

# 1 Complexity Zoo

## 1.1 P

Informally: all problems that can be solved in polynomial time.

**Definition 1.1.**

$$\mathbf{P} = \bigcup_{k \geq 0} \text{TIME}[n^k]$$

Descriptive Complexity definitions:

**Definition 1.2.**

$$\mathbf{P} = \text{FO}(\text{LFP})$$

(First Order logic extended with the Least Fixed Point operator, with successor)

**Definition 1.3.**

$$\mathbf{P} = \text{SO}(\text{Horn})$$

(Second Order logic restricted with Horn)

Circuit Complexity definition:

**Definition 1.4.**

$\mathbf{P}$  = Set of problems that can be solved by a polynomial-time uniform family of boolean circuits

Notable Problems in  $\mathbf{P}$ :

- 2-SAT
- 2-Colourability
- Reachability



**1.2 NP**  
**1.3 FPT**  
**1.4 W[1]**  
**1.5 FPTAS**  
**1.6 PTAS**  
**1.7 L**  
**1.8 NL**  
**1.9 PSPACE**  
**1.10 coNP**  
**1.11  $\Sigma_2^P$**   
**1.12  $\Sigma_i^P$**   
**1.13  $\Pi_2^P$**   
**1.14  $\Pi_i^P$**   
**1.15 PH**  
**1.16  $P^{SAT}$**   
**1.17  $NP^{SAT}$**   
**1.18 P/poly**  
**1.19 P-Uniform**  
**1.20 EXP**  
**1.21 NC**  
**1.22  $NC_0$**   
**1.23  $NC_1$**   
**1.24  $NC_2$**   
**1.25  $NC_i$**   
**1.26  $AC_i$**   
**1.27  $AC_0$**   
**1.28  $AC_1$**   
**1.29 BPP**  
**1.30 RP**  
**1.31 co-RP**  
**1.32 ZPP**  
**1.33 APX**  
**1.34 PO**