# Mathematical Induction

Use mathematical induction to prove that
$1+2+3 \ldots\ldots\ldots\ldots + = n(n+1)/2$, for all n greater or equals to 1.

Sol: Let $P(n) = 1+2+3 \ldots\ldots\ldots\ldots +n = n(n+1)/2 \ldots\ldots\ldots\ldots (A)$

## 1. Basic steps: P(1) is true

L.H.S = n, where P(1) means n=1
so L.H.S = 1

R.H.S = $n(n+1)/2 = 1(1+1)/2 = 1$
L.H.S = R.H.S, Hence P(1) is True

## 2. Inductive Step:

Suppose P(k) is true for some integers $k \geq 1$

So (A) implies, Replace "n" with k

So $P(k) = 1+2+3+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots +k = k(k+1)/2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$
To prove P(k+1) is true,
So Eq 1, imples........
$P(k+1) = 1+2+3+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + k+1 = k+1(k+1+1)/2$
$P(k+1) = 1+2+3+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + k+1 = (k+1)(k+2)/2 \ldots\ldots\ldots\ldots\ldots\ldots (2)$
Solving eq (2)

| | |
|---|---|
| L.H.S of eq 2 = $1+2+3 +\ldots\ldots\ldots\ldots +k+1$<br><br>L.H.S = $1+2+3+\ldots\ldots\ldots\ldots +k+k+1$<br>Using eq 1<br><br>L.H.S = $k(k+1)/2 + k+1$<br>$= \{k(k+1) + 2(k+1)\}/ 2$<br>$= (k+1)(k+2)/2$<br>= R.H.S | Hence<br><br>L.H.S = R.H.S<br><br>Hence Proved |

Q 2: Use Mathematical Induction to Prove that

1+3+5 +----------+(2n-1) = (n)2, For all n>=1..(A)

Sol: Let P(n) = 1+3+5 +----------+(2n-1) = (n)2, For all n>=1

1. **Basis Step:**

P(1) is true
So, L.H.S of (A) = (2n-1)
Put n =1
L.H.S = 2(1) -1 = 1
R.H.S = (n)2 = (1)2 = 1
So L.H.S = R.H.S, Hence P(1) is True.

2. **Inductive Step**

Suppose P(k) is true
Eq (A) becomes
P(k) = 1+3+5 +----------+(2k-1) = $(k)^2$, For all k>=1....(1)
To prove P(k+1) is true, Then
Eq (1) becomes
P(k+1) = 1+3+5 +----------+(2k +1) = $(k+1)^2$, For all k>=1--------(2)

Now Take L.H.S of Eq -2,
L.H.S = 1+3+5 +----------+(2k +1)
= 1+3+5 +----------+(2k-1) + (2k +1)
Using Eq-1

L.H.S = $k^2$ + 2k+1
= $(k+1)^2$
L.H.S = R.H.S

Hence Proved.