National University of Computer and Emerging Sciences
Lahore Campus

Operations Research (MT 4031)

QUIZ 2-BCS -7A

Total Marks: 10
weightage: 3

Date: 09-10-2024

Course Instructor(s) Dr. Uzma Bashir,

Q1: [4]
Consider the following LP model:

$$\text{Maximize } z = 4x_1 + 14x_2$$

$$\begin{aligned} 2x_1 + 7x_2 + x_3 \quad &= 21 \\ 7x_1 + 2x_2 \quad + x_4 &= 21 \\ x_{1,} x_{2,} x_{3,} x_4 &\geq 0 \end{aligned}$$

Check the optimality and feasibility of the basic solution given as:

$$\text{Basic variables} = (x_2, x_1), \text{Inverse} = \begin{pmatrix} \frac{7}{45} & -\frac{2}{45} \\ -\frac{2}{45} & \frac{7}{45} \end{pmatrix}$$

SOLUTION:

Feasibility: (1 mark)

$$\binom{x_2}{x_1} = \begin{pmatrix} 7/45 & -7/45 \\ -2/45 & 7/45 \end{pmatrix} \times \binom{21}{21} = \binom{7/3}{7/3}$$

Thus, the given solution is feasible.
The dual is constructed as: **(1 mark)**

$$\text{minimize } w = 21y_1 + 21y_2$$

$$\text{subject to} \quad \begin{aligned} 2y_1 + 7y_2 &\geq 4 \\ 7y_1 + 2y_2 &\geq 14 \\ y_1 &\geq 0 \\ y_2 &\geq 0 \end{aligned}$$

Solution of dual: **(1 mark)**

$$(y_1 \quad y_2) = (14 \quad 4) \times \begin{pmatrix} 7/45 & -7/45 \\ -2/45 & 7/45 \end{pmatrix} = (2 \quad 0)$$

Objective function coefficient of $x_3 = y_1 - 0 = 2 - 0 = 2 > 0$
Objective function coefficient of $x_4 = y_2 - 0 = 0 - 0 = 0$ **(1 mark)**
So, the given solution is optimal as well.

Q2: [6]

Solve the following LP using dual simplex method.

$$\text{maximize } z = x_1 + 2x_2 + 3x_3$$

subject to

$$x_1 - x_2 + x_3 \geq 4$$
$$x_1 + x_2 + 2x_3 \leq 8$$
$$x_2 - x_3 \geq 2$$
$$x_1, x_2, x_3 \geq 0$$

SOLUTION:

$$\text{maximize } z = x_1 + 2x_2 + 3x_3$$

subject to

$$-x_1 + x_2 - x_3 \leq -4$$
$$x_1 + x_2 + 2x_3 \leq 8$$
$$-x_2 + x_3 \leq -2$$
$$x_1, x_2, x_3 \geq 0$$

Putting in tableau form

Basic	x_1	x_2	x_3	x_4	x_5	x_6	Sol
z	--1	-2	-3	0	0	0	0
x_4	-1	1	-1	1	0	0	-4
x_5	1	1	2	0	1	0	8
x_6	0	-1	1	0	0	1	-2

Basic	x_1	x_2	x_3	x_4	x_5	x_6	Sol
z	0	-3	-2	-1	0	0	4
x_1	1	-1	1	-1	0	0	4
x_5	0	2	1	1	1	0	4
x_6	0	-1	1	0	0	1	-2

Basic	x_1	x_2	x_3	x_4	x_5	x_6	Sol
z	0	0	-5	-1	0	-3	10
x_1	1	0	0	-1	0	-1	6
x_5	0	0	3	1	1	2	0
x_2	0	1	-1	0	0	-1	2

Basic	x_1	x_2	x_3	x_4	x_5	x_6	Sol
z	0	0	0	2/3	5/3	1/3	10
x_1	1	0	0	-1	0	-1	6
x_3	0	0	1	1/3	1/3	2/3	0
x_2	0	1	0	1/3	1/3	1/3	2

The optimal and feasible solution.

$$x_1 = 6, x_2 = 2, x_3 = 0, z = 10.$$