Question no:1 [CSP... map coloring problem]

Consider the constraint statisfication problem of following map coloring problem where the state space is represented as

varaibles: DJ, SO, ET, KE, UG, TA, RW, B, U

Domain: Di : red; green; blue

Constraint: adjacent region have different colors

- $DJ \neq SO$
- $(DJ; SO) \in$ [(red; green); (red, blue); (green, red); (green, blue)...]

How many solutions are there above mentioned map coloring problem?
How many solutions if four colors are allowed? Two colors?

Ans:- 1:- Three colors allowed (Red, Green, Blue)

Each adjancent region must be different colors and there are three colors avaible, we can calculate it by permutation.
There are three choices for first region, 2 choices for second, and 1 choice for third and so on. $3 \times 2 \times 1 = 6$

2:- Four colors allowed (Red, green, Blue, Yellow):
There are 4 choices for the first region 3 second, 2 for the third and 1 for the last one.
So the total number of solutions is "$4 \times 3 \times 3 \times 2 \times 2 \times 2 \times 2 \times 2 = 1152$" ~~4! which is 4x3x2x1=24~~
and there are "1152" possible solutions.

3:- Two colors allowed (Red, Green):
If only two colors are allowed, the problem becomes unsolvable because the adjacent region that must have different colors, and if there are only two colors, this constraint cannot be statisfy.

To Summarize: PS#02

- With three colors allowed, there are 6 solutions.
- With four colors allowed, there are 24 solutions.
- With two colors allowed, there are no solution.

Question no:-2

Consider the problem of placing K queen on an nxn Chessboard such that no two queen are attacking each other where K is given and $K \leq n^2$?

1. Choosing a CSP formulation. In your formulation, what are varibles?

In this problem of placing of k queens or on an nxn chessboard such no two queens attacking each other.

Varaibles:

The varaibles represent the positions of the queen on the chessboard. Each varaible crosspands to a row of the Chessboard. Since we placed at most one queen in each row, we have n varaibles, one for each row.

Each varaibles can take on values from 1 to n, representing the column in which the queen is placed in that particular row. So, the varaibles can be denoted as $X_1, X_2, \ldots X_n$ where X_i represents the column position of queen in row.

For example:- If we have 4x4 chessboard, the varaibles could be X_1, X_2, X_3, X_4 represeting the column position of queens in rows 1, 2, 3 and 4 respectively.

2) What are possible values of each varaible?

In the CSP ~~function~~ formulation for placing ~~keyword~~ k queens on a nxn chessboard, where each vareible represent the position of a queen in a particular row, the possible value of each varcible (column position) are integers 1 to n.

For example:- If we have nxn chess board, the possible values for each varaible representing the column position of a queen in a row would be; 1, 2, 3 ... n

This means that each queen can be placed in any column of its repestive row, from 1 to column n.

3) What sets of varaibles are constrained and how?

In the CSP formulation for placing k queens are nxn chessboard cretain sets of varaible are constraint to ensure that no two queens are attacking each other.

1) Row Constraint

Varaibles:- All varaibles X1, X2, ...Xn representing the column positions of queens in their respective rows.

Constraint: Each varaibles must have a different values from each other varaible. This ensure that no queen are placed each other.

2) Column Constraint

Varaibles:- All varaibles X1, X2...Xn representing the column position of queens in their respective rows.

Constraint:- Each varaible must have different values form every other varaible. This ensure that no queens are placed in the same direction.

3) Diagonal Constraint

Varaibles:- Pairs of varaibles representing the positions of queen that are diagonally related.

Constraint:- The absolute difference between the values of any two varaibles must not equal this crossponding row difference. This ensure that two queen are placed diagnolly adjacent each other.

Constraint with an example of 4x4 chessboard

Row Constraint: $X_1 != X_2 != X_3 != X_4$

Column Constraint: For a 4x4 board, this constraint is automitcally statisfied due to each ~~other~~ varaible representing different row.

Diagonal Constraint: This pairs of varaibles that are diagonally related are (1,2), (1,3), (1,4), (2,3), (2,4), and (3,4)

For example, the constraint between varaibles X_1 and X_3 would be $|X_1 - X_3| \neq |1-3|$ i.e., $|X_1 - X_3| != 2$, ensuring that no two queens are diagonally adjacent.

Question 4:-

(a) Decide weather each of the following setence is valid, unstatisfiable or neither with some

Smoke	Fire	¬ Smoke	¬ Fire	Smoke->Fire	¬Smoke => ¬Fire	(Smoke => Fire) => (¬Smoke => ¬Fire)
T	T	F	F	T	T	T
T	F	F	T	F	T	T
F	T	T	F	T	F	F
F	F	T	T	T	T	T

So, there is one false so the sentence is satisfy ii Smoke $\vee$ Fire $\vee \neg$Fire

Smoke	Fire	$\neg$ Smoke	$\neg$ Fire	Smoke $\vee$ Fire $\vee \neg$ Fire
T	T	F	F	T
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

Answer is tatology so the setence is valid

iii $((\text{Smoke} \wedge \text{Heat}) \Rightarrow \text{Fire}) \Leftrightarrow ((\text{Smoke} \Rightarrow \text{Fire}) \vee (\text{Heat} \Rightarrow \text{Fire}))$

Smoke	Heat	Fire	Smoke $\wedge$ Heat $\Rightarrow$ Fire	Smoke $\Rightarrow$ Fire	Heat $\Rightarrow$ Fire	(Smoke $\wedge$ Heat) $\Rightarrow$ Fire $\Leftrightarrow$ (Smoke $\Rightarrow$ Fire) $\vee$ (Heat $\Rightarrow$ Fire)
T	T	T	T	T	T	T
T	T	F	F	F	F	T
T	F	T	T	T	T	T
T	F	F	T	F	T	T
F	T	T	T	T	T	T
F	T	F	T	T	F	T
F	F	T	T	T	T	T
F	F	F	T	T	T	T

Answer is tatology so the setence is valid.

Food	Drinks	Party	(Food => Party)	(Drinks => Party)	(Food => Party) v (Drinks => Party)
T	T	T	T	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

(Food ∧ Drinks)	[(Food ∧ Drinks) => Party]	[(Food => Party) v (Drinks => Party)] => [(Food ∧ Drinks) => Party]
T	T	T
T	F	T
F	T	T
F	T	T
F	T	T
F	T	T
F	T	T
F	T	T

Since all are true so the answer and sentence is valid.

iv (Smoke => Fire) => ((Smoke ∧ Heat) => Fire)

Smoke	Heat	Fire	Smoke => Fire	(Smoke ∧ Heat) => Fire	(Smoke => Fire) => ((Smoke ∧ Heat) => Fire)
T	T	T	T	T	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	T	T
F	F	T	T	T	T
F	F	F	T	T	T

Answer is tatology so the sentence is valid

v Big ∨ Dumb ∨ (Big => Dumb)

Big	Dumb	Big => Dumb	Big ∨ Dumb	Big ∨ Dumb ∨ (Big => Dumb)
T	T	T	T	T
T	F	F	T	T
F	T	T	T	T
F	F	T	F	T

Answer is tatology so the setence is valid

(b) Consider the following sentence

[(Food => Party) ∨ (Drinks => Party)] => [(Food ∧ Drinks) => Party)]

i Determine, using enmuerations, wether this setence is valid Satisfiable (but not valid), or Statisfiable) ii Convert the left-hand and right-hand sides of implication into CNF.

The left-hand side:

- (Food => Party) v (Drinks => Party)

Using equivalence $\neg p \vee q$ for $q \Rightarrow q$

- ($\neg$Food v Party) v ($\neg$Drink v Party)

This can be converted into CNF by distributing the disjunction over the disjunction

- ($\neg$Food v $\neg$ Drinks v Party)

Using equvalence $\neg(p \wedge q) \vee r$ for $(p \wedge q) \Rightarrow r$

- $\neg$(Food $\wedge$ Drinks) v Party

By De-Morgen's law

- ($\neg$Food v $\neg$Drinks v Party)

Both sides in the same CNF : ($\neg$ Food v $\neg$ Drink v Party).

Since both sides are equivalent the implication is valid.

By Resolution

Negation of above sentence is :-

(Food ∧ Drinks ∧ ~party) ∧ (~Food ∧ ~Drinks ∧ Party)

1. Food from (Food ∧ Drinks ∧ Party) ∧ (~Food ∧ ~Drinks ∧ Party)
2. Drinks from (Food ∧ Drinks ∧ ~Party)
3. ~Party from (Food ∧ Drinks ∧ ~Party)
4. ~Food from (~Food ∧ ~Drinks ∧ Party)
5. ~Drinks from (~Food ∧ ~D ∧ Party)
6. ~Party from 1 and 4
7. Empty from 5 and 6

Question No: 5

Consider a vacabulary withe only four propositions A, B, C and D

How many models are there for following sentences?

(a) B∨C

A	B	C	D	B∨C
T	T	T	T	T
T	T	T	F	T
T	T	F	T	T
T	T	F	F	T
T	F	T	T	T
T	F	T	F	T
T	F	F	T	F
T	F	F	F	F
F	T	T	T	T
F	T	T	F	T
F	T	F	T	T
F	T	F	F	T
F	F	T	T	T
F	F	T	F	T
F	F	F	T	F
F	F	F	F	F

Out of 16 possible combination, B∨C is true in 12 of them. So, there are 12 model for B∨C sentence.

(b) $\neg A \vee \neg B \vee \neg C \vee \neg D$

A	B	C	D	$\neg A \vee \neg B \vee \neg C \vee \neg D$
F	F	F	F	T
F	F	F	T	T
F	F	T	F	T
F	F	T	T	T
F	T	F	F	T
F	T	F	T	T
F	T	T	F	T
F	T	T	T	T
T	F	F	F	T
T	F	F	T	T
T	F	T	F	T
T	F	T	T	T
T	T	F	F	T
T	T	F	T	T
T	T	T	F	T
T	T	T	T	F

So, out of 16 possible combinations $\neg A \vee \neg B \vee \neg C \vee \neg D$ is true of them. So, there are 15 models of the sentence.

A	B	C	D	$A \Rightarrow B$	$A \wedge \neg B \wedge (C \wedge D)$	$(A \Rightarrow B) \wedge A \wedge \neg B \wedge (C \wedge D)$
F	F	F	F	T	F	F
F	F	F	T	T	F	F
F	F	T	F	T	F	F
F	F	T	T	T	F	F
F	T	F	F	T	F	F
F	T	F	T	T	F	F
F	T	T	F	T	F	F
F	T	T	T	T	F	F
T	F	F	F	F	F	F
T	F	F	T	F	F	F
T	F	T	F	F	F	F
T	F	T	T	F	F F	F
T	T	F	F	T	T	F
T	T	F	T	T	F	F
T	T	T	F	T	F	F
T	T	T	T	T	F F	F

So out of 16 possible combinations, $(A \Rightarrow B) \wedge A \wedge \neg B \wedge (C \wedge D)$

is false there are 0 modules for this sentence

Q6:- Wampus World:-
(a) Imference laws:-

Prove : Pit does not exists in (1,2) i.e $\sim P_{1,2}$

R_2 states that of breeze is in (1,1), then pit will exist in either (1,2) or (2,1)

$R_1: \sim P_{1,1}$

$R_2: B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$

$R_3: B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$

$R_4: \sim B_{1,1}$

$R_5: B_{2,1}$

Changing biconditional on R_2 into implies:-

$R_6: B_{1,1} \rightarrow (P_{1,2} \vee P_{2,1}) \wedge (P_{1,2} \vee P_{2,1}) \rightarrow B_{1,1}$

$R_7: (P_{1,2} \vee P_{2,1}) \rightarrow B_{1,1}$

$\sim B_{1,1}$

$R_8: \sim(P_{1,2} \vee P_{2,1})$

Applying De Morgan's Law on R_3

$R_9: \sim P_{1,2} \wedge \sim P_{2,1}$

With And elimination, we will get the result of not having a pit at (1,2)

So

$R_{10}: \sim P_{1,2}$

(b) Resolution laws

Converting R_2 into CNF:-

$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$

$(B_{1,1} \rightarrow (P_{1,2} \vee P_{2,1})) \wedge ((P_{1,2} \vee P_{2,1}) \rightarrow B_{1,1})$ Expanding Bidirectional

$(\sim B_{1,1} \vee (P_{1,2} \vee P_{2,1})) \wedge (\sim(P_{1,2} \vee P_{2,1}) \vee B_{1,1})$ (Implication law)

$(\sim B_{1,1} \vee (P_{1,2} \vee P_{2,1})) \wedge (\sim P_{1,2} \vee B_{1,1}) \wedge (\sim P_{2,1} \vee B_{1,1}))$

$R_6 : \sim B_{1,1}$

$R_7 : P_{1,2}$

$R_8 = B_{1,1} \vee \sim P_{1,2}$

$R_9 = \sim P_{2,1} \vee B_{1,1}$

$R_{10} : \sim P_{2,1}$ Applying resolution therom 10 from 6 and 9