Question

a) Use any
Converges
method to
determine if
the

Series

or

E ni 2n

130

η

diverges

Solution

Here

An =

n! 2n

nn

No

$$A_{n+1} = \frac{(n+1)!\ 2^{n+1}}{(n+1)^{n+1}}$$

Ant!

$$\frac{(n+1)!\ 2^{n+1}}{n^n}$$

an

$$\frac{(n+1)^{n}\,n!}{n!\ 2^n}$$

八

lim I am

712

An

$$(n+1)\ n!\ 2^n \times 2$$

$$(n+1)^n\ (n+1)$$

2nn

(n+1)n

lim

7-72

nn

n! gn

2nn

2

nn (1+)/n)"

(1+)

2/10/1

(1+1)^

<1

IV

So by Ratio Test, the

absolutey

2

Series Converges

E n!2"

nn

Converges

b) Find

interval

Fourier series of f on the

f(x)=

{

1

0 4X47

given

Solution: The Fourier series of a

defined

function f

interval (-P,p) is
given *by*

an

bn

Ilxis of = 2. (on cosmax
+ by simona)

where

90=

-P

An=

nai

So
franda

So f(x)

cosnax de

ア

P

bn b2 =

P

-P

x

frel sinna P

dx

An=

Here

a.:

-

P = a

== Sr fix dx

dx

= [5ader

["2de]

+

x

$= \{(x = 0)]$

=

+5"

f(x)

Cosn T

x

dx

flxl cosnx

dx

= 1

o dx + (

Cosne de}

[5° o

dx

ース

= [["

Cosnx

dx]

Sinna

( )

n

=(

-IK

Sinna

Sino

4

bn

11

M

- /K

(0-0) = 0

= (a flx, Sinna x

dx

=

(* Fix) Sinnx

dx

-7

dx +

*[Sooda Sa

sinna da

= [{ "

Sinnx dx}

二六

(

-15

(

六(

=

nπ

-

Cosnx

0

Cosna

n

Coso)

$(-1)" + 2)$

n

$(1-(-1)^)$

So Fourier Series

√(x) =

18

八

ق/-

is

そ

Σ (0 + =

(1-(-))") Sinna

Sinnix)

Ž

n=1

1-(-1)n Sinnx

n7

(J+yx)dx + x2dy =

M(xy) = y2+yx

M(txty)= ty2+ ty- tx

So

degree 2.

So suppose

t2 (y2+yx) = t2

M(x,y)

N (xty) = t2 Neig

both Mand N are homogeneous

functions of

dy

Tx

y=ux

u+xdu

Fre

[dy = udx + xdu

becomes,

(x2+ux^2)dx+x

+ x2 (dx + xdu) = 0

x2 (u2+u) dx + x2udx + x3 du = 。

(n't 2u) dx + x du = .

x

-fdx = +fdu

二十

u(u+2)

-Sox =+ √ (--) du

u+ 2

-hx == (mu -

bu+z)) me == b (

12) + hx

H

A + B

LA u(4+2) utz

LL

1 = A (u) + B

(U+2)

4-o

B = 1/2

4--2

A = -1/2

he = h

jux2

Cz 4x2

4+2

C = y x2

+22

ydr = 4 fa+y6) dy

dr

dy

5180

dy

4/x+

4y =

475

IF = e

S - 옵 dy

чy

= yu Ly - 4xy

S = 4y

=)

dy

al (x yu)

312

=

чy

ny" = 2y2 + c

ну

y1=1

1424

e

y

-4

1=2+ C =) C = -1

y()=(!) = 1 = 2 + C

xy" = 2y2 = 1

xy

√ x =

2y6_y4

гув-уч

Solution

(२)

Let

bacteria culture be P

the size of

A $ given in

question

Integratin
g

-

12

dp ap

ap

dt

=====

KP

dt

dp

K dt.

S

dp

Sk

k dt.

In P

K t + c1

c=e'

$P = ($

$kt.$

ce

The above equation shows

at

any

time t.

Conditions

At $t = 0$

$P = Pu$

$P = 2\ Po$

$t=4$

lising 1st condition in

$Po = ce$

egr

$k(0)$

$P = P2$

ekt

=>

size of bacteria

C = Po

Scanned with
CamScanner

Using and condition

2 P

=

Poe

Po e

4k

4K

2

Now after two

e

4K = ln z

K

==

ln2/4

conditions

0.1732t

P = P2 e

P=

10 P_o = Po e Poe

0.1732 t

介

= 0.1732.

0.1732t.

10

e

t

In 10

13.29

0.1732

It takes

13 days for the bacteria culture to.

grow 10 times

to its initial sizeo

b

Solve the following Cauchy Euler

Equation

23

x3 d3y +

2x2 d2y +

dx3

dxz

+2y=0

Using substitution

y

= xm

dy

m- 1

mx =

dre

dy

m (m-1)xm-2

dxz

d3y

dx3

in above diff egy

ear

m-3

m (m - 1)(m-2) xm

m(m-1)(m-2) xm + 2 m (m-1) xm + 2xm

x

m

m

[(m2-m) (m-2) + 2m2-
2m+2] =

= 0

[m3_2yh2 _ m2+2n+21/2 =
2/2+2] = [m3_2yh2_m2+2x/n

-

=0

= 0

In this

case

we say

that

y=

=xm will be a.

Solution of diff eql for

m3_m2 + 2 = 0

i-e

m=-1 2 m=I+i

2

y = c, x2 + x[C2

1

Cos lnx + Cz sin lnx

C3

Method of Undetermined Coefficient

Date: L

Day:

4

Golden

Cosmetics

Review

Grevine)

W. Grexcuse) Pearl

4sinx $y(0) = 2$ Jy

cor=

y-y

Set

J. Complementary solution

The Aciated

q y

θ

3

2

The Associated Homogeneous form

2

y-y

$-1=0$

$=0$

$(m-1)(m+1)=0$

$m_1 = 1$

m

2

$M2 ==$

Real And Distinct Roots 2

$Y_1 = Ge + \quad Gex$

Yo

Date:

L

Day:

For particular Solution yo

Ypa

Golden Pearl

Cosmetic
s

g(x)

=

x + sind

Yp

Yp=

(Ax +B) + (Ccosx +

Dsinx)

-

a2 = A + (-C sinx + Deosx)

of

yp

=

-Ccosx

-

Dsinx

-

yo" - Yp = atsing

(Ccosx-D sinx)

-

Ax+B) +

((cosx+1)sind

x+sina

Ccosx D sinx-An-B- Ccosx -D Simex sin

-2C cosx-2D sinx-Ax-B

=

x + sina

ʿ(

V

a

Compaing Coefficients of "x and sinx

-2D = 1

$1D = 1/2$

$-A =$

A

$= -1$

20

$[C$

$=$

$\$$

$= 0$

$B=0$

B

Day:

$\cdot\ (\alpha + 0) + (o\c{c}osa + (-$

1/, sinx))

-x

-

тp

Isina

श्

The General Solution is 1.

y= Ye + Yp

y

=

ce

yp

e+ ce

2

e

x

-α

Isina

Given conditions are,

$

0)

-

2

$y(0) = 2 +$

$y'co)=3$

R

We obtain

e

P

-0-1 sign (0)

$y(0) = c, \quad e + c2e$ -0-1

11

2

=

B

C

· C1 + C = 2

ट

+ C2

2

Date:

Day:

y

(0)

=

70

Ge-c2e

Golden Pearl
Cosmetics

1-10

cosx

I cos(9)

c, e-ce°

де

ے

G - C -1 -1 (1)

3

32

4-

Cz

2

2

4 - Cz

G-62

G-Cz

2

=

=

9

2

3+1++

6+2+1

2

New Solving simultaneously,

+

2

41/42 =

9/2

२८, - 2+%,

26

4+9

2

2

we

we get

Date:

Day:

2 C

구

c

$C1 + C2 = 2$

13

$+ \underline{C2}$

Cz

13

2

13

4

2

Golde

Pearl

Cosmeti
cs

4

긑

2

13

4

4-8

G

4

5

4

13

The general solution becomes

13e-5 e-t

=

4

4

-α-1sinx 2

xlution

Using Method of Separation of Variables

Solution of p.de

O

is of the form.

$u(x,t) = X(x).\ T(E)$

$= X''T$

au

)

эка

$= XT$

'

at

Using above assumption in heat eg.

X" J

=

XT

K

X"

XX

I' KT

A function of

m of

X

can be equal to a function

of

I if only we

we

equate bath of them.

equal to same

constant say -1, where

-d is separation constant and arbitrary.

XY FX

X"

TAKT = 0

Od

Q

has solutions

for JA

X

C1 + C2x

fox [1

λ

X

C2 coshax + C1 Sinhax

for [λ= x2 >o

X

CE COS XX

+C Sinax

The

boundary

hansformed

conditions will also be

using

method

of product
sotul.com

X(0) I(t) = 0

T(0) FO

[X (0)=0

u (L,t) = 0

for nontrivial sol

Similarly |X (L) =0

Using the derived boundary conditions

in the

solutions of O·de 2,

particular
solutions.

for 1=0.

ше

O.d·e 2, to find.

-get c1 = (2 =0 si-e trivial

solutia.

for λ=-22 <0

Wie

2

get

for 1=02>0

X(0) = 0

X(x)

C2-C1 =0

G3 = C1 = 0, i-e trivial solubica

=

Cs cos xx

+ Co Sin xx.

O

=

Cs to

=> G=0

$X(x) = \underline{(6\ }\sin ax$

$X(L) = 0$

$X(k) = \underline{C1\ }\text{Sin}\ \&\ L = 0$

$C.\ \text{Sin}\ a\ L$

$= 0$

$\underline{C\ }$

$\text{Sin}\ \sqrt{\ } L = 0$

So to for non trivial solution.

Sin A L

Sin L

= Sin nÃ

चत

= nλ

n

λ= (n_^)2

n=1,2,3,-.

L

It means the only

non trivial

solution for o.de @ with satisfied

boundary condition

IS

X(x) = C2 Sin nλ x

η

n = 1, 2, .

(3)

For solution of ode C

J' takT = 0

dT

-√ KI

at

dT

√ & dr = fade

dt

INT = -λk t

(Seperable)

+ a1

-Akt a

I

e

e

-Akt

T

-

a e

-

mr) kt

I (t)

=

= a e

M=1, 2, 3,

It follows that

$Un\ (x, t) = X2\ (x)\ Tn\ (t)$

Xn

(1)

$Um\ (x,t) = An\ e$

Sin nt

m2 x mx 1.2.3%

L

(4

$(L(x,\ 0) = f(x)$

SLAY

= f(x) = An sin

Sin T

λ

x

L

L

Sin min

mtx f(x)dx_

JAn sin ntv sin

m2 xde

L

L

As

L

(sin

Sin nx

nt

L

x

=>

A

Sin mix da

o

n#m

=

4/2

n=m

L

2 S Sin nt

γι

L

x

feur che

= 12

As

2

$$f(x) = x(L - X)$$

An -

2

L

L

Sin $\underline{n\tilde{A}x}$ (XL - x) *de*

L

L

L

x dx

2

-

L

2

x

Cos nix

L

L

+

L

nī

2

L

x Cosnt x

L

L+2L

nã

nλ

L

x2 Sin nix dx.

L

cos nλx dr

fco

s

L

corn T x x dx

L

Work

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2

XL COS MIX + X Sia ntx

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x2L Cos nã x + 2L{ x L Sin nπ x

nī

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DANA

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-- Sinnt ~ dr)

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cos nλ + ↳ Sinnt

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+24

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n3x3

Cos nã

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Cos nλ x + 24 x SinnTx

n2x2

L

Cosnλ + 223 Sinnt + 2L3 cosnt

3

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n2x2

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n3ñ3

← 2 (-1)

(-1)+1 L

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+ 412

n3x3

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n=1

(nt)2KL

Sinnt

Sin nt x

e

Sin ni x

L