THE KARNAUGH MAP

Digital logic design

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Karnaugh Map

- Simplification of Boolean Expressions:

1. Boolean algebra rules
   - *Doesn't guarantee simplest form of expression*
   - *Terms are not obvious*
   - *Skills of applying rules and laws*
2. K-map provides a systematic method
   - *An array of cells*
   - *Used for simplifying 2, 3, 4 and 5 variable expressions*

Karnaugh Maps (K-map)

- A K-map is a collection of squares
  - *Each square represents a minterm*
  - *The collection of squares is a graphical representation of a Boolean function*
  - *Adjacent squares differ in the value of one variable*

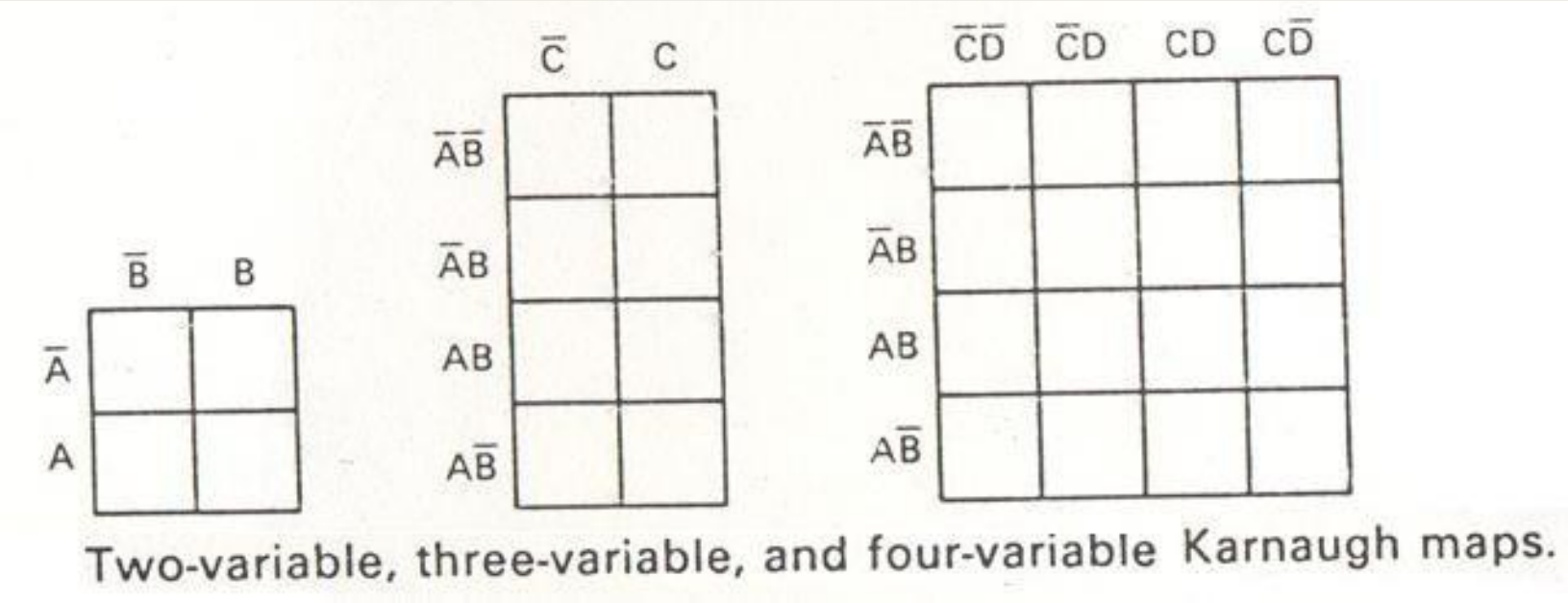
- The K-map can be viewed as
  - *A reorganized version of the truth table*

K-Map Format

- Each minterm in a truth table corresponds to a cell in the K-Map.
- K-Map cells are labeled so that both horizontal and vertical movement differ only in one variable.
- Once a K-Map is filled (0's & 1's) the SOP expression for the function can be obtained by OR-ing together the cells that contain 1's.
- Since the adjacent cells differ by only one variable, they can be grouped to create simpler terms in the sum-of-product expression.

The Karnaugh Map

- A two-variable map will require 4 cells. A three-variable map will require 8 cells and four-variable map will require 16 cells.
- The three different K-maps are shown:



Two-variable, three-variable, and four-variable Karnaugh maps.

K-Map Simplification of SOP Expressions

- Each group must contain either *1, 2, 4, 8 or 16* cells.
- Always include the largest possible number of 1s in a group in accordance with rule 1
- Each 1 on the map must be included in at least one group. The 1s already in a group can be included in another group as long as the overlapping groups include non-common 1s

Goal: **Maximize the size of the groups & Minimize the number of groups**

Two Variable Maps

- A 2-variable Karnaugh Map:
  - *Note that minterm m0 and minterm m1 are "adjacent" and differ in the value of the variable y*
  - *Similarly, minterm m0 and minterm m2 differ in the x variable.*
  - *Also, m1 and m3 differ in the x variable as well.*
  - *Finally, m2 and m3 differ in the value of the variable y*

	y = 0	y = 1
x = 0	$m_0 = \bar{x}\bar{y}$	$m_1 = \bar{x}y$
x = 1	$m_2 = x\bar{y}$	$m_3 = xy$

K-Map and Truth Tables

- The K-Map is just a different form of the truth table.
- Example – Two variable function:
  - *Simplification using Boolean algebra rules*
  - *F=X'Y'+X'Y+XY= X'+XY=(X'+Y) SOP form*
  - *Simplification using Boolean K-map*

Function Table

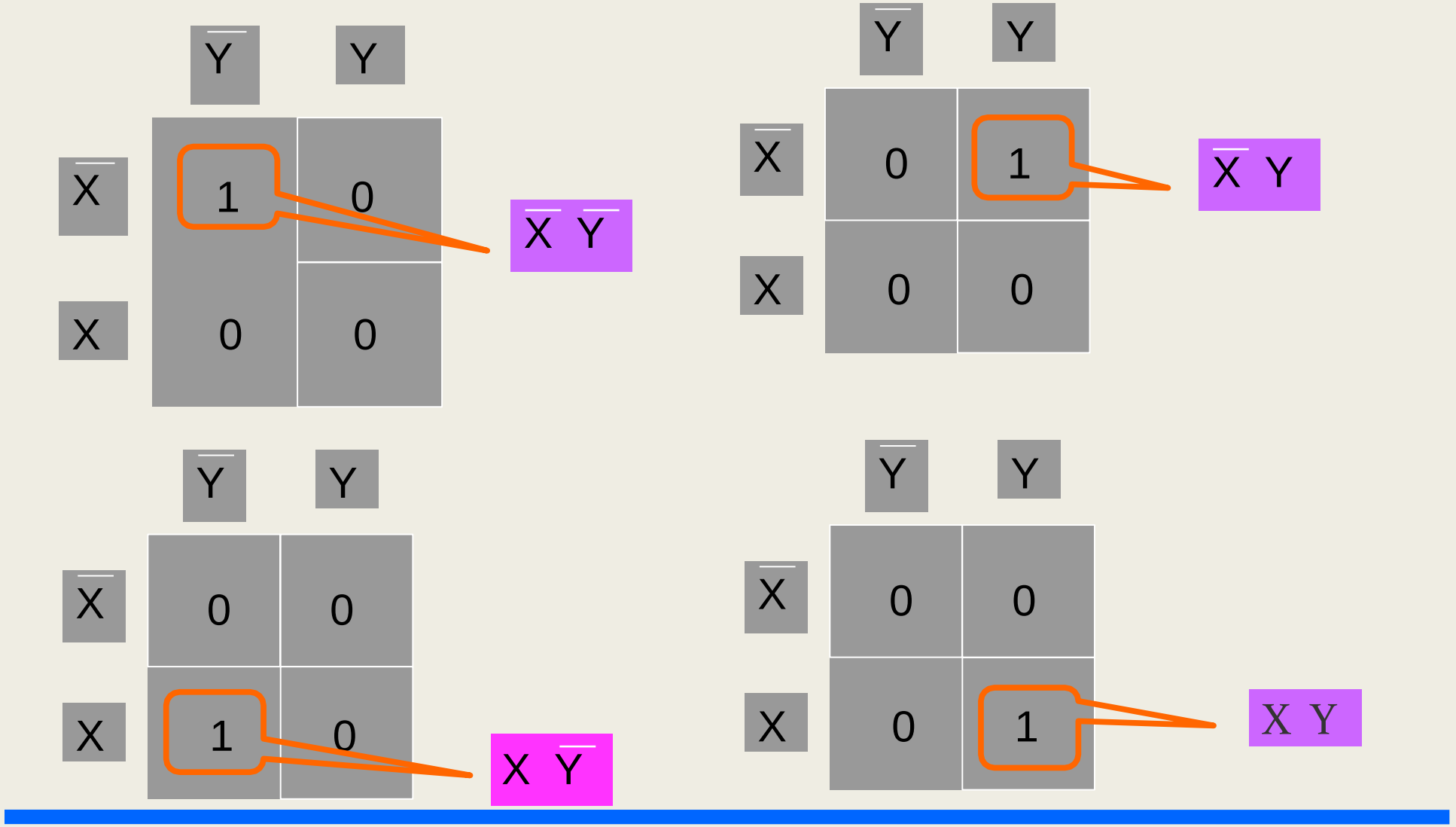
Input Values (x,y)	Function Value F(x,y)
0 0	1
0 1	1
1 0	0
1 1	1

K-Map

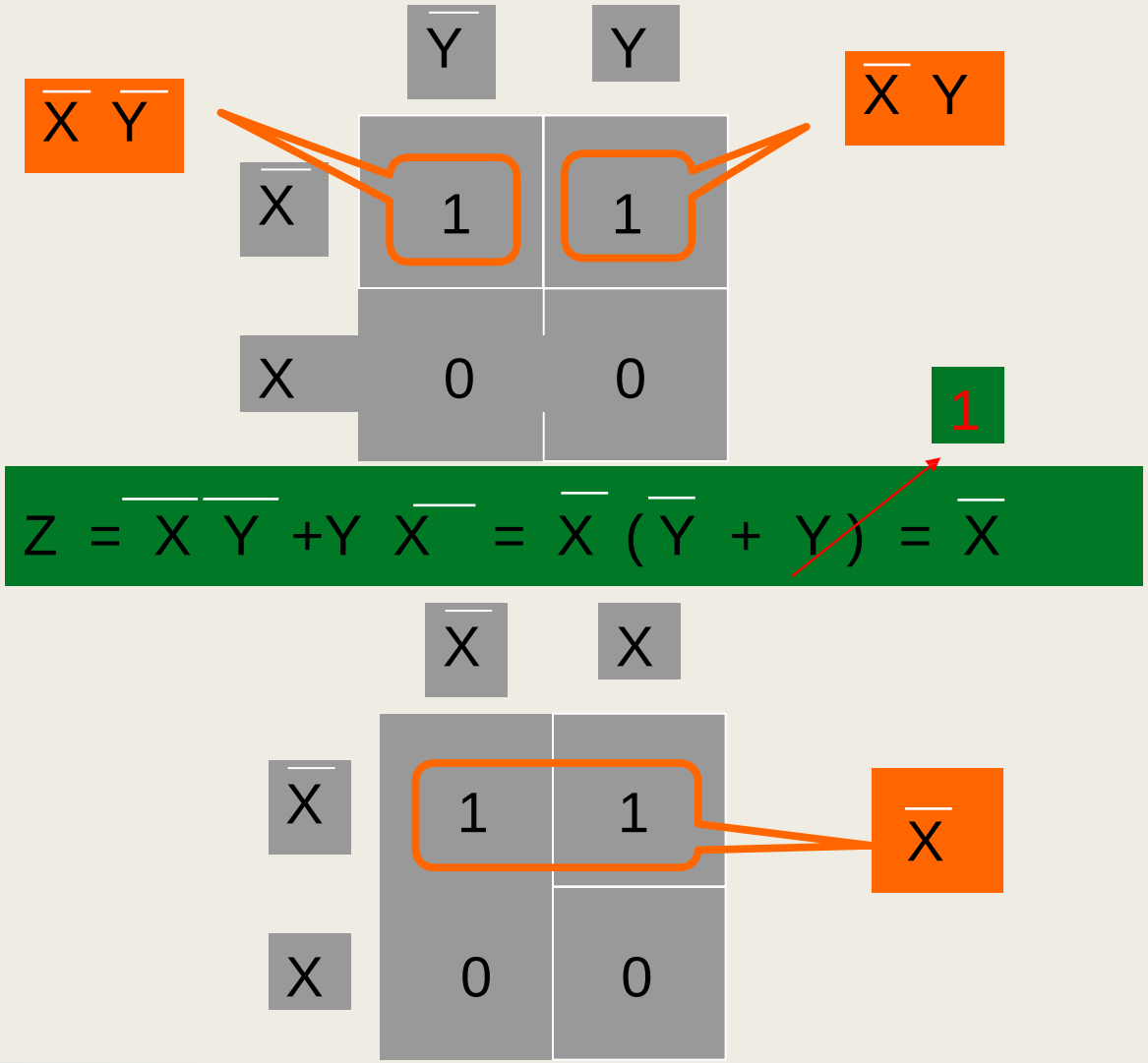
	y = 0	y = 1
x = 0	1	1
x = 1	0	1

F=x'+y

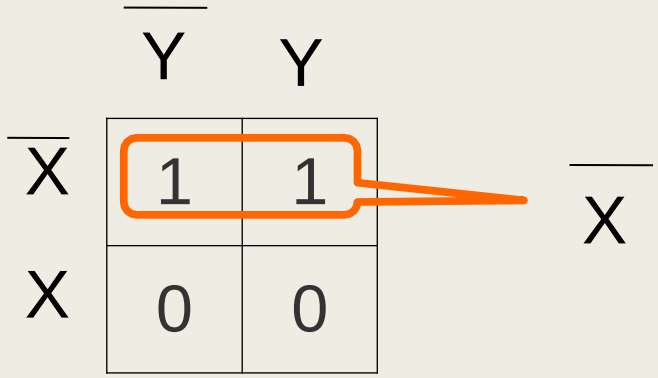
2 Variable K-Map : Groups of One

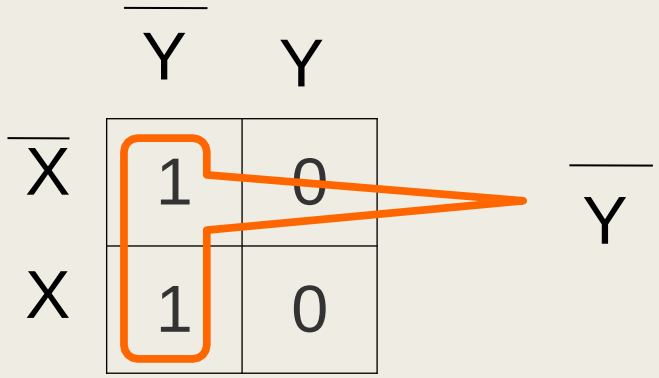


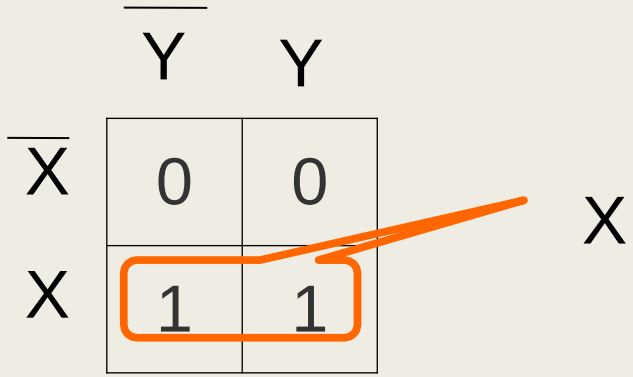
Adjacent Cells

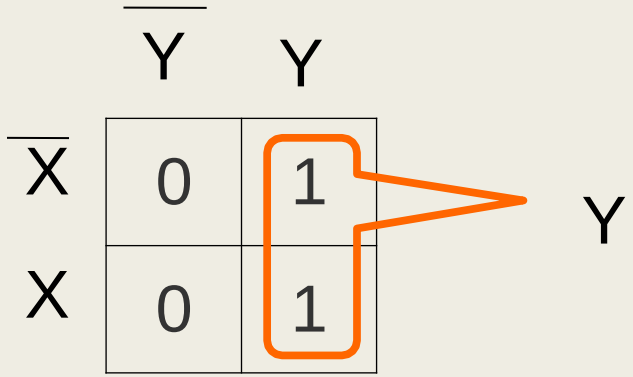


2 Variable K-Map : Groups of Two



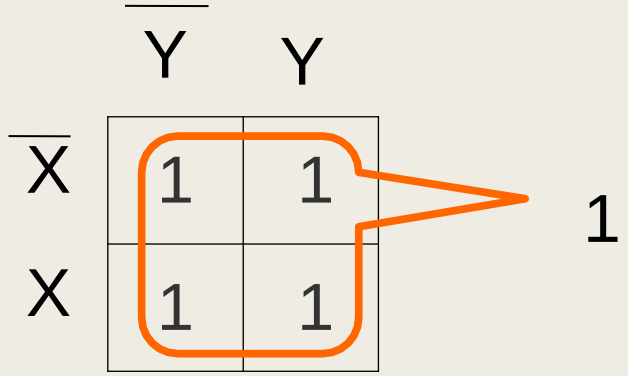






2 Variable K-Map : Groups of four

NOTE:There is no group of three



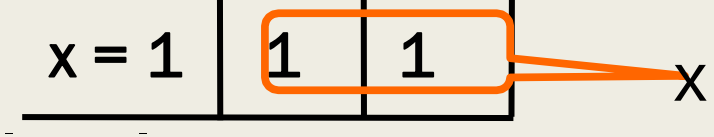
Example of 2 variable K-Map

Example:

- By using k-map:

F(x,Y) = x

F = x	Y= 0	Y = 1
x = 0	0	0
x = 1	1	1



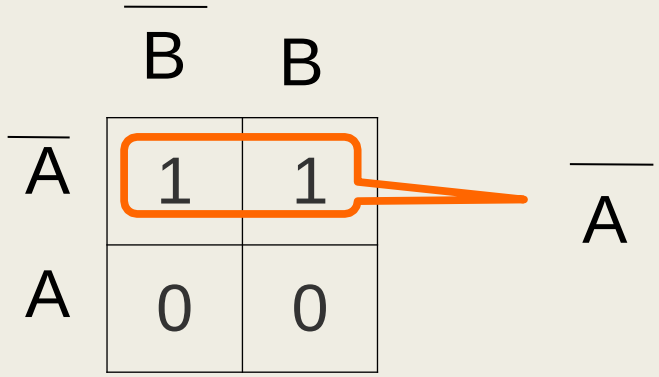
- By using Boolean algebra rules:

$$F(x,Y) = x\overline{Y} + xY = x$$

Example of 2 variable K-Map (SOP)

- Truth Table for a function F
- From truth table the SOP expression is F = A'B' + A'B=m0+m1
- Reducing the expression by using k-map technique, we get
- F =A'

A	B	F
0	0	1
0	1	1
1	0	0
1	1	0



Another Example of 2 variable K-Map

- Example: G(x,y) = x + y
- Truth table is as shown
- The Sum of minterm expression is

$$G(x,y) = x\,y' + x'\,y + x\,y$$

- The Sum of product term expression using k-map

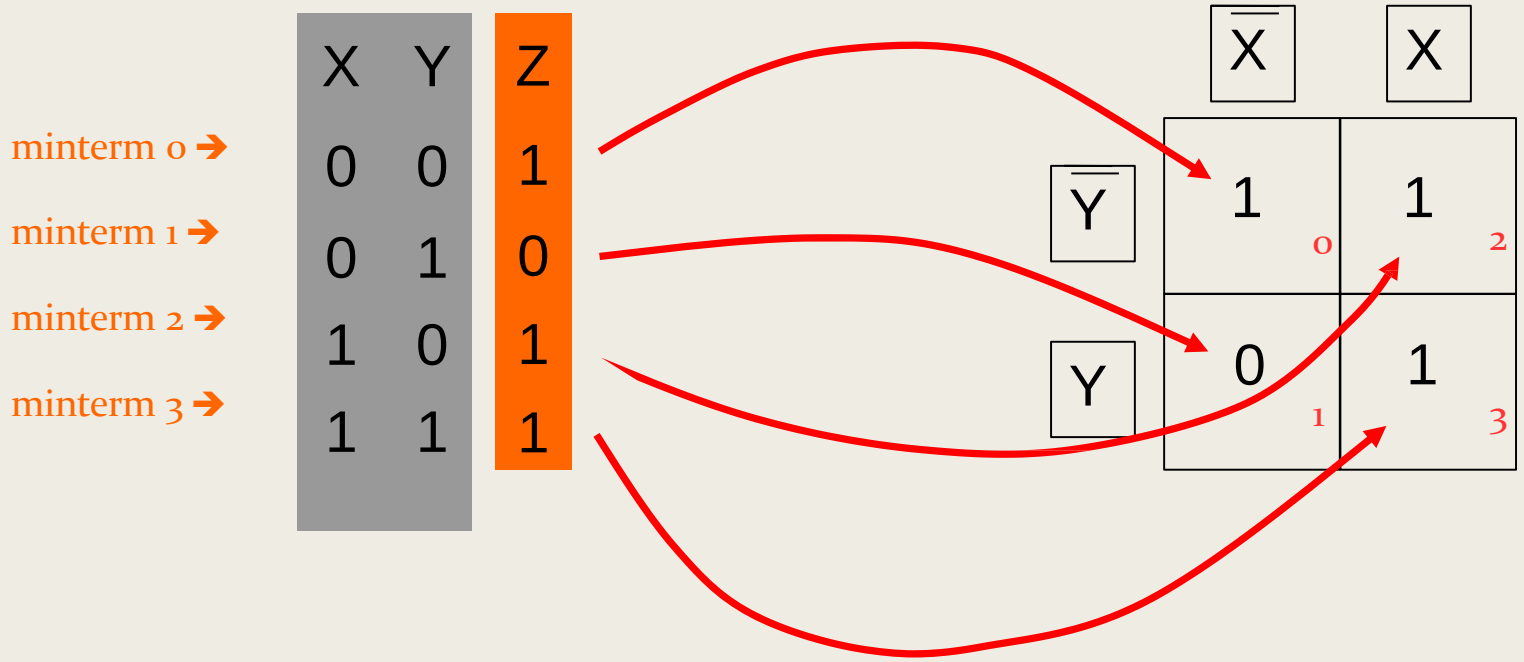
$$G(x,y) = x + y$$

x	y	G
0	0	0
0	1	1
1	0	1
1	1	1

For G(x,y), two pairs of adjacent cells containing 1's can be combined using the Minimization Theorem:

G = x+y	y = 0	y = 1
x = 0	0	1
x = 1	1	1

Truth Table -to- K-Map:If we change the order of the variable



Thanks