# THE CUTTING–PLANE ALGORITHM (Cont...)

| Contribution Per Unit | $C_j$ | 5 | 8 | 0 | 0 | 0 | |
|---|---|---|---|---|---|---|---|
| $C_{Bi}$ | Basic Variables (B) | $X_1$ | $X_2$ | $S_1$ | $S_2$ | $S_3$ | SOLUTION |
| 8 | $X_2$ | 0 | 1 | 1 | 0 | - 1/2 | 7/2 |
| 5 | $X_1$ | 1 | 0 | -1 | 0 | 1 | 1 |
| 0 | $S_2$ | 0 | 0 | 3 | 1 | -7/2 | 5/2 |
| Total Profit ($Z_j$) | | 5 | 8 | 3 | 0 | 1 | 33 |
| Net Contribution ($C_j - Z_j$) | | 0 | 0 | -3 | 0 | -1 | |

The Solution is still non-integer. So, develop a fractional cut. The Basic variables $X_2$ and $S_2$ are not integers.

**STEP #4:**

**Summary of Integer & Fractional Parts**

| Basic Variable in the above Optimal table | $b_i$ | $[b_i] + f_i$ |
|---|---|---|
| $X_2$ | 7/2 | 3 + 1/2 |
| $S_2$ | 5/2 | 2 + 1/2 |

**STEP # 5:** Here, the fractional parts are the same for $X_2$ & $S_2$. But, we preferred the fractional part of the $X_2$. So, Select the Row "$X_2$" as the Source row for developing Cut.

# THE CUTTING–PLANE ALGORITHM (Cont...)

$7/2 = X_2 + S_1 - 1/2S_3 \rightarrow (3 + 1/2) = (1+0)X_1 + (1+0)S_1 + (-1+1/2)S_3$

The Corresponding fractional cut is:

$-f_i = S_i -$ Summation $((f_i)(\text{Non–Basic Variable}))$

$$\mathbf{-1/2 = S_4 - 1/2S_3}$$

**STEP # 6:** This cut is added to the above table; and further solved using dual simplex method.

| | $C_j$ | 5 | 8 | 0 | 0 | 0 | 0 | |
|---|---|---|---|---|---|---|---|---|
| $CB_i$ | Basic variable | $X_1$ | $X_2$ | $S_1$ | $S_2$ | $S_3$ | $S_4$ | Solution |
| 8 | $X_2$ | 0 | 1 | 1 | 0 | −1/2 | 0 | 7/2 |
| 5 | $X_1$ | 1 | 0 | −1 | 0 | 1 | 0 | 1 |
| 0 | $S_2$ | 0 | 0 | 3 | 1 | −7/2 | 0 | 5/2 |
| 0 | $S_4$ | 0 | 0 | 0 | 0 | $\boxed{-1/2}$ | 1 | −1/2* |
| | $Z_j$ | 5 | 8 | 3 | 0 | 1 | 0 | 33 |
| | $C_j - Z_j$ | 0 | 0 | −3 | 0 | −1* | 0 | |

For ENTERING Variable;

Ratio = $(C_j - Z_j)$ / (Pivot Row <0)

| | | | | | |
|---|---|---|---|---|---|
| -- | -- | -- | -- | 2 | -- |

The smallest positive ratio is "2" and the corresponding variable is "$S_3$". So, the variable "$S_3$" enters the basis.

# THE CUTTING–PLANE ALGORITHM (Cont...)

| Contribution Per Unit $C_j$ | | 5 | 8 | 0 | 0 | 0 | 0 | |
|---|---|---|---|---|---|---|---|---|
| $C_{Bi}$ | Basic Variables (B) | $X_1$ | $X_2$ | $S_1$ | $S_2$ | $S_3$ | $S_4$ | SOLUTION |
| 8 | $X_2$ | 0 | 1 | 1 | 0 | 0 | -1 | 4 |
| 5 | $X_1$ | 1 | 0 | -1 | 0 | 0 | 2 | 0 |
| 0 | $S_2$ | 0 | 0 | 3 | 1 | 0 | -7 | 6 |
| 0 | $S_3$ | 0 | 0 | 0 | 0 | 1 | -2 | 1 |
| Total Profit $(Z_j)$ | | 5 | 8 | 3 | 0 | 0 | 2 | 32 |
| Net Contribution $(C_j - Z_j)$ | | 0 | 0 | -3 | 0 | 0 | -2 | |

So, The values of all the basic variables are integers. So, the optimality is reached and the corresponding results are summarized as follows:

**$X_1 = 0, X_2 = 4$ and $Z$ (Optimum) $= 32$**