ME-491A Project Report

Global instability analysis

Onset of natural convection in heated cavities

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Abstract

Linear stability analysis of fluid flow problems involves the determination of eigenvalues of very large matrices due to the large number of variables corresponding to the nodes of the computational grid. It has been successful in predicting several flow phenomena, like boundary layer transition. This project aims to develop a large-scale eigenvalue solver based on the subspace iteration method to obtain the eigenvalues of the resulting linearized perturbation matrices (Flux Jacobians). Two instability problems are considered: 1.) Critical Reynolds number for steady to unsteady laminar transition in flow past a circular cylinder and 2.)Onset of natural convection in an open heated cavity. The base solutions of these problems is obtained using the SIMPLE algorithm implementation in the open source software OpenFOAM. Also, the effect of the variation in Prandtl number is studied to further categorize the relationships between the onset of natural convection at the critical Rayleigh number and the Prandtl number.

Keywords: **Linear Stability Analysis, Large Scale Eigenvalue Solver, Onset of Natural Convection, Effect of Prandtl Number on convection**

CONTENTS

1. Introduction and literature review

Stability analysis is an essential part in the design of dynamic systems besides being an active field for academic research. It finds applications in problems involving disruption of an equilibrium state through induced perturbations. The concept of stability of an equilibrium state refers to the response of the state to disturbances that shift the position from the equilibrium state. If the disturbance is amplified by the system, it moves away from its equilibrium state and hence the equilibrium is considered unstable. However, if the disturbances die out and the system returns to the equilibrium state, then that state is considered stable.

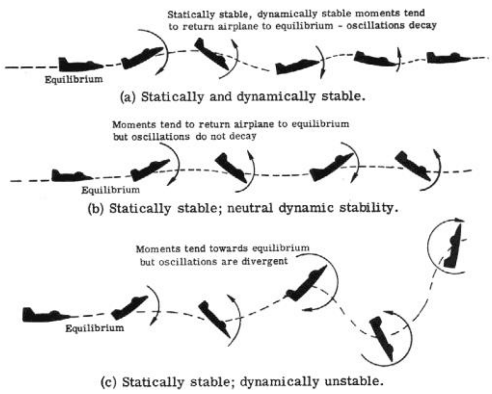


Fig. 1 Stability Concept

The formulation of the stability problem often allows the change of system state to be expressed as a function of its existing state. This formulation yields an eigenvalue equation as shown in Section 2. The stability of the system can then be expressed in terms of the eigenvalues (which are often complex). If all eigenvalues have a negative real part, then the disturbances are not allowed to grow and the system is stable. However, if even one of the eigenvalues has a positive real part, the system is unstable in at least one mode. Therefore, for large-scale problems, it is often useful to find only the eigenvalues with the largest real part and judge the stability on the basis of its sign.

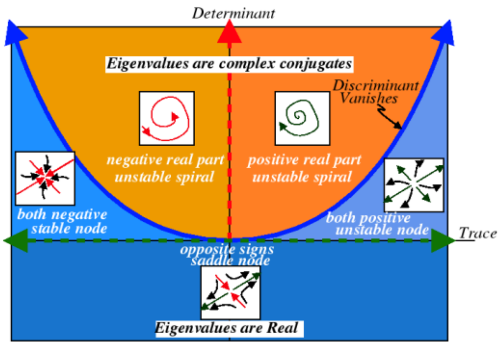


Fig. 2 Eigenvalue criteria for stability

Gaps in existing literature:

1. Existing studies avoid the use of Large Eigenvalue solvers for linear stability analysis. Instead, approximate methods are used to save computational effort.
2. Existing studies are available for varying Rayleigh number while the Prandtl number is fixed.
3. Recent developments have begun in using Open Source solvers for linear stability analysis however, these are few in number (e.g. [9])

Novelty of the work-

1. The effect of varying Prandtl number is also studied in determining the critical Rayleigh number. Very little emphasis is placed in existing studies on varying Prandtl number.
2. Different fluids can behave differently for the onset of convection which could be utilised to improve heat transfer characteristics.
3. It is hoped that a trend between Ra_{cr} and Pr can be determined to predict the convection characteristics of any fluid.
4. Linear stability analysis using Large Eigenvalue Solvers is uncommon in literature and its use can be hoped to set benchmarks for future studies while validating existing studies

2. Problem Formulation

The linear stability theory is based on the concept of linearity of perturbations and base flow kinetics. In linear stability analysis, a perturbation is directly superimposed upon the base flow and the resulting equation is solved to predict the stability of the base state. Fig. 1 shows the general procedure.

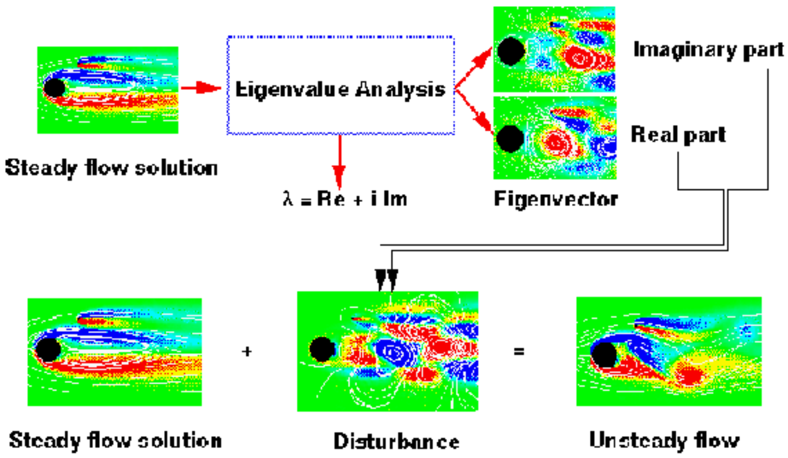


Fig. 3 Global instability analysis of flow past a circular cylinder [3]

The equations of dynamic equilibrium of a system may be expressed conveniently in the form

$$\frac{d}{dt}\begin{pmatrix}\eta_1(t)\\ \eta_2(t)\\ \vdots\\ \eta_N(t)\end{pmatrix} = \begin{pmatrix} \left.\frac{\partial F_1}{\partial y_1}\right|_{\vec{y}_{\text{fixed}}} & \left.\frac{\partial F_1}{\partial y_2}\right|_{\vec{y}_{\text{fixed}}} & \cdots & \left.\frac{\partial F_1}{\partial y_N}\right|_{\vec{y}_{\text{fixed}}} \\ \left.\frac{\partial F_2}{\partial y_1}\right|_{\vec{y}_{\text{fixed}}} & \ddots & & \\ \vdots & & & \vdots \\ \left.\frac{\partial F_N}{\partial y_1}\right|_{\vec{y}_{\text{fixed}}} & \cdots & \cdots & \left.\frac{\partial F_N}{\partial y_N}\right|_{\vec{y}_{\text{fixed}}} \end{pmatrix} \begin{pmatrix}\eta_1\\ \eta_2\\ \vdots\\ \eta_N\end{pmatrix}$$

This formulation expresses the rate of change of the solution variables as a function of the variables themselves and is related by the Jacobian of the function w.r.t the solution variables. Fig. 2 illustrates the same concept applied to a single variable system.

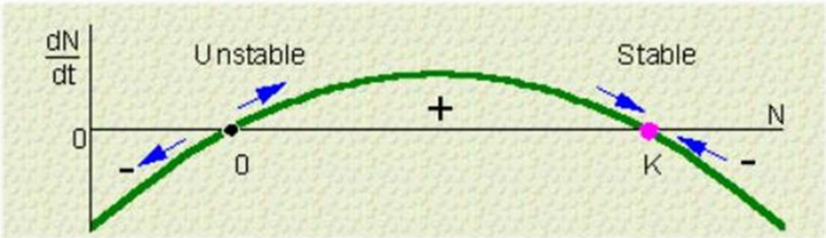


Fig. 4 Stability problem formulation

In this example, the rate of change of the variable 'N' is expresses as a function of N. It helps us identify the states of equilibrium of the system. It may be observed that the system exhibits an unstable equilibrium at N=0 because a slight disturbance to this state takes the system away from the state ($\frac{dN}{dt} > 0$ for N>0 and $\frac{dN}{dt} < 0$ for N<0). However, the equilibrium of the system at N = K is stable because any change from this point ring the system back to the same state ($\frac{dN}{dt} > 0$ for N< K and $\frac{dN}{dt} < 0$ for N> K).

A similar approach may be taken for flow stability analysis. The governing equations for fluid flows are

$$\nabla \cdot \boldsymbol{u} = 0$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\nabla p + Re^{-1} \nabla^2 \boldsymbol{u}$$

For stability analysis, we assume a disturbance u' imposed upon the base flow. Therefore, the total velocity is written as

$$u = \bar{u} + u'$$

Substituting for u in the governing equation and simplifying (by subtracting the steady state equation) gives

$$\nabla \cdot \boldsymbol{u}' = 0$$

$$\frac{\partial \boldsymbol{u}'}{\partial t} + \overline{\boldsymbol{u}} \cdot \nabla \boldsymbol{u}' + \boldsymbol{u}' \cdot \nabla \overline{\boldsymbol{u}} = -\nabla p' + Re^{-1} \nabla^2 \boldsymbol{u}'$$

This is expressed compactly as

$$\frac{\partial \boldsymbol{u}'}{\partial t} = \boldsymbol{A}\boldsymbol{u}'$$

where A is the flux Jacobian

Now, we assume a form of u' which is suitably expresses different kinds of perturbations. The exponential function is a good choice because it may be used for both oscillatory and non-oscillatory forms of perturbations with a variety of intensities. Thus, we have

$$\boldsymbol{u}'(x, y, z) = \hat{\boldsymbol{u}}(x, y, z)\mathrm{e}^{\lambda t} + \mathrm{c.c.}$$

Substituting this in the compact form of the perturbation equation gives

$$\lambda \boldsymbol{u}' = \boldsymbol{A}\boldsymbol{u}'$$

which is the generalized eigenvalue problem. The eigenvalues obtained from this equation are used to predict the stability of the flow. Since, matrix size is too large for complete solution, only the largest eigenvalues are used and are computed using numerical techniques such as those discussed in section 3.

2.1 Flow past a circular cylinder

This problem has been selected to validate our large-scale eigenvalue solver. Since the flow is known to turn unstable at Re = 47, we have a solution that we can compare with

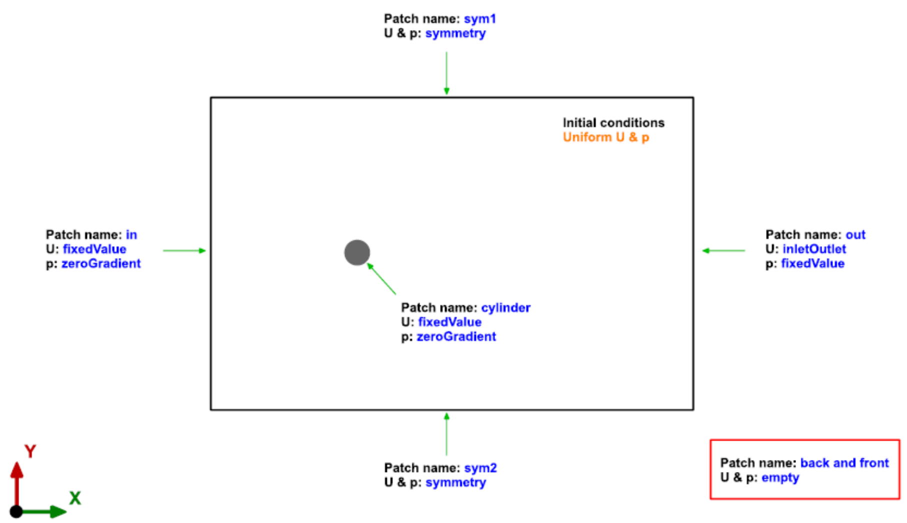


Fig. 5 Problem description: Flow past a circular cylinder

2.2 Natural convection in an open heated cavity

- The flow in the cavity is incompressible and laminar according to previous studies [1].
- Also, the Boussinesq approximation is used according to previous studies [1].
- The Rayleigh number is varied by changing the height of the cavity.

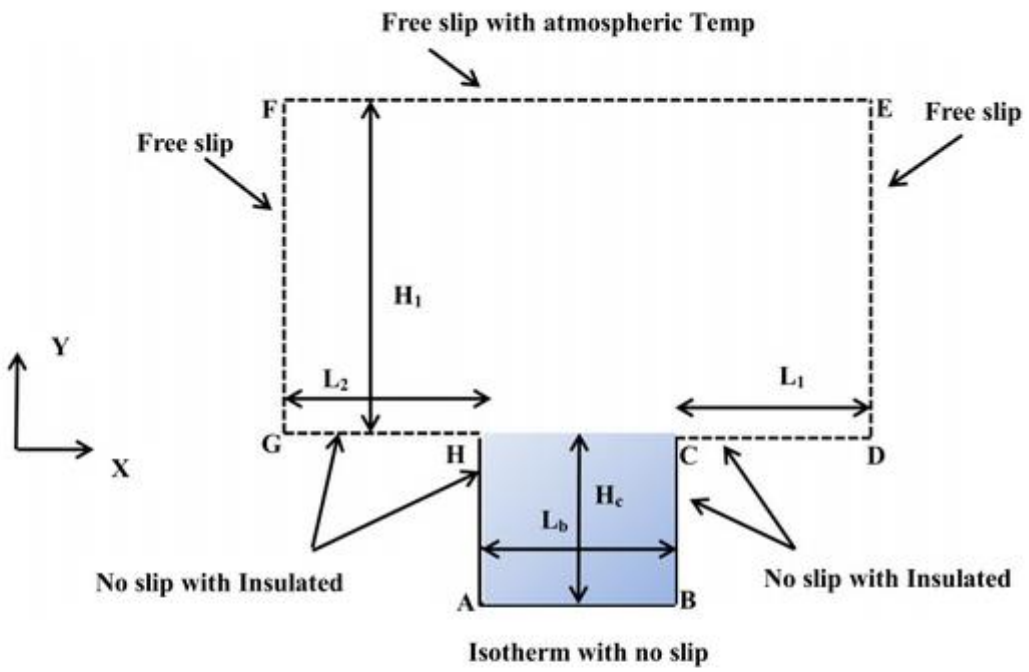
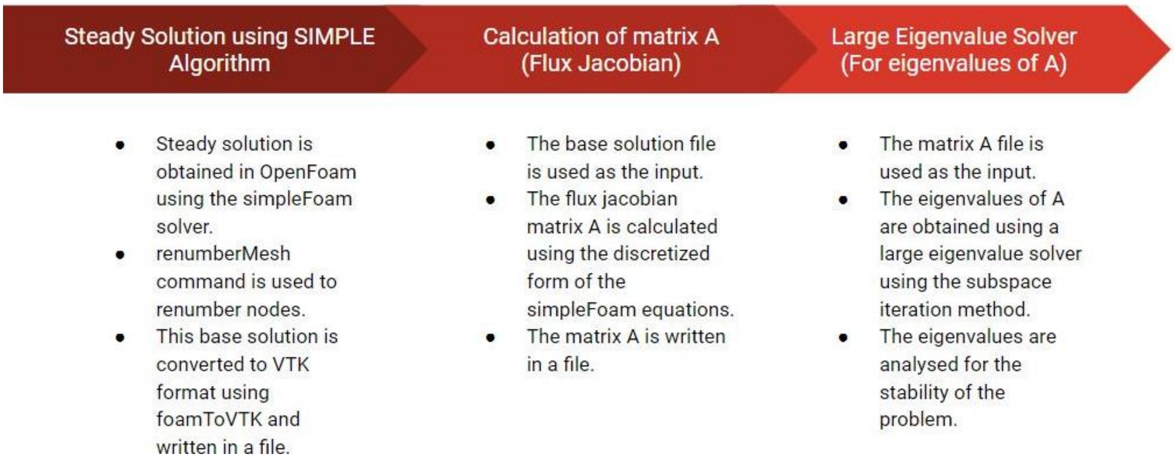


Fig. 6 Problem description: Flow in an open heated cavity

3. Methodology

Workflow



SIMPLE algorithm - Implementation in OpenFOAM

- simpleFoam - Steady-state solver for incompressible, turbulent flow, using the SIMPLE algorithm [2].
- SIMPLE stands for Semi Implicit Method for Pressure Linked Equations.

- The equations and steps involved in the SIMPLE algorithm as used in the simpleFoam solver

PIMPLE algorithm - Implementation in OpenFOAM

- pimpleFoam - Large time step transient solver for incompressible, turbulent flow of Newtonian fluids, with optional mesh motion and mesh topology changes [2].
- The PIMPLE algorithm combines the PISO (Pressure Implicit with Splitting of Operator) and SIMPLE algorithms.
- The advantage of using the PIMPLE algorithm is that we can use large Courant numbers (Co > 1) and thus the time steps can be increased considerably [7].

Subspace Iteration Algorithm:

Below is the algorithm for the implementation of the subspace iteration method to find the dominating eigenvalues of a matrix.

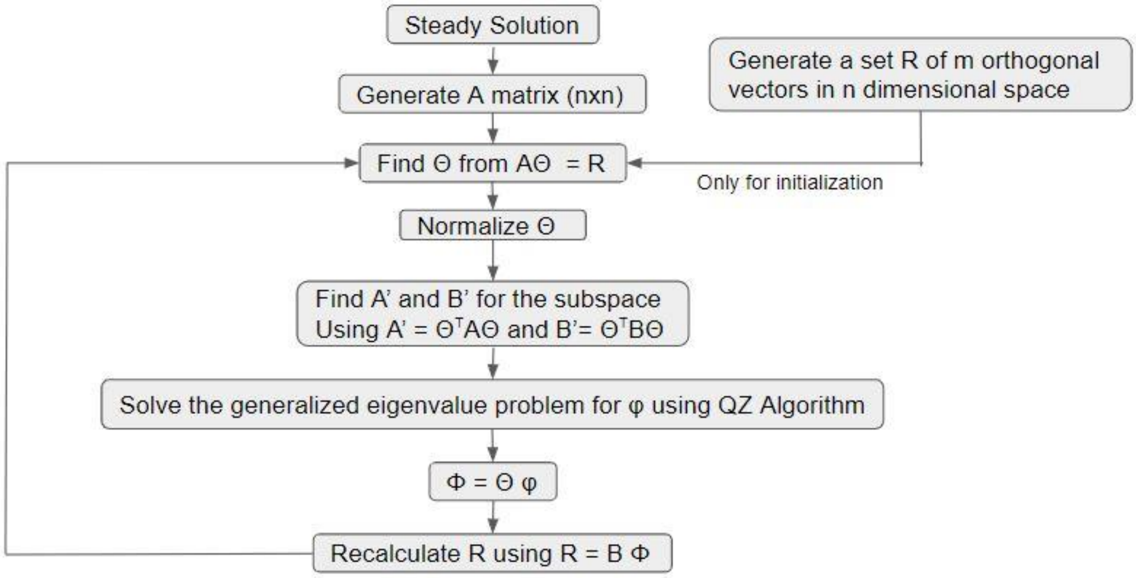


Fig. 7 Subspace Iteration – Algorithm

Gram-Schmidt Method

This method is used to generate orthogonal basis vectors from any set of basis vectors. Let x be a set of linearly independent vectors in n-dimensional space. Then, the orthogonal bases (v) in n-dimensional space can be calculated as

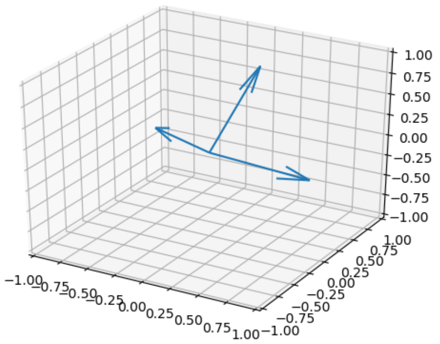


Fig. 8 Orthogonal vectors generated by our code

Some LAPACK (Linear Algebra PACKage) modules that would be used for writing the code-

DGETRI - It computes the inverse of a matrix using the LU factorization

DHGEQZ - It computes the eigenvalues of a real matrix pair (H,T), where H is an upper Hessenberg matrix and T is upper triangular, using the double-shift QZ method. Matrix pairs of this type are produced by the reduction to generalized upper Hessenberg form of a real matrix pair (A,B) using: A = Q1*H*Z1**T, B = Q1*T*Z1**T

4. Results

4.1 Flow past a circular cylinder

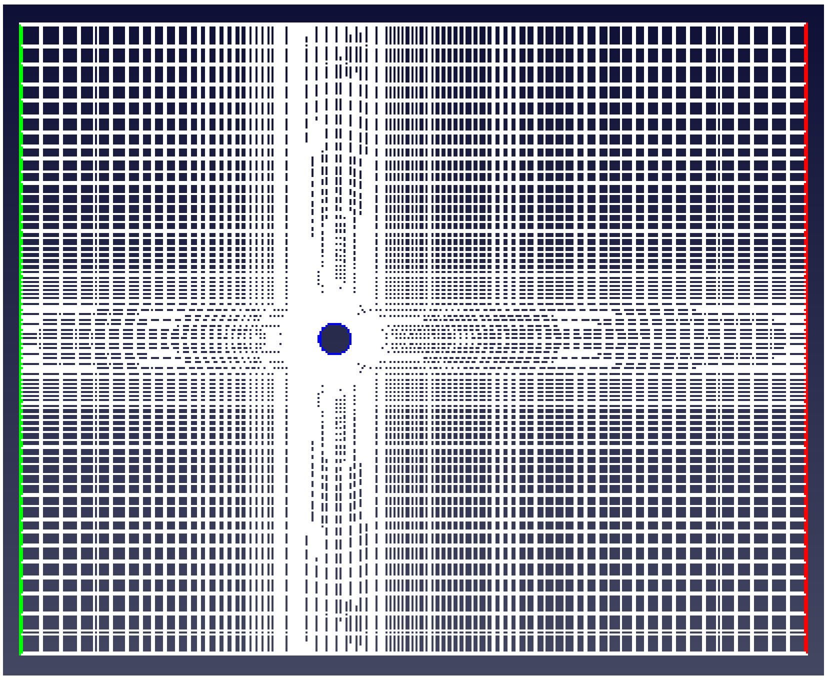
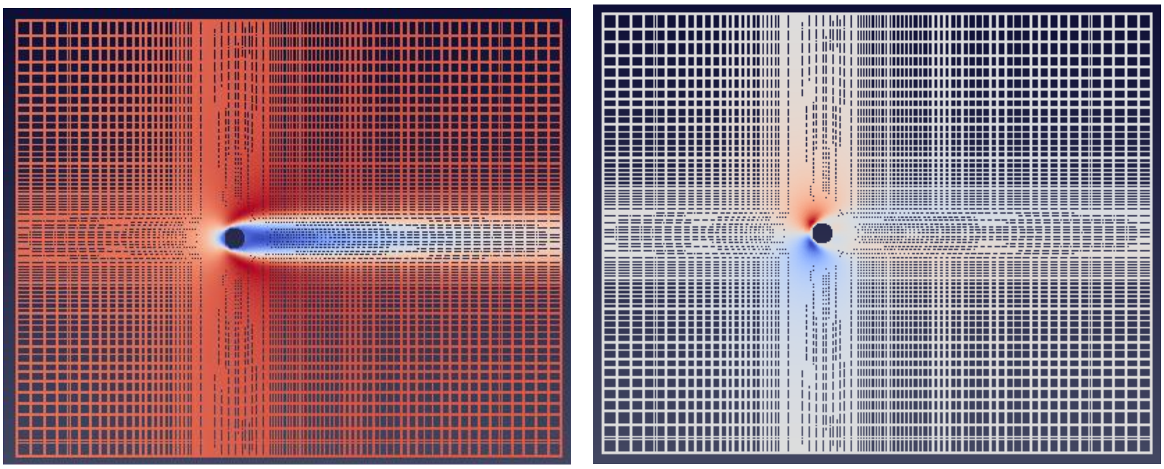


Fig. 9 Base solution: Pressure Field at Re = 40



(a) Base solution: U_x Field at Re = 40 **(b) Base solution: U_y Field at Re = 40**

Fig. 10 Velocity fields at Re = 40

4.2 Onset of natural convection in heated cavity

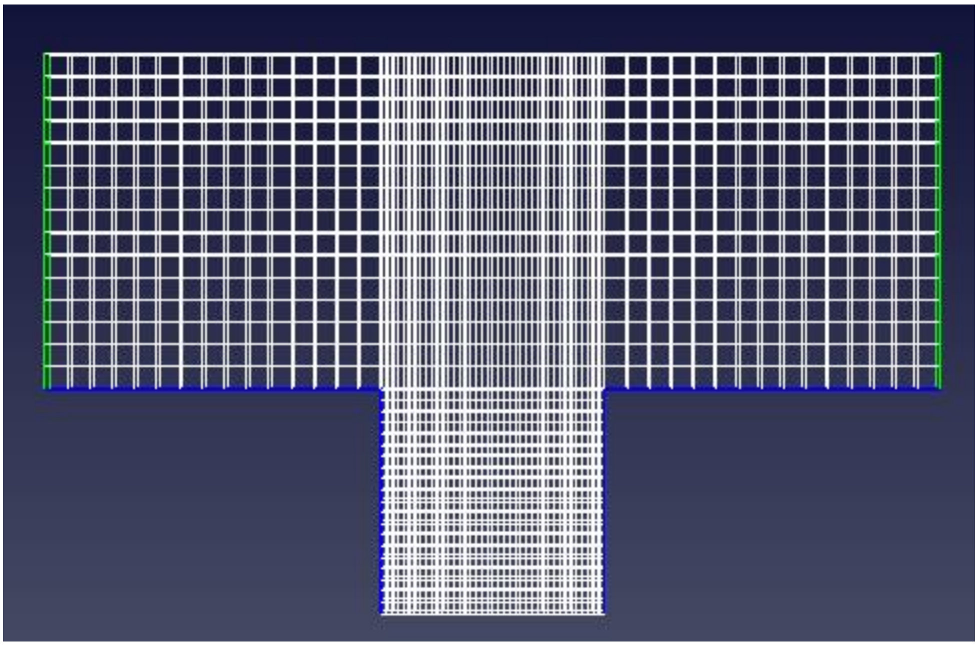


Fig. 11 Computational mesh for Natural convection problem

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