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26. $x - 4y + z = 6$
$4x - y + 2z = -1$
$2x + 2y - 3z = -20$

Solution: $x - 4y + z = 6$
$4x - y + 2z = -1$
$2x + 2y - 3z = -20$

Find the determinat, D by using the x, y, and z values from the problem

$$\begin{array}{ccccc} & & 2 & 4 & 48 \end{array}$$

$$D = \begin{vmatrix} 1 & -4 & 1 \\ 4 & -1 & 2 \\ 2 & 2 & 3 \end{vmatrix} = \begin{vmatrix} 1 & -4 & 1 & 1 & -4 \\ 4 & -1 & 2 & 4 & -1 \\ 2 & 2 & -3 & 2 & 2 \end{vmatrix}$$

$$\begin{array}{ccc} 3 & -16 & 8 \end{array}$$

$= -5 - 50$

$= -55$

$$D_x = \begin{vmatrix} 6 & -4 & 1 \\ -1 & -1 & 2 \\ -20 & 2 & -3 \end{vmatrix} = \begin{vmatrix} 6 & -4 & 1 & 6 & -4 \\ -1 & -1 & 2 & -1 & -1 \\ -20 & 2 & -3 & -20 & 2 \end{vmatrix}$$

$-20 \quad 24 \quad -12$

$18 \quad -160 \quad -2$

$$= -144 - 32 = 176$$

$$D_y = \begin{vmatrix} 1 & 6 & 1 \\ 4 & -1 & 2 \\ 2 & -20 & -3 \end{vmatrix} = \begin{vmatrix} 1 & 6 & 1 & 1 & 6 \\ 4 & -1 & 2 & 4 & -1 \\ 2 & -20 & -3 & 2 & -20 \end{vmatrix}$$

$-2 \quad -40 \quad -72$

$3 \quad 24 \quad -80$

$$= -53 + 114 = 61$$

$$D_z = \begin{vmatrix} 1 & -4 & 6 \\ 4 & -1 & -1 \\ 2 & 2 & -20 \end{vmatrix} = \begin{vmatrix} 1 & -4 & 6 & 1 & -4 \\ 4 & -1 & -1 & 4 & -1 \\ 2 & 2 & -20 & 2 & 2 \end{vmatrix}$$

$-12 \quad -2 \quad -320$

$-20 \quad 8 \quad 48$

$$= 36 + 334 = 370$$

use cramer's rule to find the valu

of the x, y, z

$$x = \frac{D_x}{D} = \frac{176}{-55} = \frac{-16}{5}$$

$$y = \frac{D_y}{D} = \frac{61}{-55} = -\frac{61}{55}$$

$$z = \frac{D_z}{D} = \frac{370}{-55} = -\frac{74}{11}$$

Generally the ans. is written as an order triple $\left(-\frac{16}{5}, -\frac{61}{55}, -\frac{74}{11}\right)$

Ans:

27. $x_1 - 3x_2 + x_3 = 4$

$2x_1 - x_2 = -2$

$4x_1 - 3x_3 = 0$

Solution:

$$D = \begin{vmatrix} 1 & -3 & 1 \\ 2 & -1 & 0 \\ 4 & 0 & -3 \end{vmatrix} = \begin{vmatrix} 1 & -3 & 1 & 1 & -3 \\ 2 & -1 & 0 & 2 & -1 \\ 4 & 0 & -3 & 4 & 0 \end{vmatrix}$$

$-4 \quad 0 \quad 18$

$3 \quad 0 \quad 0$

$= 3 - 14 = -11$

$$D_x = \begin{vmatrix} 4 & -3 & 1 \\ -2 & -1 & 0 \\ 0 & 0 & -3 \end{vmatrix} = \begin{vmatrix} 4 & -3 & 1 & 4 & -3 \\ -2 & -1 & 0 & -2 & -1 \\ 0 & 0 & -3 & 0 & 0 \end{vmatrix}$$

$$= 12 + 18 = 30$$

$$D_y = \begin{vmatrix} 1 & 4 & 1 \\ 2 & -2 & 0 \\ 4 & 0 & 3 \end{vmatrix} = \begin{vmatrix} 1 & 4 & 1 & 1 & 4 \\ 2 & -2 & 0 & 2 & -2 \\ 4 & 0 & 3 & 4 & 0 \end{vmatrix}$$

$$= -6 - 16 = -22$$

$$D_z = \begin{vmatrix} 1 & -3 & 4 \\ 2 & -1 & -2 \\ 4 & 0 & 0 \end{vmatrix} = \begin{vmatrix} 1 & -3 & 4 & 1 & -3 \\ 2 & -1 & -2 & 2 & -1 \\ 4 & 0 & 0 & 4 & 0 \end{vmatrix}$$

$$= -24 + 16 = -8$$

use creman's rules.

$x_1, x_2\ x_3$

$$x_1 = \frac{D_{x1}}{D} = \frac{30}{-11} = \frac{30}{-11}$$

$x_2 = \frac{Dx_2}{D} = \frac{-22}{-11} = 2$

$x_3 = \frac{Dx_3}{D} = \frac{-8}{-11} = \frac{8}{11}$

Ans. written on order triple $\left(\frac{30}{-11}, 2, \frac{8}{11}\right)$

$\underline{\underline{28}}$ $-x_1 - 4x_2 + 2x_3 + x_4 = -32$

$2x_1 - x_2 + 7x_3 + 9x_4 = 19$

$-x_1 + x_2 + 3x_3 + x_4 = -11$

$x_1 - 2x_2 + x_3 + 4x^4 = -4$

$$D = \begin{vmatrix} -1 & -4 & 2 & 1 \\ 2 & -1 & 7 & 9 \\ -1 & 1 & 3 & 1 \\ 1 & -2 & 1 & 4 \end{vmatrix} = \begin{vmatrix} -1 & 4 & 2 & 9 & -1 & -4 & 2 \\ 2 & -1 & 7 & 9 & 2 & -1 & 7 \\ -1 & 1 & 3 & 1 & -1 & 1 & 3 \\ 1 & -2 & 1 & -4 & 1 & -2 & 1 \end{vmatrix}$$

7 54 -8 -8

-12 -28 36 4

$= 0 - 45 = -45$

$$Dx = \begin{vmatrix} -32 & -4 & 2 & 1 \\ 1 & & & \end{vmatrix}$$

$$D_x = \begin{vmatrix} -32 & -4 & 2 & 1 \\ 14 & -1 & 7 & 9 \\ 11 & 1 & 3 & 1 \\ -4 & -2 & 1 & -4 \end{vmatrix} =$$

$-28 \quad 1728 \quad -96 \quad 171$

$-384 \quad 112 \quad -396 \quad 14$

$$= 654 - 1692 = -2346$$

$$D_y = \begin{vmatrix} -1 & -32 & 2 & 1 \\ 2 & 14 & 7 & 9 \\ -1 & 11 & 3 & 1 \\ 1 & -4 & 1 & -4 \end{vmatrix}$$

$77 \quad 108 \quad -69 \quad 122$

$168 \quad 229 \quad 92 \quad -32$

$$= 38 - 233 = 195$$

$$D_{x_3} = \begin{vmatrix} -1 & -4 & -32 & 1 \\ 2 & -1 & 14 & 9 \\ -1 & 1 & 11 & 1 \\ 1 & -2 & -4 & -4 \end{vmatrix} =$$

$14 \quad 198 \quad 32 \quad 12$

$-44 \quad -40 \quad -288$

$$= 396 - 372 = -768$$

$$D_{x_4} = \begin{vmatrix} -1 & -9 & 2 & -32 \\ 2 & -1 & 7 & 19 \\ -1 & 1 & 3 & 21 \\ 1 & -2 & 1 & -9 \end{vmatrix} =$$

$$= \left|\begin{matrix} -1 & -9 & 2 & -32 \\ 2 & -1 & 7 & 19 \\ -1 & 1 & 3 & 21 \\ 1 & -2 & 1 & -9 \end{matrix}\right| \begin{matrix} -1 & -9 & 2 \\ 2 & -1 & 7 \\ -1 & 1 & 3 \\ 1 & -2 & 1 \end{matrix}$$

-229, 9, -98, -8

-306, 12, 56, -6

$= -328 + 326 = -92$

use cramer's rules,

x_1, x_2, x_3, x_4

$$\therefore x_1 = \frac{D_{x_1}}{D} = \frac{-2346}{-45} = \frac{782}{15}$$

$$x_2 = \frac{D_{x_2}}{D} = \frac{195}{-45} = \frac{13}{3}$$

$$x_3 = \frac{D_{x_3}}{D} = \frac{-768}{-45} = \frac{256}{15}$$

$$x_4 = \frac{D_{x_4}}{D} = \frac{-92}{-45} = \frac{92}{45}$$

Ans. is

$$\left(\frac{782}{15},\ \frac{13}{3},\ \frac{256}{15},\ \frac{92}{45}\right)\ A.$$

21. $A = \begin{bmatrix} 2 & -3 & 5 \\ 0 & 1 & -3 \\ 0 & 0 & 2 \end{bmatrix}$

$$A = \begin{vmatrix} 2 & -3 & 5 \\ 0 & 1 & -3 \\ 0 & 0 & 2 \end{vmatrix} = \begin{vmatrix} 2 & -3 & 5 & 2 & -3 \\ 0 & 1 & -3 & 0 & 1 \\ 0 & 0 & 2 & 0 & 0 \end{vmatrix}$$

$0 \quad 0 \quad 0$

$4 \quad 0 \quad 0$

$= 4 - 0$

$= 4$

The cafactor of A are,

$$C_{11} = 2\begin{vmatrix} 1 & -3 \\ 0 & 2 \end{vmatrix} = 2\times(2-0) = 4$$

$$C_{12} = 3\begin{vmatrix} 0 & -3 \\ 0 & 2 \end{vmatrix} = 3(0-0) = 0$$

$C_{13} = 5\begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 5(0-0) = 0$

$C_{21} = 0\begin{vmatrix} -3 & 5 \\ 0 & 2 \end{vmatrix} = 0(-6-0) = 0$

$C_{22} = 1\begin{vmatrix} 2 & 5 \\ 0 & 2 \end{vmatrix} = 1(4-0) = 4$

$C_{23} = 3\begin{vmatrix} 2 & -3 \\ 0 & 0 \end{vmatrix} = 3(0-0) = 0$

$C_{31} = 0\begin{vmatrix} -3 & 5 \\ 1 & -3 \end{vmatrix} = 0(-9-5) = 0$

$C_{32} = -0\begin{vmatrix} 2 & 5 \\ 0 & -3 \end{vmatrix} = 0(15-0) = 0$

$C_{33} = 2\begin{vmatrix} 2 & -3 \\ 0 & 1 \end{vmatrix} = 2(2-0) = 4$

matrix of cofactor is

$$\begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

A) Adjoint is.

$$\text{adj}(A) = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det(A)}\,\text{adj}(A) = \frac{1}{4}\begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ Ans:}$$

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$$A = \begin{bmatrix} 2 & 0 & 0 \\ 8 & 1 & 0 \\ 5 & 3 & 6 \end{bmatrix}$$

$$\begin{array}{ccccc} & & 0 & 0 & 0 \\ \end{array}$$

$$\text{Det}(A) = \left|\begin{array}{ccc|cc} 2 & 0 & 0 & 2 & 0 \\ 8 & 1 & 0 & 8 & 1 \\ -5 & 3 & 6 & -5 & 3 \end{array}\right|$$

$$\begin{array}{ccc} 12 & 0 & 0 \end{array}$$

$= 12 - 0 = 12$

cofactor of A are,

$$C_{11} = 2\begin{vmatrix} 1 & 0 \\ 3 & 6 \end{vmatrix} = 2(6-0) = 12$$

$C_{12} = 0 \begin{vmatrix} 8 & 0 \\ -5 & 6 \end{vmatrix} = 0(48-0) = 0$

$C_{13} = 0 \begin{vmatrix} 8 & 1 \\ -5 & 3 \end{vmatrix} = 0(24-5) = 0$

$C_{21} = 8 \begin{vmatrix} 0 & 0 \\ 3 & 6 \end{vmatrix} = -8(0-0) = 0$

$C_{22} = 1 \begin{vmatrix} 2 & 0 \\ -5 & 6 \end{vmatrix} = 1(12-0) = 12$

$C_{23} = 0 \begin{vmatrix} 2 & 0 \\ -5 & 3 \end{vmatrix} = 0(6-0) = 0$

$C_{31} = 5 \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 5(0-0) = 0$

$C_{32} = -3 \begin{vmatrix} 2 & 0 \\ 8 & 0 \end{vmatrix} = -3(0-0) = 0$

$C_{33} = 6 \begin{vmatrix} 2 & 0 \\ 8 & 1 \end{vmatrix} = 6(2-0) = 12$

matrix is

$$\begin{bmatrix} 12 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & 12 \end{bmatrix}$$

Adjoint is

$$Adj(A) = \begin{bmatrix} 12 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & 12 \end{bmatrix}$$

Then:

$$A^{-1} = \frac{1}{\det(A)} \cdot Adj(A) = \frac{1}{12}$$

$$\begin{bmatrix} 12 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & 12 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Ans:

20) $$A = \begin{bmatrix} 1 & 3 & 1 & 1 \\ 2 & 5 & \cancel{8}2 & 2 \\ 1 & 3 & 8 & 9 \\ 1 & 3 & 2 & 2 \end{bmatrix}$$

6 48 108 10

$$Det(A) = \begin{bmatrix} 1 & 3 & 1 & 1 & 1 & 3 & 1 \\ 2 & 5 & 2 & 2 & 2 & \cancel{8}5 & \cancel{8}2 \\ 1 & 3 & 8 & 9 & 1 & 3 & 8 \\ 1 & 3 & 2 & 2 & 1 & 3 & 2 \end{bmatrix}$$

80 54 6

$$= 140 - 172 = -32$$

$$\det\begin{bmatrix}1 & 3 & 1 & 1\\ 2 & 5 & 2 & 3\\ 1 & 3 & 8 & 9\\ 1 & 3 & 2 & 2\end{bmatrix} \Rightarrow C_4 = C_4 - C_3$$

$$= \det\begin{bmatrix}2 & 5 & 2\\ 1 & 3 & 8\\ 1 & 3 & 2\end{bmatrix}$$

cofactor of A are,

$$C_{11} = 2\begin{vmatrix}3 & 8\\ 3 & 2\end{vmatrix} = 2(6-24) = -36$$

$$C_{12} = -5\begin{vmatrix}1 & 8\\ 1 & 2\end{vmatrix} = -5(2-8) = -30$$

$$C_{13} = 2\begin{vmatrix}1 & 3\\ 1 & 3\end{vmatrix} = 2(3-3) = 0$$

$$C_{21} = 1\begin{vmatrix}5 & 2\\ 3 & 2\end{vmatrix} = -1(10-6) = 4$$

$$C_{22} = 3\begin{vmatrix}2 & 2\\ 1 & 2\end{vmatrix} = 3(4-2) = 6$$

$$C_{23} = 3\begin{vmatrix}2 & 5\\ 1 & 3\end{vmatrix} = -8(6-5) = -8$$

$$C_{31} = 1\begin{vmatrix}5 & 2\\ 3 & 8\end{vmatrix} = 1(40-6) = 34$$

$$C_{32} = 3\begin{vmatrix} 2 & 2 \\ 1 & 8 \end{vmatrix} = -3(16-2) = -42$$

$$C_{33} = 2\begin{vmatrix} 2 & 5 \\ 1 & 3 \end{vmatrix} = 2(6-5) = 2$$

matrix

$$\begin{bmatrix} 36 & 30 & 0 \\ -4 & 6 & -8 \\ 34 & -42 & 2 \end{bmatrix}$$

Adjoin is

$$Adj(A) = \begin{bmatrix} 36 & -4 & 34 \\ 30 & 6 & -42 \\ 0 & -8 & 2 \end{bmatrix}$$

Then:

$$A^{-1} = \frac{1}{\det(A)} \cdot Adj(A) = \frac{1}{-32}$$

$$\begin{bmatrix} 36 & -4 & 34 \\ 30 & 6 & -42 \\ 0 & -8 & 2 \end{bmatrix} = \begin{bmatrix} \frac{36}{31} & \frac{-4}{-32} & \frac{34}{-32} \\ \frac{30}{-32} & \frac{6}{-32} & \frac{-42}{32} \\ \frac{0}{-32} & \frac{-8}{-32} & \frac{2}{-32} \end{bmatrix}$$

Ar:

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$$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 9 & 3 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & 3/9 & 0 & 1/9 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 1/9 & -3/9 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

Thus.

$$A^{-1} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1/4 & 3/9 \\ 0 & 0 & 1 \end{bmatrix}$$ An:

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$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{array}\right]$$

since we have a row of zeros on the right side, A is not intertible Ans: