National University of Computer and Emerging Sciences, Lahore Campus				
	Course:	**Diff. Eq.(Cal-II)**	**Course Code:**	**MT-224**
	Program:	**BS(CS)**	**Semester:**	**Spring-20**
	Duration:	**3.5 Hours**	**Total Marks:**	**130**
	Paper Date:	**26-06-20**	**Weight**	**50%**
	Section:	**All Sections**	**Page(s):**	**3**
	Exam:	**Final Exam**	**Roll No:**	
Instruction/Notes:	**Attempt All Questions/Read all the instructions very carefully.**			

1. **Convert your answers into a single PDF and email within 20-25 minutes to your course instructor. Failure to submit within due time will result zero credit.**
2. **Make sure you all have video cameras and microphones on during the exam.**
3. **Instructor will keep the record of all those whose cameras and microphones will be on/off.**
4. **Attendance will be marked by the instructor.**
5. **Don't forget to write your name and roll no on each page. Single PDF file name should be your Roll # (e.g. 19L-2020).**
6. **Detailed complete working solution will be rewarded.**
7. **Instructors have a right to take viva (after exams) of such students whose performance has a significant improvement as compared to semester work, therefore, be honest and avoid using any unfair means, consequences are already known to you.**

Q#1[10]: Determine for which value(s) of x the given series:

1. Converges absolutely(5);
2. Converges conditionally(3);
3. and diverges (2).

$$\sum_{n=1}^{\infty} \frac{x^n}{2^n \ln n}$$

Q#2(a)[5]: Discuss the convergence/divergence of the following series.

$$\sum_{n=1}^{\infty} ne^{-n}$$

[b](5): Find the Maclaurin's series for the function $f(x) = \ln(1+x)$ (write at least first three to four terms including nth term of the series). Using the answer also find Maclaurin's series of $\frac{\ln(1+x)}{x}$.

Q#3[10]: A swimming pool containing 60,000 gallon of water has been contaminated by 5 kg of a nontoxic dye that leaves a swimmer's skin an unattractive green. The pool's filtering system can take water from pool, remove the dye, and return the water to the pool at a flow rate of 200gal/min.

Formulate and solve the initial value problem for the filtering process by assuming **Q(t)** as the amount of dye in the pool at any time t.

Q#4[10]: Show that if a and λ are any positive constants, and b is any real number, then every solution of the equation

$$y' + ay = be^{-\lambda x}.$$

has the property that $y \to 0 \quad as \quad x \to \infty$.

Q#5[10]: Show that the following ODE is not exact. Use the integrating factor $\mu(x,y) = \dfrac{1}{xy(2x+y)}$, to convert the ODE into exact form and then solve.

$$(3xy + y^2) + (x^2 + xy)\frac{dy}{dx} = 0.$$

Q#6[10]: Solve the following ODE, if $y_p(x) = x$, is one of its Particular solution.

$$\frac{dy}{dx} = 1 + x^2 - 2xy + y^2$$

Q#7[10]: Given that $y_1(x) = x^{-1}$, is a solution of

$$2x^2y'' + 3xy' - y = 0, \quad x > 0.$$

Find a second linearly independent solution **Without Using Direct Formula.** Verify that the solutions obtained are linearly independent.

Q#8[10]: If $m = 0,\ 0,\ \pm i,\ \pm i,\ +1, -1$ are the roots of the **Auxilary/Characteristics Equation** of a constant co-efficient homogeneous ODE then determine the following:

a. Formulate Homogeneous Constant Co-efficient ODE related to these roots;
b. Linearly independent solutions of the ODE obtained in **part (a)**.
c. General Solution of the ODE obtained in **part (a)**.
d. Are the solutions obtained in **part (c)** linearly independent?

Q#9[10]: Find a suitable form of the particular solution of the given ODE and then solve.

$$y'' + 2y' = 3 + 4\sin x$$

Q#10[10]: For the following set of four ODE's determine **Suitable form of the Particular Solution ONLY.** Do not evaluate co-efficient.

1. $y'' + 3y' = 2x^4 + x^2e^{-3x} + \sin 3x$, **Using Method of Undetermined Coefficients (Annihilator approach).**
2. $y'' + 2y' + 2y = 3e^{-x} + 2e^{-x}\cos x + 4e^{-x}x^2\sin x$, **Using Method of Undetermined Coefficients (Superposition approach).**

Q#11[10]: For the following ODE use an appropriate substitution to find:

$$ax^3 y''' + bx^2 y'' + cxy' + dy = g(x)$$

a) Auxiliary /Characteristic Equation.
b) Choose a = 1, b = 4, c = 1 and d = -1 to find the **Complementary Solution**.
c) Choose g(x) = x^2 to find the **Particular Solution** and hence find the **General Solution**.

Q#12[10]: Find the Fourier Series of the function

$$f(x) = \begin{cases} -\pi, & -2\pi < x < -\pi \\ x, & -\pi \le x < \pi \\ \pi, & \pi \le x < 2\pi \end{cases}$$

b) Suppose f_1, f_2, f_3 are continuous functions on the interval $[a, b]$, use definition of the inner product of two functions show that $< f_1 + f_2, f_3 > = < f_1, f_3 > + < f_2, f_3 >$.

Q#13[10]: Use **Method of Separation** of variables to solve the given PDE.

$$k\frac{\partial^2 u(x,t)}{\partial x^2} - u = \frac{\partial u(x,t)}{\partial t}$$

Subject to $u(0,t) = 0, \quad u(L,t) = 0, \quad t > 0$

$$u(x,0) = f(x), \quad 0 < x < L.$$

Good Luck