



FOUR-VARIABLE K-MAP

DLD



Four-Variable K-Map

- The map
 - 16 minterms
 - Combinations of 2, 4, 8, and 16 adjacent squares

| | | | |
|----------|----------|----------|----------|
| m_0 | m_1 | m_3 | m_2 |
| m_4 | m_5 | m_7 | m_6 |
| m_{12} | m_{13} | m_{15} | m_{14} |
| m_8 | m_9 | m_{11} | m_{10} |

(a)

| | | yz | | y | |
|----|----|------------|-----------|----------|-----------|
| | | 00 | 01 | 11 | 10 |
| wx | 00 | $w'x'y'z'$ | $w'x'y'z$ | $w'x'yz$ | $w'x'yz'$ |
| | 01 | $w'xy'z'$ | $w'xy'z$ | $w'xyz$ | $w'xyz'$ |
| | 11 | $wxy'z'$ | $wxy'z$ | $wxyz$ | $wxyz'$ |
| | 10 | $wx'y'z'$ | $wx'y'z$ | $wx'yz$ | $wx'yz'$ |
| | | z | | | |

(b)

Figure 3.8 Four-variable Map

Simplification of SOP expressions using K-map

■ 4-variable K-map

- *1 cell group yields a 4 variable product term*
- *2 cell group yields a 3 variable product term*
- *4 cell group yields a 2 variable product term*
- *8 cell group yields a 1 variable product term*
- *16 cell group yields a value of 1 for function*

K-Map Examples

Four Variable K-Map: Example-1

| | A | B | C | D | Z |
|--------------|---|---|---|---|---|
| minterm 0 → | 0 | 0 | 0 | 0 | 1 |
| minterm 1 → | 0 | 0 | 0 | 1 | 0 |
| minterm 2 → | 0 | 0 | 1 | 0 | 1 |
| minterm 3 → | 0 | 0 | 1 | 1 | 0 |
| minterm 4 → | 0 | 1 | 0 | 0 | 1 |
| minterm 5 → | 0 | 1 | 0 | 1 | 0 |
| minterm 6 → | 0 | 1 | 1 | 0 | 1 |
| minterm 7 → | 0 | 1 | 1 | 1 | 0 |
| minterm 8 → | 1 | 0 | 0 | 0 | 0 |
| minterm 9 → | 1 | 0 | 0 | 1 | 0 |
| minterm 10 → | 1 | 0 | 1 | 0 | 1 |
| minterm 11 → | 1 | 0 | 1 | 1 | 0 |
| minterm 12 → | 1 | 1 | 0 | 0 | 1 |
| minterm 13 → | 1 | 1 | 0 | 1 | 1 |
| minterm 14 → | 1 | 1 | 1 | 0 | 0 |
| minterm 15 → | 1 | 1 | 1 | 1 | 0 |

Truth table is given and you have to drive the expression and simplify it by using k-map method

Four Variable K-Map : Example-1

Drive the expression from truth table

| | A | B | C | D | Z |
|--------------|---|---|---|---|---|
| minterm 0 → | 0 | 0 | 0 | 0 | 1 |
| minterm 1 → | 0 | 0 | 0 | 1 | 0 |
| minterm 2 → | 0 | 0 | 1 | 0 | 1 |
| minterm 3 → | 0 | 0 | 1 | 1 | 0 |
| minterm 4 → | 0 | 1 | 0 | 0 | 1 |
| minterm 5 → | 0 | 1 | 0 | 1 | 0 |
| minterm 6 → | 0 | 1 | 1 | 0 | 1 |
| minterm 7 → | 0 | 1 | 1 | 1 | 0 |
| minterm 8 → | 1 | 0 | 0 | 0 | 0 |
| minterm 9 → | 1 | 0 | 0 | 1 | 0 |
| minterm 10 → | 1 | 0 | 1 | 0 | 1 |
| minterm 11 → | 1 | 0 | 1 | 1 | 0 |
| minterm 12 → | 1 | 1 | 0 | 0 | 1 |
| minterm 13 → | 1 | 1 | 0 | 1 | 1 |
| minterm 14 → | 1 | 1 | 1 | 0 | 0 |
| minterm 15 → | 1 | 1 | 1 | 1 | 0 |

$$F = m_0 + m_2 + m_4 + m_6 + m_{10} + m_{12} + m_{13}$$

$$F = A'B'C'D' + A'B'CD' + A'BC'D' + A'BCD' + AB'CD' + ABC'D' + ABCD'$$

| | | y | | | |
|----|----|------------|-----------|----------|-----------|
| | | yz | | 11 | 10 |
| wx | 00 | $w'x'y'z'$ | $w'x'y'z$ | $w'x'yz$ | $w'x'yz'$ |
| | 01 | $w'xy'z'$ | $w'xy'z$ | $w'xyz$ | $w'xyz'$ |
| | 11 | $wxy'z'$ | $wxy'z$ | $wxyz$ | $wxyz'$ |
| | 10 | $wx'y'z'$ | $wx'y'z$ | $wx'yz$ | $wx'yz'$ |

Four Variable K-Map : Example-1

Simplification by using k-map method

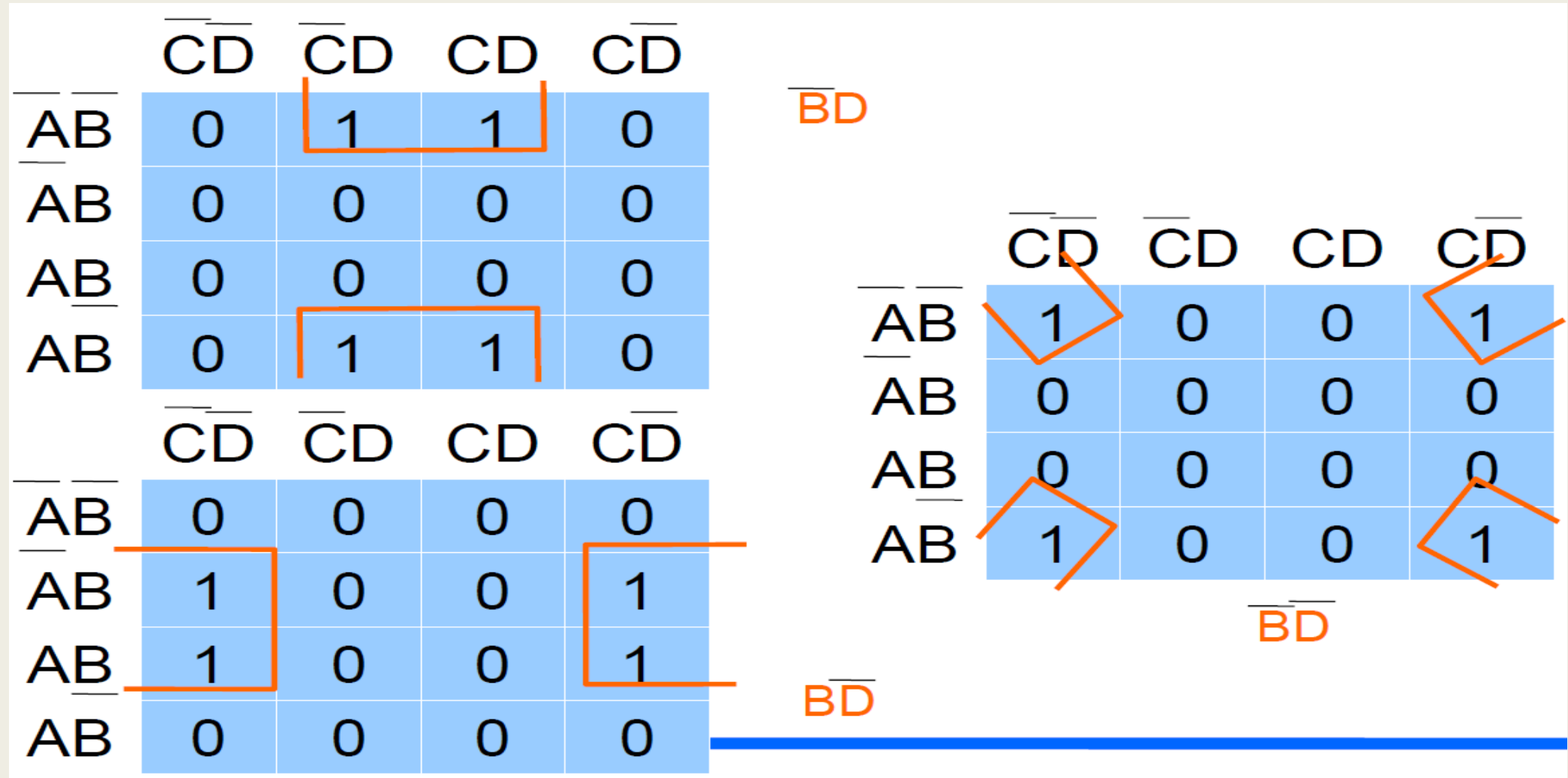
$$F = A'B'C'D' + A'B'CD' + A'BC'D' + A'BCD' + AB'CD' + ABC'D' + ABCD'$$

| | A | B | C | D | Z |
|--------------|---|---|---|---|---|
| minterm 0 → | 0 | 0 | 0 | 0 | 1 |
| minterm 1 → | 0 | 0 | 0 | 1 | 0 |
| minterm 2 → | 0 | 0 | 1 | 0 | 1 |
| minterm 3 → | 0 | 0 | 1 | 1 | 0 |
| minterm 4 → | 0 | 1 | 0 | 0 | 1 |
| minterm 5 → | 0 | 1 | 0 | 1 | 0 |
| minterm 6 → | 0 | 1 | 1 | 0 | 1 |
| minterm 7 → | 0 | 1 | 1 | 1 | 0 |
| minterm 8 → | 1 | 0 | 0 | 0 | 0 |
| minterm 9 → | 1 | 0 | 0 | 1 | 0 |
| minterm 10 → | 1 | 0 | 1 | 0 | 1 |
| minterm 11 → | 1 | 0 | 1 | 1 | 0 |
| minterm 12 → | 1 | 1 | 0 | 0 | 1 |
| minterm 13 → | 1 | 1 | 0 | 1 | 1 |
| minterm 14 → | 1 | 1 | 1 | 0 | 0 |
| minterm 15 → | 1 | 1 | 1 | 1 | 0 |

| | $\overline{C}\overline{D}$ | $\overline{C}D$ | CD | $C\overline{D}$ |
|----------------------------|----------------------------|-----------------|-----------------|-----------------|
| $\overline{A}\overline{B}$ | 1 ₀ | 0 ₁ | 0 ₃ | 1 ₂ |
| $\overline{A}B$ | 1 ₄ | 0 ₅ | 0 ₇ | 1 ₆ |
| $A\overline{B}$ | 1 ₁₂ | 1 ₁₃ | 0 ₁₅ | 0 ₁₄ |
| AB | 0 ₈ | 0 ₉ | 0 ₁₁ | 1 ₁₀ |

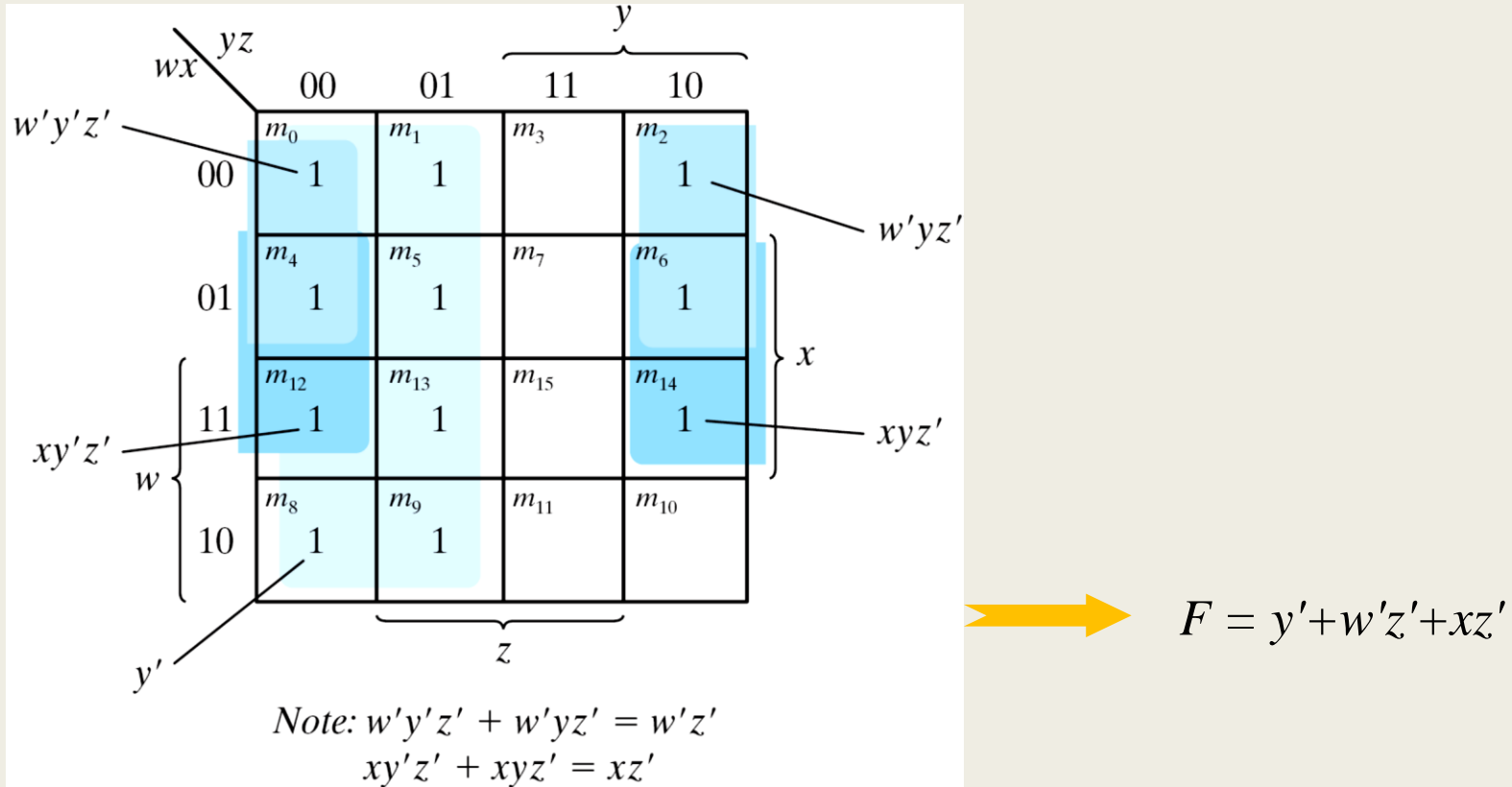
$$F = A'D' + ABC' + B'CD'$$

Four Variable K-Map



Example 2

- Example 3.5: simplify $F(w, x, y, z) = \Sigma(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$



$$F(w, x, y, z) = \Sigma(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14) = y' + w'z' + xz'$$

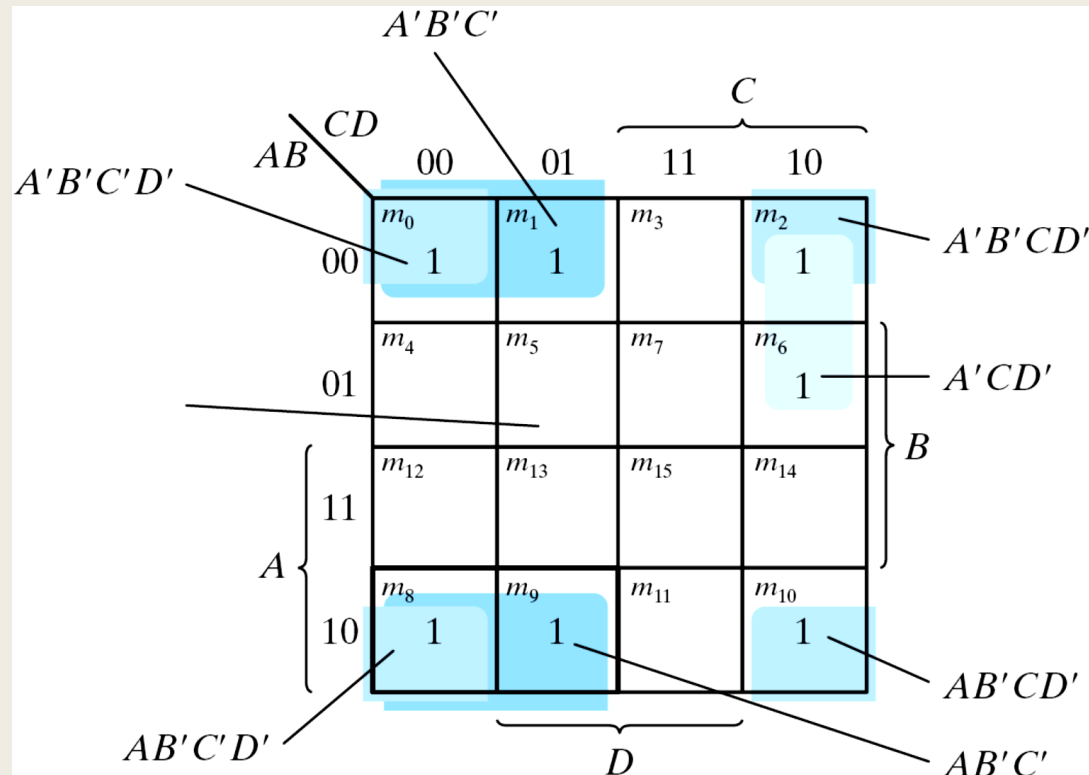
Example 3

$$\begin{matrix} A'B'C'D \\ A'B'C'D' \end{matrix}$$

- Example 3-6: simplify $F = A'B'C' + B'CD' + A'B'CD' + AB'C'$

$$F = A'B'C'(D+D') + B'CD'(A+A') + A'B'CD' + AB'C'(D+D')$$

$$F = A'B'C'D + A'B'C'D' + AB'CD' + A'B'CD' + A'B'CD' + AB'C'D + AB'C'D'$$



Note: $A'B'C'D' + A'B'CD' = A'B'D'$
 $AB'C'D' + AB'CD' = AB'D'$
 $A'B'D' + AB'D' = B'D'$
 $A'B'C' + AB'C' = B'C'$

$$F = A'B'C' + B'CD' + A'B'CD' + AB'C' = B'D' + B'C' + A'CD'$$

Four Variable K-Map: Example 4

| | A | B | C | D | Z |
|--------------|---|---|---|---|---|
| minterm 0 → | 0 | 0 | 0 | 0 | 1 |
| minterm 1 → | 0 | 0 | 0 | 1 | 0 |
| minterm 2 → | 0 | 0 | 1 | 0 | 0 |
| minterm 3 → | 0 | 0 | 1 | 1 | 0 |
| minterm 4 → | 0 | 1 | 0 | 0 | 1 |
| minterm 5 → | 0 | 1 | 0 | 1 | 1 |
| minterm 6 → | 0 | 1 | 1 | 0 | 0 |
| minterm 7 → | 0 | 1 | 1 | 1 | 1 |
| minterm 8 → | 1 | 0 | 0 | 0 | 1 |
| minterm 9 → | 1 | 0 | 0 | 1 | 1 |
| minterm 10 → | 1 | 0 | 1 | 0 | 0 |
| minterm 11 → | 1 | 0 | 1 | 1 | 0 |
| minterm 12 → | 1 | 1 | 0 | 0 | 0 |
| minterm 13 → | 1 | 1 | 0 | 1 | 1 |
| minterm 14 → | 1 | 1 | 1 | 0 | 1 |
| minterm 15 → | 1 | 1 | 1 | 1 | 1 |

| | $\overline{C}\overline{D}$ | $\overline{C}D$ | CD | $C\overline{D}$ |
|----------------------------|----------------------------|-----------------|-----------------|-----------------|
| $\overline{A}\overline{B}$ | 1 ₀ | 0 ₁ | 0 ₃ | 0 ₂ |
| $\overline{A}B$ | 1 ₄ | 1 ₅ | 0 ₇ | 1 ₆ |
| $A\overline{B}$ | 0 ₁₂ | 1 ₁₃ | 1 ₁₅ | 1 ₁₄ |
| AB | 1 ₈ | 1 ₉ | 0 ₁₁ | 0 ₁₀ |

Four Variable K-Map: Example 4

| | A | B | C | D | Z |
|--------------|---|---|---|---|---|
| minterm 0 → | 0 | 0 | 0 | 0 | 1 |
| minterm 1 → | 0 | 0 | 0 | 1 | 0 |
| minterm 2 → | 0 | 0 | 1 | 0 | 0 |
| minterm 3 → | 0 | 0 | 1 | 1 | 0 |
| minterm 4 → | 0 | 1 | 0 | 0 | 1 |
| minterm 5 → | 0 | 1 | 0 | 1 | 1 |
| minterm 6 → | 0 | 1 | 1 | 0 | 0 |
| minterm 7 → | 0 | 1 | 1 | 1 | 1 |
| minterm 8 → | 1 | 0 | 0 | 0 | 1 |
| minterm 9 → | 1 | 0 | 0 | 1 | 1 |
| minterm 10 → | 1 | 0 | 1 | 0 | 0 |
| minterm 11 → | 1 | 0 | 1 | 1 | 0 |
| minterm 12 → | 1 | 1 | 0 | 0 | 0 |
| minterm 13 → | 1 | 1 | 0 | 1 | 1 |
| minterm 14 → | 1 | 1 | 1 | 0 | 1 |
| minterm 15 → | 1 | 1 | 1 | 1 | 1 |

| | $\overline{C}\overline{D}$ | $\overline{C}D$ | CD | $C\overline{D}$ |
|----------------------------|----------------------------|-----------------|-----------------|-----------------|
| $\overline{A}\overline{B}$ | 1 ₀ | 0 ₁ | 0 ₃ | 0 ₂ |
| $\overline{A}B$ | 1 ₄ | 1 ₅ | 0 ₇ | 1 ₆ |
| $A\overline{B}$ | 0 ₁₂ | 1 ₁₃ | 1 ₁₅ | 1 ₁₄ |
| AB | 1 ₈ | 1 ₉ | 0 ₁₁ | 0 ₁₀ |

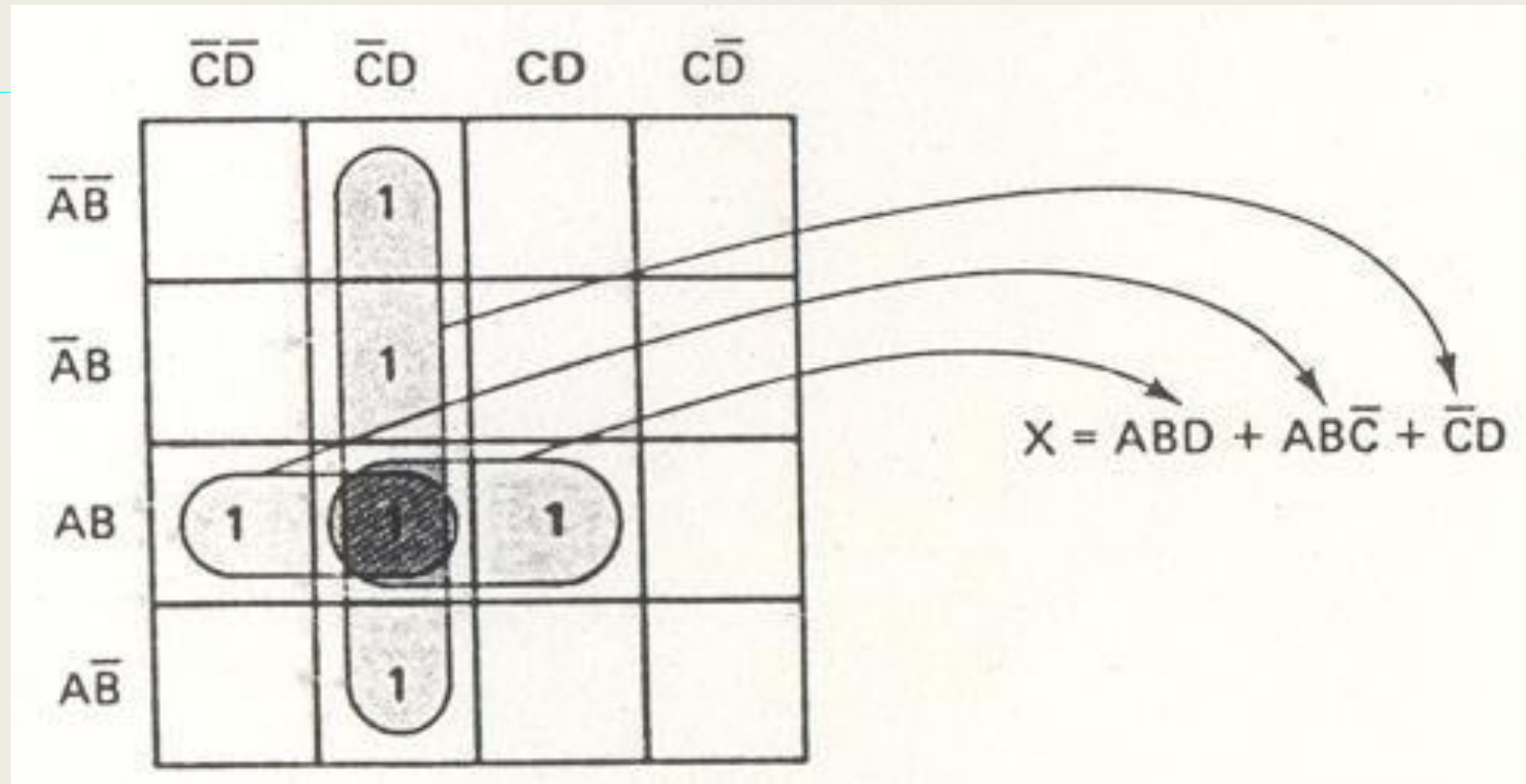
$$F = AB'D' + A'C'D' + BCD' + BC'D + ABD$$

$$F = AB'D' + A'C'D' + BCD' + BC'D + ABC$$

Example 5:k-map

Simplify the following equation using the Karnaugh mapping procedure:

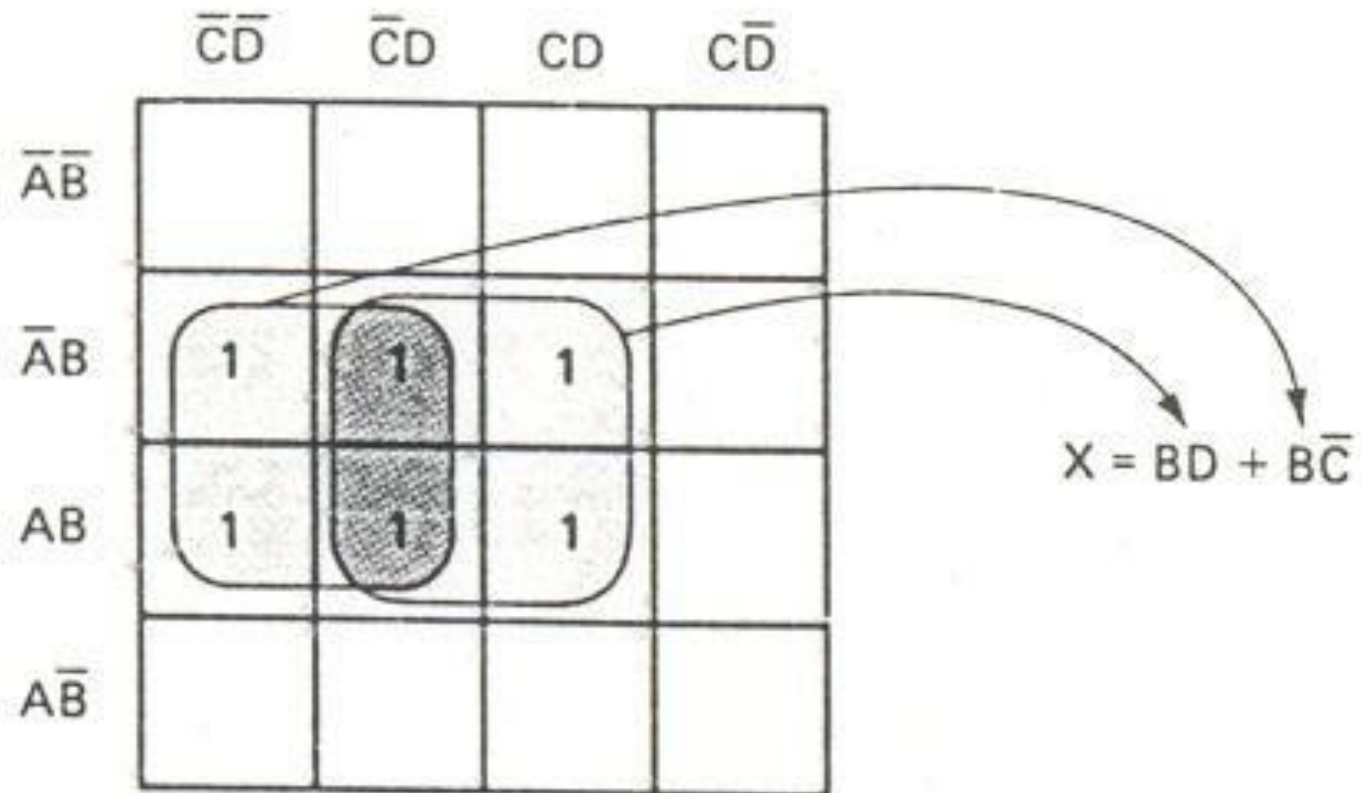
$$X = \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}CD + A\bar{B}\bar{C}D + A\bar{B}C\bar{D} + ABCD$$



Example 6:k-map

Simplify the following equation using the Karnaugh mapping procedure:

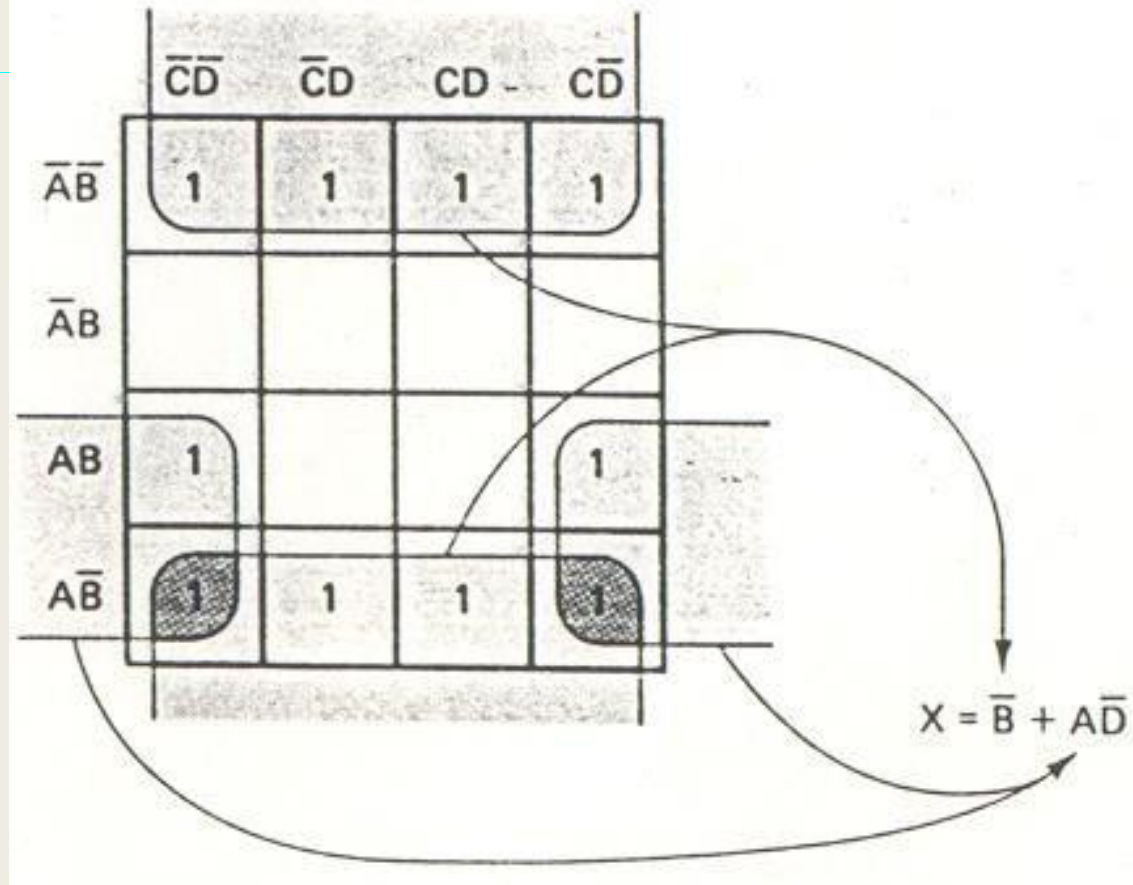
$$X = B\bar{C}\bar{D} + \bar{A}B\bar{C}D + AB\bar{C}D + \bar{A}BCD + ABCD$$



Example 7:K-Map Wraparound feature

Simplify the following equation using the Karnaugh mapping procedure:

$$X = \overline{A}\overline{B}\overline{C} + A\overline{C}\overline{D} + A\overline{B} + ABC\overline{D} + \overline{A}\overline{B}C$$



Example 8:k-map

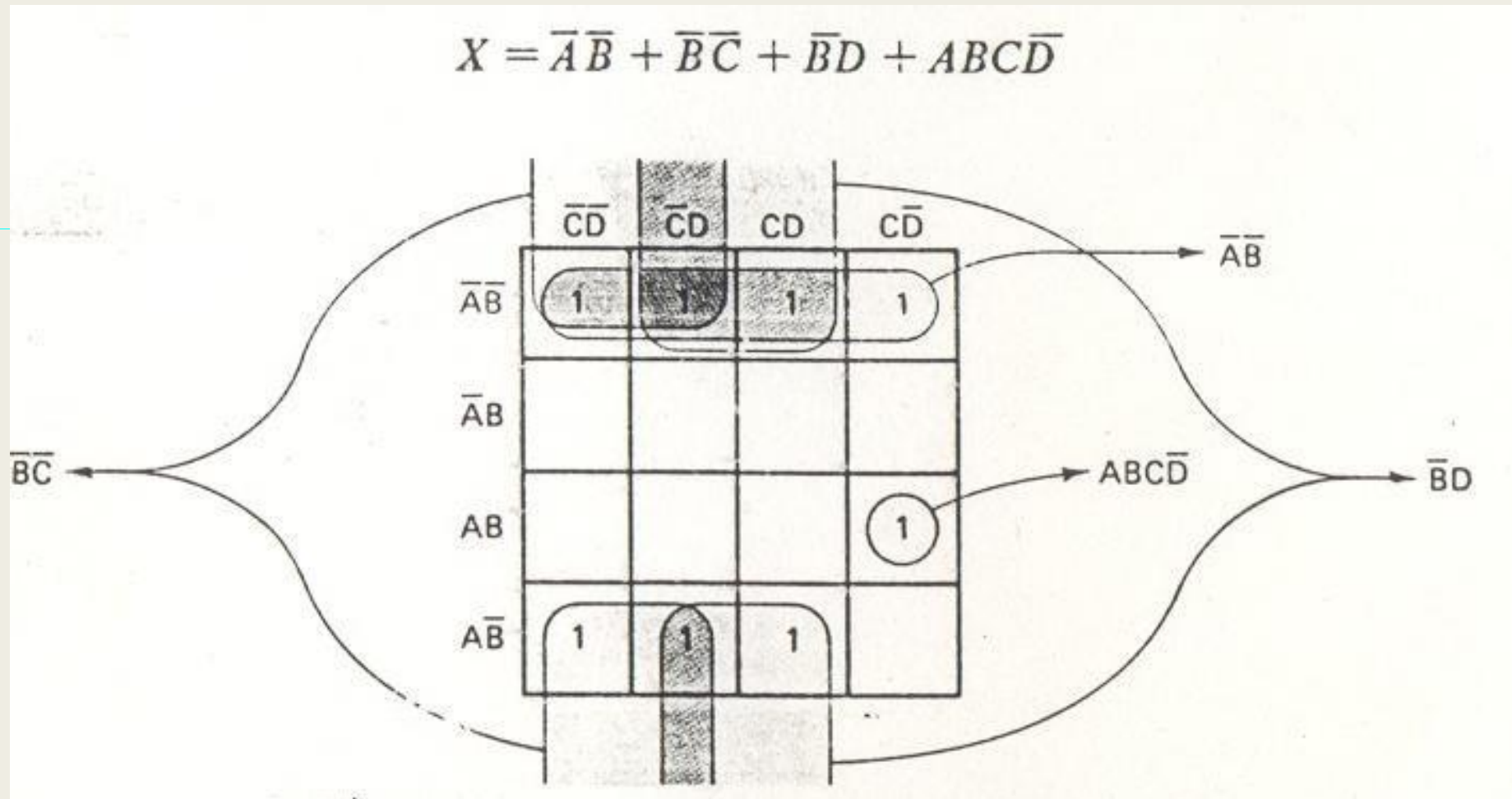
Simplify the following equation using the Karnaugh mapping procedure:

$$X = \bar{B}(CD + \bar{C}) + C\bar{D}(\bar{A} + \bar{B} + AB)$$

Solution: Before filling in the K-map, an SOP expression must be formed:

$$\begin{aligned} X &= \bar{B}CD + \bar{B}\bar{C} + C\bar{D}(\bar{A}\bar{B} + AB) \\ &= \bar{B}CD + \bar{B}\bar{C} + \bar{A}\bar{B}C\bar{D} + ABC\bar{D} \end{aligned}$$

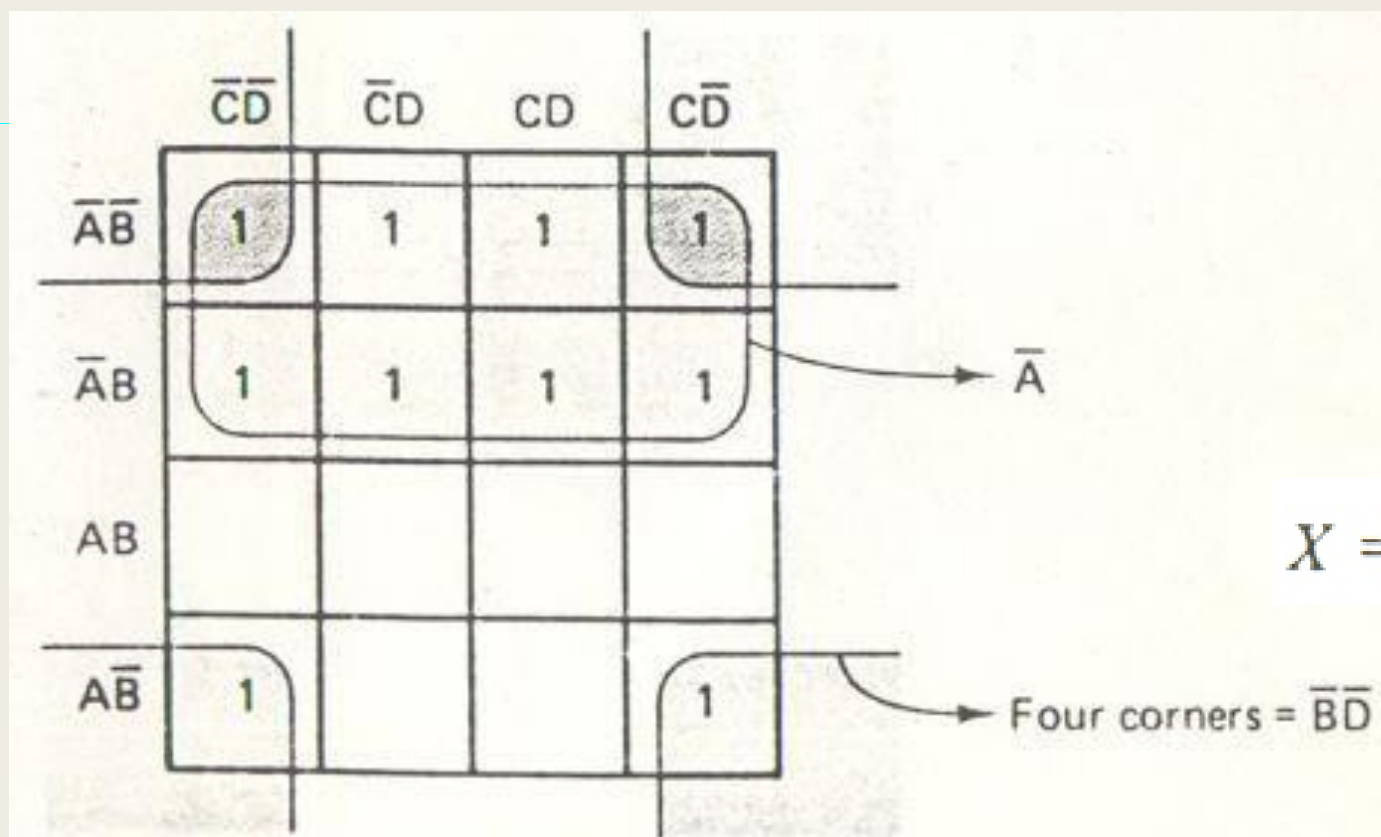
Example 9:k-map



Example 10:k-map

Simplify the following equation using the Karnaugh mapping procedure:

$$X = \bar{A}\bar{D} + A\bar{B}\bar{D} + \bar{A}\bar{C}D + \bar{A}CD$$

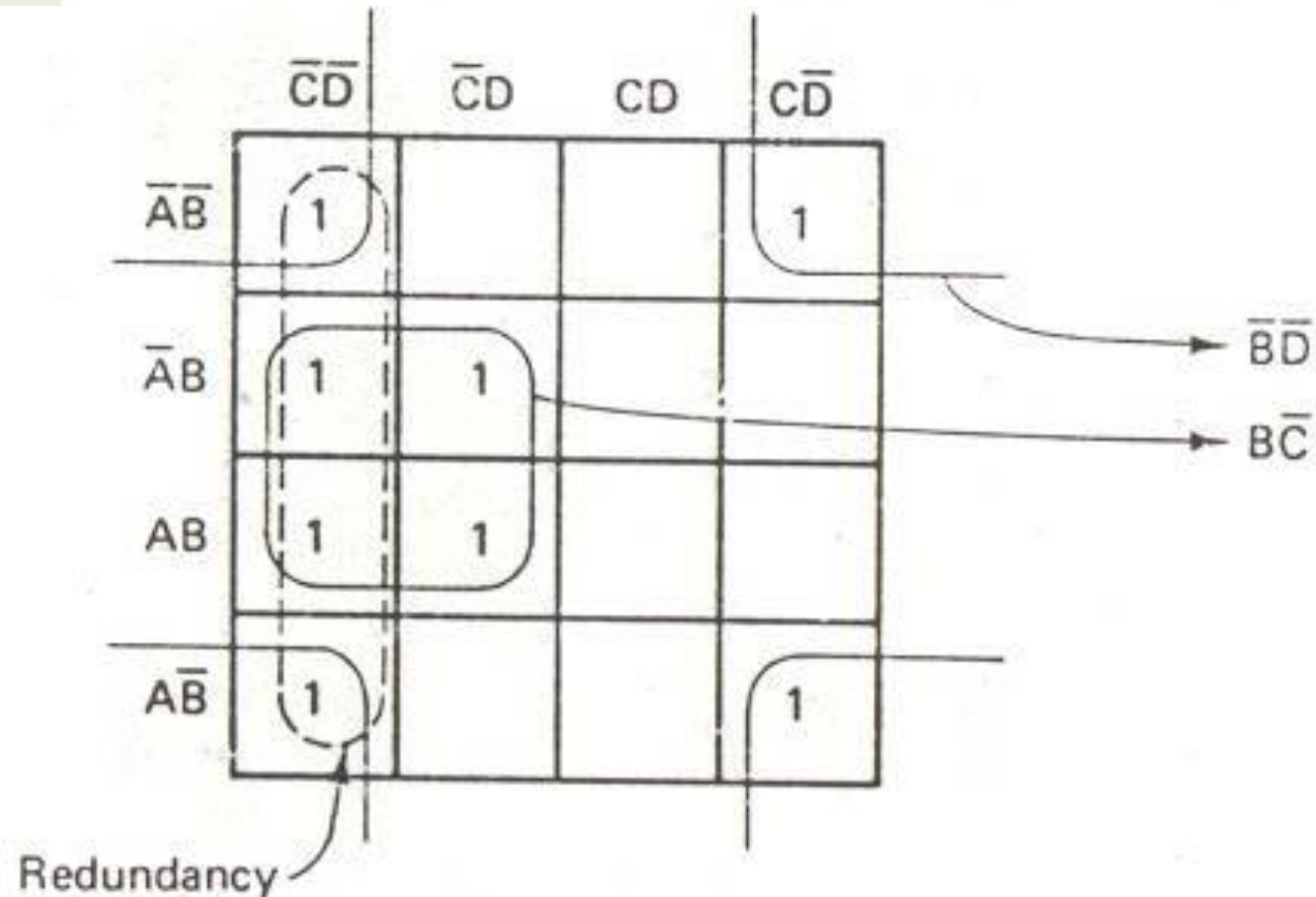


$$X = \bar{A} + \bar{B}\bar{D}$$

Example 11:k-map

Simplify the following equation using the Karnaugh mapping procedure:

$$X = \bar{A}\bar{B}\bar{D} + A\bar{C}\bar{D} + \bar{A}B\bar{C} + AB\bar{C}D + A\bar{B}C\bar{D}$$

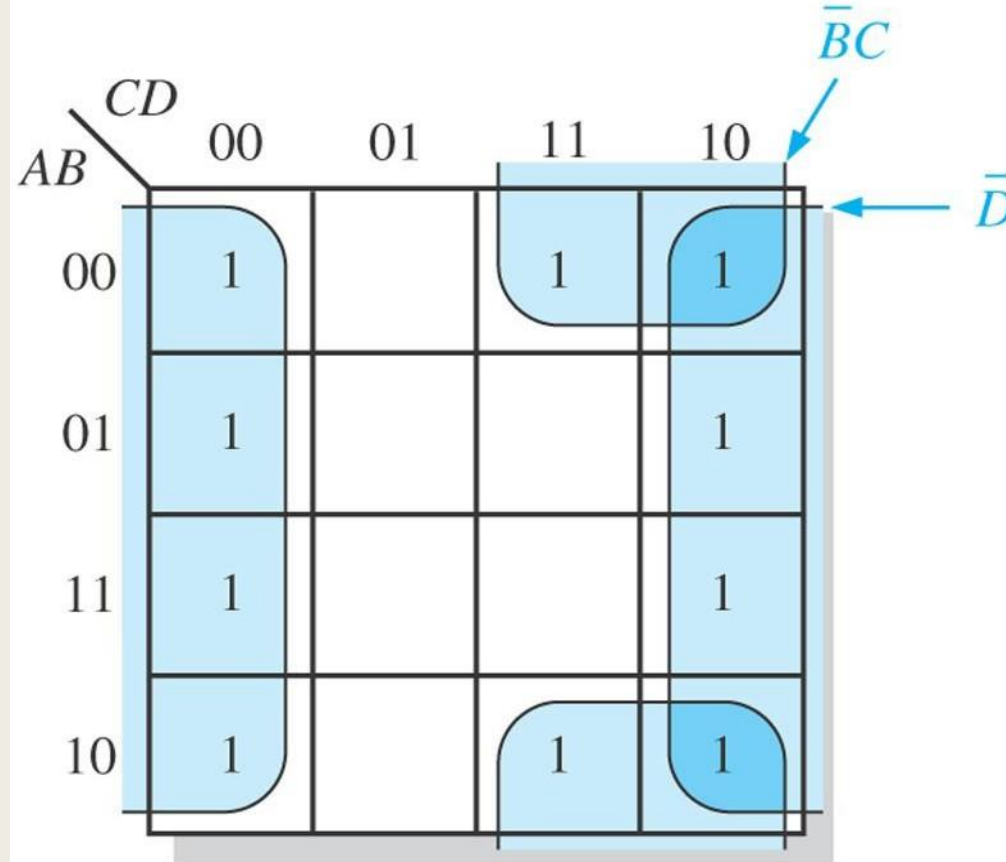


$$X = \bar{B}\bar{D} + B\bar{C}$$

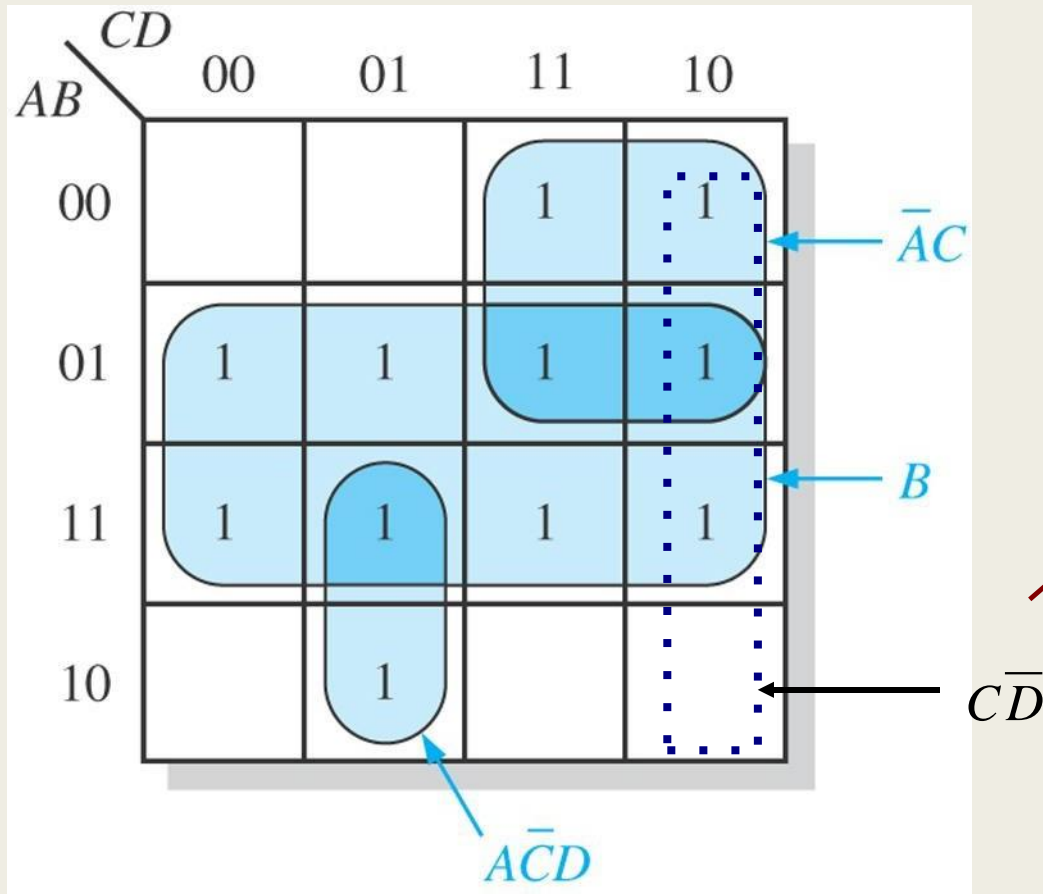
Example 12: Use a K-Map to minimize the following standard SOP expression

$$\overline{B}\overline{C}\overline{D} + \overline{A}\overline{B}\overline{C}\overline{D} + A\overline{B}\overline{C}\overline{D} + \overline{A}\overline{B}C\overline{B}D + A\overline{B}C\overline{D} + \overline{A}\overline{B}C\overline{D} + \overline{A}B\overline{C}\overline{D} + A\overline{B}C\overline{D} + A\overline{B}C\overline{D}$$

$$\overline{D} + \overline{B}C$$

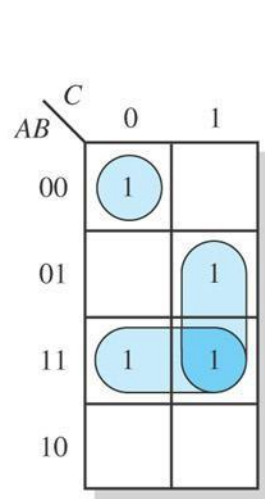


Example 13: Find the product term for the K- Map & write the minimum SOP form

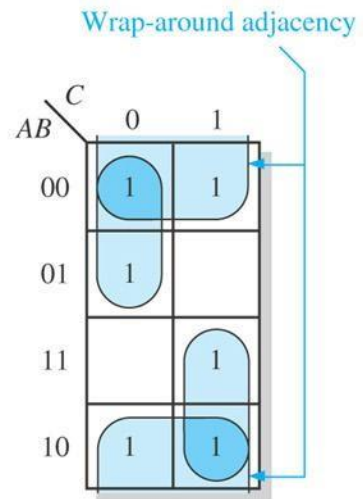


$$B + \bar{A}C + A\bar{C}D$$

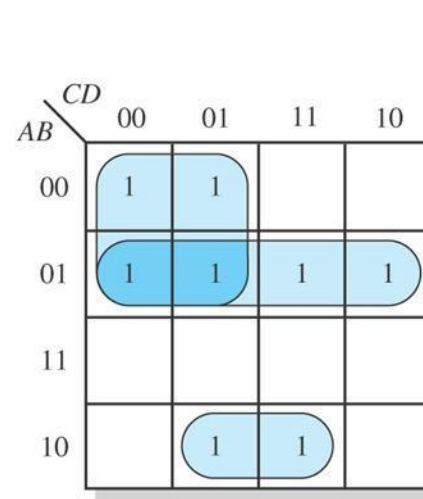
$$B + \bar{A}C + A\bar{C}D + C\bar{D}$$



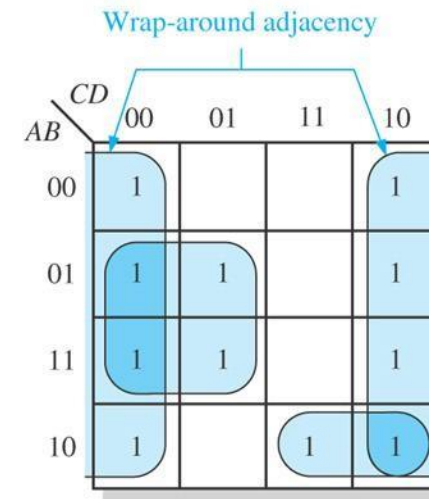
(a)



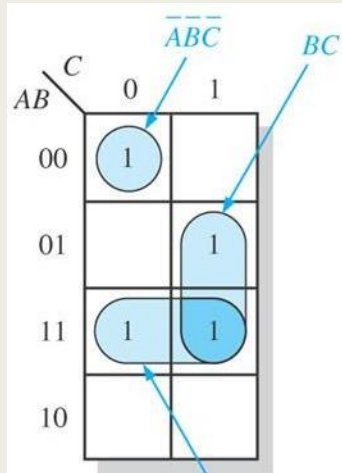
(b)



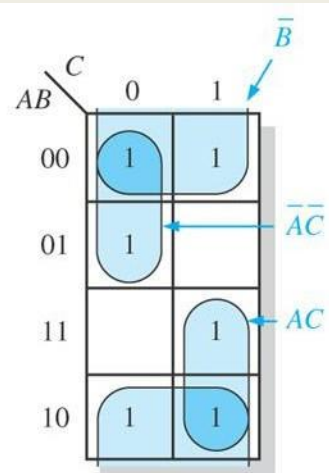
(c)



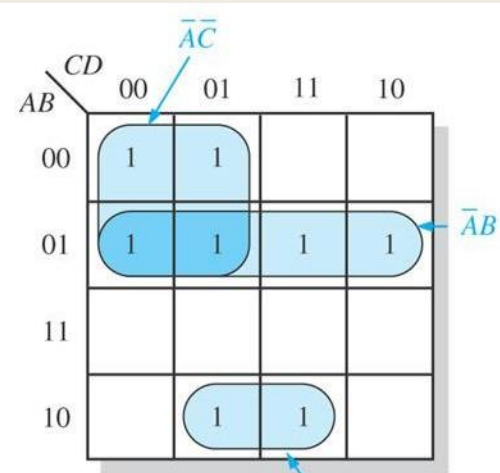
(d)



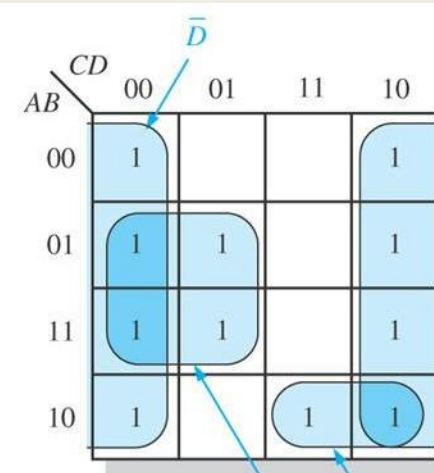
(a)



(b)



(c)



(d)



K-MAP SIMPLIFICATION IN POS FORM

Digital logic design



Product of Sums Simplification

- *Simplified F' in the form of sum of products*
- *Apply DeMorgan's theorem $F = (F')'$*
- *F' : sum of products $\rightarrow F$: product of sums*

Example 1:

- Example-simplify $F = \Sigma(0, 1, 2, 5, 8, 9, 10)$ into (a) sum-of-products form, and (b) product-of-sums form:

| AB \ CD | $\bar{C}\bar{D}$ | $\bar{C}D$ | $C\bar{D}$ | CD |
|------------------|--------------------------------|--------------------------|--------------------------|--------------------|
| $\bar{A}\bar{B}$ | $\bar{A}\bar{B}\bar{C}\bar{D}$ | $\bar{A}\bar{B}\bar{C}D$ | $\bar{A}\bar{B}C\bar{D}$ | $\bar{A}\bar{B}CD$ |
| $\bar{A}B$ | $\bar{A}B\bar{C}\bar{D}$ | $\bar{A}B\bar{C}D$ | $\bar{A}B C\bar{D}$ | $\bar{A}B CD$ |
| $A\bar{B}$ | $A\bar{B}\bar{C}\bar{D}$ | $A\bar{B}\bar{C}D$ | $A\bar{B}C\bar{D}$ | $A\bar{B}CD$ |
| AB | $AB\bar{C}\bar{D}$ | $AB\bar{C}D$ | $AB C\bar{D}$ | $AB CD$ |

| AB \ CD | 00 | 01 | 11 | 10 |
|---------|----------|----------|----------|----------|
| 00 | m_0 | m_1 | m_3 | m_2 |
| 01 | m_4 | m_5 | m_7 | m_6 |
| 11 | m_{12} | m_{13} | m_{15} | m_{14} |
| 10 | m_8 | m_9 | m_{11} | m_{10} |

a) Combine the 1's:

$$F(A, B, C, D) = \Sigma(0, 1, 2, 5, 8, 9, 10) = B'D' + B'C' + A'C'D$$

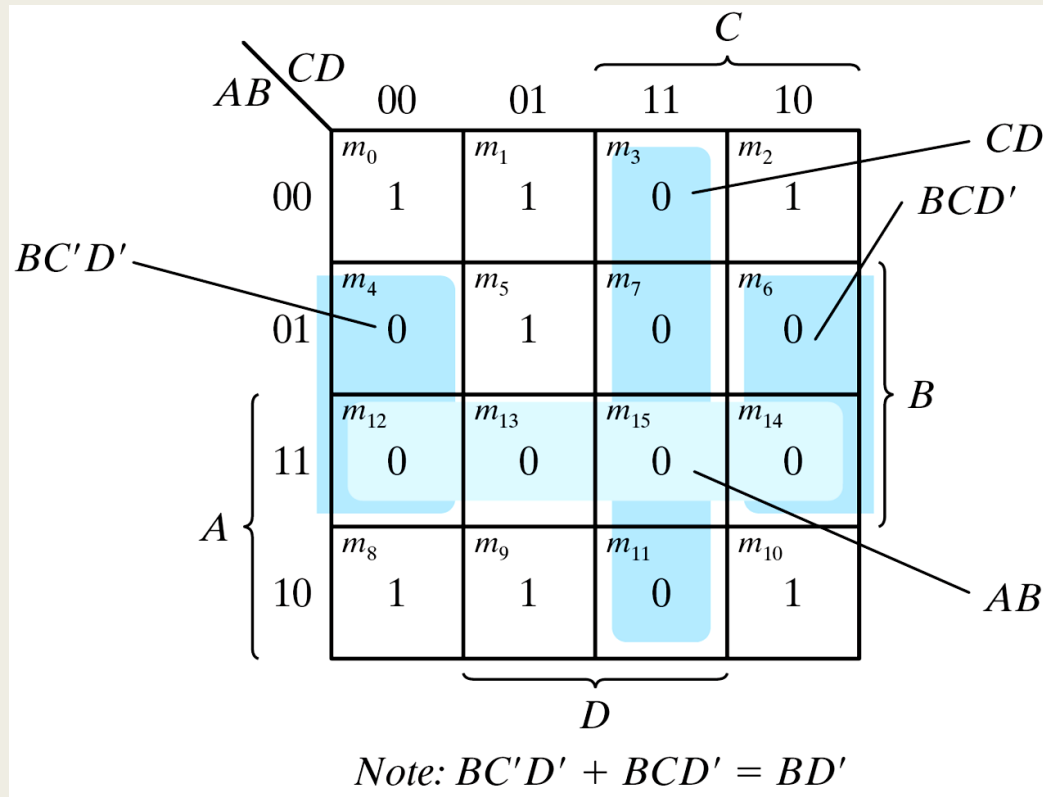
b) Combine the 0's:

$$F' = AB + CD + BD' \quad \text{Apply DeMorgan's theorem;}$$

$$F = (A' + B')(C' + D')(B' + D) \quad \text{Or think in terms of maxterms}$$

Example 1: Continue..

- ▣ Example-simplify $F = \Sigma(0, 1, 2, 5, 8, 9, 10)$ into (a) sum-of-products form, and (b) product-of-sums form:



a) Combine the 1's:

$$F(A, B, C, D) = \Sigma(0, 1, 2, 5, 8, 9, 10) = B'D' + B'C' + A'C'D$$

b) Combine the 0's:

$$F' = AB + CD + BD' \quad \text{Apply DeMorgan's theorem;}$$

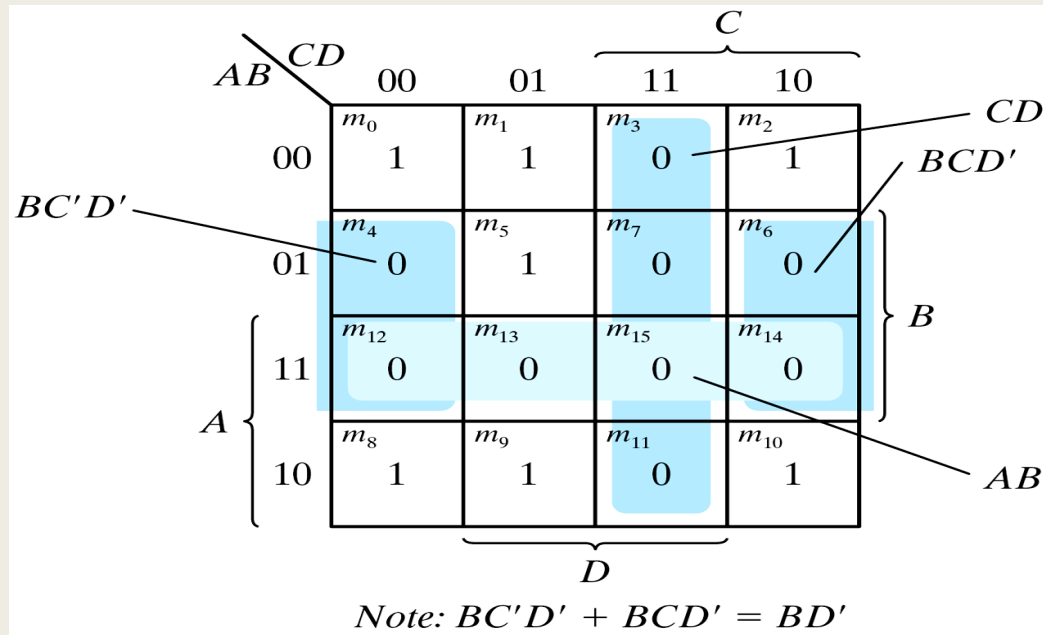
$$F = (A' + B')(C' + D')(B' + D) \quad \text{Or think in terms of maxterms}$$

Figure 3.14 Map for Example 3.8, $F(A, B, C, D) = \Sigma(0, 1, 2, 5, 8, 9, 10) = B'D' + B'C' + A'C'D$

Example 1: Continue..

- Example-simplify $F = \Sigma(0, 1, 2, 5, 8, 9, 10)$ into (a) sum-of-products form, and (b) product-of-sums form:

This is for F function



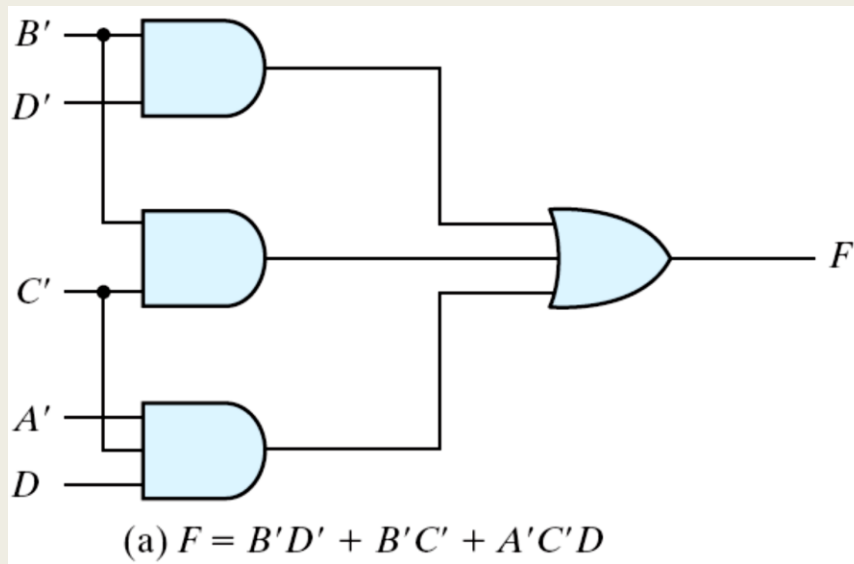
This is for F' function

| AB \ CD | CD | | | |
|---------|----|----|----|----|
| | 00 | 01 | 11 | 10 |
| 00 | 0 | 0 | 1 | 0 |
| 01 | 1 | 0 | 1 | 1 |
| 11 | 1 | 1 | 1 | 1 |
| 10 | 0 | 0 | 1 | 0 |

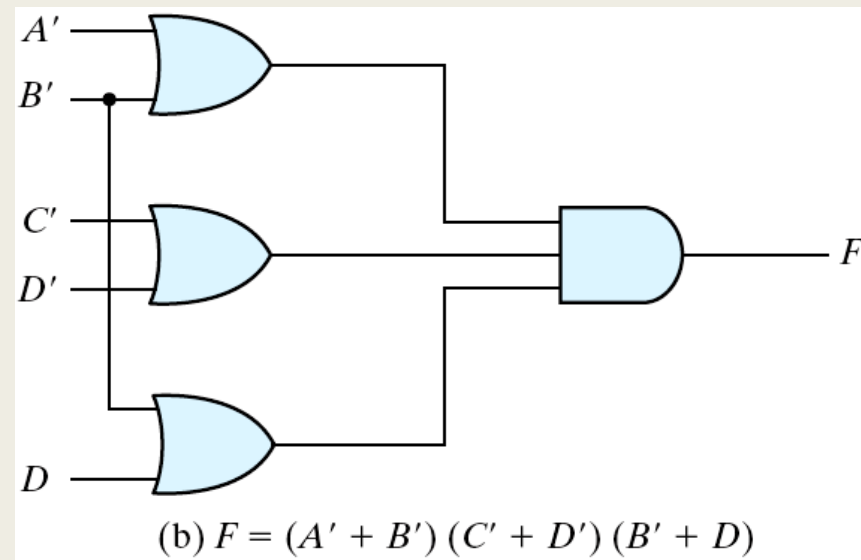
Figure 3.14 Map for Example 3.8, $F(A, B, C, D) = \Sigma(0, 1, 2, 5, 8, 9, 10) = B'D' + B'C' + A'C'D$

Example 1 (cont.)

- Gate implementation of the function of Example1



Sum-of products form



Product-of sums form

Sum-of-Minterm Procedure

- Consider the function $F = m_1 + m_3 + m_4 + m_6$.

OR $F = m_0 + m_2 + m_5 + m_7$

- Combine the 1's:

$$F(x, y, z) = x'z + xz'$$

- Combine the 0's:

$$F(x, y, z) = xz + x'z'$$

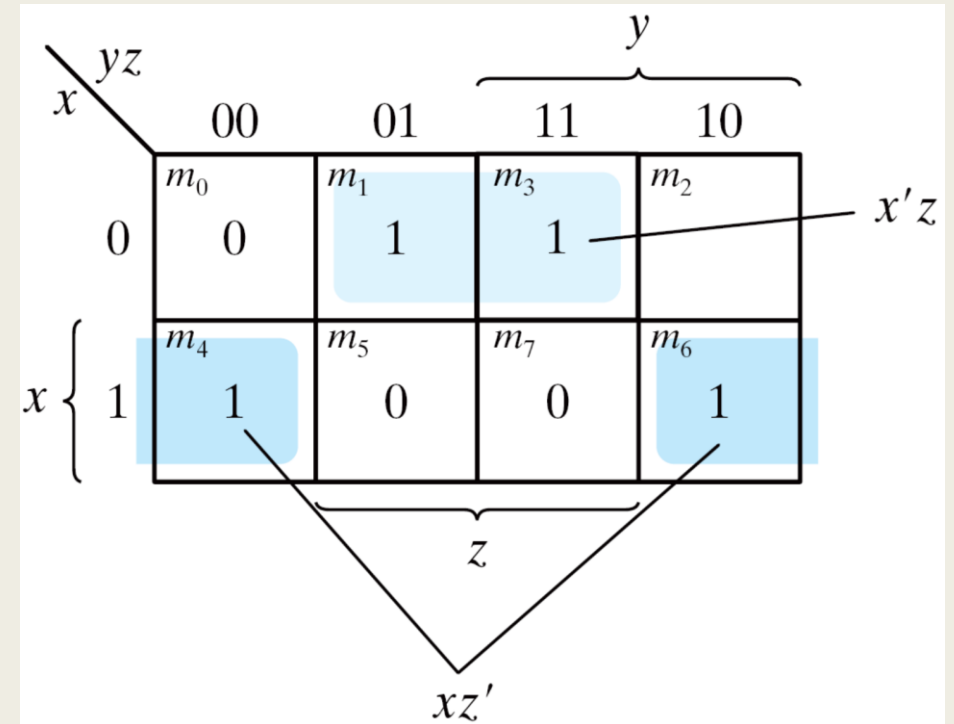


Figure 3.16 Map for the function of Table 3.2

Thank You