
<u>Notice</u>: In the grading of the exercises special attention will be paid to check that the answers are substantiated and that the procedure of calculations is well documented. When results are achieved using a calculator or a pc it should be noted in the solution. All six problems are weighted equally in the grading. Your solution to the problem set can be written in Danish or English as you prefer.

Problem 1

Let the matrix M and vectors $\boldsymbol{x}$ and $\boldsymbol{b}$ be given by:

$$M=\begin{bmatrix} 4 & 9 & 5 & -4 & 4 \\ -2 & -3 & 1 & 2 & -1 \\ 1 & 6 & 10 & -10 & 8 \\ 3 & 0 & -12 & 0 & -3 \end{bmatrix}; \qquad \boldsymbol{x}=\begin{bmatrix} 10 \\ -6 \\ 2 \\ 1 \\ 2 \end{bmatrix}; \qquad \boldsymbol{b}=\begin{bmatrix} 4 \\ 1 \\ 3 \\ 2 \end{bmatrix}$$

a) Determine the pivot-columns of the matrix M.

b) Is $\boldsymbol{x}$ in $Nul\ M$?

c) Is $\boldsymbol{b}$ in $Col\ M$?

Problem 2

Let the matrices B and C be given by:

$$B=\begin{bmatrix} -3 & 4 & 2 \\ 0 & b & 1 \\ 2 & 0 & -1 \end{bmatrix} \quad \text{and} \quad C=\begin{bmatrix} 1 & 4 & 1 \\ 3 & 0 & c \\ 2 & 2 & -3 \end{bmatrix}$$

a) By using cofactor expansion determine b, so $det\ B = 1$.

b) Determine all values of c, so $Rank\ C = 2$.

Problem 3

$$A=\begin{bmatrix} -2 & 2 \\ 0 & 4 \\ 3 & -1 \end{bmatrix} \text{ and } \boldsymbol{b}=\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

a) Determine a least-squares solution $\hat{\boldsymbol{x}}$ to $A\boldsymbol{x}=\boldsymbol{b}$.

b) Determine the least-squares error.

Problem 4

A real $n \times n$ matrix A have the eigenvalue $\lambda = -2 - 4j$.
For the statements given below, state whether they are true or false and justify your answer for each statement:

a) The eigenvector $\boldsymbol{v}$ related to the eigenvalue $\lambda = -2 - 4j$ is complex (that is, $\boldsymbol{v} = \boldsymbol{v}_a + j\boldsymbol{v}_b$, where $\boldsymbol{v}_a, \boldsymbol{v}_b \in \mathbb{R}^n$ and $\boldsymbol{v}_b \neq \boldsymbol{0}$).

b) We know for sure that $\lambda = 2 - 4j$ is also an eigenvalue.

c) A cannot have any real eigenvalue.

Problem 5

In case 1 cubic spines was introduced. This problem is about the less complex quadratic splines. We which to spline two quadratic functions to a smooth curve passing through the following datapoints:

t	1	3	5
x	0	5	10

The quadratic spline is defined by:

$$x(t) = \begin{cases} a_0 + a_1 t + a_2 t^2; & 1 \leq t \leq 3 \\ b_0 + b_1 t + b_2 t^2; & 3 \leq t \leq 5 \end{cases}$$

The spline function must be continuous and have equal derivatives $(x'(t))$ at $t = 3$. Further the derivative must be 0 at $t = 1$ and $t = 5$.

a) Write down the equations needed to determine the coefficients a_0, a_1, a_2, b_0, b_1 and b_2.

b) Solve the above equations, write down the full expression for the quadratic spline and draw the spline graph.

Problem 6

A basis for $\mathbb{P}_2$ (second order polynomials) is $\{1, t, t^2\}$.

Let an inner product on $\mathbb{P}_2$ be given by: $\langle f, g\rangle = f(0)\cdot g(0) + f(1)\cdot g(1) + f(2)\cdot g(2)$

a) Determine $\langle 1, t\rangle, \langle 1, t^2\rangle$ and $\langle t, t^2\rangle$.

b) Determine an orthogonal basis for $\mathbb{P}_2$.

c) Find the coordinates of $g(t) = 3(t^2 - 1) + 2t$ with respect to the orthogonal basis.