In the grading of the exercises special attention will be paid to check that answers are substantiated and that the procedure of calculations is well documented. When results are achieved using a calculator or a pc it should be noted in the paper. All 6 problems are weighted equally in the grading. Your solution to the problem set can be written in Danish or English as you prefer.

PROBLEM 1.

Let the matrix A and the vector $\mathbf{b}$ be given by

$$A = \begin{bmatrix} 1 & 2 & 2 & -1 \\ 3 & 7 & 8 & -3 \\ -1 & -3 & -3 & 2 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}.$$

1. Calculate the solutions to the matrix equation $A\mathbf{x} = \mathbf{b}$.

PROBLEM 2.

Let the following matrix and vector be given

$$A = \begin{bmatrix} 4 & -14 & 8 \\ 1 & -5 & 4 \\ 1 & -7 & 6 \end{bmatrix}, \qquad \mathbf{v}_1 = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$$

1. Show that $\mathbf{v}_1$ is an eigenvector of A and find the corresponding eigenvalue.
2. Find the other eigenvectors and eigenvalues of A.
3. Can any vector in $\mathbb{R}^3$ be written as a linear combination of the eigenvectors?

PROBLEM 3.

If A is a diagonalizable and positive semidefinite $n \times n$ matrix, then the square root of A can be defined as $A^{1/2} = PD^{1/2}P^{-1}$ where

$$D = \begin{bmatrix} \sqrt{\lambda_1} & 0 & \dots & \dots & 0 \\ 0 & \sqrt{\lambda_2} & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \sqrt{\lambda_n} \end{bmatrix} \quad \text{and } P = [\mathbf{v}_1 \dots \mathbf{v}_n]$$

where λ_i and $\mathbf{v}_i$ are corresponding eigenvalues and eigenvectors of A.

1. Show that $(A^{1/2})^2 = A$

Let a 3×3 matrix be given as

$$A = \begin{bmatrix} 4 & -3 & 3 \\ 0 & 1 & 8 \\ 0 & 0 & 9 \end{bmatrix}$$

2. Calculate $A^{1/2}$.

PROBLEM 4.
For the statements given below, state whether they are true or false and justify your answer for each statement.

1. For a $m \times n$ matrix with $m < n$ the set of row vectors form a basis for the row space.
2. The polynomial space $\mathbb{P}_2$ is a subspace of $\mathbb{P}_3$.
3. The vectors $f(t) = e^{-2t}$ and $g(t) = e^{-3t}$ are linearly independent.

PROBLEM 5.
Assume that corresponding values of time t and output y have been measured for a system as shown in this table

t	y
0	1
1	3
2	6
3	6
4	8
5	11

It is assumed that the system can be fitted to a linear model of the form $y_1(t) = \beta_0 + \beta_1 t^2$.

1. Write up the observation vector and the design matrix for the problem.
2. Determine the two model parameters.
3. Comment on whether the assumed model is appropriate.

PROBLEM 6.
This problem is based on case 5: "Error-Detecting and Error-Correcting Codes". All calculations must therefore be done using $\mathbb{Z}_2$ arithmetics . Let the matrix A be given by

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}.$$

1. Determine a basis for the column space of A, a basis for the null space of A and the rank of A.