AARHUS UNIVERSITET
INGENIØRHØJSKOLEN

Aarhus University School of Engineering

Electronic- & Information Technology- & Electrical Power- & Healtcare-Engineering

Term:	**Q4 reexam – Summer 2017**
Test in:	**ETSMP – Stochastic Modelling and Processing**
Date:	August 15, 2017
Duration:	3 hours
Supervisor:	Lars Mandrup

Aarhus University School of Engineering will hand out:
2 covers plus paper for draft and fair copy will be handed out. The student must fill out and hand in **2** covers. The student should only upload / hand in **1** paper.

Digital Exam
This examination is part of "Digital Exam". This means that the assignment will be distributed and submitted via the digital exam platform. Hand written parts of the answers must, however, be submitted in the covers.

If you submit everything by hand, you **MUST** upload a document via the digital exam platform, stating that you have submitted your answers by hand.

You will receive an electronics delivery receipt as soon as you have handed in. Remember to submit on time, as otherwise a dispensation application must be submitted.

Answers submitted via the digital exam platform must be submitted in PDF format. Please remember to write your name and student number on **ALL** pages and in the document title / filename.

Aids:
All materials are permitted, including the internet as an encyclopedia, and it is **NOT** allowed to communicate with others electronically or otherwise during the exam.

Remarks:
At assessment, all part-assignments will be weighted the same.

All electronic answers must be submitted in pdf format. If the answer is made In Mathcad Prime, you must also submit it as an attachment as Mathcad Prime.

Term:	Q4 reexam – Summer 2017
Test in:	ETSMP – Stochastic Modelling and Processing
Date:	August 15, 2017

Assignment 1

A continuous stochastic variable X has the following cumulative distribution function (cdf) $F_X(x)$:

$$F_X(x) = \begin{cases} 0 & for\ x < 0 \\ a \cdot x^2 & for\ 0 \leq x \leq 2 \\ 1 & for\ x > 2 \end{cases}$$

a) Determine a, so $F_X(x)$ is an allowed distribution function.

b) Determine and sketch the probability density function (pdf) $f_X(x)$ for X.

c) Calculate the probability for $1 \leq X \leq 2$.

d) Calculate the expectation value and variance of X.

Assignment 2

A delivery of resistors includes:

1000 pcs. 1 kΩ; 800 pcs. 10 kΩ; 400 pcs. 100 kΩ

The resistors are delivered both as 5% resistors and 1% resistors.
The 1% resistors represent ¼ of the 1 kΩ, ½ of the 10 kΩ and ¾ of the 100 kΩ resistors.
The resistors have unfortunately been mixed up and are delivered as one mixed delivery.

One resistor is randomly picked up from the delivery.

a) What are the probabilities that the selected resistor is respectively1 kΩ, 10 kΩ or 100 kΩ?

b) What is the probability that the resistor is a 10 kΩ 1% resistor?

c) What is the probability that it is a 1% resistor regardless of the resistance?

d) If the selected resistor is a 5% resistor, what is the probability that it has a resistance of 100 kΩ?

Assignment 3

A discrete stochastic process X(n) is given by:

$$X(n) = 2 \cdot W(n) - 1$$

where W(n) are i.i.d. and distributed according to:

w(n)	0	1	2	3
$f_{w(n)}(w(n))$	1/4	1/4	1/4	1/4

a) Sketch 10 samples (n=1, 2,…,10) of one realization of the process X(n).

b) Calculate the mean and variance for one realization of X(n).

c) Calculate the ensemble mean and variance for the process X(n).

d) State whether the process is WSS (wide sense stationary), and whether it is ergodic. State the reason behind your answers.

Assignment 4

For a particular type of battery, the supplier specifies that the lifetime T for the batteries is normally distributed with a mean lifetime of 3000 hours with a standard deviation of 100 hours: T ~ $\mathcal{N}$ (3000 hours; (100 hours)^2).

There are randomly picked up 12 batteries of the type. In a test, the lifetime of these are measured as:

Battery nr.	1	2	3	4	5	6	7	8	9	10	11	12
Lifetime T (hours)	3148	2956	2803	2933	2869	3111	2789	2995	2909	2929	3148	2867

a) Setup a hypothesis test to determine whether the mean lifetime for the batteries corresponds to the suppliers specification.

b) Estimate the mean lifetime for the batteries.

c) Use a z-test to test your hypothesis. Can the NULL-hypothesis be rejected with a significance level of 0,05? State a reason for your answer.

d) Calculate the 95% confidence interval for the mean lifetime T of the batteries.