Homework Assignment No.2

Digital Signal Processing (module no. 24505)

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Task 1 (6 Points)
Discrete Fourier Transform (DFT)

Two 8-point sequences, $x[k]$ and $h[k]$ are shown below.

$$\text{DFT} : X[\mu] = \sum_{k=0}^{N-1} x[k] \cdot \mathrm{e}^{-\mathrm{j}\frac{2\pi}{N}k\mu} \tag{1}$$

(a) Compute the 8-point DFT of $x[k]$ using (1) and draw the resulting spectrum.

(b) Find $X[\mu]$ by recognizing that it can be obtained by sampling the transform of $x[k] = \delta[k+1] + \delta[k] + \delta[k-1]$ at eight points.

(c) Find the sequence $y[k]$ defined as the inverse DFT of $Y[\mu] = X[\mu]H[\mu]$. You do not have to compute $Y[\mu]$ to solve this problem, why?

Task 2 (3 Points)
Properties of circular convolution

The sequence $x[k] = [1, 2, 3, 4, 5, 0, 0, 6]$ is given.

Sketch the circular shift $y[k] = x[(k - k_0)_8]$ for the following values of k_0:

(a) $k_0 = 2$

(b) $k_0 = -2$

(c) $k_0 = 1068$

Task 3 (4 Points)
Cyclic convolution

Consider the DFT spectra $X[\mu]$, $H[\mu]$ with a length of $N = 4$:

$$\begin{aligned} X[\mu] &= 4\,\delta[\mu] + & 3\,\delta[\mu-1] - 1\,\delta[\mu-2] & & +3\,\delta[\mu-3] \\ H[\mu] &= & 2\,\mathrm{j}\,\delta[\mu-1] + 5\,\delta[\mu-2] & & -2\,\mathrm{j}\,\delta[\mu-3] \end{aligned}$$

Compute the cyclic convolution $y[k] = x[k] \circledast_N h[k]$.

Task 4 (4 Points)
Discrete Fourier Transform

A sequence $x[k]$ of length $N = 4$ has DFT $X[\mu] = \delta[\mu] + 2\delta[\mu-1] + 3\delta[\mu-2] + 4\delta[\mu-3]$.

A new sequence is created, $y[k] = x[k]e^{-j1.5\pi k}$. Find $Y[\mu]$.