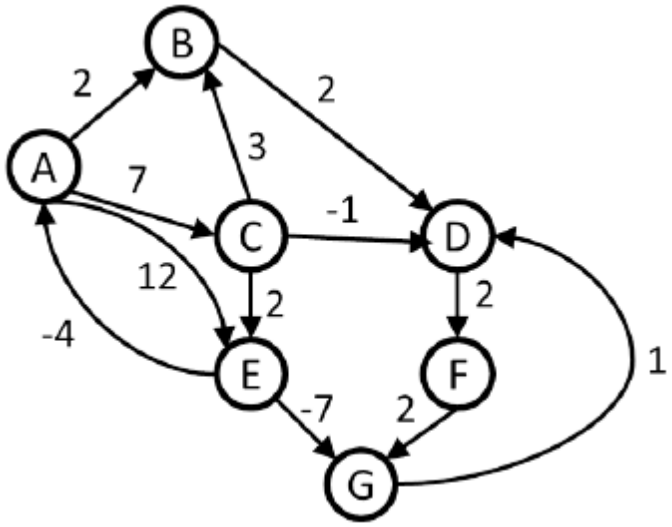
Design and Analysis of Algorithms

HW6

Total Marks = 40

Q1) Consider the following directed, weighted graph: Even though the graph has negative weight edges, step through Dijkstra's algorithm to calculate supposedly shortest paths from A to every other vertex. Show your steps in the table below. Cross out old values and write in new ones, from left to right within each cell, as the algorithm proceeds. Also list the vertices in the order which you marked them known.



Solution:

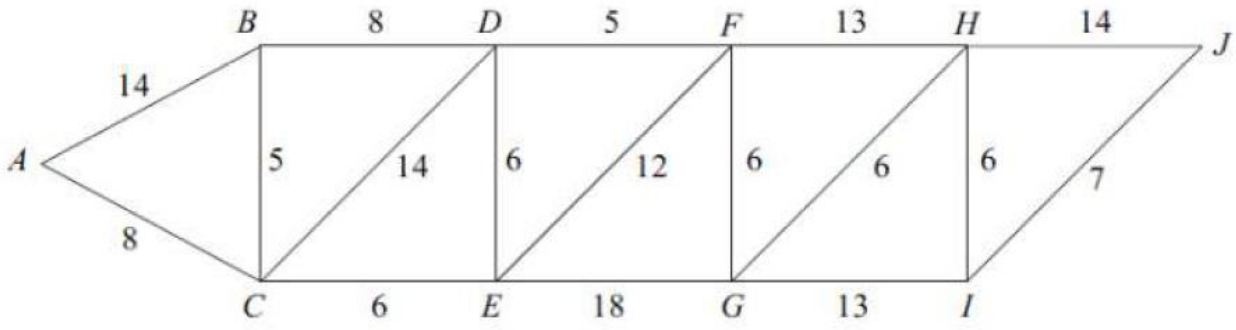
Known vertices (in order marked known): [4 Marks]

Vertex	Shortest Path	Predecessor in Shortest Path

(b) Dijkstra's algorithm found the wrong path to some of the vertices. For just the vertices where the wrong path was computed, indicate both the path that was computed and the correct path. [2 Marks]

(c) What single edge could be removed from the graph such that Dijkstra's algorithm would happen to compute correct answers for all vertices in the remaining graph? [2 Marks]

Q2) The following network show times, in minutes, to travel between 10 towns. [3+2 Marks]



a) Use Dijekstra's algorithm on above figure to find minimum travel time from A to J.

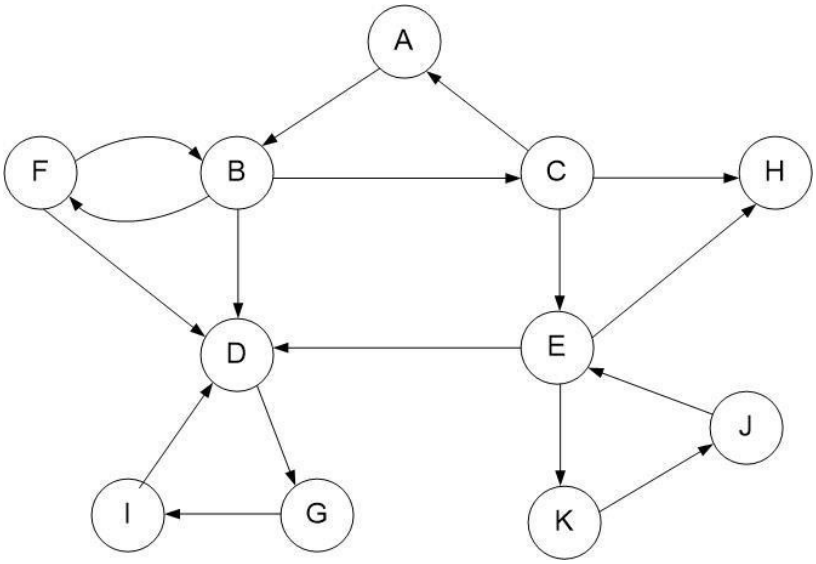
b) State the corresponding route.

Q3) CM of Punjab has decided to build a new road in Lahore. He has a choice of k roads from which he can only build one. However, secretly, CM is only interested in two places in the city: s and t (his office and home). He wishes to build the road which, when added into the city, reduces the cost of the shortest path from s to t the most. He has hired you to write a computer program to tell him which of the k roads to build for this purpose. The input to your program is going to be a directed weighted graph and list of k weighted edges not already in the graph.

(a)Write an algorithm that does the job in O(k(|E|)lg|V|).Note that since k itself is O($|V|^2$)this is a pretty slow algorithm. [4 Marks]

(b) CM is in a hurry. He wants you to write an algorithm that works in O (k+|E|lg|V|), only. Can you do it? [8 Marks]

Q4) Determine the strongly connected components of the graph using the algorithm discussed in class. Show all workings. Show finish times of both iterations of DFS. [8 Marks]



Q5) Suppose a manager of a company wants to find out the best order of execution of different processes for production. Some processes have constraints such that a particular process should be completed before another can be started. Given all possible such constraints among particular processes, give an efficient algorithm (pseudocode) to find the best order of execution of processes which will not violate any constraint. There are n processes and each constraint is expressed as follows: (p_1, p_2) means p_1 should be completed before p_2. [7 Marks]

Input to algorithm: n processes and constraints. For each process p_i, you are given a list of processes such that p_i should be completed before those processes.