In the grading of the exercises special attention will be paid to check that answers are substantiated and that the procedure of calculations is well documented. When results are achieved using a calculator or a pc it should be noted in the paper. All 6 problems are weighted equally in the grading. Your solution to the problem set can be written in Danish or English as you prefer.

PROBLEM 1.

Consider the following matrix and vector

$$A = \begin{bmatrix} 1 & -2 & 1 & -1 & 4 \\ 2 & -2 & 5 & 3 & 6 \\ 1 & 3 & 1 & 4 & -1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 7 \\ 1 \\ -3 \end{bmatrix}.$$

1. Show that $\mathbf{x} = \begin{bmatrix} 2 & -1 & -4 & 1 & 2 \end{bmatrix}^T$ is a solution of $A\mathbf{x} = \mathbf{b}$.
2. Compute the general solution of $A\mathbf{x} = \mathbf{b}$ and write the solution in parametric form.

PROBLEM 2.

Consider the matrix A and three vectors $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ given by

$$A = \begin{bmatrix} 5 & -1 & -3 \\ 3 & 1 & -3 \\ 1 & -1 & 1 \end{bmatrix}, \quad \mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

1. Show that the three vectors $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ form a basis for $\mathbb{R}^3$.
2. Show that the three vectors are all eigenvectors for A and determine the eigenvalue corresponding to each eigenvector.

Let a vector be given by

$$\mathbf{y} = \begin{bmatrix} 4 \\ 5 \\ 1 \end{bmatrix}.$$

3. Find the coordinates of $\mathbf{y}$ in the eigenvector basis $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

PROBLEM 3.

For the statements given below, state whether they are true or false and justify your answer for each statement.

1. if A is an $n \times n$ matrix and $\mathbf{b}$ is an $n \times 1$ vector and $A\mathbf{x} = \mathbf{b}$ is inconsistent then $A\mathbf{x} = \mathbf{0}$ have both trivial and nontrivial solutions.
2. If A is an $n \times n$ matrix and all singular values of A are greater than zero, then $A\mathbf{x} = \mathbf{b}$ is consistent for all values of $\mathbf{b}$.
3. If $\mathbf{x}_1(t) = \mathbf{v}_1 e^{\lambda_1 t}$ and $\mathbf{x}_2(t) = \mathbf{v}_2 e^{\lambda_2 t}$ are both solutions of the system of differential equations $\mathbf{x}' = A\mathbf{x}$, then the sum $\mathbf{x}_1(t) + \mathbf{x}_2(t) = \mathbf{v}_1 e^{\lambda_1 t} + \mathbf{v}_2 e^{\lambda_2 t}$ is also a solution of $\mathbf{x}' = A\mathbf{x}$.

PROBLEM 4.

Let a quadratic form be given as

$$Q(\mathbf{x}) = 4x_1^2 - 6x_1x_2 - 10x_1x_3 - 10x_1x_4 - 6x_2x_3 - 6x_2x_4 - 2x_3x_4.$$

As usual, a quadratic form can also be written as $Q(\mathbf{x}) = \mathbf{x}^T A\mathbf{x}$.

1. Find the matrix A of the quadratic form.
2. Determine whether A is positive definite, negative definite or indefinite.
3. Find the maximum and minimum values of $Q(\mathbf{x})$ subject to the constraint $\mathbf{x}^T\mathbf{x} = 1$.

PROBLEM 5.

Let the vector space $\mathbb{P}_2$ have the inner product defined by evaluation at -2, -1, 1 and 2. Let $p_0(t) = 1$, $p_1(t) = t$ and $p_2(t) = t^2$.

1. Compute the distance between p_0 and p_2.
2. Compute the orthogonal projection of p_2 onto the subspace spanned by p_0 and p_1.

Another polynomium is given by $p_a(t) = 1 + c$, where c is a real number.

3. Determine c so that p_2 and p_a are orthogonal.

PROBLEM 6.

In the case *Computer Graphics in Automotive Design*, homogeneous coordinates were introduced. In this problem, homogeneous coordinates in $\mathbb{R}^2$ are used. Consider an arrow-like object $\mathcal{O}$ with nodes $n_1, n_2, \ldots, n_5$ with coordinates

$$C = \{(2,0), (4,2), (4,3), (3,3), (1,1)\}$$

and adjacency matrix

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

1. Sketch the object.
2. Find a translation matrix that moves the object so the tip of the arrow is at the center of the coordinate system.
3. Find a transformation matrix that will keep the tip of the arrow of the object at its original position, but make the arrow point in the opposite direction.