Graphs

Graph:

- **Graph** is a mathematical representation of a network and it describes the relationship between lines and points.
- Graph is a way of expressing information in pictorial form.
- A graph is a collection of nodes and edges. A graph is also called a network.
- A node is whatever you are interested in: person, city, team, project, computer, etc.
- An edge represents a relationship between nodes.

Daily life examples of Graphs

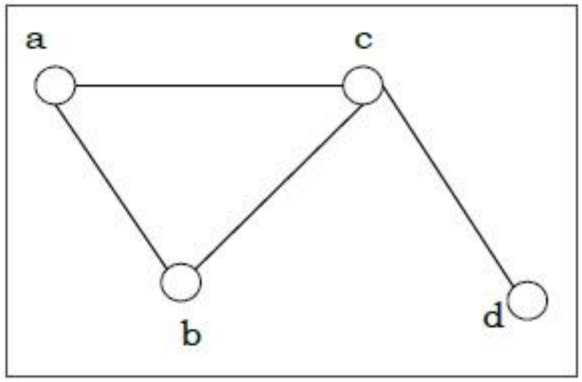
- Map of the world, or world
  - Pakistan
  - Provinces
  - Islamabad
  - Sector H-9
  - NUML
  - Ghazali Block
- Internet
- WAN, LAN
- Our Knowledge
- Our friendship circle
- Human body

And many more…..

What is a Graph?

Definition − A graph (denoted as $G=(V,E)$ $G=(V,E)$) consists of a non-empty set of vertices or nodes V and a set of edges E.

Example − Let us consider, a Graph is $G=(V,E)$ $G=(V,E)$ where $V=\{a,b,c,d\}$ $V=\{a,b,c,d\}$ and $E=\{\{a,b\},\{a,c\},\{b,c\},\{c,d\}\}$



Degree of a Vertex − The degree of a vertex V of a graph G (denoted by deg (V)) is the number of edges incident with the vertex V.

Vertex	Degree	Even / Odd
a	2	even
b	2	even
c	3	odd
d	1	odd

Even and Odd Vertex − If the degree of a vertex is even, the vertex is called an even vertex and if the degree of a vertex is odd, the vertex is called an odd vertex.

Degree of a Graph − The degree of a graph is the largest vertex degree of that graph. For the above graph the degree of the graph is 3.

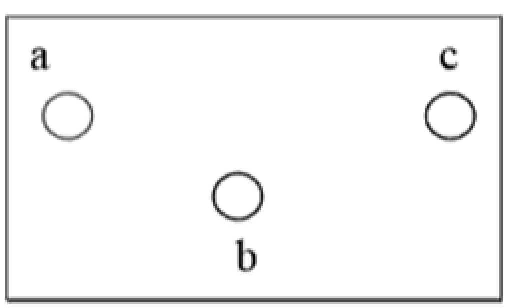
The Handshaking Lemma − In a graph, the sum of all the degrees of all the vertices is equal to twice the number of edges.

Types of Graphs

There are different types of graphs, which we will learn in the following section.

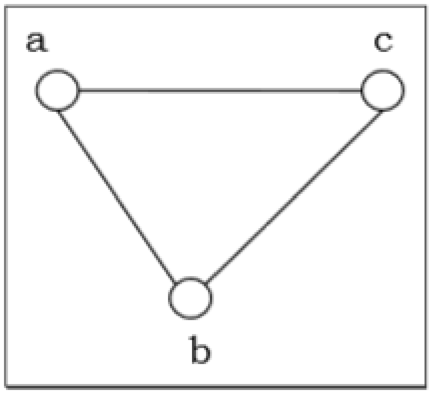
Null Graph

A null graph has no edges. The null graph of n vertices is denoted by N_n



Simple Graph

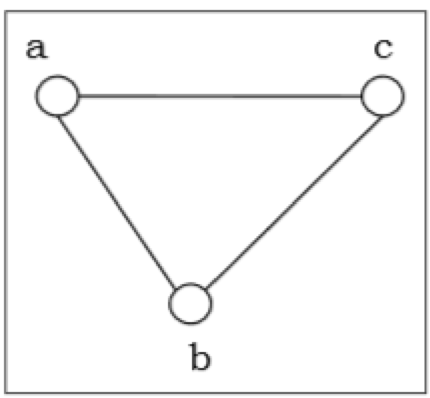
A graph is called simple graph/strict graph if the graph is undirected and does not contain any loops or multiple edges.



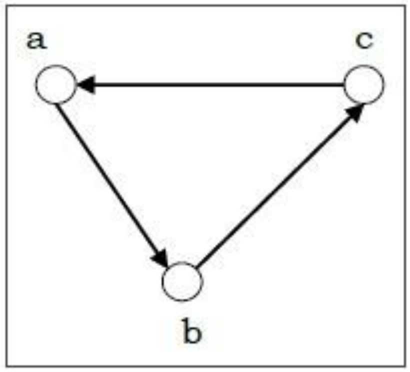
Directed and Undirected Graph

A graph $G=(V,E)$G=(V,E) is called a directed graph if the edge set is made of ordered vertex pair and a graph is called undirected if the edge set is made of unordered vertex pair (a), and if directed edges called directed graph (b)

a)

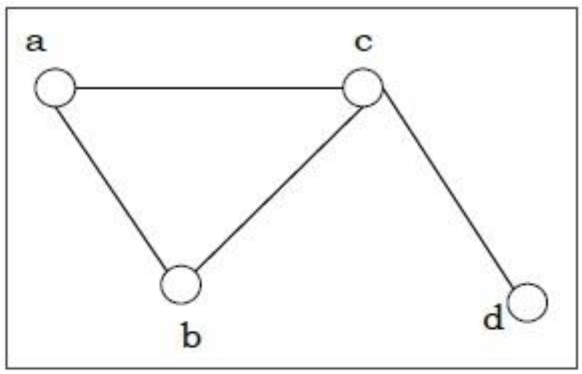


b)

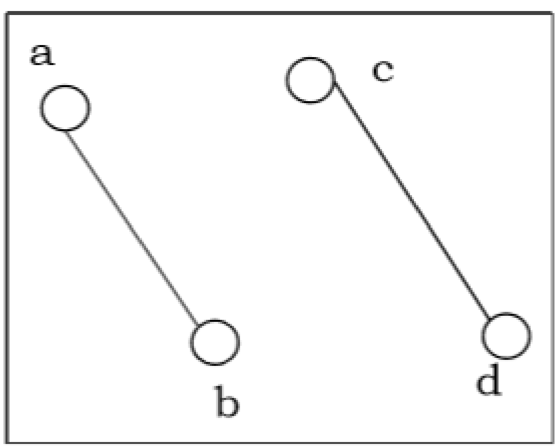


Connected and Disconnected Graph

A graph is connected if any two vertices of the graph are connected by a path; while a graph is disconnected if at least two vertices of the graph are not connected by a path. If a graph G is disconnected, then every maximal connected sub graph of G is called a connected component of the graph G.



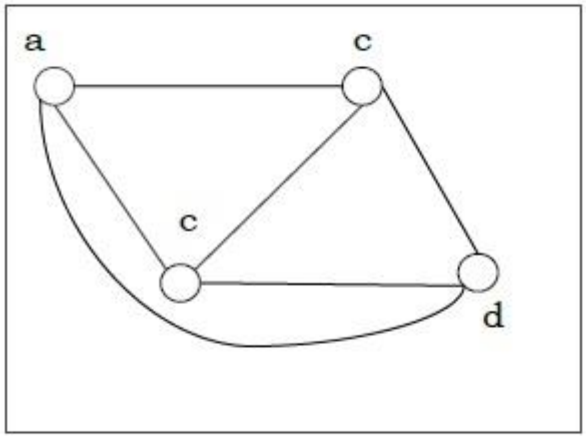
Connected Graph



Disconnected Graph

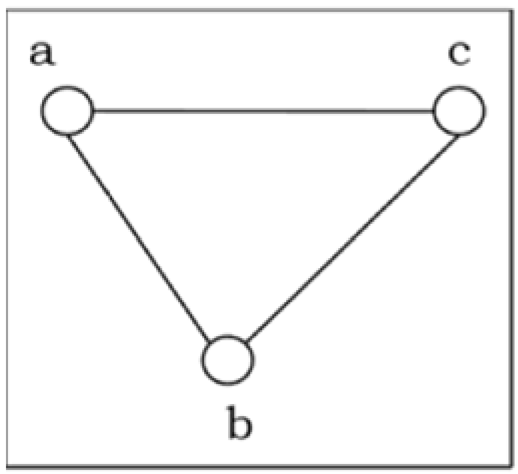
Regular Graph

A graph is regular if all the vertices of the graph have the same degree. In a regular graph G of degree r , the degree of each vertex of G is r.



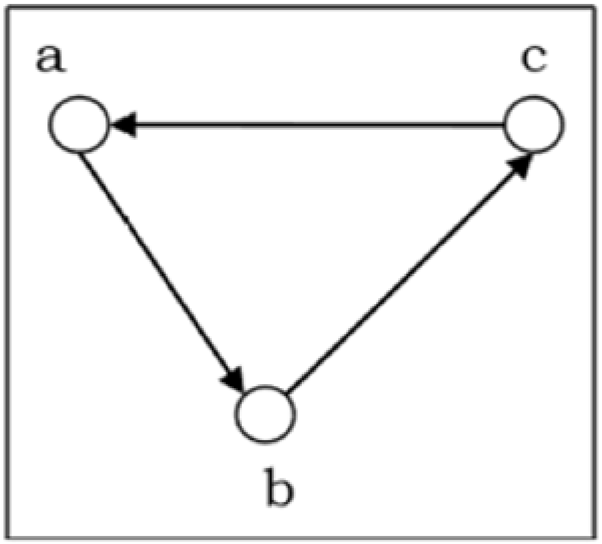
Complete Graph

A graph is called complete graph if every two vertices pair are joined by exactly one edge. The complete graph with n vertices is denoted by K_n



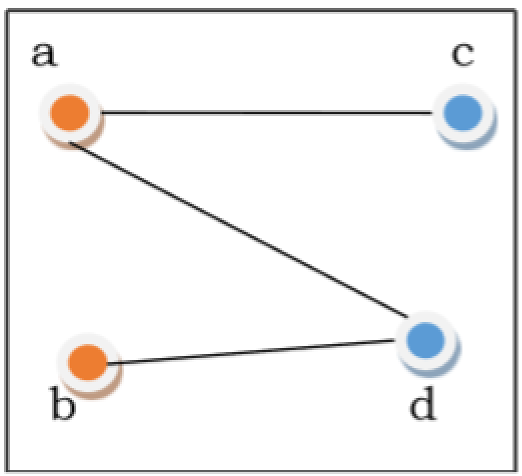
Cycle Graph

If a graph consists of a single cycle, it is called cycle graph. The cycle graph with n vertices is denoted by C_n



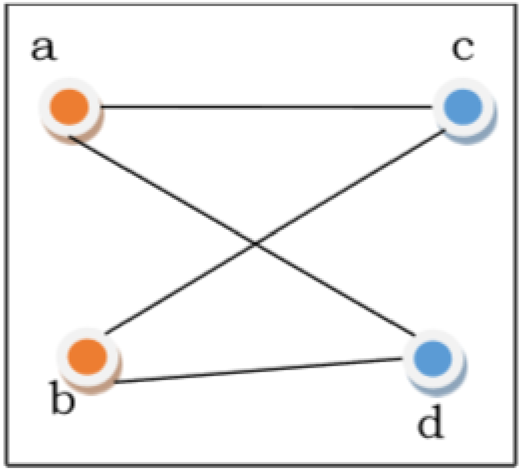
Bipartite Graph

If the vertex-set of a graph G can be split into two disjoint sets, V_1 and V_2, in such a way that each edge in the graph joins a vertex in V_1 to a vertex in V_2, and there are no edges in G that connect two vertices in V_1 or two vertices in V_2, then the graph G is called a bipartite graph.



Complete Bipartite Graph

A complete bipartite graph is a bipartite graph in which each vertex in the first set is joined to every single vertex in the second set. The complete bipartite graph is denoted by $K_{x,y}$ where the graph G contains x vertices in the first set and y vertices in the second set.



Representation of Graphs

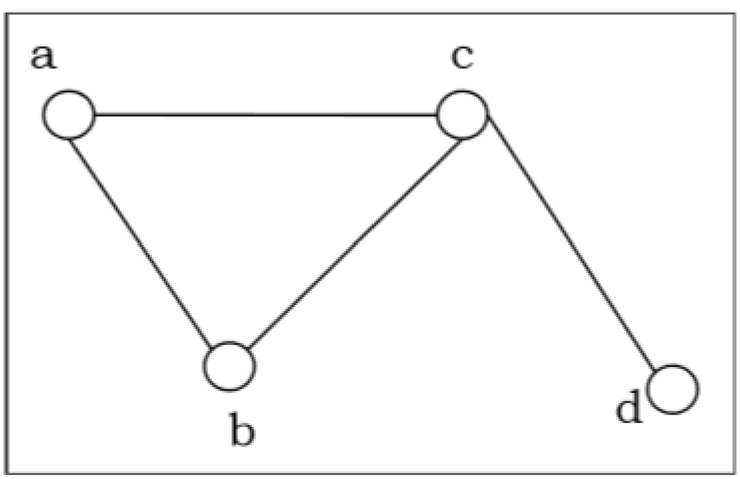
There are mainly two ways to represent a graph −

- Adjacency Matrix
- Adjacency List

Adjacency Matrix

Adjacency Matrix of an Undirected Graph

Let us consider the following undirected graph and construct the adjacency matrix –

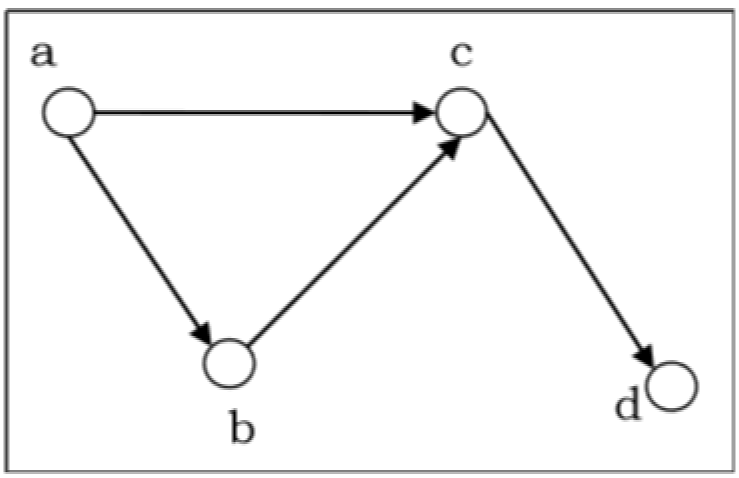


Adjacency matrix of the above undirected graph will be –

	a	**b**	**c**	**d**
a	0	1	1	0
b	1	0	1	0
c	1	1	0	1
d	0	0	1	0

Adjacency Matrix of a Directed Graph

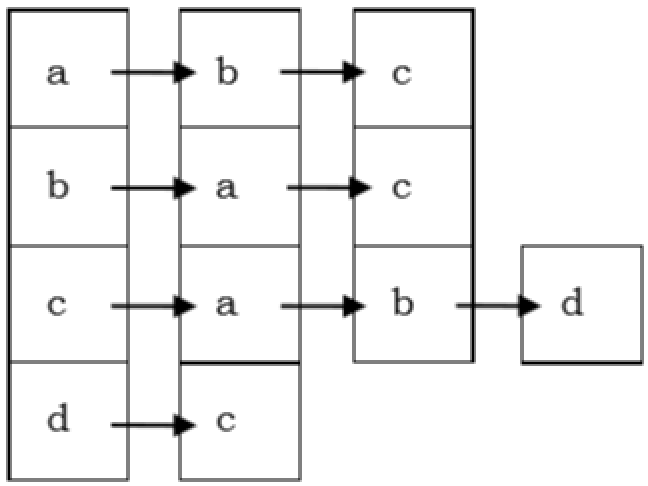
Let us consider the following directed graph and construct its adjacency matrix −



Adjacency matrix of the above directed graph will be −

	a	**b**	**c**	**d**
a	0	1	1	0
b	0	0	1	0
c	0	0	0	1
d	0	0	0	0

Adjacency List



Isomorphism

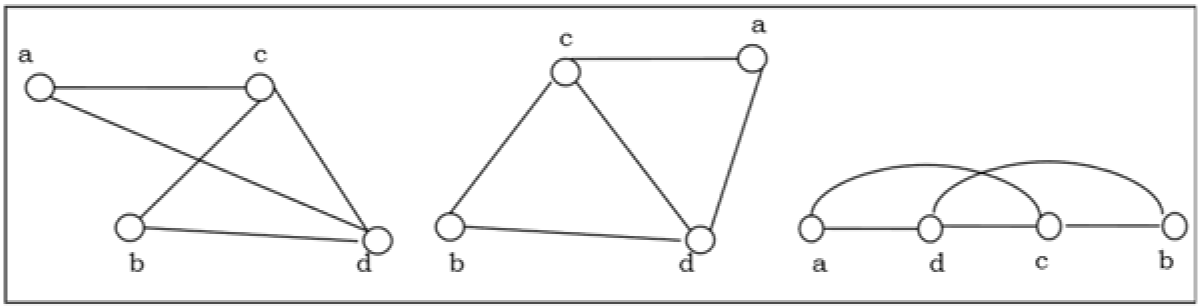
If two graphs G and H contain the same number of vertices connected in the same way, they are called isomorphic graphs (denoted by $G \cong H$G≅H).

It is easier to check non-isomorphism than isomorphism. If any of these following conditions occurs, then two graphs are non-isomorphic −

- The number of connected components are different
- Vertex-set cardinalities are different
- Edge-set cardinalities are different
- Degree sequences are different

Example

The following graphs are isomorphic −



Euler Graphs

A connected graph G is called an Euler graph, if there is a closed trail which includes every edge of the graph G. An Euler path is a path that uses every edge of a graph exactly once. An Euler path starts and ends at different vertices.
An Euler circuit is a circuit that uses every edge of a graph exactly once. An Euler circuit always starts and ends at the same vertex. A connected graph G is an Euler graph if and only if all vertices of G are of even degree, and a connected graph G is Eulerian if and only if its edge set can be decomposed into cycles.