2021

National University of Computer and Emerging Sciences, Lahore Campus

Course:	Linear Algebra	**Course Code:**	
Program:	BSE (3A)	**Semester:**	3A
Duration:		**Total Marks:**	03
Paper Date:	Time: 20 minutes 30/9	**Weight**	03
Section:	BSSE 3A1	**Page(s):**	
Exam:		**Roll No:**	
Instruction/Notes:	**Attempt All Questions**		

Q① Use parametric equations to describe the infinite solution of: $x_1+3x_2-x_3=-4$, $3x_1+9x_2-3x_3=-12$, $-2x_1-6x_2+2x_3=8$

Let $x_1 = y$, ~~$x_2 =$~~

$y+3x_2-x_3=-4 \xrightarrow{Eq.1}$ ~~$y+3z-x_3=-4$~~

$3y+9x_2-3x_3=-12 \xrightarrow{Eq.2}$ ~~$3y+9z-3x_3=+2$~~

$-2y-6x_2+2x_3=8 \xrightarrow{Eq.3}$ ~~$-2y-6z+2x_3=8$~~

0.5

using Eq 1

$x_1 = y$

$x_2 = \dfrac{-4+x_3-\cancel{3x_2}-y}{3}$

$x_3 = \dfrac{8+6x_2+2y}{2}$

$\frac{2.5}{3.0}$

Q② Find the value of k, for which the augmented matrix of a consistent system is: $\begin{pmatrix} 1 & k & -1 \\ 4 & 8 & -4 \end{pmatrix}$

1.0

~~R_2-4R_1~~

~~$\begin{bmatrix} 1 & k & -1 \\ 0 & (8-4k) & 0 \end{bmatrix}$~~

$[x+ky=-1]\times -4$

$4x+8y=-4$

$-4x-4ky=4$

$+\ 4x+8y=-4$

$(8-4k)y=0$

$8-4k=0$

$4k=8$ $\boxed{k=2}$ $\rightarrow$ so that $0=0$ and infinite solutions exist.

~~no x or y~~ ~~so no unique solut~~

IF $k \neq 2$ then $y=0, x=-1$

so for all values of k, consistent

(P.T.O)

Q③ Solve by Gauss Jordan Method

$-2b+3c=1, \quad 3a+6b-3c=-2, \quad 6a+6b+3c=5$

$$\begin{bmatrix} 0 & -2 & 3 & 1 \\ 3 & 6 & -3 & -2 \\ 6 & 6 & 3 & 5 \end{bmatrix}$$

$$\downarrow R_{12}$$

$$\begin{bmatrix} 3 & 6 & -3 & -2 \\ 0 & -2 & 3 & 1 \\ 6 & 6 & 3 & 5 \end{bmatrix}$$

$$\downarrow R_1 \times 1/3$$

$$\begin{bmatrix} 1 & 2 & -1 & -2/3 \\ 0 & -2 & 3 & 1 \\ 6 & 6 & 3 & 5 \end{bmatrix}$$

$$\downarrow R_3 - 6R_1$$

$$\begin{bmatrix} 1 & 2 & -1 & -2/3 \\ 0 & -2 & 3 & 1 \\ 0 & -6 & 9 & 9 \end{bmatrix}$$

$$\downarrow R_2 \times -1/2, \quad R_3 + 6R_2$$

$$\begin{bmatrix} 1 & 2 & -1 & -2/3 \\ 0 & 1 & -3/2 & -1/2 \\ 0 & 0 & 0 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & -1 & -2/3 \\ 0 & 1 & -3/2 & -1/2 \\ 0 & 0 & 0 & 6 \end{bmatrix}$$

$$\downarrow R_1 - 2R_2$$

$$\begin{bmatrix} 1 & 0 & 2 & 1/3 \\ 0 & 1 & -3/2 & -1/2 \\ 0 & 0 & 0 & 6 \end{bmatrix}$$

$$\downarrow R_3 \times 1/6$$

$$\begin{bmatrix} 1 & 0 & 2 & 1/3 \\ 0 & 1 & -3/2 & -1/2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1.0

$$\downarrow R_2 + 1/2 R_3$$

$$\begin{bmatrix} 1 & 0 & 2 & 1/3 \\ 0 & 1 & -3/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\downarrow R_1 - \frac{1}{3} R_3$$

$$\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -3/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

↑ Answer

$a + 2c = 0$

$b - 3/2\, c = 0$

$0 = 1$ (False) no solutions exist

(SE) 3A

National University of Computer and Emerging Sciences, Lahore Campus				
	Course:	Linear Algebra	Course Code:	
	Program:	BS SE 3A	Semester:	
	Duration:		Total Marks:	4
	Paper Date:	11/11/2021	Weight	4
	Section:	BS SE 3A	Page(s):	
	Exam:	2nd Quiz	Roll No:	
Instruction/Notes:		Attempt All Questions		

1. Find k, such that $\|k\underline{v}\| = 7$, $\underline{v} = (4, 3, -1/2)$

$|k|\,\|v\| = 7$

$|k| \sqrt{4^2 + 3^2 + (-1/2)^2} = 7$

$|k| \left(\frac{\sqrt{101}}{2}\right) = 7$

$k = \pm 1.393$

0.9

$\frac{2.9}{4.0}$

2. Find $\underline{u} \times (\underline{v} \times \underline{w})$, where $\underline{u} = (1, 1, 1)$, $\underline{v} = (-1, -1, -1)$, $\underline{w} = (4, 3, -2)$

$(u \cdot w)v - (u \cdot v)w$

$[(1,1,1)\cdot(4,3,-2)](-1,-1,-1) - [(1,1,1)\cdot(-1,-1,-1)](4,3,-2)$

$(4+3-2)(-1,-1,-1) - [(-1-1-1)(4,3,-2)]$

$(5)(-1,-1,-1) - [(-3)(4,3,-2)]$

$(-5,-5,-5) - (-12,-9,-6)$

$= (7, 4, +1)$

$= (+7, 4, \cancel{1})$

0.7

Q③ Find distance between parallel planes

$3x - y + 2z = 1$ & $9x - 3y + 6z = 2$

↳ Plane 1 ↳ Plane 2

↓

Point on Plane 1

$x = y = 0$

$z = 1/2$

$(0, 0, 1/2)$ with Plane 2 ↗ $9x - 3y - 6z - 2 = 0$

False

$$D = \frac{|9(0) + (-3)(0) + -6(1/2) - 2|}{\sqrt{9^2 + (-3)^2 + (-6)^2}}$$

$$= \frac{5}{3\sqrt{14}}$$

0.7

Q④ Find the volume of tetrahedron given by vectors $\underline{u} = (1, 2, 3)$, $\underline{V} = 2\underline{i} + 3\underline{j} + \underline{k}$

$\underline{w} = (-1, -3, -2)$,

$$\begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ -1 & -3 & -2 \end{vmatrix} \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ -1 & -3 & -2 \end{vmatrix}$$

0.6

$= |(1)(3)(-2) + (2)(1)(-1) + 3(2)(-3) - [(3)(3)(-1) + (2)(2)(-2) + (1)(1)(-3)]|$

$= |(-6 - 2 - 18) - (-20)|$

$= |-6|$

$= 6$

3A Quiz (3)

National University of Computer and Emerging Sciences, Lahore Campus

Course:	Linear algebra	Course Code:	
Program:		Semester:	
Duration:		Total Marks:	03
Paper Date:	16/12/2021	Weight	
Section:	BS SE 3A	Page(s):	
Exam:		Roll No:	[redacted]
Instruction/Notes:	Attempt All Questions		

Q① Find the characteristic equation & eigenvalues of the matrix $A=\begin{bmatrix}-2 & -7\\ 1 & 2\end{bmatrix}$

characteristic Eq.:

$\det(\lambda I - A) = 0$

$\left|\lambda\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} - \begin{bmatrix}-2 & -7\\ 1 & 2\end{bmatrix}\right| = 0$

$\left|\begin{bmatrix}\lambda & 0\\ 0 & \lambda\end{bmatrix} - \begin{bmatrix}-2 & -7\\ 1 & 2\end{bmatrix}\right| = 0$

$\begin{vmatrix}(\lambda+2) & 7\\ -1 & (\lambda-2)\end{vmatrix} = 0$

$\frac{2.6}{3.0}$

$(\lambda+2)(\lambda-2) - (-1)(7) = 0$

$\lambda^2 - 2\lambda + 2\lambda - 4 + 7 = 0$

characteristic eq. $\leftarrow \lambda^2 + 3 = 0$, $\lambda^2 = -3$

No real eigen values

1.0

Q② Find matrix P that diagonalizes A & check your work by computing $P^{-1}AP$. ; $A=\begin{bmatrix}4 & 0\\ 2 & 4\end{bmatrix}$

characteristic Eq.

$\det(\lambda I - A) = 0$

$\left|\lambda\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} - \begin{bmatrix}4 & 0\\ 2 & 4\end{bmatrix}\right| = 0$

$\left|\begin{bmatrix}\lambda & 0\\ 0 & \lambda\end{bmatrix} - \begin{bmatrix}4 & 0\\ 2 & 4\end{bmatrix}\right| = 0$

$\begin{vmatrix}(\lambda-4) & 0\\ -2 & (\lambda-4)\end{vmatrix} = 0$

$(\lambda-4)(\lambda-4) - 0 = 0$

$(\lambda-4)^2 = 0$ (characteristic equation)

$\lambda = 4, 4$ (eigen values)

(P.T.O)

For $\lambda = 4$

$Ax = \lambda x$

$$\begin{bmatrix} 4 & 0 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 4\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$4x_1 = 4x_1$

$2x_1 + 4x_2 = 4x_2$

$\hookrightarrow 2x_1 = 4x_2 - 4x_2$

$2x_1 = 0$

$\boxed{x_1 = 0}$ Let $x_2 = t$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ t \end{bmatrix} = t\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\boxed{\text{basis } P_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}}$$

~~basis $P_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$~~

1.6

$P = \cancel{\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}}$

$\downarrow$

$P^{-1} = \cancel{?}$

~~det $= 0$~~

~~inverse is not possible~~

To Find

$P^{-1} A P$