

### Assignment #3

Q1 a)

$$\begin{aligned} S &\rightarrow TU|V \\ T &\rightarrow aTb|\epsilon \\ u &\rightarrow cu|\epsilon \\ V &\rightarrow avc|w \\ w &\rightarrow bw|\epsilon \end{aligned}$$

Sol: step 1: No new starting state needed

step 2: Eliminate NULL production.

$w \rightarrow \epsilon$ :

$$\begin{aligned} S &\rightarrow TU|V \\ T &\rightarrow aTb|\epsilon \\ u &\rightarrow cu|\epsilon \\ V &\rightarrow avc|w \\ w &\rightarrow bw|b \end{aligned}$$

$V \rightarrow \epsilon$ :

$$\begin{aligned} S &\rightarrow TU|V|\epsilon \\ T &\rightarrow aTb|\epsilon \\ u &\rightarrow cu|\epsilon \\ V &\rightarrow avc|w|ac \\ w &\rightarrow bw|b \end{aligned}$$

$u \rightarrow \epsilon$ :

$$\begin{aligned} S &\rightarrow TU|V|\epsilon|T \\ T &\rightarrow aTb|\epsilon \\ u &\rightarrow cu|\epsilon \\ V &\rightarrow avc|w|ac \\ w &\rightarrow bw|b \end{aligned}$$

$T \rightarrow \epsilon$ :

$$\begin{aligned} S &\rightarrow TU|V|T|U|\epsilon \\ T &\rightarrow aTb|ab \\ u &\rightarrow cu|c \\ V &\rightarrow avc|w|ac \\ w &\rightarrow bw|b \end{aligned}$$

Step 3: Eliminate unit production

$$S \rightarrow V, S \rightarrow T, S \rightarrow U, V \rightarrow W$$



$S \rightarrow \bar{T}U | aVc | bW | b | a\bar{c} | a\bar{T}b | ab | cU | c | \epsilon$

$T \rightarrow a\bar{T}b | ab$

$U \rightarrow cU | c$

$V \rightarrow aVc | bW | b | ac$

$W \rightarrow ~~bW~~ | bW | b$

Step 4: Remove useless states:

$\{a, b, c, S, \bar{T}, U, V, W\}$

No useless states

Step 5: Add production rules:

$S \rightarrow \bar{T}U | DC | BW | cU | \epsilon B | AB | AC | b | c | \epsilon$

$T \rightarrow \epsilon B | AB$

$U \rightarrow cU | c$

$V \rightarrow DC | BW | AC | b$

$W \rightarrow BW | b$

$A \rightarrow a$

$B \rightarrow b$

$C \rightarrow c$

$D \rightarrow AV$

$\epsilon \rightarrow A\bar{T}$

B)  $S \rightarrow ASA | aB$

$A \rightarrow B | S$

$B \rightarrow b | S$

$X \rightarrow A$



Sol: Step 1: Add starting state

$$S_0 \rightarrow S$$

$$S \rightarrow ASA/aB$$

$$A \rightarrow B|S$$

$$B \rightarrow b|\epsilon$$

$$X \rightarrow A$$

Step 2: Eliminate NULL production

$B \rightarrow \epsilon$ :

$$S_0 \rightarrow S$$

$$S \rightarrow ASA/aB/a$$

$$A \rightarrow B|S|\epsilon$$

$$B \rightarrow b$$

$$X \rightarrow A$$

$A \rightarrow \epsilon$ :

$$S_0 \rightarrow S$$

$$S \rightarrow ASA/aB/a|SA|AS|S$$

$$A \rightarrow B|S$$

$$B \rightarrow b$$

$$X \rightarrow A|\epsilon$$

$X \rightarrow \epsilon$ :

$$S_0 \rightarrow S$$

$$S \rightarrow ASA/aB/a|SA|AS|S$$

$$A \rightarrow B|S$$

$$B \rightarrow b$$

$$X \rightarrow A$$

Step 3: Eliminate unit production:

$$X \rightarrow A, S \rightarrow S, S_0 \rightarrow S, A \rightarrow B, A \rightarrow S$$

$$S_0 \rightarrow ASA/aB/a|SA|AS$$

$$S \rightarrow ASA/aB/a|SA|AS$$

$$A \rightarrow b|ASA/aB/a|SA|AS$$

$$B \rightarrow b$$

$$X \rightarrow b|ASA/aB/a|SA|AS$$

Step 4: Remove useless states

$$\text{useless check} \rightarrow \{a, b, B, A, S, S_0, X\}$$



Unsearchable  $\rightarrow \{X\}$

So,  $S_0 \rightarrow ASA|aB|a|SA|AS$   
 $S \rightarrow ASA|aB|a|SA|AS$   
 $A \rightarrow b|ASA|aB|a|SA|AS$   
 $B \rightarrow b$

Step 5: Add Production Rule:

$S_0 \rightarrow XA|YB|a|SA|AS$   
 $S \rightarrow XA|YB|a|SA|AS$   
 $A \rightarrow b|~~ASA~~XA|YB|a|SA|AS$   
 $B \rightarrow b$   
 $X \rightarrow AS$   
 $Y \rightarrow a$

c)  $S \rightarrow a|aA|aB$   
 $A \rightarrow aBB| \epsilon$   
 $B \rightarrow Aa|b$

Sol: step 1: No new starting state needed.

Step 2: Eliminate Null Production

$A \rightarrow \epsilon$ :

$S \rightarrow a|aA|aB|a$

$A \rightarrow aBB$

$B \rightarrow Aa|b|a$

$\Rightarrow S \rightarrow a|aA|aB|a$  (after removing duplicate)



Step 3: Eliminate unit Production.

NO unit production

Step 4: NO useless state.

Step 5: Add production Rules.

$S \rightarrow a \mid XA \mid XB$

$A \rightarrow YB$

$B \rightarrow AX \mid b \mid a$

$X \rightarrow a$

$Y \rightarrow XB$