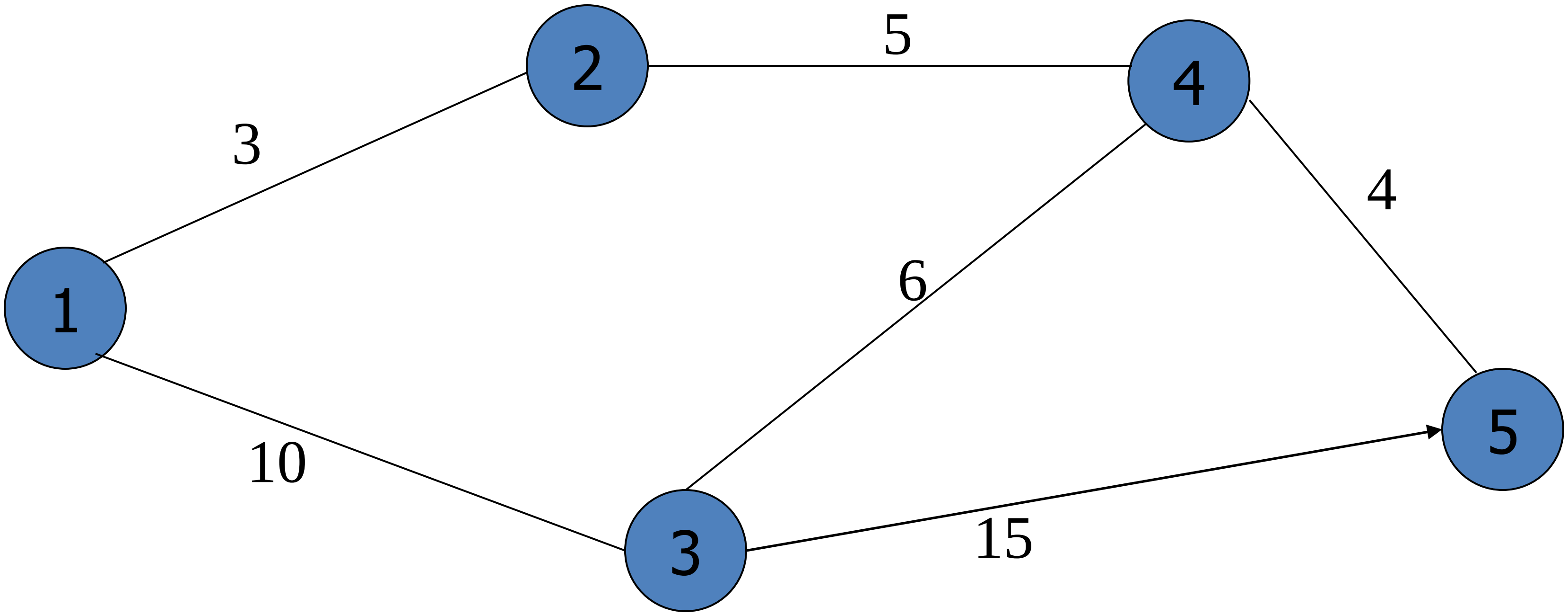
EXAMPLE:

Determine the shortest routes with their distances between node–1 & node–5. Also between node–2 & node–3 using Floyd's algorithm.



SOLUTION:

ITERATION – 0:

$$
D_0 = \begin{array}{c|ccccc}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
1 & - & 3 & 10 & \infty & \infty \\
\hline
2 & 3 & - & \infty & 5 & \infty \\
\hline
3 & 10 & \infty & - & 6 & 15 \\
\hline
4 & \infty & 5 & 6 & - & 4 \\
\hline
5 & \infty & \infty & \infty & 4 & - \\
\hline
\end{array}
, \quad
S_0 = \begin{array}{c|ccccc}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
1 & - & 2 & 3 & 4 & 5 \\
\hline
2 & 1 & - & 3 & 4 & 5 \\
\hline
3 & 1 & 2 & - & 4 & 5 \\
\hline
4 & 1 & 2 & 3 & - & 5 \\
\hline
5 & 1 & 2 & 3 & 4 & - \\
\hline
\end{array}
$$

ITERATION – 1:

- Set k = 1, thus PIVOT column – 1 and row – 1.

- Improvements can be made for d_{23} and d_{32}.

    1. Replace d_{23} by $d_{21} + d_{13} = 3 + 10 = 13$ & Set $S_{23} = 1$.
    2. Replace d_{32} by $d_{31} + d_{12} = 10 + 3 = 13$ & Set $S_{32} = 1$.

$$D_1 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & - & 3 & 10 & \infty & \infty \\ \hline 2 & 3 & - & 13 & 5 & \infty \\ \hline 3 & 10 & 13 & - & 6 & 15 \\ \hline 4 & \infty & 5 & 6 & - & 4 \\ \hline 5 & \infty & \infty & \infty & 4 & - \\ \hline \end{array}, \quad S_1 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & - & 2 & 3 & 4 & 5 \\ \hline 2 & 1 & - & 1 & 4 & 5 \\ \hline 3 & 1 & 1 & - & 4 & 5 \\ \hline 4 & 1 & 2 & 3 & - & 5 \\ \hline 5 & 1 & 2 & 3 & 4 & - \\ \hline \end{array}$$

ITERATION – 2:

- Set k = 2, thus PIVOT column – 2 and row – 2.

- Improvements can be made for d_{14} and d_{41}.

    1. Replace d_{14} by $d_{12} + d_{24} = 3 + 5 = 8$ & Set $S_{14} = 2$.
    2. Replace d_{41} by $d_{42} + d_{21} = 5 + 3 = 8$ & Set $S_{41} = 2$.

$$D_2 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & - & 3 & 10 & 8 & \infty \\ \hline 2 & 3 & - & 13 & 5 & \infty \\ \hline 3 & 10 & 13 & - & 6 & 15 \\ \hline 4 & 8 & 5 & 6 & - & 4 \\ \hline 5 & \infty & \infty & \infty & 4 & - \\ \hline \end{array}, \quad S_2 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & - & 2 & 3 & 2 & 5 \\ \hline 2 & 1 & - & 1 & 4 & 5 \\ \hline 3 & 1 & 1 & - & 4 & 5 \\ \hline 4 & 2 & 2 & 3 & - & 5 \\ \hline 5 & 1 & 2 & 3 & 4 & - \\ \hline \end{array}$$

ITERATION – 3:

- Set $k = 3$, thus PIVOT column – 3 and row – 3.
- Improvements can be made for d_{15} and d_{25}.

1. Replace d_{15} by $d_{13} + d_{35} = 10 + 15 = 25$ & Set $S_{15} = 3$.
2. Replace d_{25} by $d_{23} + d_{35} = 13 + 15 = 28$ & Set $S_{25} = 3$.

$$
D_3 = \begin{array}{c|ccccc}
 & 1 & 2 & 3 & 4 & 5 \\ \hline
1 & - & 3 & 10 & 8 & 25 \\
2 & 3 & - & 13 & 5 & 28 \\
3 & 10 & 13 & - & 6 & 15 \\
4 & 8 & 5 & 6 & - & 4 \\
5 & \infty & \infty & \infty & 4 & -
\end{array}
, \quad
S_3 = \begin{array}{c|ccccc}
 & 1 & 2 & 3 & 4 & 5 \\ \hline
1 & - & 2 & 3 & 2 & 3 \\
2 & 1 & - & 1 & 4 & 3 \\
3 & 1 & 1 & - & 4 & 5 \\
4 & 2 & 2 & 3 & - & 5 \\
5 & 1 & 2 & 3 & 4 & -
\end{array}
$$

ITERATION – 4:

- Set k = 4, thus PIVOT column – 4 and row – 4.
- Improvements can be made for d_{25}, d_{52}, d_{23}, d_{32}, d_{35}, d_{53}, d_{15} and d_{51}.

1. Replace d_{25} by $d_{24} + d_{45} = 5 + 4 = 9$ & Set $S_{25} = 4$.
2. Replace d_{52} by $d_{54} + d_{42} = 4 + 5 = 9$ & Set $S_{52} = 4$.
3. Replace d_{23} by $d_{24} + d_{43} = 5 + 6 = 11$ & Set $S_{23} = 4$.
4. Replace d_{32} by $d_{34} + d_{42} = 6 + 5 = 11$ & Set $S_{32} = 4$.
5. Replace d_{35} by $d_{34} + d_{45} = 6 + 4 = 10$ & Set $S_{35} = 4$.
6. Replace d_{53} by $d_{54} + d_{43} = 4 + 6 = 10$ & Set $S_{53} = 4$.
7. Replace d_{15} by $d_{14} + d_{45} = 8 + 4 = 12$ & Set $S_{15} = 4$.
8. Replace d_{51} by $d_{54} + d_{41} = 4 + 8 = 12$ & Set $S_{51} = 4$.

$$D_4 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & - & 3 & 10 & 8 & 12 \\ \hline 2 & 3 & - & 11 & 5 & 9 \\ \hline 3 & 10 & 11 & - & 6 & 10 \\ \hline 4 & 8 & 5 & 6 & - & 4 \\ \hline 5 & 12 & 9 & 10 & 4 & - \\ \hline \end{array} , \quad S_4 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & - & 2 & 3 & 2 & 4 \\ \hline 2 & 1 & - & 4 & 4 & 4 \\ \hline 3 & 1 & 4 & - & 4 & 4 \\ \hline 4 & 2 & 2 & 3 & - & 5 \\ \hline 5 & 4 & 4 & 4 & 4 & - \\ \hline \end{array}$$

ITERATION – 5:

- Set k = 5, thus PIVOT column–5 and row–5.
- No further Improvements are possible thus:

1. d_{15} = 12 ROUTE = 1 → 2 → 4 → 5

   " Route is 1 → 5 if S_{15} = 5 but S_{15} = 4. So, Route is 1 → 4 → 5 if S_{14} = 4 but = 2. So, Route is 1 → 2 → 4 → 5 if S_{12} = 2."

2. Route Node – 2 to Node – 3 is: 2 → 4 → 3

   " Route is 2 → 4 if S_{24} = 4. Route is 4 → 3 if S_{43} = 3. So, Route from node – 2 to node – 3 is: 2 → 4 → 3