THE KARNAUGH MAP FOR THREE-VARIABLES

Digital logic design

Iqra Chaudhary(Lecturer CS dept. NUML)

Three Variable K-Maps

- A three-variable K-map:

	$\bar{y}\bar{z}=00$	$\bar{y}z=01$	$yz=11$	$y\bar{z}=10$
$\bar{x}=0$	x'y'z' m_0	x'y'z m_1	x'yz m_3	x'yz' m_2
$x=1$	xy'z' m_4	xy'z m_5	xyz m_7	Xyz' m_6

3 Variable K-Map:

Simplify the given equation using k-map technique

Given: F=A’B’C’+A’BC’+A’BC+ABC

	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	1_0	0_1	1_3	1_2
A	0_4	0_5	1_7	0_6

	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	1_0	0_1	1_3	1_2
A	0_4	0_5	1_7	0_6

F=BC+A’C’ ➔ SOP

3 Variable K-Map:
Obtain the equation from the truth table and simplify it using k-map

Given :

	A	B	C	Y
minterm 0 →	0	0	0	1
minterm 1 →	0	0	1	0
minterm 2 →	0	1	0	1
minterm 3 →	0	1	1	1
minterm 4 →	1	0	0	0
minterm 5 →	1	0	1	0
minterm 6 →	1	1	0	1
minterm 7 →	1	1	1	0

3 Variable K-Map:
Obtain the equation from the truth table and simplify it using k-map

	A	B	C	Y
minterm 0 →	0	0	0	1
minterm 1 →	0	0	1	0
minterm 2 →	0	1	0	1
minterm 3 →	0	1	1	1
minterm 4 →	1	0	0	0
minterm 5 →	1	0	1	0
minterm 6 →	1	1	0	1
minterm 7 →	1	1	1	0

$$F = A'B'C' + A'BC' + A'BC + ABC'$$

3 Variable K-Map:

Obtain the equation from the truth table and simplify it using k-map

	A	B	C	Y
minterm 0 ➔	0	0	0	1
minterm 1 ➔	0	0	1	0
minterm 2 ➔	0	1	0	1
minterm 3 ➔	0	1	1	1
minterm 4 ➔	1	0	0	0
minterm 5 ➔	1	0	1	0
minterm 6 ➔	1	1	0	0
minterm 7 ➔	1	1	1	1

F=A’B’C’+A’BC’+A’BC+ABC

	$\bar{B}\bar{C}$	BC	BC	$B\bar{C}$
$\bar{A}$	1_0	0_1	1_3	1_2
A	0_4	0_5	1_7	0_6

F=BC+A’C’➔SOP

3 Variable K-Map:
Specification Given: Implement Adder that can add three bits (knows as full adder)

- Step 1: Number of input=3 and Number of output=2
- Step 2: Drive the truth table
- Step 3: Obtain the equation from the truth table and **simplify it**

$S = xy'z'+x'yz'+x'y'z+xyz = x \oplus y \oplus z$

x y z	S	C	minterm
0 0 0	0	0	$x'y'z'$
0 0 1	1	0	$x'y'z$
0 1 0	1	0	$x'yz'$
0 1 1	0	1	$x'yz$
1 0 0	1	0	$xy'z'$
1 0 1	0	1	$xy'z$
1 1 0	0	1	xyz'
1 1 1	1	1	xyz

K-map for S (columns: y over the last two, z under the middle two; x on the second row):

	0	1	0	1
x	1	0	1	0

$C = x'yz+xy'z+xyz'+xyz$

K-map for C (columns: y over the last two, z under the middle two; x on the second row):

	0	0	1	0
x	0	1	1	1

$C = xy + xz + yz$

Step 4: Draw the circuit diagram from simplified expression

3 Variables K-Maps : Grouping of two

	$\overline{B}\,\overline{C}$	$\overline{B}C$	BC	$B\overline{C}$
$\overline{A}$	0	0	0	0
A	0	0	1	1

AB

	$\overline{B}\,\overline{C}$	$\overline{B}C$	BC	$B\overline{C}$
$\overline{A}$	0	0	0	0
A	1	1	0	0

$A\overline{B}$

	$\overline{B}\,\overline{C}$	$\overline{B}C$	BC	$B\overline{C}$
$\overline{A}$	0	0	1	0
A	0	0	1	0

BC

	$\overline{B}\,\overline{C}$	$\overline{B}C$	BC	$B\overline{C}$
$\overline{A}$	0	0	0	0
A	0	1	1	0

AC

	$\overline{B}\,\overline{C}$	$\overline{B}C$	BC	$B\overline{C}$
$\overline{A}$	0	0	0	1
A	0	0	0	1

$B\overline{C}$

	$\overline{B}\,\overline{C}$	$\overline{B}C$	BC	$B\overline{C}$
$\overline{A}$	1	0	0	1
A	0	0	0	0

$\overline{A}\,\overline{C}$

3 Variables K-Maps : Grouping of fours

	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	0	0	1	1
A	0	0	1	1

B

	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	0	0	0	0
A	1	1	1	1

A

	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	0	1	1	0
A	0	1	1	0

C

	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	1	0	0	1
A	1	0	0	1

$\bar{C}$

3 Variables K-Maps : Grouping of eight

	$\overline{B}\,\overline{C}$	$\overline{B}C$	BC	$B\overline{C}$
$\overline{A}$	1	1	1	1
A	1	1	1	1

1

Combining Squares

- By combining squares, we reduce number of literals in a product term, reducing the literal cost, thereby reducing the other two cost criteria

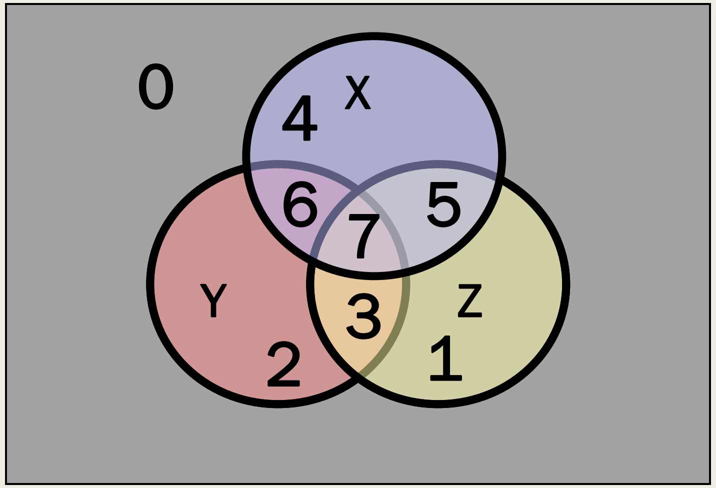
- On a 3-variable K-Map:
  - *One square represents a minterm with three variables*
  - *Two adjacent squares represent a product term with two variables*
  - *Four “adjacent” terms represent a product term with one variable*
  - *Eight “adjacent” terms is the function of all ones (no variables) = 1.*

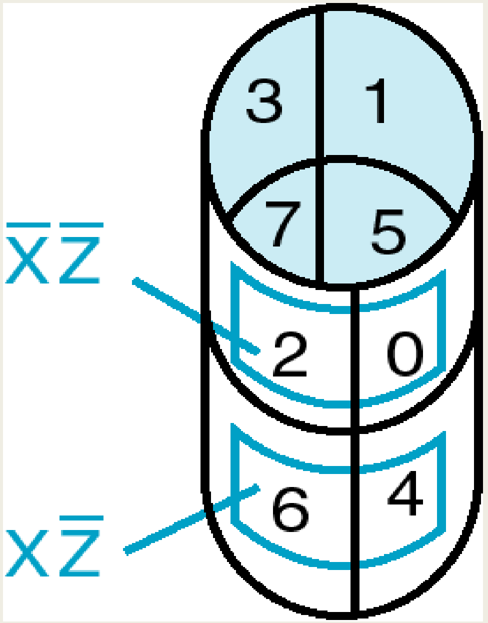
Simplification of SOP expressions using K-map

- Mapping of expression
- Forming of Groups of 1s
- Each group represents product term
- 3-variable K-map
  - *1 cell group yields a 3 variable product term*
  - *2 cell group yields a 2 variable product term*
  - *4 cell group yields a 1 variable product term*
  - *8 cell group yields a value of 1 for function*

Three-Variable Maps

- Topological warps of 3-variable K-maps that show *all* adjacencies:
  - *Venn Diagram*
  - *Cylinder*





K-Map Examples

Example 1

- Example 3.1: simplify the Boolean function $F(x, y, z) = \Sigma m(2, 3, 4, 5)$
  - $F(x, y, z) = \Sigma(2, 3, 4, 5) = x'y + xy'$

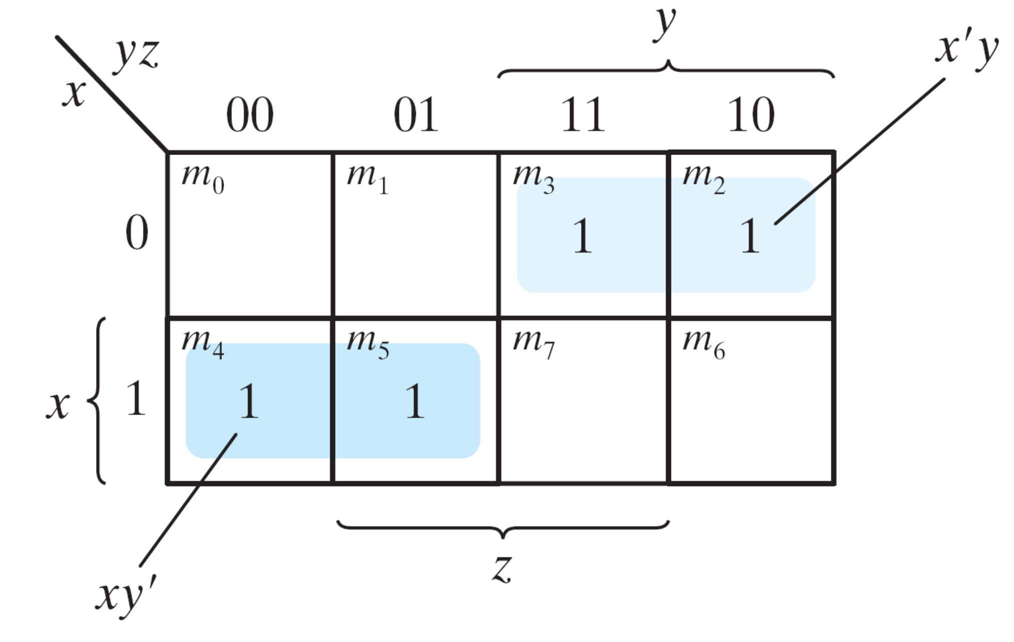


Figure 3.4 Map for Example 3.1, $F(x, y, z) = \Sigma(2, 3, 4, 5) = x'y + xy'$

Example 2

- Example 3.2: simplify $F(x, y, z) = \Sigma(3, 4, 6, 7)$
  - $F(x, y, z) = \Sigma(3, 4, 6, 7) = yz + xz'$

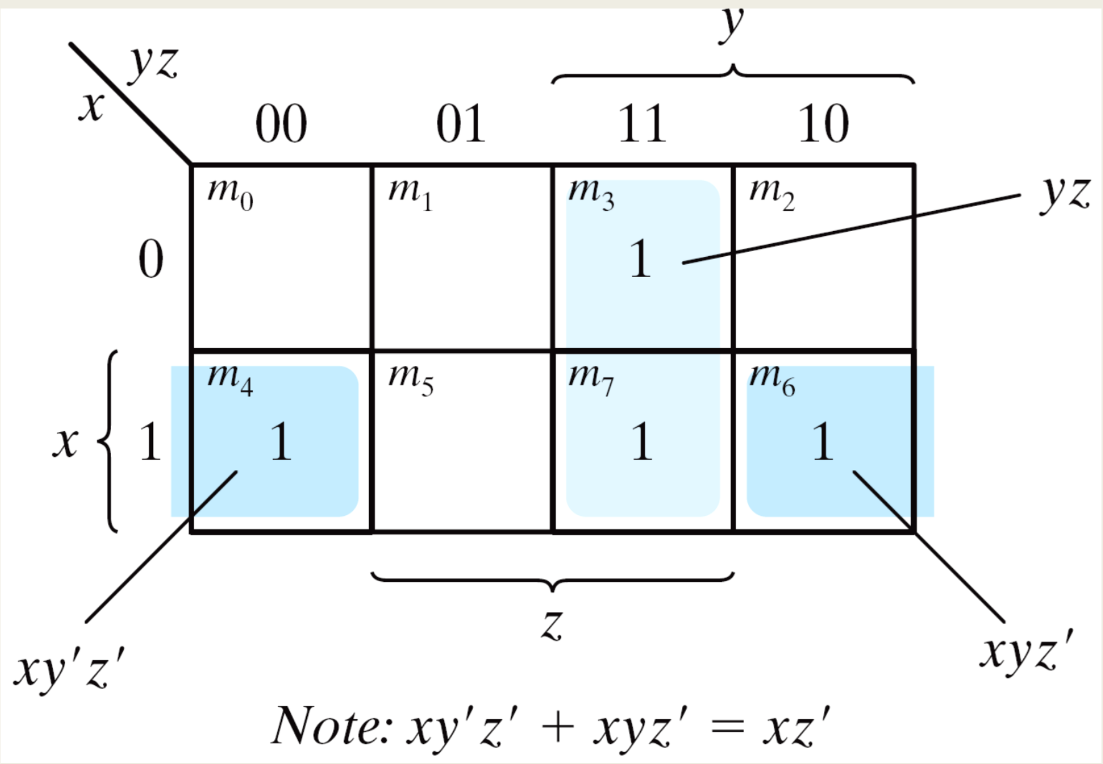


Figure 3.5 Map for Example 3-2; $F(x, y, z) = \Sigma(3, 4, 6, 7) = yz + xz'$

Example 3

- *Example 3.3: simplify F(x, y, z) = Σ(0, 2, 4, 5, 6)*
  - *F(x, y, z) = Σ(0, 2, 4, 5, 6) = z'+ xy'*

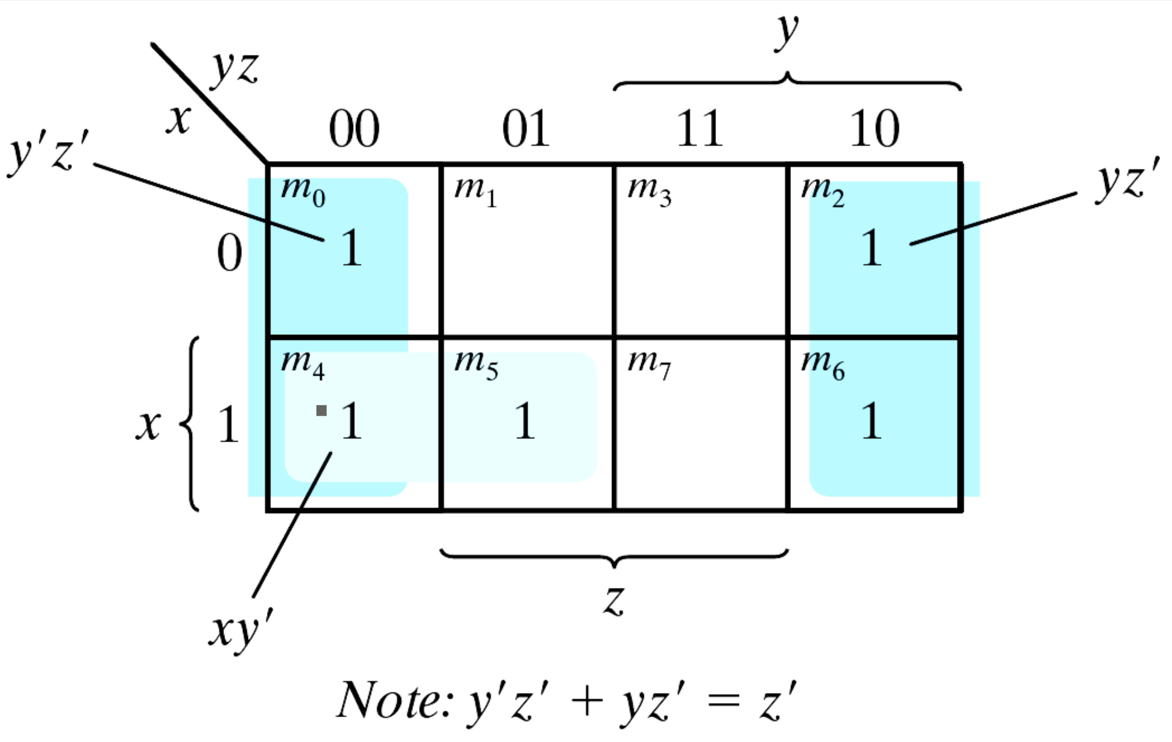


Figure 3.6 Map for Example 3-3, $F(x, y, z) = \Sigma(0, 2, 4, 5, 6) = z' + xy'$

Example 4

- Example 3.4: let $F = A'C + A'B + AB'C + BC$

  a) *Express it in sum of minterms.*

  b) *Find the minimal sum of products expression.*

  Ans:

  $F(A, B, C) = \Sigma(1, 2, 3, 5, 7) = C + A'B$

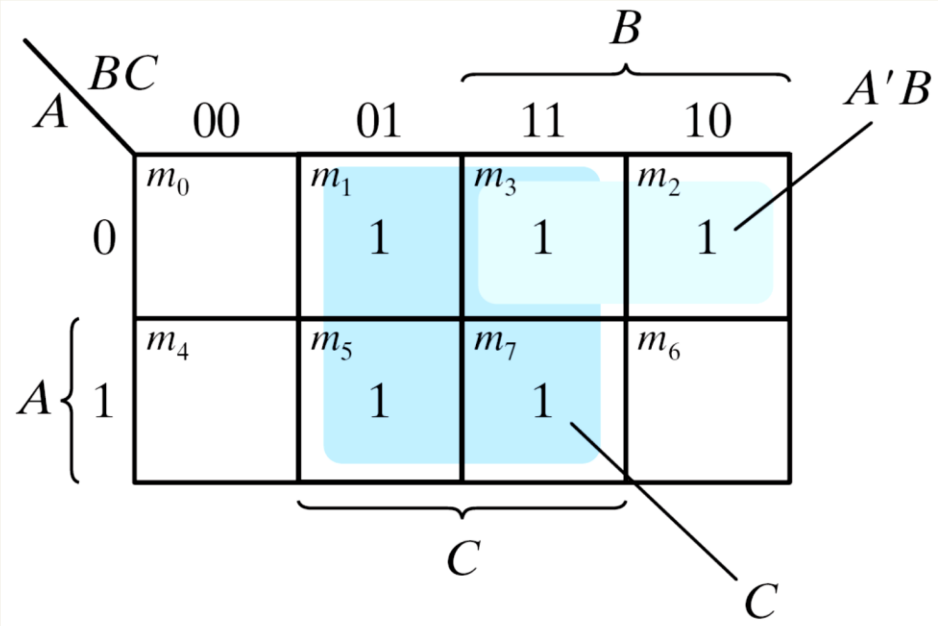


Figure 3.7 Map for Example 3.4, $A'C + A'B + AB'C + BC = C + A'B$

Example 5

$$F = \Sigma m(2,3,6,7)$$

- Example: Let

			y	y
	0	1	3 1	2 1
x	4	5	7 1	6 1
		z	z	

<u>By using k-map:</u>

F=y

<u>By using the Minimization Theorem/Boolean algebra rules:</u>

$$\begin{aligned} F(x,y,z) &= \bar{x}\,y\,z + x\,y\,z + \bar{x}\,y\,\bar{z} + x\,y\,\bar{z} \\ &= yz + y\bar{z} \\ &= y \end{aligned}$$

- Thus the four terms that form a 2 × 2 square correspond to the term "y".

Example 6: If the truth table is given

J	K	L	Z
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

	$\bar{K}\bar{L}$	$\bar{K}L$	KL	$K\bar{L}$
$\bar{J}$	1	0	1	1
J	0	1	0	0

	$\bar{L}$	L
$\bar{J}\bar{K}$	1	0
$\bar{J}K$	1	1
JK	0	0
$J\bar{K}$	0	1

$$Z = Z_{(J,K,L)} = \bar{J}\,\bar{L} + \bar{J}\,K + J\,\bar{K}\,L$$

Example 7: If the truth table is given

A	B	C	Z
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

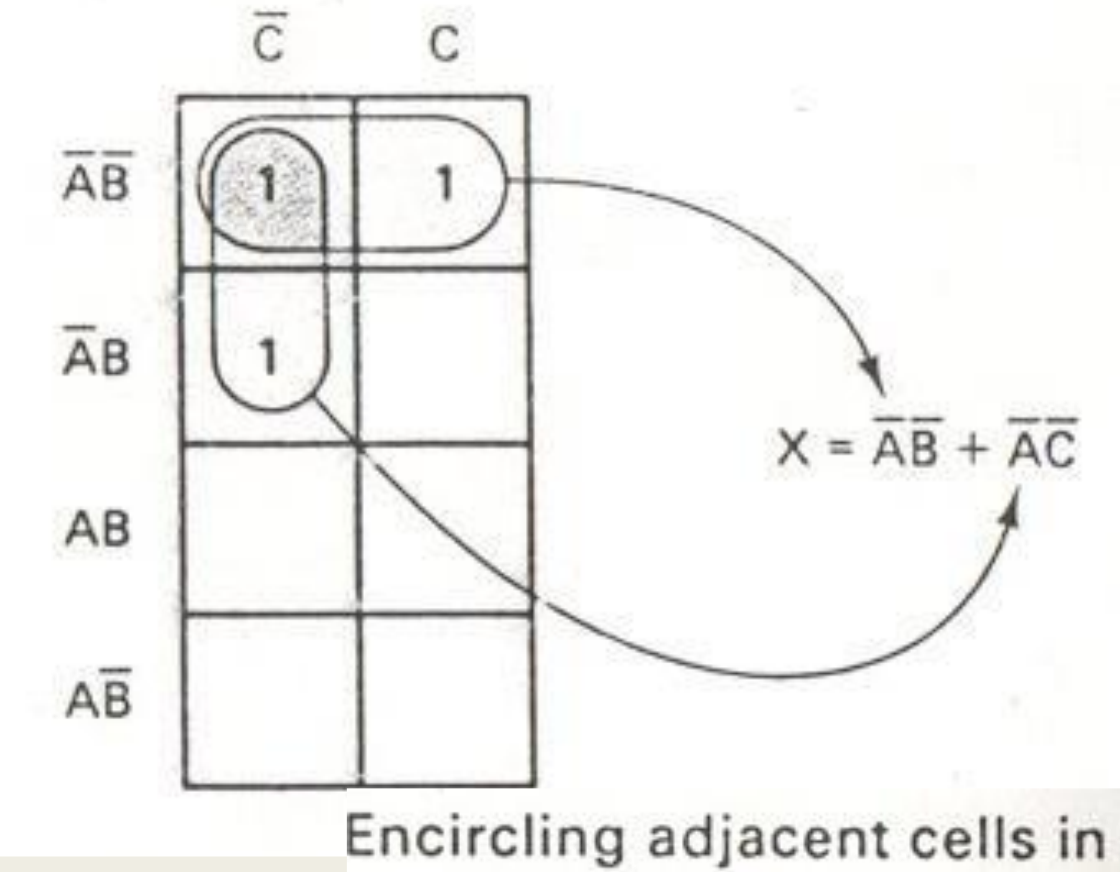
	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	1	0	0	0
A	1	1	1	0

	$\bar{C}$	C
$\bar{A}\bar{B}$	1	0
$\bar{A}B$	0	0
AB	0	1
$A\bar{B}$	1	1

$$Z = Z_{(A,B,C)} = \bar{B}\bar{C} + AC$$

Example 8:k-map

$$X = \overline{A}(\overline{B}C + \overline{B}\,\overline{C}) + \overline{A}B\overline{C}$$

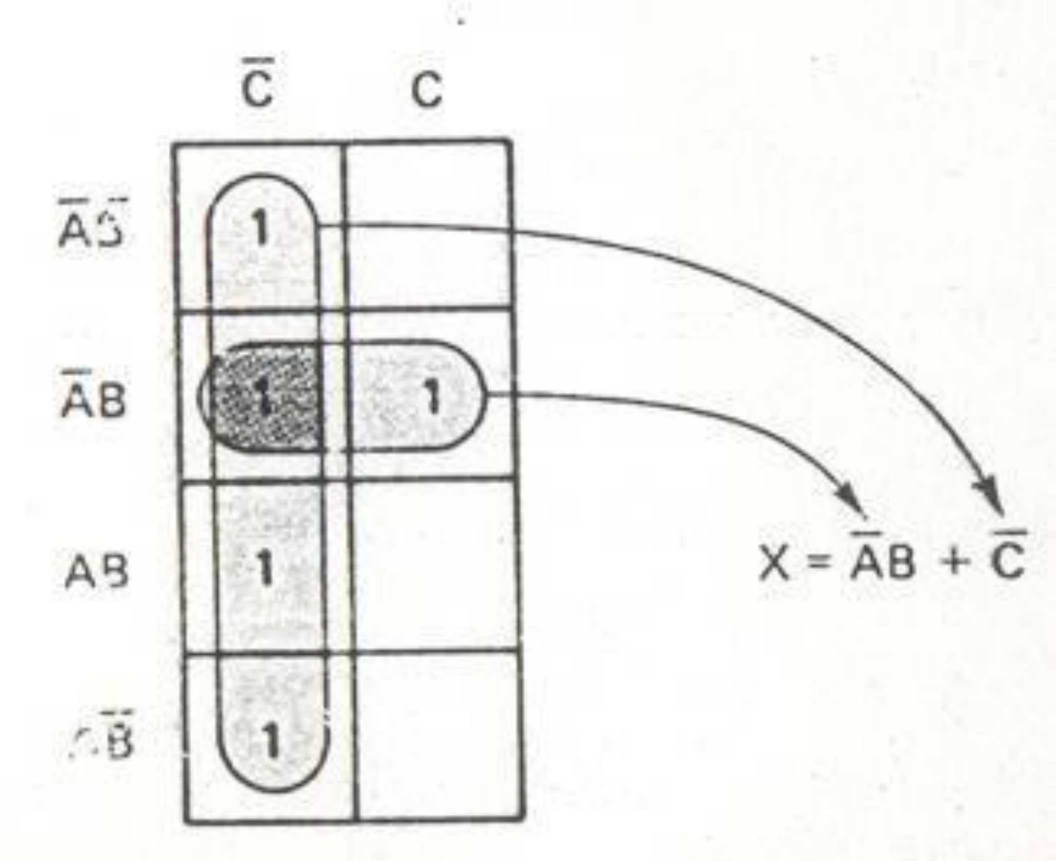


Encircling adjacent cells in a Karnaugh map.

Example 9:k-map

Simplify the following SOP equation using the Karnaugh mapping technique:

$$X = \overline{A}B + \overline{A}\,\overline{B}\,\overline{C} + AB\overline{C} + A\overline{B}\,\overline{C}$$

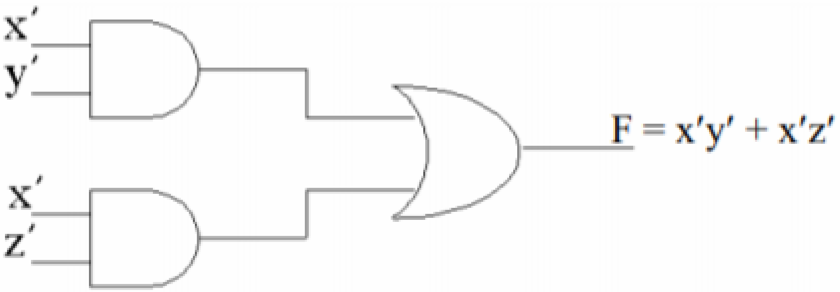


Example 10:k-map

Example: Design a combinational circuit with three inputs and one output. The output is 1 when the binary value of the inputs is less than 3. The output is 0 otherwise.

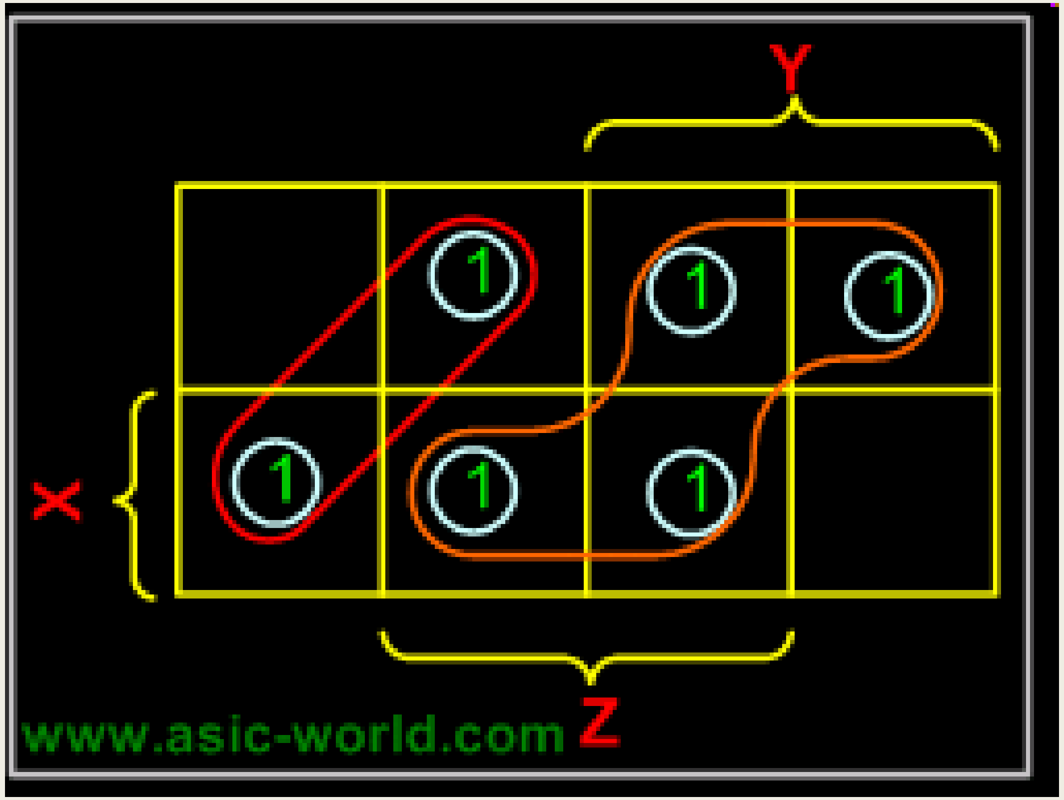
F=x’y’z’+x’y’z+x’yz’

x\yz	00	01	11	10
0	1	1		1
1				



x	y	z	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

Example of Invalid Groups



Assignment

Three-Variable Map Simplification

Task: Use a K-map to find an optimum SOP equation

A	B	C	Z
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$				
A				

	$\bar{C}$	C
$\bar{A}\bar{B}$		
$\bar{A}B$		
AB		
$A\bar{B}$		

Three-Variable Map Simplification

$$\mathbf{F(X, Y, Z) = \Sigma_m(0,1,2,4,6,7)}$$

Task:Use a K-map to find an optimum SOP equation

Thanks