In the grading of the exercises special attention will be paid to check that answers are substantiated and that the procedure of calculations is well documented. When results are achieved using a calculator or a pc it should be noted in the paper. All 6 problems are weighted equally in the grading. Your solution to the problem set can be written in Danish or English as you prefer.

**PROBLEM 1.**

1. Determine the solution(s) of the following vector equation

$$x_1 \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -3 \\ -1 \\ 11 \end{bmatrix}$$

Let $B$ be an invertible $n \times n$ matrix.

2. Reduce the expression $B^3B(B^{-1})^TB^{-1}BB^T(BB)B^{-1}(B^{-1})^2$ as much as possible and account for the rules used in each step of the reduction.

**PROBLEM 2.**
Consider the following $2 \times 2$ matrix

$$A = \begin{bmatrix} 1 & q \\ 4 & 2 \end{bmatrix}$$

where $q$ is a parameter that can be adjusted.

1. Calculate the eigenvalues of $A$ as a function of $q$.
2. Calculate $q$ so that $\lambda = 5$ is an eigenvalue of $A$.

**PROBLEM 3.**
For the statements given below, state whether they are true or false and justify your answer for each statement.

1. If $A\mathbf{x} = \mathbf{b}$ has the solution $\mathbf{x} = \mathbf{c}$, then $A\mathbf{x} = 2\mathbf{b}$ has the solution $\mathbf{x} = \frac{1}{2}\mathbf{c}$.
2. The matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ is positive definite.
3. Two vectors, $\mathbf{a}$ and $\mathbf{b}$ are linearly dependent if $\mathbf{a}^T\mathbf{b} \neq 0$.

**PROBLEM 4.**
In case 1 cubic splines was introduced. In this problem we continue this line of thought and determine the parameters of an unknown function satisfying a number of constraints. Let the function $f(x)$ be given by

$$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

The function must pass through the following data points

| $x$ | $f(x)$ |
|---|---|
| 1 | 3 |
| 3 | 2 |

Further, the first and second derivatives of the function must satisfy

$$f'(1) = 3, \quad f''(2) = -1$$

1. Write down the equations needed to determine $a_0$, $a_1$, $a_2$, and $a_3$.
2. Solve the equations and write down the full expression for $f(x)$.

**PROBLEM 5.**
Consider the following matrix and vector

$$A = \begin{bmatrix} 2 & 1 \\ 2 & 0 \\ 1 & 1 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$$

1. Make a QR decomposition of $A$ with the method from the textbook.
2. Solve the equation $A\mathbf{x} = \mathbf{b}$ using the QR decomposition, i.e. substitute $A = QR$ to obtain a simplified problem.

**PROBLEM 6.**
Let a weighted inner product between two vectors in $\mathbb{R}^3$ be given by

$$< \mathbf{x}, \mathbf{y} >= 3x_1y_1 + 2x_2y_2 + x_3y_3,$$

and let two vectors, $\mathbf{x}$ and $\mathbf{y}$ be given by

$$\mathbf{x} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \text{ and } \mathbf{y} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}.$$

1. Calculate $< \mathbf{x}, \mathbf{y} >$, $||\mathbf{x}||$ and the distance between $\mathbf{x}$ and $\mathbf{y}$, $\text{dist}(\mathbf{x}, \mathbf{y})$ using the weighted inner product.
2. Show that the triangle inequality holds for the weighted inner product and the two vectors $\mathbf{x}$ and $\mathbf{y}$.