

$$h(n-k) = \frac{1}{3^{n-k}} u(n-k)$$

$$n-k \geq 0$$

Question #01

$$x(n) = \left(\frac{1}{5}\right)^n u(n)$$

$$h(n) = 3^n u(n)$$

$$n \geq k$$

$$\Rightarrow (-\infty, n]$$

$$y(n) = x(n) * h(n)$$

$$= \sum_{k=-\infty}^{\infty} x(k) h(n-k)$$

for $n < 0$ $y(n) = 0$
(no common area)

for $n \geq 0$

$$\sum_{k=0}^n x(k) h(n-k)$$

$$\sum_{k=0}^n \left(\frac{1}{5}\right)^k 3^{n-k}$$

$$y(n) = 3^n \sum_{k=0}^n \left(\frac{1}{5 \times 3}\right)^k$$

$$y(n) = 3^n \frac{15}{14} \left[1 - \frac{1}{15^{n+1}}\right] \quad \text{... summation formula used}$$

$$y(n) = \begin{cases} 0 & n < 0 \\ \frac{15}{14} \left(1 - \frac{1}{15^{n+1}}\right) 3^n & n \geq 0 \end{cases}$$

(ii)

$$y(n) = \sum_{k=0}^n (0.6)^k (0.2)^{n-k}$$

$$= (0.2)^n \sum_{k=0}^n \left(\frac{0.6}{0.2} \right)^k = 3^k$$

$$= (0.2)^n \frac{3^{n+1} + 1}{2}$$

$$y(n) = \begin{cases} 0 & ; n < 0 \\ (0.2)^n \frac{3^{n+1} + 1}{2} & ; n \geq 0 \end{cases}$$

Question #102

$$(a) y(n) = x(n)^2$$

$$y_1(n) = x_1(n)^2$$

$$y_2(n) = x_2(n)^2$$

$$x_3(n) = \alpha x_1(n) + \beta x_2(n)$$

$$y_3(n) = x_3(n)^2$$

$$= \alpha^2 x_1(n)^2 + \alpha^2 x_2(n)^2 + 2\alpha\beta x_1(n)x_2(n)$$

$$y_3'(n) = \alpha y_1(n) + \beta y_2(n)$$

$$y_3'(n) = \alpha x_1(n)^2 + \beta x_2(n)^2$$

$$y_3(n) \neq y_3'(n)$$

non linear

$$y(n-n_0) = x_1(n-n_0)^2$$

$$y(n-n_0) = x(n^2-n_0)$$

(b) $y(n) = x(n+1)$

$$y_1(n) = x_1(n+1); y_2(n) = x_2(n+1)$$

$$x_3(n) = \alpha x_1(n) + \beta x_2(n)$$

$$y_3(n) = x_3(n+1) = \alpha x_1(n+1) + \beta x_2(n+1)$$

$$y'_3(n) = \alpha y_1(n) + \beta y_2(n)$$

$$= \alpha x_1(n+1) + \beta x_2(n+1)$$

\Rightarrow linear

$$y(n-n_0) = x(n-n_0+1)$$

$$y(n, n_0) = x(n-n_0+1)$$

$$y(n-n_0) \neq y(n, n_0)$$

\Rightarrow Not time invariant

(c) $y(n) = x(n) + \frac{1}{x(n+1)}$

$$y_1(n) = x_1(n) + \frac{1}{x_1(n+1)}; y_2(n) = x_2(n) + \frac{1}{x_2(n+1)}$$

$$x_3(n) = \alpha x_1(n) + \beta x_2(n)$$

$$y_3(n) = x_3(n) + \frac{1}{x_3(n+1)} = \alpha x_1(n) + \beta x_2(n) + \frac{1}{\alpha x_1(n+1) + \beta x_2(n+1)}$$

$$y'_3(n) = \alpha y_1(n) + \beta y_2(n)$$

$$= \alpha x_1(n) + \frac{1}{x_1(n+1)} + \beta x_2(n) + \frac{1}{x_2(n+1)}$$

\Rightarrow Non linear

$$y(n-n_0) = x(n-n_0) + \frac{1}{x(n-n_0+1)}$$

$$y(n, n_0) = x(n-n_0) + \frac{1}{x(n-n_0+1)} \quad \text{Time invariant}$$

(d) $y = x(n^2) \Rightarrow y_1(n) = x_1(n^2); y_2(n) = x_2(n^2)$

Let $x_3(n) = \alpha x_1(n) + \beta x_2(n) \Rightarrow y_3(n) = x_3(n^2)$

$y_3(n) = \alpha x_1(n^2) + \beta x_2(n^2) \Rightarrow$ Linear

$y_3'(n) = \alpha y_1(n) + \beta y_2(n) = \alpha x_1(n^2) + \beta x_2(n^2)$

$y(n-n_0) = x_1[(n-n_0)^2] \neq \Rightarrow$ Time Variant

$y(n, n_0) = x_1(n^2 - n_0)$

$\Rightarrow y_2(n) = x_2(n) + x_2(n+1)$

(e) $y(n) = x(n) + nx(n+1) \Rightarrow y_1(n) = x_1(n) + nx_1(n+1)$

$\alpha_3(n) = \alpha x_1(n) + \beta x_2(n)$

$y_3(n) = x_3(n) + nx_3(n+1) = \alpha x_1(n) + \beta x_2(n) + n[\alpha x_1(n+1) + \beta x_2(n+1)]$

$y_3'(n) = \alpha y_1 + \beta y_2 = \alpha x_1(n) + nx_1(n+1) + \alpha [x_1(n+1) + nx_1(n+2)]$

$y_3(n) = y_3'(n) \Rightarrow$ Linear

$y(n-n_0) = x(n-n_0) + (n-n_0)x(n-n_0+1)$

$y(n, n_0) = x(n-n_0) + (n-n_0)x(n-n_0+1)$

not equal \Rightarrow Time Variant

Question #03

(i) Time Domain: Represents how signal varies with respect to time

Frequency Domain Represent signal as function of frequency (How it varies with different frequencies)

Time Domain

Pros: Better visualization

Useful for transient analysis

Cons: More computation

Difficult to analyze frequency components

Frequency Domain

Pros: Less computation

easier visualization of harmonics

Cons: Loses time specific information

: Not better for visualization