BOOLEAN EXPRESSION SIMPLIFICATION

Digital logic design

Iqra Chaudhary (Lecturer CS dept. NUML)

Simplification using Boolean identities

Boolean Operator Precedence

- **The order of evaluation in a Boolean expression is:**
  1. Parentheses
  2. NOT
  3. AND
  4. OR
- **Consequence: Parentheses appear around OR expressions**
- **Example:** $F = A(B + C)(C + \overline{D})$

Example : (OR Absorption Law)

- $A + A \cdot B = A$ (Absorption Theorem)

Proof Steps	Justification (identity or theorem)
$A + A \cdot B$	
$= A \cdot 1 + A \cdot B$	$X = X \cdot 1$
$= A \cdot (1 + B)$	
$= A \cdot 1$	$1 + X = 1$
$= A$	$X \cdot 1 = X$

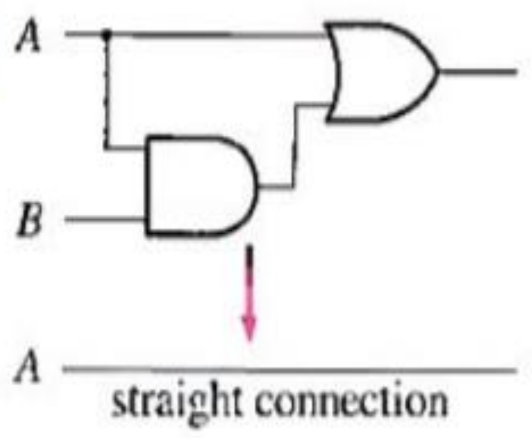
- Our primary reason for doing proofs is to learn:
  - *Careful and efficient use of the identities and theorems of Boolean algebra, and*
  - *How to choose the appropriate identity or theorem to apply to make forward progress, irrespective of the application.*

Example : OR Absorption Law

$A + A.B = A$

A	B	AB	$A + AB$
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

equal



Example:(AND Absorption Law)

- **$A(A + B) = A$**
- Proof:
- $A.A+A.B$ $\qquad\qquad$ $A.A=A$
- $A+A.B$
- $A.1+A.B$
- $A(1+B)$ $\qquad$ $1+B=1$
- $A.1$
- A

Example :Simplification using Boolean Algebra

A +A’B   = A + B

<u>Proof:</u>

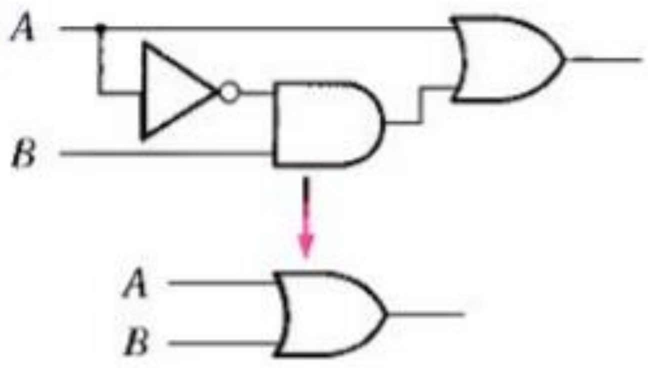
= A +A’B   x+yz=(x+y)(x+z)   Distributive law

=(A+A’)(A+B)   (As A+A’=1)

=A+B

A	B	$\bar{A}B$	$A+\bar{A}B$	$A+B$
0	0	0	0	0
0	1	1	1	1
1	0	0	1	1
1	1	0	1	1

equal



Example :Simplification using Boolean Algebra

$$x \cdot y + \bar{x} \cdot y = y$$

Proof:

=(x+x')y (x+x'=1)

=1.y (1.y=y)

=y

Example :Simplification using Boolean Algebra

$$(X+Y)(\bar{X}+Y)=Y$$

Proof:

(X+Y)(X'+Y)

XX'+XY+X'Y+YY A+A=A

=0+XY+X'Y+Y As 0+A=A

=XY+X'Y+Y

=Y(X+X'+1) A+B+1=1

=Y.1

=Y

Example :Simplification using Boolean Algebra

$(A + B).(A + C)$

$A.A + A.C + A.B + B.C$ – Distributive law

$A + A.C + A.B + B.C$ – $(A.A = A)$

$A(1 + C) + A.B + B.C$ – Distributive law

$A.1 + A.B + B.C$ – Identity OR law $(1 + C = 1)$

$A(1 + B) + B.C$ – Distributive law

$A.1 + B.C$ – Identity OR law $(1 + B = 1)$

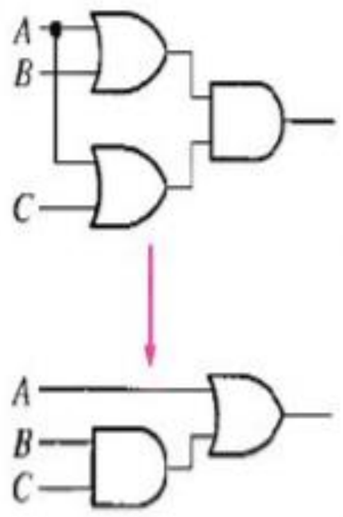
$A + (B.C)$ – Identity AND law $(A.1 = A)$

$(A + B).(A + C)= A + (B.C)$

Example :Simplification using Boolean Algebra

A	B	C	$A+B$	$A+C$	$(A+B)(A+C)$	BC	$A+BC$
0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	1	1	1	1
1	0	0	1	1	1	0	1
1	0	1	1	1	1	0	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

equal



Example :Simplification using Boolean Algebra

- $AB + A(B+C) + B(B+C)$

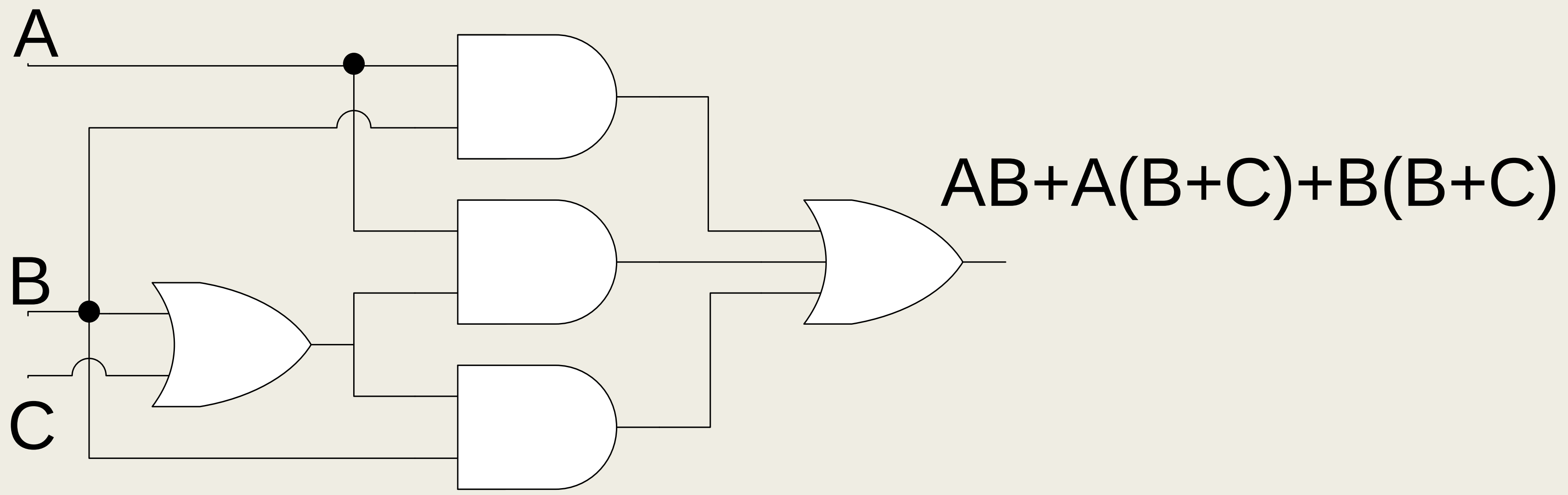
  $= AB + AB + AC + BB + BC$

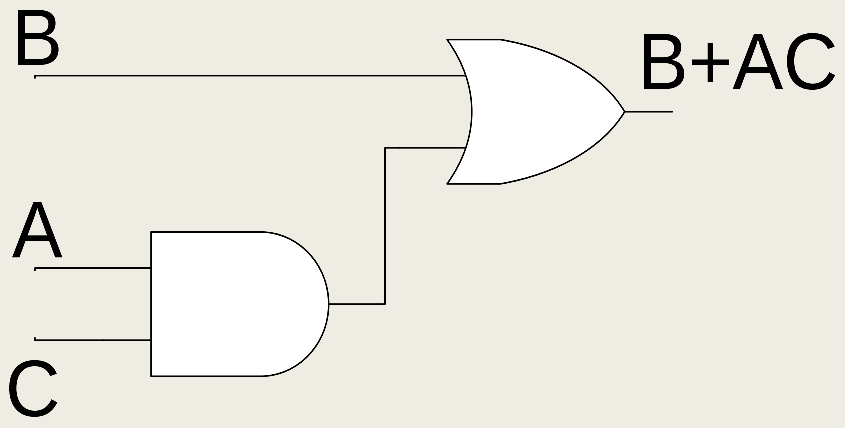
  $= AB + AC + B + BC$

  $= AB + AC + B$

  $= B + AC$

Simplified Circuit





Example :Simplification using Boolean Algebra

$F(A,B,C)=A'B+BC'+BC+AB'C'$

$=B(C+C')+A'B+AB'C'$

$=B+A'B+AB'C'$

$=B(1+A')+AB'C'$

$=B+B'AC'$ $\qquad X+YZ=(X+Y)(X+Z)$

$=(B+B')(B+AC')$

$=B+AC'$

Simplification Example

Canonical form: Sum of minterms

F=ABC+ABC'+AB'C+AB'C'+A'B'C → Canonical form: sum of minterm

F=AB(C+C')+AB'(C+C')+A'B'C

F=AB+AB'+A'B'C

F=A(B+B')+A'B'C

F=A+A'B'C

F=(A+A')(A+B'C)

F= A+B'C → Standard form: sum of product

A	*B*	*C*	*F*	minterm	design-ation
0	0	0	0	*A'B'C'*	*m0*
0	0	1	1	*A'B'C*	*m1*
0	1	0	0	*A'BC'*	*m2*
0	1	1	0	*A'BC*	*m3*
1	0	0	1	*AB'C'*	*m4*
1	0	1	1	*AB'C*	*m5*
1	1	0	1	*ABC'*	*m6*
1	1	1	1	*ABC*	*m7*

Practice problem: Simplification using Boolean Algebra

a) $ABC + A'B + ABC' = AB(C+C') + A'B = AB + A'B = B(A+A') = B$

b) $x'yz + xz = z(x'y+x)= z(x'+x)(x+y) = z(x+y)$

c) $(x+y)'x' = x'y'x'= x'y'$

d) $xy + x(wz + wz') = xy + xw(z+z') = xy + xw = x(y+w)$

e) $(BC'+A'D)(AB'+CD')= AB'BC' + AB'A'D + CD'BC' + CD'A'D = 0$

Example :Proof of (Consensus Theorem)

- $AB + \overline{A}C + BC = AB + \overline{A}C$ (Consensus Theorem)

Proof Steps Justification (identity or theorem)

$AB + \overline{A}C + BC$

$= AB + \overline{A}C + 1 \cdot BC$ $(1 \cdot X = X)$

$= AB + \overline{A}C + (\overline{A} + A) \cdot BC$ $(X + X' = 1)$

$= AB + ABC + A'C + A'BC$ $(X(Y + Z) = XY + XZ$ (Distributive Law))

$= AB \cdot 1 + ABC + A'C \cdot 1 + A'C \cdot B$ $(X \cdot 1 = X)$

$= AB(1 + C) + A'C(1 + B)$ ($X(Y + Z) = XY + XZ$ (Distributive Law))

$= AB \cdot 1 + A'C \cdot 1 = AB + A'C$ $(X \cdot 1 = X)$

Boolean algebra: Dual of Consensus Theorem

Assignment:

$$(X + Y)(X' + Z)(Y + Z) = (X + Y)(X' + Z)$$

Boolean algebra: Dual of Consensus Theorem

Proof of the consensus theorem

Consider the dual of $xy + x'z + yz = xy + x'z$

$$(x + y)(x' + z)(y + z) = (x+y)(x' + z)$$

$$(x + y)(x' + z)(y + z) = (x + y)(x' + z)(y + z + 0)$$

$$= (x + y)(x' + z)(y + z + xx')$$

Use the distributivity theorem

$$(y + z + xx') = (y + z + x)(y + z + x')$$

$$(x + y)(x' + z)(y + z) = (x + y)(x' + z)(y + z + x)(y + z + x')$$

Use the dual of $a + ab = a$, $a(a+b) = a$

$$(x + y)(x' + z)(y + z + x)(y + z + x') = (x + y)(x' + z)$$

Complementing Functions

- Use DeMorgan's Theorem to complement a function:
  1. *Interchange AND and OR operators*
  2. *Complement each constant value and literal*
- Example: Complement F = $\bar{x}y\bar{z} + x\bar{y}\bar{z}$

$$F' = (x + \bar{y} + z)(\bar{x} + y + z)$$

Example :Simplification using Boolean Algebra

- $$(\overline{X+Y})Z + X\overline{Y} = \overline{Y}(X+Z)$$

Proof Steps **Justification (identity or theorem)**

$$(\overline{X+Y})Z + X\overline{Y}$$

= X' Y' Z + X Y'	(A + B)' = A' · B' (DeMorgan's Law)
= Y' X' Z + Y' X	
= Y' (X' Z + X)	A(B + C) = AB + AC (Distributive Law)
= Y' (X' + X)(Z + X)	A + BC = (A + B)(A + C) (Distributive Law)
= Y' · 1 · (Z + X)	A + A' = 1
= Y' (X + Z)	1 · A = A, A + B = B + A (Commutative Law)

Example :Simplification using Boolean Algebra

$$\overline{\overline{A + B\overline{C}} + \overline{D(\overline{E + \overline{F}})}}$$

$$\overline{\overline{A + B\overline{C}} + \overline{D(\overline{E + \overline{F}})}} = (\overline{\overline{A + B\overline{C}}})\,(\overline{D(\overline{E + \overline{F}})})$$

$$(\overline{\overline{A + B\overline{C}}})\,(\overline{D(\overline{E + \overline{F}})}) = (A + B\overline{C})(\overline{D(\overline{E + \overline{F}})})$$

$$(A + B\overline{C})(\overline{D(\overline{E + \overline{F}})}) = (A + B\overline{C})(\overline{D} + (\overline{\overline{E + \overline{F}}}))$$

$$(A + B\overline{C})(\overline{D} + \overline{\overline{E + \overline{F}}}) = (A + B\overline{C})(\overline{D} + E + \overline{F})$$

$$\overline{\overline{A + B\overline{C}} + D\overline{\left(E + \overline{F}\right)}}$$

- $(A'.B'+C+D(E'.F))'$
- $(A+BC')(D'+E+F')$

Practice problem: Find the complement of the following expressions:

a) **$[xy'+x'y]'$** $= (xy')' + (x'y)' = (x'+y).(x+y') = xx' + yy'$

b) **$[(AB'+C)D'+E]'$** $= [(AB'+C)D']'.E' = [(AB'+C)'+D] \;.\; E'=$

$=[(A'+B).C' +D].E' = (A'+B+D).(C'+D).E'$

c) **$[(x+y'+z)(x'+z')(x+y)]'$** $= (x+y'+z)'+(x'+z')'+(x+y)'= x'yz' + xz + x'y'$

Practice problem: Find the complement of the following expressions

Example

Apply DeMorgan's theorems to each of the following expressions:

(a) $\overline{(A + B + C)D}$ (b) $\overline{ABC + DEF}$ (c) $\overline{A\overline{B} + C\overline{D} + EF}$

Thank You