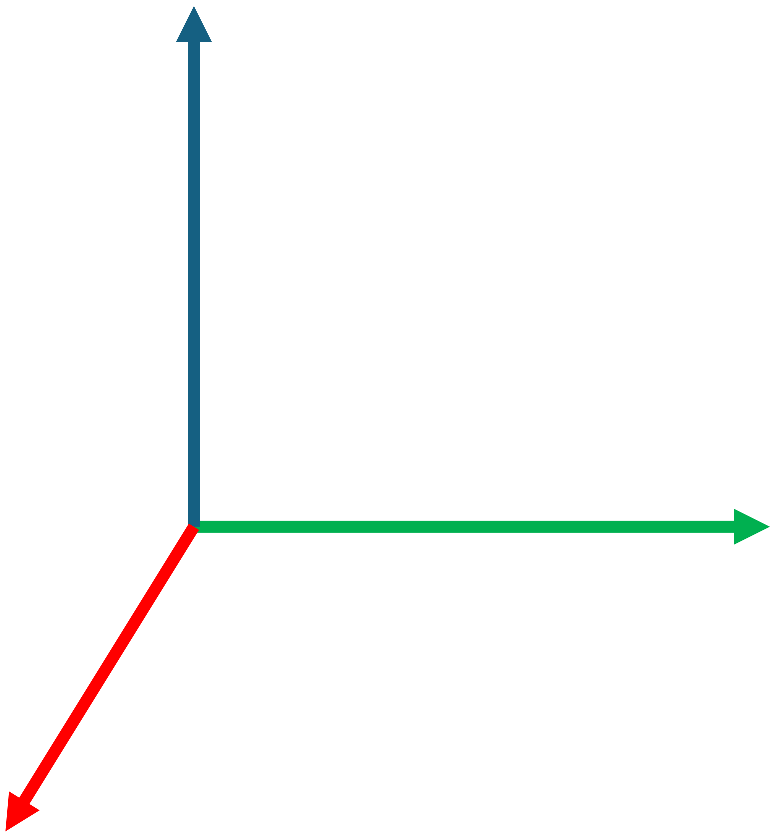
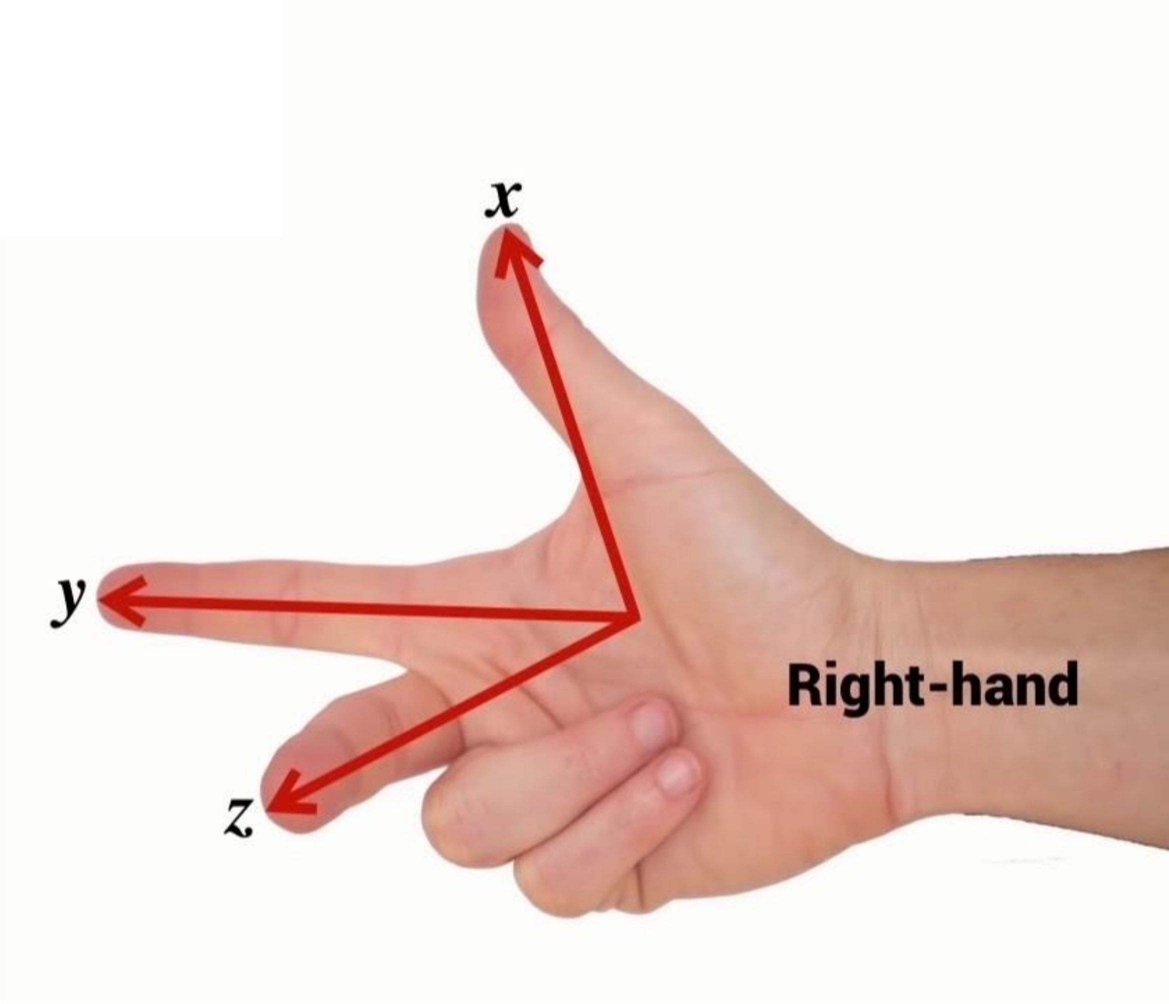
Right Hand Rule

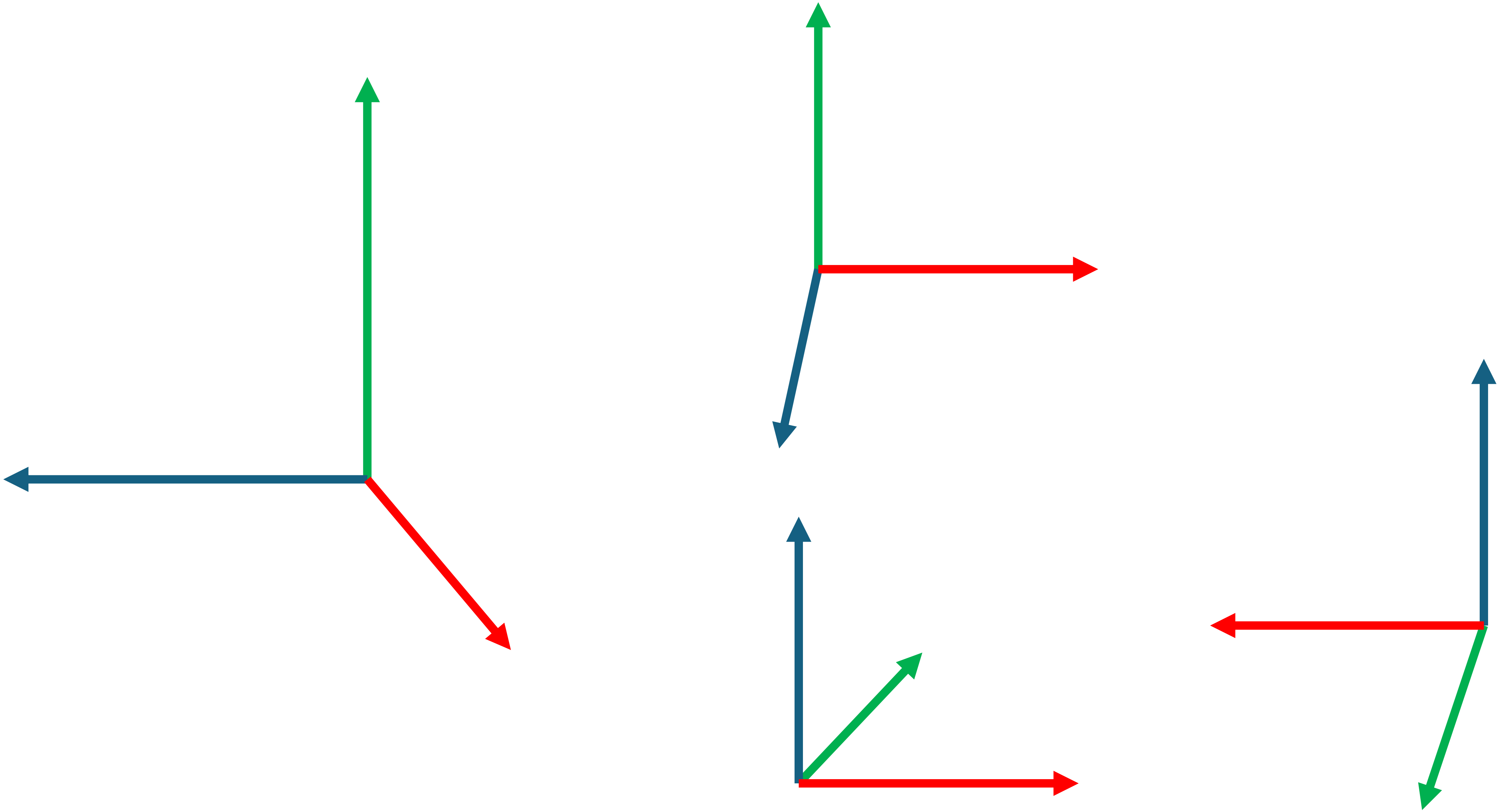
Z X Y





Right Hand Rule

Z X Y



Description of a Frame

- The information needed to completely specify the whereabouts of the manipulator hand is a position and an orientation

- For convenience, the point whose position we will describe is chosen as the origin of the body-attached frame

- The situation of a position and an orientation pair arises so often in robotics that we define an entity called a **frame**, which is a set of four vectors giving position and orientation information

- Note that a frame is a coordinate system where, in addition to the orientation, we give a position vector which locates its origin relative to some other embedding frame to define any frame wrt to any reference frame we need to its rotation and translation of it wrt ro refene

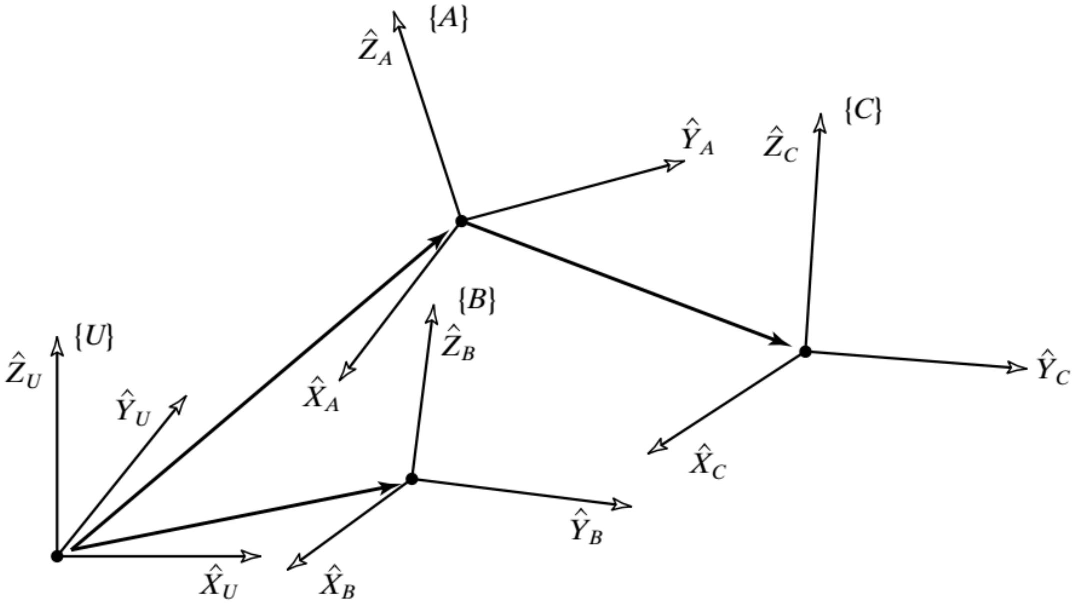
Description of a Frame

position of origin of B with respect to frame A

$$\{B\} = \{{}^A_B R, {}^A P_{BORG}\}$$

- There are three frames that are shown along with the universe coordinate system.

- Frames $\{A\}$ and $\{B\}$ are known relative to the universe coordinate system, and frame $\{C\}$ is known relative to frame $\{A\}$



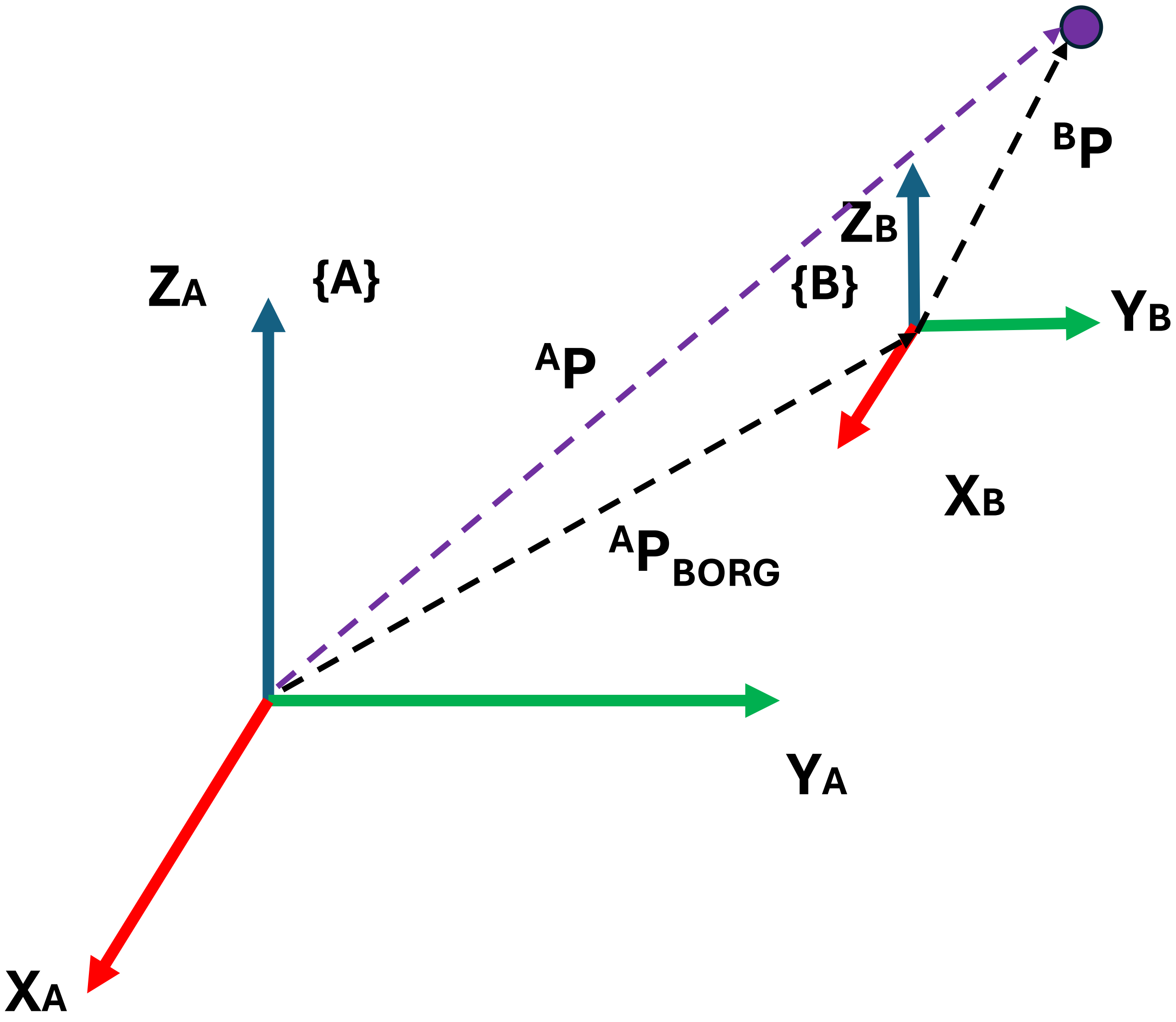
Mappings: Changing Descriptions from Frame to Frame

- In Robotics, we are concerned with expressing the same quantity in terms of various reference coordinate systems.

- The previous section introduced descriptions of positions, orientations, and frames

- We now consider the mathematics of mapping in order to change descriptions from frame to frame

Mappings Involving Translated Frames



position of P wrt frame A

$$^{A}P = {}^{B}P + {}^{A}P_{BORG}$$

In this simple example, we have illustrated mapping a vector from one frame to another.

The quantity itself (here, a point in space) is not changed; only its description is changed.

Point described by ^{B}P is not translated, but remains the same, and instead we have computed a new description of the same point, but now with respect to system $\{A\}$

Mappings Involving Rotated Frames

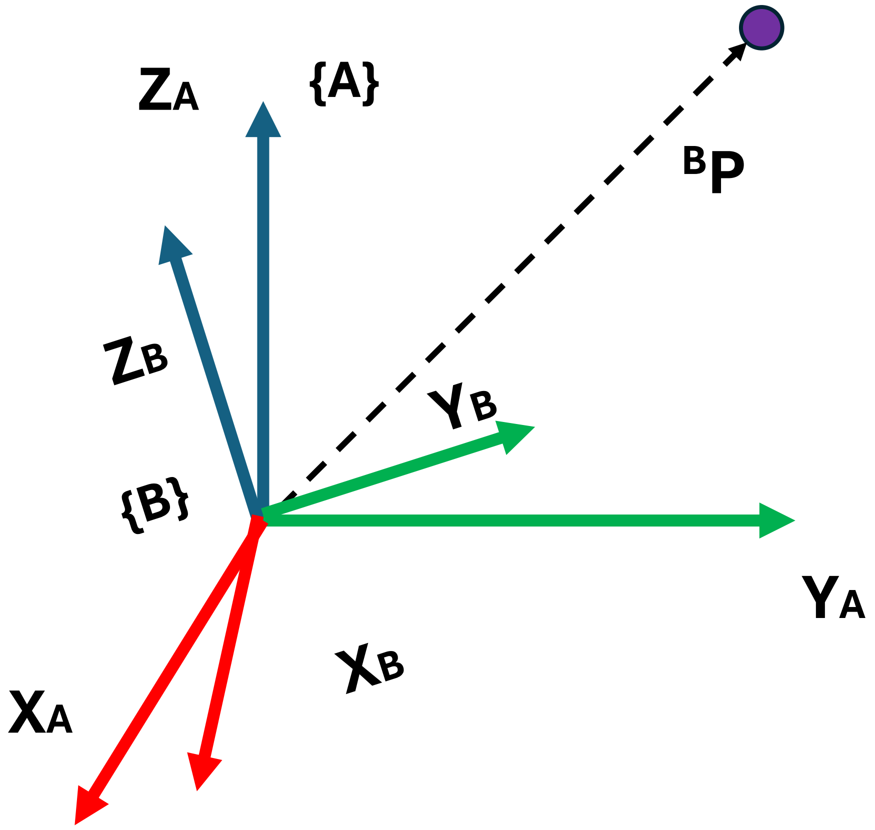
- The rotation matrix describes frame {*B*} relative to frame {*A*}, it was named with ${}^{A}_{B}R$

$$ {}^{A}_{B}R = {}^{B}_{A}R^{-1} = {}^{B}_{A}R^{T} $$

- Because the columns of ${}^{A}_{B}R$ are the unit vectors of {*B*} written in {*A*}, the rows of ${}^{A}_{B}R$ are the unit vectors of {*A*} written in {*B*}

$$ {}^{A}_{B}R = \left[\ {}^{A}\hat{X}_{B} \quad {}^{A}\hat{Y}_{B} \quad {}^{A}\hat{Z}_{B}\ \right] = \begin{bmatrix} {}^{B}\hat{X}_{A}^{T} \\ {}^{B}\hat{Y}_{A}^{T} \\ {}^{B}\hat{Z}_{A}^{T} \end{bmatrix} $$

Mappings Involving Rotated Frames

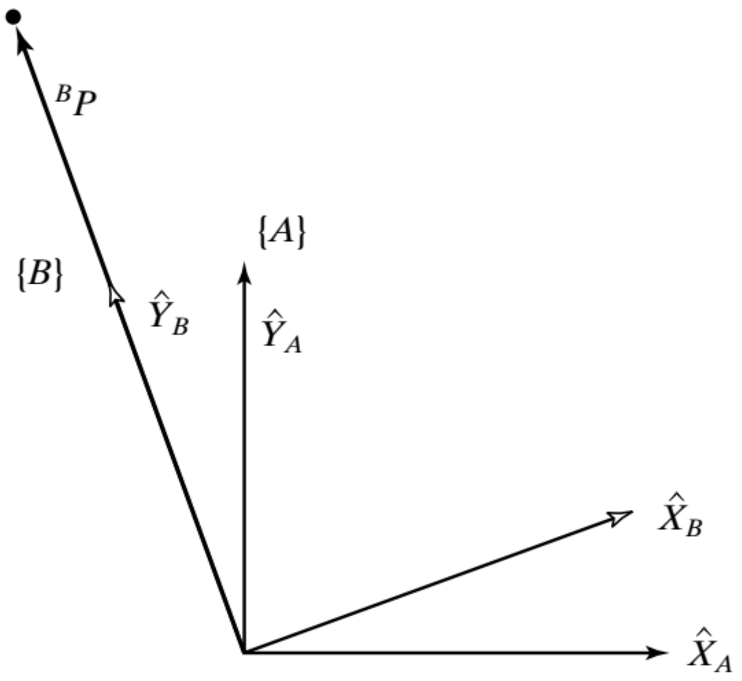


$$^AP = {}^A_BR\,{}^BP$$

In order to calculate ^AP, we note that the components of any
vector are simply the projections of that vector onto the unit
directions of its frame

Example

A frame $\{B\}$ that is rotated relative to frame $\{A\}$ about $\hat{Z}$ by 30 degrees. Here, $\hat{Z}$ is pointing out of the page.



$$
{}^{A}_{B}R = \begin{bmatrix} 0.866 & -0.500 & 0.000 \\ 0.500 & 0.866 & 0.000 \\ 0.000 & 0.000 & 1.000 \end{bmatrix}
$$

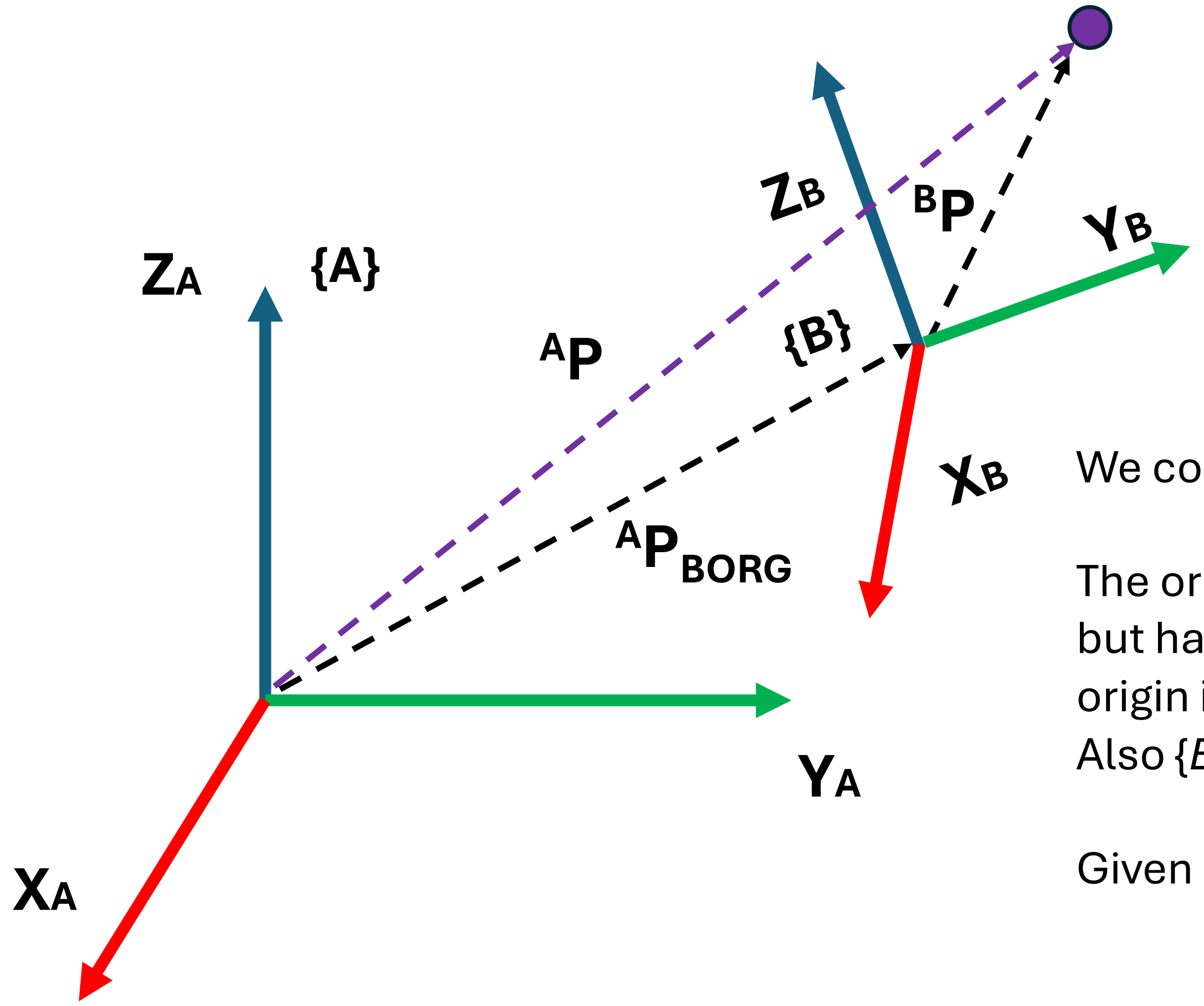
Given

$$
{}^{B}P = \begin{bmatrix} 0.0 \\ 2.0 \\ 0.0 \end{bmatrix},
$$

we calculate ${}^{A}P$ as

$$
{}^{A}P = {}^{A}_{B}R \, {}^{B}P = \begin{bmatrix} -1.000 \\ 1.732 \\ 0.000 \end{bmatrix}
$$

Mappings Involving General Frames

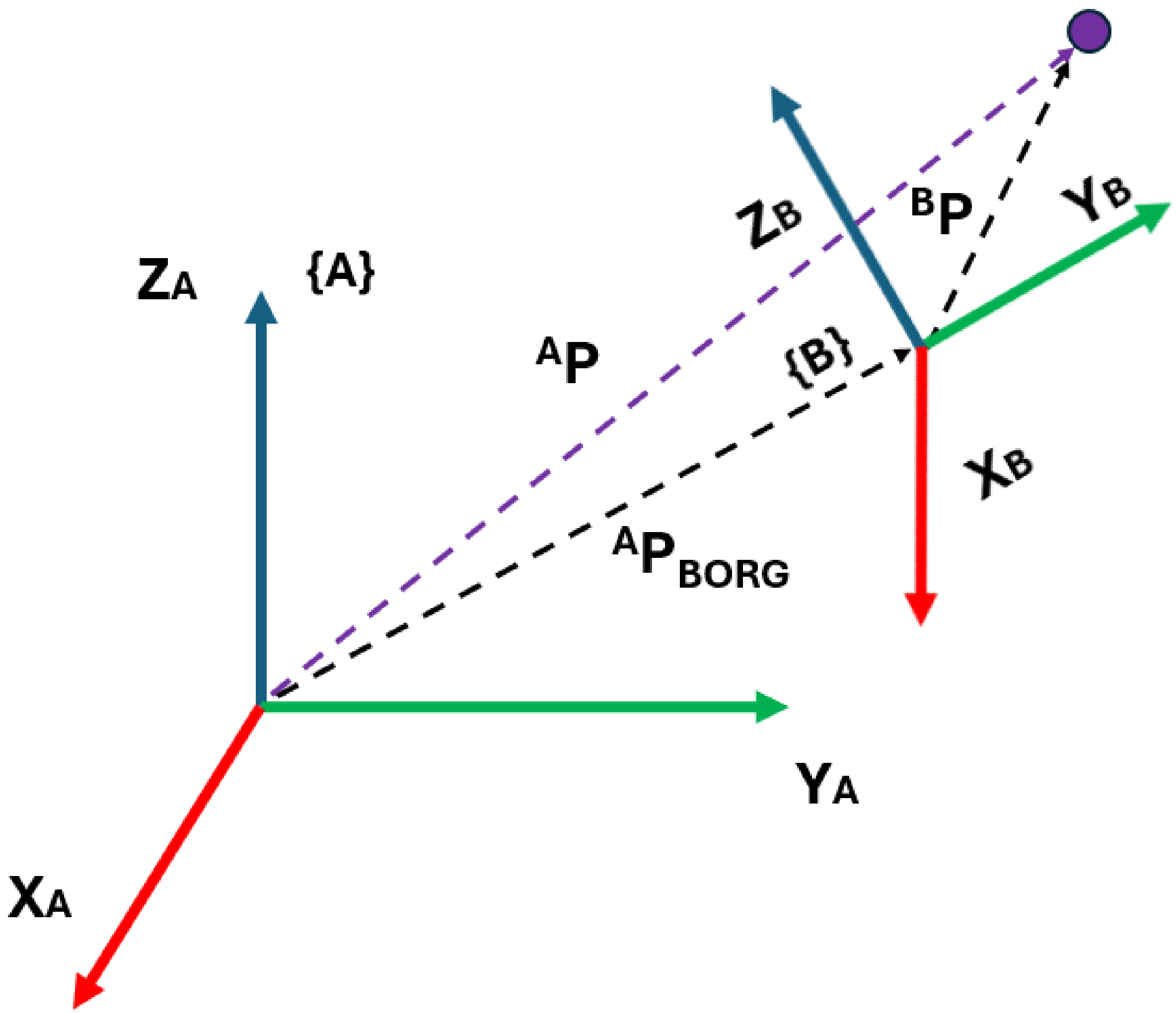


We consider the general case of mapping.

The origin of frame $\{B\}$ is not coincident with that of frame $\{A\}$ but has a general vector offset. The vector that locates $\{B\}$'s origin is called ${}^{A}P_{BORG}$.
Also $\{B\}$ is rotated with respect to $\{A\}$, as described by ${}^{A}_{B}R$

Given ${}^{B}P$, we wish to compute ${}^{A}P$

Mappings Involving General Frames



$${}^{A}P = {}^{A}_{B}R\,{}^{B}P + {}^{A}P_{BORG}$$

$${}^{A}P = {}^{A}_{B}T\,{}^{B}P$$

$$\begin{bmatrix} {}^{A}P \\ 1 \end{bmatrix} = \left[\begin{array}{ccc|c} & {}^{A}_{B}R & & {}^{A}P_{BORG} \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \begin{bmatrix} {}^{B}P \\ 1 \end{bmatrix}$$

Homogeneous Transformation matrix

Mappings Involving General Frames

$$\left[\begin{array}{ccc|c} & {}^{A}_{B}R & & {}^{A}P_{BORG} \\ \hline 0 & 0 & 0 & 1 \end{array}\right] = \begin{bmatrix} Rotation\ matrix & Position\ vector \\ Perspective\ Transformation & Scaling\ factor \end{bmatrix} = \begin{bmatrix} R_{3x3} & P_{3x1} \\ f_{1x3} & S_{1x1} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & Cos\theta & -Sin\theta & 0 \\ 0 & Sin\theta & Cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} Cos\theta & 0 & Sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -Sin\theta & 0 & Cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} Cos\theta & -Sin\theta & 0 & 0 \\ Sin\theta & Cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Basic Homogeneous Rotation matrices

$$\begin{bmatrix} 1 & 0 & 0 & P_x \\ 0 & 1 & 0 & p_y \\ 0 & 0 & 1 & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Basic Homogeneous Translation matrix

Example

Figure 2.8 shows a frame $\{B\}$, which is rotated relative to frame $\{A\}$ about $\hat{Z}$ by 30 degrees, translated 10 units in $\hat{X}_A$, and translated 5 units in $\hat{Y}_A$. Find ${}^AP$, where ${}^BP = [3.07.00.0]^T$.

$$
{}^A_BT = \begin{bmatrix} 0.866 & -0.500 & 0.000 & 10.0 \\ 0.500 & 0.866 & 0.000 & 5.0 \\ 0.000 & 0.000 & 1.000 & 0.0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

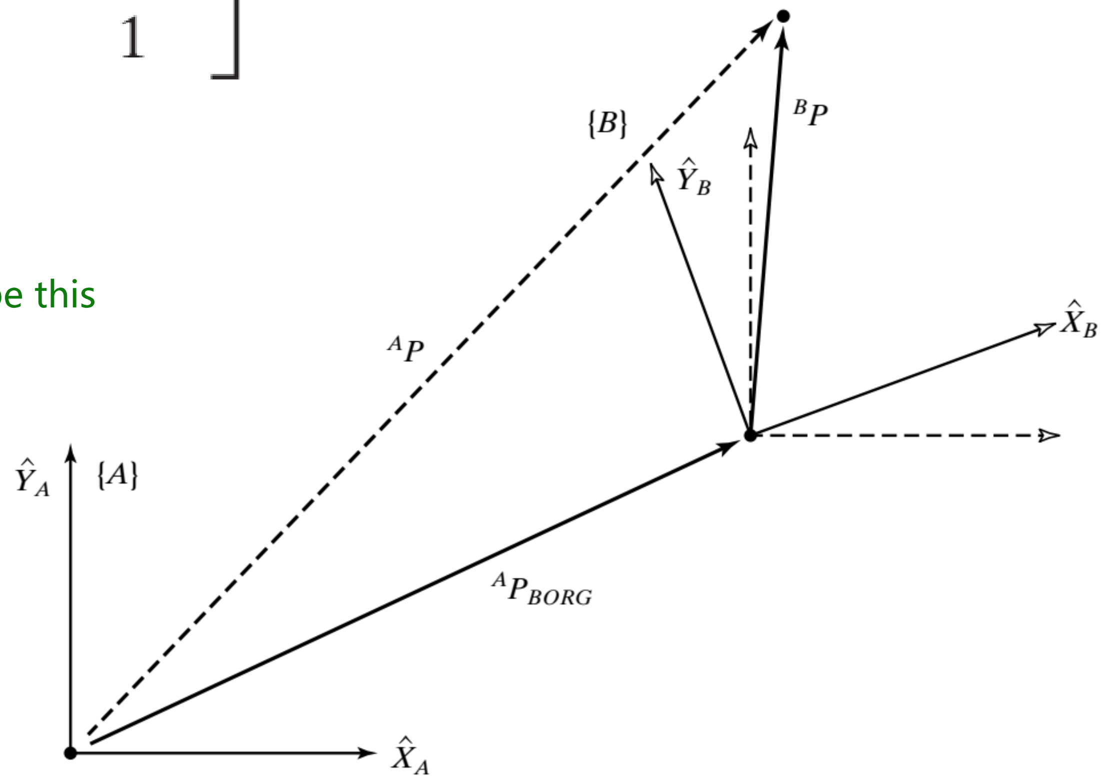
for multiplicaton only

$$
{}^BP = \begin{bmatrix} 3.0 \\ 7.0 \\ 0.0 \end{bmatrix}
$$

vector would only be this

$$
{}^AP = {}^A_BT\,{}^BP = \begin{bmatrix} 9.098 \\ 12.562 \\ 0.000 \end{bmatrix}
$$

1



Operators: Translations, Rotations, And Transformations

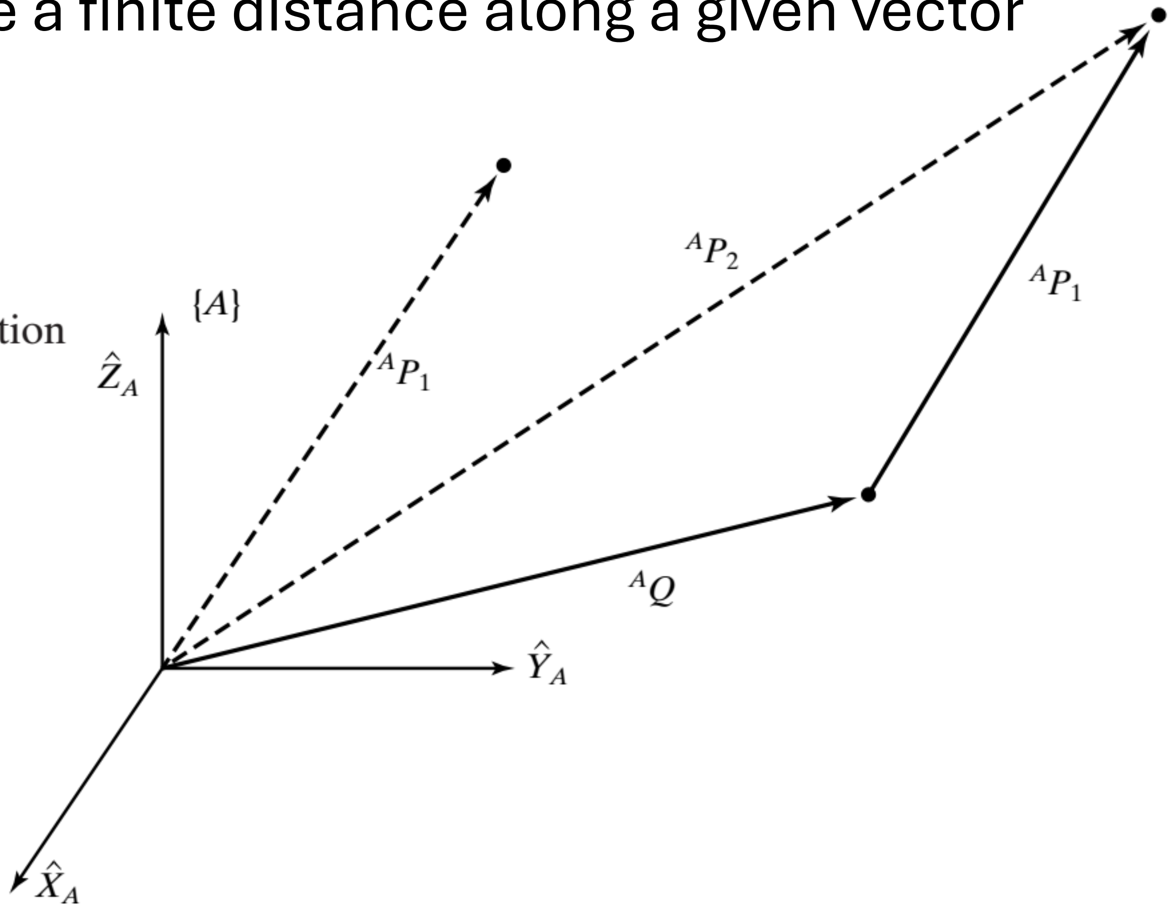
- **Translational Operator**
  - A translation moves a point in space a finite distance along a given vector direction

$$^{A}P_2 = {}^{A}P_1 + {}^{A}Q.$$

To write this translation operation as a matrix operator, we use the notation

$$^{A}P_2 = D_Q(q)\ {}^{A}P_1,$$

$$D_Q(q) = \begin{bmatrix} 1 & 0 & 0 & q_x \\ 0 & 1 & 0 & q_y \\ 0 & 0 & 1 & q_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

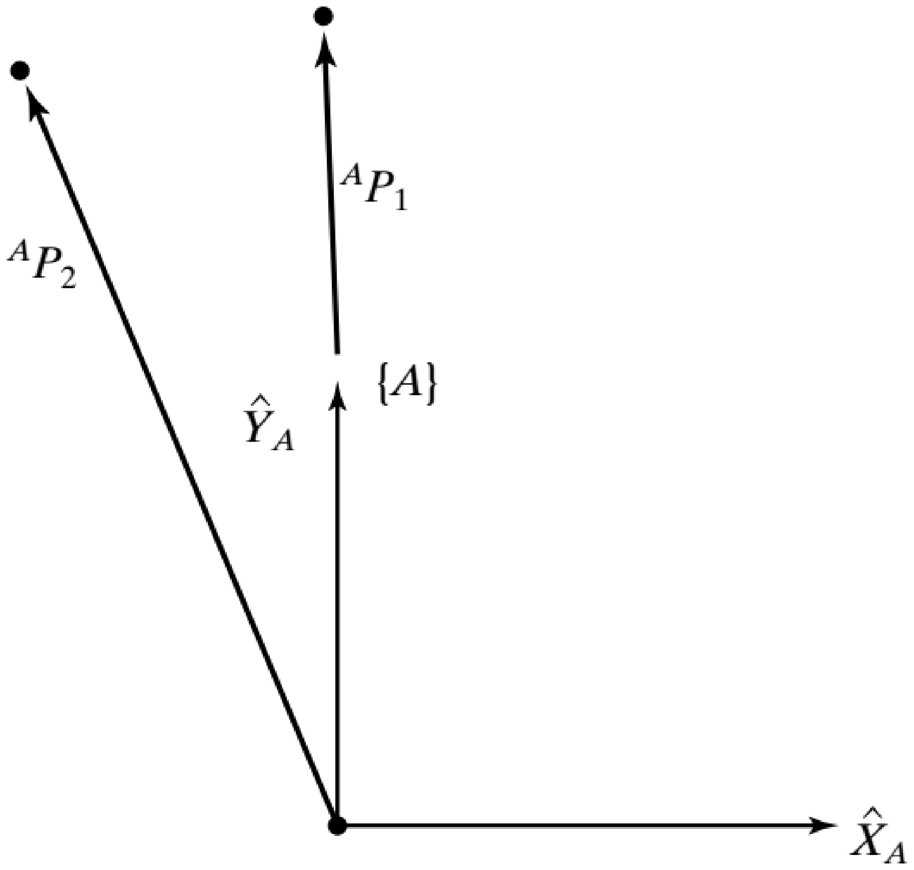


Rotation Operator

$$^{A}P_2 = R\ ^{A}P_1$$

$$^{A}P_2 = R_K(\theta)\ ^{A}P_1$$

$$R_z(\Theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Transformation Operator

$${}^{A}P_2 = T\,{}^{A}P_1.$$

The transform that rotates by R and translates by Q is the same as the transform

that describes a frame rotated by R and translated by Q relative to the reference frame

Figure 2.11 shows a vector ${}^{A}P_1$. We wish to rotate it about $\hat{Z}$ by 30 degrees and translate it 10 units in $\hat{X}_A$ and 5 units in $\hat{Y}_A$. Find ${}^{A}P_2$, where ${}^{A}P_1 = [3.0\ 7.0\ 0.0]^T$.

Example

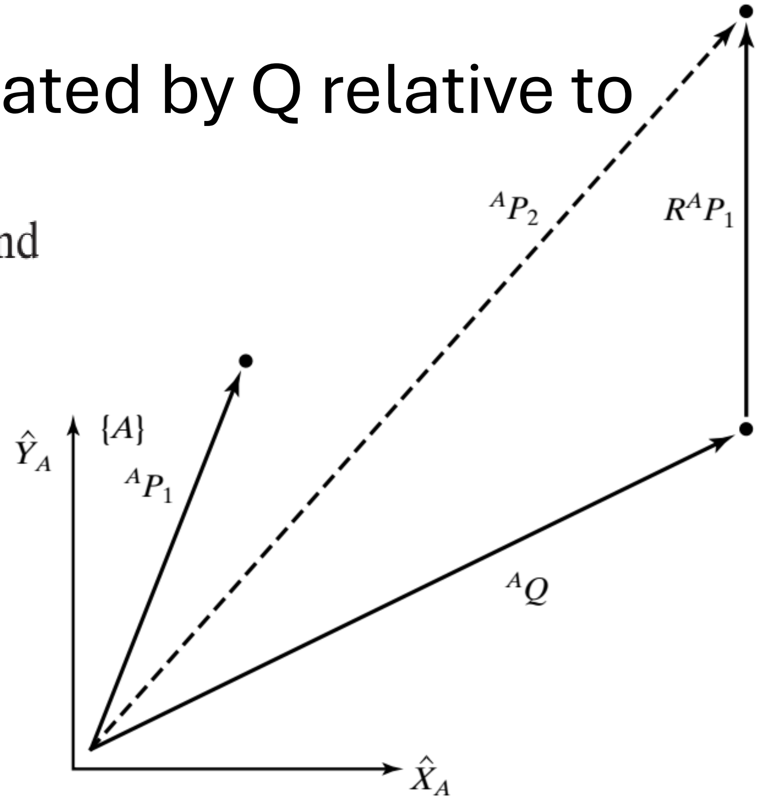
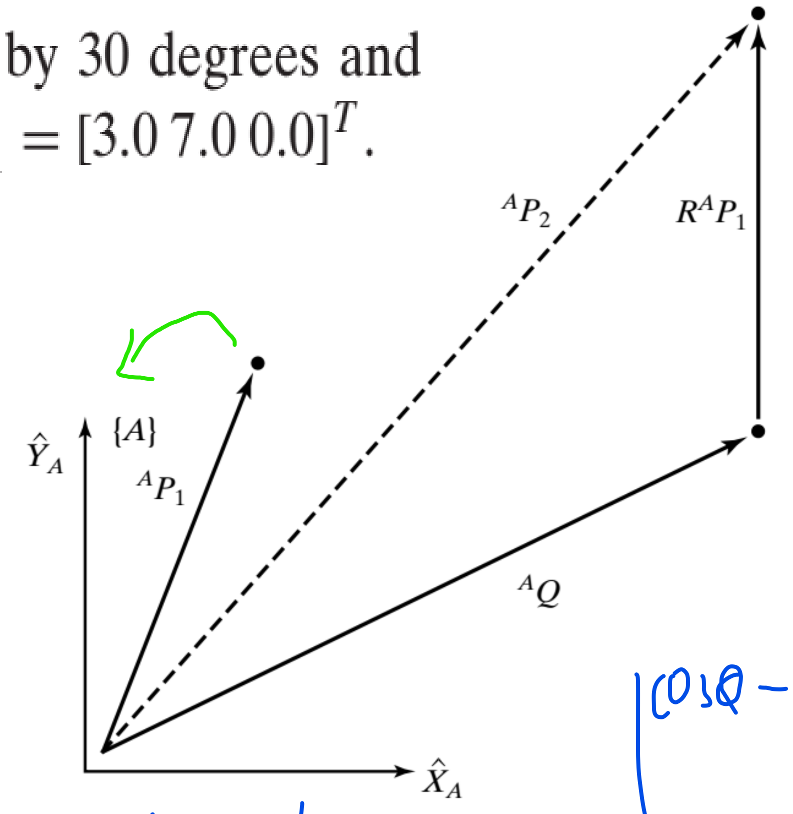


Figure 2.11 shows a vector ${}^{A}P_1$. We wish to rotate it about $\hat{Z}$ by 30 degrees and translate it 10 units in $\hat{X}_A$ and 5 units in $\hat{Y}_A$. Find ${}^{A}P_2$, where ${}^{A}P_1 = [3.0\ 7.0\ 0.0]^T$.

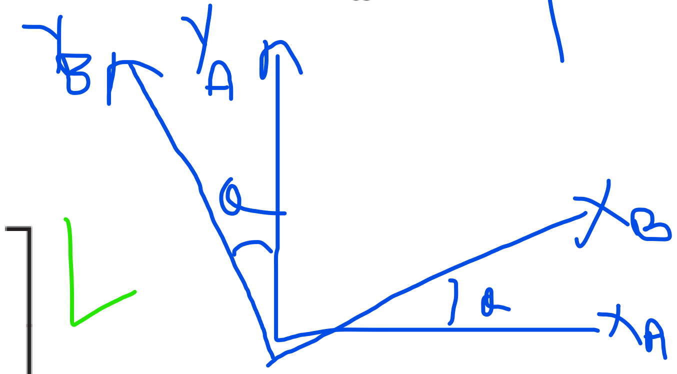


$$T = \begin{bmatrix} 0.866 & -0.500 & 0.000 & 10.0 \\ 0.500 & 0.866 & 0.000 & 5.0 \\ 0.000 & 0.000 & 1.000 & 0.0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos\theta & -\sin\theta \end{bmatrix}$$

Given

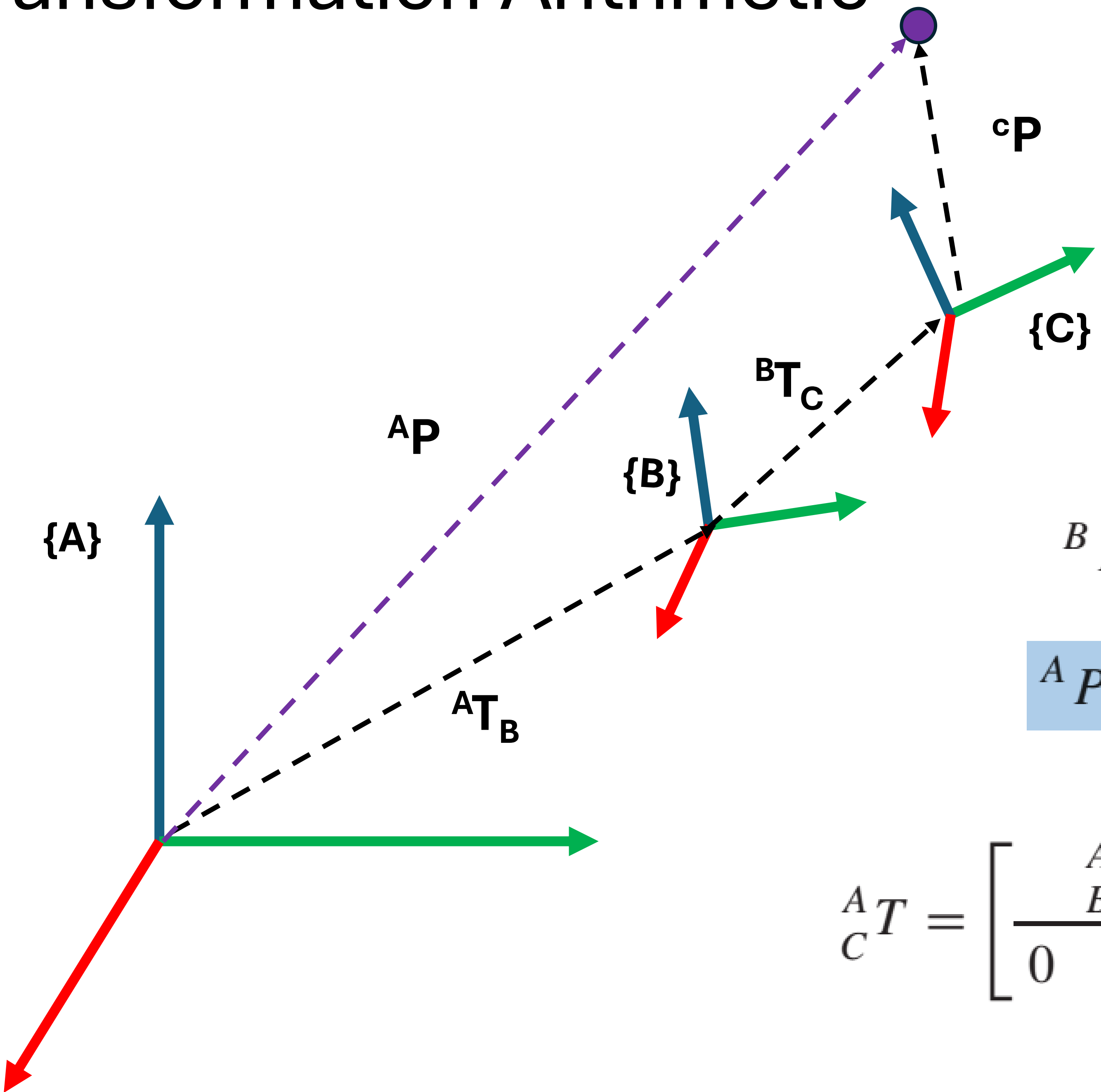
$${}^{A}P_1 = \begin{bmatrix} 3.0 \\ 7.0 \\ 0.0 \end{bmatrix}$$



we use T as an operator:

$${}^{A}P_2 = T\ {}^{A}P_1 = \begin{bmatrix} 9.098 \\ 12.562 \\ 0.000 \end{bmatrix}$$

Transformation Arithmetic



Compound Transformations

We have ^CP and wish to find ^AP

Frame $\{C\}$ is known relative to frame $\{B\}$ and frame $\{B\}$ is known relative to frame $\{A\}$. We can transform ^CP into ^BP as

then we can transform ^BP into ^AP as

$$^BP = {}^B_CT\,{}^CP; \qquad {}^AP = {}^A_BT\,{}^BP$$

$$^AP = {}^A_BT\,{}^B_CT\,{}^CP \qquad {}^A_CT = {}^A_BT\,{}^B_CT$$

$$^A_CT = \left[\begin{array}{ccc|c} & {}^A_BR\,{}^B_CR & & {}^A_BR\,{}^BP_{CORG} + {}^AP_{BORG} \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$