Fundamentals of Statistics
WEEK 2 Tutorial answers

1. For the following exercises, decide whether the statement makes sense (or is clearly true) or does not make sense (or is clearly false).

a. When randomly selecting a day the week, it is certain that you will select a day containing the letter y, so P (y) = 1. TRUE
b. If there is a 0.9 probability that it will rain sometime today, then there is a probability of 0.1 that it will not rain sometime today. TRUE
c. An insurance company states that the probability that a particular car will be involved in a car crash this year is 0.6 and the probability that the car will not be involved in a car crash this year is 0.3. FALSE
d. Jack estimates that the subjective probability of his being struck by lightning sometime next year is 1/2. FALSE
e. The probability of rolling 7 on a die is $\frac{1}{6}$. False
f. Let A be the event that you throw a six, and B be the event that you throw a three from rolling a die. Then the events A and B are independent. False
g. If events A and B are independent, then $P(A \cap B) = P(A) + P(B)$. False
h. When rolling a die, the probability of getting a number no higher than 6 is 1. True
i. If two events, A and B, are independent, then the conditional probability of A given B, denoted $P(A|B)$, is always equal to 1. False
j. The number of ways to select 3 objects from a set of 5 objects without a replacement is smaller when using combinations than when using permutations. True

2. Solution:

a. $P(A) = 3/4$	e. $P(A^c) = 1/4$
b. $P(B) = 12/20$	f. $P(B^c) = 8/20$
c. $P(A \cup B) = 19/20$	g. $P(A \cup A^C) = 1$
d. $P(A \cap B) = 2/5$	h. $P(A^C \cap B) = 1/5$

3. Suppose that 80% of the employees of a company received cash or company stock as a bonus at the end of the year. If 120 employees (60% of the total employees) received a cash bonus and 30% received stock, then how many employees received neither cash nor stock as a bonus?

N = 200

Cash ∪ Stock = 160 => $(\text{Cash} \cup \text{Stock})^c = 200 - 160 = 40$

4. You work in a small company with 10 employees, 3 of whom are close friends of yours. If the directors of this company are chosen at random, what is the probability that you are named director and one of your friends is named deputy director?

$_{10}P_2 = 90$ ways to select the directors
Using multiplicative rule, there is one choice for the director and 3 choices for deputy, and 1*3 = 3 choices of interest.
Answer: $\frac{3}{90} = \frac{1}{30}$

5. A personnel officer has 8 candidates to fill 4 similar positions. 5 candidates are men, and 3 are women. If, in fact, every combination of candidates is equally likely to be chosen, what is the probability that no women will be hired?

Solution.
The total number of possible combinations: $_8C_4 = \frac{8!}{4!*4!} = 70$
In order for no women to be hired, it follows that the 4 successful candidates must come from the available 5 men. The number of such combinations is as follows:
$_5C_4 = \frac{5!}{1!*4!} = 5$
Answer: 5/70 = 1/14

6. Suppose that there are 7 students in your classroom who were born in the month of October. Compute the probability that at least two of them share the same birthday.

$1 - \frac{_{31}P_7}{31^7} = 0.518$

7. In an audit Ali analyses 70% of the audit items and Ahmed analyses the remaining items. Ali's error rate is 5.5% and Ahmed's error rate is 3.4%. Suppose an item is sampled at random.

Solution.

a. P(E) = 0.7*0.055 + 0.3*0.034 = 0.0487
b. P(Ali|E) = $\frac{0.7*0.055}{0.0487}$ = 0.79
c. P(Ahmed|E) = 1 – 0.79 = 0.21

8. The successful operation of three separate switches is needed to control a machine. If the probability of failure of each switch is 0.1 and the failure of any switch is independent of any other switch, what is the probability that the machine will break down?
Solution. The machine works if all three switches work, its probability is 0.9*0.9*0.9 = 0.729. So, the probability it will break down is 1 – 0.729 = 0.271.

9. Which Is Safer: Flying or Driving? (work in groups of 3-4 students to discuss this task)

For the period 1990 through 2010, the average (mean) number of deaths in commercial airplane accidents in the United States was roughly 60 per year. (The actual number varies significantly from year to year.) As of 2010, airplane passengers in the United States travel a total of about 8 billion miles per year. Use these numbers to calculate the death rate per mile of air travel.

The figure below shows the number of automobile fatalities and the total number of miles driven (among all Americans) for each year over a period of more than four decades.

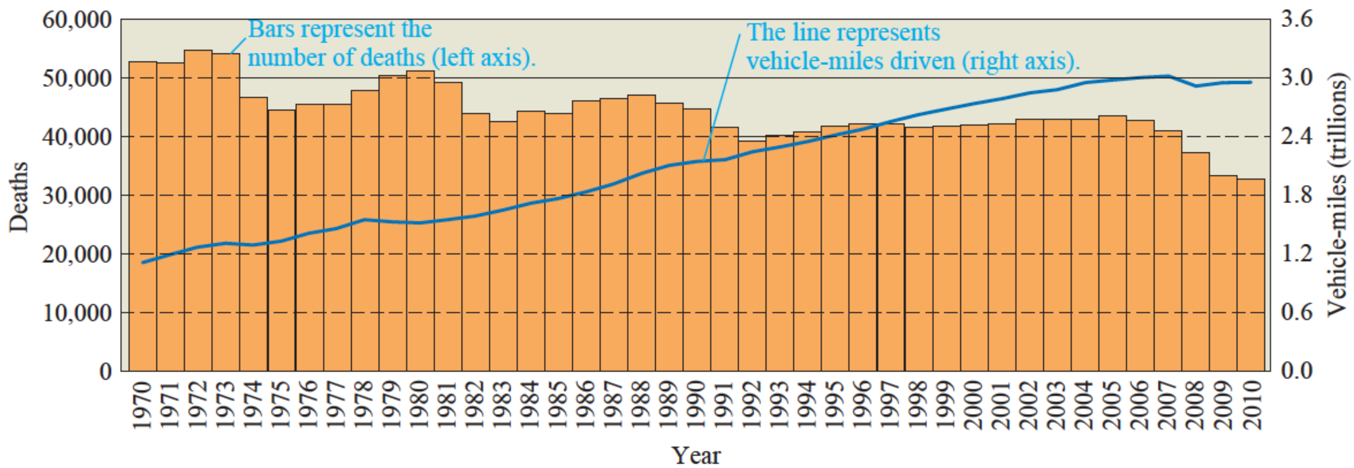

Figure 9 This graph shows the annual number of deaths (bars) and the annual number of vehicle-miles driven (line) in the United States from 1970 to 2010. *Source:* National Transportation Safety Board.

Compare the risk of flying to the risk of driving (you can compare year 2010 for driving and flying since this was the safest driving year according to the figure).

Solution.

Death rate of driving per mile:

$$\text{2010:} \quad \frac{33{,}000 \text{ deaths}}{3 \times 10^{12} \text{ miles}} \approx 1.1 \times 10^{-8} \text{ death per mile}$$

Death rate of flying per mile:

$$\frac{60 \text{ deaths}}{8 \times 10^{9} \text{miles}} \approx 0.75 \times 10^{-8} \text{ death per mile}$$

So, flying is safer.

HOMEWORK

10. In a study of checkout scanning systems, samples of purchases were used to compare the scanned prices to the posted prices. Table 1 summarizes results for a sample of 819 items.

TABLE 1 Scanner Accuracy

	Regular-priced items	Advertised-special items
Undercharge	20	7
Overcharge	15	29
Correct price	384	364

Based on these data,

a. what is the probability that a regular-priced item has a scanning error? 35/419

b. what is the probability that a randomly selected item is either advertised-special or overcharged item? (7+ 29 + 364 + 15) / 819 = 0.507

11. Let event A = learning Spanish. Let event B = learning German. Then A∩B = learning Spanish and German. Suppose P(A) =0.4 and P(B) = 0.2 . P(A∩B) = 0.08 . Are events A and B independent?

P(A∩B) = 0.08 = P(A)*P(B) = 0.4*0.2 = 0.08, A and B are independent events

12. Suppose you randomly select a family with three children. Assume that births of boys and girls are equally likely. What is the probability that the family has each of the following?

a. Three girls
b. Two boys and a girl
c. A girl, a boy, and a boy, in that order
d. At least one girl

Solution. For a family of 3: 2*2*2 = 8 possible outcomes

BBB, BBG, BGG, BGB, GGG, GGB, GBB, GBG
a. Three girls. 1/8
b. Two boys and a girl. 3/8
c. A girl, a boy, and a boy, in that order.1/8
d. At least one girl. 7/8

13. In the past, Energy construction's main competitor, Skyline Company, has submitted bids 75% of the time. If Skyline does not bid on a job, then there is 0.70 probability that Energy will get the job. If Skyline submits a bid, then the probability that Energy gets the job drops to 0.40.

a. What is the probability that Energy gets the job for the next bid?

b. If Energy gets the job, what is the probability that Skyline made a bid?

c. If Energy did not get the job, what is the probability that Skyline did not make a bid?

14. At a local high school, 80% of the students took IELTS test, and 15% of the students took both IELTS and SAT. Based on the information provided, which of the following calculations are not possible, and why? Here, IELTS = A, SAT = B.

a. $P(B \mid A)$ - possible

b. $P(A \mid B)$ - not possible

c. $P(A \cup B)$ – not possible

15. Ahmed is in a dark room selecting socks from his drawer. He has only six socks in his drawer, a mixture of black and white. If he chooses two socks, the chances that he picks a white pair is 2/3. Then what are the chances that he selects a black pair?

Solution. Let w – number of white socks.

Choosing a white pair: P(W and W) = $\frac{w}{6} * \frac{w-1}{5} = \frac{2}{3}$

$\frac{w*(w-1)}{30} = \frac{2}{3}$ => $w*(w-1) = 20$ => $w = 5$

P(B and B) = 0, because there is only one black sock.