

## Linear Algebra (MT1004)

Date: May 27<sup>th</sup> 2024

## Course Instructors

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## Final-Term Exam

Total Time: 3 Hours

Total Marks: 85

Total Questions: 8

22i-2505 SE-4A

Roll No

Section

JKS

Student Signature

Attempt all the questions. Attach question paper with answer sheet.

**CLO # 1:** Convert real life problems into system of linear equations and perform elementary row operations to solve them. (10 marks)

**Q. No 1:** Zainab, Ayesha, and Maya go to a jewelry store. Zainab buys 3 necklaces, 1 ring, and 2 bracelets and spends \$17. Ayesha buys 1 necklace, 4 rings, and 5 bracelets and spends \$31. Maya buys 6 necklaces, 2 rings, and 1 bracelet and spends \$19. Determine the system of linear equations from the given information where the unknown variables  $x$ ,  $y$  and  $z$  represents necklaces, ring and bracelets respectively. How much does each type of jewelry cost? Use the Gauss Elimination method.

**CLO #4:** Analyse vectors and their properties in 2-space, 3-space and n-space. (10 marks)

**Q. No 2:** Determine whether the given planes are parallel. Also find the distance between them.  
 $2x - y + z = 1$  and  $2x - y + z = -1$

**CLO #5:** Recognize vector spaces and/or subspaces and compute their bases and its dimension. (5+5 marks)

**Q. No 3:** Given the set of vectors  $v_1 = (1, 2, 3)$ ,  $v_2 = (-4, 5, 6)$ ,  $v_3 = (7, -8, 9)$ .

a. Show that  $\{v_1, v_2, v_3\}$  forms a basis for  $R^3$ .

b. Find the coordinate vector of  $w = (5, -12, 3)$  relative to the basis  $S = \{v_1, v_2, v_3\}$ .

**CLO #6:** Eigen Value analysis and use it to Diagonalize a matrix and find its powers. (10 marks)

**Q. No 4:** Find a matrix  $P$  that diagonalizes  $A$ , and check your work by computing  $P^{-1}AP$ .

$$\begin{bmatrix} 1 & 3 \\ 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix}$$

and compute  $A^{10}$ .

**CLO #7:** Identify inner product spaces. (10 marks)

**Q. No 5:** Let  $p = x + x^3$  and  $q = 1 + x^2$ . Find  $\|p\|$  and  $d(p, q)$  relative to the evaluation inner product  $P_3$  at the stated sample points.  $x_0 = -2, x_1 = -1, x_2 = 0, x_3 = 1$ .

CLO #7: Identify inner product spaces and/or perform Gram Schmidt process. (10 marks)

Q. No 6: Let  $R^3$  have the Euclidean inner product and use the Gram-Schmidt process to find the basis  $\{u_1, u_2, u_3\}$  into an orthonormal basis where  $u_1 = (1, 0, 0)$ ,  $u_2 = (3, 7, -2)$ ,  $u_3 = (0, 4, 1)$ .

CLO #8: Express a linear transformation using matrices and to solve problems. (10 marks)

Q. No 7: Consider the basis  $S = \{v_1, v_2, v_3\}$  for  $R^3$ ,  $v_1 = (1, 2, 1)$ ,  $v_2 = (2, 9, 0)$ , and  $v_3 = (3, 3, 4)$ , and let  $T: R^3 \rightarrow R^2$  be the linear transformation for which

$$T(v_1) = (1, 0), T(v_2) = (-1, 1), T(v_3) = (0, 1).$$

Find a formula for  $T(x_1, x_2, x_3)$ , and use that formula to find  $T = (7, 13, 7)$ .

CLO #8: Express a linear transformation using matrices and to solve problems. (5+5+5 marks)

Q. No 8: Given the standard matrix  $A = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & 2 \\ -1 & 1 & 0 \end{bmatrix}$  of a matrix transformation  $T_A : R^3 \rightarrow R^3$ .

- Determine  $T_A$  is an isomorphism.
- Find the range and Kernel of the transformation.
- Also find the dimension of row space and null space of  $A$ .

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