

# Aarhus University School of Engineering

Electronic- & Information Technology- & Electrical Power Engineering

Term:	Q2 exam - Winter 2016-17
Test in:	ETSMP
Date:	21/12 -2016
Duration:	3 timer
Supervisor:	Gunvor Elisabeth Kinkelund
Aarhus University School of Engineering will hand out:  2 covers plus paper for draft and fair copy will be handed out. The student must fill out and hand in 2 covers. The students should only upload / hand in 1 paper.	
<b>Practical information:</b>  <b>Digital Exam</b> This examination is part of "Digital Exam". This means that the assignment will be distributed and submitted via the digital exam platform. Hand written parts of the answers must, however, be submitted in the covers. Answers submitted via the digital exam platform must be submitted in PDF format.  If you submit everything by hand, you <b>MUST</b> upload a document via the digital exam platform, stating that you have submitted your answers by hand.  Please remember to write your name and student number on <u>ALL</u> pages and in the document title / filename.	
<b>Remarks:</b> All materials are permitted, including the internet as an encyclopedia, and it is <b>NOT</b> allowed to communicate with others electronically or otherwise during the exam.  <b>Special Notifications:</b> At the assessment of the assignment, the used method will be assessed, and it is important that the used mathematical methods are clear from your answers. Furthermore, the reason behind all	

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answers should be stated clearly. If a result is obtained with the help of a calculator or computer, this has to be stated in your answers as well.

At assessment, all part-assignments will be weighted the same.

## Assignment 1: Random Variables

A discrete random variable  $X$  has the following probability mass function (pmf):

x	-1	1	7
$f_X(x)$	$k$	$\frac{3}{4}$	$\frac{1}{8}$

1) For which value of  $k$  is  $f_X(x)$  a valid probability mass function? State the reason for your answer.

2) Assume that  $k = \frac{1}{8}$ , find the cumulative distribution function (cdf)  $F_X(x)$  for  $X$ . Sketch  $F_X(x)$ .

3) Use  $f_X(x)$  to find the expected value  $E[X]$  and the standard deviation  $\sigma_X$  for  $X$ . Assume that  $k = \frac{1}{8}$ .

4) If a function is defined as  $g(X = x) = 3 \cdot x^2$ . Find the expected value  $E[g(X = x)]$ . Assume that  $k = \frac{1}{8}$ .

5) State which values  $X$  can take.

**The assignments are continued on the next page**

## Assignment 2: Stochastic Processes

A continuous stochastic process is given by:

$$X(t) = w(t)$$

Where  $w(t)$  is i.i.d. uniformly distributed according to  $w(t) \sim U(-2, -1)$ .

1) The stochastic process  $X(t)$  is sampled every second, sketch 6 samples from 0 - 5 s of one realization. State how the realization is made, use a random number generator, e.g. `rand()` in matlab.

2) Find the ensemble mean value and the ensemble variance for the process  $X(t)$ .

3) Write the formula to determine the timely mean value for the process  $X(t)$ .

4) State whether the process  $X(t)$  is WSS (wide sense stationary), and whether it is ergodic. State the reason behind your answers.

**The assignments are continued on the next page**

## Assignment 3: Probability Theory

An HIV test based on saliva is positive in 92% of the cases, given that the person is HIV infected. The same test is negative in 98% of the cases, given that the person is not HIV infected. Of the entire population, 0,1% are HIV infected.

- 1) What is the probability that a person from the population is both HIV infected and has a positive test?
- 2) What is the total probability that a person from the population has a positive test?
- 3) If a person from the population has a positive test, what is the probability that he is HIV infected?
- 4) Are the events: “to have a positive test” and “to be HIV infected” independent? State the reason for your answer.

**The assignments are continued on the next page**

## Assignment 4: Statistics

The number of patients who died of AIDS in DK between 1985 – 1994 are stated in the table. The number of deaths are given by "Number" and the year is given by "Year"<sup>1</sup>.

<b>Number:</b>	28	46	44	63	104	148	172	187	223	236
<b>Year:</b>	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994

- 1) What is the empirical mean value and the empirical variance for the number of dead AIDS patients?
- 2) Plot the data from the table. Use linear regression to determine a model for the data, state how the parameters of the model are calculated (the slope of the linear model and the intersection with the y-axis). Draw also the linear model on the plot.
- 3) Make a residual drawing for the model from 2) on a graph. State also how the residuals are calculated.
- 4) Calculate a 95% confidence interval for the slope.

**The assignments are continued on the next page**

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<sup>1</sup> Source: <http://www.faktalink.dk/titelliste/aids/aidsidan>

5) Based on your answer in 3) and 4), would you conclude that the assumption of linearity between the number and the year is reasonable? State the reason behind your answer.

6) Are there any years, where the linear model cannot be used?