Lecture 2 :Proposition &Predicate Logic

BOOK CHAPTER 6

LOGIC

- Logic is study of reasoning and validity of arguments
- Valid arguments are truth preserving
- Valid Deductive arguments ,the conclusion will always be true if premises are true.

LOGIC

- Logic is primarily a language to
  - model the system (program)
  - reason about the correctness or incorrectness of the system's properties

PHILOSOPHICAL LOGIC

- Logic dealt with reasoning of arguments in the natural language used by humans.
- Abstract study of propositions and statements

- Example: Valid Argument
  - All IIUI students are good at studies.
  - Ayesha is a IIUI student
  - Therefore, Ayesha is good at studies.

PHILOSOPHICAL LOGIC

1. If the train arrives late and there are no taxis at the station, then John is late for his meeting.
2. John is not late for his meeting.
3. The train did arrive late.
4. Therefore, there were taxis at the station.

- Valid or Invalid?
  - Valid (Combine 1, 2 and 3 and then use

PHILOSOPHICAL LOGIC

- Natural languages are very ambiguous.

- Example:
- Tom hates Jim and he likes Mary.
  - Tom likes Mary, or
  - Jim likes Mary

- Thus, we need a more mathematical language for logical reasoning

PROPOSITIONAL LOGIC

- A proposition - a sentence that can be either true or false.
  - -Mr. Abid is teaching Formal Methods in this term
  - - 5 is greater than 4

PROPOSITION

- A **proposition** is a declarative sentence that is either true or false.
- Examples of propositions:
  - The Moon is made of green cheese.
  - Trenton is the capital of New Jersey.
  - Toronto is the capital of Canada.
  - $1 + 0 = 1$
  - $0 + 0 = 2$
- Examples that are **not** propositions.
  - Sit down!
  - What time is it?
  - $x + 1 = 2$
  - $x + y = z$

Activity: Identify Propositions

May fortune come your way. **F**

Jane reacted violently to Jack's accusations. **T**

Every even natural number > 2 is the sum of two prime numbers. **T**

Ready, steady, go. **F**

Please pass me the salt and pizza. **F**

Connectives in Propositional Logic

- ∧ and (conjunction):
- ∨ OR Disjunction

- **a ∧ b**: Both a and b are true.
- **a ∨ b**: at least one of a or b are true

Connectives in Propositional Logic

- $\neg$ not (negation)
  - $\neg$ **a**: a is not true
- $\rightarrow$ implication
  - **a $\rightarrow$ b**: if a then b *(a: assumption, b: conclusion)*
- $\leftrightarrow$ equivalent to
  - **a $\leftrightarrow$ b**: a is equivalent to b, i.e., a $\rightarrow$ b $\wedge$ b $\rightarrow$ a
- therefore

$\perp$, T

False, True

The Negation Operator

The unary negation operator "¬" (NOT) transforms a prop. into its logical negation.

If p = "I have brown hair."

then ¬p = "I do not have brown hair."

Negation of p

Let p be a proposition. The statement "It is not the case that p" is also a proposition, called the "negation of p" or $\neg p$ (read "not p").

p = The sky is blue.

$\neg p$ = The sky is not blue.

The Truth Table for the Negation of a Proposition

p	$\neg p$
T	F
F	T

Negation

For any proposition

- $(\neg\neg p) \Leftrightarrow p$
- $(\neg\neg \text{true}) \Leftrightarrow \text{true}$
- $(\neg\neg \text{false}) \Leftrightarrow \text{false}$

Simple Exercise

Calculate the truth values of following propositions

- $\neg(0<1)$
- $\neg(1+1=2)$
- ¬(The earth revolves around the moon)

Conjunction Operator

The binary conjunction operator "$\wedge$" (AND) combines two propositions to form their logical conjunction.

If

p="I will have salad for lunch."

q="I will have steak for dinner."

then

$p \wedge q$="I will have salad for lunch and I will have steak for dinner."

Conjunction of p and q

Let p and q be propositions.

The proposition "p and q," denoted by $p \wedge q$ is true when both p and q are true and is false otherwise This is called the conjunction of p and q.

The Truth Table for the Conjunction of two propositions

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Truth Table

- The truth table for $p \land (\neg q)$

p	q	$\neg q$	$p \land (\neg q)$
True	True	False	False
True	False	True	True
False	True	False	False
False	False	True	False

The Disjunction Operator

The binary disjunction operator "$\vee$" (OR) combines two propositions to form their logical disjunction.

p="That car has a bad engine."

q="That car has a bad carburetor."

$p \vee q$="Either that car has a bad engine, or that car has a bad carburetor."

Disjunction of p and q

Let p and q be propositions.

The proposition "p or q," denoted by p∨q, is the proposition that is false when p and q are both false and true otherwise.

The Truth Table for the Disjunction of two propositions		
p	q	p∨q
T	T	T
T	F	T
F	T	T
F	F	F

Symbols in Propositional Logic

- Each proposition is assigned a symbol
- ‘x is greater than y.’: p
- ‘Mr. Abid is teaching Formal Methods this term’: q
- ‘I won a gold medal in last years Sports gala.’: r

Activity: Modeling with Propositional Logic

1. If the train arrives late and there are no taxis at the station, then John is late for his meeting.
2. John is not late for his meeting.
3. The train did arrive late.
4. Therefore, there were taxis at the station.

Activity: Modeling with Propositional Logic

1. If the train arrives late (p) and there are no taxis at the station (q), then John is late for his meeting (r).
   - $(p \land (\neg q)) \rightarrow r$
2. John is not late for his meeting.
   - $\neg r$
3. The train did arrive late.
   - p
4. Therefore, there were taxis at the station.
   - Q

Activity

- If a request occurs, then either it will eventually be acknowledged, or the requesting process won't ever be able to make progress.

p : "A request occurs."

q : "The request will eventually be acknowledged."

r : "The requesting process will eventually make progress."

The formula representing the declarative sentence is then

$$p \to (q \vee \neg r) .$$