

# Aarhus University School of Engineering

Electronic- & Information Technology- & Electrical Power Engineering

Examination Term:	Q4 – re-exam Summer 2016
Test in:	ETSMP
Date:	17 August 2016
Duration:	3 hours
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<p>Aarhus University School of Engineering will hand out:</p> <p><b>2</b> covers plus paper for draft and fair copy will be handed out. The student must fill out and hand in <b>2</b> covers. The students should only upload / hand in 1 paper.</p>	
<p>For this examination, electronic submission is <b>possible</b>. The paper must be uploaded in PDF-format. Please remember to indicate on the cover whether you hand in your paper handwritten, electronic or both.</p> <p>Remember to write your name and student number on all pages and in the document/file name.</p> <p>All aids are allowed at this examination, including the internet as an encyclopedia, and it is <b>NOT</b> allowed to communicate with others electronically or otherwise during the exam.</p>	
<p>Remarks:</p> <p>At the assessment of the assignment, the used method will be assessed, and it is important that the used mathematical methods are clear from your answers. Furthermore, the reason behind all answers should be stated clearly. If a result is obtained with the help of a calculator or computer, this has to be stated in your answers as well.</p> <p>At assessment, all part-assignments will be weighted the same.</p>	

## Assignment 1: Random Variables

A discrete random variable  $X$  has the following probability mass function (pmf):

$x$	$-3$	$0$	$2$	$4$	$7$	$10$	$12$
$f_X(x)$	$k$	$k$	$k$	$k$	$k$	$k$	$k$

1) For which value of  $k$  is  $f_X(x)$  a valid probability mass function? State the reason for your answer.

2) Sketch the probability mass function.

3) Find the expected value and the variance of  $X$  given  $f_X(x)$ . State also which formulas you used to find the values. Assume that  $k = \frac{1}{7}$ .

4) Calculate the probabilities  $\Pr(x \geq 2)$  and  $\Pr(x > 2)$ . Assume that  $k = \frac{1}{7}$ .

5) Find the cumulative distribution function (cdf) for  $X$ . State also the intermediate calculations. Assume that  $k = \frac{1}{7}$ .

## Assignment 2: Stochastic Processes

A continuous stochastic process is given by:

$$X(t) = w(t)$$

Where  $w(t)$  is i.i.d. (independent and identically distributed) and Gaussian distributed according to  $w(t) \sim N(t, 1)$ .

- 1) Sketch one realization of the process  $X(t)$ , where it is sampled at the times:  $t = [0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9]$ . Use a Gaussian random number generator, it can e.g. be the build in generator in matlab, *randn()*. State how the realization is generated.
- 2) Find the ensemble mean value and the ensemble variance for the process  $X(t)$ .
- 3) What do you expect the timely mean value of a random realization of  $X(t)$  in the time interval  $t = [0; 100]$  to be? State the reason for your answer.
- 4) State whether the process  $X(t)$  is WSS (wide sense stationary), and whether it is ergodic. State the reason behind your answers.
- 5) Construct the equation to determine the autocorrelation  $R_{X(t_1)X(t_2)}(t_1 = 1, t_2 = 2)$  and calculate the value.

## Assignment 3: Probability Theory

A study shows that if a child age 14 has moved more than once in a year, the child will with a probability of 0,06 commit a felony crime in the next 10 years. For children that moved once or less in a year, the probability was 0,03.

31% of the children in the study belonged to the group that moved more than once.

- 1) What is the total probability that one of the children in the study committed a felony in the next 10 years?
- 2) If a child in the study had committed a felony in the 10 years following, what is the probability that the child belonged to the group that had moved more than once?

## Assignment 4: Statistics

In a study of the expected lifespan of cetaceans, the cause of death of individual cetaceans was registered. In the study, 10 cetaceans that died in captivity were registered and 10 cetaceans that died in the wild were registered.

Died in the wild (age in years)	Died in captivity (age in years)
50	7
43	2
11	1
35	3
7	15
62	6
70	14
67	1
25	5
1	9

1) Set up a NULL hypothesis and an alternative hypothesis to determine whether the mean values are the same for the two groups.

2) Should the test carried out be paired or unpaired? State the reason for your answer.

- 3) Estimate the mean values for both groups of cetaceans.
- 4) Estimate the variances for both groups and the pooled variance.
- 5) Use an un-paired t-test for hypothesis test of your hypothesis. Can the NULL hypothesis be rejected with a significance level of 0.05? State the reason for your answer.
- 6) Construct the equation and find the 95% confidence interval for the difference in the mean values. State the formula used for the calculations.