National University of Computer and Emerging Sciences 0220

Lahore Campus

Multivariable Calculus (MT1008) Final Exam

Date: Jun 5th 2024

Total Time (Hrs.): 3

Total Marks: 100

Total Questions: 10

Course Instructor(s)

Dr. Mazhar Hussain (Moderator)

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Attempt all the questions. The last question is bonus question, if you someone attempts this question he/she will get extra benefit of it

1. **(a) Suppose that $w = f(x,y,z)$, $x = g(r,s)$, $y = h(r,s)$, and $z = k(r,s)$. If all four functions are differentiable, then w has partial derivatives with respect to r and s,write the formulas of chain rule for $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$. Also sketch the dependency diagrams for both equations.**

   (b) Suppose that $F(x,y)$ is differentiable and that the equation $F(x,y) = 0$ defines y as a differentiable function of x. Then at any point where $F_y \neq 0$, prove that

   $$\frac{dy}{dx} = -\frac{F_x}{F_y}$$

   [6+4]

2. **Find the volume of the "ice cream cone" D cut from the solid sphere $\rho \leq 1$ by the cone $\varphi = \pi/3$.** **[10]**

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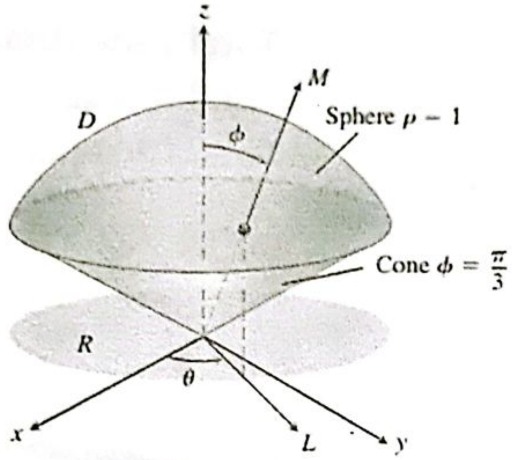

3. State the tangential form of Green's theorem and verify it (tangential form) for the vector field $F = (2x - y)i + (x + 3y)$ and the region R bounded by the ellipse C: $x^2 + 4y^2 = 4$ [10]

4. (a) Write only the formulas for surface integral of a scalar function, when the surface is defined parametrically, implicitly and explicitly.

   (b) Integrate $G(x, y, z) = x$ over the surface given by
   $z = x^2 + y$ for $0 \le x \le 1, -1 \le y \le 1$

   [4+6]

5. Let S be the portion of the cylinder $y = \ln x$ in the first octant whose projection parallel to the y-axis onto the xz-plane is the rectangle $R_{xz}: 1 \le x \le e, 0 \le z \le 1$. Let n be the unit vector normal to S that points away from the xz-plane. Find the flux of $F = 2yj + zk$ through S in the direction of $\vec{n}$.

   [10]

6. State Stokes theorem and use the surface integral in Stokes' Theorem to calculate the circulation of the field F around the curve C in the indicated direction.

$$F = x^2y^3i + j + zk$$

   C: The intersection of the cylinder $x^2 + y^2 = 4$ and the hemisphere $x^2 + y^2 + z^2 = 16, z \ge 0$, counterclockwise when viewed from above.

   [10]

7. Use Laplace transform to solve the given initial-value problem

$$\frac{dy}{dt} + 3y = 13\, sin2t, \qquad y(0) = 6.$$

   [10]

8. (a). Show that the given set of functions is orthogonal on the indicated interval.

$$\left\{1, cos\frac{n\pi}{p}x\right\}, n = 1, 2, 3, \dots, \ [0, p].$$

   (b). Expand $\begin{cases} 0, & -\pi < x < 0 \\ \pi - x, & 0 \le x < \pi \end{cases}$

in a Fourier series.

[3+7]

9. Find the Fourier transform of

$$f(x) = e^{-a|x|}$$

[10]

10. For the given power series, investigate the following

$$\sum_{n=1}^{\infty} \frac{(4x-5)^{2n+1}}{n^{3/2}}$$

(a) Interval of absolute convergence
(b) Conditionally convergence if any
(c) Interval of convergence
(d) Radius and center of convergence

[10]

Bonus question

11. (a) Define Jacobian determinant of the coordinate transformation $x = g(u,y), y = h(u,y)$ and discuss its role in the integration theory.

(b) Evaluate the following double integral.

$$\int_0^4 \int_{x=y/2}^{x=(y/2)+1} \frac{2x-y}{2}\,dx\,dy$$

(c) Use the transformations $x = r\cos\theta, y = r\sin\theta$ to prove that

$$\iint_R f(x,y)\,dx\,dy = \iint_G f(r\cos\theta, r\sin\theta)\,r\,dr\,d\theta.$$

[10]