

Aarhus University School of Engineering

Electronic- & Information Technology- & Electrical Power Engineering

Examination Term: Q2 re-exam – winter 2015-16

Test in: ETSMP

Date: 29/3-2016

Duration: 3 Hours

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Aarhus University School of Engineering will hand out:

2 covers plus paper for draft and fair copy will be handed out.

The student must fill out and hand in **2** covers.

The students should only upload / hand in 1 paper.

For this examination, electronic submission electronically is **possible**. The paper must be uploaded in PDF-format. Please remember to indicate on the cover whether you hand in your paper handwritten, electronic or both.

Remember to write your name and student number on all pages and in the document/file name.

All aids are allowed at this examination, including the internet as an encyclopedia, and it is **NOT** allowed to communicate with others electronically or otherwise during the exam.

Remarks:

At the assessment of the assignment, the used method will be assessed, and it is important that the used mathematical methods are clear from your answers. Furthermore, the reason behind all answers should be stated clearly. If a result is obtained with the help of a calculator or computer, this has to be stated in your answers as well.

At assessment, all part-assignments will be weighted the same.

Assignment 1: Random Variables

A continuous stochastic variable X has the following cumulative distribution function (cdf):

$$F_X(x) = \begin{cases} k \cdot e^x, & -\infty < x \leq 1 \\ 1, & 1 < x \end{cases}$$

1) Show that the probability density function (pdf) is given by:

$$f_X(x) = \begin{cases} k \cdot e^x, & -\infty < x \leq 1 \\ 0, & 1 < x \end{cases}$$

2) For which value of k , is $f_X(x)$ a valid probability density function?

State the reason for your answer.

3) Use $F_X(x)$ to calculate the probabilities of $\Pr(x < 0,4)$ and $\Pr(0,1 \leq x < 0,4)$. Assume that $k = \frac{1}{e}$.

4) Find the expected value and the variance of X given $f_X(x)$. Assume that $k = \frac{1}{e}$.

Assignment 2: Stochastic Processes

A discrete stochastic process is given by:

$$X(n) = w(n) + 4$$

Where a sample n of w is an i.i.d Gaussian distributed stochastic random variable $w(n) \sim N(0,1)$.

- 1) Sketch 10 samples ($n = 1, 2, \dots, 10$) of a realization of the process $X(n)$.
- 2) Find the ensemble mean value and the ensemble variance for the process $X(n)$.
- 3) State whether the process is WSS (wide sense stationary), and whether it is ergodic. State the reason behind your answers.

Assignment 3: Probability Theory

You are playing cards in a casino. There are 52 cards in one deck. You draw seven cards.

- 1) If event A is that King of Hearts is among the seven cards. What is the probability of event A?
- 2) If event B is that Ace of Spades is among the seven cards. What is the simultaneous probability of the events A and B?
- 3) Are the probabilities for event A and B independent? State the reason for your answer.
- 4) How many different combinations of 7 cards can be drawn from a deck of 52 cards?

Assignment 4: Statistics

The average age for 1st time married men in Denmark is given by the following table:

Age:	25,2	26,5	27,9	29,2	30,2	31,7	32,8	34,0	34,3
Year:	1971	1976	1981	1986	1991	1996	2001	2006	2011

Age is the average age of men, being married for the first time in the year given by Year¹:

- 1) Plot data from the table. Use linear regression to determine a model for the data. State how the parameters of the model have been calculated (state also the formulas used for the calculation). Draw also the linear model on the plot.
- 2) Make a residual plot on a graph. State how the residuals on the plot have been calculated (state also the formulas used for the calculation).
- 3) Assume that the number of weddings, where a man is married for the first time is 20.000 pr. year. Assume also that data is Gaussian distributed. If you have to compare the two mean values from the years 1971 and 2011, which statistical test would you use? State the reason behind your answer.

¹ Source: <http://www.statistikbanken.dk>