National University of Computer and Emerging Sciences, Lahore Campus

| | | | |
|---|---|---|---|
| Course: | Discrete Structures | Course Code: | CS1005 |
| Program: | BSE | Semester: | Spring 22 |
| Duration: | 180 mins | Total Marks: | 30 |
| Paper Date: | 24-03-2022 | Weightage | 15 |
| Section: | 2A, 4A, 4B | Page(s): | 01 |
| Exam: | Mid-I | Roll No: | 20L-1080 |
| Instruction/Notes: | Attempt All Questions | | |

1. Whether $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$ is tautology or not? Prove or disprove it. (5)

2. Translate each of these statements into logical expressions using predicates, quantifiers and logical connectives. Let the domain consist of all people. (6)
   a. Someone in your class can speak Urdu.
   b. Everyone in your class is friendly.
   c. There is a person in your class who was not born in Pakistan.

3. Explain the rules of inference and draw conclusion. (5)

   "If I eat spicy foods, then I have strange dreams." "I have strange dreams if there is thunder while I sleep." "I did not have strange dreams."

4.
   a. Find a counterexample, if possible, to this universally quantified statement, where the domain for all variables consists of all integers. (4)

   $\forall x \exists y (y^2 - x < 1000)$

   b. Negate the following statement:
   If n is even, then $\frac{n}{2}$ is an integer.

5. Prove the distributive law $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ without using membership table. (6)

6. Translate following by using logical connectives: (4)
   a. You get an A on the final, but you don't do every exercise in this book nevertheless, you get an A in this class.
   b. The difference of two negative integers is not necessarily negative.

***GOOD LUCK***

National University of Computer and Emerging Sciences, Lahore Campus

| | | | | |
|---|---|---|---|---|
| | Course: | Discrete Structures | Course Code: | CS1005 |
| | Program: | BSE | Semester: | Spring 22 |
| | Duration: | 60 mins | Total Marks: | 30 |
| | Paper Date: | 09-05-2022 | Weightage | 15 |
| | Section: | 2A,4B,4A | Page(s): | |
| | Exam: | Mid-II | Roll No: | |
| Instruction/Notes: | Attempt All Questions(Each question carry five marks) | | | |

1. *Show that $2^{2n}-1$ is divisible by 3 by using mathematical induction where n is positive integer.*
   a) *What is the statement P(1)?*
   b) *Show that P(1) is true, completing the basis step of the proof.*
   c) *What is the inductive hypothesis?*
   d) *What do you need to prove in the inductive step?*
   e) *Complete the inductive step.* $2^{3 \cdot 4k} - 1$

2. *Prove that $\sqrt{2}$ is irrational number.* contradiction.

3. *Let m be an integer with m > 1. Show that the relation $R = \{(a, b) \mid a \equiv b \pmod m\}$ is an equivalence relation on the set of integers.*

4. Let $R$ and $S$ be relations on a set $A$ represented by the matrices

$$M_R = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \quad M_S = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$= m^2 + n^2$
$= 2^2 + 2^2$
$= 4 + 4$
$= 8$

Find a) $R \circ S$ b) $R \cup S$

5. Determine whether the function $f: \mathbf{Z} \times \mathbf{Z} \rightarrow \mathbf{Z}$ is bijective if

   a) $f(m, n) = m + n$ b) $f(m, n) = m^2 + n^2$

6. Given the matrix representing a relation on a finite set, Write code fragment to determine whether the relation is symmetric and/or antisymmetric.

***GOOD LUCK***