FUNDAMENTALS OF STATISTICS (4ECON013C)

TUTORIAL 3 ANSWERS

TASKS

1. For the following exercises, decide whether the statement makes sense (or is clearly true) or does not make sense (or is clearly false).

a. (True) The probability mass function (pmf) of a discrete random variable must sum to 1.
b. (False) The probability of any single outcome in a discrete random variable is always greater than zero.
c. (False) The expected value of a discrete random variable is always one of its possible outcomes.
d. (True) The binomial distribution can be used to model the number of times a coin lands on heads in a sequence of 10 flips.
e. (False) The geometric distribution can be used to model the height of a student.
f. (False) The probability of getting at least 1 head in 3 coin flips is 0.5.
g. (False) The binomial distribution is a memoryless distribution.
h. (False) The larger the success probability p, the longer we expect to wait for the first success.

2. An investment company has produced the following table, which shows the probabilities of various percentage profits on money invested over a period of 1 year.

Profit (%), x	-3	0	1	5	8	12
Probability, p(x)	0.10	0.10	0.20	0.30	2k	k

a. Find the value of k.

k = 0.10

b. What is the probability of completing the year with a positive profit?

P(x > 0) = 0.8

c. Calculate the expected profit on an investment of $50,000.

E(x) = 4.2%   E(Profit) = 50000*4.2% = **$2,100**

d. If $\sum x^2 p(x) = 35.8$, then compute the standard deviation of profit rate.

$\sigma = \sqrt{\sum x^2 p(x) - \mu^2} = \sqrt{35.8 - 17.64} = 4.26\%$

3. A small business sells three products: Product A, Product B and Product C. The probabilities of selling each product on any given day are as follows:

- P(A) = 0.4
- P(B) = 0.35
- P(C) = k

The profits from each product sale are:

- Product A: $50
- Product B: $40
- Product C: $C

Given that mean profit per product is $45, do the following:

a. Construct the discrete probability distribution table for the profits from selling a product.

b. Determine the standard deviation of the profit distribution.

Solution.

a. k = 0.25
   50*0.4 + 40*0.35 + C*0.25 = 45
   C = $44

b. σ^2 = 19, σ = $4.36

X	P(X)
$40	0.35
$44	0.25
$50	0.40

4. Two 4-sided dice, labelled 1, 2, 3, 4 are rolled. The product and the sum of the two numbers obtained are calculated. The score awarded, S, is equal to the absolute (i.e. non-negative) difference between the product and the sum. For example, if 4 and 2 are rolled, then
S = | (4 × 2) − (4 + 2) |= 2.

a. Provide the probability distribution of S.

	1	**2**	**3**	**4**
1	1	1	1	1
2	1	0	1	2
3	1	1	3	5
4	1	2	5	8

S = {0, 1, 2, 3, 5, 8}

The probability distribution of S is as follows:

S	0	1	2	3	5	8
P(S)	1/16	9/16	2/16	1/16	2/16	1/16

b. Calculate E(S).   E(S) = 2.125

5. On average, 15% of the WIUT graduates are awarded a first-class degree. Groups of 20 graduates are selected at random.

a. How many candidates in each group are not expected to be awarded a first-class?

If x – number of first-class graduates, then $E(x) = 0.15*20 = 3$

$20-3 = 17$ are not expected to be awarded a first-class.

b. Calculate the variance of the number of first classes in the groups of 20.

$Var(x) = 20*0.15*0.85 = 2.55$.

c. Find the probability that:

i. four graduates in a group of 20 are awarded first-class.

$P(x = 4) = 0.182$

ii. at least three graduates in a group of 20 are awarded first-class.

$P(x \geq 3) = 1 - P(x \leq 2) = 0.595$

6. In a particular country, 90% of both females and males drink tea. Of those who drink tea, 40% of the females and 60% of the males drink it with sugar. Find the probability that in a random selection of two females and two males:

a. all four people drink tea.

$P(M = 2 \cap F = 2) = [{}_2C_2*0.90^2*0.10^0]*[{}_2C_2*0.90^2*0.10^0] = 0.81^2 = 0.6561$

b. at most one male and at most one female drink tea.

$P(M \leq 1 \cap F \leq 1) = 0.19^2 = 0.0361$

c. an equal number of females and males drink tea with sugar (do not include 0).

The probability of drinking with sugar is (multiplicative rule):

$P(M_s) = 0.90*0.60 = 0.54$, $P(F_s) = 0.90*0.40 = 0.36$

$P(M_s = 1 \cap F_s = 1) + P(M_s = 2 \cap F_s = 2) = 0.4968*0.4608 + 0.2916*0.1296 = 0.267$

7. A computer generates random numbers using any of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. The numbers appear on the screen in blocks of five digits, such as $\boxed{50119}$ $\boxed{16317}$ $\boxed{40068}$ ....

Find the probability that:

a. there are no 7s in the first block.

$P(X > 5) = 0.9^5 = 0.59$

b. the first zero appears in the first block.

$P(X \leq 5) = 1 - 0.59 = 0.41$

c. the first 9 appears in the second block.

$P(5 < X \leq 10) = P(X \leq 10) - P(X \leq 5) = (1-0.9^{10}) - (1 - 0.9^5) = 0.242$

8. A microfinance bank issues 4 independent business loans in one week. Each loan has a default probability of 0.2. If a loan defaults, the bank loses \$1,000. If the loan is repaid, the bank earns \$300. Let X = "number of defaults among the 4 loans" and Y="net profit from the 4 loans".

a. Identify the probability distribution of X and write its probability mass function (pmf).

$X \sim Bin(4, 0.2)$

The probability mass function: $P(X = x) = {}_4C_x\, 0.2^x\, 0.8^{(4-x)}$, x = 0, 1, 2, 3, 4

b. Construct the discrete probability distribution of Y.

$Y = -1000*x + 300*(4-x) = 1200 – 1300x$

Once we replace x with possible values, we get the probability distribution of Y:

y	-4000	-2700	-1400	-100	1200
P(Y = y)	0.0016	0.0256	0.1536	0.4096	0.4096

The probabilities are found from P(X = x) where x = 0, 1, 2, 3, 4.

HOMEWORK

9. John Doe sells new cars for Pelican Ford. John usually sells the largest number of cars on Saturday. He has developed the following probability distribution for the number of cars he expects to sell on a particular Saturday.

Number of Cars Sold, *x*	Probability, *P*(*x*)
0	.1
1	.2
2	.3
3	.3
4	.1
	1.0

a. On a typical Saturday, how many cars does John expect to sell?

b. What is the probability of selling at least 2 cars?

c. What is the probability of selling no cars or at most 2?

d. What is the standard deviation of the distribution?

Solution.

a. $E(x) = \mu = \sum x * p(x) = 0*0.1 + 1*0.2 + 2*0.3 + 3*0.3 + 4*0.1 = 2.1$

b. $P(x \geq 2) = 0.3 + 0.3 + 0.1 = 0.7$

c. $P(x = 0 \cup x \leq 2) = P(x \leq 2) = 0.6$

d. Variance = $\sigma^2 = \sum (x - \mu)^2 * p(x) = (0-2.1)^2*0.1 + (1-2.1)^2*0.2 + (2-2.1)^2*0.3 + (3-2.1)^2*0.3 + (4-2.1)^2*0.1 = 1.29$

Standard deviation = $\sigma = \sqrt{1.29} = 1.136$

10. The probability distribution table for the random variable X is given.

x	0	1	2	3
P(X = x)	1-2k	2-4k	3-6k	4-8k

a. Find the value of the constant k.

$1\text{-}2k + 2 - 4k + 3 - 6k + 4 - 8k = 1$

$k = 0.45$

x	0	1	2	3
P(X = x)	0.1	0.2	0.3	0.4

b. Show that E(X)=2.

$E(X) = 0.2+0.6+1.2 = 2$

c. Find $Var(X) = (0.2 + 1.2 + 3.6) - 2^2 = 1$

11. The Census Bureau reports that 27% of California residents are foreign-born. Suppose that you choose three Californians at random, so that each has probability 0.27 of being foreign-born and the three are independent of each other. Let the random variable W be the number of foreign-born people you chose.

a. What are the possible values of W?
b. What is the probability of all three being foreign born?
c. What is the probability that at least one of them is foreign born?
d. What is the probability that at most one of them is foreign born?

Solution.

a. W: {0, 1, 2, 3}

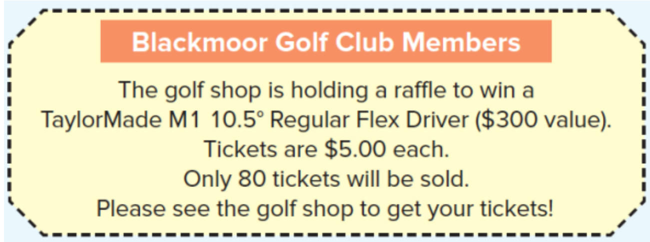
b. $p = 0.27, n = 3$

$P(W = 3) = {}_3C_3 * 0.27^3 * 0.73^0 = 0.0196$

c. $P(W \geq 1) = 1 - P(W < 1) = 1 - {}_3C_0 * 0.27^0 * 0.73^3 = 0.611$

d. $P(W \leq 1) = P(W = 0) + P(W=1) = {}_3C_0 * 0.27^0 * 0.73^3 + {}_3C_1 * 0.27^1 * 0.73^2 = 0.389 + 0.432 = 0.821$

12. The following notice appeared in the golf shop at a Myrtle Beach, South Carolina, golf course.



What is the expected gain from playing this raffle?

Solution.

Outcome	Probability
$295	0.0125 or 1/80
-$5	0.9875 or 79/80

E(x) = 295*0.0125 + (-5)*0.9875 = -1.25
On average people who buy a ticket lose $1.25 for every ticket bought.

13. An FBI survey shows that about 80% of all property crimes go unsolved. Suppose that in your town 4 such crimes are committed and they are each deemed independent of each other.

a. What is the probability that 1 of 4 of these crimes will be solved?
b. What is the probability that at least 1 of 4 of these crimes will be solved?
c. Suppose total of 5985 crimes were reported to FBI during the month of September 2019.

Find the mean and standard deviation of solved crimes.

Solution. Probability of solved crimes = p = 0.2

a. $P(X = 1) = {}_4C_1 * 0.2^1 * 0.8^3 = 0.4096$

b. $P(X \geq 1) = 1 - P(X = 0) = 1 - {}_4C_0 * 0.2^0 * 0.8^4 = 0.59$

c. $\mu = np = 5985*0.2 = 1197$

$\sigma = \sqrt{np(1-p)} = \sqrt{5985 * 0.2 * 0.8} = 30.945$

14. A biased 4-sided die is numbered 1, 3, 5 and 7. The probability of obtaining each score is proportional to that score.

Total weight = 1 + 3 + 5 + 7 = 16.

a. Find the expected number of times that the die will be rolled, up to and including the roll on which the first non-prime number is obtained.

Only 1 is non-prime here and its probability is 1/16.

P(1) = 1/16

X - number of times that the die will be rolled to get 1.

$E(X) = \frac{1}{p} = 16$

b. Find the probability that the first prime number is obtained on the third roll of the die.

$P(X = 3) = (\frac{1}{16})^2 * \frac{15}{16} = 0.0037$