Prof. Dr.-Ing. A. Bruhn
Institute for Visualization and Interactive Systems
Computer Vision Group
University of Stuttgart

Assignment C2 – Classroom Work

Problem 1 (Properties of the Continuous Fourier Transform)

Prove the following properties of the continuous Fourier transform for 1-D signals:

(a) Linearity: $\mathcal{F}[af + bg] \;=\; a\mathcal{F}[f] + b\mathcal{F}[g] \qquad \forall a, b \in \mathbb{R}$

(b) Shift Theorem: $\mathcal{F}[f(x - x_0)](u) \;=\; e^{-i2\pi u x_0}\mathcal{F}[f](u)$

(c) Convolution Theorem: $\mathcal{F}[f * g](u) = \mathcal{F}[f](u) \cdot \mathcal{F}[g](u)$

Problem 2 (Discrete Fourier Transform)

For complex vectors $\boldsymbol{f} = (f_i)_{i=0}^{M-1}$ and $\boldsymbol{g} = (g_i)_{i=0}^{M-1}$, one defines their Hermitian inner product as $\langle \boldsymbol{f}, \boldsymbol{g} \rangle := \sum\limits_{m=0}^{M-1} f_m \bar{g}_m$ where $\bar{g}_m$ is the complex conjugate of g_m.

Show that with respect to this inner product, the M vectors

$$\boldsymbol{v}_p := \frac{1}{\sqrt{M}} \left(\exp\left(\frac{2\pi i\, p\, 0}{M}\right), \exp\left(\frac{2\pi i\, p\, 1}{M}\right), ..., \exp\left(\frac{2\pi i\, p\, (M-1)}{M}\right)\right)^\top$$

with $p = 0,...,M-1$ form an orthonormal basis of the M-dimensional complex vector space $\mathbb{C}^M$.

(This property allows to interpret the DFT as a change of basis.)

Assignment H2 – Homework

Problem 1 (Continuous Fourier Transform of a Discrete Filter) (6 points)

Let f be a 2-D continuous signal and g be defined as

$$\begin{aligned} g(x,y) \quad := \quad & \frac{1}{8}\Big(-f\Big(x-1,y-1\Big)+f\Big(x+1,y-1\Big)-2f\Big(x-1,y\Big) \\ & +2f\Big(x+1,y\Big)-f\Big(x-1,y+1\Big)+f\Big(x+1,y+1\Big)\Big) \end{aligned}$$

Compute the Fourier transform $\mathcal{F}[g](u,v)$ and express it in terms of $\mathcal{F}[f](u,v)$.

Problem 2 (Image Pyramids) (8 points)

Let the following 1-D signal be given:

$$f := (20, 16, 24, 6, 18, 8, 16, 8)^\top$$

(a) Calculate the Gaussian pyramid of the signal.

(b) Calculate the Laplacian pyramid of the signal.

(c) Reconstruct the initial signal from the Laplacian pyramid.

(d) The Laplacian pyramid requires more pixels than the original signal.
Where is the redundancy hidden?

Problem 3 (Interpretation of the Fourier Spectrum) (6 points)

Please download the archive `is21_ex02.tgz` from ILIAS into your own directory. You can unpack them with the command `tar xvzf is21_ex02.tgz`.

In this archive you can find the programme `fourierspectrum`. It computes the logarithmically transformed Fourier spectrum $c\ln(1+\hat{f}(u,v))$ of an image $f(x,y)$ by means of the FFT. The lowest frequencies have been shifted towards the centre of the image.

Apply this programme to the images `pattern.pgm`, `gauss1.pgm`, `gauss2.pgm`, `gauss3.pgm` and `tile.pgm` by typing `./fourierspectrum`, and visualise them by using `display image_name.pgm &`.

(a) Why do you observe a three-point spectrum for `pattern.pgm` and why it is located this way?

(b) Why is the DFT of `gauss3.pgm` not rotationally symmetric?

(c) Can you find aliasing artifacts in the spectrum of `tile.pgm`? If you can, describe them.
(This image has been downsampled with `display` to half its size.)

Submission

Please remember that up to four people from the same tutorial group can work and submit their results together. The theoretical problems 1 and 2 have to be submitted in a single pdf file, which can be either digitally created or contain a scanned document. For the practical problem 3 you have to submit files as follows: Rename the main directory `Ex02` to `Ex02_<your_name>` and use the command

```
tar czvf Ex02_<your_name>.tgz Ex02_<your_name>
```

to pack the data. The directory that you pack and submit should contain the following files:

- a text file README that contains answers to all questions of problem 3 as well as information on all people working together for this assignment.

Please make sure that only your final version of the programmes and images are included.
Submit the file via ILIAS.
(Remark: Please do **not** use the button "Upload Multiple Files as Zip-Archive".)

Deadline for submission: Tuesday, June 1st, 23:59