**STATISTICAL PATTERN RECOGNITION & LEARNING**
**FALL 2016**
**EXERCISES FOR DIMENSIONALITY REDUCTION**

**QUESTION 1**
Suppose the estimator S for covariance matrix is given by:

$$S = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

If we want to do a spectral decomposition of S then what is the diagonal matrix?

**QUESTION 2**
Suppose we have a two class problem with the following information:

$$S_1 = \begin{bmatrix} 4 & 2 & 5 \\ 2 & 1/2 & 1 \\ 5 & 1 & 2 \end{bmatrix} \qquad S_2 = \begin{bmatrix} 5 & 1 & 1 \\ 1 & 1/2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\text{matrix } S_T = \begin{bmatrix} 19 & 3 & 6 \\ 3 & 31 & 2 \\ 6 & 2 & 54 \end{bmatrix}$$

The number of example points in both classes is the same. Suppose matrix $S_T$ is given by:

$S_T = S_1 + S_2 + S_W$
and
$S_B = S_1 + S_2$
Find the weight vector required to make the transformation of data in LDA. Transform the following points:
(1, 2, 3), (2, 2, 2), (1, 1, 1), (0, 1, 2), (0,0,0)

**QUESTION 3**
Given the following two points:
(0,2,2),(2,4,0)
Apply MDS to the above data points and transform to 1D and 2D space.

**QUESTION 4**
Apply LLE to the following data points using k=3. Note: you can use here Matlab's functions for computing eigen values and eigen vectors. Transform to 2D and 1D space.
(1,1,0,1,2), (0,0,0,0,0),(1,1,2,1,3),(1,3,1,4,1),(2,2,2,1,1)

**QUESTION 5**
Find the weight matrix for applying LLE to the following data points using k=1. (how would you do this without using a calculator?)
(1,1,0,1,2), (0,0,0,0,0),(1,1,2,1,3),(1,3,1,4,1),(2,2,2,1,1)

**QUESTION 6**
Given the following matrix: What is the spectral decomposition of this matrix?

$$\begin{bmatrix} 4 & 2 & 1 \\ 2 & 9 & 1 \\ 1 & 1 & 8 \end{bmatrix}$$

Suppose the above matrix represents the distance square between pairs of instances. Apply MDS to find the mapping of points from original space to a 2D space.

**QUESTION 7**
Apply LDA to the following data:

| $x_1$ | $x_2$ | label |
|---|---|---|
| 1 | 1 | +1 |
| 0 | 1 | +1 |
| 2 | 1 | +1 |
| 2 | 2 | -1 |
| -1 | -1 | +1 |
| 3 | 1 | -1 |
| 1 | 7 | -1 |

What is the mapping of points in the new space?

**QUESTION 8**
Given the following points:
(0,1,1),(2,4,0),(0,0,0),(1,-1,1),(1,2,1),(1,1,0)
Apply MDS to the above data points and transform to 1D and 2D space. Now repeat by using any distance matrix of your choice as the distance measure

**QUESTION 9**
Apply LLE to the following data points using k=2. Transform to 2D and 1D space.
(0,1,1),(2,4,0),(0,0,0),(1,-1,1),(1,2,1),(1,1,0)
For finding the nearest neighbors you have to use the similarity matrix given by the dot product between two vectors. Compute the W matrix by hand and then use Matlab as a tool to compute its eigen values and eigen vectors. What is the transformation in 1D and 2D space.

Suppose that the first three points are classified as +1 and last three points are classified as -1. Use 1-nearest neighbor algorithm to classify the transformed points. What is your training data's balanced error rate, precision, recall, specificity and sensitivity?