

(Q1)

(a) Using the hypergeometric distribution.

$$P(x) =$$

$$\frac{\binom{a}{x} \binom{N-a}{n-x}}{\binom{N}{n}}$$

$$P(x=2) =$$

$$0.6122$$

$$a = 4$$

$$N =$$

$$n =$$

Using the Binomial distribution

$$P(x) = \binom{n}{x} p^x q^{n-x}$$

$$p = 0.6122$$

$$q = 1 - p = 0.3878$$

$$n = 20$$

$$x = 2$$

For atleast 2:

$$P(x \geq 2) = 1 - P(x < 2)$$

$$x < 2$$

$$P(x < 2) = P(0) + P(1)$$

$$P(0) = \binom{20}{0} p^0 q^{20} = (1)(0.6122)^0 (0.3878)^{20} = 5.9 \times 10^{-9}$$

$$P(1) = \binom{20}{1} p^1 q^{19} = (20)(0.6122)^1 (0.3878)^{19} = 1.86 \times 10^{-9}$$

$$5.9 \times 10^{-9} + 1.86 \times 10^{-7} = 1.927 \times 10^{-7}$$

$$P(X < 2) = 1.927 \times 10^{-7}$$

$$1 - P(X < 2) = 1 - 1.927 \times 10^{-7}$$

$$= 0.9999$$

$$P(X \geq 2) = 99.99\%$$

(b)

$$E(X) = np$$

$$= (20)(0.6122)$$

$$E(X) = 12.244$$

$$\sigma^2 = \text{variance} = npq$$

$$= (20)(12.244)(0.3878)$$

$$\sigma^2 = 4.7482$$

$$\sigma = 2.1790$$

Question: 03

$$f(x) = \begin{cases} cx^2, & 1 \leq x \leq 4 \\ 0, & \text{otherwise} \end{cases}$$

(a) Value of c

we know that

$$\int_{-\infty}^{\infty} f(x) dx = 1$$

$$\int_1^4 cx^2 dx = 1$$

$$\Rightarrow [c] \frac{x^3}{3} \Big|_1^4$$

$$\Rightarrow c \left[\frac{(4)^3}{3} - \frac{(1)^3}{3} \right]$$

$$\Rightarrow c \left[\frac{64}{3} - \frac{1}{3} \right]$$

$$\Rightarrow c \left[\frac{63}{3} \right]$$

$$\Rightarrow c [21] = 1$$

$$c = \frac{1}{21}$$

$$\begin{aligned} F(x) &= \int_x^4 \frac{x^2}{21} = \frac{x^3}{21} \Big|_x^4 \\ &= \frac{(4)^3}{21} - \frac{x^3}{21} \\ &= \frac{1}{21} (64 - x^3), \quad x \leq 4 \end{aligned}$$

$$F(x) = \begin{cases} \frac{1}{21} (64 - x^3), & x \leq 4 \\ 0, & x \leq 1 \end{cases}$$

0.5

Question. 02

X = num of accept. circuits

Y = num of succes. tests by first reject.

SG = { aa, ar, ra, rr }

(a)

Y

X	Y		
	0	1	2
0	0.01	0	0
1	0.09	0.09	X 0
2	0.0	0.0	0.81

(b) Marginal prob distributions

(y next page)

X	0	1	2
g(x)	0.01	0.18	0.91

(a) working

$$(a) P(Y) = \binom{n}{y} (0.9)^y (0.1)^{n-y}$$

$$aa \text{ ; } X=2, Y=2$$

$$ar \text{ ; } X=1, Y=1$$

$$ra \text{ ; } X=1, Y=0$$

$$rr \text{ ; } X=0, Y=0$$

$$P(aa) = 0.9 \times 0.9$$

$$P(ar) = 0.9 \times 0.1$$

$$P(ra) = 0.1 \times 0.9$$

$$P(rr) = 0.1 \times 0.1$$

(b) Marginal

y	0	1	2
p(y)	0.1	0.09	0.81

$$(c) \text{cov}(X, Y) = \frac{\text{cov}(XY)}{\sigma_X \sigma_Y}$$

$$\text{cov}(X, Y) = E(XY) - E(X) \cdot E(Y)$$

$$E(XY) = (0)(0)(0.01) + \dots (0)$$

$$(1)(1)(0.09) + (2)(1)(0) + (1)(2)(0) + (2)(2)(0.81)$$

$$= 0.09 + 4(0.81)$$

$$E(XY) = 3.33$$

$$E(X) = 0.18 + 2(0.81)$$

$$E(X) = 1.8$$

$$E(Y) = 0.09 + 1.62$$

$$E(Y) = 1.71$$

$$\text{cov}(X, Y) = 3.33 - (1.8)(1.71)$$

$$= 3.33 - 3.078$$

$$\text{cov}(X, Y) = 0.252$$

$$\sigma_X^2 = 0.18$$

$$\sigma_X = 0.4242$$

$$\sigma_X^2 = E(X)^2 - (E(X))^2$$

$$\sigma_Y^2 = E(Y)^2 - (E(Y))^2$$

$$\sigma_X^2 = 0.18 + 4(0.81) - (1.8)^2$$

$$\sigma_X^2 = 3.42$$

$$\sigma_X = 1.84$$

$$\sigma_Y^2 = E(Y) - (E(Y))^2$$

$$= 0.09 + 3.24$$

$$= 3.33 - 2.9241$$

$$\sigma_Y^2 = 0.4059$$

$$\sigma_Y = 0.6371$$

$$\text{cov}(X, Y) = \frac{(0.252)}{(0.4242)(0.6371)} = 0.27029$$

formula!

Q / Part No.

$$e = 0.9323$$

(1)

interpretation?