

1 Complexity Zoo

1.1 P

Informally: all problems that can be solved in polynomial time.

Definition 1.1.

$$\mathbf{P} = \bigcup_{k \geq 0} \text{TIME}[n^k]$$

Descriptive Complexity definitions:

Definition 1.2.

$$\mathbf{P} = \text{FO}(\text{LFP})$$

(First Order logic extended with the Least Fixed Point operator, with successor)

Definition 1.3.

$$\mathbf{P} = \text{SO}(\text{Horn})$$

(Second Order logic restricted with Horn)

Circuit Complexity definition:

Definition 1.4.

\mathbf{P} = Set of problems that can be solved by a polynomial-time uniform family of boolean circuits

Notable Problems in \mathbf{P} :

- 2-SAT
- 2-Colourability
- Reachability

1.2 NP

Informally: all problems that can be solved in nondeterministic polynomial time.

Definition 1.5.

$$\mathbf{NP} = \bigcup_{k \geq 0} \text{NTIME}[n^k]$$

In terms of a verifier:

Informally: The set of decision problems where a solution can be verified in polynomial time.

Descriptive Complexity Definition:

Definition 1.6.

$$\mathbf{NP} = \text{SO}\exists$$

(Existential Second Order)

Notable Problems in \mathbf{P} :

- SAT
- 3-Colourability
- TSP
- Subset sum

1.3 **FPT**
1.4 **W[1]**
1.5 **FPTAS**
1.6 **PTAS**
1.7 **L**
1.8 **NL**
1.9 **PSPACE**
1.10 **coNP**
1.11 Σ_2^p
1.12 Σ_i^p
1.13 Π_2^p
1.14 Π_i^p
1.15 **PH**
1.16 P^{SAT}
1.17 NP^{SAT}
1.18 **P/poly**
1.19 **P-Uniform**
1.20 **EXP**
1.21 **NC**
1.22 NC_0
1.23 NC_1
1.24 NC_2
1.25 NC_i
1.26 AC_i
1.27 AC_0
1.28 AC_1
1.29 **BPP**
1.30 **RP**
1.31 **co-RP**
1.32 **ZPP**
1.33 **APX**
1.34 **PO**
1.35 **PCP**