CS 557: STATISTICAL PATTERN RECOGNITION AND LEARNING
Fall 2016
Exercises: Hidden Markov Models

Problem 1
When playing ludo, Imran normally hides two die in his pocket. Both of them are biased dies
and he switches them unseen from his opponents. Die 1 has the probability of the numbers 1..6
as {1/6,1/6,1/6,1/6,2/9,1/9}. Die 2 has the probability of the numbers 1..6 as {1/12, 2/12, 1/12,
1/12, 2/12, 5/12}. Initially the probability of selecting any one die is 0.5. The probability of
switching from die 1 to die 2 is 0.3 and the probability of switching from die 2 to die 1 is 0.6.

a. Draw the state transition diagram for the above process and also completely specify all the
parameters of an HMM, i.e., (A,B,π).
b. What is the probability of choosing die1 three times and then switching to die 2.
c. What is the probability of this sequence of numbers: 1,6,6,1
d. If we observe the sequence 6,6,6 then what is the most likely sequence of selection of dies.
d. What is the probability of getting the sequence 1,3,2 given that only die1 is selected and rolled
3 times

Problem 2
Suppose there are 5 possible observation symbols {1,2,3,4,5}. We are given the following
HMM parameters:
$\pi = [0,1,0]^{T}$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0.5 & 0.5 & 0 \end{pmatrix}$$

$$B = \begin{pmatrix} .3 & 0 & 0 \\ 0.7 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.3 \\ 0 & 0 & 0.7 \end{pmatrix}$$

a. Write down all the possible state sequences for 4 time steps, along with their probabilities.
b. What is the probability of generating the sequence 3,3,3,3 ?
c. What is the probability of generating the sequence 1,2,3,4 ?
d. What is the probability of generating the sequence 3,1,2,1 ?

PROBLEM 3
Construct a Markov chain with two states (and define all its parameters) so that generally state S_1
repeats itself three times and S_2 repeats itself 4 times in a given observed sequence.

(Hint: look in Rabiner's paper for expected number of time steps in which the system stays in the same state)