National University of Computer and Emerging Sciences, Lahore Campus

Course:	Theory of Automata	Course Code:	CS-3005
Program:	BS (Computer Science)	Semester:	Fall 2022
Duration:	180 Minutes	Total Marks:	80
Paper Date:	17-December-2022	Weight	40 %
Section:	ALL	Page(s):	16
Exam:	Final Term	Roll No.	

Instruction/Notes:
1. Answer in the space provided, showing all the work.
2. Rough Sheets are not allowed.
3. In case of confusion or ambiguity make a reasonable assumption.
4. Attempt all Questions

Section 1: (Short Question Answers) [25 Marks]

Q1: What is the cardinality of L? [3 Marks]

$L = \{ w \text{ over } \Sigma \mid |w| > 5 \text{ and } |w| \leq 10 \}$.

$\Sigma = \{0,1,2\}$

Note: Cardinality means the total number of elements in the given set.

88209

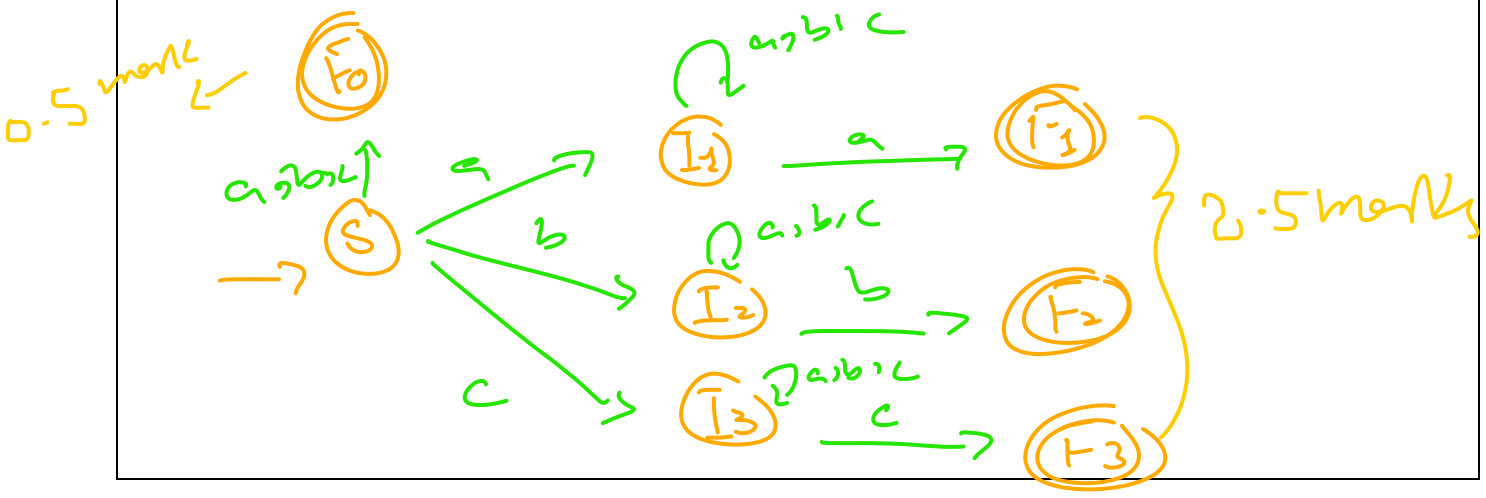
3 marks

$\Sigma^6 + \Sigma^7 + \Sigma^8 + \Sigma^9 + \Sigma^{10} = 3$

Q2: Design a Finite Automaton (DFA or NFA) for the following language. [3 Marks]

$L = \{ x \mid x \text{ over } \{a, b, c\} \text{ x starts and ends with same alphabet} \}$

Note: Pick a suitable sub-category from FA and design the machine accordingly.



Q3: What will be the language of the following grammar? [7 Marks]

$$L \rightarrow ALB \mid AABB$$

$$A \rightarrow aAb \mid aaabbb$$

$$B \rightarrow ccBd \mid \wedge$$

Note: You are required to write answer in a proper format. For Example, see Q1 statement. You are expected to write a proper answer based on CFG given above. Lengthy Statements are not required here.

$L = \{ (a^i b^i)^{12} (c^{2j} d^j)^K ; i>3; j>0; K>2 \}$

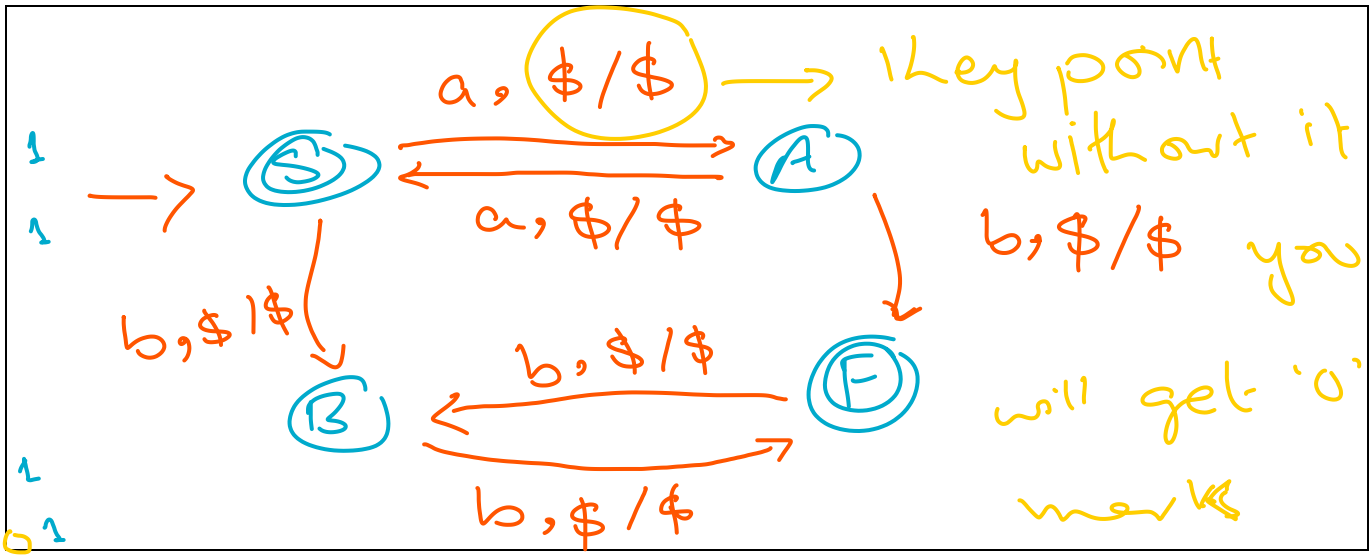
Q4: Write a Regular Expression for the following Language. [4 Marks]

$$L = \{ x \mid x \text{ over } \{a, b\}\ x \text{ contains 'aba' and 'bab' as a substring} \}$$

R.E = abab + baba + (a+b)*aba(a+b)*bab(a+b)* + (a+b)*bab(a+b)*aba(a+b)*

Q5: Design the transition diagram of a PDA for the following Language? [4 Marks]

$$L = \{a^n b^m ;\ n + m = \text{even}\}$$



Q6: What will be the Regular Expression for the following Finite Automaton? [4 Marks]

Start State = A & Final State = {A,C}

States(q)	δ(q,a)	δ(q,b)
A	B	A
B	B	C
C	D	C
D	D	D

Note: Use State Elimination Method for extraction of Regular Expression. Write Final Regular Expression in the space provided below. Delete the given states in the following order, first State A then B then C & then D.

Regular Expression:

$b^* + b^* a^+ b^+ \,/\, b^*(\lambda + a^+ b^+)$

1 — Exact marks (4) — 3

No partial marks

Section 2: (Long / Detailed Solving Question Answers) [55 Marks]

Q1: Develop 3 multi-tape TM having 2 inputs X and Y (X and Y ε {0,1}*) [15 Marks]

X is on tape 1 and Y is on tape 2. Y slides over the X with the step of 1. Each time it computes the exclusive nor (XNOR) of the corresponding overlapping bits and note down the number of 1's (only) in tape 3 as shown below:

A	B	A XNOR B
0	0	1
0	1	0
1	0	0
1	1	1

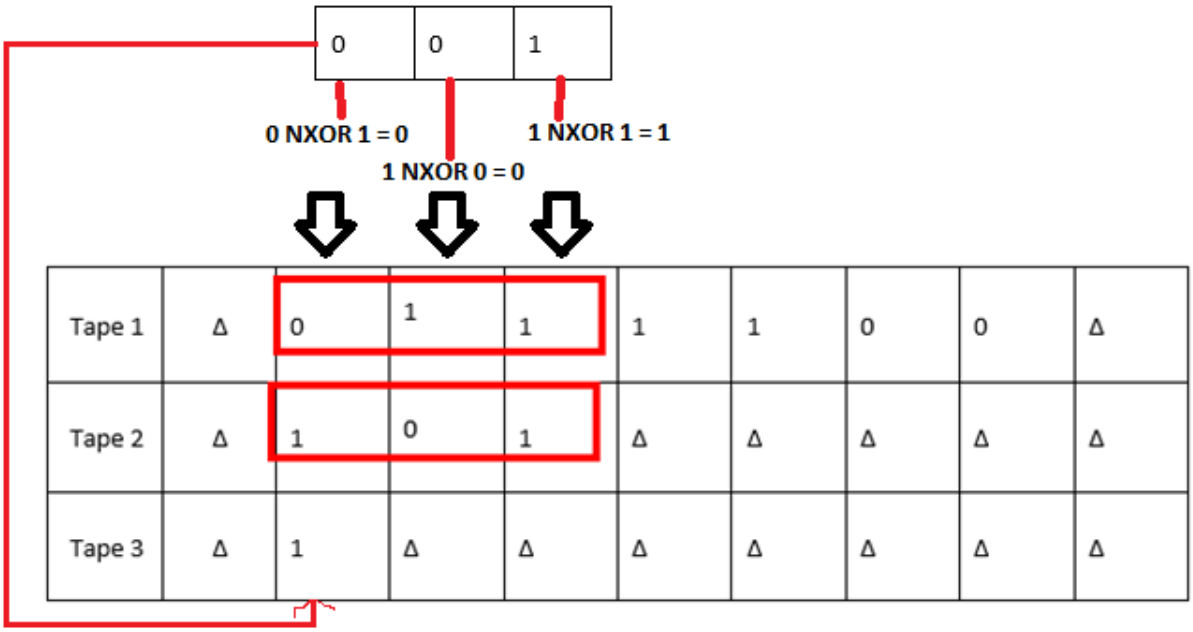
Truth Table for XNOR for inputs A and B

Initial configuration of 3 multi-tape TM

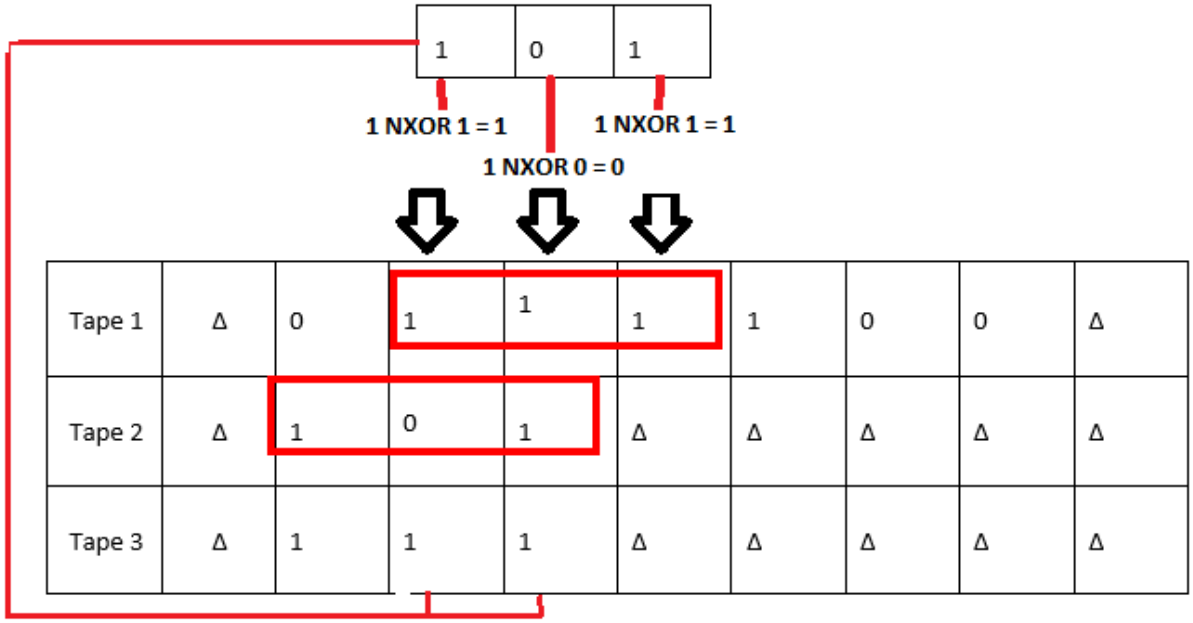
Tape 1: X	Δ	0	1	1	1	1	0	0	Δ
Tape 2: Y	Δ	1	0	1	Δ	Δ	Δ	Δ	Δ
Tape 3: Output	Δ	Δ	Δ	Δ	Δ	Δ	Δ	Δ	Δ

Y will slide 5 times on X (in this example)

First time (first slide)



Second time (second slide)



Tape 1	Δ	0	1	1	1	1	0	0	Δ
Tape 2	Δ	1	0	1	Δ	Δ	Δ	Δ	Δ
Tape 3	Δ	1	1	1	Δ	Δ	Δ	Δ	Δ

.

.

.

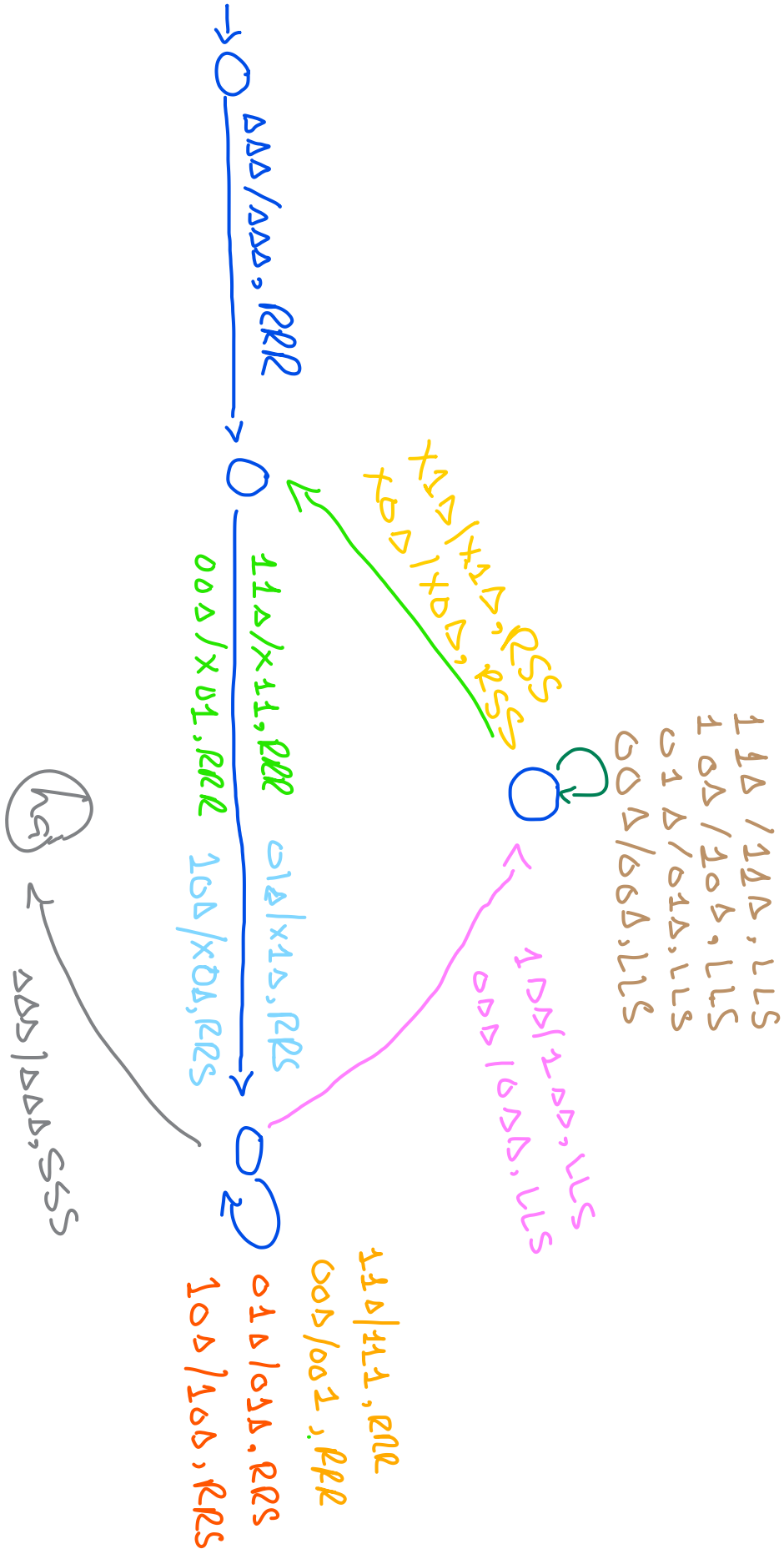
(last and 5th slide) Eventually Output will be

Tape 1	Δ	0	1	1	1	1	0	0	Δ	Δ
Tape 2	Δ	1	0	1	Δ	Δ	Δ	Δ	Δ	Δ
Tape 3	Δ	1	1	1	1	1	1	1	1	Δ

Provide the algorithm first that will explain your logic in simple statement and then draw TM:

Note: ***Be clear and to the point***. Clearly mention where your pointers are. **No marks if algorithm is incorrect.**

Algorithm:


Δ Δ Δ / Δ Δ Δ , R R
1 1 Δ / 1 1 Δ , L L
1 0 Δ / 1 0 Δ , L L
0 1 Δ / 0 1 Δ , L L
0 0 Δ / 0 0 Δ , L L
X 1 Δ / X 1 Δ , R S S
0 0 Δ / X 0 1 , R R
1 1 Δ / X 1 1 , R R
0 1 Δ / X 1 Δ , R S
1 0 Δ / X 0 Δ , R S
1 Δ Δ / 1 Δ Δ , L L S
0 Δ Δ / 0 Δ Δ , L L S
1 1 Δ / 1 1 1 , R R
0 0 Δ / 0 0 1 , R R
0 1 Δ / 0 1 Δ , R S
1 0 Δ / 1 0 Δ , R S
Δ Δ Δ / Δ Δ Δ , S S S

Q2: Dry run the single-tape Turing machine on page 10 and give the content of the tape after running it (When TM halts). [15 Marks = 10 + 5]

The initial configuration of the TM is given below

Δ	1	0	1	1	Δ	0	0	1	0	Δ	Δ	.....	Δ

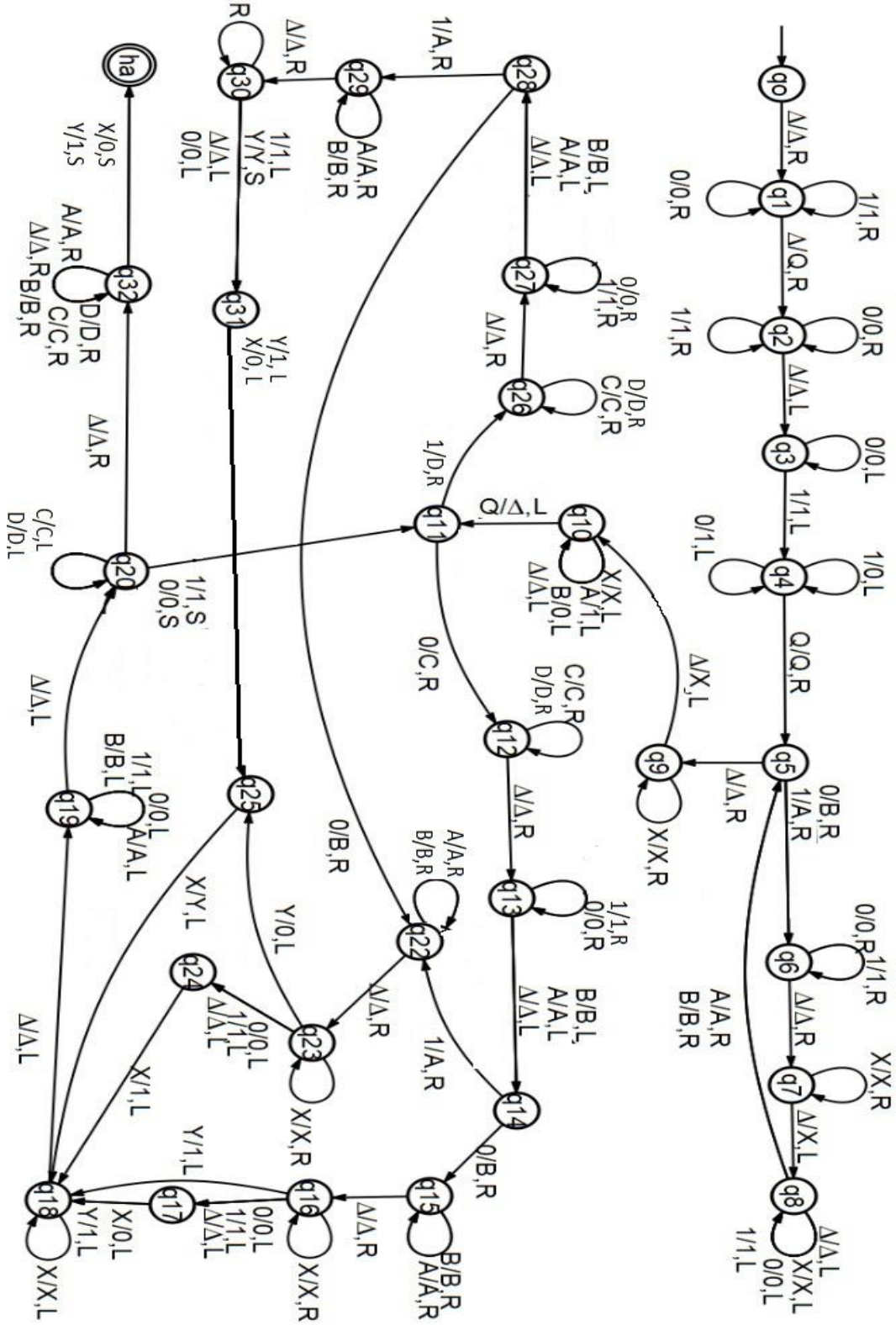
head/pointer

Answer:

Δ	O	C	D	D	Δ	A	A	A	B	Δ	1	1	0	0	1	Δ			

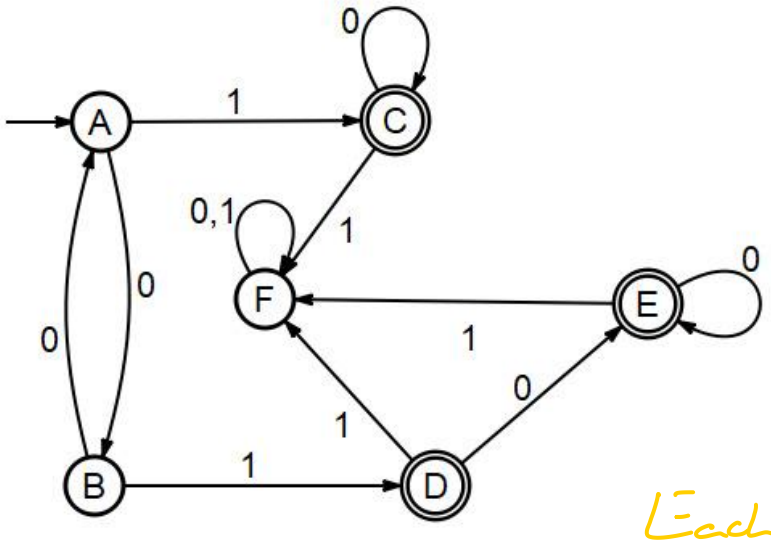
Clearly show where will be the head/pointer when TM halts

What is TM doing? (Explain in not more than 2 lines. Be brief and to the point. No mark for stories)


q0
q1
q2
q3
q4
q5
q6
q7
q8
q9
q10
q11
q12
q13
q14
q15
q16
q17
q18
q19
q20
q21
q22
q23
q24
q25
q26
q27
q28
q29
q30
q31
q32
ha
Δ/Δ,R
1/1,R
0/0,R
Δ/Q,R
Δ/Δ,L
0/0,L
1/1,L
1/0,L
0/1,L
Q/Q,R
Δ/X,L
X/X,R
0/B,R
1/A,R
A/A,R
B/B,R
Δ/X,R
0/0,R
1/1,R
Δ/Δ,L
X/X,L
0/0,L
1/1,L
B/B,L
A/A,L
Δ/Δ,L
1/D,R
Q/Δ,L
X/X,L
A/1,L
B/0,L
Δ/Δ,L
0/C,R
C/C,R
D/D,R
Δ/Δ,R
1/A,R
0/0,R
1/1,R
B/B,L
A/A,L
Δ/Δ,L
0/B,R
Δ/Δ,R
B/B,R
A/A,R
X/X,R
Y/1,L
X/1,L
0/0,L
1/1,L
Δ/Δ,L
X/0,L
Y/1,L
X/X,L
Δ/Δ,L
1/1,L
B/B,L
0/0,L
A/A,L
X/Y,L
Y/0,L
0/B,R
1/1,S
0/0,S
Δ/Δ,R
C/C,L
D/D,L
1/A,R
A/A,R
B/B,R
Δ/Δ,R
D/D,R
C/C,R
Y/1,L
X/0,L
1/1,L
Y/Y,S
Δ/Δ,L
0/0,L
Δ/Δ,R
X/X,R
X/0,S
Y/1,S
A/A,R
Δ/Δ,R
B/B,R
D/D,R
C/C,R
Δ/Δ,R

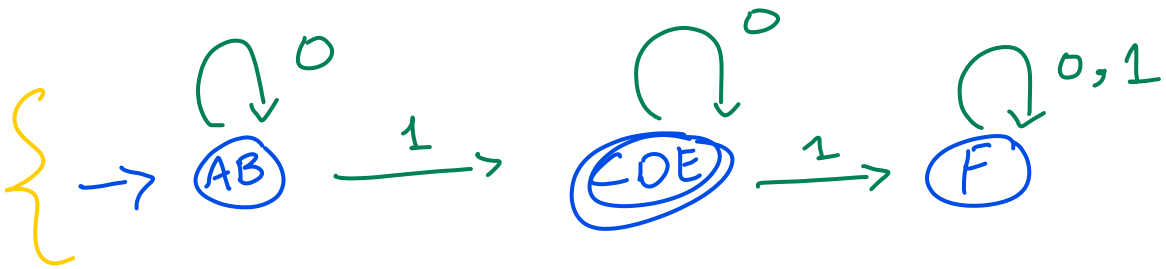
Q3. For the DFA pictured in the figure below, use the minimization algorithm discussed in the class to find a minimum-state DFA recognizing the same language. [10 Marks]

DFA:



Each row = 1 mark

Minimized DFA:



4 marks

$R.E = \{ 0^* 1 0^* \}$

	A	B	C	D	E	F
A	-					
B		-				
C	✓	✓	-			
D	✓	✓		-		
E	✓	✓			-	
F	✓	✓	✓	✓	✓	-

A, B

C, D

C, E
D, E

→ Possible Grouping

1 x 6 = 6

Note: Use only the cell required

Q4. Let G be the following CFG:

3 marks

$S \rightarrow AaB \mid aB$
$A \rightarrow a \mid Aa$
$B \rightarrow b \mid C$
$C \rightarrow bC \mid a$

X -> a
Y -> b
Z -> AX
S -> ZB | XB
A -> AX | a
B -> YC | a | b
C -> YC | a

CNF Grammar

Determine whether the string "abba" is a member of L(G) using CYK Algorithm. [10 Marks]

j=4	S	-	-	-
j=3	-	B,C	-	-
j=2	S	-	B,C	-
j=1	X,A,B,C	Y,B	Y,B	X,A,B,C
	a	b	b	a

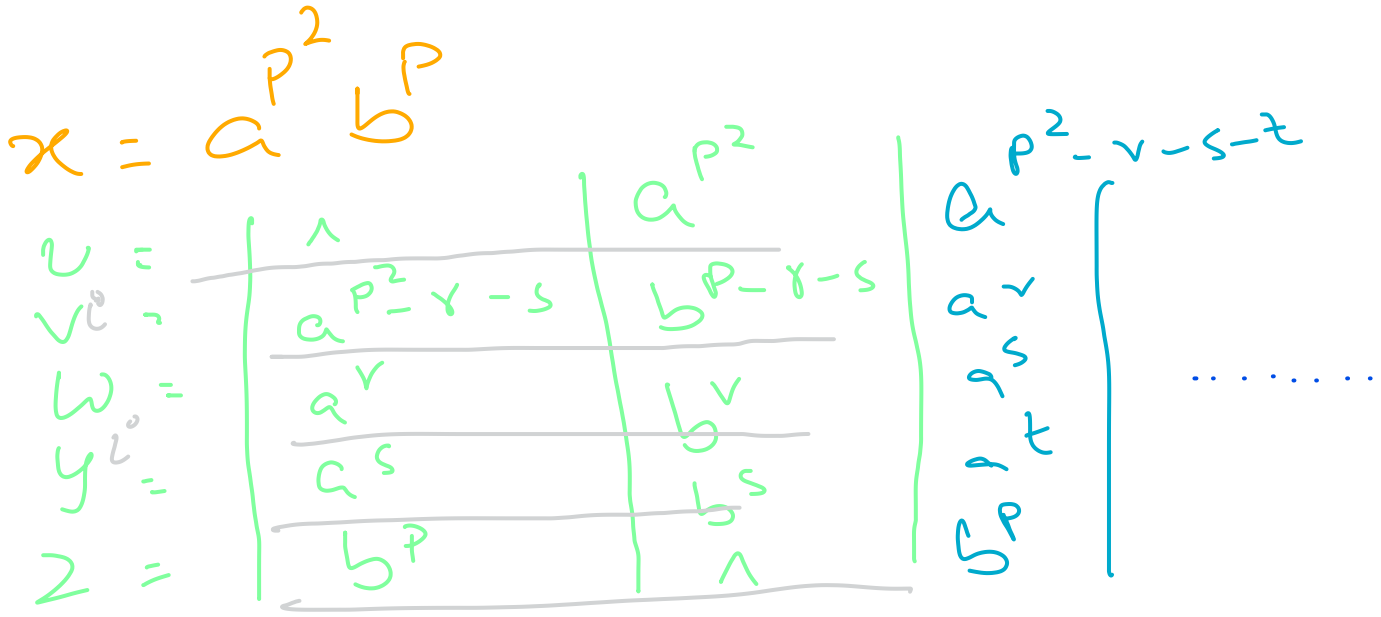
Each row = 1.5 marks = 1.5x4 = 6

Note: Use only the cell required

'abba' belongs to Language. => 1 marks

Q5. Tell whether the following Language is context free (CFL) or non- context free (non- CFL). If it is CFL provide PDA else prove it using Pumping Lemma

$$L = \{a^{m^2} b^m \mid m \geq 0\}$$ [5 Marks]



Schedy story = 1 marks

Lusem S = 2 marks

Proof = 2 marks