In the grading of the exercises special attention will be paid to check that answers are substantiated and that the procedure of calculations is well documented. When results are achieved using a calculator or a pc it should be noted in the paper. All 6 problems are weighted equally in the grading. Your solution to the problem set can be written in Danish or English as you prefer.

PROBLEM 1.

Consider the following matrix and vector

$$A = \begin{bmatrix} 2 & 1 & 0 & 3 \\ -1 & 4 & 1 & 7 \\ 3 & 2 & 1 & 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 2 \\ 7 \\ 3 \end{bmatrix}.$$

1. Is $\mathbf{x} = \begin{bmatrix} 1 & 2 & 1 & 1 \end{bmatrix}^T$ a solution of $A\mathbf{x} = \mathbf{b}$?
2. Determine the general solution of $A\mathbf{x} = \mathbf{b}$.
3. How many vectors are there in the solution set?

PROBLEM 2.

Let a matrix be given by

$$A = \begin{bmatrix} 4 & -3 \\ 1 & 0 \end{bmatrix}.$$

1. Find the characteristic equation.
2. Calculate, *by hand*, the eigenvalues and eigenvectors of A.
3. Show that the vector $\mathbf{y} = \begin{bmatrix} 9 \\ 1 \end{bmatrix}$ can be written as a linear combination of A's eigenvectors.

PROBLEM 3.

For the statements given below, state whether they are true or false and justify your answer for each statement.

1. The determinant of $\begin{bmatrix} 2a & 2b \\ c & a \end{bmatrix}$ is twice as big as the determinant of $\begin{bmatrix} a & b \\ c & a \end{bmatrix}$.
2. If $\mathbf{x}(t) = \mathbf{v}_1 e^{\lambda_1 t}$ solves $\mathbf{x}' = A\mathbf{x}$ with $\mathbf{v}_1$ and λ_1 being an eigenvector and corresponding eigenvalue of A, then $\mathbf{x}(t) = 2\mathbf{v}_1 e^{\lambda_1 t}$ also solves $\mathbf{x}' = A\mathbf{x}$.
3. If one or more of the singular values of an $n \times n$ matrix A is equal to zero, then $A\mathbf{x} = \mathbf{b}$ will be inconsistent for all $\mathbf{b}$.

PROBLEM 4.

The following data points have been measured in an experiment

x	y
1	7
2	7
3	8
4	8
5	9

It is assumed that the points lie on a straight line.

1. Write up the necessary vectors, matrices and equations to fit the linear model $y = \beta_0 + \beta_1 x$ to data.
2. Calculate the model parameters for the linear model by solving the equation using the least squares method.

Assume that it is later realized that the last two data points might contain errors and should only be weighted half as much as the first three data points.

3. Calculate a new linear model using these assumptions.

PROBLEM 5.

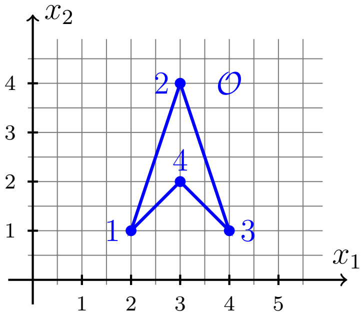
Consider a plane in $\mathbb{R}^3$ given by

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + t \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} + s \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}, \quad t, s \in \mathbb{R}$$

1. Does the point $\mathbf{x} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T$ lie on the plane?
2. Is the plane a subspace of $\mathbb{R}^3$?

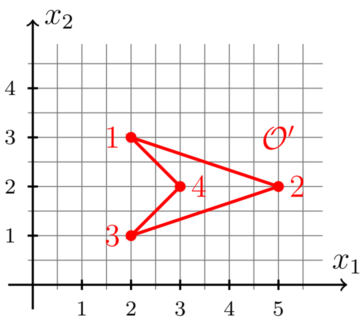
PROBLEM 6.

In the case *Computer Graphics in Automotive Design*, homogeneous coordinates were introduced. In this problem, homogeneous coordinates in $\mathbb{R}^2$ are used. Consider the following arrow-like object, denoted $\mathcal{O}$:



1. Determine the data matrix and the adjacency matrix for $\mathcal{O}$.

In the figure below $\mathcal{O}$ has transformed into the new object $\mathcal{O}'$ plotted in red.



2. Determine the transformation matrix that transforms $\mathcal{O}$ into $\mathcal{O}'$.