INTRODUCTION TO CONIC AND ITS PARTS

- INTRODUCTION OF EACH AND EVERY PARTS.
- EXAMPLE QUESTION WITH SOLUTION
- PRACTICE HOMEWORKS QUESTIONS

INTRODUCTION TO CONIC AND ITS PARTS

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TOPICS:

- PARABOLA
- CIRCLE
- ELLIPSE
- HYPERBOLA

PARABOLA

PARABOLA:-

- ***DEFINITION:-***
  - Parabola is a locus of a point p(x,y) which moves such that it's distance from the fixed point is same as that of its distance from the fixed line.

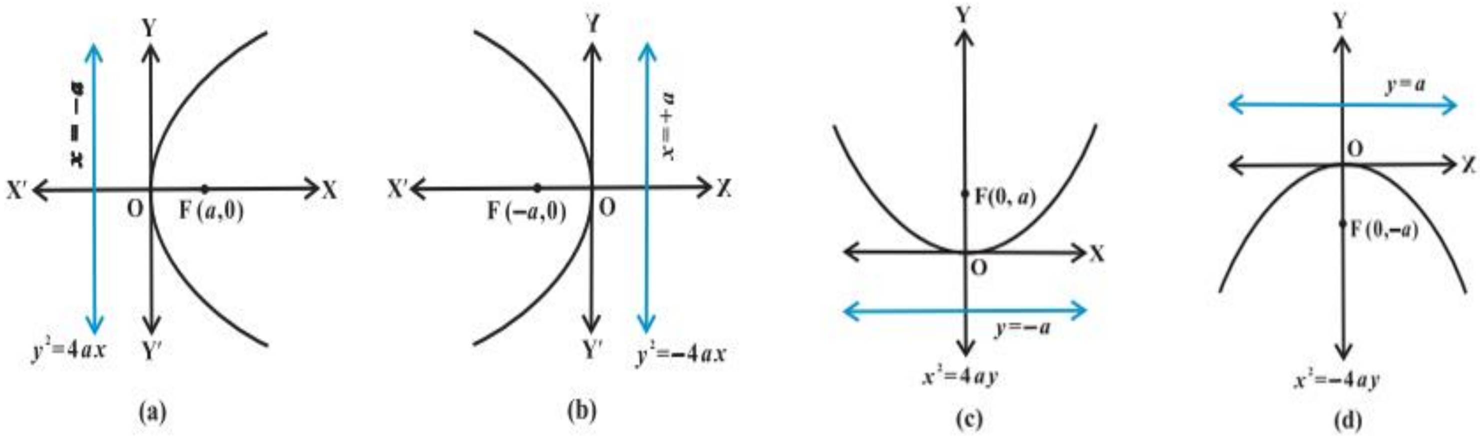
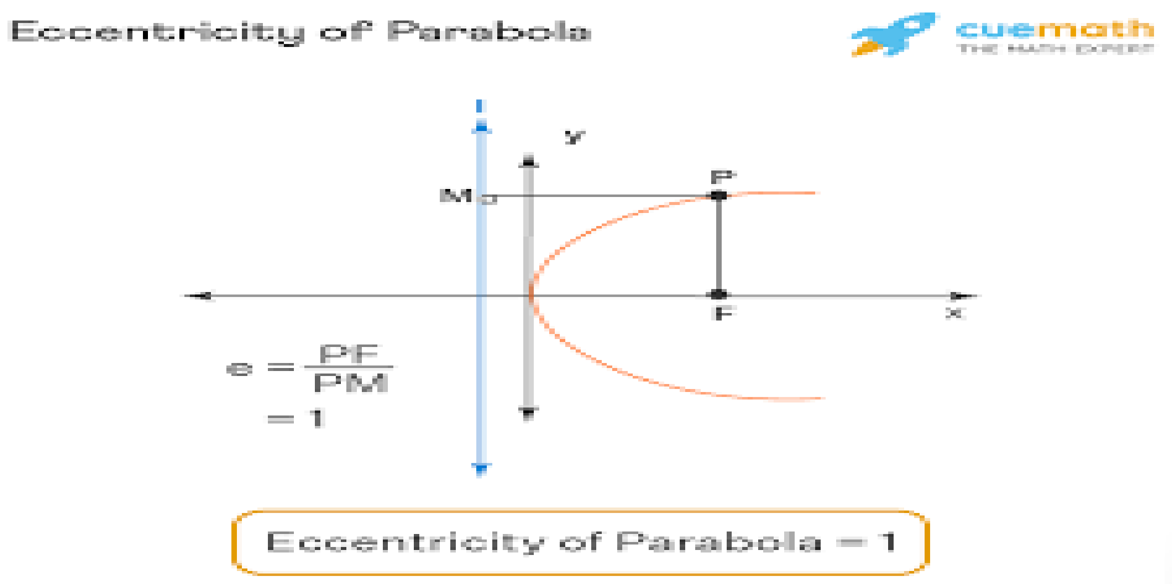


Fig. 2

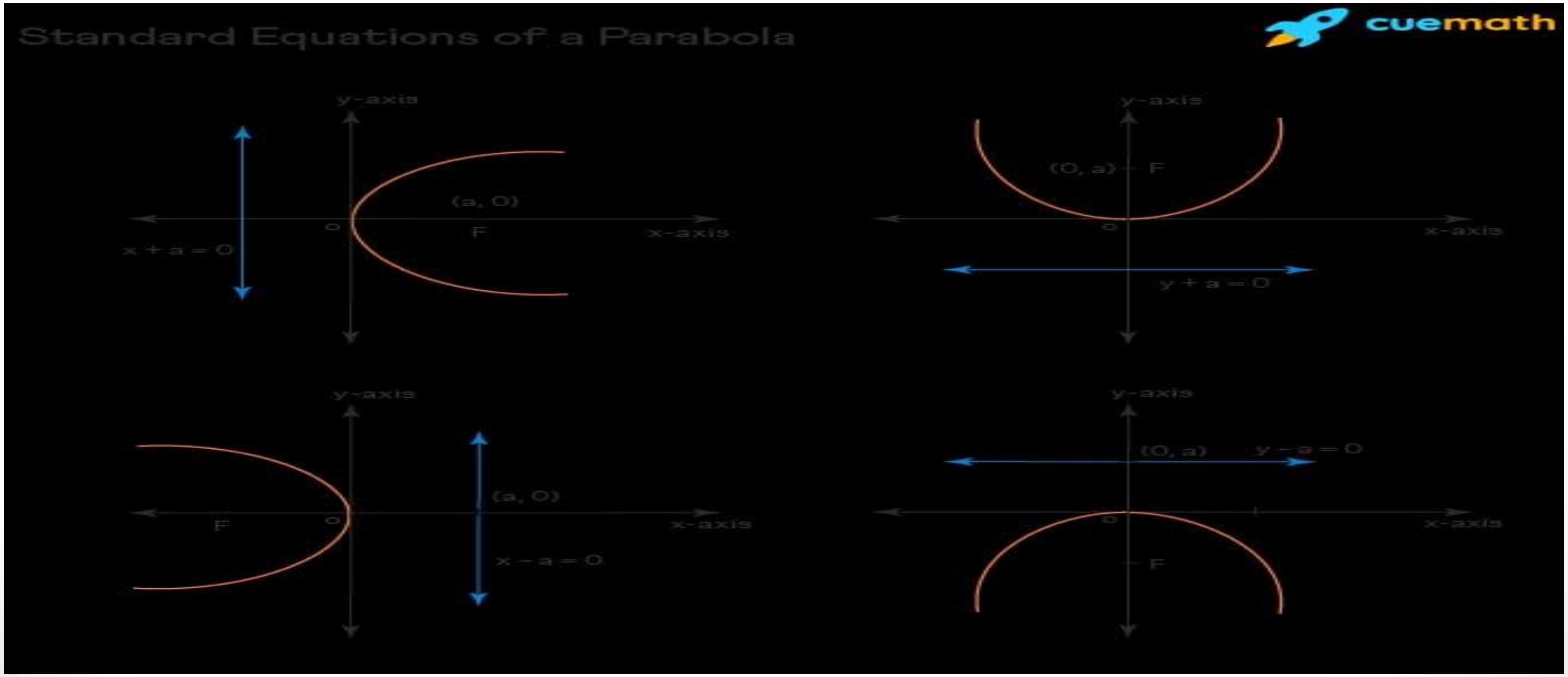
ECCENTRICITY:-

- ***DEFINITION:-***
  - ***Ratio between distance of point to focus and distance of point to directrix is called eccentricity.PF/PL=1.***



STANDARD PARABOLA:-

- ***POINTS:-***
- ***A parabola whose focus is at (a,0)and directrix is x+a=0.Note:Vertex is at origin.***
- ***Focus of standard parabola is=(a,0)Directrix of standard parabola is (x+a)=0.***



QUESTION:-

- ***CONSIDER THE PARABOLA (y^2=2x),find its focus and directrix.***

- ***SOLUTION:-***
  - ***Y^2=2x***
  - ***Y^2=4ax***
  - ***4a=2***
  - ***a=1/2***
  - ***Focus→(a,0)(-a,0)→(1/2,0)(-1/2,0)***
  - ***Vertex→(0,0)***
  - ***Directrix→x=-a→x=-1/2***

PRACTICE QUESTION:-

- ***QUESTION NO 01:-***
  - FIND THE VERTEX,FOCUS,LENGTH MAJOR AXIX AND MINOR AXIS,ECCENTRICITY,LACTUS RECTUM OF THE FOLLOWING EQUATION:-

1. y^2=8ax
2. X^2=32ax
3. Y^2-36ax=0
4. -49ax=-x^2

CIRCLE

CIRCLE:

Circle is a round shaped figure that has no edge of corners.

The shape or object which has 360 degree is called circle.

DEFINITION:

As we know that circle is a round figure whose boundary (the circumference) consists of points equidistant From a fixed point (the center)

PROPERTIES:

There are three main properties of circle:

1. **CIRCUMFERENCE:**The circumference is the distance around the outer edge of a circle.

2. **DIAMETER:** A straight line passing from side to side through the centre of a body or figure, especially a circle or sphere.

3. **RADIUS:**The radius is the distance from the center of a circle to any point on its edge.

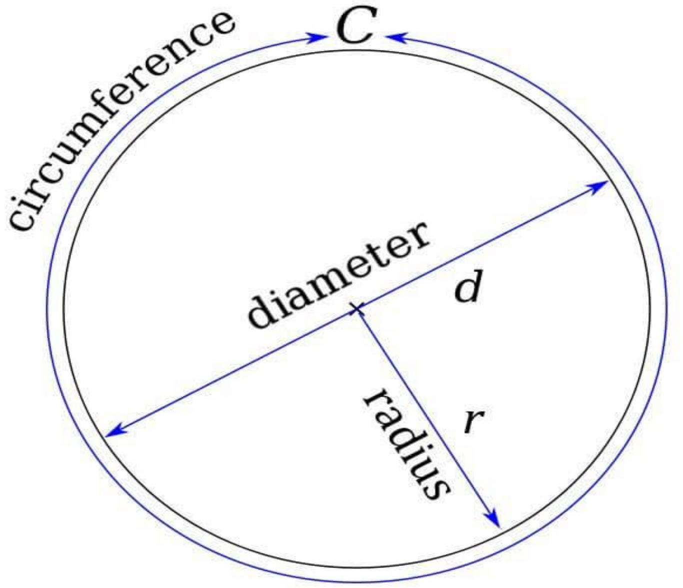
POINTS:-

- WHAT IS CHORD AND TANGENT LINE?
- 1. CHORD: Chord is a line segment that connects two points on a curve.
- 2. TANGENT: A tangent line is a line that touches a curve at a single point.

- FORMULA’S OF CIRCLE:
- 1. Area: $A = \pi \times r2$
- 2. Circumference Of Circle: $C = 2 \times \pi \times r$
- 3. Diameter: $D = 2r$
- 4. Radius: $r=d/2$

QUESTION:-

- GENERAL EQUATION OF A CIRCLE: (x – h)2 + (y – k)2 = r2
- Given center: (h, k) = (3, -2)
- Given radius: r = 5
- · $(x - 3)^2 + (y - (-2))^2 = 5^2$
- · $(x - 3)^2 + (y + 2)^2 = 25$
- · $x^2 - 6x + 9 + y^2 + 4y + 4 = 25$
- · $x^2 + y^2 - 6x + 4y + 13 = 25$
- · $x^2 + y^2 - 6x + 4y - 12 = 0$
- · $x^2 + y^2 - 6x + 4y - 12 = 0$
- This equation represents all the points that are 5 units away from the center (3, -2), forming a circle with the given properties.

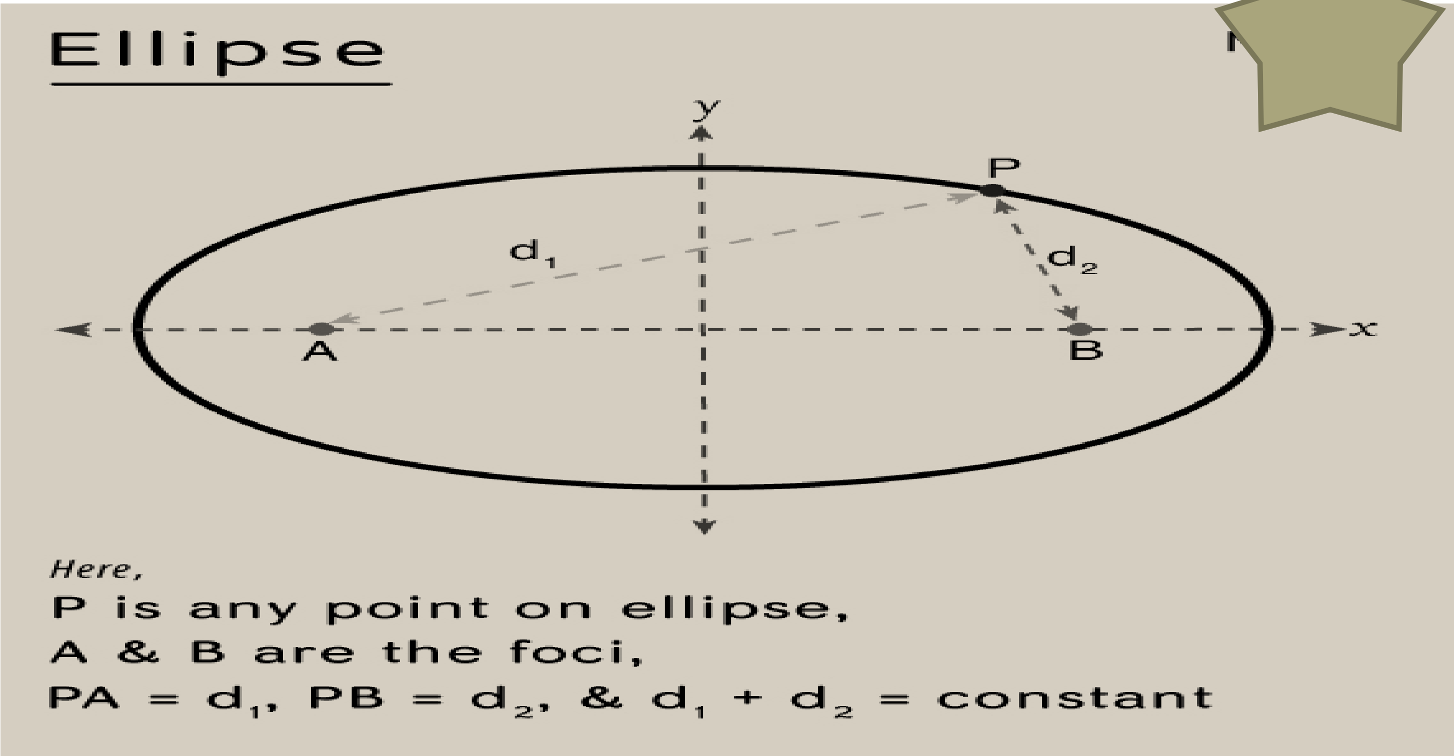


ELLIPSE

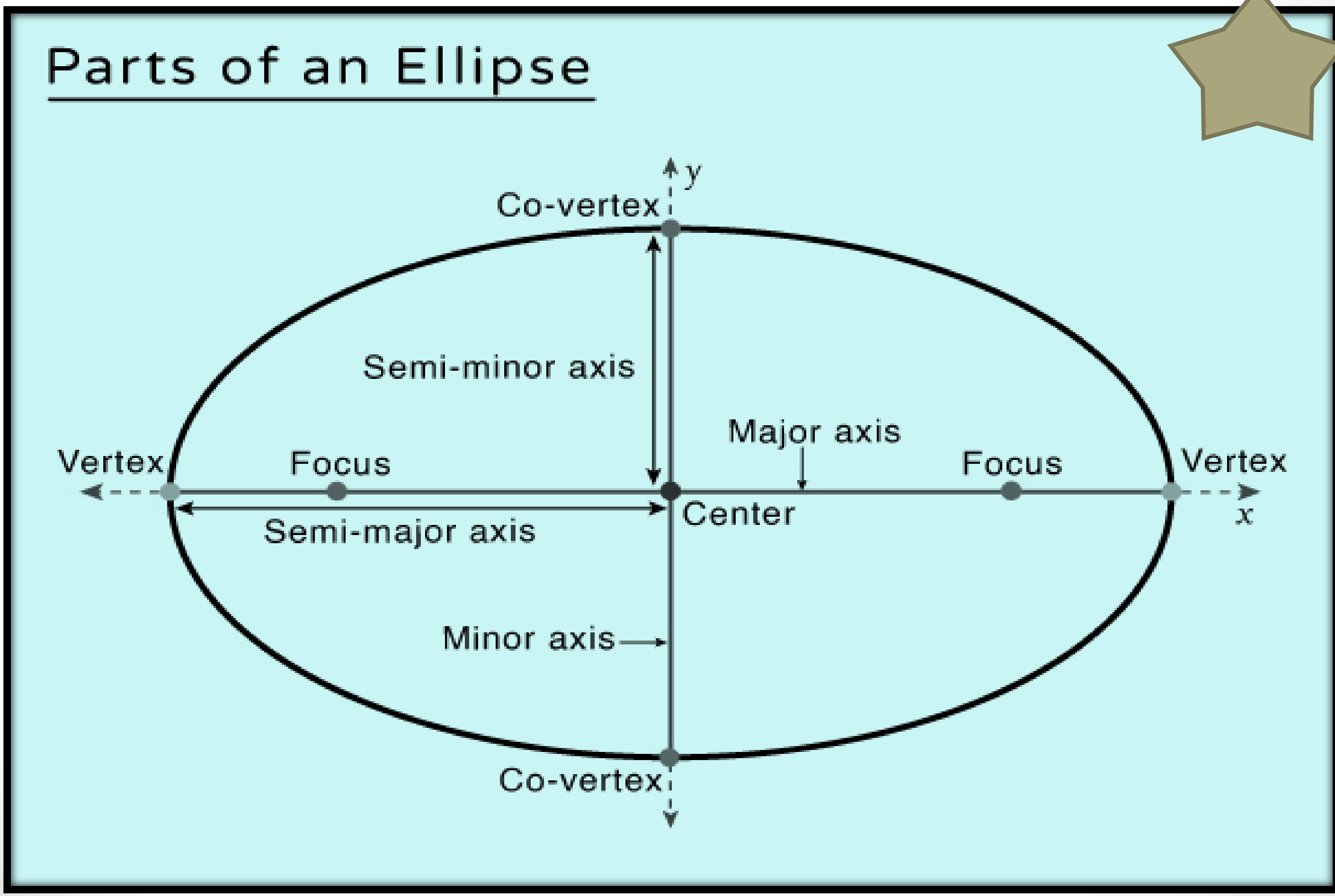
ELLIPSE

ELLIPSE:

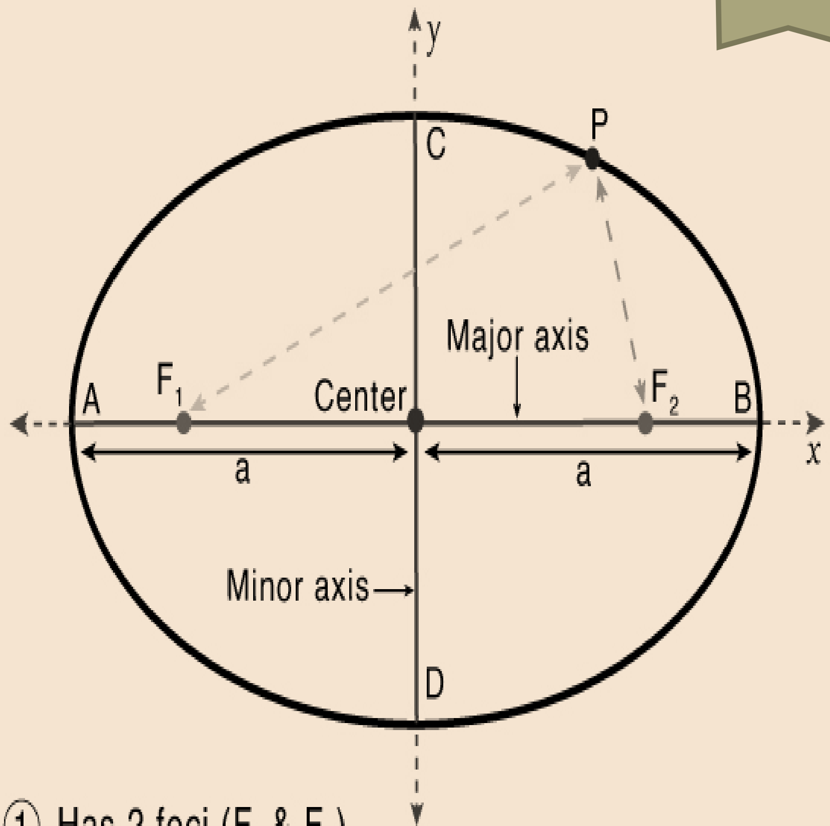
An ellipse is the set of all points in a plane such that the sum of the distances from two fixed points (foci) is constant. An ellipse can be formed by intersecting a cone with a plane that is not parallel to the base or the apex of the cone..



PARTS OF ELLIPSE

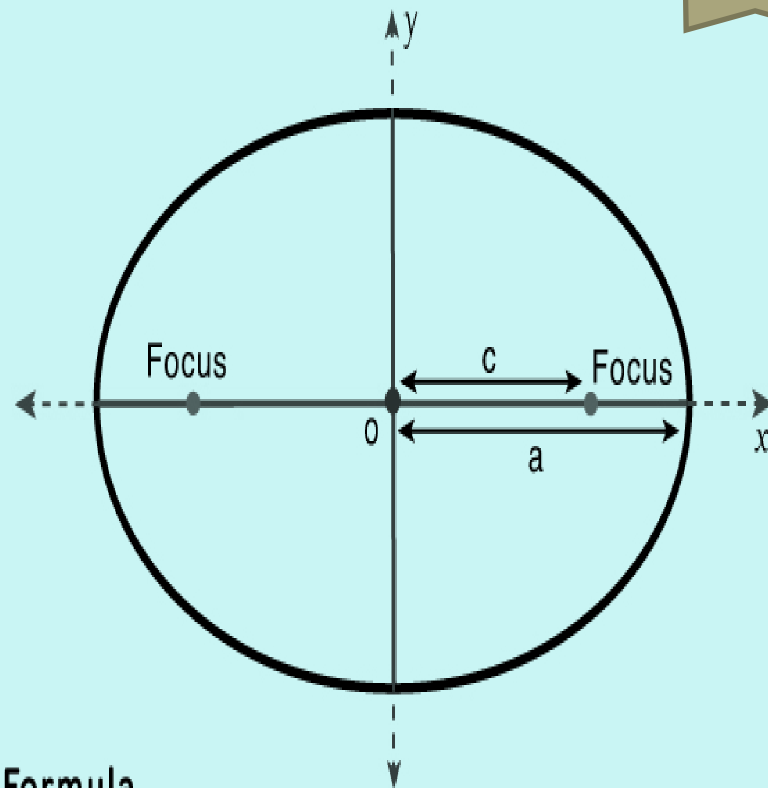


Properties of an Ellipse



1. Has 2 foci (F_1 & F_2)
2. Has a center and 2 axes - major (AB) & minor (CD)
3. Sum of the distances from the 2 foci to any point on the ellipse is constant and is equal to the length of the major axis (2a)
4. Eccentricity $e<1$

Eccentricity of an Ellipse



Formula

$$\text{Eccentricity (e)} = \frac{c}{a}$$

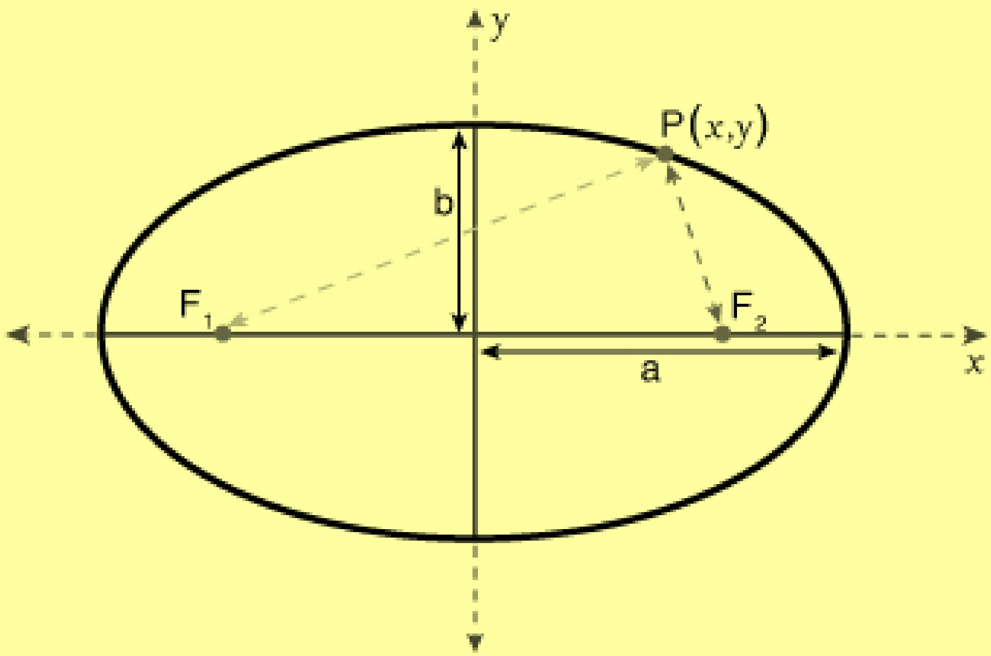
here,

$c = \frac{1}{2}$ the distance between the 2 foci,

a = semi-major axis

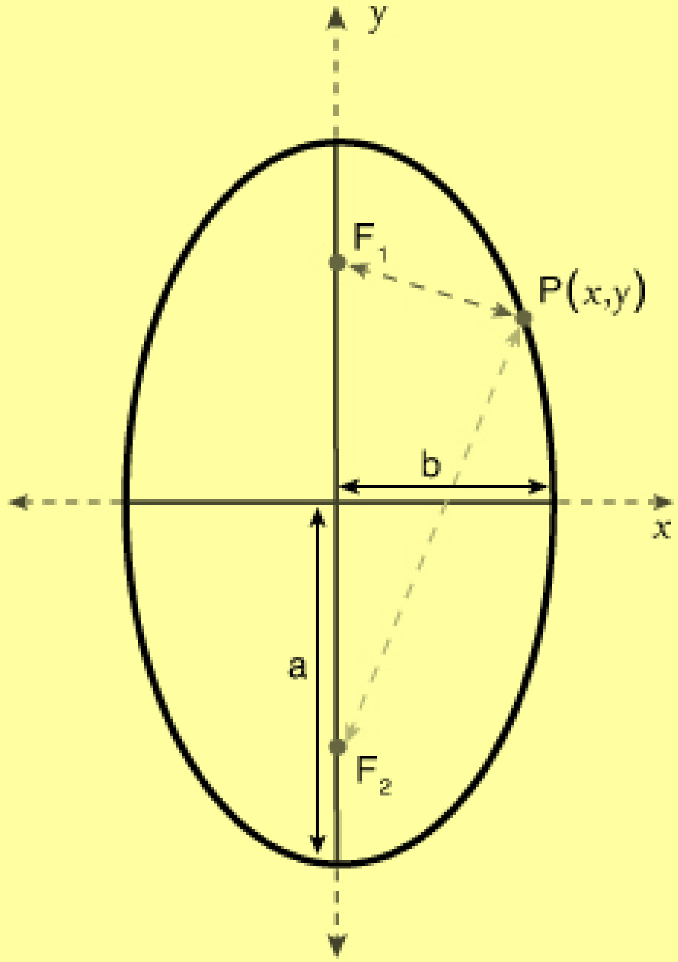
Equation of an Ellipse

Foci on x-axis



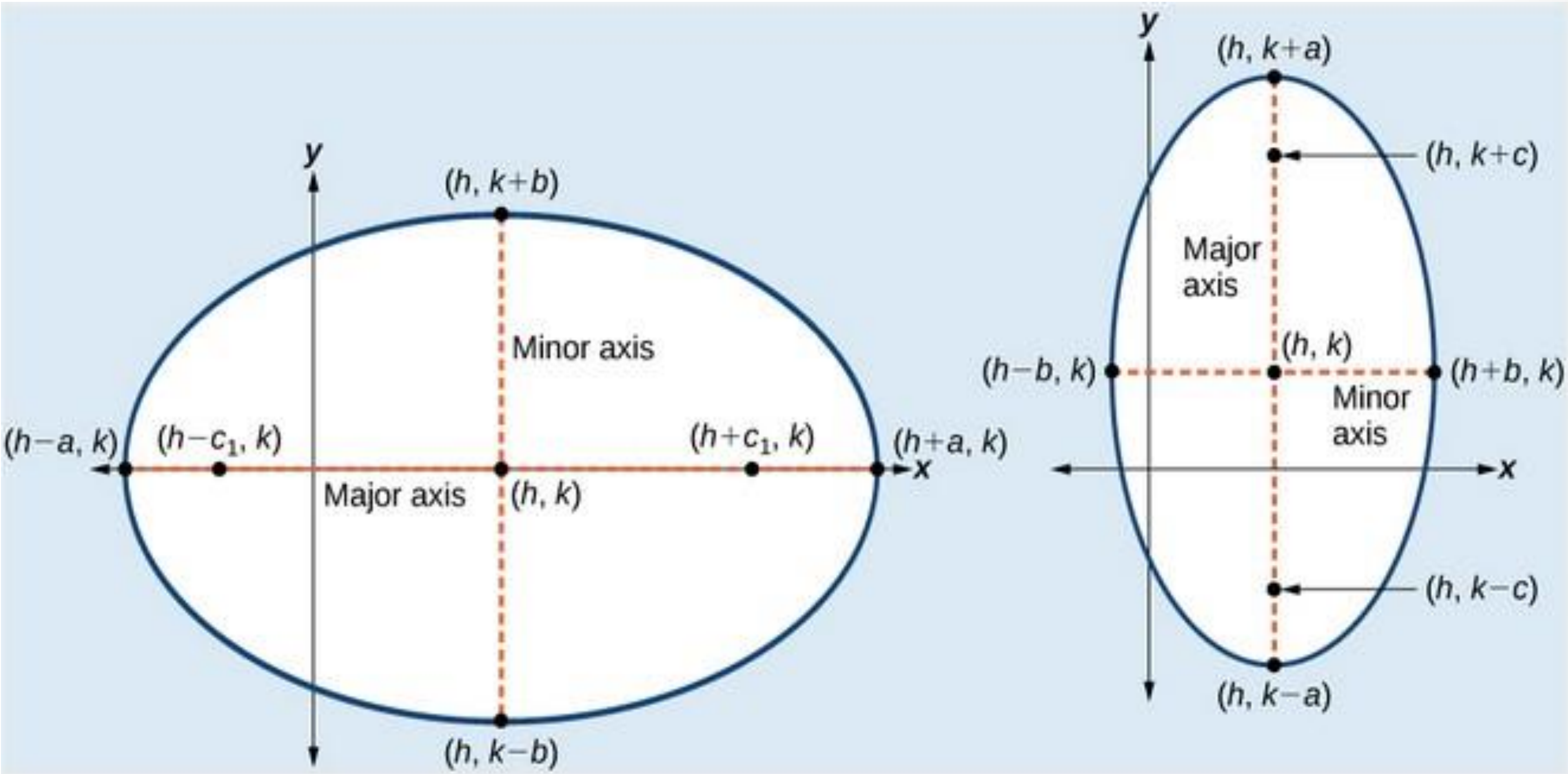
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Foci on y-axis



$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

EQUATION OF AN ELLIPSE (NOT AT ORIGIN)



General formula for Translation of ellipse by h,k $(T_{h,k})$

Vertical Major Axis

$$\boxed{\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1}$$

Horizontal Major Axis

$$\boxed{\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1}$$

ELLIPSE WHOSE CENTRE AT ORIGIN:

Properties	along x-axis	along y-axis
Centre	$(0,0)$	$(0,0)$
Foci	$(\pm c, 0)$	$(0, \pm c)$
Vertex	$(\pm a, 0)$	$(0, \pm a)$
end point of minor axis	$(0, \pm b)$	$(\pm b, 0)$
Directrix	$x = \pm a/e$	$y = \pm a/e$
eq of Ellipse	$x^2/a + y^2/b = 1$	$x^2/b^2 + y^2/a^2 = 1$
Length of Major axis	$2a$	$2a$
Length of Minor axis	$2b$	$2b$
semi major axis	a	a
semi minor axis	b	b
Lectum Rectum	$2b^2/a$	$2b^2/a$

ELLIPSE WHOSE CENTRE NOT AT ORIGIN;

Properties	along x-axis	along y-axis
Centre	(h, k)	(h, k)
foci	$(h \pm c, k)$	$(h, k \pm c)$
Vertex	$(h \pm a, k)$	$(h, k \pm a)$
end point of minor axis	$(h, k \pm b)$	$(h \pm b, k)$
eq of Directrix	$x = h \pm a/e$	$y = k \pm a/e$
eq of ellipse	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2}$	$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2}$

EXAMPLE QUESTION:-

- ***FIND ALL THE PARTS OF THE GIVEN EQUATION.(x^2+25y^2=25)***

$$\frac{x^2}{25} + \frac{25y^2}{25} = \frac{25}{25}$$

$$\frac{x^2}{25} + \frac{y^2}{1} = 1$$

Comparing it with;

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

i.e

$$a^2 = 25$$

$$\boxed{a = 5}$$

$$b^2 = 1$$

$$\boxed{b = 1}$$

$$\because b^2 = a^2 - c^2$$

$$c^2 = a^2 - b^2$$

$$c^2 = 25 - 1$$

$$c^2 = 24$$

$$\boxed{c = 2\sqrt{6}}$$

Centre: $(0,0)$: $(0,0)$

Vertices: $(\pm a, 0)$: $(\pm 5, 0)$

Foci: $(\pm c, 0)$: $(\pm 2\sqrt{6}, 0)$

eccentricity: $\pm \frac{c}{a}$: $\pm \frac{2\sqrt{6}}{5}$

Semi major axis: 5

semi minor axis: 1

EXAMPLE QUESTION:-

- FIND THE ECCENTRICITY OF AN ELLIPSE WHOSE LATUS RECTUM IS EQUAL TO HALF OF ITS MAJOR AXIS.

Q11 :~

The given information are:

latus rectum $= \frac{1}{2}(2a)$

$$\frac{2b^2}{a} = \frac{1}{\cancel{2}}(\cancel{2}a)$$

$$\frac{2b^2}{a} = a$$

$$2b^2 = a^2$$

$$2(a^2 - c^2) = a^2 \qquad [b^2 = a^2 - c^2]$$

$$2a^2 - 2c^2 = a^2$$

$$2a^2 - 2e^2a^2 - a^2 = 0 \qquad [c = ea]$$

$$a^2 - 2e^2a^2 = 0$$

$$a^2(1 - 2e^2) = 0$$

$$1 - 2e^2 = 0$$

$$\boxed{e = 1/\sqrt{2}} \qquad \text{ANS;}$$

PRACTICE QUESTIONS:-

- ***QUESTION NO 01:-***
  - FIND THE ECCENTRICITY OF AN ELLIPSE WHOSE LATUS RECTUM IS EQUAL TO HALF OF ITS MAJOR AXIS.
- ***QUESTION NO 02:-***
  - THE LENGTH OF THE MAJOR AXIS OF AN ELLIPSE IS 25 AND ITS FOCI ARE THE POINTS (5,0)(-5,0);FIND THE EQUATION OF THE ELLIPSE.
- ***QUESTION NO 03:-***
  - FIND THE ECCENTRICITY OF THE ELLIPSE WHOSE AXES ARE 12 AND 24.

Hyperbola

Difference b/w two fixed point is known as Hyperbola

Eccentricity = Greater than 1…

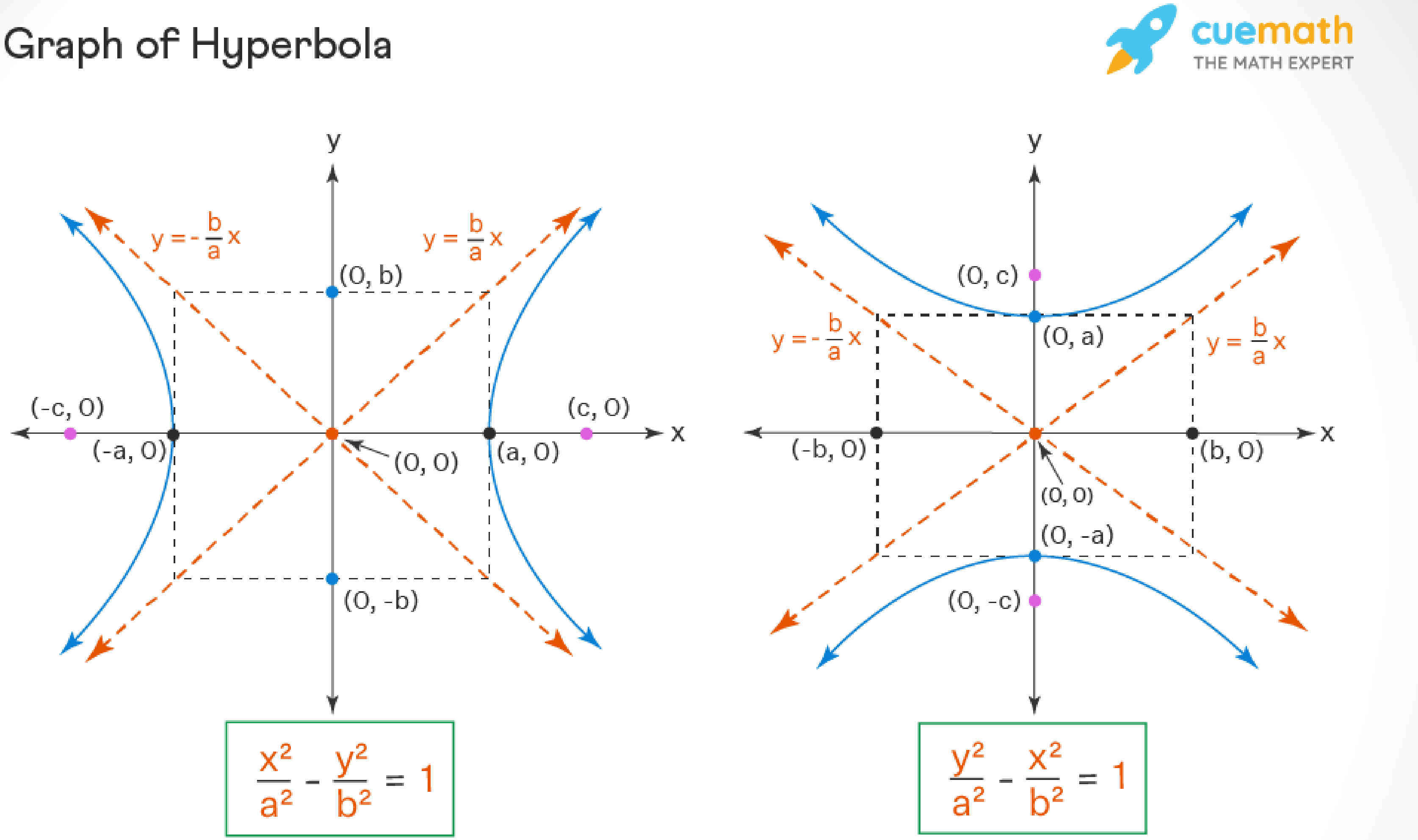
- ***Along X Axis:***

- Center (0,0)
- Foci (+c,0), (-c,0)
- Vertex (+a,0), (-a,0)
- Asymptotes: b/a (x), -b/a (x)
- Eq of hyperbola : x^2/a^2 – y^2/b^2 = 1

- ***Along Y Axis:***

- Center (0,0)
- Foci (0,c), (0,-c)
- Vertex (0,a), (0,-a)
- Asymptotes: a/b (x), -a/b (x)
- Eq of hyperbola : y^2/a^2 – x^2/b^2 = 1

- As we know that: c^2=a^2+b^2



HORIZONTAL HYPERBOLA **VERTICAL HYPERBOLA**

1. Graph the hyperbola. Identify the coordinates of the center, vertices, and foci. Write the equations of the asymptotes.

$$\frac{x^2}{4} - \frac{y^2}{9} = 1$$

$a^2 = 4$ $\quad V(\pm a, 0)$

$a = 2$ $\quad \boxed{V(\pm 2, 0)}$

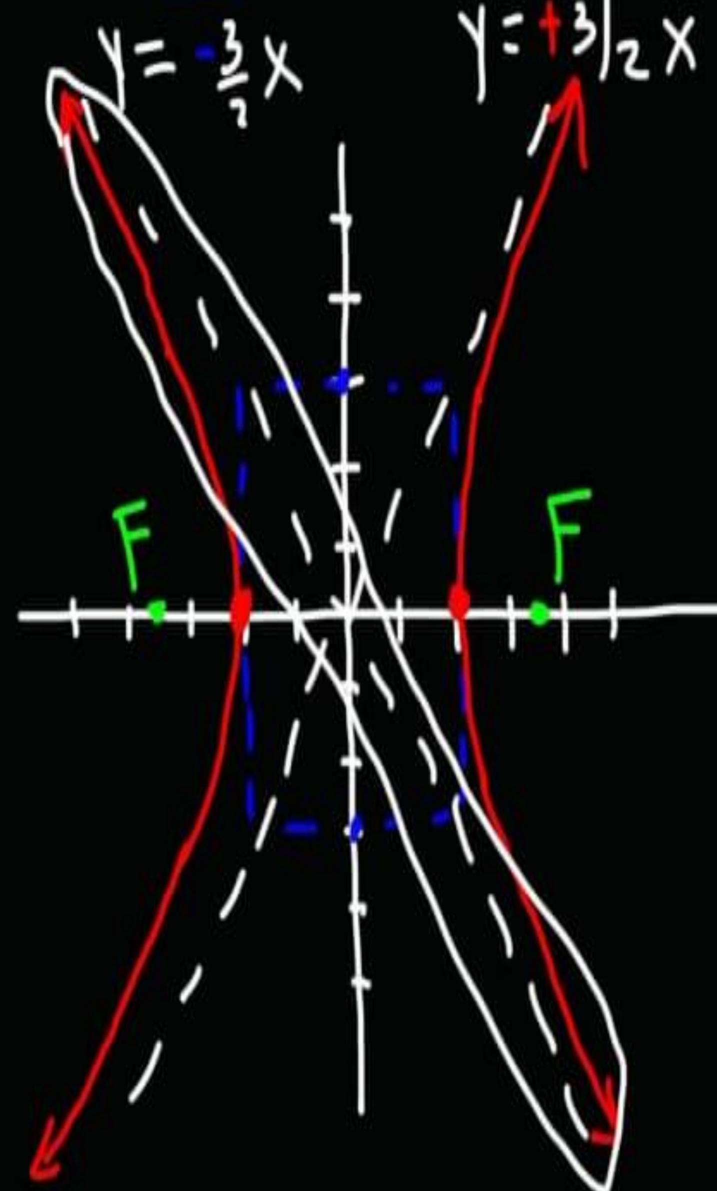
$b^2 = 9$ $\quad \boxed{F(\pm\sqrt{13}, 0)}$

$b = 3$

$c = \sqrt{13}$ $\quad y = \pm\frac{b}{a}x$

$$\boxed{y = \pm\frac{3}{2}x}$$

$C(0,0)$



2. Graph the hyperbola. Identify the coordinates of the center, vertices, and foci. Write the equations of the asymptotes.

$$\frac{y^2}{16} - \frac{x^2}{9} = 1$$

$\boxed{C(0,0)}$

$\boxed{F(0,\pm 5)}$

$a^2 = 16$

$a = 4$

$b^2 = 9$

$b = 3$

$c = 5$

$\boxed{V(0,\pm 4)}$

$y = \pm m x$

$y = \pm \frac{a}{b} x$

$\boxed{y = \pm \frac{4}{3} x}$

Along x – axis:

- Center (h,k)
- Foci (h+c,k), (h-c,k)
- Vertex (h+a,k), (h-a,k)
- Asymptotes: y-k=b/a (x-h), -b/a (x-h)
- Eq of hyperbola: (x-h)^2/a^2 – (y-k)^2/b^2 = 1

Along y – axis:

- Center (h,k)
- Foci (h,k+c), (h,k-c)
- Vertex (h,k+a), (h,k-a)
- Asymptotes: y-k=a/b (x-h), -a/b (x-h)
- Eq of hyperbola: -(x-h)^2/b^2 + (y-k)^2/a^2 = 1

- As we know that : c^2=a^2+b^2

3. Graph the hyperbola. Identify the coordinates of the center, vertices, and foci. Write the equations of the asymptotes.

$$\frac{(x-3)^2}{4} - \frac{(y+2)^2}{9} = 1$$

Asymptotes

$h \quad k$

$\boxed{C(3,-2)}$

$$y-k = \pm\frac{b}{a}(x-h)$$

$$\boxed{y+2 = \pm\frac{3}{2}(x-3)}$$

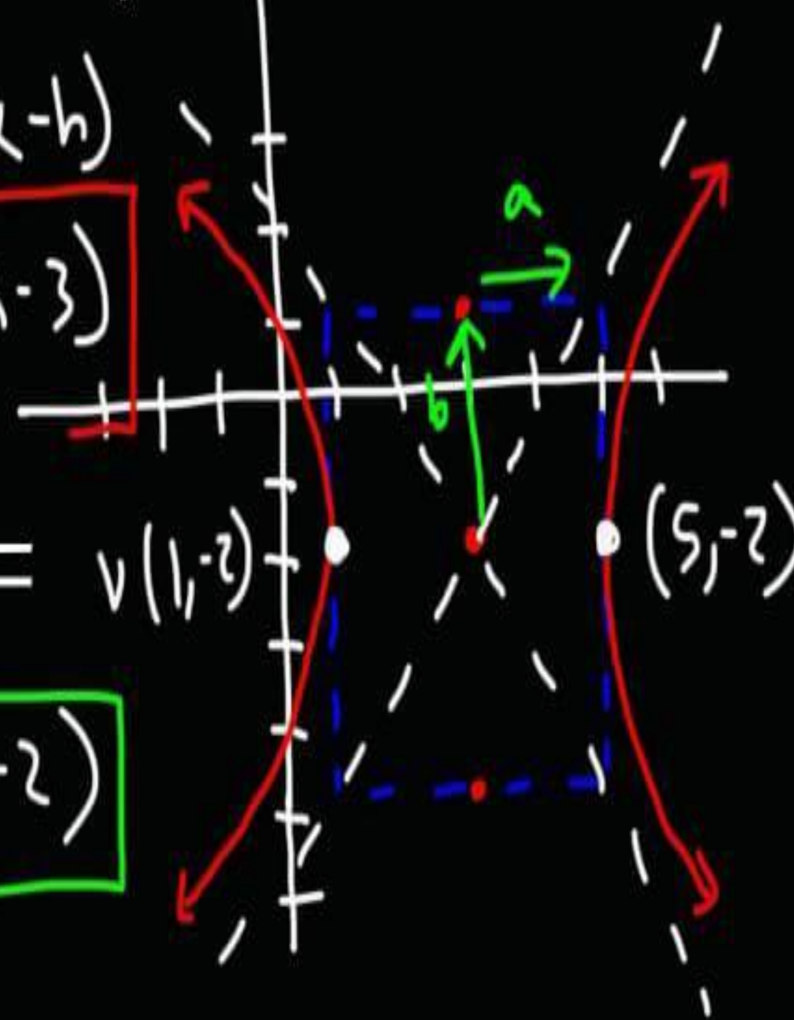
$a = 2$

$b = 3$

$c = \sqrt{13}$

$V(h+a,k) =$

$F(h \pm c, k) = \boxed{F(3 \pm \sqrt{13}, -2)}$



4. Graph the hyperbola. Identify the coordinates of the center, vertices, and foci. Write the equations of the asymptotes. Find the domain and range of the graph.

$$+\frac{(y-1)^2}{9} - \frac{(x-2)^2}{16} = 1$$

$a^2 = 9$ $\quad F(h, k \pm c)$

$a = 3$ $\quad F(2, 1 \pm 5)$

$b^2 = 16$

$b = 4$

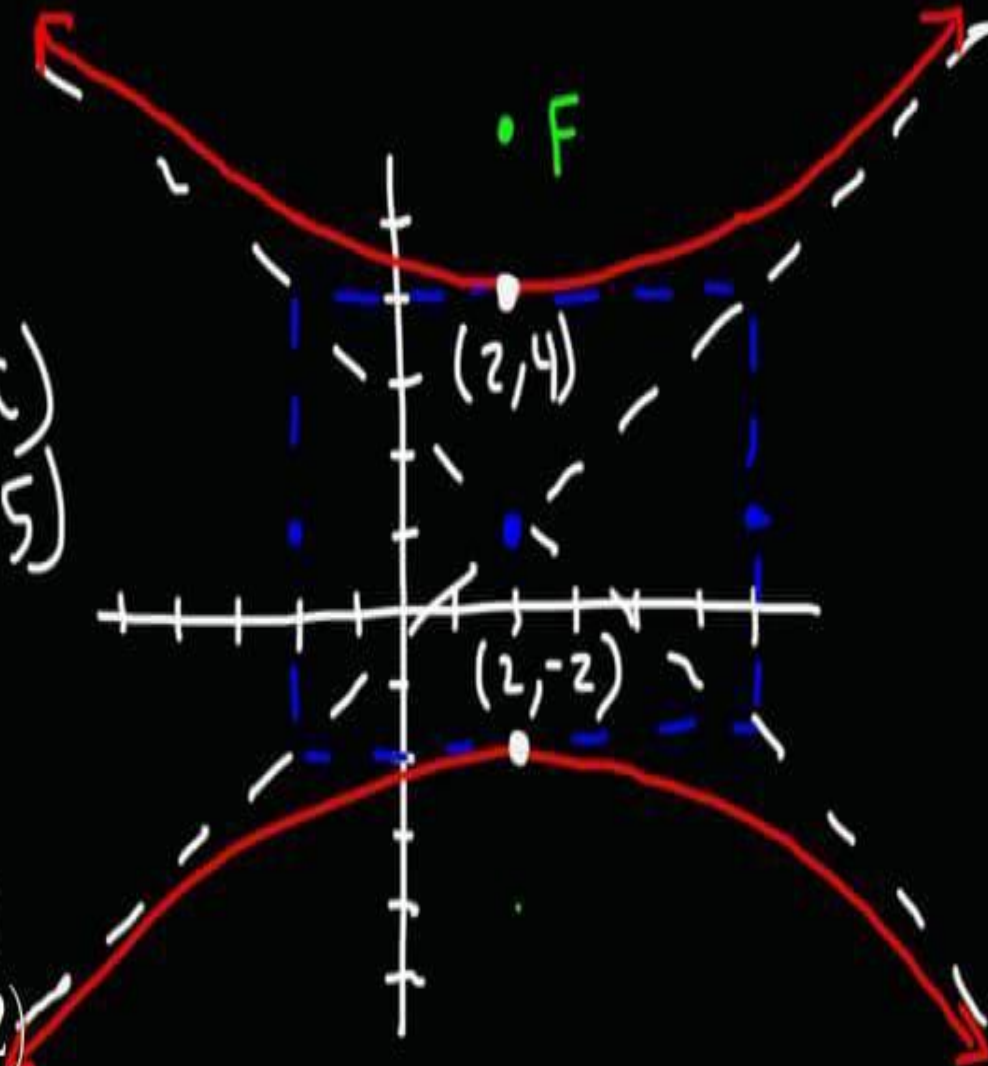
$c = 5$

$C(\underset{h}{2}, \underset{k}{\underline{1}})$

Asymptotes

y-k=a/b(x-h)

y-1=3/4(x-2)



- ***Solve the following questions :***
- Q1. Find foci, vertices and the following hyperbola.
- $(x ^ 2)/4 - (y ^ 2)/5 = 1$

- Q2. Find center, foci, vertices and Asymptotes of the following hyperbola.
- $(y ^ 2)/4 - (x^2)/5 = 1$

CONCLUSION

- A more unique conclusion of conic sections and their parts is that they are all closely related to each other. In fact, they can all be generated from a single equation:
- Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0
- This equation is called the general conic equation, and it can be used to represent any type of conic section, depending on the values of the coefficients A, B, C, D, E, and F.
- Another unique conclusion is that conic sections can be used to model a wide variety of natural phenomena. For example, the orbits of planets and comets are conic sections, as are the paths of light rays through lenses and mirrors. Conic sections are also used in the design of many man-made objects, such as bridges, telescopes, and satellite dishes.
- In short, conic sections are a beautiful and powerful tool that can be used to understand and model the world around us.
- Here is a more creative conclusion:
- Imagine a universe where the laws of physics are different, and conic sections no longer exist. What would happen?
- Without conic sections, the orbits of planets and comets would be unpredictable. Light rays would not travel in straight lines. And many of the man-made objects that we rely on every day would not be possible.
- In short, our universe would be a very different place without conic sections.
- I hope this is more unique and interesting conclusion for you.

THANKS FOR YOUR TIME

THANKS YOU ALL