# Aarhus University School of Engineering

Electronic-, Information Technology-, Electrical Power & Healthcare Engineering

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| Term: | Q2 re-exam -- Winter 2016/17 |
| Test in: | ETSMP |
| Date: | March 23, 2017 |
| Duration: | 3 timer |
| Supervisor: | Gunvor Elisabeth Kirkelund/Lars Mandrup |

Aarhus University School of Engineering will hand out:

**2** covers plus paper for draft and fair copy will be handed out.
The student must fill out and hand in **2** covers.
The students should only upload / hand in 1 paper.

**Practical information:**

**Digital Exam**
This examination is part of "Digital Exam". This means that the assignment will be distributed and submitted via the digital exam platform. Hand written parts of the answers must, however, be submitted in the covers. Answers submitted via the digital exam platform must be submitted in PDF format.

If you submit everything by hand, you **MUST** upload a document via the digital exam platform, stating that you have submitted your answers by hand.

Please remember to write your name and student number on **ALL** pages and in the document title / filename.

Remarks:
All materials are permitted, including the internet as an encyclopedia, and it is **NOT** allowed to communicate with others electronically or otherwise during the exam.

**Special Notifications:**
At the assessment of the assignment, the used method will be assessed, and it is important that the used mathematical methods are clear from your answers. Furthermore, the reason behind all answers should be stated clearly. If a result is obtained with the help of a calculator or computer, this has to be stated in your answers as well.

At assessment, all part-assignments will be weighted the same.

# Assignment 1: Random Variables

A continuous random variable $X$ has the following probability density function (pdf):

$$f_X(x) = \begin{cases} A \cdot x + B, & -2 \leq x \leq 3 \\ 0, & otherwise \end{cases}$$

1) A sketch of $f_X(x)$ is shown on figure 1, and it can be noticed that $f_X(3) = 0$. For which value of $f_X(-2) = k$, is $f_X(x)$ a valid probability density function? State the reason for your answer.

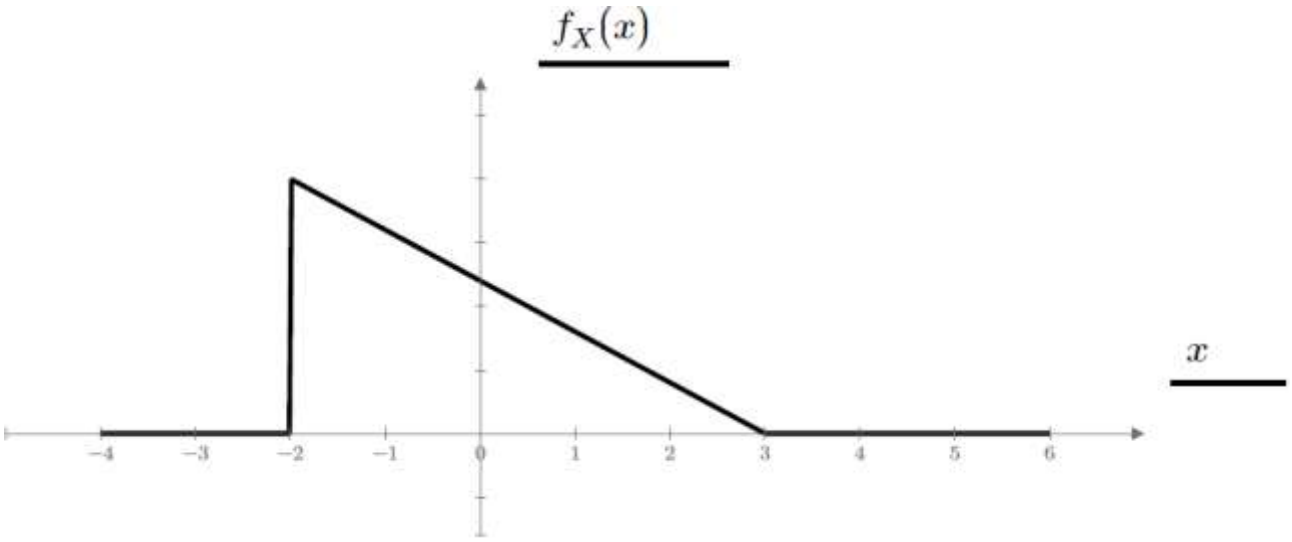


*Figure 1*

**The assignments are continued on the next page**

2) Show that the cumulative distribution function (cdf) $F_X(x)$ for $X$ is given by:

$$F_X(x) = \begin{cases} 0 & x < -2 \\ \frac{A}{2} \cdot x^2 + B \cdot x + C & -2 \leq x \leq 3 \\ 1 & 3 < x \end{cases}$$

Assume that $A = -\frac{2}{25}$ and $B = \frac{6}{25}$ and $C = \frac{16}{25}$.

3) Use $f_X(x)$ to find the expected value $E[X]$ and the variance ${\sigma_X}^2$ for $X$.

Assume that $A = -\frac{2}{25}$ and $B = \frac{6}{25}$.

4) What is the probability $\Pr(X < 0)$? State the reason for your answer.

Assume that $A = -\frac{2}{25}$ and $B = \frac{6}{25}$ and $C = \frac{16}{25}$.

**The assignments are continued on the next page**

# Assignment 2: Stochastic Processes

A discrete stochastic process is given by the $n$´th sample:

$$X(n) = W(n) + 0{,}7$$

where $W(n)$ are i.i.d. and distributed according to:

| $w(n)$ | -1 | 0 | 1 |
|---|---|---|---|
| $f_{W(n)}(w(n))$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ |

1) Sketch 11 samples from 0 – 10 of one realization of $X(n)$. State how the realization is made, use a random number generator, e.g. unidrnd() in matlab.

2) Find the ensemble mean value and the ensemble variance for the process $X(n)$.

3) Write the formula to determine the autocorrelation function for $X(n)$.

4) State whether the process $X(n)$ is WSS (wide sense stationary), and whether it is ergodic. State the reason behind your answers.

**The assignments are continued on the next page**

# Assignment 3: Probability Theory

In a train network, it is stated that delays can be caused by leaves on the tracks, errors in the signals or staff shortage. The events are independent and not mutual exclusive. If a delay happens, there are leaves on the track $\frac{1}{4}$ of the times, there are errors in the signals $\frac{1}{2}$ of the times, and there is staff shortage $\frac{1}{4}$ of the times.

1) Sketch the Venn diagram (event diagram for the sample space) for the three events: event A is that there are leaves on the track, event B is when there are errors in the signals, and event C is staff shortage.

2) Find the probability $\Pr(A \cap B)$. This is the probability that there are both leaves on the tracks and errors in the signals.

3) Find the probability $\Pr(A \cup B)$. This is the probability that there are either leaves on the tracks or errors in the signals.

4) Find the probability $\Pr(A \cup B \cup C)$. This is the probability that a delayed train is due to leaves on the tracks, errors in the signals or staff shortage.

**The assignments are continued on the next page**

# Assignment 4: Statistics

In a study of a recognized weight loss method, 10 patients are examined before treatment and one year after treatment.

| Patient no. | Weight before (kg) | Weight after (kg) |
|---|---|---|
| 1 | 140 | 130 |
| 2 | 138 | 121 |
| 3 | 110 | 127 |
| 4 | 154 | 101 |
| 5 | 125 | 92 |
| 6 | 169 | 170 |
| 7 | 142 | 143 |
| 8 | 162 | 170 |
| 9 | 131 | 134 |
| 10 | 122 | 85 |

1) Setup a NULL and an Alternative hypothesis to determine whether the treatment has changed the patient's weights.

2) Should the test carried out be paired or unpaired? State the reason for your answer.

3) Estimate the mean for the difference in the patient's weights before and after the treatment.

**The assignments are continued on the next page**

4) Estimate the variance for the difference in the patient's weights before and after the treatment.

5) Use a paired t-test to test your hypothesis. Can the NULL hypothesis be rejected with a significance level of 0,05? State the reason for your answer.

6) Set up and find the 95% confidence interval for the difference before and after the treatment. State which formula you have used.