

# Aarhus University School of Engineering

Electronic- & Information Technology- & Electrical Power Engineering

Examination Term:	Q4 – Summer 2016
Test in:	ETSMP
Date:	9 <sup>th</sup> of June 2016
Duration:	3 hours
Supervisor:	Gunvor Elisabeth Kirkelund
<p>Aarhus University School of Engineering will hand out:          2 covers plus paper for draft and fair copy will be handed out.          The student must fill out and hand in 2 covers.          The students should only upload / hand in 1 paper.</p>	
<p><b>Practical information:</b></p> <p><b>Digital Exam</b>          This examination is part of "Digital Exam". This means that the assignment will be distributed and submitted via the digital exam platform.          Hand written parts of the answers must, however, be submitted in the covers. Answers submitted via the digital exam platform must be submitted in PDF format.</p> <p>If you submit everything by hand, you <b>MUST</b> upload a document via the digital exam platform, stating that you have submitted your answers by hand.</p> <p>Please remember to write your name and student number on <u><b>ALL</b></u> pages and in the document title / filename.</p>	
<p><b>Remarks:</b></p> <p>At the assessment of the assignment, the used method will be assessed, and it is important that the used mathematical methods are clear from your answers. Furthermore, the reason behind all answers should be stated clearly. If a result is obtained with the help of a calculator or computer, this has to be stated in your answers as well.</p> <p>At assessment, all part-assignments will be weighted the same.</p>	

## Assignment 1: Random Variables

A continuous random variable  $X$  has the following cumulative distribution function (cdf):

$$F_X(x) = \begin{cases} 0, & 2 \geq x \\ k \cdot x - \frac{2}{3}, & 2 < x \leq 5 \\ 1, & 5 < x \end{cases}$$

1) Show that the probability density function (pdf) is given as:

$$f_X(x) = \begin{cases} 0, & 2 \geq x \\ k, & 2 < x \leq 5 \\ 0, & 5 < x \end{cases}$$

2) For which value of  $k$ , is  $f_X(x)$  a valid probability density function?

State the reason for your answer.

3) Sketch the probability density function and state the name of the distribution function.

4) Use  $F_X(x)$  to calculate the probability  $\Pr(x \geq 3)$ . Assume that  $k = \frac{1}{3}$ .

5) Find the expected value and the variance of  $X$  given  $f_X(x)$ . State also which formulas you used to find the values. Assume that  $k = \frac{1}{3}$ .

## Assignment 2: Stochastic Processes

A continuous stochastic process is given by:

$$X(t) = w + 4$$

Where  $w$  is Gaussian distributed after  $w \sim N(5,1)$ .

- 1) Sketch five realizations of the process  $X(t)$  between  $t \in [0; 7]$ . Use a Gaussian random number generator, it can e.g. be the build in generator in matlab, *randn()*. State how the five realizations are generated.
- 2) Find the ensemble mean value and the ensemble variance for the process  $X(t)$ .
- 3) Select one of the five realizations, and decide the mean value and the variance for that realization.
- 4) State whether the process  $X(t)$  is WSS (wide sense stationary), and whether it is ergodic. State the reason behind your answers.

## Assignment 3: Probability Theory

Event A: is that a pregnant woman gave birth to a girl in 2012.

Event B: is that she gave birth to a boy.

Event C: is that she gave birth to a child that had a birth weight above 4000g.

20,2% of newborn boys in 2012 had a birth weight above 4000g. 12,8% of newborn girls had a birth weight above 4000g.

1) If there were born 29.785 boys and 28.131 girls in 2012, what is the probability for event A?

2) What is the total probability for event C?

3) What is the probability that the woman had a girl, if her child had a birth weight above 4000g?

## Assignment 4: Statistics

We measure the height of students in a class; the class consists of 19 women and 35 men. The heights are assumed to be Gaussian distributed. The mean value for women in the class is  $\widehat{\mu}_1 = 1,68m$ , with an estimated variance of  $s_1^2 = 0,10$ . The mean value for men in the class is  $\widehat{\mu}_2 = 1,78m$ , with an estimated variance of  $s_2^2 = 0,20$ .

- 1) Set up a hypothesis test, to determine whether the mean value for men and women in the class are the same.
- 2) State which statistic test, you would use to perform the hypothesis test. State the reason behind your answer.
- 3) Estimate the difference in the mean values  $\delta$  and the standard deviation  $\sigma$  for the difference.
- 4) Use a t-test for hypothesis testing. Can the NULL hypothesis be rejected with a significance level of 0,05? State the reason behind your answer.
- 5) Set up and find the 95% confidence interval for  $\delta$ . State which formula you used.

