National University of Computer and Emerging Sciences, Lahore Campus

Course Name:	Calculus and Analytical Geometry	Course Code:	MT 1003
Degree Program:	[redacted]	Semester:	Fall 2022
Exam Duration:	3 hours	Total Marks:	90
Paper Date:	24- 12- 22	Weight	50%
Section:	ALL	Page(s):	
Exam Type:	Final Exam	[redacted]	

Instruction/Notes: Attempt all questions. Programmable calculators are not allowe[redacted]

Question#1: CLO-4 [10+5]

a) A dynamite blast blows a heavy rock straight up with a launch velocity of 160 ft / sec. It reaches a height of $s = 160t - 16t^2$ ft after t sec.

i. How high does the rock go?
ii. What are the velocity and speed of the rock when it is 256 ft above the ground on the way up? On the way down?
iii. What is the acceleration of the rock at any time t during its flight (after the blast)?
iv. When does the rock hit the ground again?

b) Show that the point (2, 4) lies on the curve $x^3 + y^3 - 9xy = 0$. Then find the tangent and normal to the curve.

Question#2: CLO-3 [10+5]

a) For the given function

$$f(x) = \begin{cases} x^2 - 1, & -1 \le x \le 0, \\ 2x, & 0 < x < 1, \\ 1, & x = 1, \\ -2x + 4, & 1 < x < 2, \\ 0, & 2 < x < 3. \end{cases}$$

i. Does $f(-1)$ exist?
ii. Does $\lim_{x \to -1^+} f(x)$ exist?
iii. Does $\lim_{x \to -1^+} f(x) = f(-1)$?
iv. Is f continuous at $x = -1$?
v. Is f defined at $x = 3$?
vi. Is f continuous at $x = 3$?
vii. At what value of x, f is continuous?
viii. What value should be assigned to $f(2)$ to make the extended function continuous at $x = 2$?

b) Find the horizontal asymptote of $\frac{x^3-2}{|x|^3+1}$.

Question#3: CLO-5 [10+5]

a) An open-top box is to be made by cutting small congruent squares from the corners of a 12-in.-by-12-in. sheet of tin and bending up the sides. How large should the squares cut from the corners be to make the box hold as much as possible?

b) Evaluate the integral $\int \frac{cos\sqrt{x}}{\sqrt{x}\, sin^2\sqrt{x}} dx.$

Question#4: CLO-6 [10+5]

a) Use **Shell method** to find the volume of the solid generated by revolving the region bounded by the curve and line about the y-axis.

$$y = \frac{3}{(2\sqrt{x})}, \quad y = 0, x = 1, x = 4.$$

b) Find the length of curve $x = \int_0^y \sqrt{sec^4 t - 1}\, dt, \quad -\frac{\pi}{4} \le y \le \frac{\pi}{4}.$

Question#5: CLO-6 [5+5+5]

a) Use L-Hopital's rule to find the limit $\lim_{x\to(\frac{\pi}{2})^-} (x - \frac{\pi}{2})\, secx.$

b) Express the integrand as a sum of partial fractions and evaluate the integral

$$\int \frac{s^4+81}{s(s+9)^2} ds$$

c) Evaluate the integral $\int 8sin^3 y\, cos^2 y\, dy$

Question#6: CLO-6 [7+8]

Evaluate the improper integrals and determine whether the given improper integral converges or diverges

a) $\int_0^2 \frac{s+1}{\sqrt{4-s^2}} ds$

b) $\int_0^{\ln 2} x^{-2} e^{-1/x}\, dx$