STATISTICAL PATTERN RECOGNITION & LEARNING
FALL 2016
PRACTICE EXERCISES 2: PROBABILITY/RANDOM VARIABLES

Question 1
Given the following data:

No.	x_1	x_2	Label
1	0	0	-1
2	0	0	+1
3	5	0	+1
4	5	0	+1
5	5	5	-1
6	5	5	-1
7	5	5	+1
8	5	5	+1
9	5	5	+1
10	5	5	+1
11	0	5	-1
12	0	5	-1
13	0	5	-1
14	0	5	+1
15	0	5	+1
16	0	5	+1
17	0	5	+1
18	0	5	+1
19	0	5	+1
20	0	5	+1

a. Determine the class of $[0\ 5]T$ $[5\ 5]^T$ $[5\ 0]^T$ and $[0\ 0]^T$ when using MAP and ML estimators using naive Bayes' with Laplacian smoothing for estimating probabilities.

b. Determine the probability of $[0\ 5]T$ $[5\ 5]^T$ $[5\ 0]^T$ and $[0\ 0]^T$ assuming a multivariate Bernoulli distribution of data and ignoring the class variable

c. Determine the class of $[0\ 5]T$ $[5\ 5]^T$ $[5\ 0]^T$ and $[0\ 0]^T$ when using MAP and ML estimators WITHOUT using naive Bayes' assumption or the assumption of a multivariate Bernoulli distribution's feature independence. Do you need smoothed estimates of probability?

d. Determine $P([0\ 0]^T|class = +1)$, $P(x_1=0|class=+1,x_2=5)$, $P(class=+1|\ x_2=5)$ using any method for estimating the probability. Clearly state which method/assumptions you are using.

Question 2

Find an expression of var(X) in terms of E(X) and E(X^2). Also, show that:
$$var(aX) = a^2 Var(X)$$

Question 3
Find an expression for var(X+Y) in terms of var(X), var(Y) and cov(X,Y)

Question 4
Find an expression for cov(X,Y) in terms of E(XY), E(X) and E(Y).

Question 5
Find the covariance of X,Y and individual variance of X and Y for the following data.
Check your answer in Matlab.

X	Y
3	3
2	7
5	8
1	5
2	9

Question 6

X	Y	Z
1	3	0
2	0	1
-1	2	2
1	2	1
2	1	0

Find the mean vector and covariance matrix for the above data. Also find the correlation matrix (correlation matrix is derived from covariance matrix by replacing the covariances with correlations).