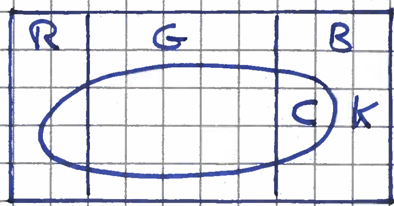
SMP

Opg. 1 S19

Handelser: R = rød, G = gul, B = blå $R+G+B=S$

C = chili, K = kaffe $C+K=S$



Givet: $Pr(R)=0.12$, $Pr(G)=0.63$

$Pr(K|R)=0.59$, $Pr(C|B)=0.32$

$Pr(G\cap C)=0.15$

a) $\underline{\underline{Pr(B)}} = 1-Pr(R)-Pr(G) = 1-0.12-0.63 = \underline{\underline{0.25 = 25\%}}$

b) $\underline{\underline{Pr(C|R)}} = 1-Pr(K|R) = 1-0.59 = \underline{\underline{0.41 = 41\%}}$ (da $\bar{C}=K$)

c) $\underline{\underline{Pr(C)}} = Pr(C\cap R)+Pr(C\cap G)+Pr(C\cap B)$

$= Pr(C|R)\cdot Pr(R) + Pr(G\cap C) + Pr(C|B)\cdot Pr(B)$

$= 0.41\cdot 0.12 + 0.15 + 0.32\cdot 0.25$

$= 0.0492 + 0.15 + 0.08$

$= \underline{\underline{0.2792 = 27.9\%}}$

d) $p = Pr(C\cap R) = Pr(C|R)\cdot Pr(R) = 0.41\cdot 0.12 = 0.0492$

Binomialfordeling: Succes = rød og chili smag $(R\cap C)$

Fiasko = Alt andet

$\underline{\underline{Pr(\text{Præcis } 2\ (R\cap C) \text{ ud af } (8))}} = B(8,2) = \frac{8!}{2!(8-2)!}\cdot 0.0492^2\cdot(1-0.0492)^6$

$= 28\cdot 0.00242\cdot 0.7388$

$= \underline{\underline{0.0501 = 5.01\%}}$

$$B(n,k) = \frac{n!}{k!(n-k)!}\cdot p^k\cdot(1-p)^{n-k}$$

SMP

Opg. 2 S19

$$f_X(x) = \begin{cases} 1/2 & ; 1 \leq x < 2.5 \\ 1/4 & ; 5 \leq x < 6 \\ 0 & \text{ellers} \end{cases} \qquad F_X(x) = \begin{cases} 0 & ; x < 1 \\ 1/2\,x - 1/2 & ; 1 \leq x < 2.5 \\ 3/4 & ; 2.5 \leq x < 5 \\ 1/4\,x - 1/2 & ; 5 \leq x < 6 \\ 1 & ; x \geq 6 \end{cases}$$

a) $\underline{\underline{Pr(2 < x < 3)}} = F_X(3) - F_X(2) = \frac{3}{4} - \left(\frac{1}{2} \cdot 2 - \frac{1}{2}\right) = \frac{3}{4} - \frac{1}{2} = \underline{\underline{\frac{1}{4}}}$

b) $F_X(x) = \int_{-\infty}^{x} f_X(x)\,dx$

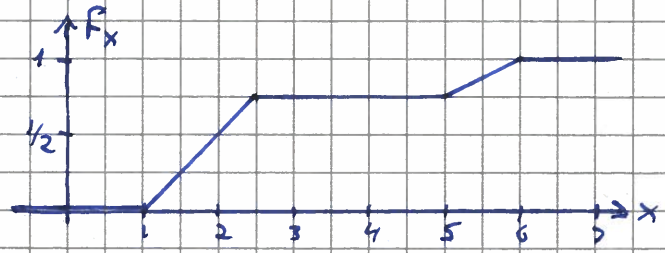
$x < 1: \quad \underline{\underline{F_X(x)}} = \int_{-\infty}^{x} 0\,dx \underline{\underline{= 0}}$

$1 \leq x < 2.5: \quad \underline{\underline{F_X(x)}} = \int_{-\infty}^{1} f_X(x)\,dx + \int_{1}^{x} f_X(x)\,dx = F_X(1) + \int_{1}^{x} \frac{1}{2}\,dx = 0 + \left[\frac{1}{2}x\right]_1^x = \underline{\underline{\frac{1}{2}x - \frac{1}{2}}}$

$2.5 \leq x < 5: \quad \underline{\underline{F_X(x)}} = F_X(2.5) + \int_{2.5}^{x} 0\,dx = F_X(2.5) = \frac{1}{2} \cdot \frac{5}{2} - \frac{1}{2} = \underline{\underline{\frac{3}{4}}}$

$5 \leq x < 6: \quad \underline{\underline{F_X(x)}} = F_X(5) + \int_{5}^{x} \frac{1}{4}\,dx = \frac{3}{4} + \left[\frac{1}{4}x\right]_5^x = \frac{3}{4} + \frac{1}{4}x - \frac{5}{4} = \underline{\underline{\frac{1}{4}x - \frac{1}{2}}}$

$x \geq 6: \quad \underline{\underline{F_X(x)}} = F_X(6) + \int_{6}^{x} 0\,dx = F_X(6) = \frac{1}{4} \cdot 6 - \frac{1}{2} = \frac{3}{2} - \frac{1}{2} = \underline{\underline{1}}$



c) $\underline{\underline{E[X]}} = \int_{-\infty}^{\infty} x \cdot f_X(x)\,dx = \int_{1}^{2.5} \frac{1}{2}x\,dx + \int_{5}^{6} \frac{1}{4}x\,dx = \left[\frac{1}{4}x^2\right]_1^{2.5} + \left[\frac{1}{8}x^2\right]_5^6 = \frac{25/4 - 1}{4} + \frac{36-25}{8} = \underline{\underline{\frac{43}{16}}} = 2.69$

$E[X^2] = \int_{-\infty}^{\infty} x^2 f_X(x)\,dx = \int_{1}^{2.5} \frac{1}{2}x^2\,dx + \int_{5}^{6} \frac{1}{4}x^2\,dx = \left[\frac{1}{6}x^3\right]_1^{2.5} + \left[\frac{1}{12}x^3\right]_5^6 = \frac{125/8 - 1}{6} + \frac{216-125}{12} = \frac{481}{48} = 10.02$

$\underline{\underline{Var(X)}} = E[X^2] - E[X]^2 = \frac{481}{48} - \left(\frac{43}{16}\right)^2 = \frac{481}{48} - \frac{1849}{256} = \frac{7696 - 5547}{768} = \underline{\underline{\frac{2149}{768}}} = 2.798$

d) $\left.\begin{array}{l} \underline{\underline{Pr(2)}} = \int_{2}^{2} f_X(x)\,dx \underline{\underline{= 0}} \\ \underline{\underline{Pr(6)}} = \int_{6}^{6} f_X(x)\,dx \underline{\underline{= 0}} \end{array}\right\}$ $f_X(x)$ er kontinuert pdf.

SMP

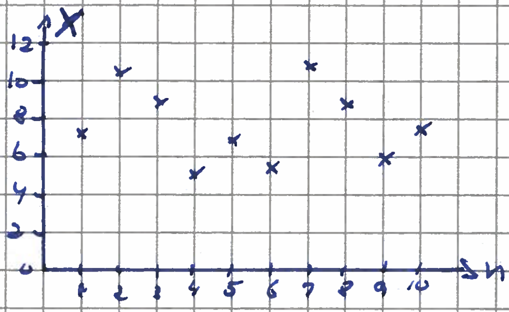
Opg. 3 S19

$X: \quad x[n] = w[n] + z \quad ; \; w[n] \sim \mathcal{U}(1,7)$, $z \sim \mathcal{N}(4,0) = 4$

iid. (kont. uniform)

a) 10 samples af 1 realisation af X:

```
Matlab: w = (7-1)·rand(1,10)+1;
        z = 0·randn(1,10)+4;
        x = w + z;
        plot(1:10, x, 'x')
```



$1+4 = 5 \leq X \leq 11 = 7+4$

b) $\mu_X = E[w+z] = E[w] + E[z] = \frac{7+1}{2} + 4 = 4+4 = \underline{\underline{8}}$

$\underline{\underline{\sigma_X^2}} = Var(w+z) = Var(w) + Var(z) = \frac{(7-1)^2}{12} + 0 = \frac{36}{12} = \underline{\underline{3}}$

c) Da μ_X og σ_X^2 er uafhængige af n (tiden), er $\underline{\underline{X \text{ WSS}}}$.

$\langle \mu_X \rangle_T = \langle \mu_w \rangle_T + \langle \mu_z \rangle_T = \frac{7+1}{2} + 4 = 4+4 = 8 = \mu_X$

$\langle \sigma_X^2 \rangle_T = \langle \sigma_w^2 \rangle_T + \langle \sigma_z^2 \rangle_T = \frac{(7-1)^2}{12} + 0 = \frac{36}{12} = 3 = \sigma_X^2$

Da $\langle \mu_X \rangle_T = \mu_X$ og $\langle \sigma_X^2 \rangle_T = \sigma_X^2$, er $\underline{\underline{X \text{ ergodisk}}}$

d) Auto-covarians: $C_{XX}(\tau) = E[X[n] \cdot X[n+\tau]] - E[X[n]] \cdot E[X[n+\tau]]$

$\underline{\underline{C_{XX}(0)}} = E[X[n]^2] - E[X[n]]^2 = Var(X[n]) = \underline{\underline{3}}$

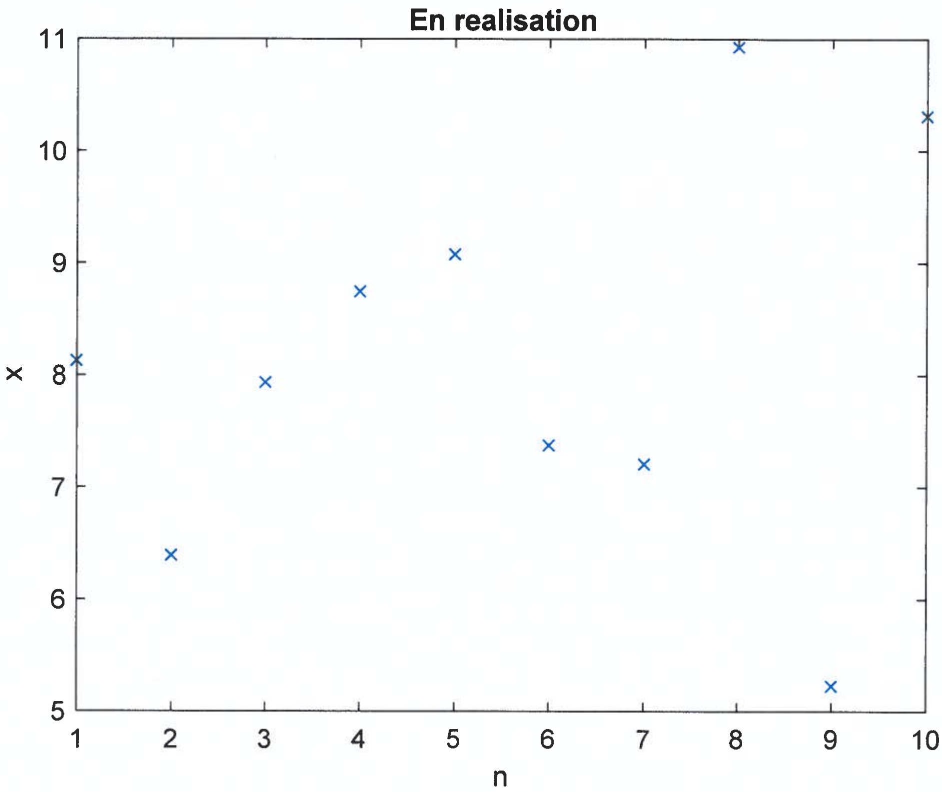
$\underline{\underline{C_{XX}(1)}} = E[X[n] \cdot X[n+1]] - E[X[n]] \cdot E[X[n+1]]$

$= E[X[n]] \cdot E[X[n+1]] - E[X[n]] \cdot E[X[n+1]]$, da $X[n]$ og $X[n+1]$ er iid.

$= \underline{\underline{0}}$

Opgave 3 S19

a) En realization:




```
%%Opg3_S19
w=6*rand(1,10)+1;
z=0*randn(1,10)+4;
x=w+z;
plot(1:10,x,'x')
title('En realisation')
xlabel('n')
ylabel('x')
```

SAP

Opg. 4 S19

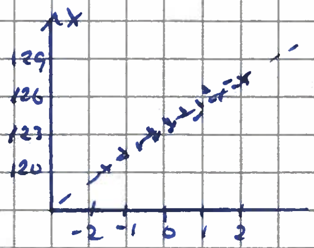
Målinger X : 126.1 ; 122.6 ; 126.9 ; 123.8 ; 122.0 ; 122.9 ; 125.3 ;
(n=14) 125.5 ; 124,2 ; 124,4 ; 124.9 ; 121.1 ; 123.2 ; 123.6

a) $\underline{\hat{\mu}_x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1736.5}{14} = \underline{\underline{124,036}}$

b) $\underline{s_x^2} = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \hat{\mu}_x)^2 = \frac{34.1721}{13} = \underline{\underline{2,629}}$

c) Q-Q plot - se bilag (Matlab: qqplot(X))

$\Leftrightarrow$ ~ret linje $\Rightarrow$ X kan antages normalfordelt



d) Null-hypotese: H_0 : $\mu_0 = 125$ (vægten er som ønsket)

Alternativ hypotese: H_1 : $\mu_0 \neq 125$ (vægten er ikke som ønsket)

e) Data normalfordelte - Varians ukendt $\rightarrow$ student t-test

f) $t = \frac{\hat{\mu}_x - \mu_0}{\sqrt{s_x^2/n}} = \frac{124.036 - 125}{\sqrt{2.629/14}} = -2.225$

p-value $= 2\cdot(1 - t_{cdf}(|t|, n-1)) = 2\cdot(1 - t_{cdf}(2.225, 13))$

$= 2\cdot(1 - 0.978) = 2\cdot 0.022 = 0.044 < 0.05$

Da p-value < 0.05 (signifikansniveau) forkastes Null hypotesen.

Dvs. filetenings maskinen er ikke indstillet korrekt.

e) $t_0 = inv.t_{cdf}(0.975, 13) = 2.16$

95% konfidens interval : $\mu_- = \hat{\mu}_x - t_0 \cdot \sqrt{\frac{s_x^2}{n}} = 124.036 - 2.16\cdot\sqrt{\frac{2.629}{14}} = 123.1$

$\mu_+ = \hat{\mu}_x + t_0 \cdot \sqrt{\frac{s_x^2}{n}} = 124.036 + 2.16\cdot\sqrt{\frac{2.629}{14}} = 124.972$

$\Downarrow$

$\underline{\underline{[\mu_- ; \mu_+] = [123.1 ; 124.972]}}$

Da $\hat{\mu}_0 = 125$ ikke ligger i intervallet, forkastes Null-hypotesen.

```
%%Opg4_S19

X=[126.1  122.6  126.9  123.8  122.0  122.9  125.3  125.5  124.2  124.4
124.9  121.1  123.2  123.6];

MeanX=mean(X)
VarX=var(X)

qqplot(X)

mu0=125

t=(MeanX-mu0)/sqrt(VarX/length(X))
pval=2*(1-tcdf(abs(t),length(X)-1))

t0=tinv(0.975,length(X)-1)

mu_min=MeanX-t0*sqrt(VarX/length(X))
mu_max=MeanX+t0*sqrt(VarX/length(X))
```

MeanX = 124.0357

VarX = 2.6286

mu0 = 125

t = -2.2254

pval = 0.0444

t0 = 2.1604

mu_min = 123.0996

mu_max = 124.9718