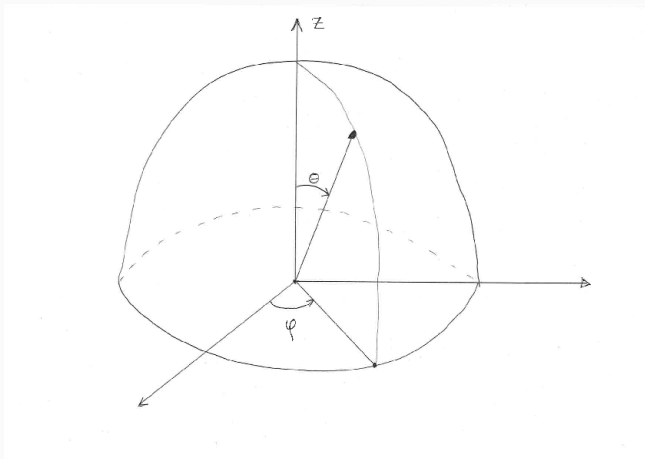
Radiosity the **exitant** radiant power on a surface per unit surface area

- $B(x)$ - [Watt/m²]

Radiance the radiant power per unit projected area per unit solid angle

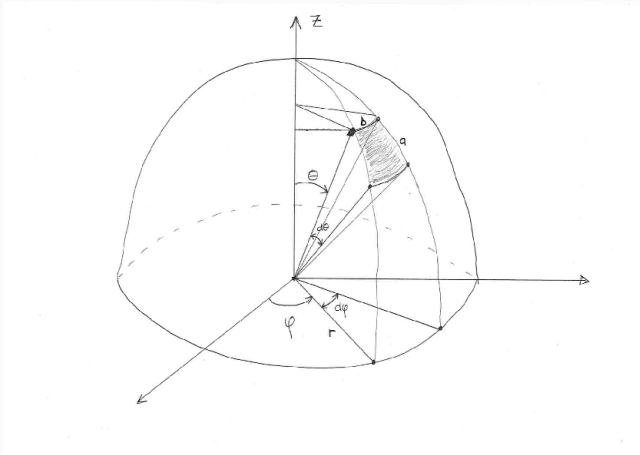
- $L(x)$ - [Watt/steradian · m²]

Solid Angles



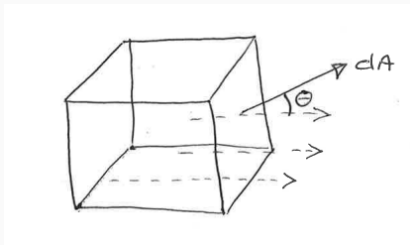
$d\omega$

Solid Angles



$$d\omega = r^2 \sin(\theta) d\phi d\theta$$

Quantification of Light



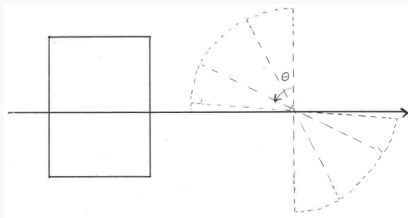
Radiance

the radiant power per unit projected area per unit solid angle

- $L(x)$ - [Watt/steradian $\cdot m^2$]

$$L = \frac{d^2\Phi}{d\omega dA^\perp} = \frac{d^2\Phi}{d\omega \cos(\theta) dA}$$

Quantification of Light



- Number of particles/photons that passes through surface dA in time dt

$$N = p(x, \omega, \lambda) \underbrace{cdtdA\cos(\theta)}_{dV} d\omega d\lambda$$

- Flux is energy per unit time.

$$\Phi = E \cdot p(x, \omega, \lambda) cdA\cos(\theta) d\omega d\lambda,$$

Quantification of Light

- We can write the radiance,

$$L(x, \omega, \lambda) = \frac{d^2\Phi}{d\omega dA \cos(\theta)} = p(x, \omega, \lambda) h \frac{c}{\lambda} d\lambda$$

Quantification of Light

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- Flux, Irradiance & Radiosity

$$\Phi(x) = \int \int L(x \leftarrow \Theta) \cos(\theta) d\omega_{\Theta} dA_x$$

$$E(x) = \frac{d\Phi}{dA_x} = \int L(x \leftarrow \Theta) \cos(\theta) d\omega_{\Theta}$$

$$B(x) = \frac{d\Phi}{dA_x} = \int L(x \rightarrow \Theta) \cos(\theta) d\omega_{\Theta}$$

Quantification of Light

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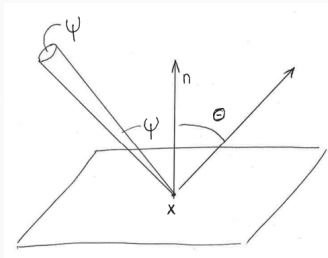
$$B(x) = \frac{d\Phi}{dA_x} = \int L(x \rightarrow \Theta) \cos(\theta) d\omega_{\Theta}$$

- Now we have everything in radiance

$$\begin{aligned}
 f_r(x, \Psi \rightarrow \Theta) &= \frac{dL(x \rightarrow \Theta)}{dE(x \leftarrow \Psi)} \\
 &= \left\{ E(x) = \int L(x \leftarrow \Theta) \cos(\theta) d\omega_{\Theta} \right\} \\
 &= \frac{dL(x \rightarrow \Theta)}{L(x \leftarrow \Psi) \cos(\mathbf{n}_x, \Psi) d\omega_{\Psi}}
 \end{aligned}$$

Definition (BRDF)

ratio of the differential radiance reflected in an exitant direction Θ and the differential irradiance incident through a differential solid angle Ψ



$$f_r(x, \psi \rightarrow \Theta) = \frac{dL(x \rightarrow \Theta)}{dE(x \leftarrow \psi)} = \frac{dL(x \rightarrow \Theta)}{L(x \leftarrow \psi) \cos(\mathbf{n}_x, \psi) d\omega_\psi}$$

$$f_r(x, \Psi \rightarrow \Theta) = \frac{dL(x \rightarrow \Theta)}{L(x \leftarrow \Psi) \cos(\mathbf{n}_x, \Psi) d\omega_\Psi}$$

- Incident & reflected Radiance: *the BRDF is independent of irradiance from other incident angles. This means it is linear (i.e. additive) with respect to all incident directions*

$$f_r(x, \Psi \rightarrow \Theta) = \frac{dL(x \rightarrow \Theta)}{L(x \leftarrow \Psi) \cos(\mathbf{n}_x, \Psi) d\omega_\Psi}$$

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- Incident & reflected Radiance: *the BRDF is independent of irradiance from other incident angles. This means it is linear (i.e. additive) with respect to all incident directions*
- *Relating incoming to outgoing radiance!*

Properties of BRDF

$$f_r(x, \Psi \rightarrow \Theta) = \frac{dL(x \rightarrow \Theta)}{L(x \leftarrow \Psi) \cos(\mathbf{n}_x, \Psi) d\omega_\Psi}$$

$$dL(x \rightarrow \Theta) = f_r(x, \Psi \rightarrow \Theta) L(x \leftarrow \Psi) \cos(\mathbf{n}_x, \Psi) d\omega_\Psi$$

$$L(x \rightarrow \Theta) = \int_{\Omega_x} f_r(x, \Psi \rightarrow \Theta) L(x \leftarrow \Psi) \cos(\mathbf{n}_x, \Psi) d\omega_\Psi$$

- Incident & reflected Radiance: *the BRDF is independent of irradiance from other incident angles. This means it is linear (i.e. additive) with respect to all incident directions*
- *Relating incoming to outgoing radiance!*

- The BRDF tells us how to represent reflected radiance in terms of incoming radiance

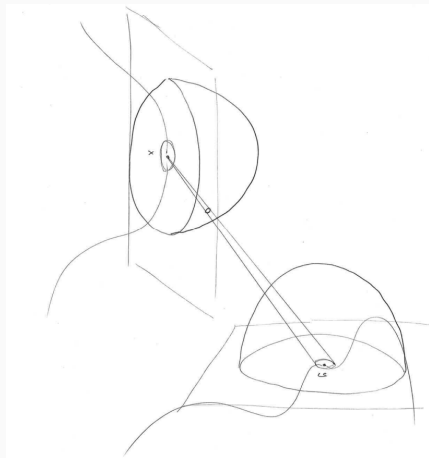
$$L_r(x \rightarrow \Theta) = \int_{\Omega_x} f_r(x, \Psi \rightarrow \Theta) L(x \leftarrow \Psi) \cos(\mathbf{n}_x, \Psi) d\omega_\Psi$$

- Add the emitted light at point

$$\begin{aligned} L(x \rightarrow \Theta) &= L_e(x \rightarrow \Theta) \\ &+ \int_{\Omega_x} f_r(x, \Psi \rightarrow \Theta) L(x \leftarrow \Psi) \cos(\mathbf{n}_x, \Psi) d\omega_\Psi \end{aligned}$$

- We want to solve for the radiance $L(\forall x \rightarrow \forall \theta)$ for the whole scene, why is this complicated?

Rendering Equation



$$L(x \rightarrow \Theta) = L_e(x \rightarrow \Theta) + \int_{\Omega_x} f_r(x, \Psi \rightarrow \Theta) L(x \leftarrow \Psi) \cos(\mathbf{n}_x, \Psi) d\omega_\Psi$$

Cornell Box



Transport Problem

$$L(x \rightarrow \Theta) = L_e(x \rightarrow \Theta) + \int_{\Omega_x} f_r(x, \Psi \rightarrow \Theta) L(x \leftarrow \Psi) \cos(\mathbf{n}_x, \Psi) d\omega_\Psi$$

$$\begin{aligned}L(x \rightarrow \Theta) &= L_e(x \rightarrow \Theta) \\&\quad + \int_{\Omega_x} f_r(x, \Psi \rightarrow \Theta) L(x \leftarrow \Psi) \cos(\mathbf{n}_x, \Psi) d\omega_\Psi \\&= L_e(x \rightarrow \Theta) + \mathcal{T}(L(x \rightarrow \Theta))\end{aligned}$$

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$$\mathcal{T}(L(x \rightarrow \Theta)) = \int_{\Omega_x} f_r(x, \Psi \rightarrow \Theta) L(x \leftarrow \Psi) \cos(\mathbf{n}_x, \Psi) d\omega_\Psi$$

$$\mathbf{x}^T \mathbf{a} = \sum_{i=1}^D x_i \cdot a_i, \quad \langle f, g \rangle = \int f(x)g(x)dx$$

- Rendering Equation formulates recursive transport
- Transport is a linear operator in radiance

$$\langle F^{\rightarrow}, G^{\leftarrow} \rangle = \int_A \int_{\Omega} F(x \rightarrow \Theta) G(x \leftarrow \Theta) \cos(\mathbf{n}_x, \Theta) d\omega_{\Theta} dA$$

Inner product formulation

- Transport problem can be written as the inner-product of two functions

$$L(x \rightarrow \Theta) = L = L_e + \langle T, L \rangle$$

$$\begin{aligned} L(x \rightarrow \Theta) &= L = L_e + \langle T, L \rangle \\ &= L_e + \langle T, L_e \rangle + \langle T, TL \rangle \end{aligned}$$

$$\begin{aligned}L(x \rightarrow \Theta) &= L = L_e + \langle T, L \rangle \\&= L_e + \langle T, L_e \rangle + \langle T, TL \rangle \\&= L_e + \langle T, L_e \rangle + \langle T, TL_e \rangle + \langle T, TTL_e \rangle + \dots\end{aligned}$$

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- Adjoint operator:

$$\forall F, G \in V : \langle \mathcal{O}_1 F, G \rangle = \langle F, \mathcal{O}_2 G \rangle$$

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- Adjoint operator:

$$\forall F, G \in V : \langle \mathcal{O}_1 F, G \rangle = \langle F, \mathcal{O}_2 G \rangle$$

- *Think raytracer!*
- Computing light transport back to light source or from light source are adjoint operators
- Remember dual formulation of raytracing

How to proceed

1. **Input:** geometry and emitted light

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How to proceed

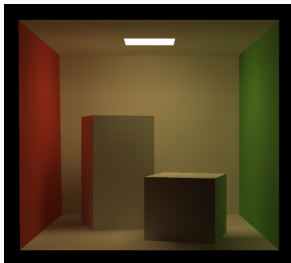
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How to proceed

1. **Input:** geometry and emitted light
2. **Output:** radiance for position & solid angle
3. What is the set of points & solid angles?
 - what is the perceptual importance of each part of the geometry?
4. How do we perform series expansion?
 - Perceptual importance?
5. We know that energy is balanced in the scene

Radiosity

- Comparing two Global Illumination Models
- Radiosity
- Equation Compendium

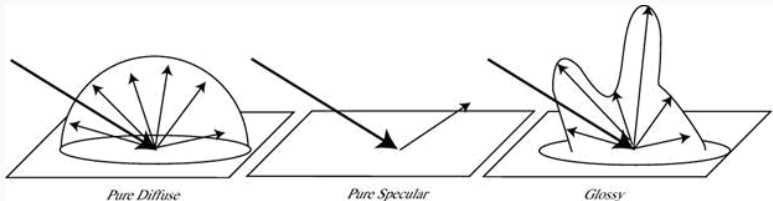


- Initially developed for solving heat transfer
- Formulation for Computer Graphics¹
- Game engine Geomerics²

¹Goral, C. M., Torrance, K. E., Greenberg, D. P., & Battaile, B. (1984).

²<http://www.geomerics.com/>

Radiosity



- Diffuse reflections means that reflection is the **same** in all directions
- This means radiance is the same in each direction
- This simplifies the problem rendering problem hugely
- Classic Radiosity assumes all surfaces are perfectly diffuse

Radiosity the **exitant** radiant power on a surface per unit surface area

- $B(x) - [Watt/m^2]$

Radiance the radiant power per unit projected area per unit solid angle

- $L(x \leftrightarrow \theta) - [Watt/steradian \cdot m^2]$

$$L(x \rightarrow \Theta) = L_e(x \rightarrow \Theta) + \int_{\Omega_x} f_r(x, \Psi \rightarrow \Theta) L(x \leftarrow \Psi) \cos(\mathbf{n}_x, \Psi) d\omega_\Psi$$

- Angles does not matter if we have perfectly diffuse surfaces

$$\begin{aligned}L(x \rightarrow \Theta) &= L_e(x \rightarrow \Theta) \\&+ \int_{\Omega_x} f_r(x, \Psi \rightarrow \Theta) L(x \leftarrow \Psi) \cos(\mathbf{n}_x, \Psi) d\omega_\Psi \\&= L_e(x) + \int_{\Omega_x} f_r(x) L(x \leftarrow \Psi) \cos(\mathbf{n}_x, \Psi) d\omega_\Psi\end{aligned}$$

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- Angles does not matter if we have perfectly diffuse surfaces
- Write the rendering equation in terms of **surface patches**

- If we have only perfectly diffuse surfaces we don't need full BRDF

$\rho(x)$ – reflectivity at point x

- The light that arrives at point x from surface y only depends on visibility

$K(x, y)$ – how much of radiance leaving point y arrives at x

$$\frac{1}{A_i} \int_{S_i} B(x) dA_x = \frac{1}{A_i} \int_{S_i} B_e(x) dA_x + \frac{1}{A_i} \int_{S_i} \rho(x) \int_S K(x, y) B(y) dA_y dA_x$$

Assumption the radiosity is **constant** across a patch

$$\forall x \in S_i : B(x) = \frac{B_i}{A_i}$$

Assumption the reflectivity is **constant** across $\forall x \in S_i : \rho(x) = \rho_i$

$$\frac{1}{A_i} \int_{S_i} B(x) dA_x = \frac{1}{A_i} \int_{S_i} B_e(x) dA_x + \frac{1}{A_i} \int_{S_i} \rho(x) \int_S K(x, y) B(y) dA_y dA_x$$
$$B_i = B_{ei} + \rho_i \sum_j \frac{1}{A_i} \int_{S_i} \int_{S_j} K(x, y) B(y) dA_y dA_x$$

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$$B_i = B_{ei} + \rho_i \sum_j F_{ij} B_j$$

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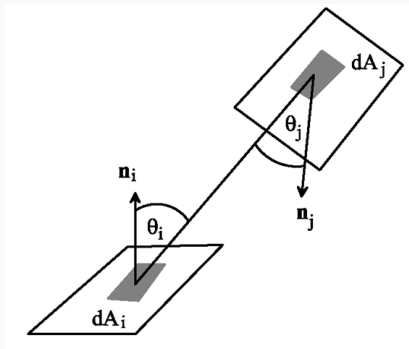
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Assumption the reflectivity is **constant** across $\forall x \in S_i : \rho(x) = \rho_i$

$$\begin{aligned} B_i &= B_{ei} + \rho_i \sum_j \frac{1}{A_i} \int_{S_i} \int_{S_j} K(x, y) B(y) dA_y dA_x \\ &= B_{ei} + \rho_i \sum_j F_{ij} B_j \\ F_{ij} &= \frac{1}{A_i} \int_{S_i} \int_{S_j} K(x, y) dA_y dA_x \end{aligned}$$

- Discretise the scene into patches
- $K(x, y)$ constant over patch
- F_{ij} is depends only on the geometry
- referred to as a **form factor**

Form Factor



- "How large portion of the *view* from *i* is blocked by *j*"

$$B_i = B_{ei} + \rho_i \sum_j F_{ij} B_j$$

$$B_{ei} = B_i - \rho_i \sum_j F_{ij} B_j$$

- The radiosity of patch B_i is a linear combination of radiosities of all other patches B_j
- Importantly the coefficients of this system depends **only** on geometry which is known and self emitted radiosity which is **known**

$$B_i = \frac{1}{A_i} \int_{S_i} \int_{\Omega_x} L(x \rightarrow \Theta) \cos(\mathbf{n}_x, \Theta) d\omega_{\Theta} dA_x$$

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 &= \frac{1}{A_i} \int_{S_i} L(x) dA_x \underbrace{\int_{\psi=0}^{2\pi} d\psi}_{2\pi} \underbrace{\int_{\theta=0}^{\theta=\frac{\pi}{2}} \cos(\mathbf{n}_x, \Theta) \sin(\mathbf{n}_x, \Theta) d\theta}_{\frac{1}{2}}
 \end{aligned}$$

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 &= \frac{1}{A_i} \int_{S_i} L(x) \pi dA_x
 \end{aligned}$$

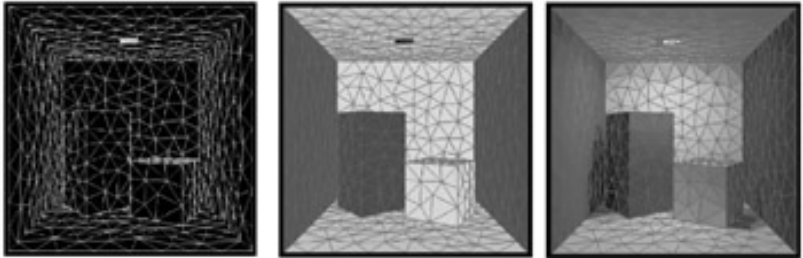
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 &= \frac{1}{A_i} \int_{S_i} B(x) dA_x
 \end{aligned}$$

$$B_{ei} = B_i - \rho_i \sum_j F_{ij} B_j$$

$$\begin{bmatrix} B_{e1} \\ B_{e2} \\ \vdots \\ B_{en} \end{bmatrix} = \begin{bmatrix} 1 - \rho_1 F_{11} & -\rho_1 F_{12} & \dots & \rho_1 F_{1n} \\ -\rho_2 F_{21} & 1 - \rho_2 F_{22} & \dots & \rho_2 F_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ -\rho_n F_{n1} & -\rho_n F_{n2} & \dots & 1 - \rho_n F_{nn} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix}$$

- We know B_{ei} and we can compute F_{ij}
- Solve for B_i

Radiosity



- Compute form factors
- Solve system of equations
- Render image (Raycaster)



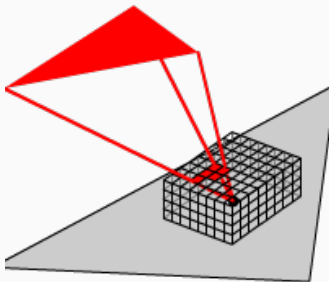
Discretisation Computational and storage cost

Form Factor Computations 1 Cubic storage in number of patches

Form Factor Computations 2 Complicated integral

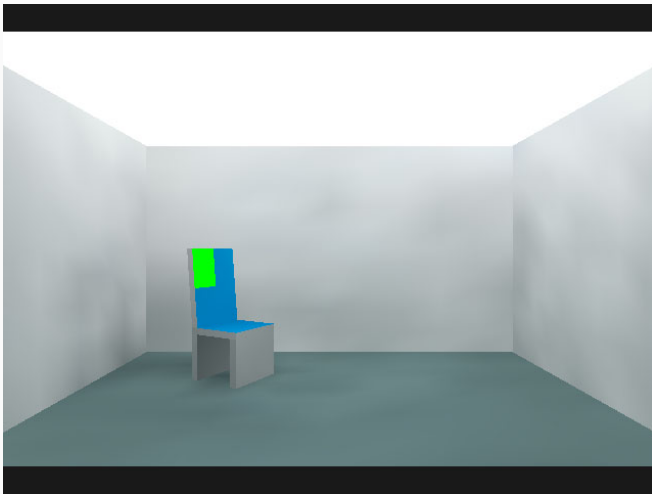
Numerical Solution Structure in coefficient to exploit

Hemicube approximation



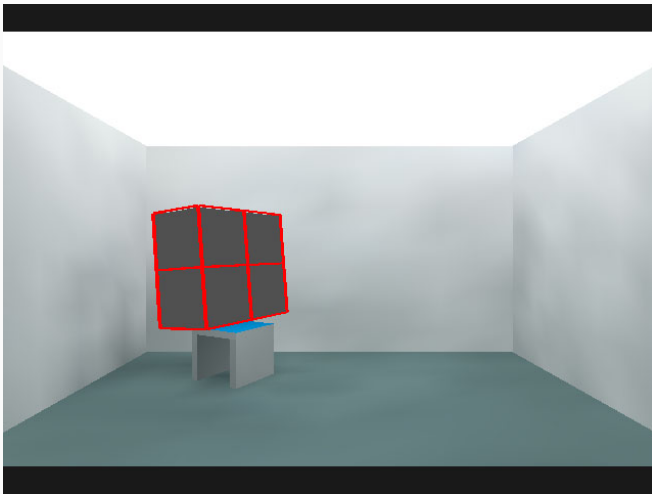
- Form factor computations are expensive
- Approximate with projections onto sphere

Hemicube approximation³



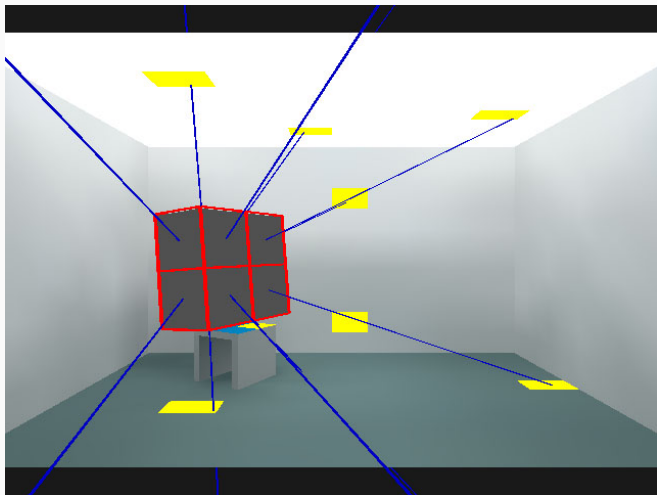
³<http://www.deluxerender.com/2014/11/hemicube-form-factors/>

Hemicube approximation³



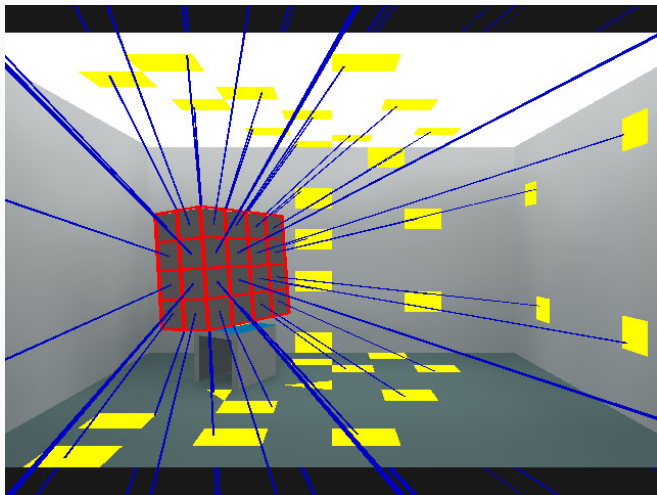
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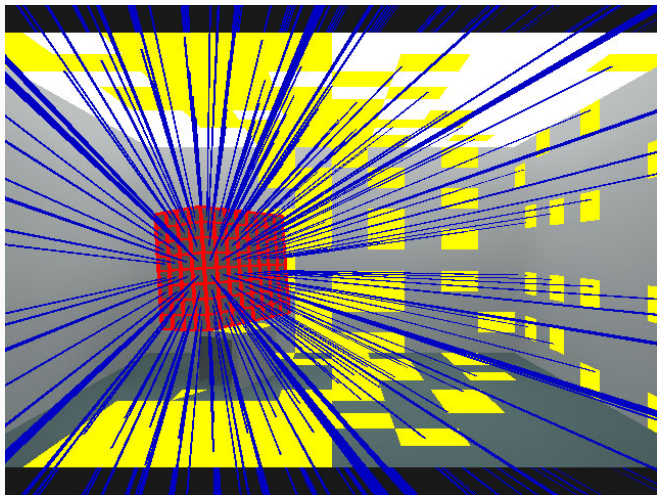
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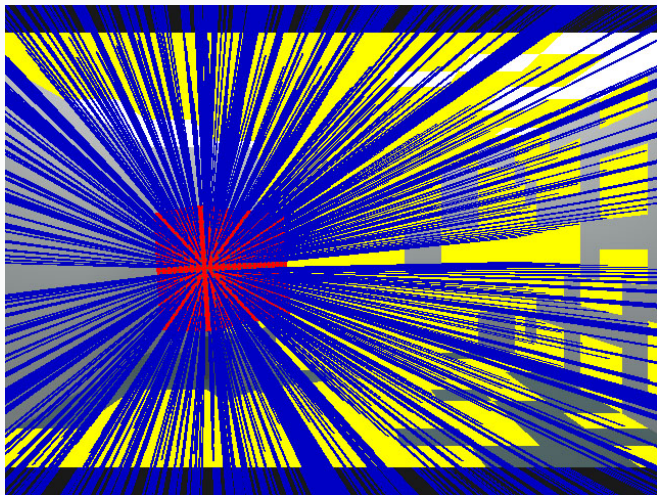
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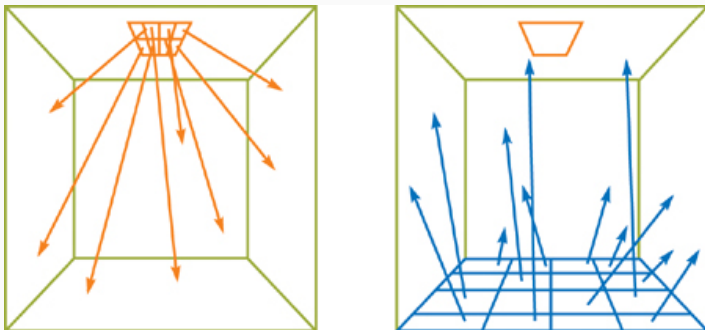
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Hemicube approximation³



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Progressive Radiosity



- Solve equation iteratively
- Each patch stores, **accumulated** and **residual** energy
- Iterate until residual energy low

Summary

- Intuition into rendering equation
- Cast raytracing as a solution to the rendering equation
- Classic Radiosity
 - very clear what assumptions are
 - images look very "radiosity" ;-)

Lecture Friday 29th of March

- Path Tracing (Stochastic Methods)

Lecture Monday 1st of April

- Photon Mapping (Caustics)

Lecture Friday 5th of April

- Simon will talk about Geometry

Lecture Friday 10th of May

- Final lecture
- Wrap up the unit

eof