Agenda

- **Convolutional & pooling layers**
- **Convolutional neural networks**
- **Feature visualization**
- **Applications**

Images as 2D Arrays

- Grayscale image is a 2D array of pixel values

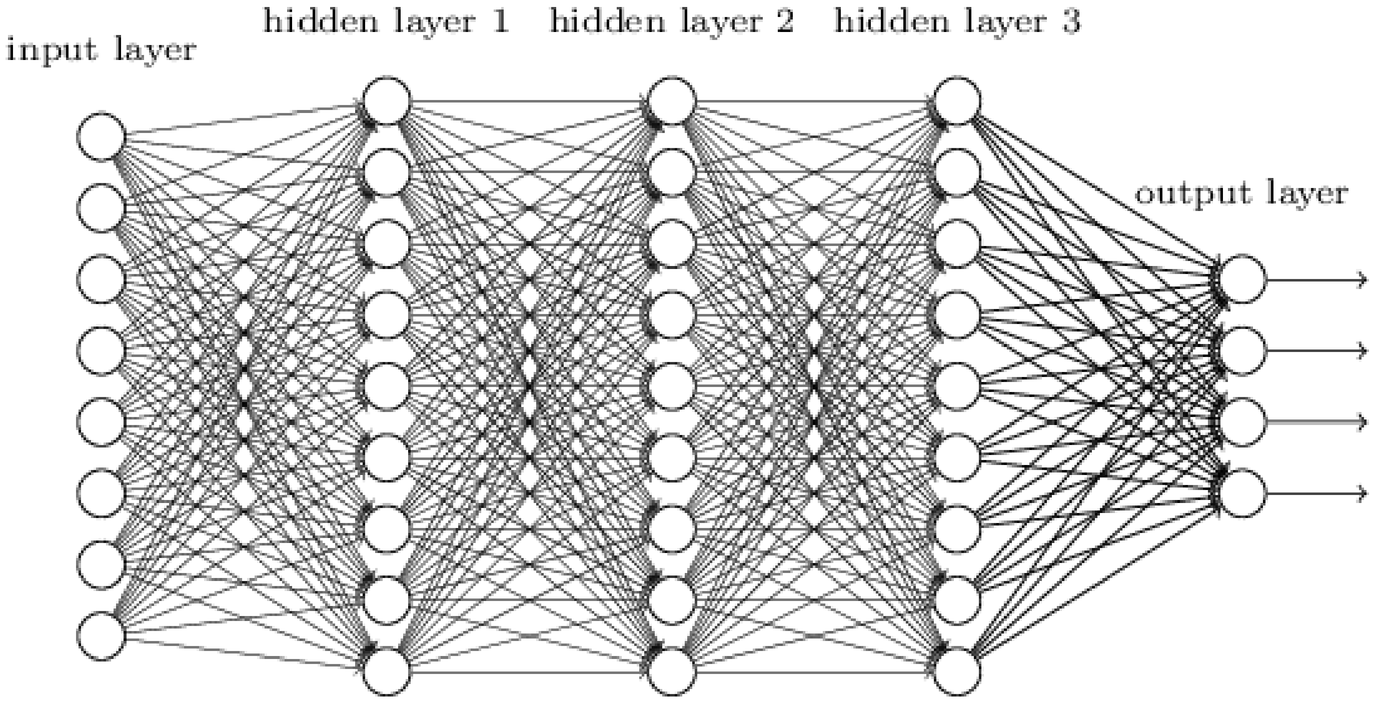
- Color images are 3D array
  - 3rd dimension is color (e.g., RGB)
  - Called "channels"

0	3	2	5	4	7	6	9	8
3	0	1	2	3	4	5	6	7
2	1	0	3	2	5	4	7	6
5	2	3	0	1	2	3	4	5
4	3	2	1	0	3	2	5	4
7	4	5	2	3	0	1	2	3
6	5	4	3	2	1	0	3	2
9	6	7	4	5	2	3	0	1
8	7	6	5	4	3	2	1	0

Source: S. Narasimhan, S. Lazebnik

Convolution Neural Network: CNN

We know it is good to learn a small model.
From this fully connected model, do we really need all the edges?
Can some of these be shared?

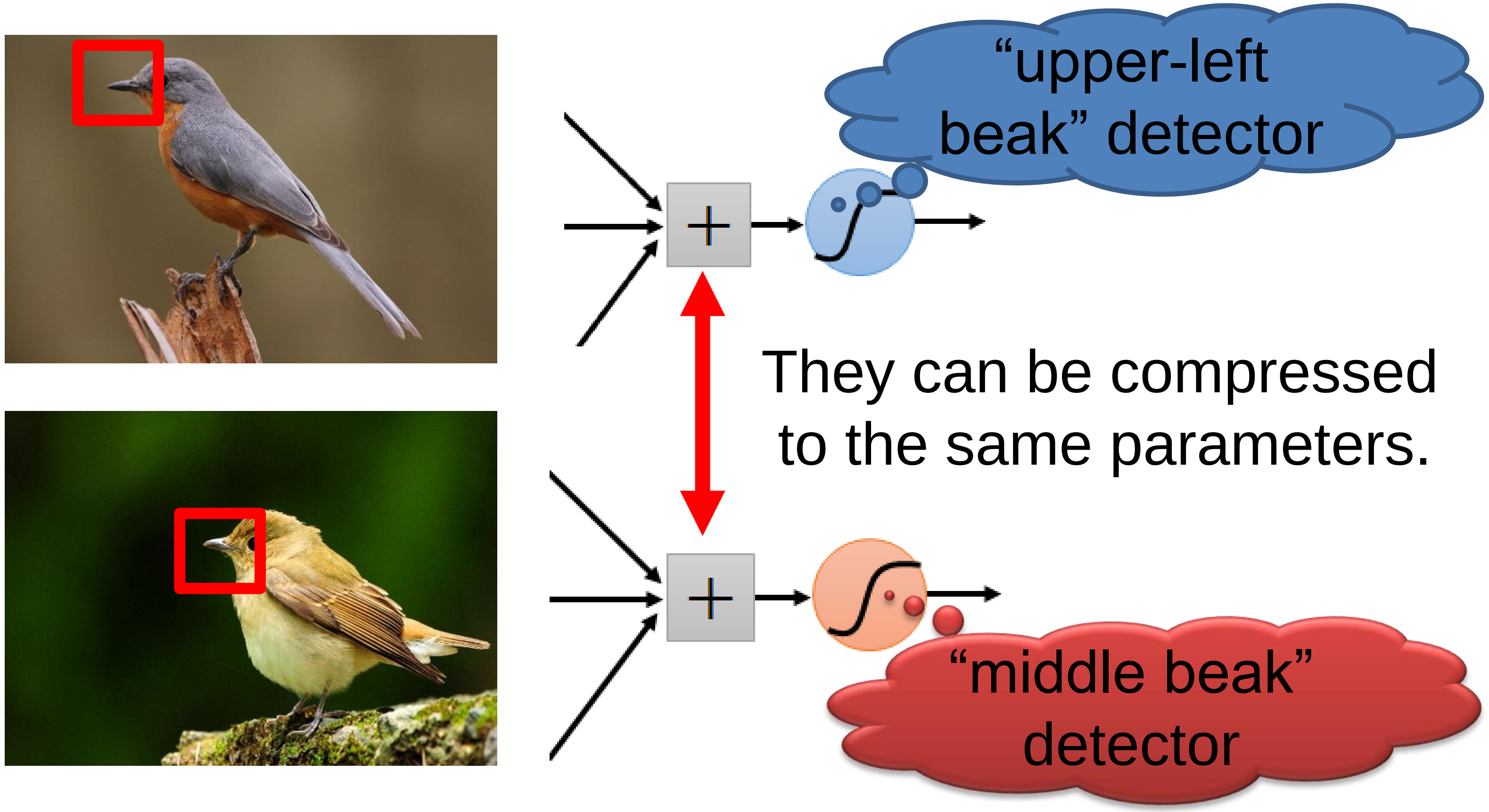


Consider learning an image:

Some patterns are much smaller than the whole image

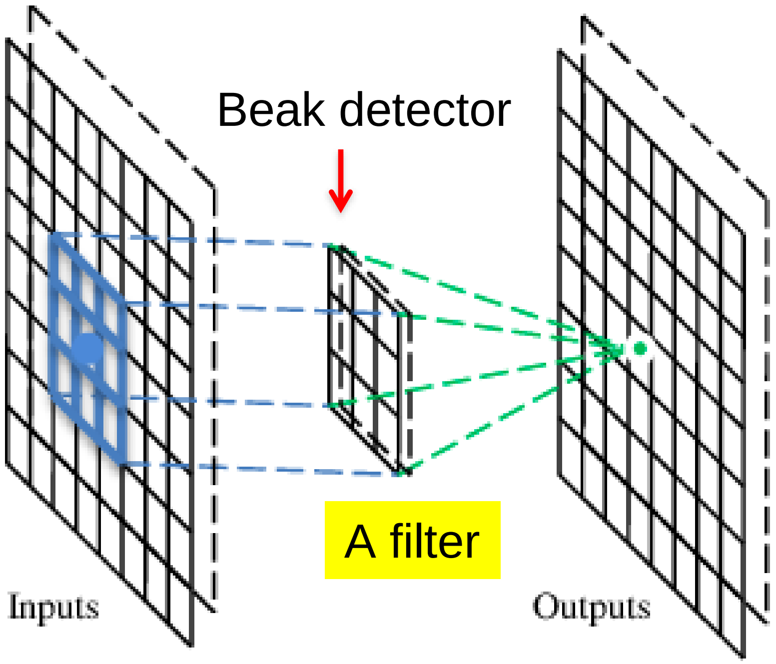
Can represent a small region with fewer parameters

Same pattern appears in different places:
They can be compressed!
What about training a lot of such "small" detectors and each detector must "move around".



A convolutional layer

A CNN is a neural network with some convolutional layers (and some other layers). A convolutional layer has a number of filters that does convolutional operation.

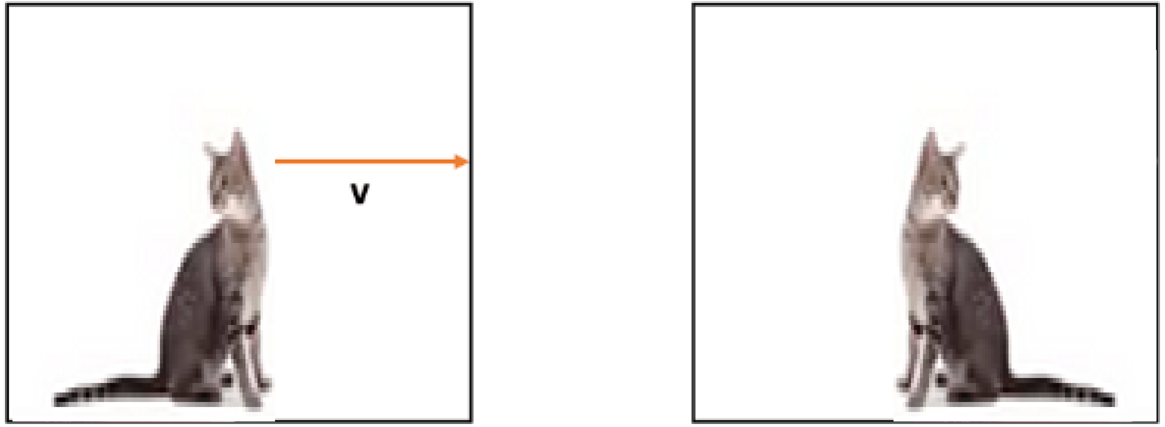


What Is Translation?

A **translation** is a geometric transformation that shifts all points in a given direction and by the same distance. Alternatively, it can be interpreted as sliding the origin of the coordinate system by the same amount but in the opposite direction.

The translation can be expressed mathematically as the vector sum of a constant vector $\mathbf{v}$ to each point $\mathbf{x}$:

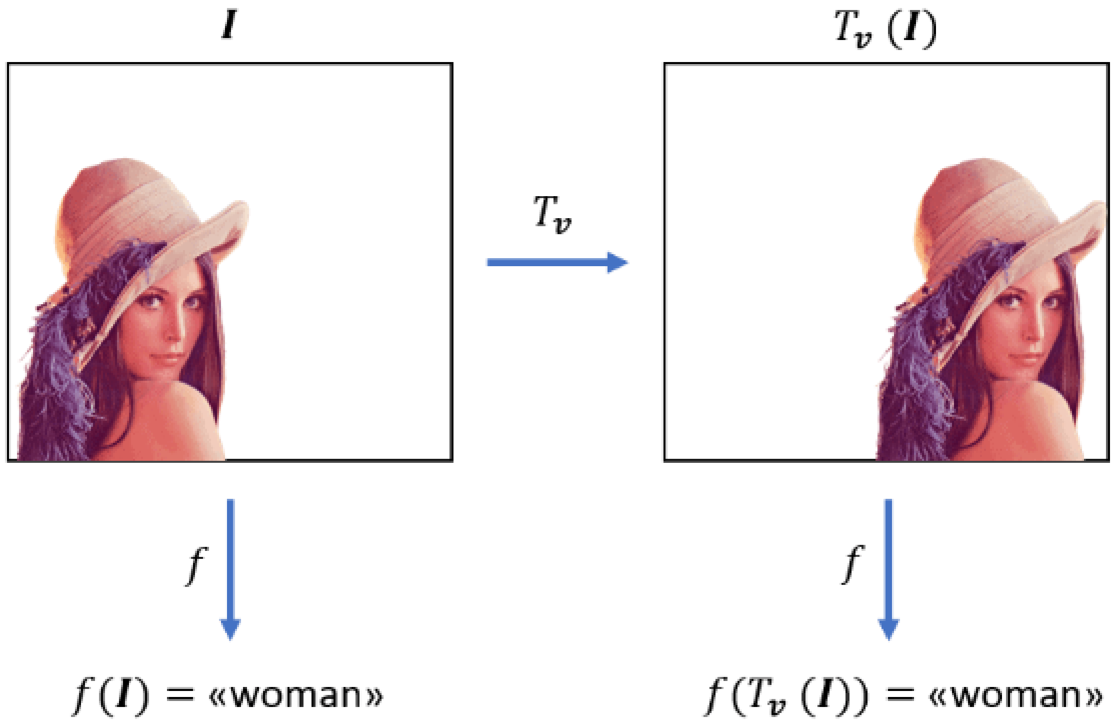
$$T_{\mathbf{v}}(\mathbf{x}) = \mathbf{x} + \mathbf{v}$$



(a) (b)

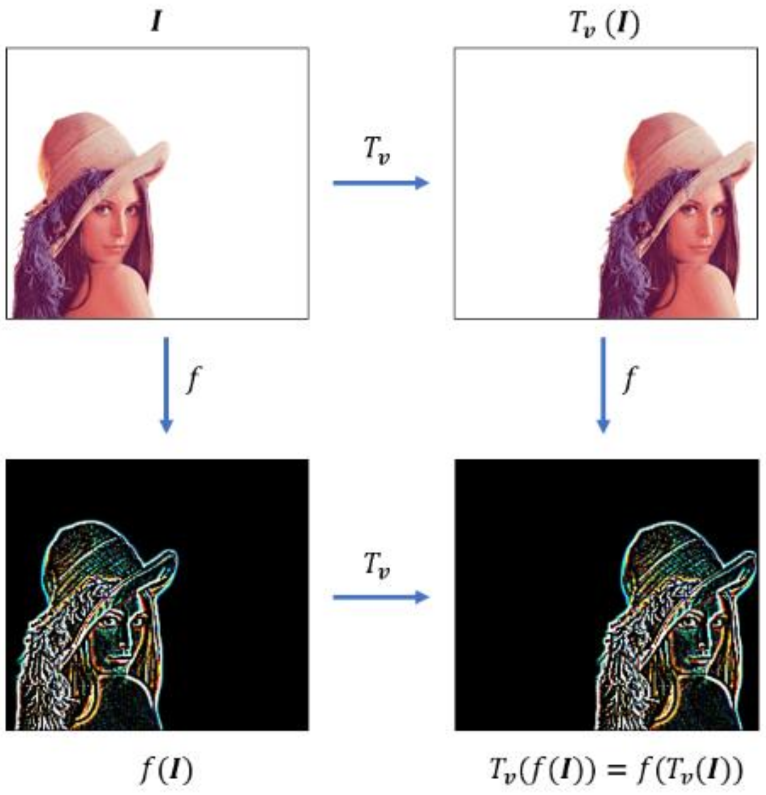
Translation Invariance

- **Translation invariance -** is a property of a system or model where its performance or output does not change when the input data is translated or shifted.



Translation Equivariance

Translation equivariance - is a property of a function or an operation where the output's structure does not change even if the input is translated.



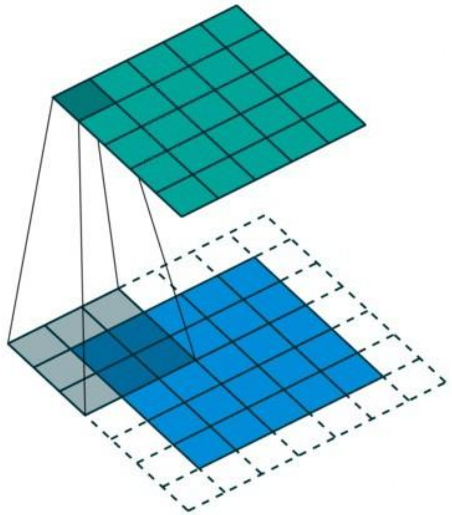
A function f is said equivariant to a function g if and only if:

$$f(g(x)) = g(f(x))$$

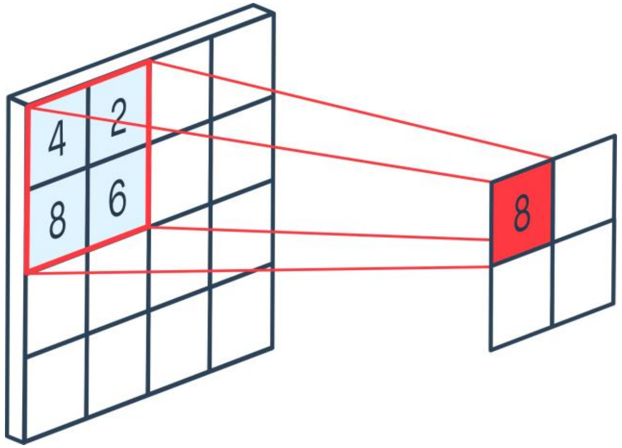
In other words, f is equivariant to g if the order of application does not change the result of the composite function.

Structure in Images

- Use layers that capture structure



Convolution layers
(Capture equivariance)

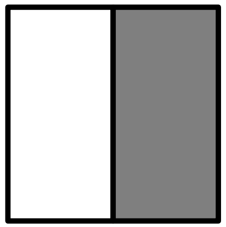


Pooling layers
(Capture invariance)

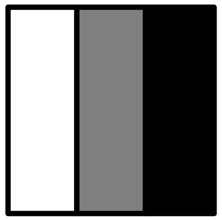
https://towardsdatascience.com/types-of-convolutions-in-deep-learning-717013397f4d
https://peltarion.com/static/2d_max_pooling_pa1.png

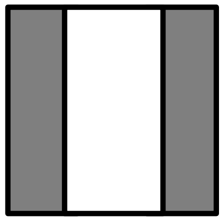
Vertical edge detection

$$
\begin{array}{|c|c|c|c|c|c|}
\hline
10 & 10 & 10 & 0 & 0 & 0 \\ \hline
10 & 10 & 10 & 0 & 0 & 0 \\ \hline
10 & 10 & 10 & 0 & 0 & 0 \\ \hline
10 & 10 & 10 & 0 & 0 & 0 \\ \hline
10 & 10 & 10 & 0 & 0 & 0 \\ \hline
10 & 10 & 10 & 0 & 0 & 0 \\ \hline
\end{array}
\;*\;
\begin{array}{|c|c|c|}
\hline
1 & 0 & -1 \\ \hline
1 & 0 & -1 \\ \hline
1 & 0 & -1 \\ \hline
\end{array}
\;=\;
\begin{array}{|c|c|c|c|}
\hline
0 & 30 & 30 & 0 \\ \hline
0 & 30 & 30 & 0 \\ \hline
0 & 30 & 30 & 0 \\ \hline
0 & 30 & 30 & 0 \\ \hline
\end{array}
$$

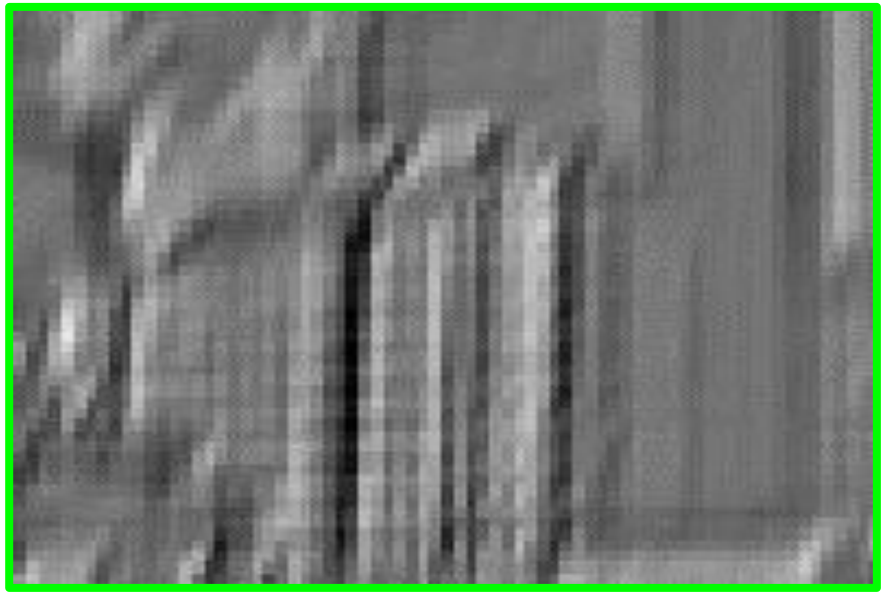


*





Convolution Filters



$$\text{output}[i, j] = \sum_{\tau=0}^{k-1} \sum_{\gamma=0}^{k-1} \text{filter}[\tau, \gamma] \cdot \text{image}[i + \tau, j + \gamma]$$

2D Convolution Filters

- **Given:**
  - A 2D input x
  - A 2D $h \times w$ kernel k

- The 2D convolution is:

$$y[s,t] = \sum_{\tau=0}^{h-1} \sum_{\gamma=0}^{w-1} k[\tau,\gamma] \cdot x[s+\tau, t+\gamma]$$

2D Convolution Filters

-1	-1	-1
2	2	2
-1	-1	-1

Horizontal lines

-1	2	-1
-1	2	-1
-1	2	-1

Vertical lines

-1	-1	2
-1	2	-1
2	-1	-1

45 degree lines

2	-1	-1
-1	2	-1
-1	-1	2

135 degree lines

Example Edge Detection Kernels

Result of Convolution with Horizontal Kernel

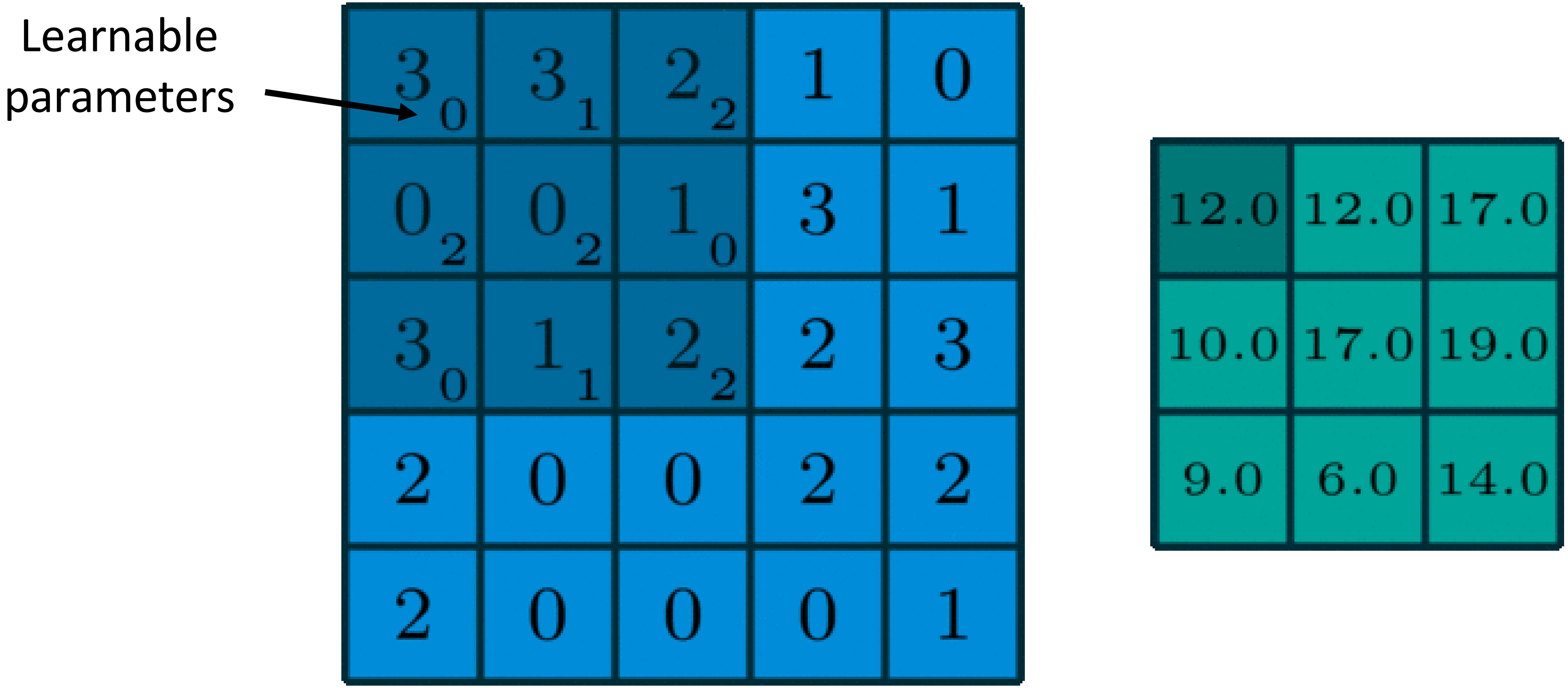
https://aishack.in/tutorials/image-convolution-examples/

2D Convolution Filters

- Historically (until late 1980s), kernel parameters were handcrafted
  - E.g., “edge detectors”

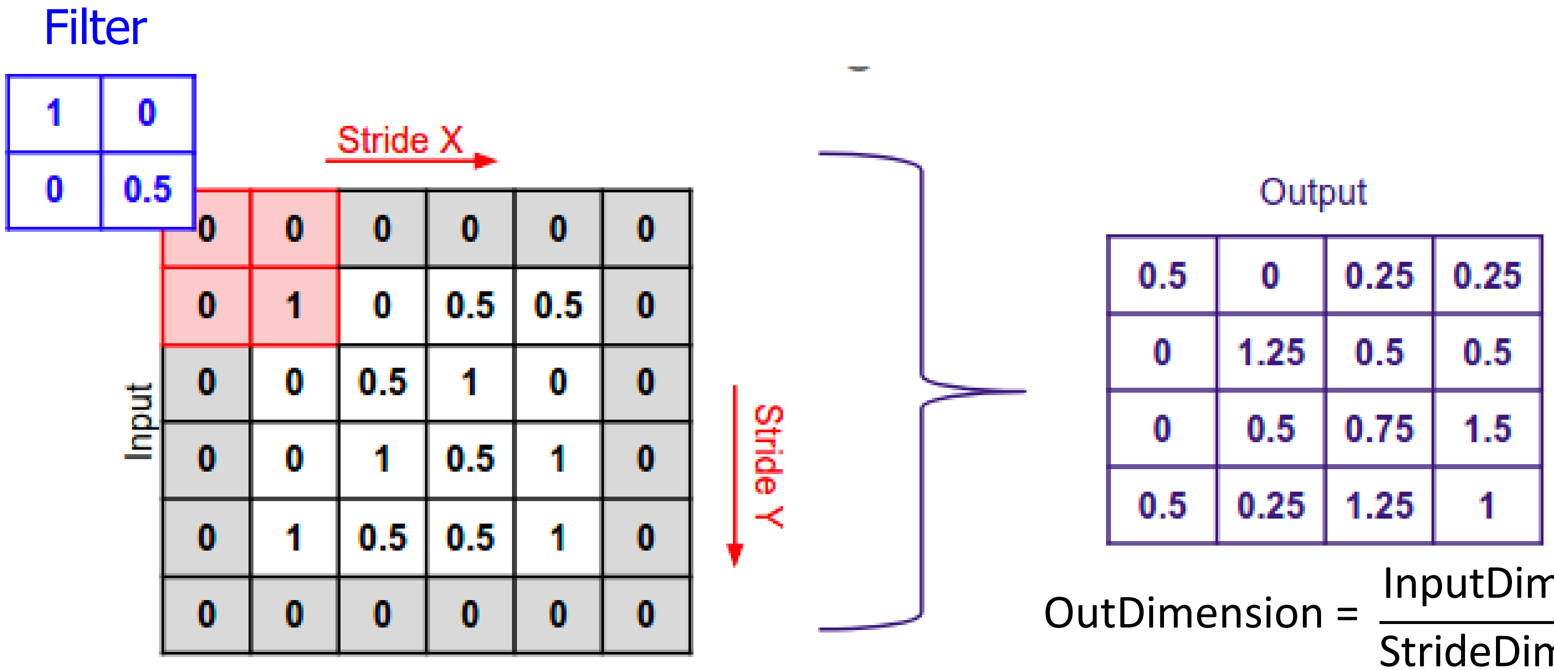
- In convolutional neural networks, they are learned
  - Essentially a linear layer with fewer “connections”
  - Backpropagate as usual!

Convolution Layers



Convolution Layer Parameters

- **Stride:** How many pixels to skip (if any)
  - **Default:** Stride of 1 (no skipping)

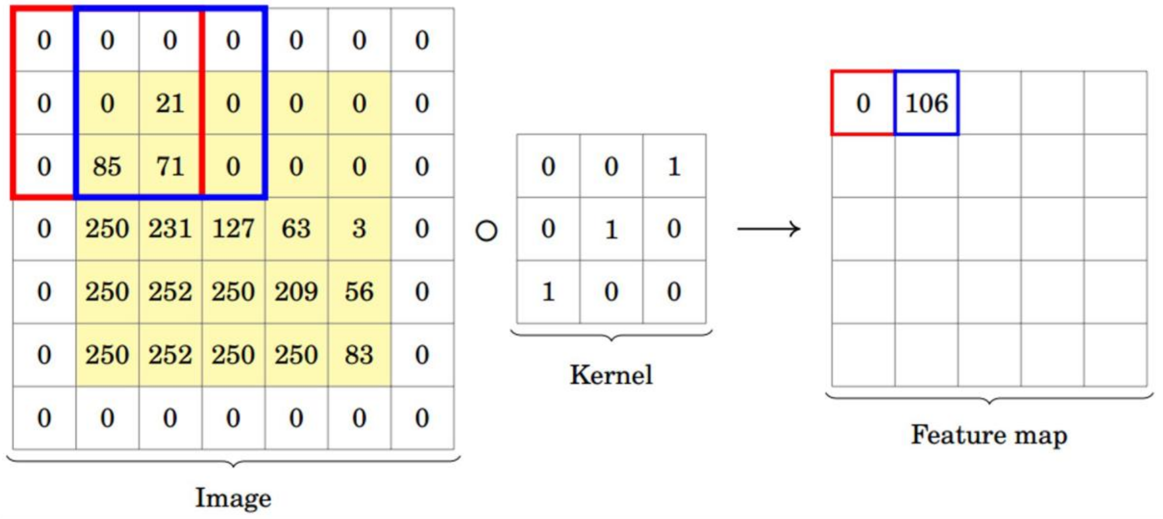


$$\text{OutDimension} = \frac{\text{InputDimension}}{\text{StrideDimension}}$$

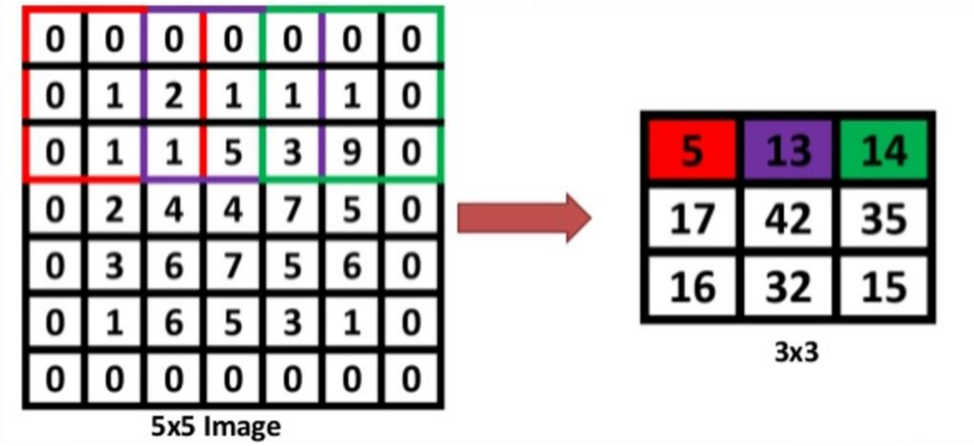
https://medium.com/@ayeshmanthaperera/what-is-padding-in-cnns-71b21fb0dd7

Convolution Layer Parameters

- **Padding:** Add zeros to edges of image to capture ends
  - **Default:** No padding



stride = 1, zero-padding = 1



stride = 2, zero-padding = 1

https://medium.com/@ayeshmanthaperera/what-is-padding-in-cnns-71b21fb0dd7

Convolution Layer Parameters

- **Summary:** Hyperparameters
  - Kernel size
  - Stride
  - Amount of zero-padding
  - Output channels

- Together, these determine the relationship between the input tensor shape and the output tensor shape

- Typically, also use a single bias term for each convolution filter

Convolution Layers

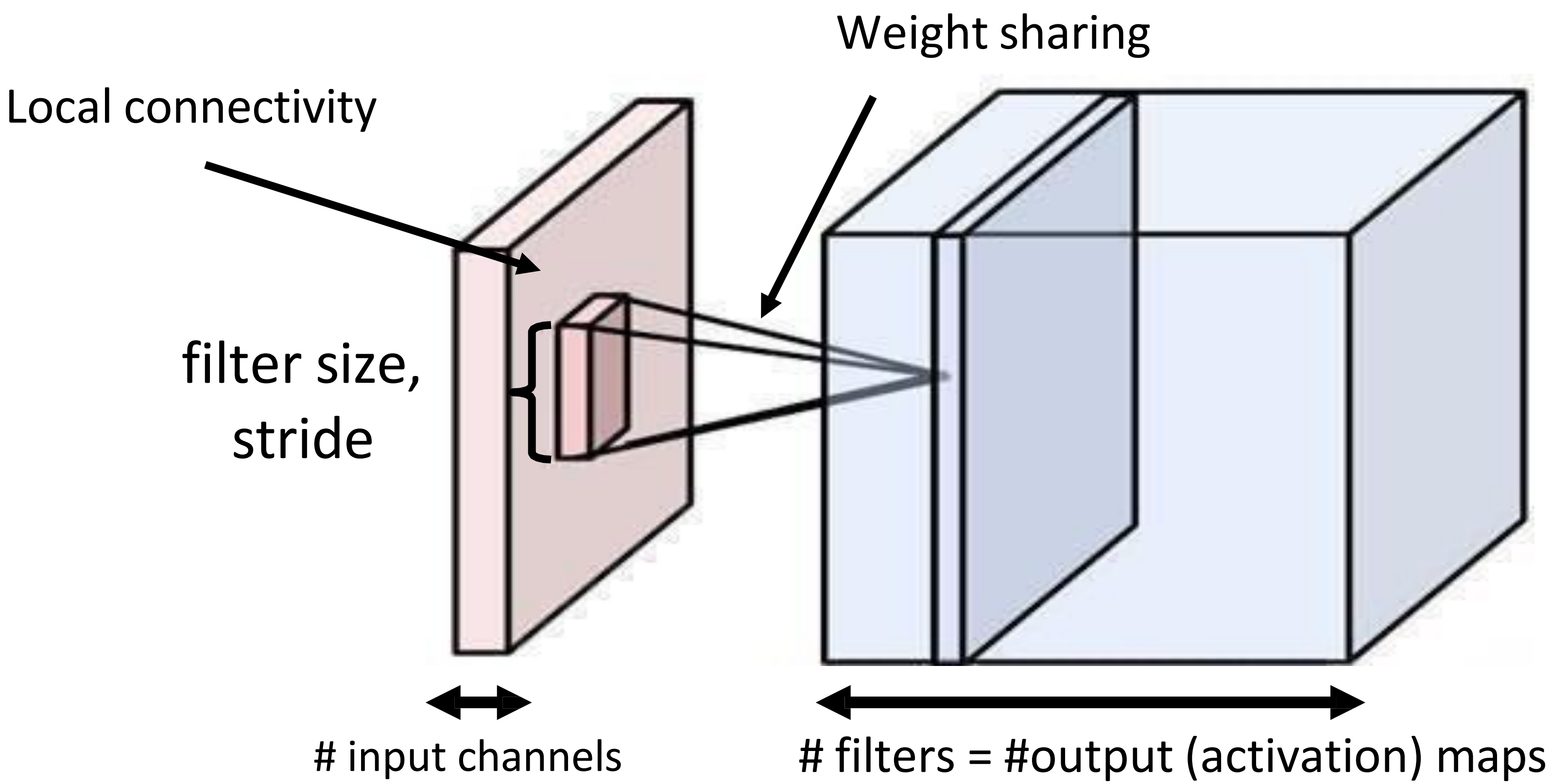
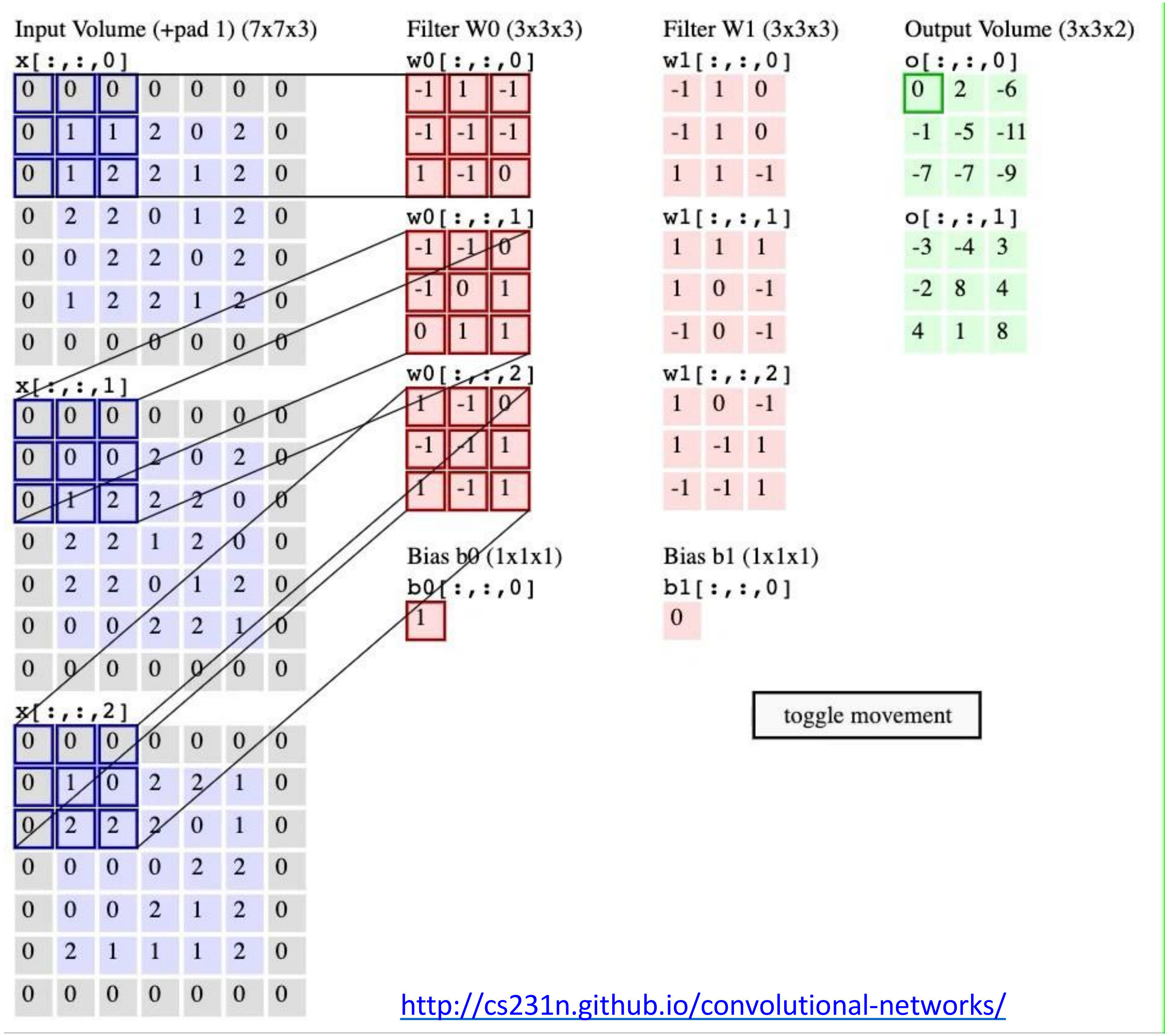


Image credit: A. Karpathy Slide credit: Jia-Bin Huang

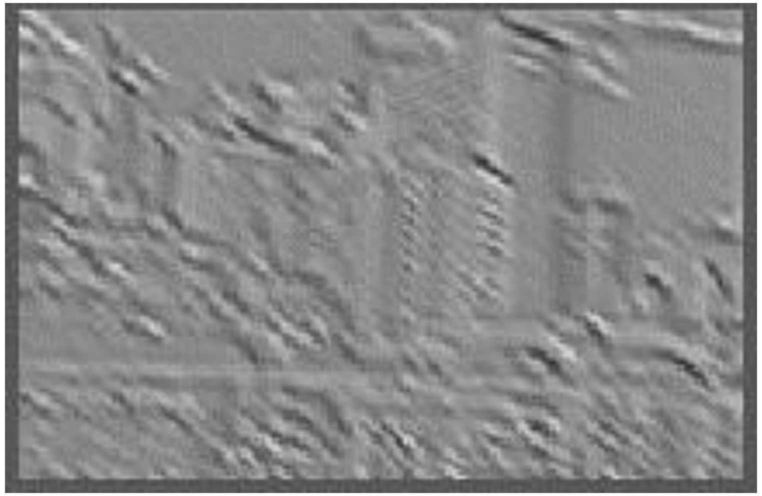
Example

- Kernel size 3, stride 2, padding 1
- 3 input channels
  - Hence kernel size 3×3×3
- 2 output channels
  - Hence 2 kernels
- Total # of parameters:
  - $(3\times3\times3 + 1)\times2 \ = \ 56$

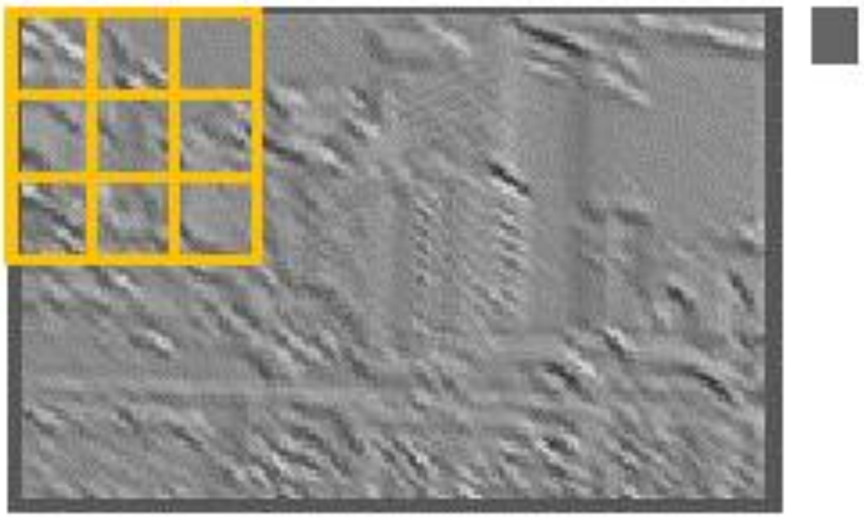


http://cs231n.github.io/convolutional-networks/

Pooling Layers

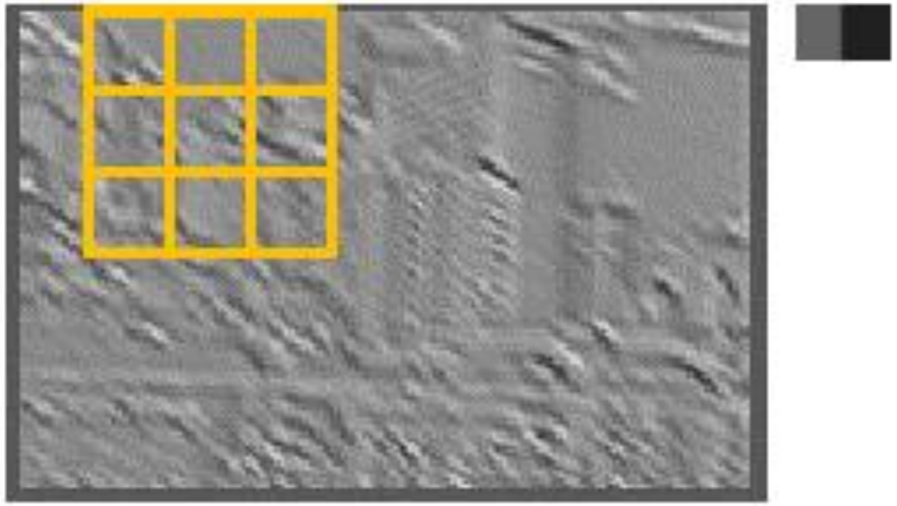


Pooling Layers



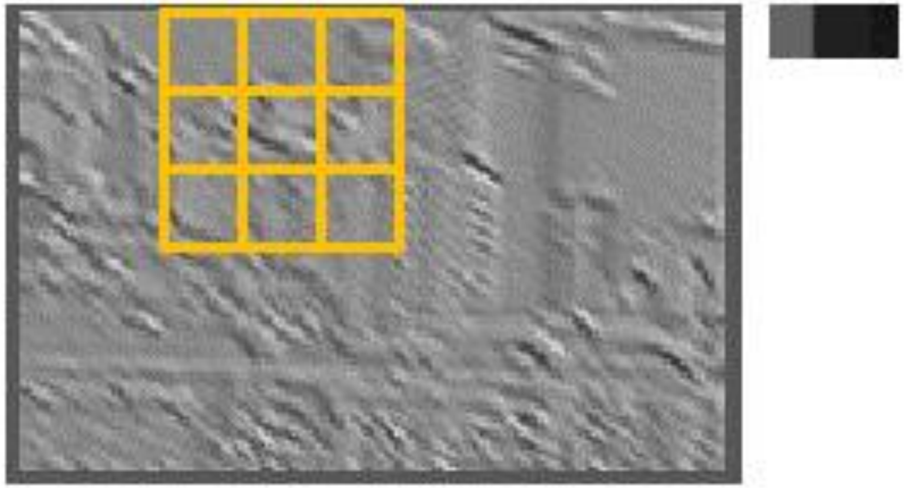
$$\text{output}[0,0] = \max_{0 \le \tau < k} \max_{0 \le \gamma < k} \text{image}[0 + \tau, 0 + \gamma]$$

Pooling Layers



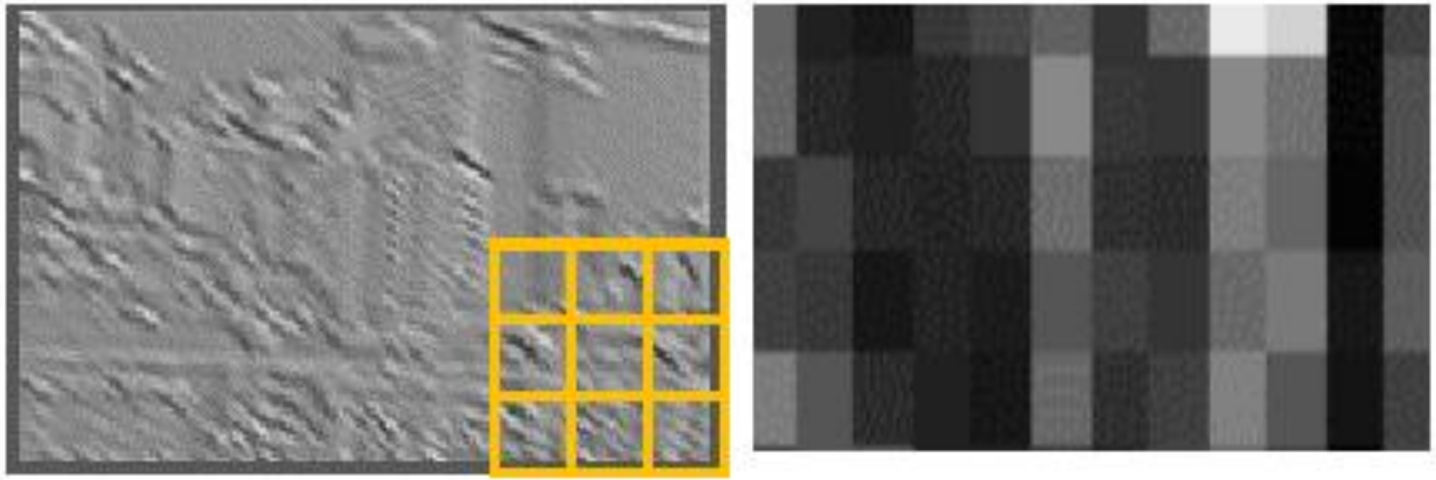
$$\text{output}[0,1] = \max_{0\le\tau<k} \max_{0\le\gamma<k} \text{image}[0+\tau, 1+\gamma]$$

Pooling Layers



$$\text{output}[0,2] = \max_{0 \le \tau < k} \max_{0 \le \gamma < k} \text{image}[0 + \tau, 2 + \gamma]$$

Pooling Layers



$$\text{output}[i,j] = \max_{0 \le \tau < k} \max_{0 \le \gamma < k} \text{image}[i+\tau, j+\gamma]$$

Pooling Layers

- **Summary:** Hyperparameters
  - Kernel size
  - Stride (usually >1)
  - Amount of zero-padding
  - Pooling function (almost always “max”)

- Together, these determine the relationship between the input tensor shape and the output tensor shape

- **Note:** Unlike convolution, pooling operates on channels separately
  - Thus, n input channels → n output channels

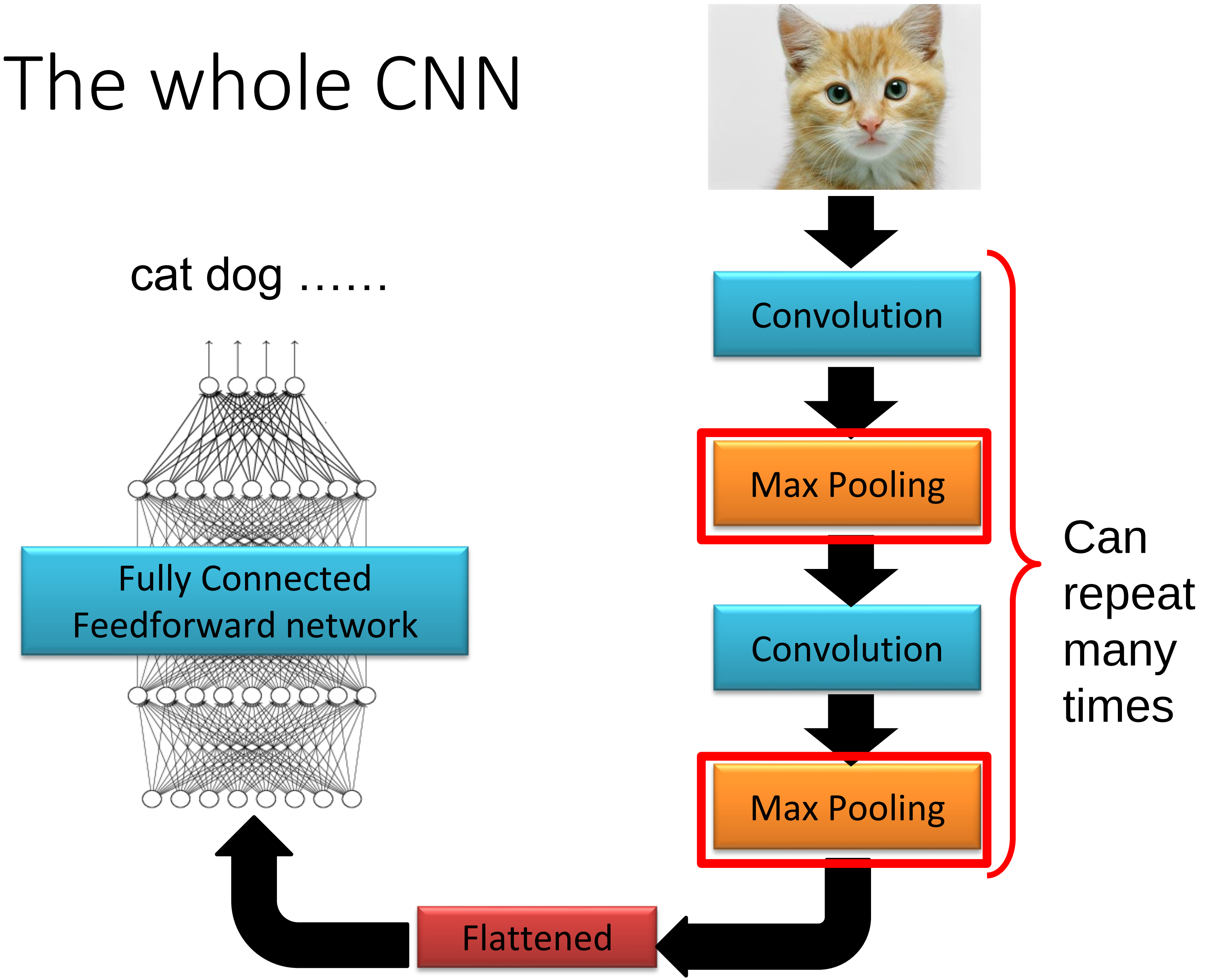
Summary: Convolution vs. Pooling

- **Convolution layers:** Translation equivariant
  - If object is translated, convolution output is translated by same amount
  - Produce “image-shaped” features that retain associations with input pixels

- **Pooling layers:** Translation invariant
  - Binning to make outputs insensitive to translation
  - Also reduces dimensionality

- Combined in modern architectures
  - Convolution to construct equivariant features
  - Pooling to enable invariance

The whole CNN

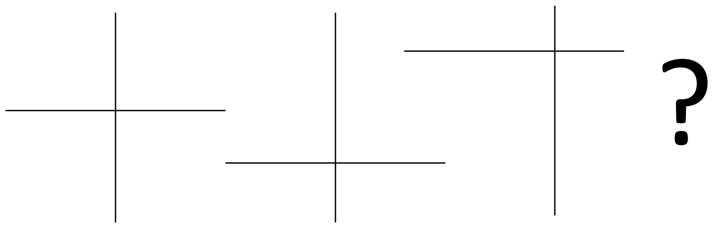


Example

- Suppose we want to predict whether an image depicts Cartesian axes

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & -2 & 0 & -2 \end{bmatrix}$$

input image



target (binary) label

Example

- **Step 1:** Convolve the image with two filters
  - No padding, stride 1

- **Step 2:** Run max pooling

$$\begin{bmatrix} -\frac{1}{2} & 1 & -\frac{1}{2} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \end{bmatrix}, \begin{bmatrix} -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ 1 & 1 & 1 \\ -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

convolution filters

Example

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & -2 & 0 & -2 \end{bmatrix} \quad \begin{bmatrix} -\frac{1}{2} & 1 & -\frac{1}{2} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \end{bmatrix}$$

Example

$$\begin{bmatrix} 0 & 1 & 0 & \\ 0 & 1 & 0 & \\ 1 & 1 & 1 & \\ & & & 2 \end{bmatrix} \quad \begin{bmatrix} -\frac{1}{2} & 1 & -\frac{1}{2} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & \cdot \\ \cdot & \cdot \end{bmatrix}$$

$$\left(0 \times \frac{-1}{2}\right) + (1 \times 1) + \left(0 \times \frac{-1}{2}\right)$$
$$\left(0 \times \frac{-1}{2}\right) + (1 \times 1) + \left(0 \times \frac{-1}{2}\right)$$
$$\left(1 \times \frac{-1}{2}\right) + (1 \times 1) + \left(1 \times \frac{-1}{2}\right) = 2$$

Example

$$
\begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 2 & 0 & 2 \end{bmatrix}
\quad
\begin{bmatrix} -\frac{1}{2} & 1 & -\frac{1}{2} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \end{bmatrix}
\quad \Rightarrow \quad
\begin{bmatrix} 2 & -2 \\ . & . \end{bmatrix}
$$

Example

$$\begin{bmatrix} & & & \\ 0 & 1 & 0 & \\ 1 & 1 & 1 & \\ 0 & -2 & 0 & 2 \end{bmatrix} \quad \begin{bmatrix} -\frac{1}{2} & 1 & -\frac{1}{2} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & -2 \\ -1 & \cdot \end{bmatrix}$$

Example

$$\begin{bmatrix} 0 & & & \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & -2 & 0 & -2 \end{bmatrix} \quad \begin{bmatrix} -\frac{1}{2} & 1 & -\frac{1}{2} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & -2 \\ -1 & 1 \end{bmatrix}$$

Example

$$
\begin{tikzcd}
 & \begin{bmatrix} -\frac{1}{2} & 1 & -\frac{1}{2} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \end{bmatrix} \arrow[r] & \begin{bmatrix} 2 & -2 \\ -1 & 1 \end{bmatrix} \\
\begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & -2 & 0 & -2 \end{bmatrix} \arrow[ur] \arrow[dr] & & \\
 & \begin{bmatrix} -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ 1 & 1 & 1 \\ -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix} \arrow[r] & \begin{bmatrix} -1 & -\frac{1}{2} \\ \frac{7}{2} & 4 \end{bmatrix}
\end{tikzcd}
$$

Input image Convolution Features

Example

$$
\begin{tikzcd}
 & \begin{bmatrix} -\frac{1}{2} & 1 & -\frac{1}{2} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \end{bmatrix} \arrow[r] & \begin{bmatrix} 2 & -2 \\ -1 & 1 \end{bmatrix} \arrow[dr] & \\
\begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & -2 & 0 & -2 \end{bmatrix} \arrow[ur] \arrow[dr] & & & \begin{bmatrix} 2 \\ 4 \end{bmatrix} \\
 & \begin{bmatrix} -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ 1 & 1 & 1 \\ -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix} \arrow[r] & \begin{bmatrix} -1 & -\frac{1}{2} \\ \frac{7}{2} & 4 \end{bmatrix} \arrow[ur] &
\end{tikzcd}
$$

Input image Convolution Features Pooling

Example

$$
\begin{tikzcd}
 & \begin{bmatrix} -\frac{1}{2} & 1 & -\frac{1}{2} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \end{bmatrix} \arrow[r] & \begin{bmatrix} 2 & -2 \\ -1 & 1 \end{bmatrix} \arrow[dr] & \\
\begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & -2 & 0 & -2 \end{bmatrix} \arrow[ur] \arrow[dr] & & & \begin{bmatrix} 2 \\ 4 \end{bmatrix} \\
 & \begin{bmatrix} -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ 1 & 1 & 1 \\ -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix} \arrow[r] & \begin{bmatrix} -1 & -\frac{1}{2} \\ \frac{7}{2} & 4 \end{bmatrix} \arrow[ur] &
\end{tikzcd}
$$

Clear vertical line

Clear horizontal line

Input image Convolution Features Pooling

$n \times n$ image $\qquad f \times f$ filter

padding $p \qquad$ stride s

$$\left\lfloor \frac{n+2p-f}{s} + 1 \right\rfloor \times \left\lfloor \frac{n+2p-f}{s} + 1 \right\rfloor$$

Agenda

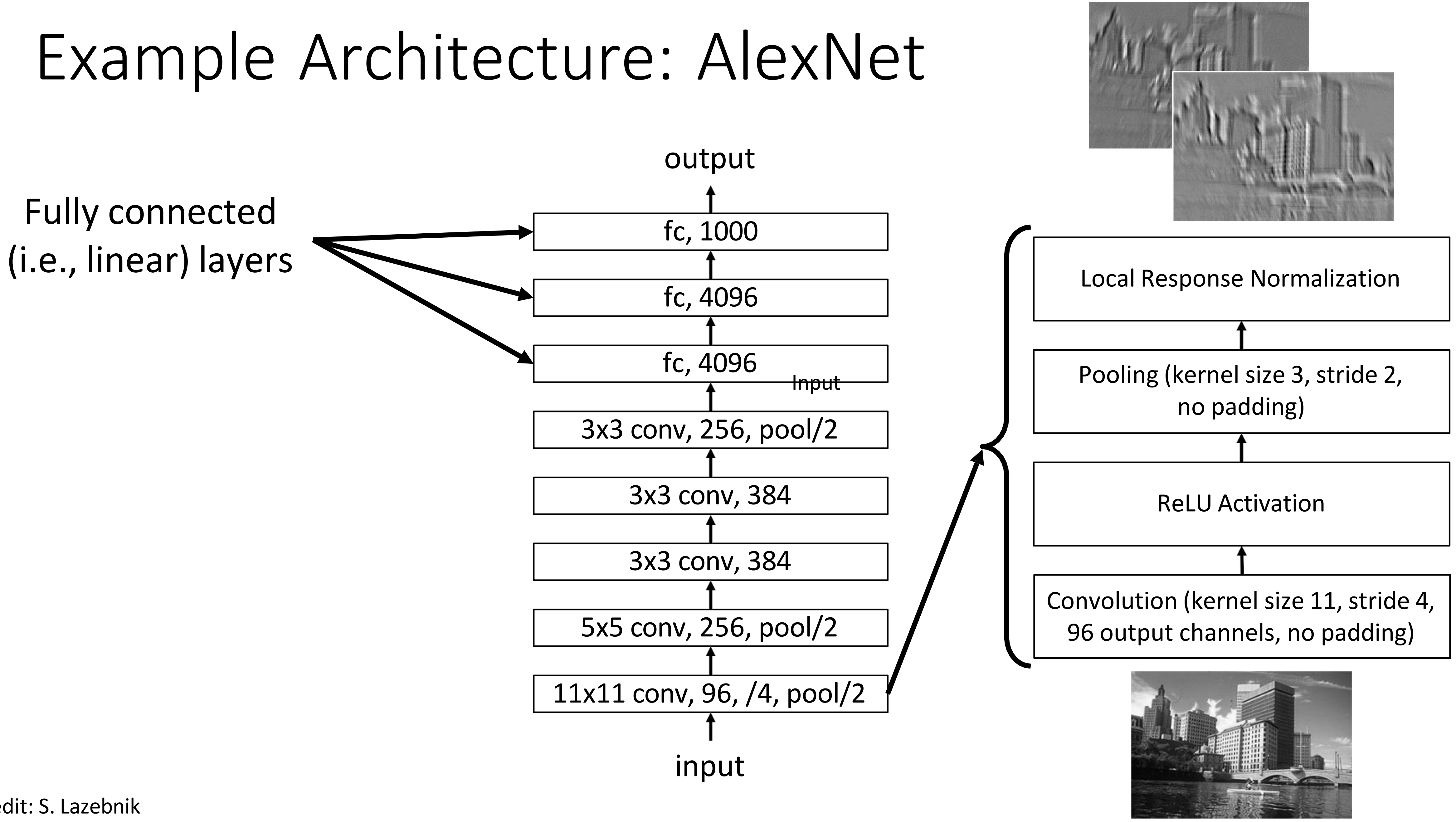
- **Convolutional & pooling layers**
- **Convolutional neural networks**
- **Feature visualization**
- **Applications**

Example Architecture: AlexNet

- **ImageNet dataset**
  - 1000 class image classification problem (e.g., grey fox, tabby cat, barber chair)
  - >1M image-label pairs gathered from internet and crowdsourced labels

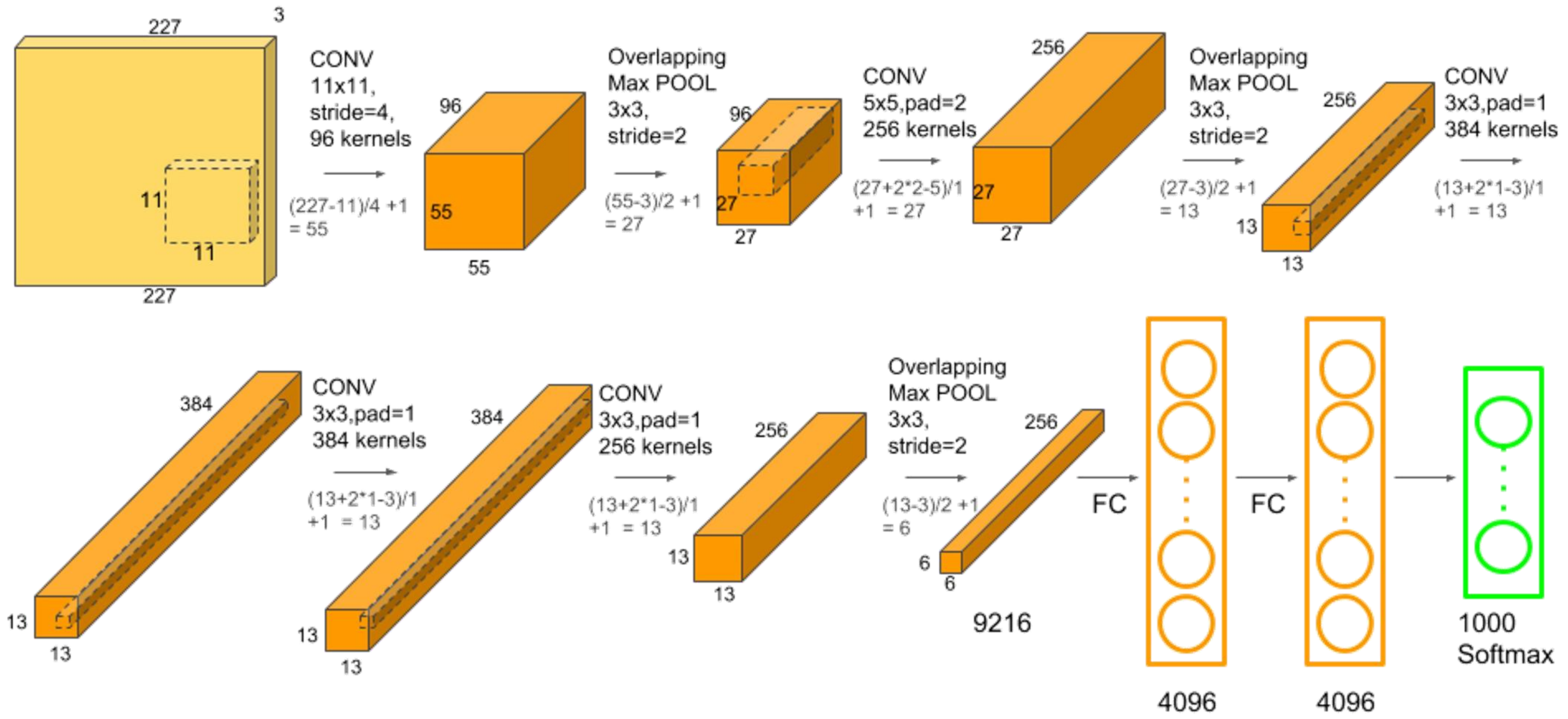
- **AlexNet Architecture (Krizhevsky 2012)**
  - Historically important architecture
  - Image classification network (~60M parameters)
  - Trained using GPUs on ImageNet dataset
  - Huge improvement in performance compared to prior state-of-the-art

Example Architecture: AlexNet



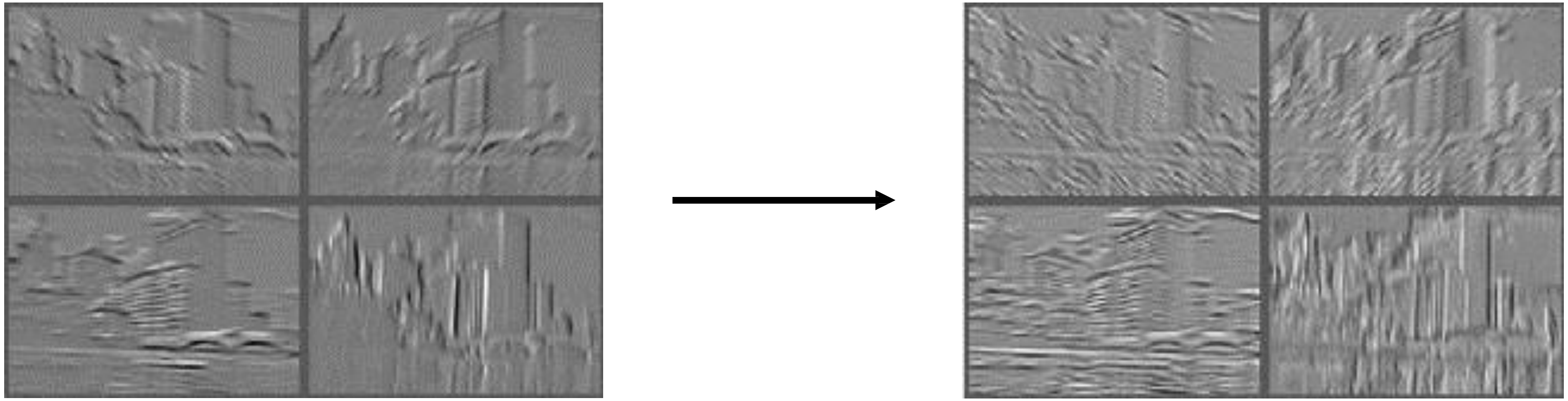
slide credit: S. Lazebnik

Example Architecture: AlexNet



Aside: Local Response Normalization

- Highlights areas where the feature maps change
- Historically a standard layer, but no longer used
- Also called “contrastive normalization”

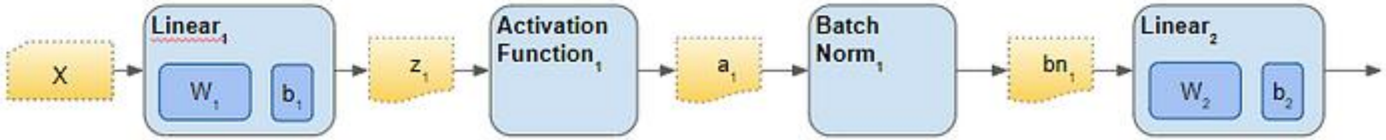


Convolutional Neural Networks

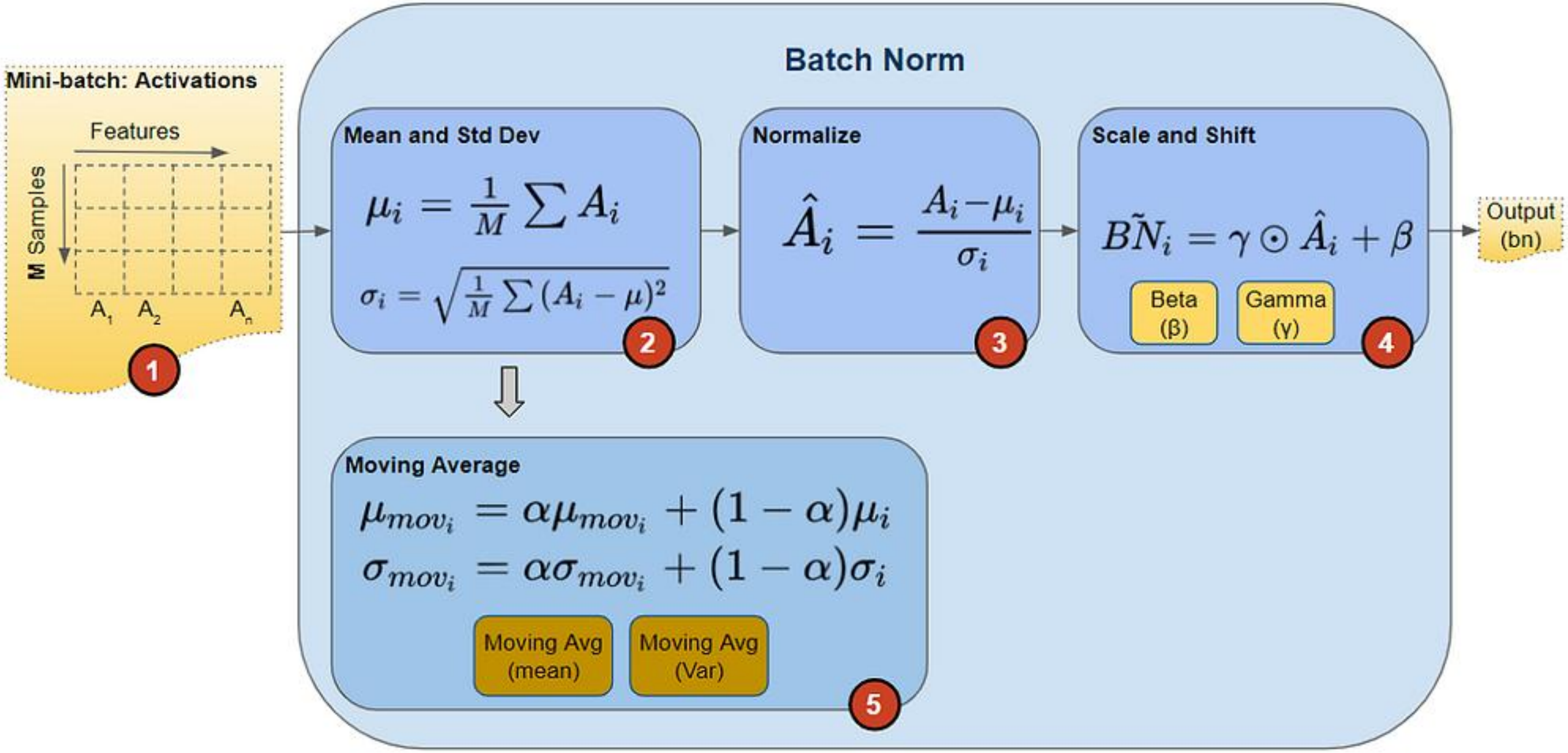
- "Convolutional layer" often refers to sequence of layers

- **Modern sequence of layers**
  - Convolution -> Batch Normalization -> Pooling -> ReLU
  - Convolution -> Batch Normalization -> ReLU -> Pooling

- Can also omit pooling (especially for very deep neural networks)



https://towardsdatascience.com/batch-norm-explained-visually-how-it-works-and-why-neural-networks-need-it-b18919692739



Mini-batch: Activations
Features
M Samples
A_1 A_2 A_n
1
Batch Norm
Mean and Std Dev
$\mu_i = \frac{1}{M}\sum A_i$
$\sigma_i = \sqrt{\frac{1}{M}\sum (A_i - \mu)^2}$
2
Normalize
$\hat{A}_i = \frac{A_i - \mu_i}{\sigma_i}$
3
Scale and Shift
$B\tilde{N}_i = \gamma \odot \hat{A}_i + \beta$
Beta (β)
Gamma (γ)
4
Output (bn)
Moving Average
$\mu_{mov_i} = \alpha\mu_{mov_i} + (1-\alpha)\mu_i$
$\sigma_{mov_i} = \alpha\sigma_{mov_i} + (1-\alpha)\sigma_i$
Moving Avg (mean)
Moving Avg (Var)
5

Residual Connections

- **Challenges with deeper networks**
  - Overfitting?
  - No, 56 layer network underfits!

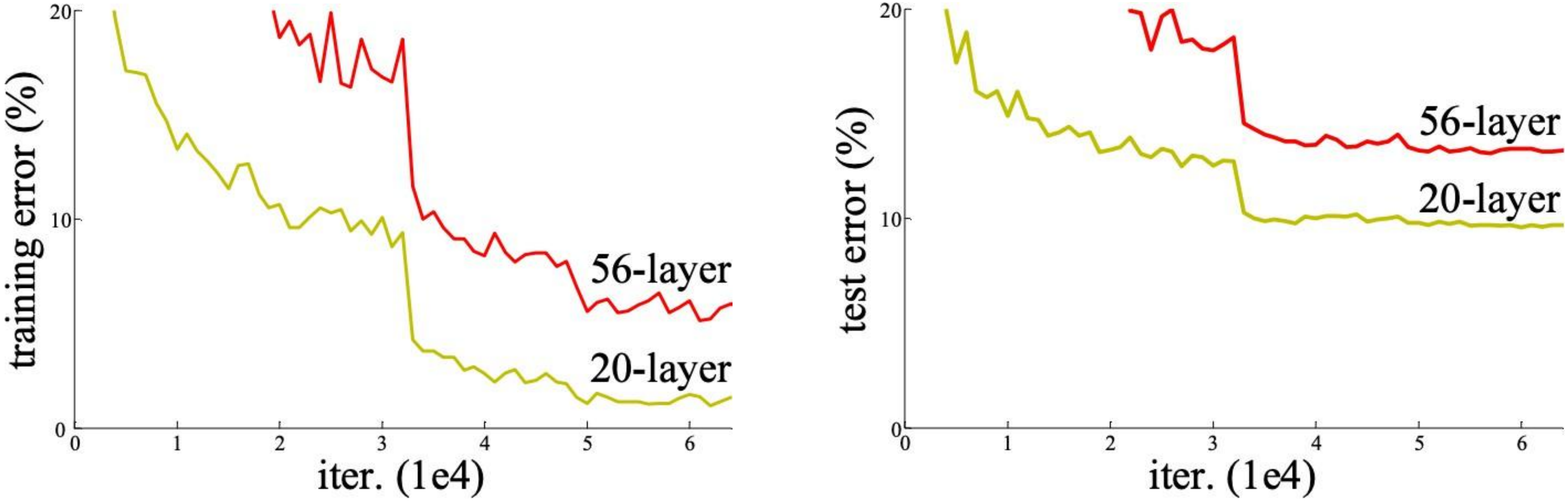


Image credit: He et al, Residual Nets, 2015

Residual Connections

- **Challenges with deep networks**
  - Overfitting?
  - No, 56 layer network underfits!
- **Optimization/representation**
  - Difficulty representing the identity function!
- **Solution:** “Skip” connections
  - Facilitate direct feedback from loss
  - Easy to represent identity function

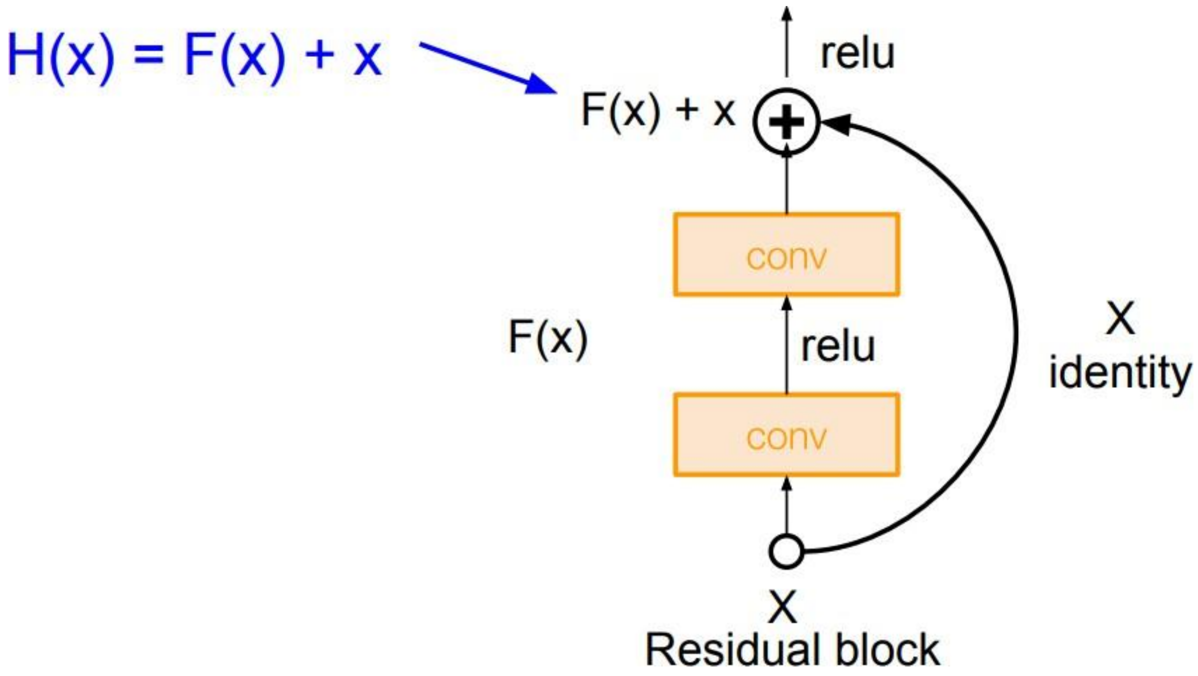


Image credit: Fei-Fei Li, Justin Johnson, Serena Yeung

Residual Connections

- **Residual layers:** Given any convolutional layer $F(x)$, use

$$H(x) = F(x) + x$$

- Two views of residual connections:
  - **View 1:** Providing shortcuts to gradients on the backward pass
  - **View 2:** Allow each "residual block" to fit the residual error (boosting!)

$$F(x) = H(x) - x$$

Residual Networks

- Stack lots of residual blocks!
  - Kernel size 3, no padding, stride 1, no pooling
  - Reduce feature dimensions by using stride 2 once every K blocks
  - Maintains feature size to build very deep nets

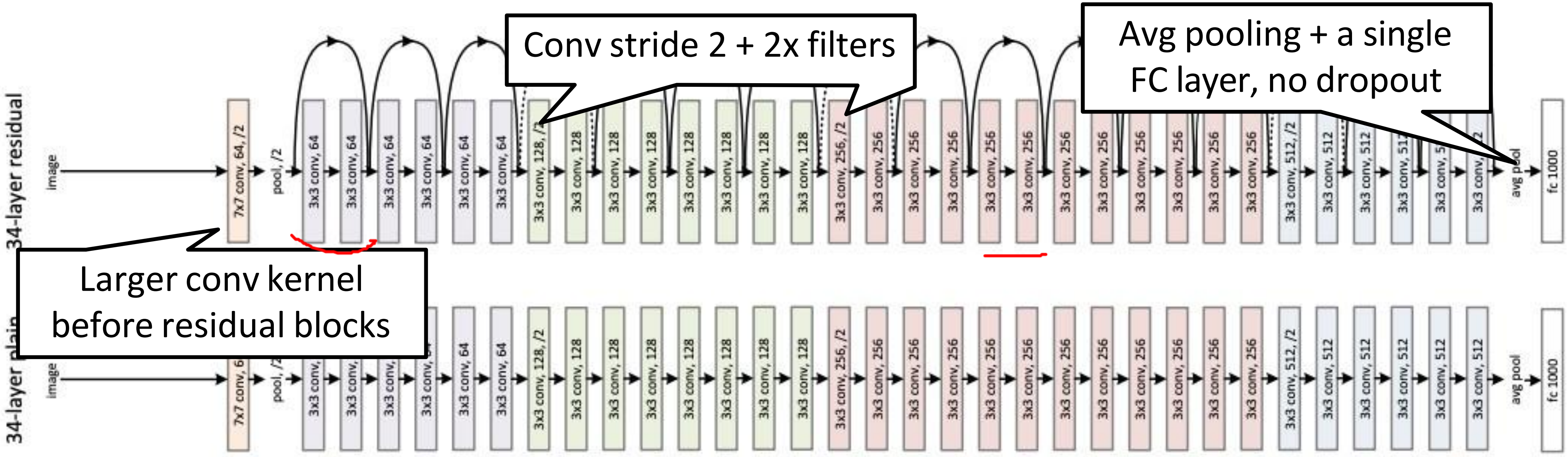


Image credit: He et al, Residual Nets, 2015

Residual Networks

- For deeper networks, improve efficiency through 1x1 convolutions
- Many other improvements since 2015!
  - E.g., “ResNeXt”, “Identity Mappings”, “ConvNeXt” etc.

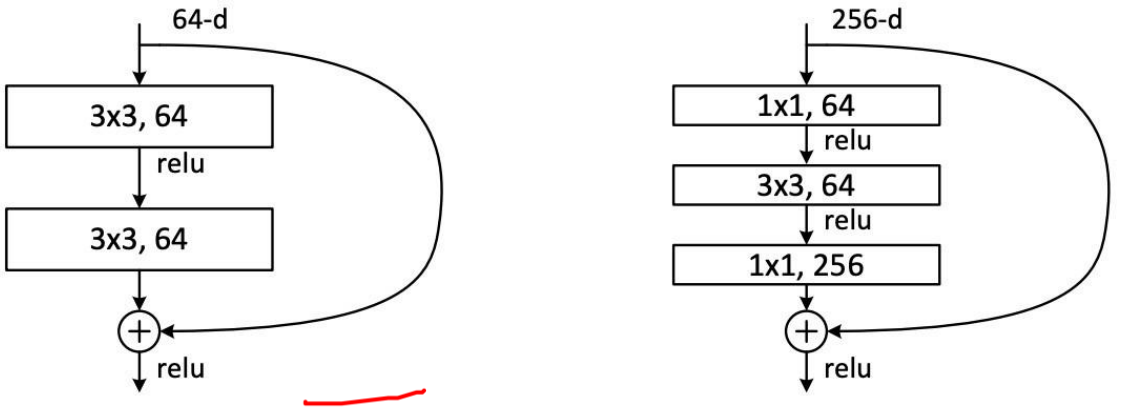
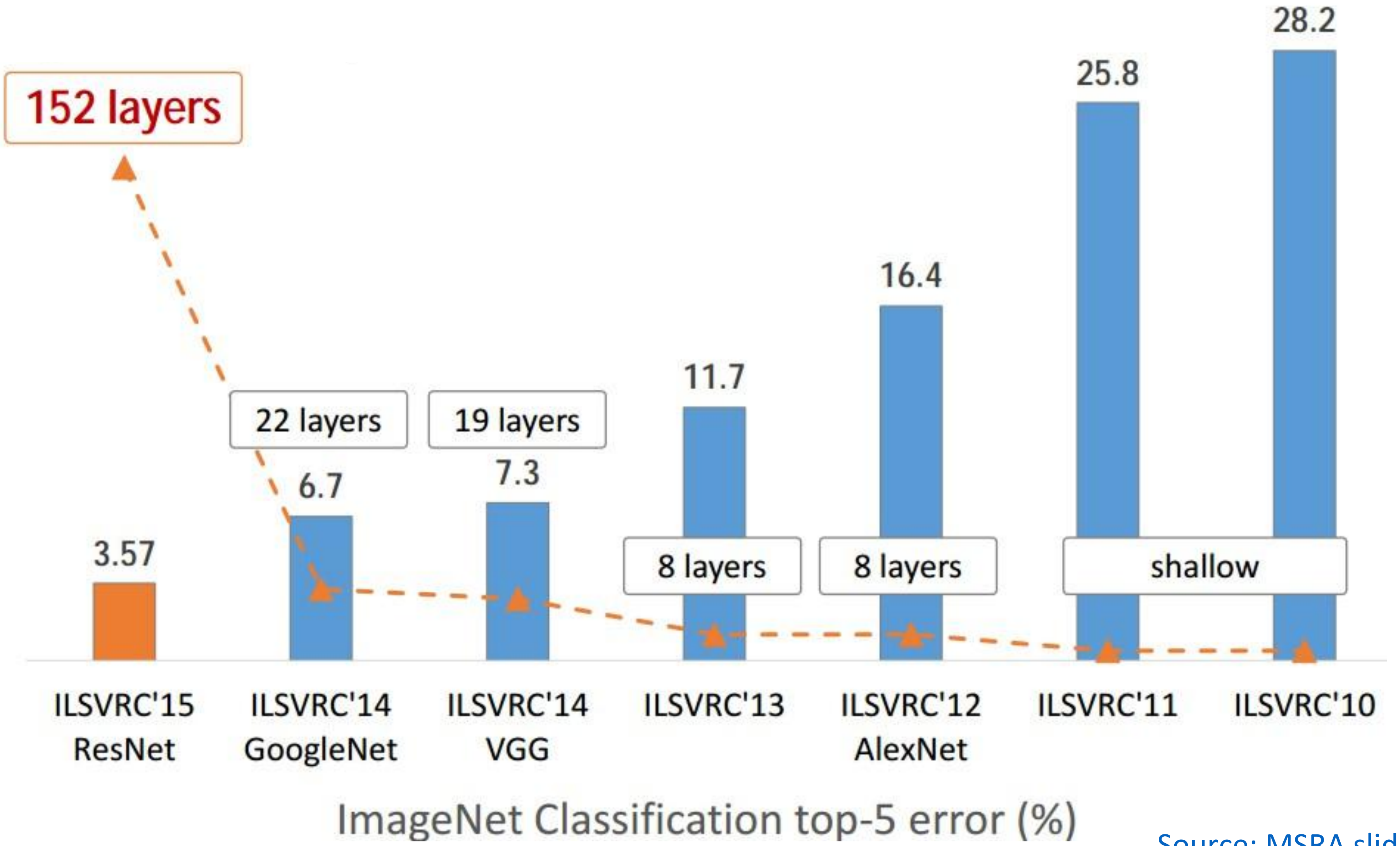


Figure 5. A deeper residual function $\mathcal{F}$ for ImageNet. Left: a building block (on 56×56 feature maps) as in Fig. 3 for ResNet-34. Right: a “bottleneck” building block for ResNet-50/101/152.

Image credit: He et al, Residual Nets, 2015

Evolution of Neural Networks



[Source: MSRA slides at ILSVRC15]()

Evolution of Neural Networks

AlexNet, 8 layers
(ILSVRC 2012)
~60M params

VGG, 19 layers
(ILSVRC 2014)
~140M params

ResNet, 152 layers
(ILSVRC 2015)

Less computation
in forward pass
than VGGNet!

Back to 60M params

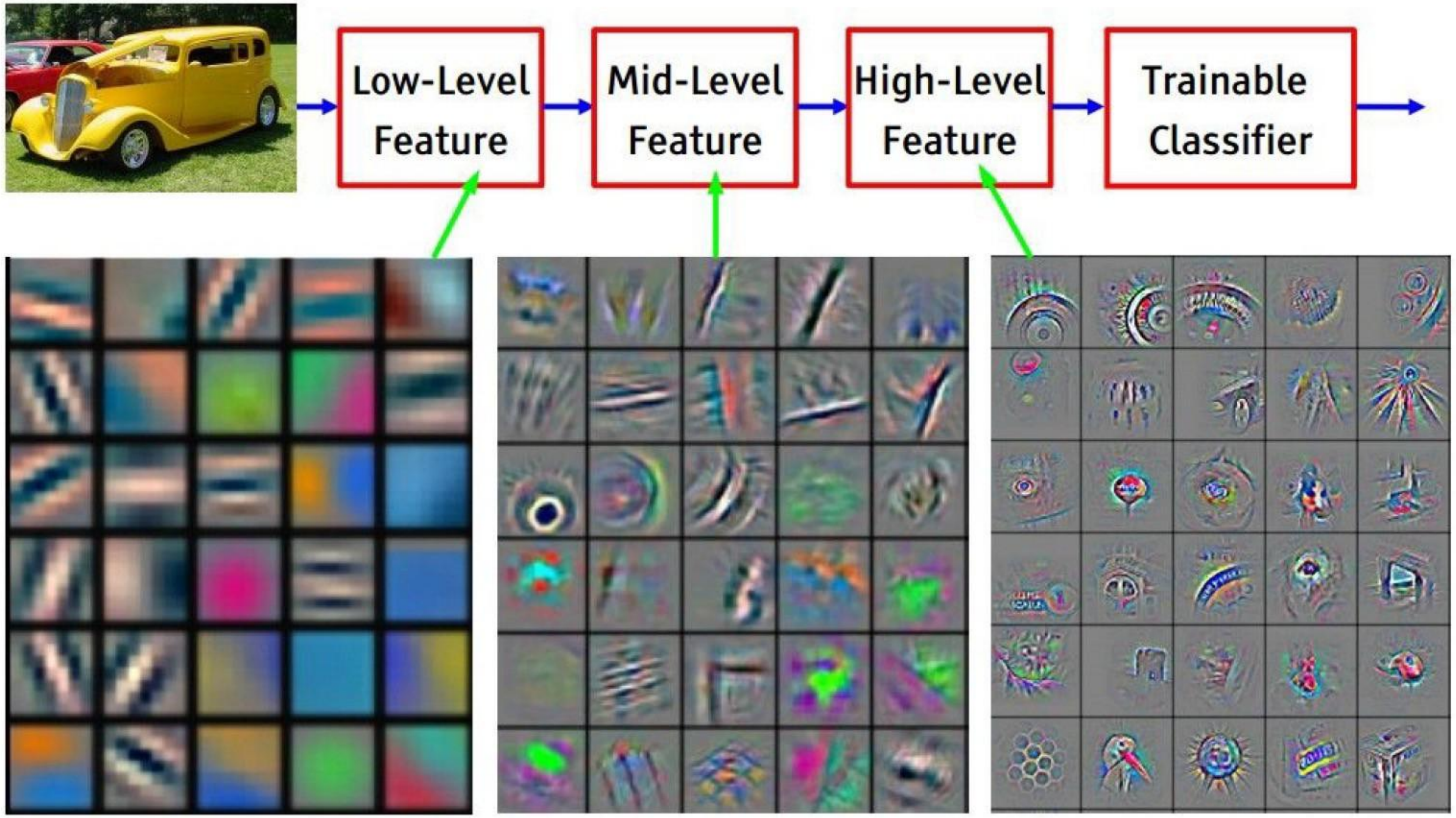
GoogleNet, 22 layers
(ILSVRC 2014)
~5M params

[Source: MSRA slides at ILSVRC15]

Agenda

- **Convolutional & pooling layers**
- **Convolutional neural networks**
- **Feature visualization**
- **Applications**

Feature Visualization



Feature visualization of convolutional net trained on ImageNet from [Zeiler & Fergus 2013]

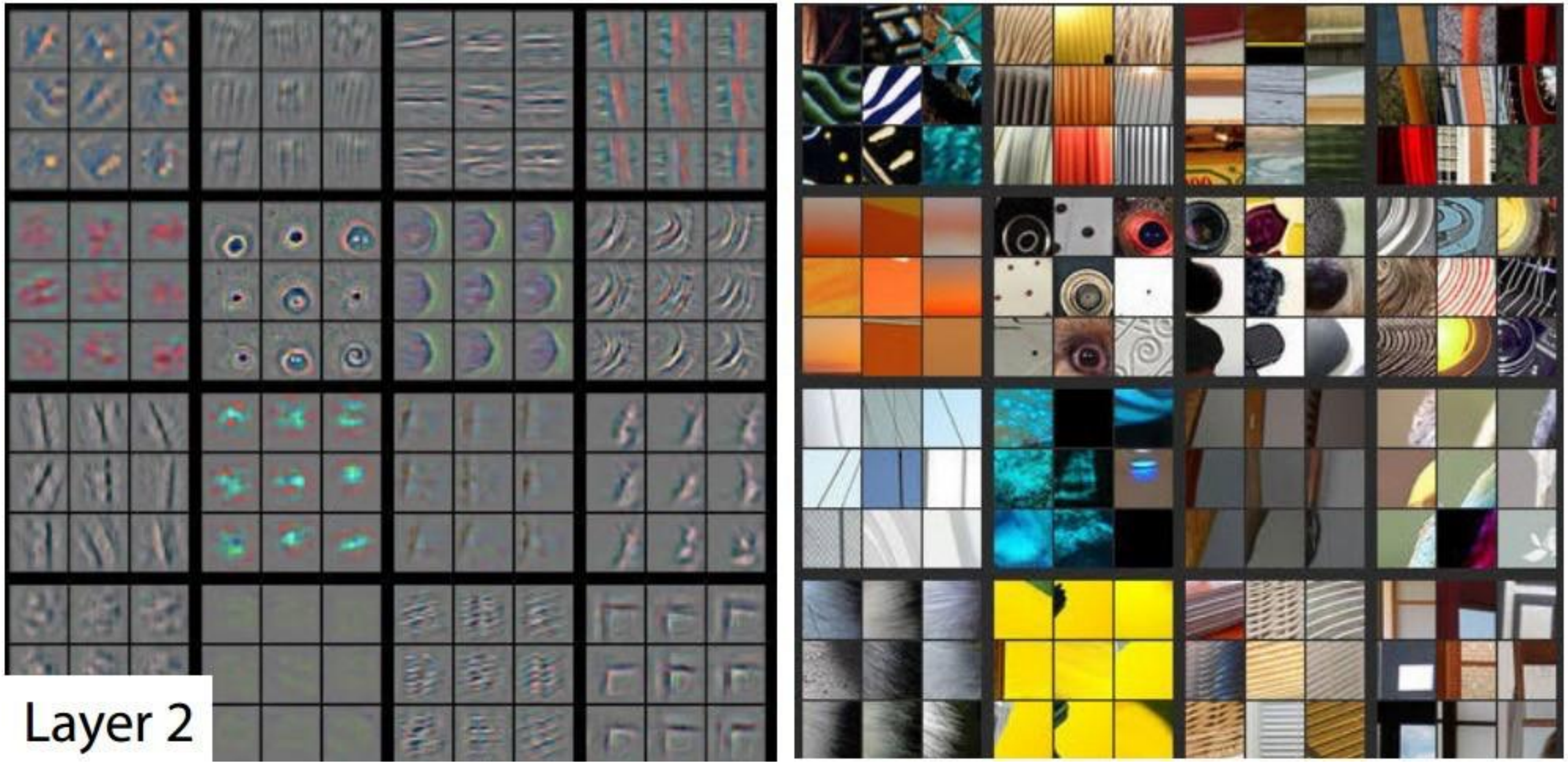
Slide credit: Yann LeCun

Layer 1

Visualizing and Understanding Convolutional Networks [Zeiler and Fergus, ECCV 2014]

Slide credit: Jia-Bin Huang

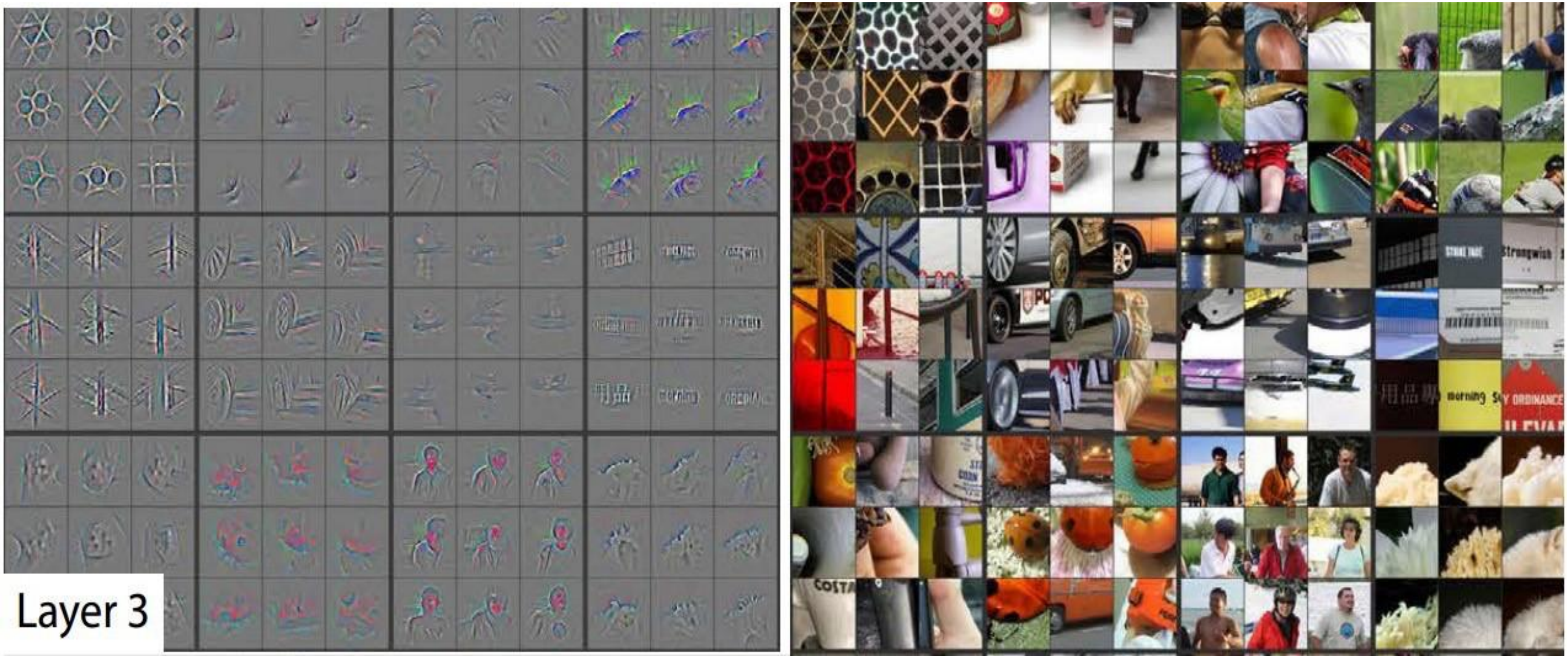
Layer 2



Visualizing and Understanding Convolutional Networks [Zeiler and Fergus, ECCV 2014]

Slide credit: Jia-Bin Huang

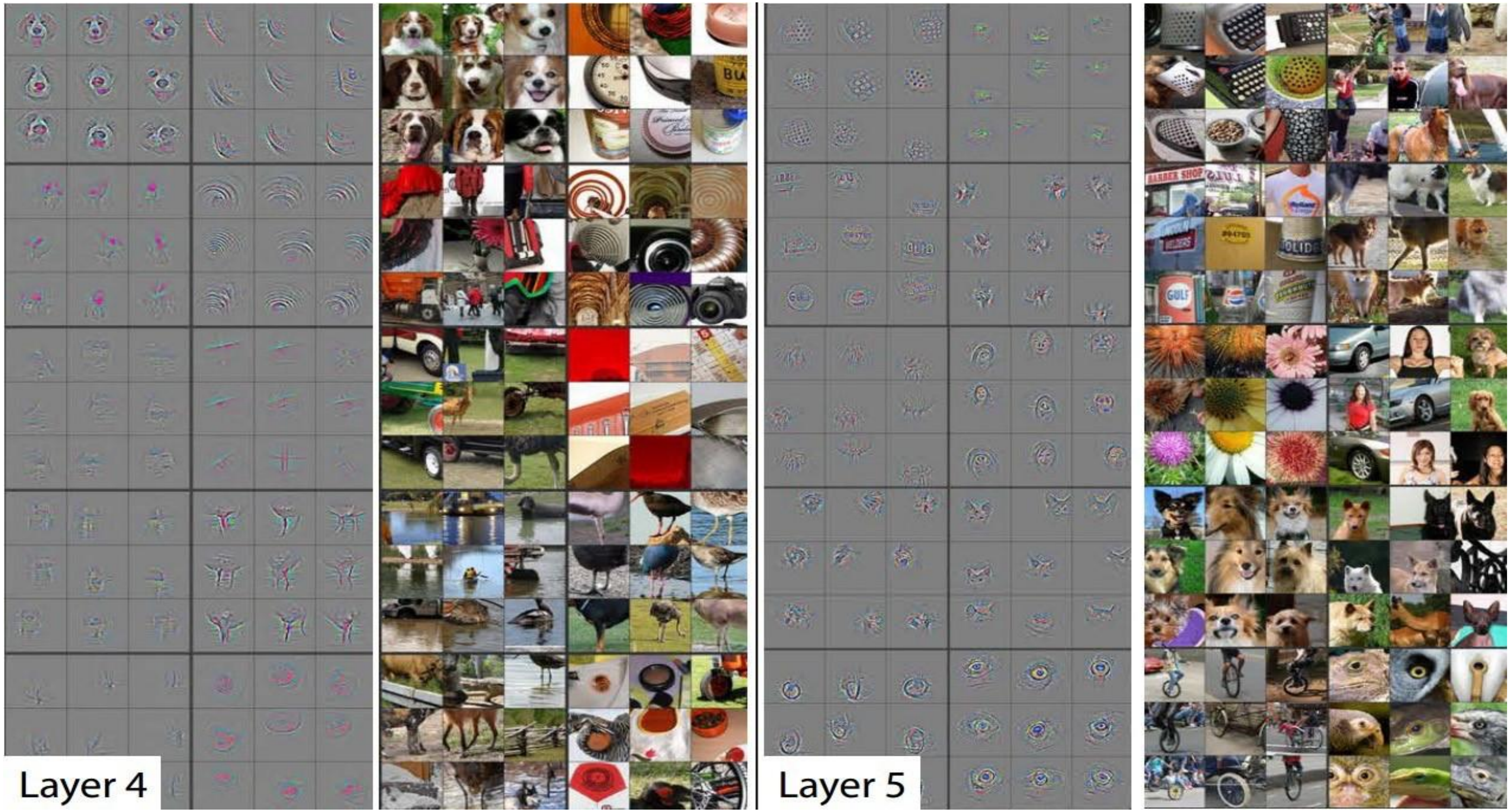
Layer 3



Visualizing and Understanding Convolutional Networks [Zeiler and Fergus, ECCV 2014]

Slide credit: Jia-Bin Huang

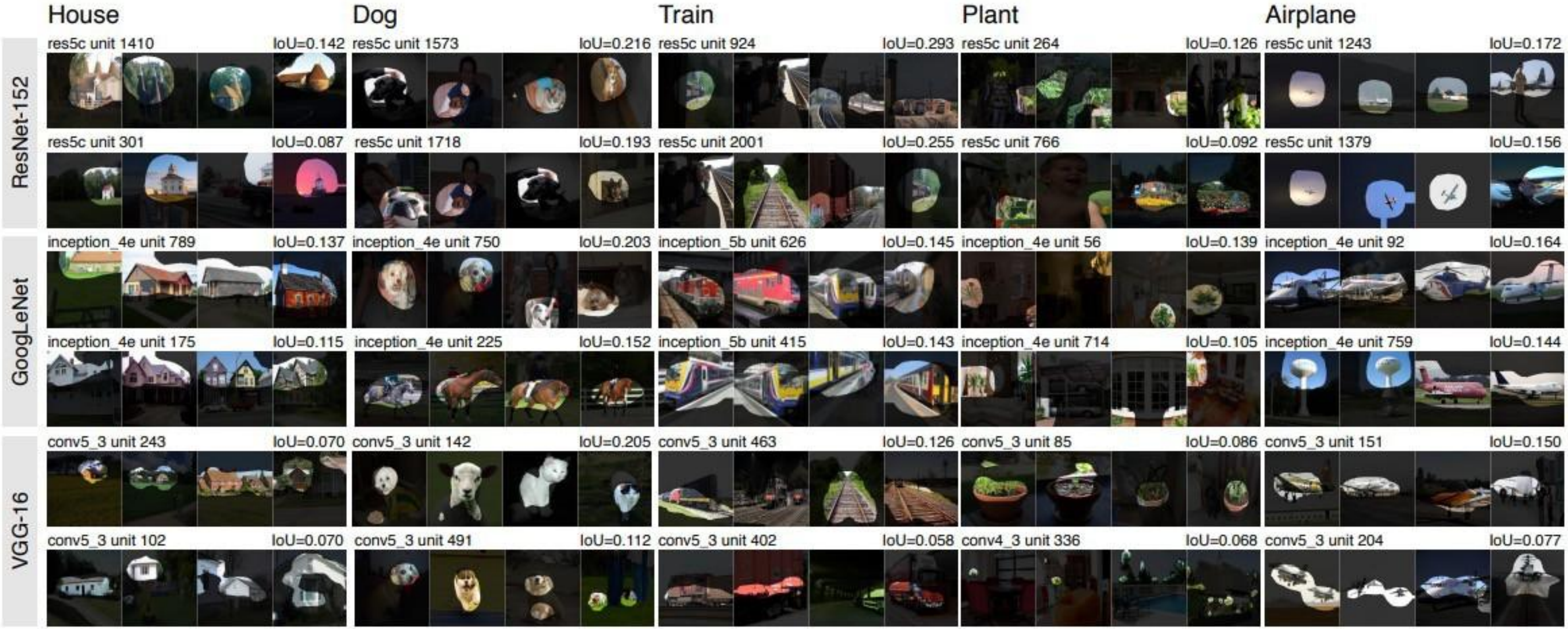
Layer 3



Visualizing and Understanding Convolutional Networks [Zeiler and Fergus, ECCV 2014]

Slide credit: Jia-Bin Huang

Neural Network Dissection



http://netdissect.csail.mit.edu/

What About Small Datasets?

- **Transfer learning:** We can reuse trained concepts!
  - Since CNNs trained on ImageNet appear to learn general features
  - We can reuse these models in some way to perform new tasks
- **Strategy 1:** Feature extraction
  - Remove final (softmax) layer and replace with a new one
  - Train only the new layer
- **Strategy 2:** Finetuning
  - Do the same thing but train end-to-end

What About Small Datasets?

- **New dataset is similar to the original dataset**
  - Can use very small datasets
  - Both strategies work
- **New dataset is different from original dataset**
  - Transfer learning still works!
  - Moderate-sized datasets
  - Finetune end-to-end
  - **Examples:** Medical images, audio spectrograms, etc.

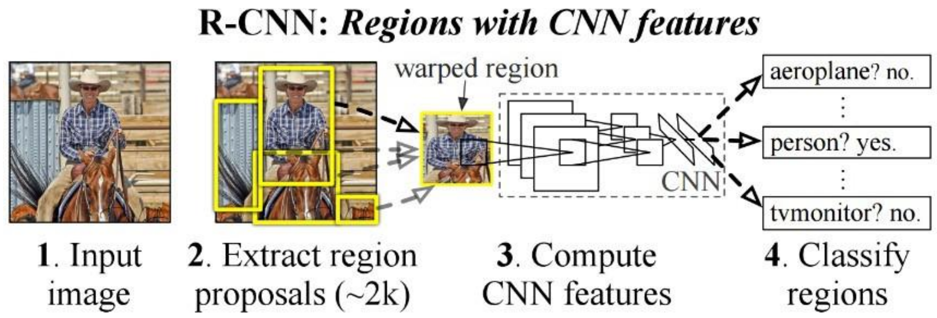
Agenda

- **Convolutional & pooling layers**
- **Convolutional neural networks**
- **Feature visualization**
- **Applications**

Applications

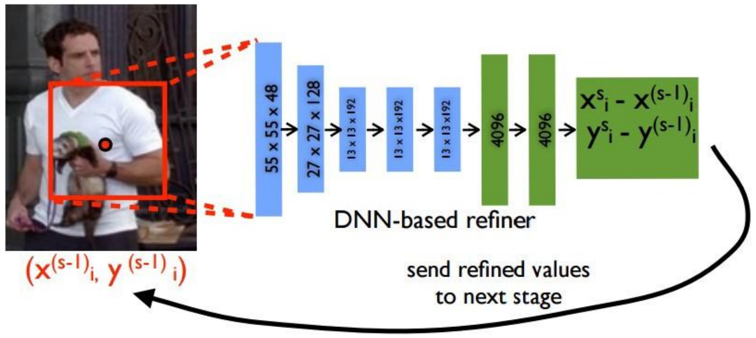
Object detection

[Girshick et al. CVPR14]



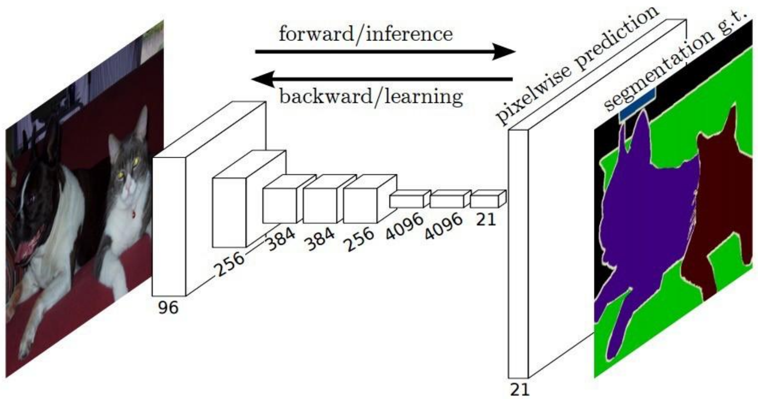
Pose detection (regression)

[Toshev et al. CVPR14]



Semantic segmentation

[Long et al. CVPR15]

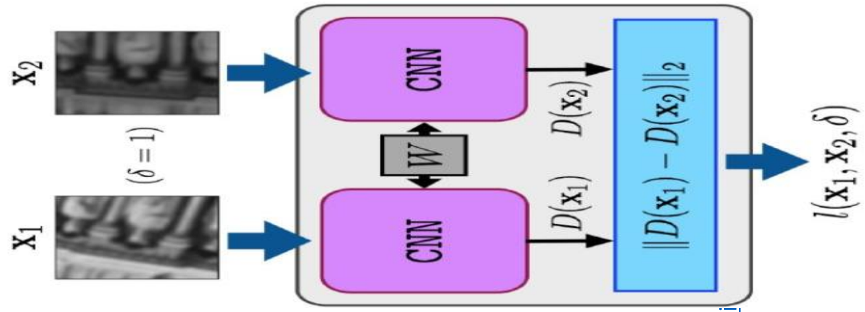


Examples courtesy Jia-Bin Huang

Applications

Similarity metric learning

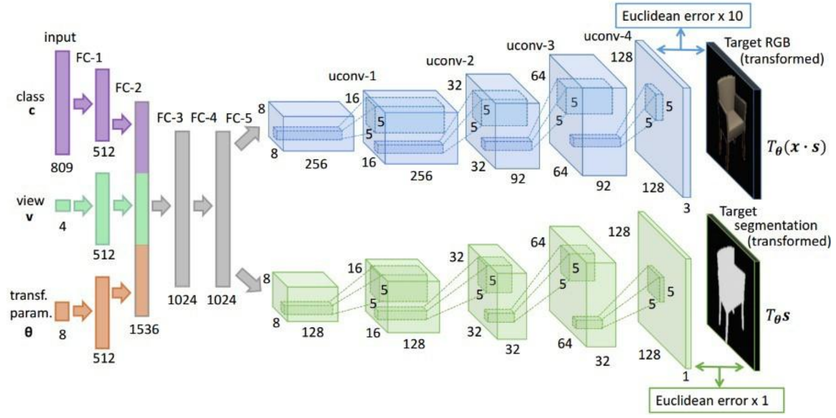
[Chopra et al. CVPR05]



Picture source [Girshi

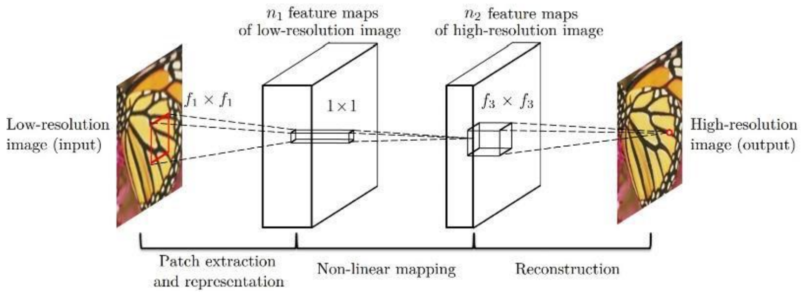
Image generation

[Dosovitskiy et al. CVPR15]



Low-level image processing: (superresolution, deblurring, image quality etc.)

[Dong et al. ECCV 2014]



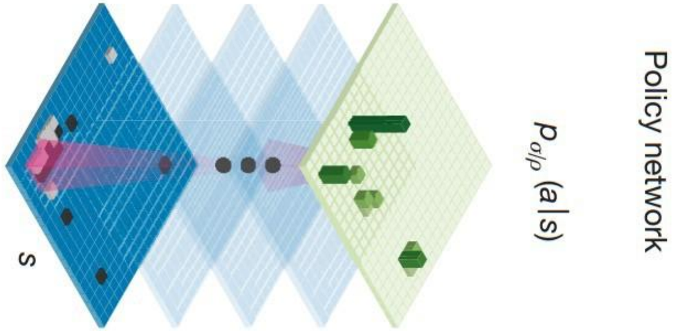
Examples courtesy Jia-Bin Huang

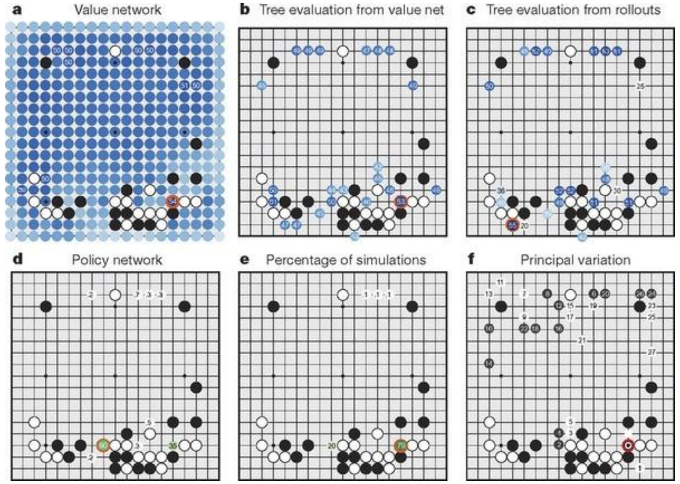
Applications: Game Playing

CNN + Reinforcement learning

[Mnih et al, Nature' 15]

[Silver et al, Nature '16]





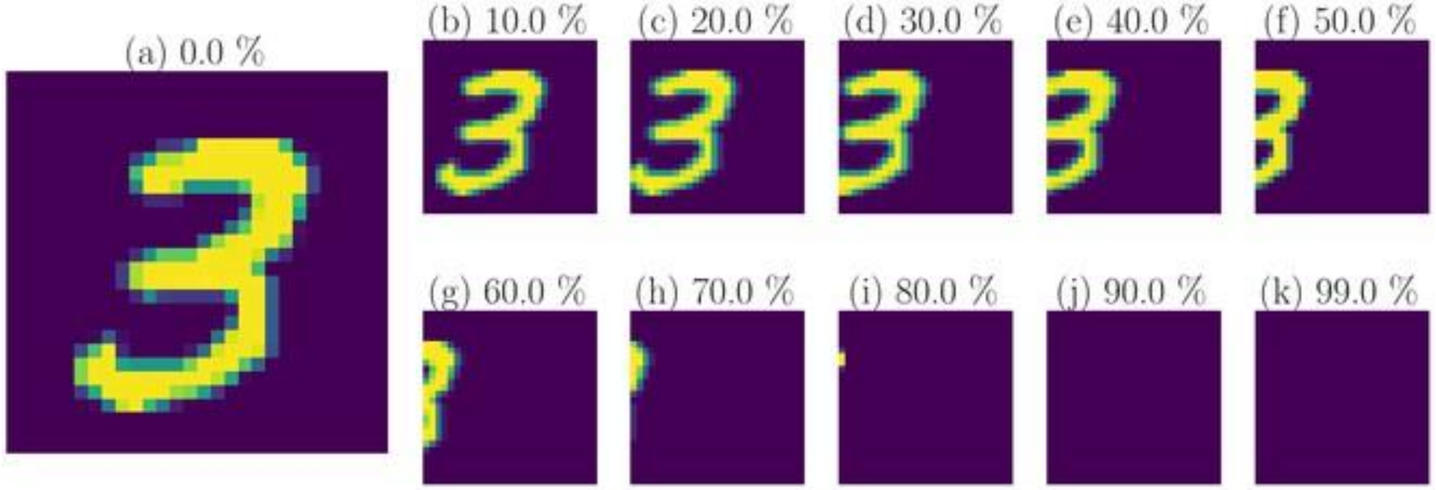
Applications: Art Generation

See if you can tell artist originals from machine style imitations at: http://turing.deepart.io/

Paper: Gatys et al, “Neural ... Style”, arXiv ‘15
Code (torch): https://github.com/jcjohnson/neural-style

Structure in Images

- **Translation invariance -** is a property of a system or model where its performance or output does not change when the input data is translated or shifted.
  - Consider image classification (e.g., labels are cat, dog, etc.)
  - **Invariance:** If we translate an image, it does not change the category label



Source: Ott et al., Learning in the machine: To share or not to share?

Structure in Images

- **Translation equivariance -** is a property of a function or an operation where the output's structure does not change even if the input is translated.
  - Consider object detection (e.g., find the position of the cat in an image)
  - **Equivariance:** If we translate an image, the the object is translated similarly