


| National University of Computer and Emerging Sciences, Lahore Campus | | | | |
|---|-----------|--------------------|--------------|-------------|
|  | Course: | Diff. Eq. (Cal-II) | Course Code: | MT-1006 |
| | Program: | BS(CS)/BS(DS) | Semester: | Spring 2022 |
| | Duration: | 3 hours | Total Marks: | 70 |
| | Date: | 10/06/22 | Weight | 50% |
| | Section: | All | Page(s): | 2 |
| | Exam: | Final | Roll No: | |
| Name: | | | | |

Note: Attempt all questions. Use of programmable calculators is not allowed. Exchange of stationary is strictly prohibited. Best of luck!

Question no. 1: (CLO-01) (10 marks)

a) Use ratio test to determine if the series

$$\sum_{n=2}^{\infty} \frac{3^{n+2}}{\ln n}$$

converges or diverges.

b) Ayesha puts 1 coin on the first square of an 8×8 chess board. Then she puts double the amount of coins in each successive square thereafter. How many coins would be on the 64th square?

Question no. 2: (CLO-02) (10 marks) Solve the given initial value problem.

$$L \frac{dy}{dt} + Ry = E, y(0) = y_0$$

where L, R, E and y_0 are constants.

Question no. 3: (CLO-02) (10 marks) Suppose that in winter the daytime temperature in a certain office building is maintained at 70°F . The heating is shut off at 10 P.M. and turned on again at 6 A.M. On a certain day the temperature inside the building at 2 A.M. was found to be 65°F . The outside temperature was 50°F at 10 P.M. and had dropped to 40°F by 6 A.M. What was the temperature inside the building when the heat was turned on at 6 A.M.?

$$\begin{aligned} u &= x \\ du &= 1 \\ v &= \sin x \\ &= \cos x \end{aligned}$$

$$\begin{aligned} u &= x^2 \\ du &= 2x \\ dv &= \sin x \\ v &= -\cos x \end{aligned}$$

$$\begin{aligned} u &= x \\ du &= 1 \\ dv &= \cos x \\ u &= \sin x \end{aligned}$$

Question no. 4: (CLO-03) (10 marks) Determine the solution of the given differential equation

$$y'' + y' = x + \sin 2x$$

using undetermined coefficients superposition principle.

Question no. 5: (CLO-03) (10 marks) Solve the Cauchy Euler equations using variation of parameters

$$x^2 y'' + xy' - y = x^2 e^{2x}$$

Question no. 6: (CLO-04) (10 marks) Find the Fourier series for the function

$$f(x) = \begin{cases} x^2 & 0 \leq x < \pi \\ -x^2 & -\pi < x < 0 \end{cases}$$

Question no. 7: (CLO-04) (10 marks) Solve the wave equation subject to given conditions:

$$u(0, t) = 0, u(2, t) = 0; t > 0$$

$$u(x, 0) = x(2 - x); 0 < x < 2$$

$$\frac{\partial u}{\partial t} = 0 \text{ at } t = 0$$

$$y'' + \frac{y'}{x} - \frac{y}{x^2} = e^{2x}$$

$$y_p = u_1 y_1 + u_2 y_2$$

$$y = x^m$$

$$y_p = \frac{1}{3} (D^2 - 2D + 1) \int_0^{\pi} x^3 dx$$

$$y = c_1 e^{2x} + c_2 e^{-1x} = c_1 + c_2 e^{-x}$$

$$m^2 + \frac{m}{x} + \frac{1}{x^2} = 0$$

$$y'' - \frac{y}{x^2}$$

$$y_p = u_1 y_1 + u_2 y_2$$

$$y = y_1 + y_2$$

$$1 + m^2$$

$$A x^2 + B \cos 2x + C \sin 2x$$

$$m^2 + m = 0$$

$$m(m+1) = 0$$

$$m = 0, m = -1$$