Do not write below this line

Attempt all the questions on Answer Book.

Statistical tables & Formula sheet are attached. You are not allowed to use your own tables.

Recreate tables/graphs (if any) on the answer book.

Don't write anything on the question paper and Do Not Attach it.

Lead Pencil work wouldn't marked or claimed for rechecking. Use permanent ink pen.

If you found any ambiguity in the data then do not ask anything to the invigilator, just make assumption and continue solving your paper.

Show the calculation/procedure of each Question/Sub-Parts properly.

Must write the final answer of each question up to 4 significant figures.

CLO 2: Use of basic counting principles & all laws of probability to analyze probabilistic experiment.

[marks: 7+8]

Q1:

An individual has 3 different email accounts. Most of her emails, in fact 70%, come into account no. 1, whereas 20% come into account no. 2 and the remaining 10% into account no. 3. It is known from past experience that 1%, 2% and 5% emails from account no 1, account no 2 and account no 3 respectively are spam.

a) What is the probability that a randomly selected email is spam?

b) If the observed email is identified as spam, to which account it is most likely associated?

CLO 3: Identify and analyze the type of random variables and its probability *distributions.*

Q2: **[marks: 6+3+11]**

Let X denote the number of software updates a company's server undergoes in a month: 1, 2, or 3 updates. Let Y denote the number of times the server experiences downtime. Their joint probability distribution is given as:

$f(x,y)$		x		
		1	2	3
y	1	0.05	0.05	0.10
	3	0.05	0.10	0.35
	5	0.00	0.20	0.10

a) Evaluate the marginal distribution of X and Y.
b) Find $P(Y = 3 \mid X = 2)$.
c) Verify whether $E(XY) = E(X)\,E(Y)$ or not?

CLO 4 & 5: Determine the type of discrete & continuous distribution and evaluate its probability distribution.

Q3: **[marks: 10+5]**

a) Each time a modem transmits one bit, the receiving modem analyzes the signal that arrives and decides whether the transmitted bit is 0 or 1. It makes an error with probability p, independent of whether any other bit is received correctly. If the probability of error is $p = 0.01$ and the modem transmits 200 bits, what is the probability that at most 3 errors will be observed?

b) A software development company releases updates to its mobile application every evening. Once the updates are released, they must be tested on various devices to ensure compatibility and functionality. The time required to test each update follows a normal distribution, with a mean of 120 minutes and a standard deviation of 20 minutes. If a manager must be present until 90% of the updates are tested, how long will the manager need to be there? Also interpret the result.

CLO 1 & 6: Compute and interpret various measures of location and variation. Apply classical hypothesis testing/confidence interval for single population.

Q4: **[marks: 10]**

The operations manager of a plant making cellular telephones has proposed rearranging the production process to be more efficient. He wants to estimate the average time to assemble the telephone using the new arrangement. For this purpose, a sample of 15 time points used to assemble the cellular telephone under the new arrangement is provided below:

Assembling Time (in minutes)														
8	8	8	8	8	10	14	15	13	8	11	13	11	10	7

What is a reasonable range of values for the population mean assembling time? Also, interpret the results.

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CLO 6 & 8: **Apply classical hypothesis testing/confidence intervals for single population and to compare two populations and draw inferences.** ***Methodologies of regression analysis for the future predictions. Able to check goodness of fit and strength of relationship between two variables.***

Q5: **[marks: 10+ (10+10) +10]**

PCW World rated four component characteristics for 10 ultraportable laptop computers: features, performance, design, and price. Each characteristic was rated using a 0-100 point scale. An overall rating, referred to as the PCW World Rating, was then developed for each laptop. The following table shows the features rating and the PCW World Rating for the 10 laptop computers.

Model	Features Rating	PCW World rating
Thinkpad X200	87	83
VGN-Z598U	85	82
U6V	80	81
Elitebook 2530p	75	78
X360	80	78
Thinkpad X360	76	78
Ideapad U110	81	77
Micro Express JFT250	73	75
Toughbook W7	79	73
HP Envy 133	68	72

a) Derive an estimated regression equation to predict PCW world ranking based on features rating.

b) Compute explained and unexplained variation? Complete the table below and comment on the overall significance of the regression model.

Source of Variation	Degrees of Freedom	Sum of Square	Mean Square	F-Ratio	p-value
Regression	$k-1$	SSR	SSR/k-1		0.006
Error	$k\ n-k$	SSE	SSE/n-k		
Total	$n-1$				
Decision/Conclusion:					

Note: *Recreate the above ANOVA table in the same sequence of entries. Use a full page in landscape orientation in your answer book for the table. Write the final answers in the table. Show the calculations on a separate page, either before or after the table.*

:) Is it reasonable to conclude that in a population, there is a positive relationship between the PCW world ranking and features rating? Test at 5% level of significance using correlation coefficient.

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FORMULA SHEET FINAL EXAM SPRING 2024

$$\bar{x} \pm Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

$$z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$$

$$\bar{x} \pm t_{(\alpha/2,\nu)}\frac{s}{\sqrt{n}};$$

$$t = \frac{\bar{x}-\mu}{s/\sqrt{n}}$$

$$b_1 = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2} = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2}$$

$$b_0 = \bar{y} - b_1\bar{x}$$

$$r = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{\sum(x-\bar{x})^2 \sum(y-\bar{y})^2}}$$

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{(n\sum x^2 - (\sum x)^2)(n\sum y^2 - (\sum y)^2)}}$$

$$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$$

$$t = \frac{b_1-\beta_1}{s_{yx}/\sqrt{S_{xx}}} \text{ or } \frac{b_1-\beta_1}{s_{b1}}$$

$$s_{yx} = \sqrt{\frac{\sum(Y-\hat{y})^2}{n-2}}$$

$$S_{xx} = \sum(X-\bar{X})^2$$

$$\text{Var(X)} = \text{E}(X^2) - [E(X)]^2$$

$$\text{Var(X)} = \frac{\sum fx^2}{\sum f} - (\frac{\sum fx}{\sum f})^2$$

$$\text{P(A} \cap B \cap C) = P(A)P(B)P(C)$$

$$\text{P(A} \cap B \cap C) = P(A)P(B/A)P(C/A \cap B)$$

$$\text{P(AUB)} = \text{P(A)+P(B)-P(A} \cap B)$$

$$F = \frac{s_1^2}{s_2^2} \text{ If } s_1^2 > s_2^2, \quad F = \frac{MSR}{MSE}$$

$$P(B_r/A) = \frac{P(B_r \cap A)}{\sum_l^k P(B_l \cap A)} = \frac{P(B_r)P(A/B_r)}{\sum_l^k P(B_l)P(A/B_l)} \quad ,r = 1,2,,k$$

$$b(x;n,p) = \binom{n}{x}p^x q^{n-x}, x = 0,1,\dots n, q = 1-p,$$

$$f(x_1,x_2,\dots,x_k;p_1,p_2,\dots,p_k;n) = \binom{n}{x_1,x_2,\dots,x_k}p_1^{x_1}p_2^{x_2},\dots,p_k^{x_k}$$

$$h(x;N,n,k) = \frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}}$$

$$p(x;\lambda t) = \frac{e^{-\lambda t}(\lambda t)^x}{x!}, \quad x = 0,1,2,\dots, \quad \mu = \lambda t$$

$$R^2 = 1 - \frac{SSE}{SST} \text{ or } \frac{SSR}{SST}$$

$$SSR = \sum(\hat{y}-\bar{Y})^2$$ explained

$$SST = \sum(Y-\bar{Y})^2 = \sum Y^2 - \frac{(\sum Y)^2}{n}$$ unexplained

$$SSE = \sum(Y-\hat{y})^2 = \sum Y^2 - b_0\sum y - b_1\sum XY$$

$$\mu = E(X)$$

$$\text{E(X)} = \sum xf(x)$$

$$t = \frac{\bar{d}-d_0}{s_d/\sqrt{n}};$$

$$\bar{d} \pm t_{(\alpha/2,\nu)}\, s_d/\sqrt{n}$$

$$z = \frac{x-\mu}{\sigma}$$

Lower limit = Q1-1.5(IQR)

Upper limit = Q3+1.5(IQR)

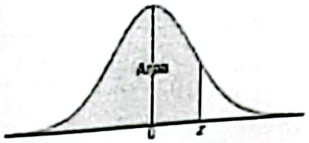

Table A.3 Areas under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Table A.3 (continued) Areas under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998