National University of Computer and Emerging Sciences, Lahore Campus				
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	Section:	A	**Page(s):**	
	Exam:	Quiz I	**Roll No:**	
Instruction/Notes	**Attempt All Questions**			

QUESTION 1: [3]

A cold drinks company has three bottling plants, located at two different places. Each plant produces three different drinks A, B and C. The capacities of three plants, in number of bottles per day are as follows:

Plants	Product A	Product B	Product C
Plant I	3000	1000	2000
Plant II	1000	1000	4000
Plant III	2000	500	3000

A market survey indicates that during a particular month there will be a demand of 24,000 bottles of A, 16,000 bottles of B and 48,800 bottles of C. The operating costs, per day, of running plants I, II and III are respectively $600, $400 and $ 500. Formulate it as a linear programming problem to find the number of days should the company run each plant during the month so that the production cost is optimized while still meeting the market demand.

SOLUTION:

Let x_1, x_2 and x_3 denote the number of days per month in which the company runs the Plants I, II and III respectively.

The LP model for the problem is:

$$\text{Minimize } Z = 600x_1 + 400x_2 + 500x_3$$

subject to

$$3000x_1 + 1000x_2 + 2000x_3 \geq 24000$$
$$1000x_1 + 1000x_2 + 500x_3 \geq 16000$$
$$2000x_1 + 4000x_2 + 3000x_3 \geq 48000$$
$$x_1, x_2, x_3 \geq 0.$$

QUESTION 2: [7]

Solve the following LP problem by using artificial variable technique and comment on the feasibility of the solution.

$$\text{Max } Z = 2x_1 + 2x_2 + 4x_3$$

subject to

$$2x_1 + x_2 + x_3 \le 2$$
$$3x_1 + 4x_2 + 2x_3 \ge 8$$
$$x_1, x_2, x_3 \ge 0.$$

SOLUTION:

Converting the model in standard form by adding slack, surplus and artificial variables.

$$\text{Max } Z = 2x_1 + 2x_2 + 4x_3$$

subject to

$$2x_1 + x_2 + x_3 + s_1 = 2$$
$$3x_1 + 4x_2 + 2x_3 - s_2 + R = 8$$
$$x_1, x_2, x_3, s_1, s_2, R \ge 0.$$

Using M method and assigning a large penalty M to artificial variable in the objective function.

$$\text{Max } Z = 2x_1 + 2x_2 + 4x_3 - MR$$

$$R = 8 - 3x_1 - 4x_2 - 2x_3 + s_2$$

Taking M=100

$$\text{Max } Z = 2x_1 + 2x_2 + 4x_3 - 100(8 - 3x_1 - 4x_2 - 2x_3 + s_2)$$

$$= 302x_1 + 402x_2 + 204x_3 - 100s_2 - 800$$

Putting in tableau form:

Basic	x_1	x_2	x_3	s_1	s_2	R	Sol
Z	-302	-402	-204	0	100	0	-800
s_1	2	1	1	1	0	0	2
R	3	4	2	0	-1	1	8

Basic	x_1	x_2	x_3	s_1	s_2	Sol
Z	-1/2	0	-3	0	-1/2	4
s_1	5/4	0	1/2	1	1/4	0
x_2	3/4	1	1/2	0	-1/4	2

Basic	x_1	x_2	x_3	s_1	s_2	Sol
Z	7	0	0	6	1	4
x_3	5/2	0	1	2	1/2	0
x_2	-1/2	1	0	-1	-1/2	2

The solution is optimal and feasible.

$$x_1 = 0, x_2 = 2, z_{max} = 4$$