Aarhus University School of Engineering

Electronic- & Information Technology- & Electrical Power Engineering

Examination Term:	Q2 exam – winter 2015-16

Test in:	ETSMP
Date:	22/12-2016
Duration:	3 Hours
Supervisor:	Gunvor Elisabeth Kirkelund

Aarhus University School of Engineering will hand out:

2 covers plus paper for draft and fair copy will be handed out.
The student must fill out and hand in **2** covers.
The students should only upload / hand in 1 paper.

For this examination, electronic submission electronicaly is **possible.** The paper must be uploaded in PDF-format. Please remember to indicate on the cover whether you hand in your paper handwritten, electronic or both.

Remember to write your name and student number on all pages and in the document/file name.

All aids are allowed at this examination, including the internet as an encyclopedia, and it is **NOT** allowed to communicate with others electronically or otherwise during the exam.

Remarks:

At the assessment of the assignment, the used method will be assessed, and it is important that the used mathematical methods are clear from your answers. Furthermore, the reason behind all answers should be stated clearly. If a result is obtained with the help of a calculator or computer, this has to be stated in your answers as well.

At assessment, all part-assignments will be weighted the same.

Assignment 1: Random Variables

Let the simultaneous probability mass function (also called simultaneous probability density function or pdf) for the two discreet random variables X and Y be given by the table:

y\x	1	2	3
5	0	$\frac{1}{12}$	0
6	$\frac{2}{12}$	0	$\frac{2}{12}$
7	$\frac{2}{12}$	$\frac{1}{12}$	$\frac{2}{12}$
8	0	$\frac{2}{12}$	0

1) Show that the marginal mass functions (also called the marginal density function) for X and Y are given as:

y	5	6	7	8
$f_Y(y)$	$\frac{1}{12}$	$\frac{4}{12}$	$\frac{5}{12}$	$\frac{2}{12}$

x	1	2	3
$f_X(x)$	$\frac{4}{12}$	$\frac{4}{12}$	$\frac{4}{12}$

2) Show that:

$$E[X] = 2, E[Y] = \frac{20}{3}, E[Y \cdot X] = \frac{40}{3}, E[X^2] = \frac{14}{3}, E[Y^2] = \frac{271}{6}$$

3) What is the correlation coefficient for X and Y?

4) Specify whether the random variables X and Y are correlated.

5) Specify whether the random variables X and Y are independent.

6) Write the conditional probability $f_{X|Y}(x|y = 6)$.

Assignment 2: Stochastic Processes

A discreet stochastic process is given by:

$$X(n) = w(n),$$

$w(n)$ is i.i.d. and distributed according to a uniform distribution according to $w(n) \sim \mathcal{U}(0,10)$.

1) Make a sketch with 10 samples of one realization of the process $X(n)$, i.e. for $n = 1, \cdots, 10$.

2) Find the ensemble mean value and the variance for the process $X(n)$.

3) Find the autocorrelation $R_{XX}(\tau)$ for the process $X(n)$ for $\tau = 0, \cdots, 3$.

4) State whether the process is WSS (wide sense stationary) and whether it is ergodic, specify the reason behind your answers.

Assignment 3: Probability Theory

Lactose intolerance is present in 20% of the Finnish population. If a Finn has lactose intolerance, a test will be positive in 90% of the cases. If a Finn does not have lactose intolerance, the test will be positive in 30% of the cases.

1) Calculate the total probability of a positive test for a Finn. It is unknown whether he have lactose intolerance.

2) If a Finn has a positive test, what is the probability that he also has the disease?

Assignment 4: Statistics

Number: is mortality of boys under 1 year in Denmark from 1901 to 1991. It is given in the following table[1]:

Number:	5562	4357	3471	3078	2309	1285	969	602	238	268
Year:	1901	1911	1921	1931	1941	1951	1961	1971	1981	1991

1) Plot the data from the table, and draw the best straight line through the points, by determining the intersection with the y-axis and the slope of the linear model. State which method was used to find the linear model.

2) Make a residual plot.

3) Calculate a 95% confidence interval for the slope.

4) Based on your answers in 2) and 3), would you conclude that the assumption of linearity between mortality and year is reasonable?

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[1] http://www.statistikbanken.dk