National University of Computer and Emerging Sciences, Lahore Campus

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Exam Type:	Final Term		

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Instruction/Notes: Attempt all questions on the answer book. Don't write anything on a question paper except your name and roll number.

Q1. (A) **Points (10)**

Let $P_3(x)$ be the interpolating polynomial for the data $(0, 0)$, $(0.5, y)$, $(1, 3)$, and $(2, 2)$. The coefficient of x^3 in $P_3(x)$ is 6. Find y.

Hint: (i) Need to find $P_3(x)$ (polynomial of degree three) from the given data and use it with the given condition to find required value "y". (ii) Data is unequal spaced so choose formula for interpolation wisely.

(B) **Points (10)**

In a circuit with impressed voltage $\mathcal{E}(t)$ and inductance L, Kirchhoff's first law gives the relationship

$$\mathcal{E}(t) = L\frac{di}{dt} + Ri$$

where R is the resistance in the circuit and i is the current. Suppose we measure the current for several values of t and obtain

t	1.00	1.01	1.02	1.03	1.04
i	3.10	3.12	3.14	3.18	3.24

where t is measured in seconds, i is in amperes, the inductance L is a constant 0.98 henries, and the resistance is 0.142 ohms. Approximate the voltage $\mathcal{E}(t)$ when $t = 1.00$.

Hint: (i) Data is equally spaced. (ii) Choose numerical differentiation formula based on time "t" when $\varepsilon(t)$ is required.

Q2 (A) **Points (10)**

Suppose that $f(0) = 1$, $f(0.5) = 2.5$, $f(1) = 2$, and $f(0.25) = f(0.75) = \alpha$. Find α if the Composite Trapezoidal rule with $n = 4$ gives the value 1.75 for $\int_0^1 f(x)\,dx$.

(B) **Points (10)**

The solid of revolution obtained by rotating the region under the curve $y = f(x)$ over the interval $a \le x \le b$ about the axis has surface area is given by the following formula:

$$S = \int_a^b 2\pi f(x)\sqrt{1 + [f'(x)]^2}\,dx$$

Approximate the surface area if $f(x) = x^3$, $0 \le x \le 1$, using composite Simpson rule with $h = 0.25$.
Note: Throughout the problem, use at least four decimal approximations.

Q3. (A) **Points (10)**

Find an approximation to $\sqrt[3]{25}$ correct to within 10^{-4} using the Bisection Algorithm.

(B) **Points (10)**

Factor the following matrix in to the LU decomposition using the LU Factorization Algorithm with $l_{ii} = 1$ for all i.

$$\begin{bmatrix} 2 & 1 & 0 & 0 \\ -1 & 3 & 3 & 0 \\ 2 & -2 & 1 & 4 \\ -2 & 2 & 2 & 5 \end{bmatrix}$$

Q4. **Points (20)**

Transform the the second-order initial-value problem

$$y'' - 2y' + 2y = e^{2t}\sin t, \qquad \text{for } 0 \le t \le 1, \quad \text{with } y(0) = -0.4, \; y'(0) = -0.6$$

into a system of first order initial-value problems and use the Runge-Kutta method **of order 2 with $h = 1$.**

Q5. **Points (20)**

Let u represent the electrostatic potential between two concentric metal spheres of radii R_1 and R_2 $(R_1 < R_2)$. The potential of the inner sphere is kept constant at V_1 volts, and the potential of the outer sphere is 0 volts. The potential in the region between the two spheres is governed by Laplace's equation, which, in this particular application, reduces to

$$\frac{d^2u}{dr^2} + \frac{2}{r}\frac{du}{dr} = 0, \quad R_1 \le r \le R_2, \quad u(R_1) = V_1, \quad u(R_2) = 0.$$

Suppose $R_1 = 2$ in., $R_2 = 4$ in., and $V_1 = 110$ volts.

(a)
Use finite difference method to approximate u when $N = 4$.

(b)
Compare the results of part (a) with the actual potential

$$u(r) = \frac{V_1 R_1}{r}\left(\frac{R_2 - r}{R_2 - R_1}\right).$$