Operations Research (MT 4031)

Sessional-I Exam

Date:

Course Instructor(s)

Dr. Misbah Irshad, Dr. Uzma Bashir, Mr. Yasir Yasin

Total Time (Hrs.): 1

Total Marks: 30

Total Questions: 3

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Attempt all the questions.

CLO #: CLO statement for question Q1 ....

Q1:

a. **Formulate the following problem as a linear mathematical model. Find its solution graphically .**

A farmer has 20 hectares to grow barley and swedes. The farmer has to decide to decide how much of each to grow. The cost per hectare for barley is $30 and swedes is $20. The farmer has budgeted $480. Barley requires 1 man-day per hectare and swedes require 2 man-days per hectare. There are 36 man-days available. The profit of barley is $100 per hectare and on swedes is $120 per hectare.

SOLUTION

$$\text{Max } z = 100x_1 + 120x_2$$

$$\begin{aligned} \text{subject to} \qquad x_1 + x_2 &\le 20 \\ 30x_1 + 20x_2 &\le 480 \\ x_1 + 2x_2 &\le 36 \\ x_1, x_2 &\ge 0. \end{aligned}$$

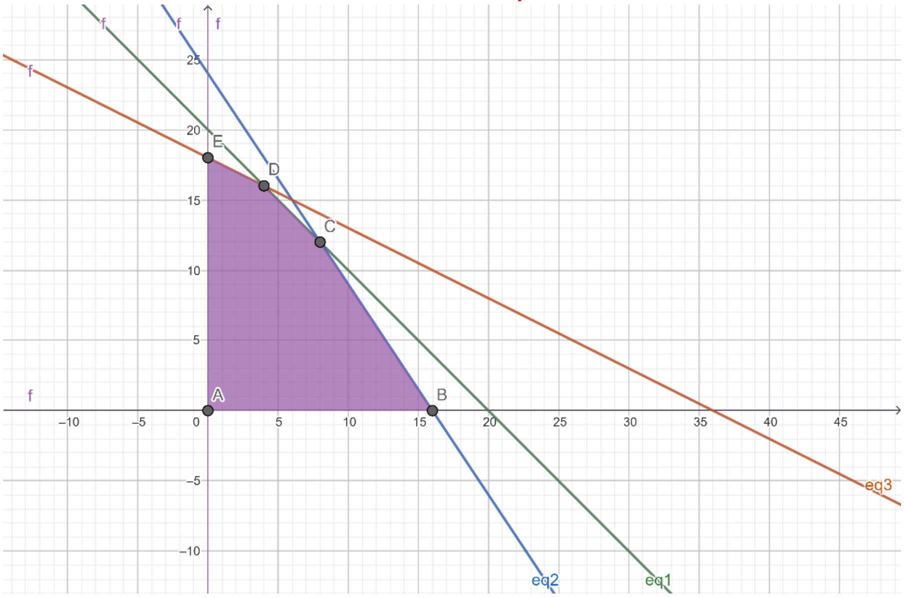
1 mark

Corner points:

A(0,0), B(16,0), C(8,12), D(4,16), E(0,18). **2 marks.**

Z(A)=0, z(B)=1600, z(C)=2240, z(D)= 2320, z(E)=2160. **1 mark**

Conclusion: z is maximum at D.



2 marks

i. **What will be the optimality range of the model.**

Consider a general objective function

$$\text{Max } Z = c_1x_1 + c_2x_2$$

The optimality range is determined by $\frac{c_1}{c_2}$

For constraint 1---- $\frac{c_1}{c_2} = 1$

For constraint 2------- $\frac{c_1}{c_2} = 3/2$

For constraint 3----- $\frac{c_1}{c_2} = 1/2$

Thus $0.5 \leq \frac{c_1}{c_2} \leq 1.5.$ **3 marks**

ii. **Suppose that the unit revenues for products barley and swedes are changed to \$150 and \$95, respectively. Will the current optimum remain the same.** **1 mark**

The solution will not remain optimal as $\frac{c_1}{c_2} = \frac{150}{95} = 1.57$ falls outside the optimality range.

CLO #: CLO statement for question Q2 …………

Q2: Solve the following model using an appropriate method.

$$\text{Max } z = 3x_1 + 2x_2 + 3x_3$$

subject to

$$2x_1 + x_2 + x_3 \leq 2$$
$$3x_1 + 4x_2 + 2x_3 \geq 8$$
$$x_1, x_2, x_3 \geq 0.$$

SOLUTION:

Expressing the problem in standard form by adding, slack, surplus and artificial variables.

$$\text{Max } z = 3x_1 + 2x_2 + 3x_3$$

subject to

$$2x_1 + x_2 + x_3 + x_4 = 2$$
$$3x_1 + 4x_2 + 2x_3 + x_5 + R = 8$$
$$x_1, x_2, x_3, x_4, x_5, R \geq 0.$$

1 mark

By using M method and assigning a penalty M to artificial variable in objective function

$$z = 3x_1 + 2x_2 + 3x_3 - MR$$

Taking M=100, and eliminating R **2 marks**

$$z = 303x_1 + 402x_2 + 203x_3 - 100x_5 - 800$$

Basic	x_1	x_2	x_3	x_5	x_4	R	sol
z	-303	-402	-203	100	0	0	-800
x_4	2	1	1	0	1	0	2
R	3	4	2	-1	0	1	8

2 marks

Basic	x_1	x_2	x_3	x_5	x_4	sol
z	-3/2	0	-2	-1/2	0	4
x_4	5/4	0	1/2	1/4	1	0
x_2	3/4	1	1/2	-1/4	0	2

2 marks

Basic	x_1	x_2	x_3	x_5	x_4	sol
z	7/2	0	0	1/2	4	4
x_3	5/2	0	1	1/2	2	0
x_2	-1/2	1	0	-1/2	-1	2

2 marks

Optimal and feasible solution is: **1 mark**

$$x_1 = 0, x_2 = 2, x_3 = 0, z = 4$$

CLO #: CLO statement for question Q3 ………….

Q3: Solve the following LPP and comment on the solution.

$$\text{Max } z = x_1 + 2x_2$$

subject to

$$x_1 + x_2 \leq 3$$
$$x_2 \leq 2$$
$$\frac{1}{2}x_1 + x_2 \leq \frac{5}{2}$$
$$x_1, x_2 \geq 0.$$

SOLUTION:

a. Expressing the problem in standard form by adding slack variables. 1 mark

$$\text{Max } z = x_1 + 2x_2$$

$$x_1 + x_2 + x_3 = 3$$
$$x_2 + x_4 = 2$$
$$\frac{1}{2}x_1 + x_2 + x_5 = \frac{5}{2}$$
$$all\ vars \geq 0.$$

Basic	x_1	x_2	x_3	x_4	x_5	sol
z	-1	-2	0	0	0	0
x_3	1	1	1	0	0	3
x_4	0	1	0	1	0	2
x_5	1/2	1	0	0	1	5/2

2 mark

Basic	x_1	x_2	x_3	x_4	x_5	sol
z	-1	0	0	2	0	4
x_3	1	0	1	-1	0	1
x_2	0	1	0	1	0	2
x_5	1/2	0	0	-1	1	1/2

2 marks

Basic	x_1	x_2	x_3	x_4	x_5	sol
z	0	0	1	1	0	5
x_1	1	0	1	-1	0	1
x_2	0	1	0	0	0	2
x_5	0	0	-1/2	-1/2	1	0

2 marks

b. In iteration 2, there is tie in the leaving variable, as a result basic variable $x_5 = 0$ in the next iteration which indicates that the solution is degenerate. **3 marks**