SEQUENCE

A sequence is just a list of elements usually written in a row.

EXAMPLES

a. 1, 2, 3, 4, 5, …
b. 4, 8, 12, 16, 20,…
c. 2, 4, 8, 16, 32, …
d. 1, 1/2, 1/3, 1/4, 1/5, …
e. 1, 4, 9, 16, 25, …
f. 1, -1, 1, -1, 1, -1, …

NOTE:
The symbol "…" is called ellipsis, and reads "and so forth"

Types of Sequence

Generally we study two types of sequence.

a. Arithmetic Sequence
b. Geometric Sequence

Arithmetic Sequence

A sequence in which the difference between two terms is same, for example,

a. 1, 2, 3, 4,……. **(Eq-1)**
b. 0, 2, 4, 6, 8, …. **(Eq-2)**
c. 1, 3, 5, 7, 9…… **(Eq-3)**

In above examples, the difference between two numbers is the same, in all three examples… so we call it, an arithmetic sequence.

Generalized Formula

$$A_n = A1 + (n - 1) * d$$

Where, A_n is the n^{th} term, A1, is the first term in sequence, "**n**" is the number in sequence, and "**d**" is the common difference.

Explanation:

Consider Eq 2.

For example if we want to find, the 10^{th} term in 1, 3, 5, …… , the value of **An** will be the term at 10^{th} position.

A1, is the first term, in this example, **A1**, is "1", the value of "n" is 10, which means 10^{th} in a sequence, and the value of "d" is 2, which means the common difference.

So **How to find the 10^{th} value in Case of Eq.2**

Is

$A_n = A_1 + (n – 1) * d$

Putting values in above equation.

$A_n = 1 + (10 – 1) * 2$

$A_n = 1 + 9*2$

$A_n = 19.$

So the 1**0th** value in sequence is **19**, it means the value of An = 19, which is 10^{th} value in sequence.

Other examples in Book.

There are three Scenarios.

Scenario 1:	**Scenario 2:**
EXAMPLE: Find the 20th term of the arithmetic sequence 3, 9, 15, 21, … **SOLUTION:** Here a = first term = 3 d = common difference = 9 - 3 = 6 n = term number = 20 a_{20} = value of 20th term = ? Since $a_n = a + (n - 1)\, d;$ $n \geq 1$ $\therefore$ $a_{20} = 3 + (20 - 1)\, 6$ $= 3 + 114$ $= 117$	**EXAMPLE:** Which term of the arithmetic sequence 4, 1, -2, …, is -77 **SOLUTION:** Here a = first term = 4 d = common difference = 1 - 4 = -3 a_n = value of nth term = - 77 n = term number = ? Since $an = a + (n - 1)\, d$ $n \geq 1$ $\Rightarrow$ $- 77 = 4 + (n - 1)\, (-3)$ $\Rightarrow$ $- 77 - 4 = (n - 1)\, (-3)$ OR $\frac{-81}{-3} = n - 1$ OR $27 = n - 1$ $n = 28$ Hence –77 is the 28th term of the given sequence.

Scenario : 3
EXERCISE:

Find the 36th term of the arithmetic sequence whose 3rd term is 7 and 8th term is 17

SOLUTION:

Let **a** be the first term and **d** be the common difference of the arithmetic sequence.

Then

$a_n = a + (n - 1)d \qquad n \geq 1$

$\Rightarrow \quad a_3 = a + (3 - 1)\, d$

and $\quad a_8 = a + (8 - 1)\, d$

Given that $a_3 = 7$ and $a_8 = 17$. Therefore

$7 = a + 2d$.....(1)

and $\quad 17 = a + 7d$.....(2)

Subtracting (1) from (2), we get,

$10 = 5d$

$\Rightarrow \quad d = 2$

Substituting $d = 2$ in (1) we have

$7 = a + 2(2)$

which gives $a = 3$

Thus, $\quad a_n = a + (n - 1)\, d$

$a_n = 3 + (n - 1)\, 2 \qquad$ (using values of a and d)

Hence the value of 36th term is

$a_{36} = 3 + (36 - 1)\, 2$

$= 3 + 70$

$= 73$

Summary:

Sequence is a list of numbers, normally written in a row, there are two standard types of sequence, i.e Arithmetic and Geometric, In Arithmetic, the difference between two consecutive terms is same, i.e a sequence in which the difference is same. If the difference is not the same then it is not an arithmetic sequence. The next type is Geometric.

Geometric Sequence

A sequence in which every term after the first is obtained from the preceding term by multiplying it with a constant number is called a geometric sequence or geometric progression (G.P.) The constant number, being the ratio of any two consecutive terms is called the common ratio of the G.P. commonly denoted by "r".

In Simple Words, if the ratio is same, we call it Sequence. Geometric

For example

a. **1, 2, 4, 8, 16,……..**
b. **1, 3, 9, 27, 81, ……**
c. **1, 4, 16, 64, …….**

In above all examples, the common **ratio** is the same, [by ratio we mean, Consider, part-a, example-1, ie.. divide 2^{nd} term by first term, i.e 2 / 1 = 2, similarly, divide 3^{rd} term by 2^{nd},

i.e 4 / 2 = 2, again the ratio is 2.

In example-2, again here we can see, the ratio is same, i.e divide the 2^{nd} term by first one, i.e

3 / 1 = 3, here ratio is 3, similarly divide 9 by 3, we get ratio 3.

So, Geometric Sequence means, if the ratio is same.

Generalized Formula

$$A_n = ar^{n-1}$$

Where, **An**, is the nth term, **a is the first term**, **r** is a common ratio, and **n** means nth term in a sequence.

For example

Consider the sequence

1, 3, 9, 27, ……

For Example we want to find the fourth number in above sequence, we already know its 27. But let we calculate it using generalized formula.

Solution:

As we know, it is Geomatric sequence, because the common ratio is same, so we will use geometric sequence formula.

A = 1 (first term)

n = 4 (because we want to find the fourth value in the given sequence)

r = 3, (common ratio)

so

An = ar^{n-1}

An = $(1)(3)^{4-1}$

An = $1 * 3^3$

An = 27

Task: Find the 7^{th} and 10^{th} value.

Other examples in book/notes.

EXAMPLE-1 (SCENARIO-I)

Find the 8th term of the following geometric sequence

4, 12, 36, 108, …

SOLUTION:

Here a = first term = 4

r = common ratio = $\frac{12}{4} = 3$

n = term number = 8

a_8 = value of 8th term = ?

Since $a_n = ar^{n-1}$; $n \geq 1$

$\Rightarrow$ $a_8 = (4)(3)^{8-1}$

$= 4\ (2187)$

$= 8748$

EXAMPLE-2 (SCENARIO-II)

Which term of the geometric sequence is 1/8 if the first term is 4 and common ratio ½

SOLUTION:

Given a = first term = 4

r = common ratio = 1/2

a_n = value of the nth term = 1/8

n = term number = ?

Since $a_n = ar^{n-1}$ $\quad n \geq 1$

$$\Rightarrow \quad \frac{1}{8} = 4\left(\frac{1}{2}\right)^{n-1}$$

$$\Rightarrow \quad \frac{1}{32} = \left(\frac{1}{2}\right)^{n-1}$$

$$\Rightarrow \quad \left(\frac{1}{2}\right)^{5} = \left(\frac{1}{2}\right)^{n-1}$$

$$\Rightarrow \quad n-1 = 5 \qquad \Rightarrow n = 6$$

Hence 1/8 is the 6th term of the given G.P.

EXAMPLE-3 (SCENARIO-III)

Write the geometric sequence with positive terms whose second term is 9 and fourth term is 1.

SOLUTION:

Let **a** be the first term and **r** be the common ratio of the geometric sequence. Then

$a_n = ar^{n-1}$ $\quad n \geq 1$

Now $a_2 = ar^{2-1}$

$\Rightarrow \quad 9 = ar$.....(1)

Also $a_4 = ar^{4-1}$

$1 = ar^3$ .....(2)

Dividing (2) by (1), we get,

$$\frac{1}{9} = \frac{ar^3}{ar}$$

$$\Rightarrow \quad \frac{1}{9} = r^2$$

$$\Rightarrow \quad r = \frac{1}{3} \qquad \left(\text{rejecting } r = -\frac{1}{3}\right)$$

Substituting r = 1/3 in (1), we get

$$9 = a\left(\frac{1}{3}\right)$$

$$\Rightarrow \quad a = 9 \times 3 = 27$$

Hence the geometric sequence is

27, 9, 3, 1, 1/3, 1/9, …