National University of Computer and Emerging Sciences, Lahore Campus

Course:	**Theory of Automata-I**	**Course Code:**	**CS301**
Program:	**BS (CS)**	**Semester:**	**Fall 2020**
Duration:	**90 Minutes**	**Total Marks:**	**50**
Paper Date:	**Oct 16, 2020**	**Weight**	**15%**
Section:	**ALL**	**Page(s):**	**7**
Exam:	**Sessional I**		

Instruction/Notes: Attempt all questions on the question paper.

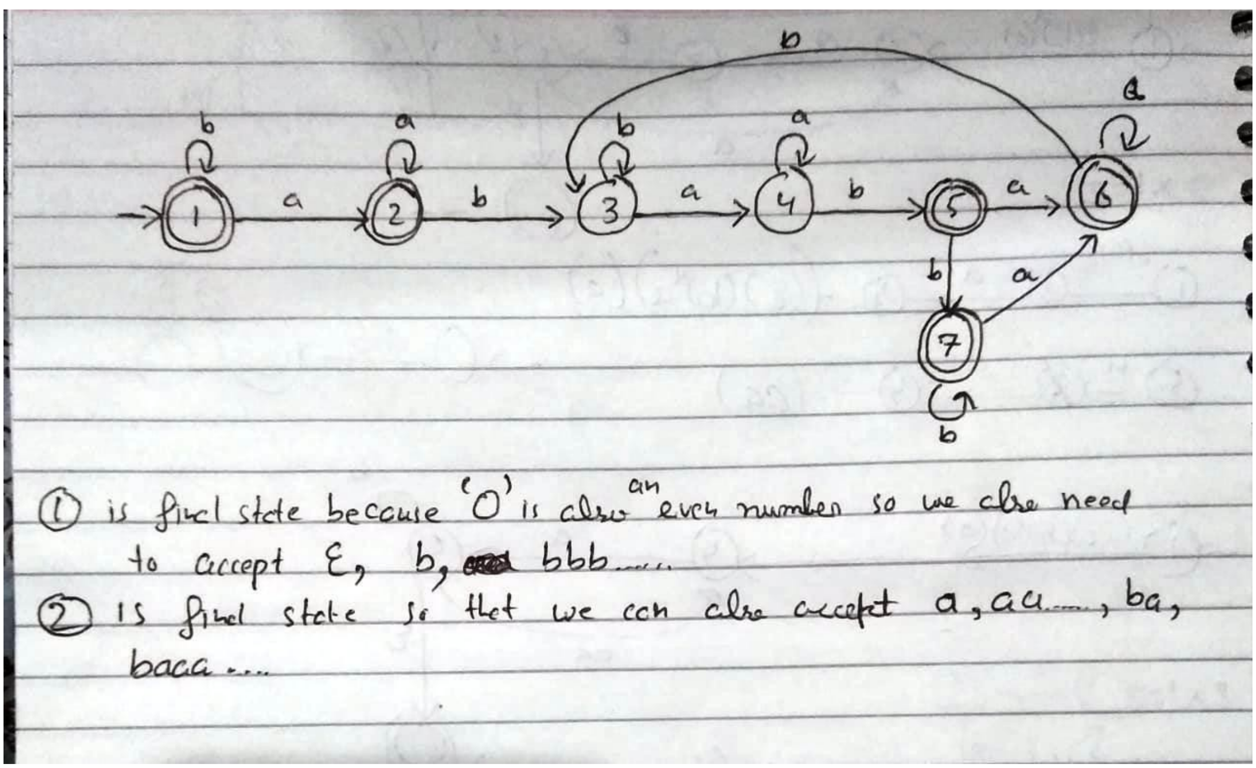
You may use rough sheet for your work but it will not be graded.

Name: ________________________________ Roll Number: _______________ Section _____

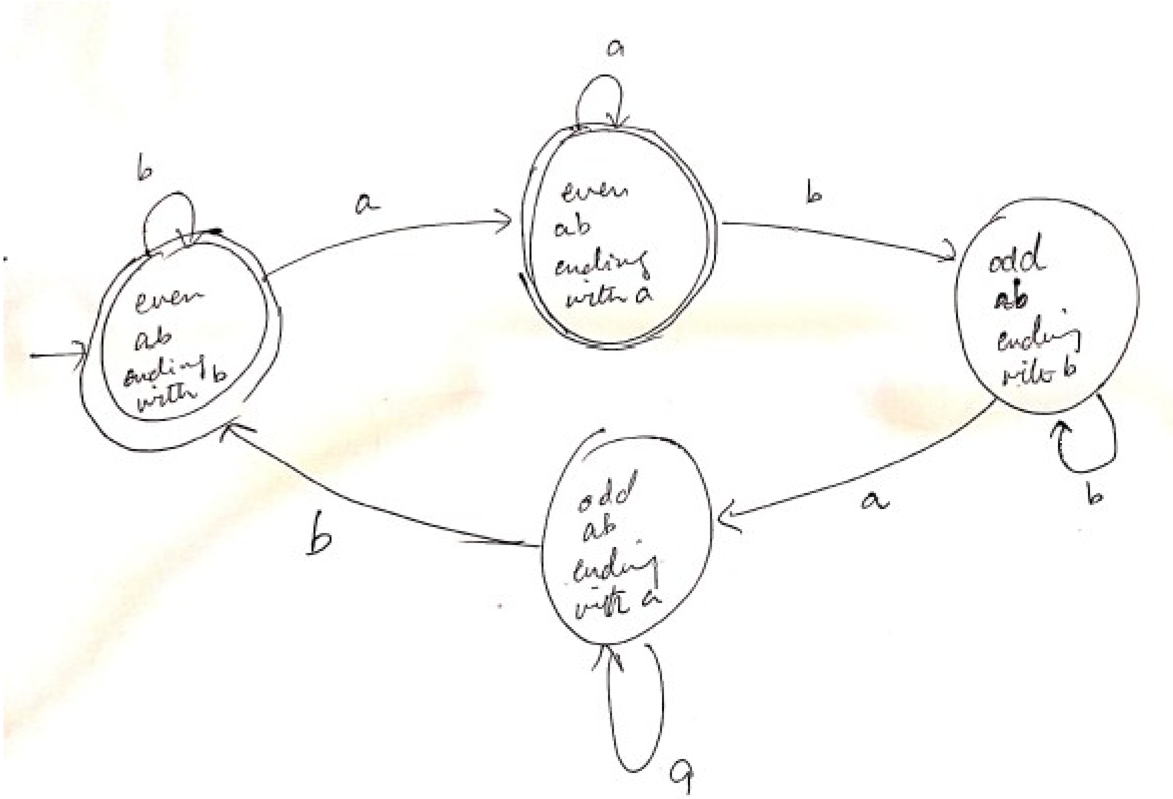
Question 1 [10]

Design a DFA (only state diagram is required) that accepts only such words in which the count (or number) of occurrences of substring **ab** should be even. $\Sigma = \{a, b\}$

SOLUTION 1



SOLUTION 2



even
ab
ending
with b
a
b
even
ab
ending
with a
a
b
odd
ab
ending
with b
b
a
odd
ab
ending
with a
a
b

Question 2 [8 + 2 = 10]

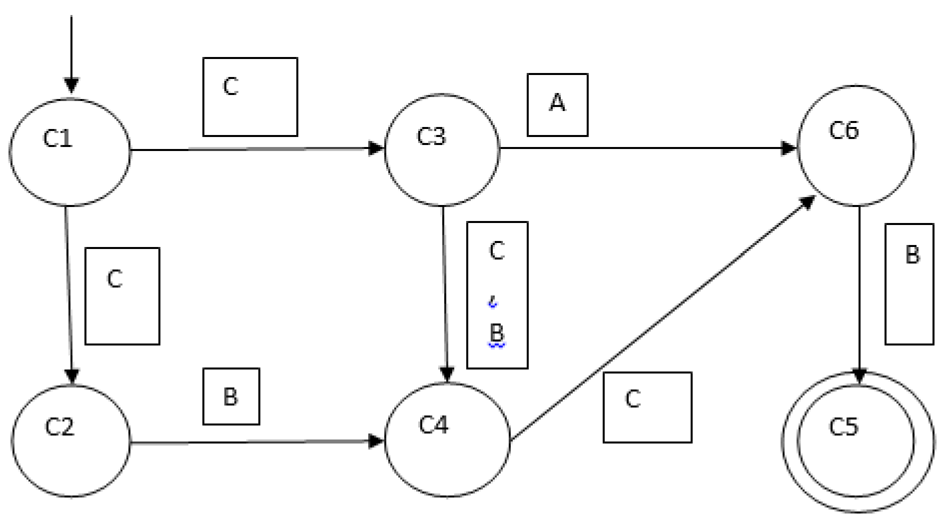
Let us suppose that there are 6 cities named as C1, C2, C3, C4, C5, and C6. A traveler wants to travel from city C1 to city C5. The traveler can either take a Car or a Bicycle to travel from one city to another or he/she can fly to some city using Airplane.

But some anonymous authority has put the restrictions on the modes of transportation that a traveler can use to travel from one city to another and they made a list of restrictions which you will find below. Also the authority was too lazy to write full names of the modes of transportation so they used abbreviations for transportation modes which are as follows. C stands for Car, B stands for Bicycle and A stands for Airplane.

The list of restrictions by the authority are given below.

[1] You can travel from C1 to C2 or to C3 using C.
[2] You can travel from C2 to C4 using B.
[3] You can travel from C3 to C4 by using either a B or a C.
[4] You can travel from C3 to C6 by using A.
[5] You can travel from C6 to C5 using B.
[6] You can travel from C4 to C6 using C.

a. Draw the NFA using elements of alphabet set and elements of set of states described above.



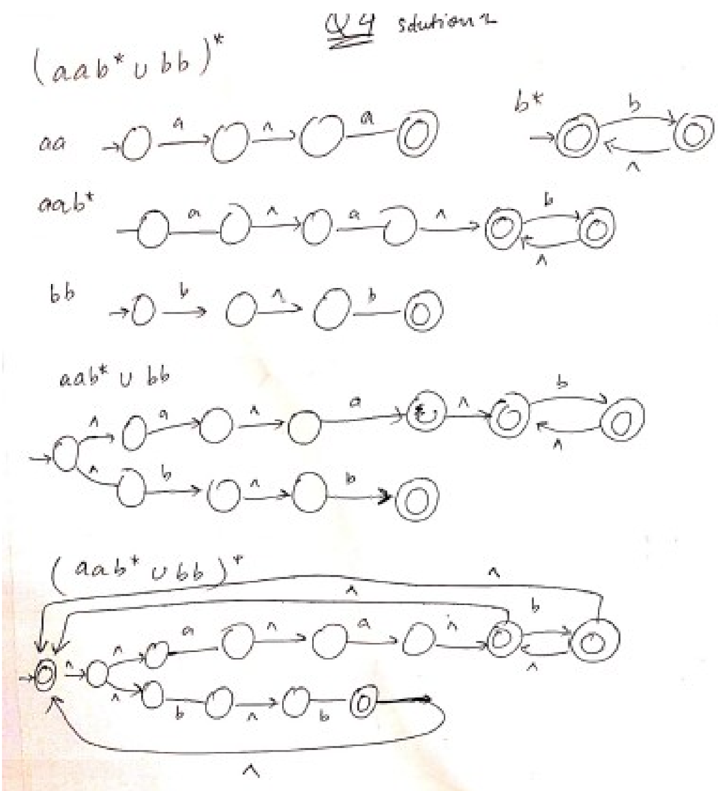
b. What are the unique sequences of transportation modes a traveler can take to reach city C5?
Answer={CAB,CCCB,CBCB}

Question 3 [10]

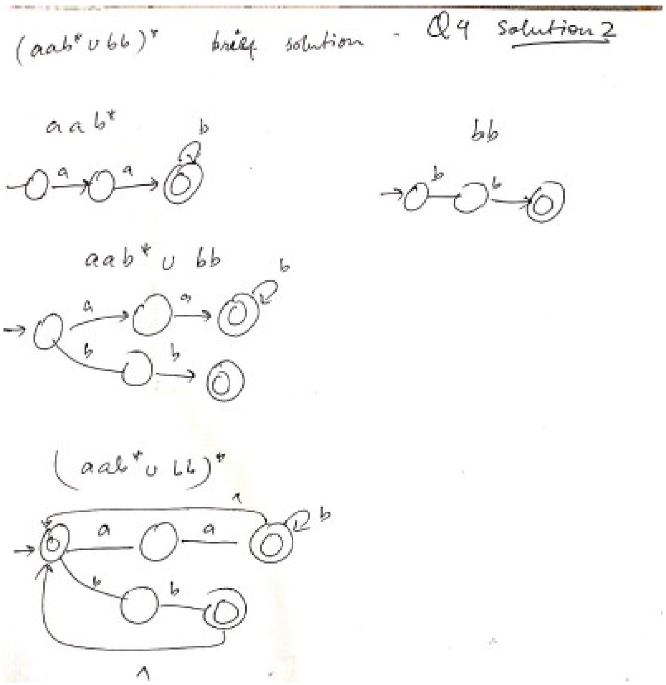
Convert the following regular expression into NFA-null using Kleene's Theorem: (aab* U bb) *

Note: U is equivalent to +

SOLUTION 1 as suggested by Ms Noshaba



SOLUTION 2 as suggested by Ms. Noshaba



Question 4 [10]

Convert Finite automaton given in Figure 1 to regular expression using State Elimination Method. States should be eliminated in alphabetical order, that is A should be removed first, followed by B and finally C. Clearly show all the working in solution.

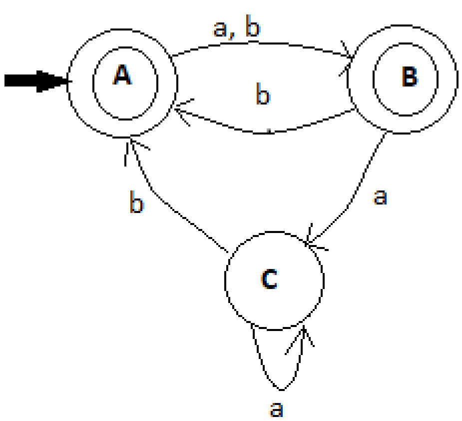
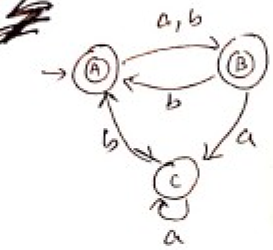
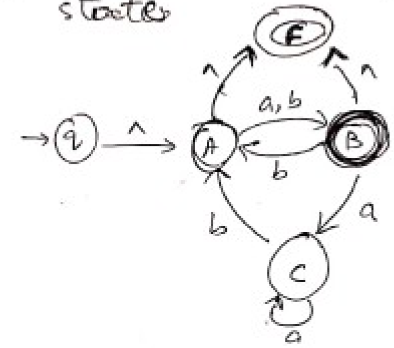


Figure 1: Finite Automaton for Question 4

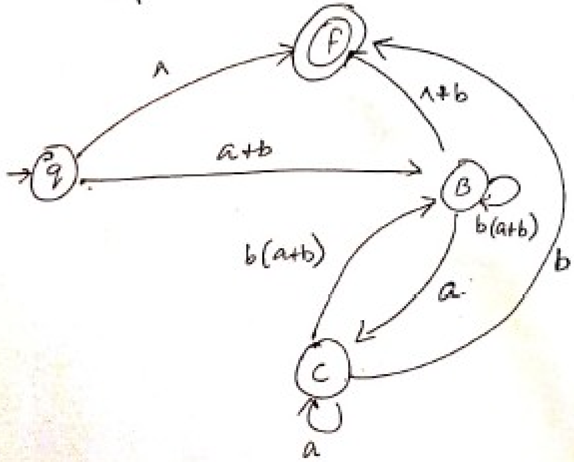
Q2



step1 add new initial & a new final states



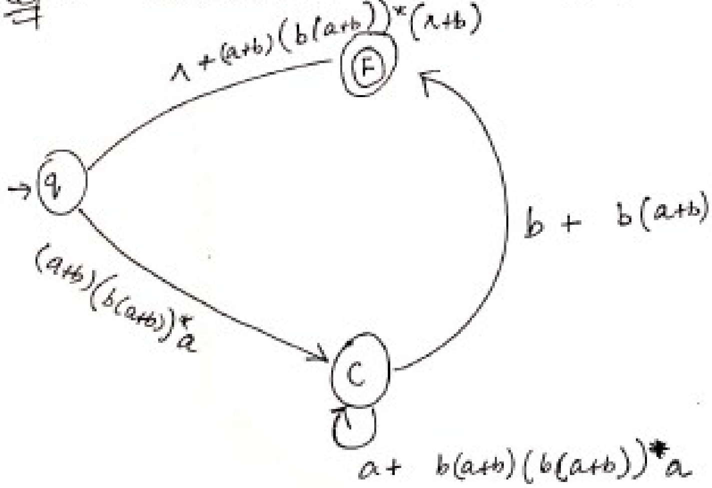
step2 eliminate state A in 3{q, C, B} out {B, F}



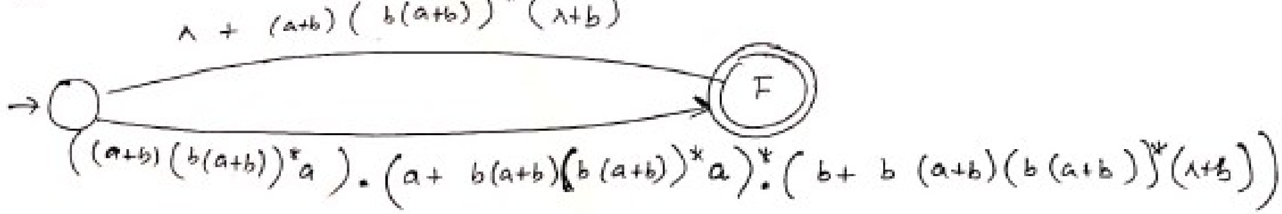
- q, B
- q, F
- C, B
- C, F
- B, B
- BF

step 3 eliminate state B . in 2 {q, c} out 2 {c, F}

✓ q c
✓ c c
✓ q F
✓ c F



step 4 eliminate state C :



Final Regular expression .

$$(\Lambda+(a+b).(b(a+b))^* . (\Lambda+b)) + ((a+b).(b(a+b))^* . a)(a + b(a+b)(b(a+b))^* a)^* . (b + b(a+b)(b(a+b))^*(\Lambda+b))$$

$$= (\Lambda + (a+b).(b(a+b))^* . (\Lambda+b)) + ((a+b).(b(a+b))^* . a).(a + (b(a+b))^+ a)^* . (b + (b(a+b))^+ . (\Lambda+b))$$

Question 5 [10]

Given the following language L = $\{0^{n/2}\ 1^n \mid n > 0\}$, prove that the language is non-regular with the help of pumping lemma. For this question, $\sum = \{0, 1\}$.
Note that both n/2 and n are valid integers.

SOLUTION 1:

Assume L is a regular language

Let $s = 0^n\ 1^{2n}$ s is valid as $|s| > n$

s = xyz

$x = 0^a$ $y = 0^b$ $z = 0^c1^{2n}$

such that a + b <= n, b > 0

According to pumping lemma, if xyz belongs to L, then xy^iz must also belong to L for $i \geq 0$

For i = 0, $s_0 = xz$ $0^{a+c}1^{2n}$ $a + c < n$

Hence s0 does not belong to L. Therefore L is not a regular language

SOLUTION 2

Assume L is a regular language

Let $s = 0^{n/2}\ 1^n$ s is valid as $|s| > n$

s = xyz

Three possible cases:

Case I: y lies in the "0" part of the string

$s = 0^{n/2}\ 1^n$
s = xyz
$x = 0^a$
$y = 0^b$
$z = 0^c\ 1^n$

such that a + b <= n and b >= 1 and a + b + c = n

If (s = xyz) belongs to L, then ($s_i = xy^iz$) must also belong to L, for all i >= 0

For i = k,
$s_k = xy^kz = 0^a\ 0^{kb}\ 0^c\ 1^n \Rightarrow 0^{a+kb+c}\ 1^n$ -→ a+kb+c > n/2 for all k > 2

Case II: y lies in the "1" part of the string

$s = 0^{n/2} 1^n$
$s = xyz$
$x = 0^{n/2}$
$y = 1^b$
$z = 1^{n-b}$

such that $n/2 + b <= n$ and $b >= 1$

If ($s = xyz$) belongs to L, then ($s_i = xy^iz$) must also belong to L, for all $i >= 0$

For $i = 0$,
$S_0 = xy^0z = 0^a 1^{n-b}$
$(n – b)$ is not twice of $n/2$ as minimum value of b is 1

Case III: y lies in both "0" and "1" part of the string

$s = 0^{n/2} 1^n$
$s = xyz$
$x = 0^a$
$y = 0^b1^c$
$z = 1^{n-c}$

such that $a + b = n/2$ and $a + b + c <= n$

If ($s = xyz$) belongs to L, then ($s_i = xy^iz$) must also belong to L, for all $i >= 0$
For $i = k$,
$s_k = xy^kz = 0^a (0^b1^c)^k 1^{n-c}$ -→ repeats (0^b1^c) k times, thus the string does not remain in the form 0^i1^j