In the grading of the exercises special attention will be paid to check that answers are substantiated and that the procedure of calculations is well documented. When results are achieved using a calculator or a pc it should be noted in the paper. All 6 problems are weighted equally in the grading. Your solution to the problem set can be written in Danish or English as you prefer.

PROBLEM 1.

Consider the following matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & -1 \\ 0 & 1 & 3 \\ 1 & 1 & 3 \end{bmatrix}.$$

1. Determine the solution to the homogeneous matrix equation $A\mathbf{x} = \mathbf{0}$.
2. Can $A\mathbf{x} = \mathbf{b}$ be solved for any $\mathbf{b}$?

PROBLEM 2.

Consider the following matrix and vector

$$A_1 = \begin{bmatrix} 4 & -3 \\ 3 & -3 \end{bmatrix}, \qquad \mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$

1. Use a graphical method to decide if $\mathbf{v}_1$ is an eigenvector of A_1 and if so, determine the corresponding eigenvalue.

Another matrix is given by

$$A_2 = \begin{bmatrix} 3 & 2 & 1 & -1 \\ 0 & 1 & -1 & 1 \\ 0 & -2 & 0 & -1 \\ 0 & 2 & 1 & 2 \end{bmatrix}.$$

2. Determine the eigenspaces and the dimensions of these.
3. Explain whether A_2 is a diagonalizable matrix.

PROBLEM 3.

For the statements given below, state whether they are true or false and justify your answer for each statement.

1. The columns of a 7×3 matrix with rank 3 are linearly independent.
2. A 2×2 matrix A with columns $\mathbf{a}_1$ and $\mathbf{a}_2$ is invertible if $\mathbf{a}_1 \neq \mathbf{a}_2$.
3. If A and B are $n \times n$ matrices and A is invertible then $A^{-1}BA = B$.

PROBLEM 4.

Consider the first four Laguerre polynomials shown here.

$$L_0(x) = 1, \qquad L_1(x) = -x + 1$$
$$L_2(x) = \frac{1}{2}x^2 - 2x + 1, \qquad L_3(x) = -\frac{1}{6}x^3 + \frac{3}{2}x^2 - 3x + 1.$$

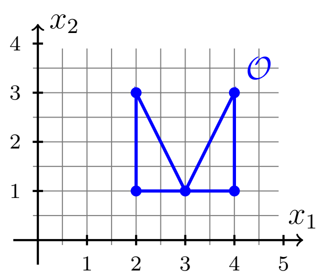
1. Show that the four Laguerre polynomials form a basis for $\mathbb{P}_3$.

Let a vector in $\mathbb{P}_3$ be given as $z(x) = x^3$.

2. Write $z(x)$ as a linear combination of the four Laguerre polynomials.

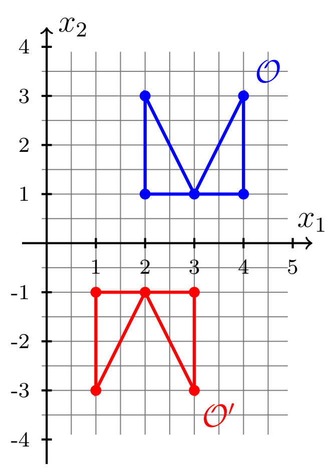
PROBLEM 5.

In the case *Computer Graphics in Automotive Design*, homogeneous coordinates were introduced. In this problem, homogeneous coordinates in $\mathbb{R}^2$ are used. Consider the following object, denoted $\mathcal{O}$:



1. Determine the data matrix and the adjacency matrix for $\mathcal{O}$.

In the figure below $\mathcal{O}$ has transformed into the new object $\mathcal{O}'$ plotted in red.



2. Determine the transformation matrix that transforms $\mathcal{O}$ into $\mathcal{O}'$.

PROBLEM 6.

Let V be the vector space of all $n \times n$ matrices and define the inner product between two matrices in V as

$$< A, B >= \mathrm{Tr}(B^T A).$$

Where Tr is the usual trace function, i.e. the sum of the diagonal elements.
Consider matrices

$$A = \begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ 2 & b \end{bmatrix}, \quad C = \begin{bmatrix} 2 & -3 \\ -1 & 4 \end{bmatrix},$$

where b is a scalar.

1. Calculate the norm of A.
2. Determine b so that A and B are orthogonal.

The matrices A and C are orthogonal.

3. Find a non-zero matrix D that is orthogonal to both A and C.