EE-379 Linear Control Systems

Week No. 2: Continuous Time System Description

- The Concept of Stability in Control Systems
- Block Diagrams
- Signal Flow Graphs
- Modeling Example: A Position Servo

System Description

Stability

- Stability is usually evaluated from two viewpoints internal and external.

Internal Stability

- For internal asymptotic stability, the **zero-input (natural)** response decays to zero as the time approaches ∞, for **all possible initial conditions**.
- The **characteristic polynomial roots** influence the response.
- Ensured if **all roots are located in the LHP**

System Description

Stability

- Stability is usually evaluated from two viewpoints internal and external.

External Stability

- For external (bounded input bounded output or BIBO) stability, the **zero-state response** is bounded as the time approaches ∞, for all bounded inputs.

- For **a bounded input** we can expect the **forced response to be bounded**.
- For the system to be BIBO stable, **the natural response of the output** should also be bounded.
- **If natural response decays to zero** as time approaches ∞ system is BIBO.
- It is ensured if the **zero-state impulse response** of the output decays to zero.

System Description

Stability – Evaluation Criterion

- If all characteristic polynomial roots are in the **LHP** then the natural response decays to zero and the system is asymptotically and BIBO stable.
- Another possibility for BIBO exists wherein suppose the characteristic polynomial contains some RHP roots (*poles*).
- RHP zeros of the *T(s)* cancel all the RHP poles from the TF and from the zero-state impulse response.
- At least **one term** in the **zero-input response** will go to ∞ , while the **zero-state impulse response** **decays to zero**. So, the system will be **asymptotically unstable but BIBO stable**.

System Description

Stability – Evaluation Criterion

- Thus, the two types of stability differ when all RHP poles are canceled by RHP zeros.
- We **evaluate BIBO stability** by examining the **zero-state impulse response** which allows for poles zero cancellation.
- We **evaluate asymptotic stability** by examining the **zero-input response of the system.**

System Description

Stability – Evaluation Criterion (simple words)

- **A system is stable** if the **natural response approaches zero** as time approaches infinity.
- **A system is unstable** if the **natural response approaches infinity** as time approaches infinity.
- **A system is marginally stable** if the **natural response neither decays nor grows** but **remains constant or oscillates**.
- **A system is stable** if every bounded input yields a bounded output.
- **A system is unstable** if any bounded input yields an unbounded output.

System Description

Stability – Example

- Consider the following three *TFs.*

$$T_1(s) = \frac{(s+1)}{(s+2)(s+3)} = \frac{2}{(s+3)} - \frac{1}{(s+2)}$$

$$T_2(s) = \frac{(s-7)}{(s-2)(s+3)} = \frac{2}{(s+3)} - \frac{1}{(s-2)}$$

$$T_3(s) = \frac{2(s-2)}{(s-2)(s+3)} = \frac{2}{(s+3)} + \frac{0}{(s-2)}$$

- The **first system** will have **zero input response**

$$y_{zi1} = K_1 e^{-3t} + K_2 e^{-2t}$$

- K_1 and K_2 are determined by initial conditions
- **Zero-state impulse response** when input *r(t)* is a unit impulse function is;

$$y_{zs,impulse1}(t) = 2e^{-3t} - e^{-2t}$$

- Which decays to zero, so the system is both asymptotically and BIBO stable

System Description

Stability – Example

- Consider the following three *TFs.*

$$T_1(s) = \frac{(s+1)}{(s+2)(s+3)} = \frac{2}{(s+3)} - \frac{1}{(s+2)}$$

$$T_2(s) = \frac{(s-7)}{(s-2)(s+3)} = \frac{2}{(s+3)} - \frac{1}{(s-2)}$$

$$T_3(s) = \frac{2(s-2)}{(s-2)(s+3)} = \frac{2}{(s+3)} - \frac{0}{(s-2)}$$

- For the **second system**

$$y_{zi2} = K_1 e^{-3t} + K_2 e^{2t}$$

$$y_{zs,impulse2}(t) = 2e^{-3t} - e^{2t}$$

- This is asymptotically unstable because e^{2t} will approach infinity and it is *BIBO* unstable due to the same term.

System Description

Stability – Example

- Consider the following three *TFs.*
- For the **third system**

$$T_1(s) = \frac{(s+1)}{(s+2)(s+3)} = \frac{2}{(s+3)} - \frac{1}{(s+2)}$$

$$T_2(s) = \frac{(s-7)}{(s-2)(s+3)} = \frac{2}{(s+3)} - \frac{1}{(s-2)}$$

$$T_3(s) = \frac{2(s-2)}{(s-2)(s+3)} = \frac{2}{(s+3)} - \frac{0}{(s-2)}$$

$$y_{zi3} = K_1 e^{-3t} + K_2 e^{2t}$$

$$y_{zs,impulse3}(t) = 2e^{-3t}$$

- This is asymptotically unstable because e^{2t} term, but *BIBO* stable due to the cancellation of *RHP zero also located at +2*

System Description

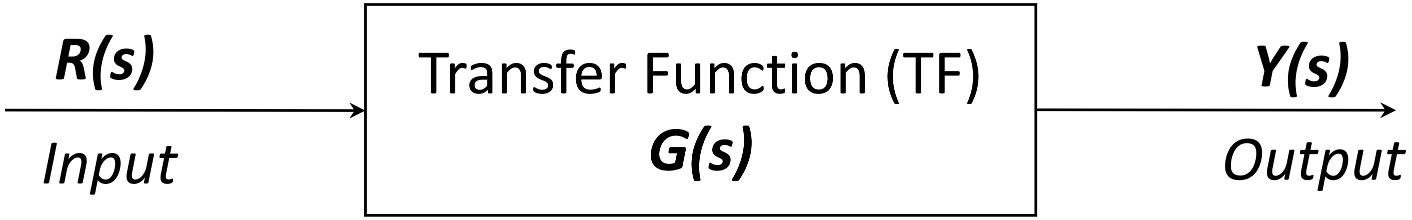
Stability Description – Example

- The most conservative approach is to demand **asymptotic stability**.
- Suppose the **zero-state response** of each of the three systems provides the **external position of an aircraft** while the zero-input response provides the **response of some internal device**.
- The first system has an acceptable response for both the aircraft and the device.
- While the second system has an unacceptable response for both the aircraft and the device.
- Although the zero-state response of the third system is stable, the electrical device is probably destroyed.
- This is unacceptable since this device may play a vital role in the future activity of the aircraft.
- Only **asymptotic stability is accepted**

System Description

Block Diagram - Uses

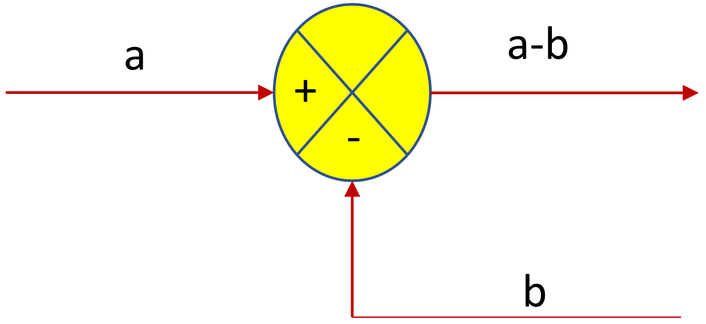
- Block diagrams are used to model all types of systems because of their simplicity and versatility.
- A block diagram can be used to describe the **composition** and **interconnection** of a system.
- Also, it can be used together with transfer functions, to describe the **cause** and **effect** relationships throughout the system.
- A block diagram consists of unidirectional, operational blocks that represent the *TF*.



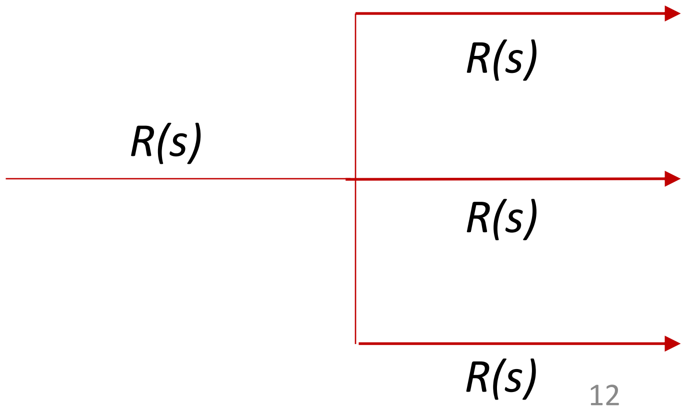
System Description

Block Diagram - Uses

- Many systems are composed of **multiple subsystems**.
- When multiple subsystems are interconnected, a few more **schematic elements** must be added to the block diagram. Like **summing junctions** and **pickoff points**.



- Summing Point: A **circle with a cross** indicates a summing operation. The plus + and minus - sign at each arrowhead indicates whether the signal is to be added or to be subtracted.



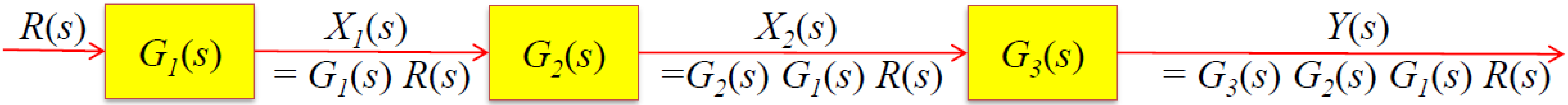
- Junction/Pick-off Point: A point from which the signal from a block goes **concurrently** to other blocks or summing points.

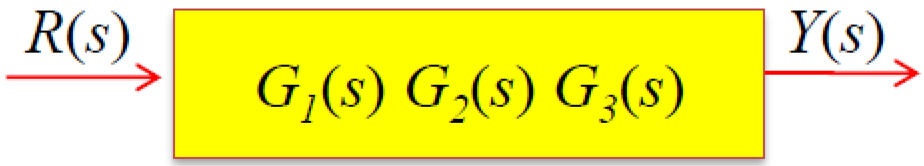
System Description

Block Diagram – Common Forms

- There are three basic forms, by which the subsystems are connected together.
    - **Cascade form**
    - Parallel form
    - Feedback form

Cascade Form



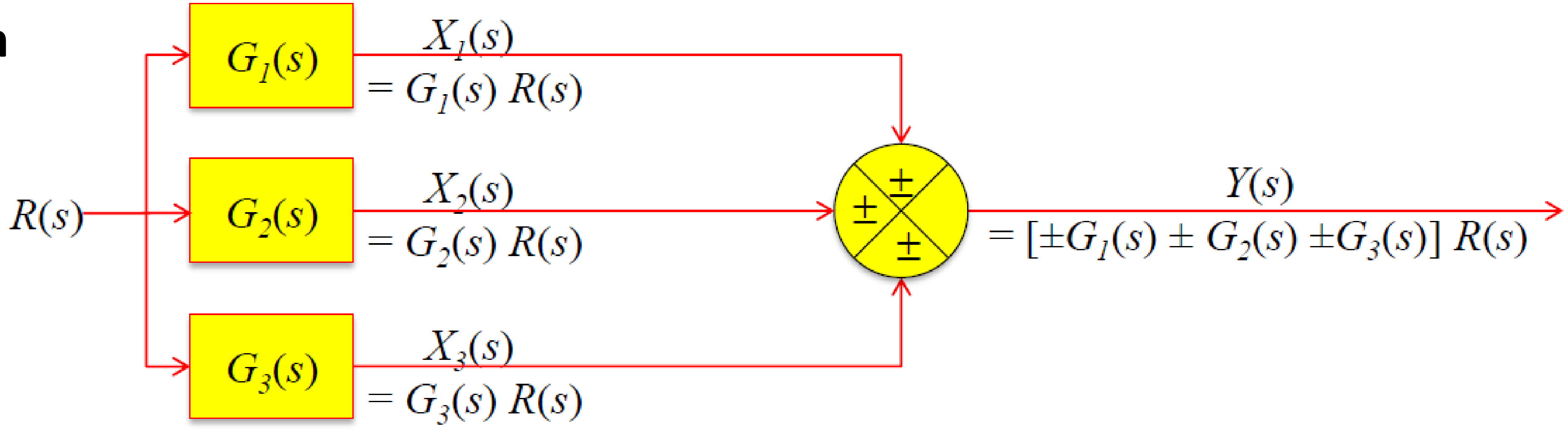


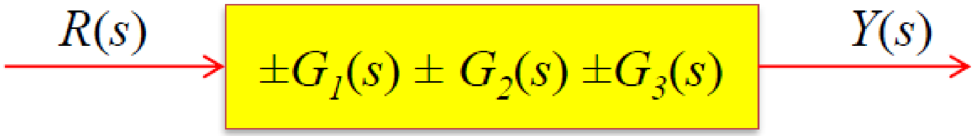
System Description

Block Diagram – Common Forms

- There are three basic common forms, by which the subsystems are connected together.
  - Cascade form
  - **Parallel form**
  - Feedback form

Parallel Form



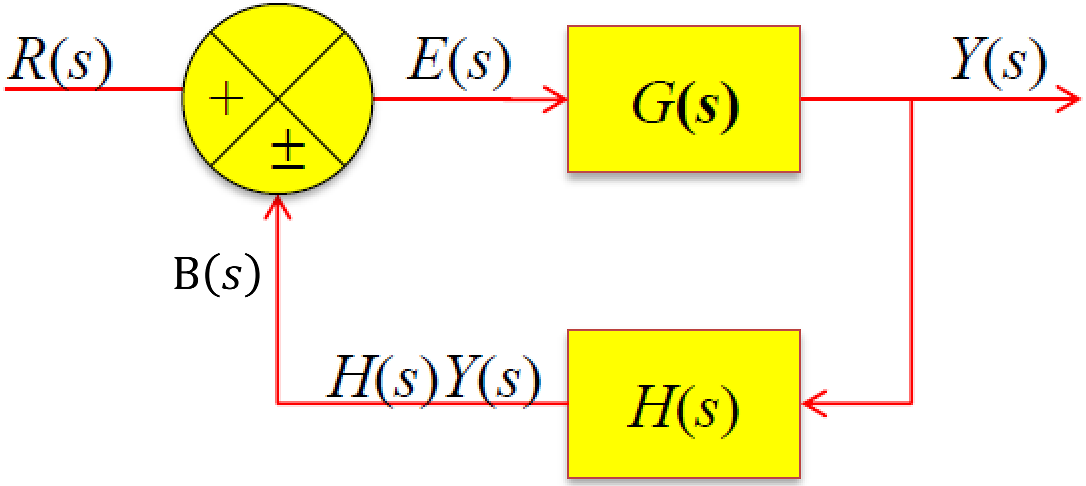


System Description

Block Diagram – Common Forms

- There are three basic common forms, by which the subsystems are connected together.
  - Cascade form
  - Parallel form
  - **Feedback form**

Feedback Form



Open-Loop Transfer Function

- The ratio of the **feedback signal** $\mathbf{H}(\boldsymbol{s})\boldsymbol{Y}(\boldsymbol{s})$ to the **actuating error signal** $\mathbf{E}(\boldsymbol{s})$ is called the open-loop transfer function.

$$T(s) = \frac{B(s)}{E(s)} = G(s)H(s)$$

Feed-forward Transfer Function

- The ratio of the **output** $\boldsymbol{Y}(\boldsymbol{s})$ to the **actuating error signal** $\mathbf{E}(\boldsymbol{s})$ is called the feedforward transfer function.

$$T(s) = \frac{Y(s)}{E(s)} = G(s)$$

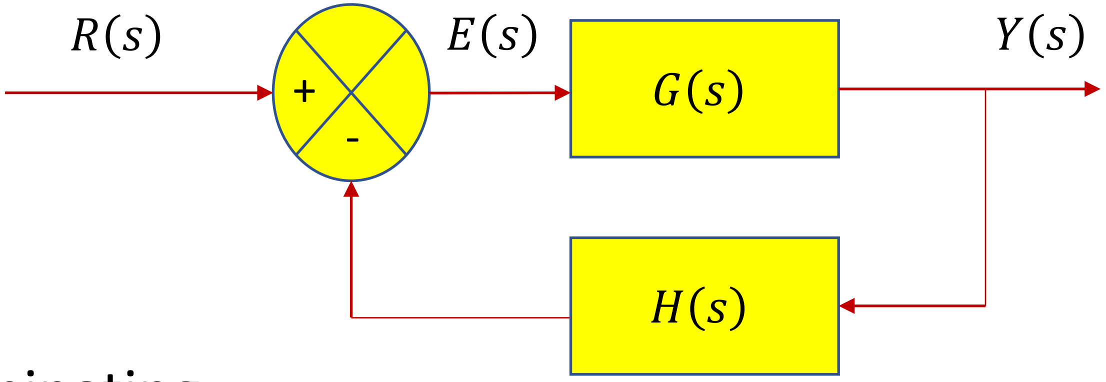
- If the **feedback transfer function** $\mathbf{H}(\boldsymbol{s})$ is **unity**, then the **open-loop transfer function** and the **feedforward transfer function** are the **same**.

System Description

Block Diagram – Common Forms – Feedback

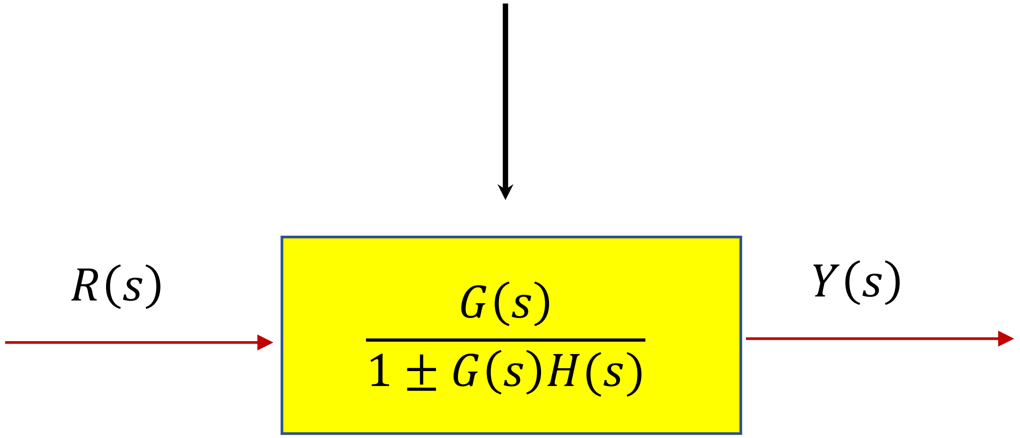
- The relationship between the signals is:

$$Y(s) = G(s)E(s)$$
$$E(s) = R(s) \pm H(s)Y(s)$$



Solving for $Y(s)$ in terms of $\mathrm{R}(s)$ and eliminating $\mathrm{E}(s)$, we get.

$$Y(s) = G(s)[R(s) \pm H(s)Y(s)]$$
$$Y(s) = G(s)R(s) \pm G(s)H(s)Y(s)$$



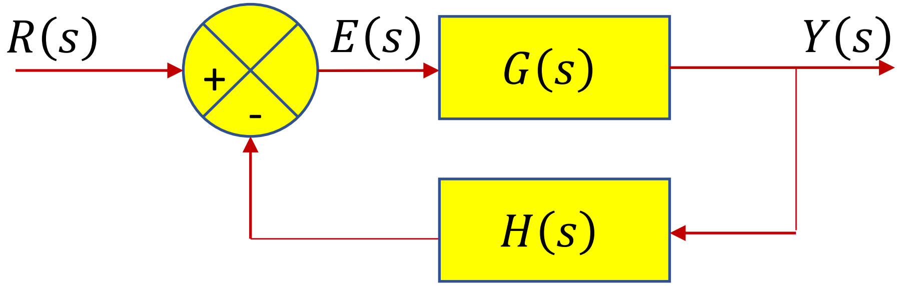
which results in

$$T(s) = \frac{Y(s)}{R(s)} = \frac{G(s)}{1 \pm G(s)H(s)}$$

Closed-loop transfer function

System Description

Block Diagram – Common Forms – Feedback



- Fundamental to control engineering as it reveals the **effect** of applying **feedback** to a system.
- Practical meaning of two possible summation (positive and negative):

  Lets imagine,
  - $G(s)$ – the combination of cruise controller and dynamics of a vehicle.
  - $H(s)$ – velocity measuring device.
  - $R(s)$ – desired velocity
  - $Y(s)$ – actual velocity
  - Say, desired velocity = 65 km/h
  - Actual velocity = 55 km/h

Using negative summation:

- Error = (65-55)km/h = 10 km/h
- This would speed up the car by 10 km/h and would be a logical choice.

Using positive summation

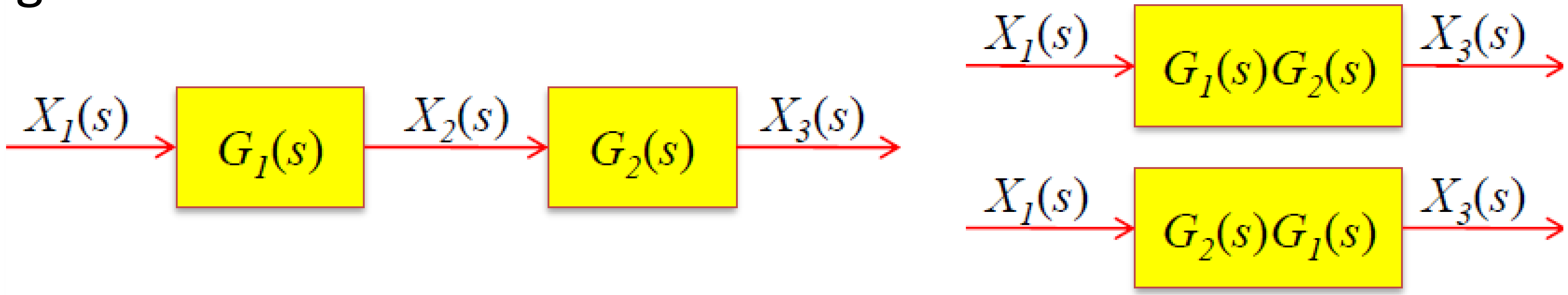
- Error = (65+55) = 110 km/h
- +'ve sign on summer introduces negative co-eff and results in RHP roots.
- This causes instability.

Therefore, negative feedback should be used.

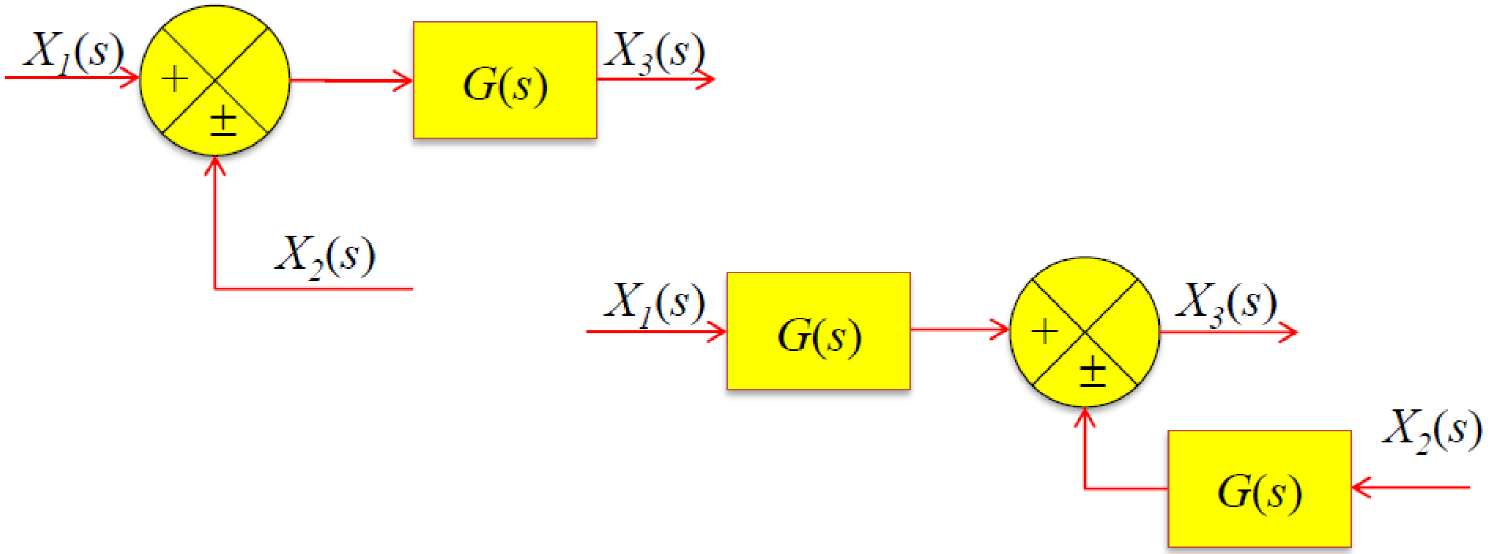
System Description

Block Diagram – Reductions (6)

1) Combining blocks in cascade



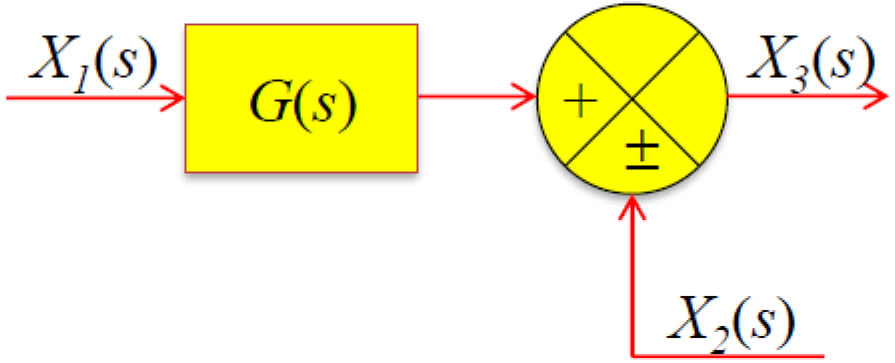
2) Moving a summing point forward

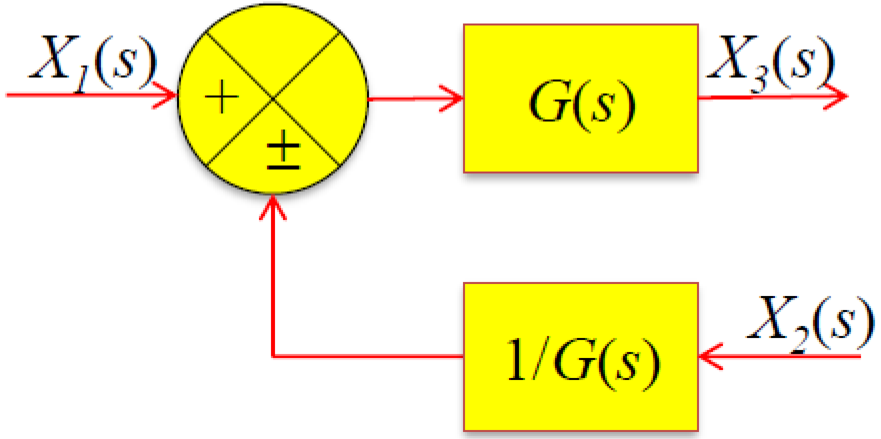


System Description

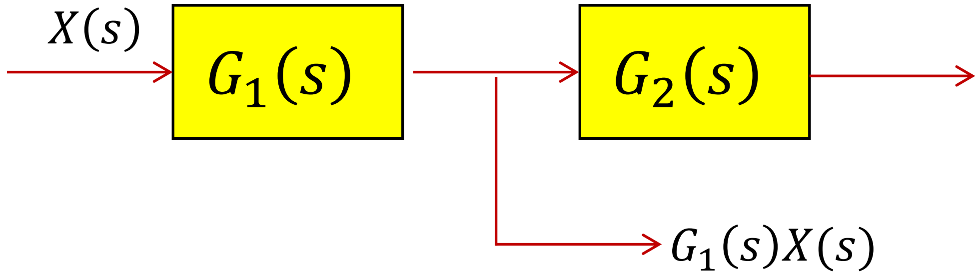
Block Diagram – Reductions (6)

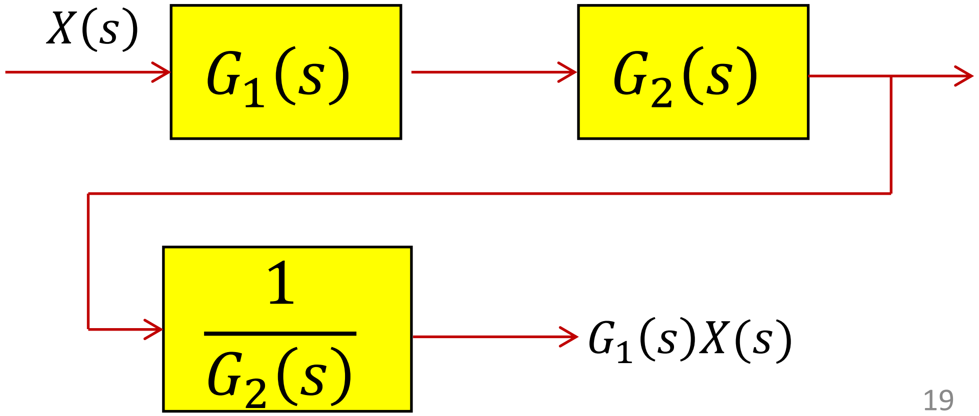
3) Moving a summing point back





4) Moving a pickoff point forward

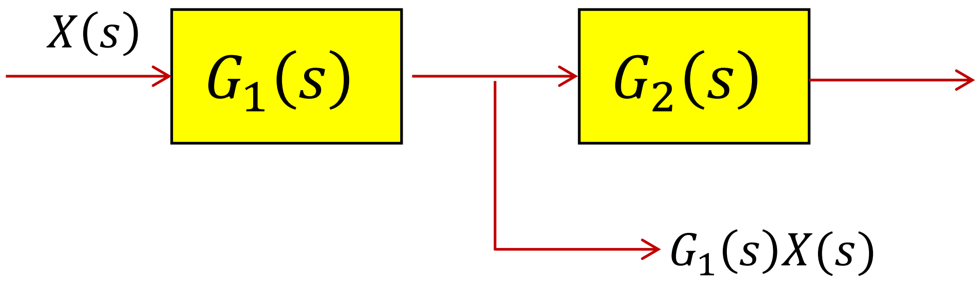


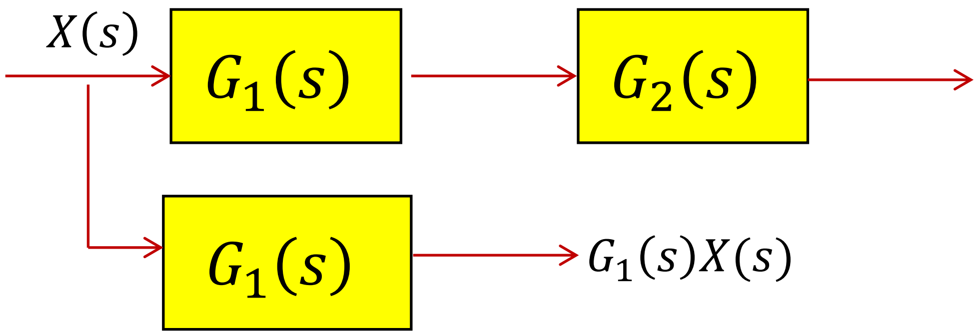


System Description

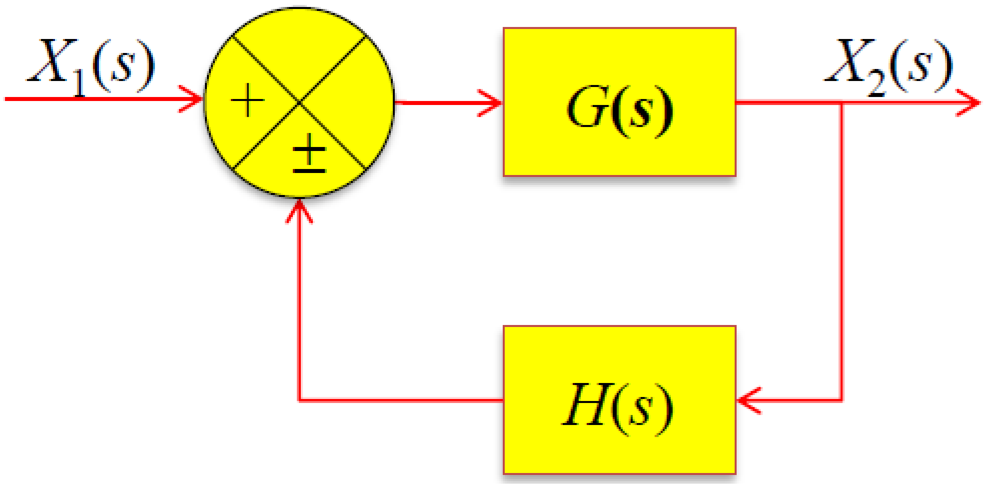
Block Diagram – Reductions (6)

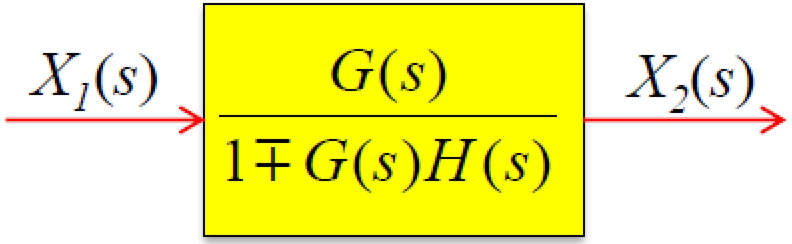
5) Moving a pickoff point back





6) Eliminating a feedback loop

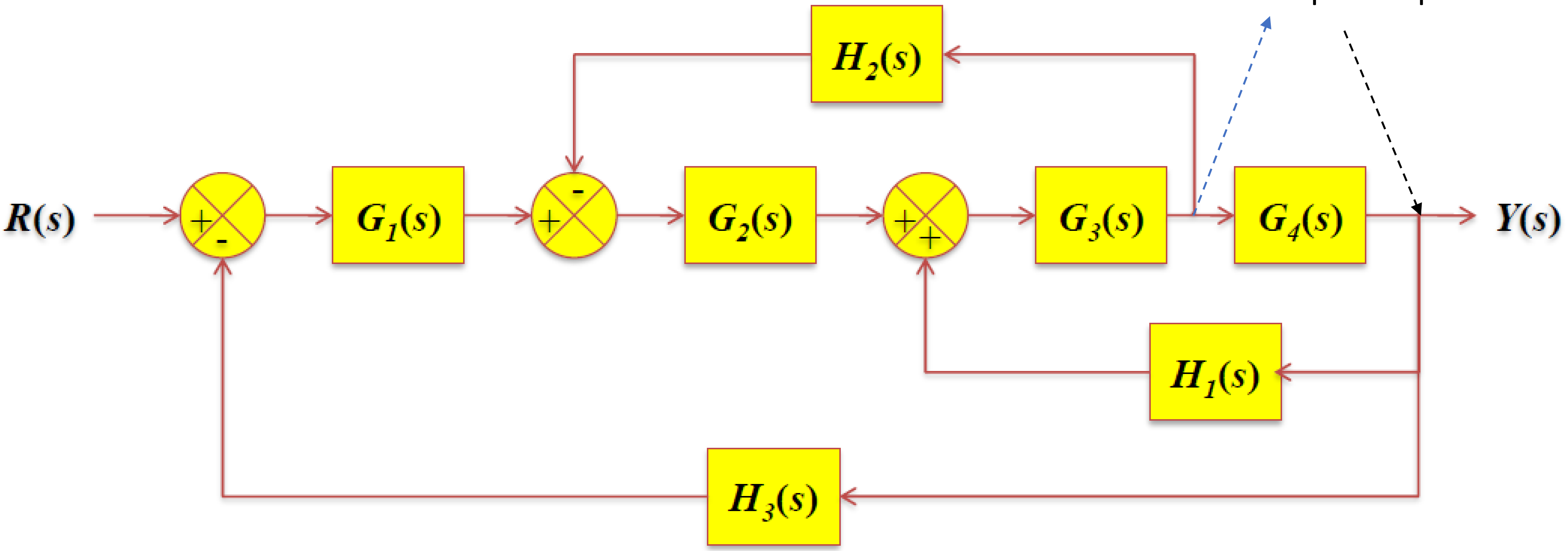




System Description

Block Diagram Reduction – Example

1. Move pickoff point

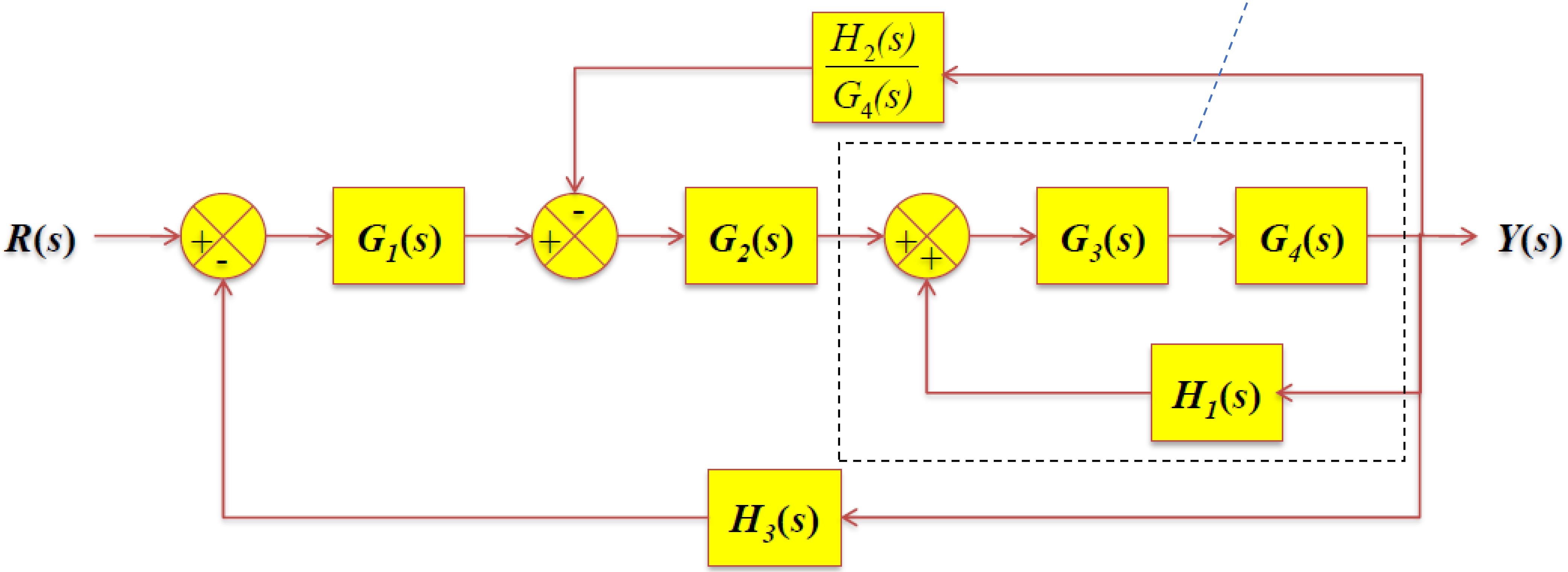


System Description

Block Diagram Reduction – Example

- Step-by-step solution

2. Calculate closed loop transfer function

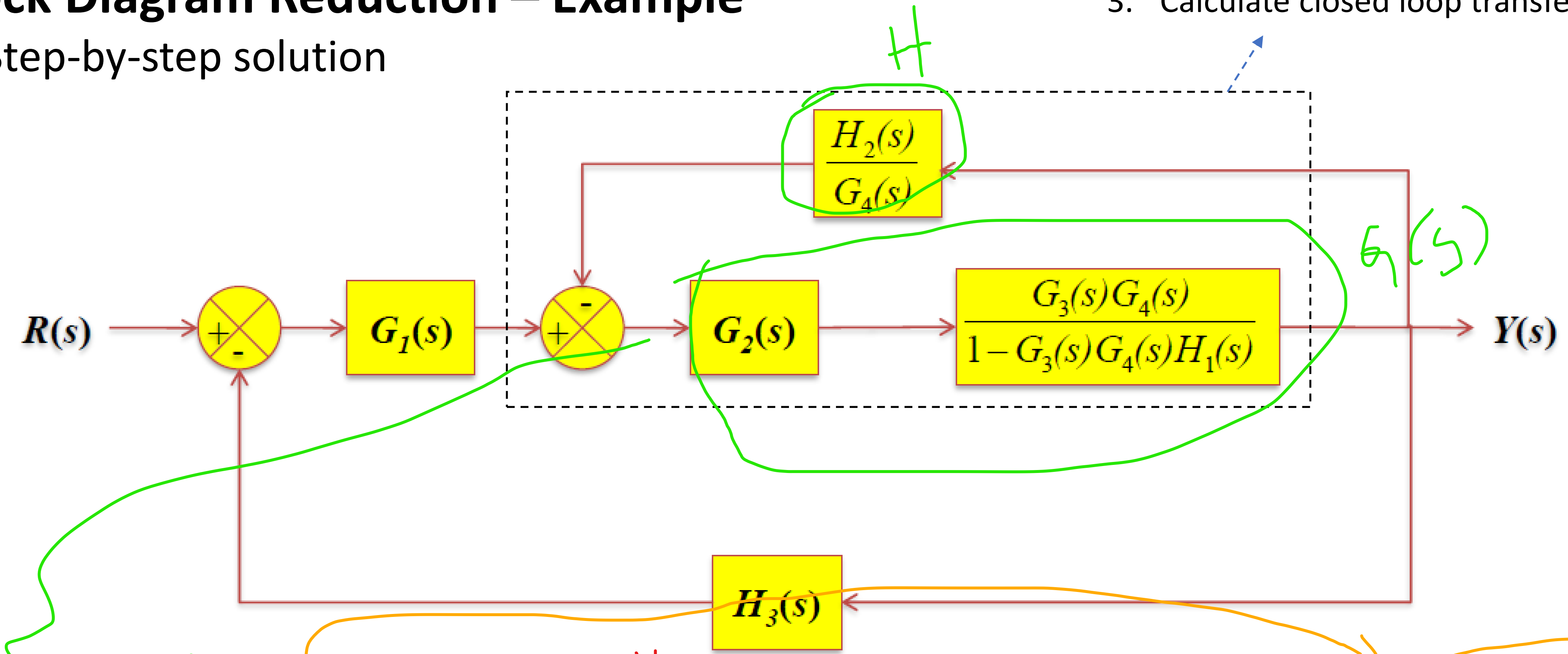


System Description

Block Diagram Reduction – Example

3. Calculate closed loop transfer function

- Step-by-step solution



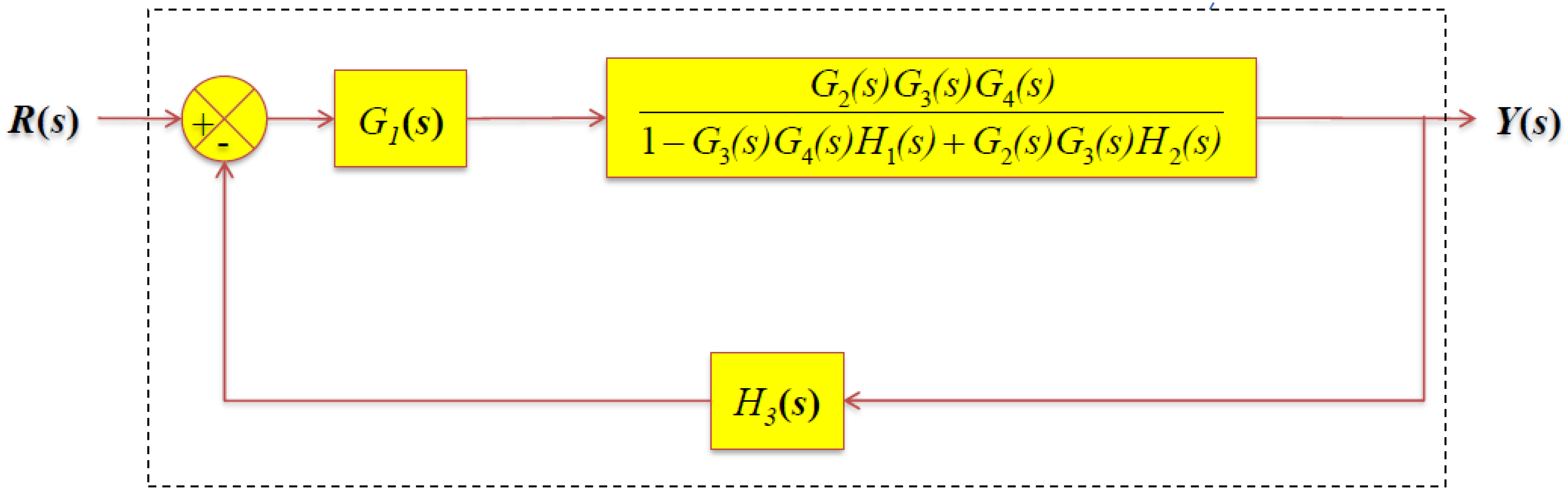
$$\frac{G}{1-GH} \qquad \frac{G_{23\cancel{4}}}{1-G_{34}H_1} \times \frac{H_2}{\cancel{G_4}} \Rightarrow 1-GH = 1-\frac{G_{23}H_2}{1-G_{34}H_1} \rightarrow \frac{1-G_{34}H_1-G_{23}H_2}{1-G_{34}H_1}$$

System Description

Block Diagram Reduction – Example

- Step-by-step solution

3. Calculate closed loop transfer function



System Description

Block Diagram Reduction – Example

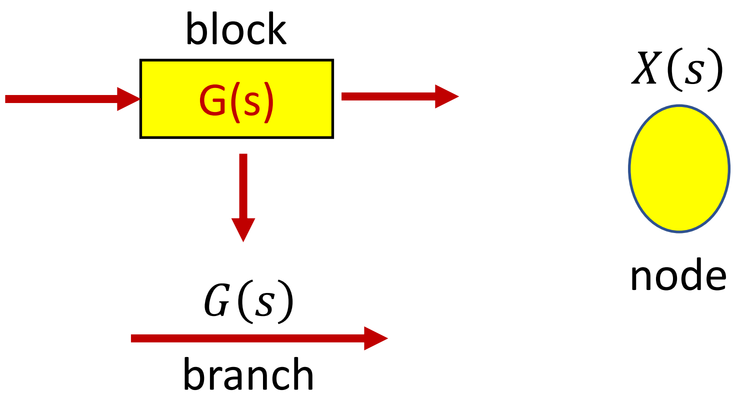
- Step-by-step solution
- **Answer**

$$R(s) \longrightarrow \boxed{\frac{G_1(s)G_2(s)G_3(s)G_4(s)}{1 - G_3(s)G_4(s)H_1(s) + G_2(s)G_3(s)H_2(s) + G_1(s)G_2(s)G_3(s)G_4(s)H_3(s)}} \longrightarrow Y(s)$$

Signal Flow Graphs

What is it?

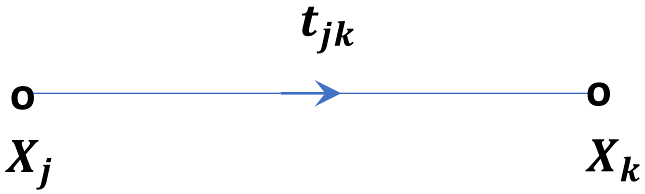
- A Signal Flow Graph SFG) is a **special type of block diagram** and directed graph solution.
- It consists of **nodes** and **branches**. Its nodes are the **variables** of a set of linear algebraic relations.
- SFG can only represent **multiplications** and **additions**.
  - **Multiplications** are represented by the **weights of the branches**.
  - **Additions** are represented by **multiple branches going into one node**.
  - It has a one-to-one relationship with a system of linear equations and can also be used to represent the signal flow in a physical system; i.e., it can represent relations of **cause and effect**.
  - **Consists of branches** (**represent system**)
  - **Nodes** (**represents signals**)



Signal Flow Graphs

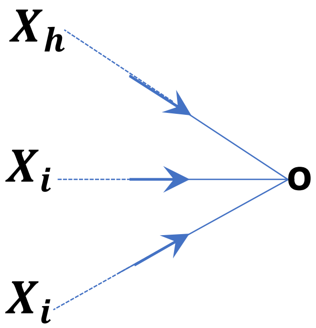
Basic Properties

- A signal flows along a branch only in the direction defined by the arrow and is multiplied by the transmittance of that branch.



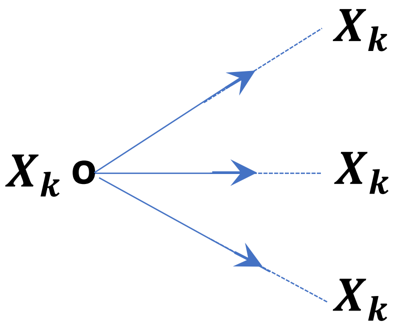
$$X_k = t_{jk}\, X_j$$

- A node signal is equal to the algebraic sum of all signals entering the pertinent node via the incoming branches.



$$X_k = X_h + X_i + X_j$$

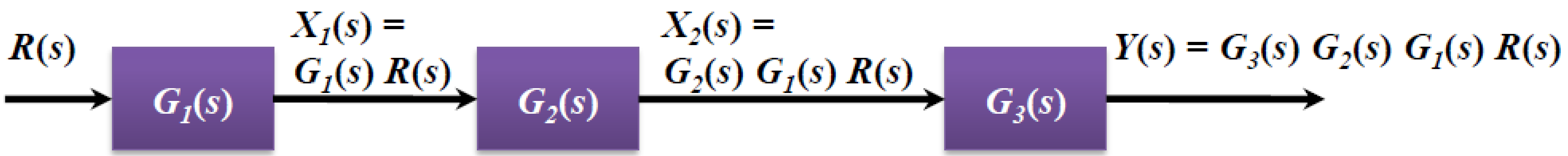
- The signal at a node is applied to each outgoing branch that originates from that node.

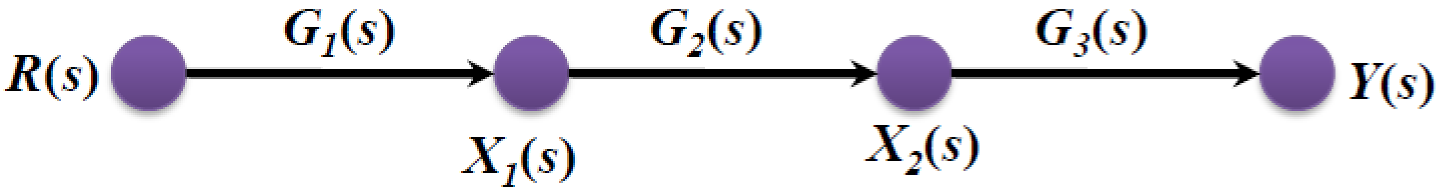


Signal Flow Graphs

Conversion between Block Diagrams and SFG

- We can convert the block diagrams in cascade, parallel, and feedback forms into signal flow diagrams.
  - **Cascade Form**

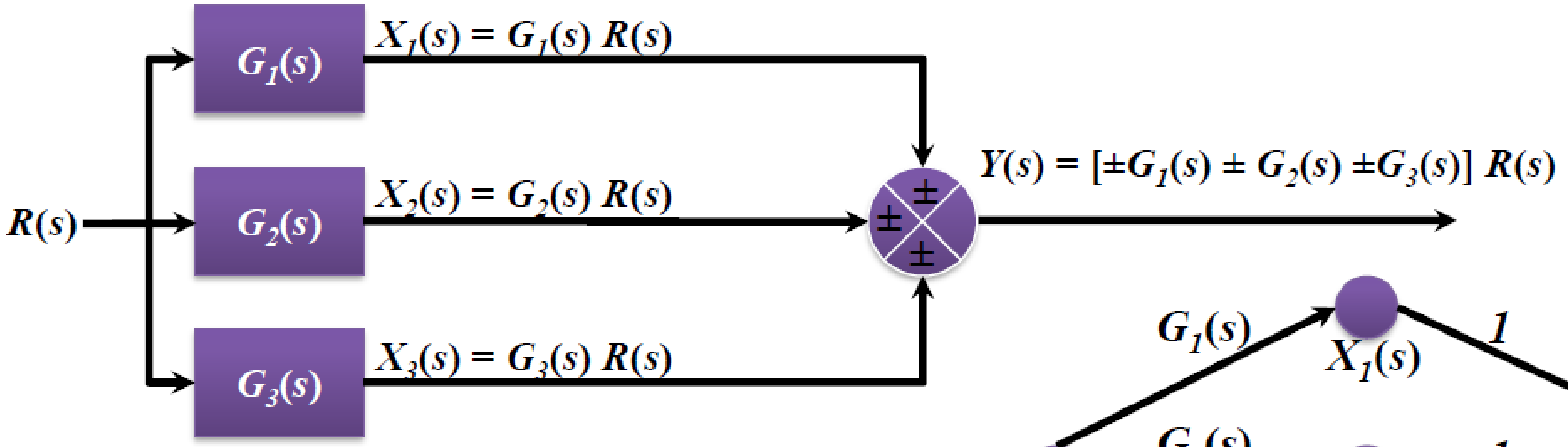


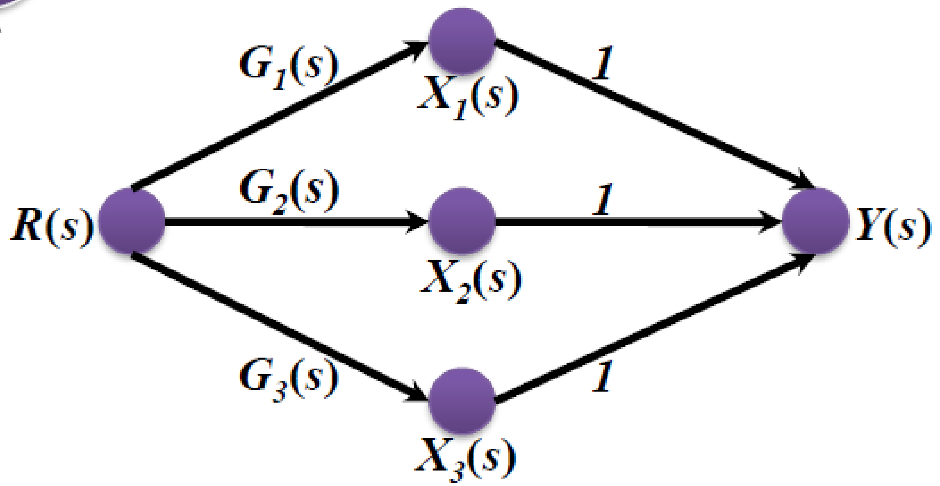


Signal Flow Graphs

Conversion between Block Diagrams and SFG

- We can convert the block diagrams in cascade, parallel, and feedback forms into signal flow diagrams.
  - Cascade Form
  - **Parallel Form**





Signal Flow Graphs

Conversion between Block Diagrams and SFG

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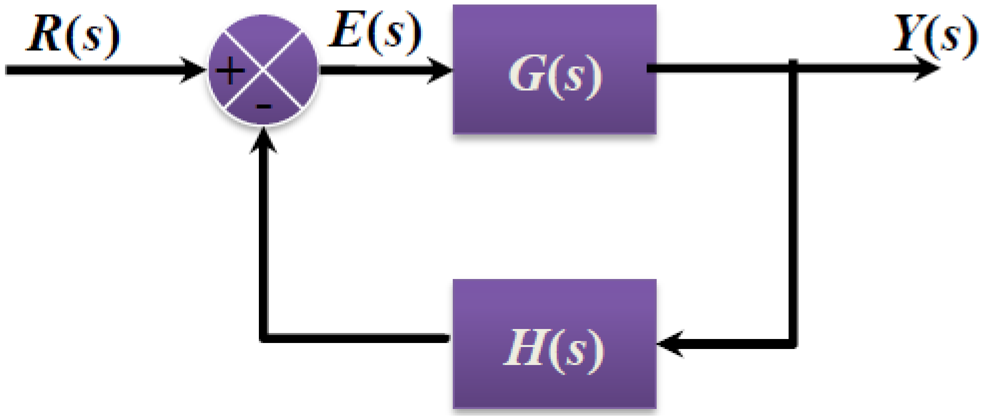
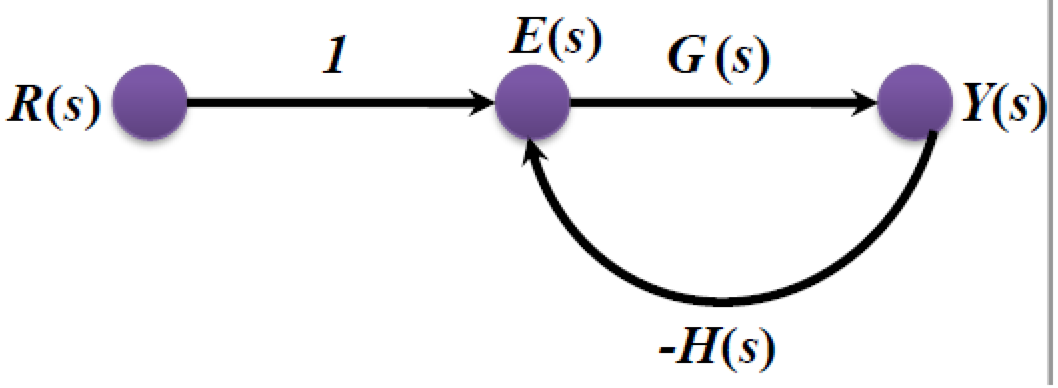
  - Cascade Form
  - Parallel Form
  - **Feedback Form**

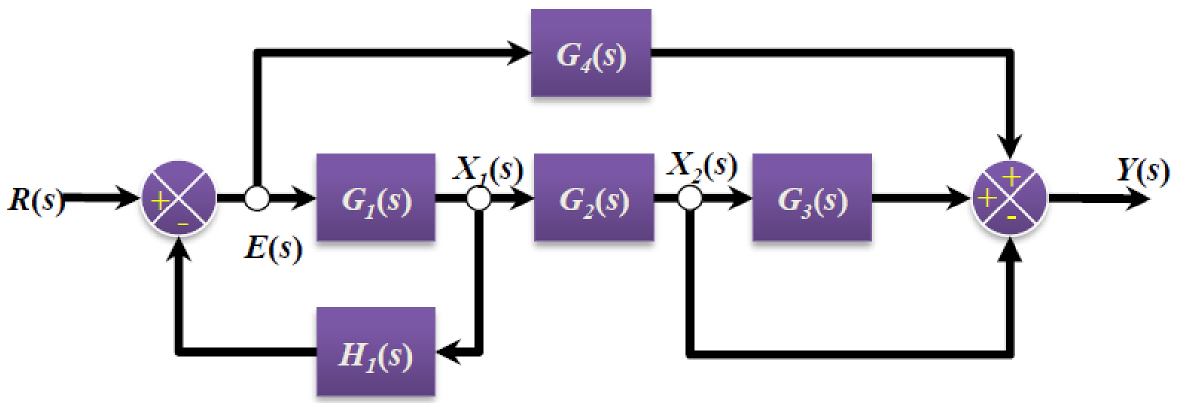


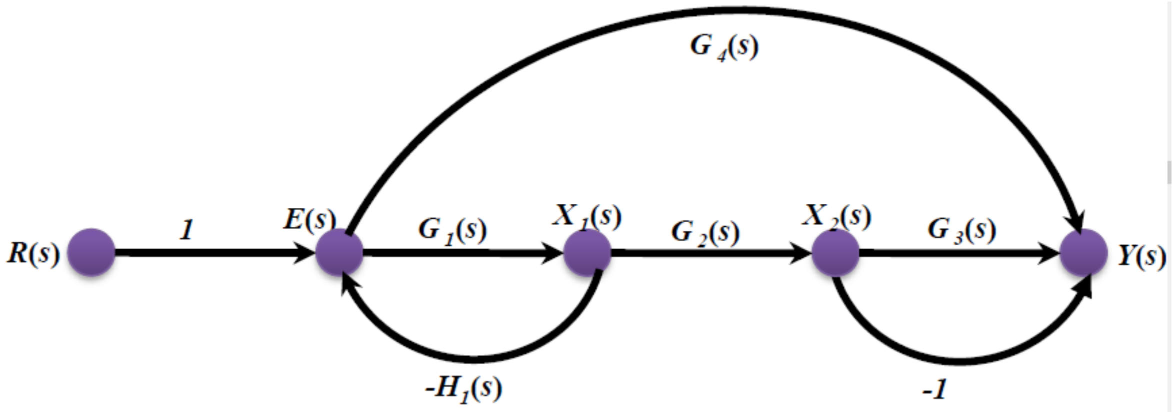


Signal Flow Graphs

Conversion: Example

- Replace every **block** with a **branch**.
- Replace each combination of **summer** and **pick-off points** with a **node** in the signal flow graph (all sums are assumed to be +ve. For -ve sums add a -ve sign
- Replace each solitary **pick-off point** (not connected to a summer) with a **label of the variable** assigned to the pick-off point.
- For each input show a node labeled with the variable assigned to the input.
- Add unity branches as needed or for clarity.



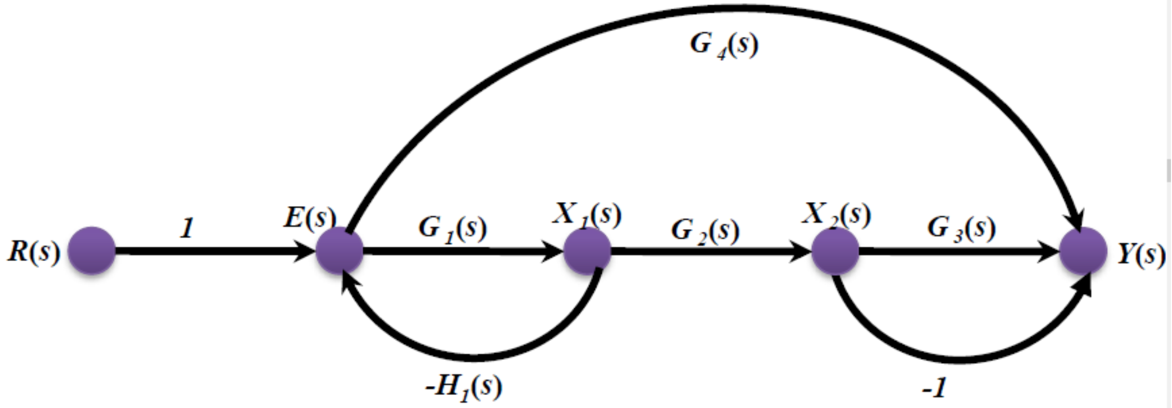


make nodes in place of
1. inputs and outputs
2. branch points
3. summing points
4. in between cascaded block

Signal Flow Graphs

Mason's Gain Formula

$$G(s) = \frac{Y(s)}{R(s)} = \frac{\sum_i^N p_i \Delta_i}{\Delta}$$



N = total number of forward paths

p_i*= gain of the ith forward path*

Δ = 1- (Σ all individual feedback loop gains including self-loops) + (Σ gain product of all possible combinations of two nontouching loops) - (Σ gain product of all possible combinations of three non-touching loops)+…….
Δ_i *= value of Δ after eliminating all loops that touch its ith forward path*

1. *Path =* p_i
2. *Loop*
3. *Touching loops*
4. *Determinant* $= \Delta$
5. *Cofactor* $= \Delta_i$

Signal Flow Graphs

1. ***Path*** = $p_i =$ *A succession of branches, from input to output in the direction of arrows, that does not pass any node more than once.*

2. ***Path gain*** = ***Product of the transmittances*** *of the branches of the path. For the ith path, the path gain is denoted by* p_i.

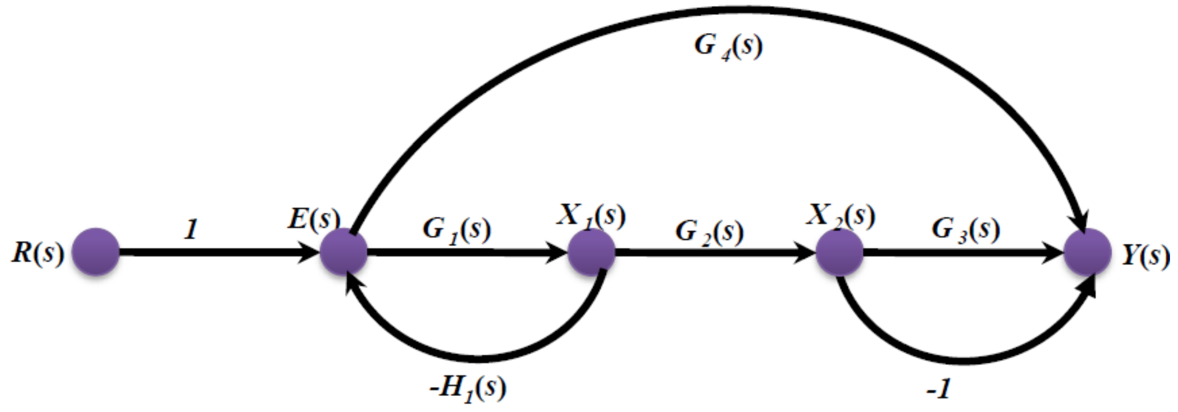
3. ***Loop*** = ***A closed succession of branches****, in the direction of the arrows, , that does not pass any node more than once.*

4. ***Loop gain*** = ***Product of the transmittances*** *of the branches of the loop.*

5. ***Touching loops*** = *Loops with one or more* ***nodes in common.***

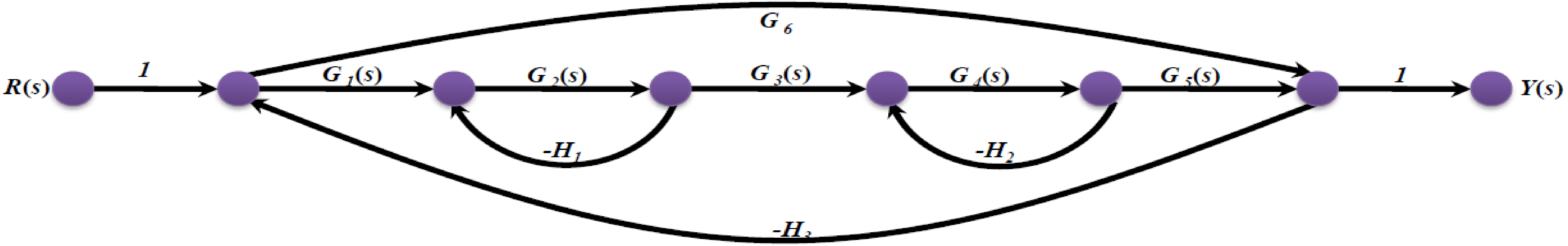
6. ***Determinant*** $= \Delta$

4. ***Cofactor*** $= \Delta_i$

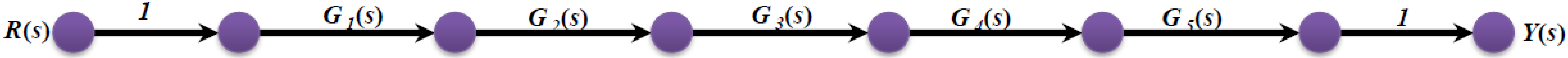


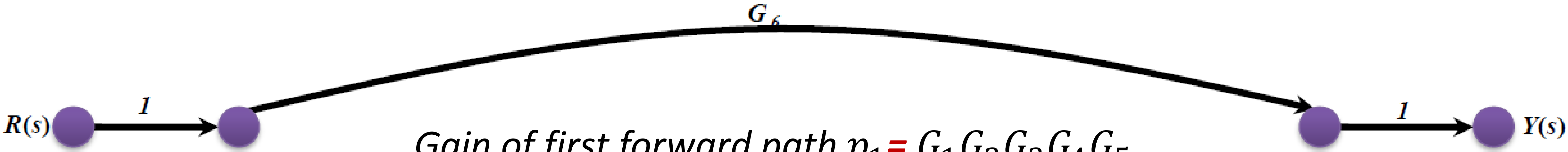
Signal Flow Graphs

Mason’s Gain Formula: Example



Step 1: *There are two forward paths as below so* **N=2**



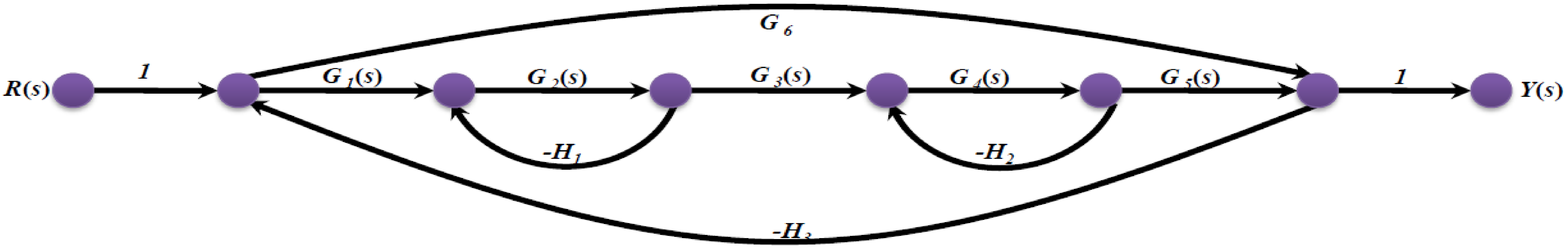


Gain of first forward path $p_1 = G_1 G_2 G_3 G_4 G_5$

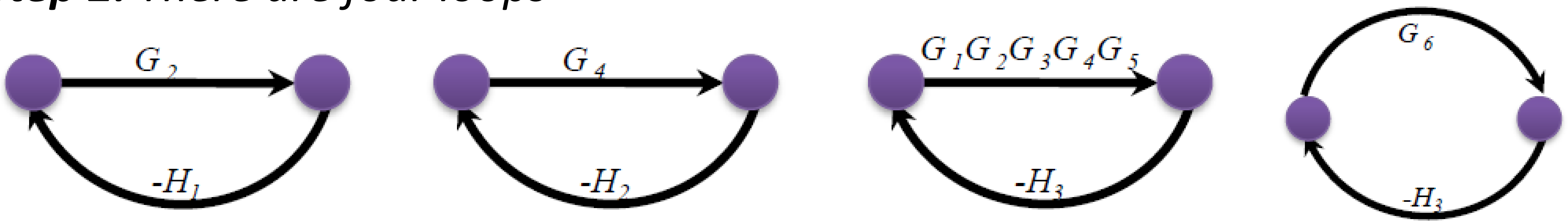
Gain of second forward path $p_2 = G_6$

Signal Flow Graphs

Mason's Gain Formula: Example



Step 2: *There are four loops*



Loop gain of first loop (L_1) **=** $-G_2H_1$

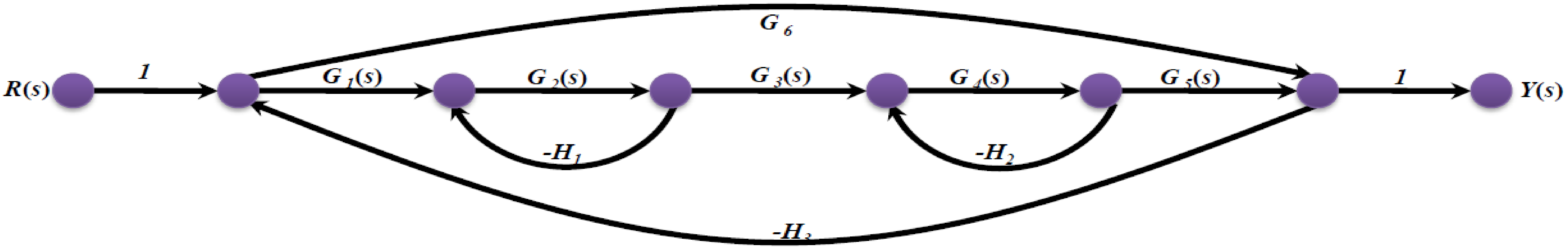
Loop gain of second loop (L_2) **=** $-G_4H_2$

Loop gain of third loop (L_3) **=** $-G_1G_2G_3G_4G_5H_3$

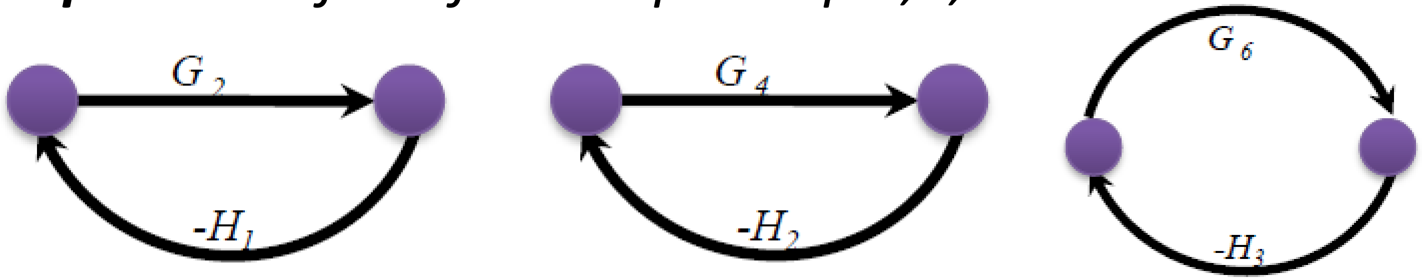
Loop gain of fourth loop (L_4) **=** $-G_6H_3$

Signal Flow Graphs

Mason's Gain Formula: Example



Step 3: *Out of the four loops: loop 1,2, and 4 are non touching*



Combinations of two non touching loops are:

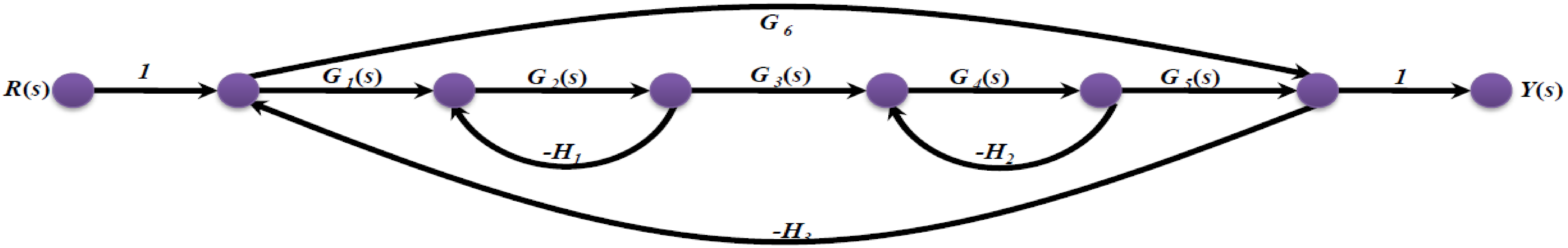
Loop 1, Loop 2: *Loop gain* (L_{12}) **=** $G_2G_4H_1H_2$

Loop 1, Loop 4: *Loop gain* (L_{14}) **=** $G_2G_6H_1H_3$

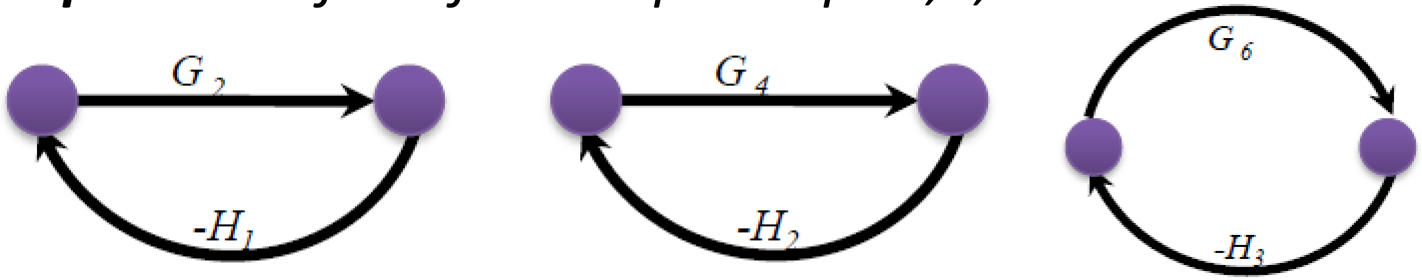
Loop 2, Loop 4: *Loop gain* (L_{24}) **=** $G_4G_6H_2H_3$

Signal Flow Graphs

Mason's Gain Formula: Example



Step 4: *Out of the four loops: loops 1,2, and 4 are non-touching*



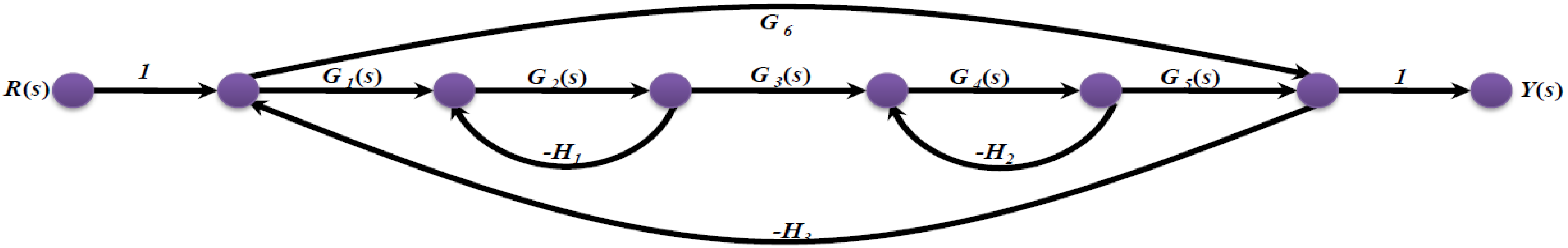
Combinations of three non touching loops are:

Loop 1, Loop 2, Loop 4: *Loop gain* (L_{124}) **=** $-G_2G_4G_6H_1H_2H_3$

Step 5: *There are no higher order non-touching loops.*

Signal Flow Graphs

Mason's Gain Formula: Example



Step 6: *Calculate* Δ

$$\Delta = \mathbf{1} - (\boldsymbol{L_1} + \boldsymbol{L_2} + \boldsymbol{L_3} + \boldsymbol{L_4}) + (\boldsymbol{L_{12}} + \boldsymbol{L_{14}} + \boldsymbol{L_{24}}) - (\boldsymbol{L_{124}})$$

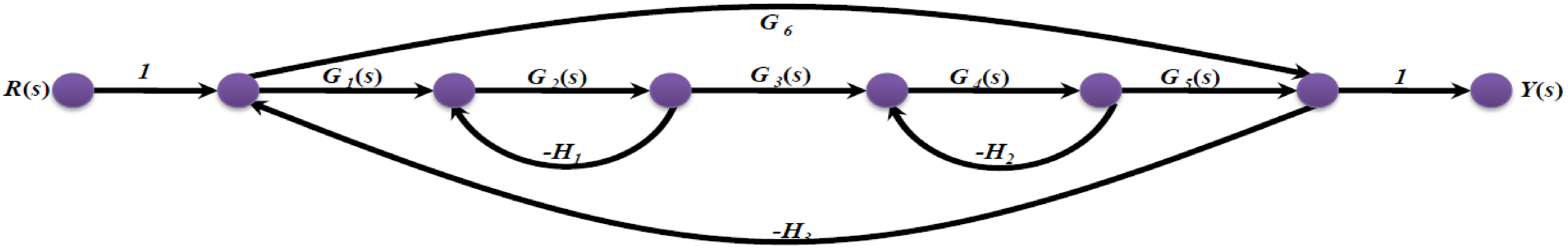
$$= \mathbf{1} + (G_2H_1 + G_4H_2 + G_1G_2G_3G_4G_5H_3 + G_6H_3)$$

$$+(G_2G_4H_1H_2 + G_2G_6H_1H_3 + G_4G_6H_2H_3)$$

$$+(G_2G_4G_6H_1H_2H_3)$$

Signal Flow Graphs

Mason's Gain Formula: Example



Step 7: *Calculate* Δ_i

We know that: $\Delta = \mathbf{1} - (\boldsymbol{L_1} + \boldsymbol{L_2} + \boldsymbol{L_3} + \boldsymbol{L_4}) + (\boldsymbol{L_{12}} + \boldsymbol{L_{14}} + \boldsymbol{L_{24}}) - (\boldsymbol{L_{124}})$

Considering path $\boldsymbol{p_1}$, $loops$ 1,2,3,4 touch it: eliminating all these from Δ

$$\Delta_{\mathbf{1}} = 1 - (0) = 1$$

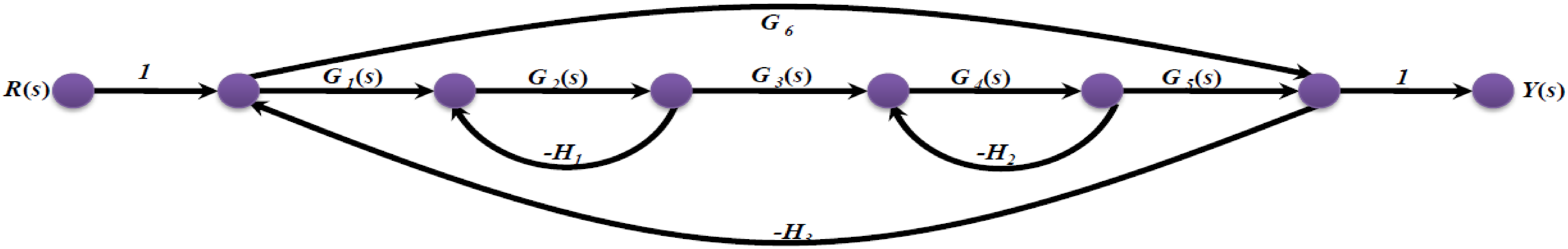
Considering path $\boldsymbol{p_2}$, $loops$ 3,4 touch it: eliminating loops 3, 4 from Δ

$$\Delta_{\mathbf{2}} = 1 - (L_1 + L_2) + (L_{12}) = 1 - (-G_2H_1 - G_4H_2) + G_2G_4H_1H_2$$

$$= 1 + G_2H_1 + G_4H_2 + G_2G_4H_1H_2$$

Signal Flow Graphs

Mason's Gain Formula: Example



Step 8: *Transfer Function*

$$\boldsymbol{G(s)} = \frac{\boldsymbol{p_1}\Delta_{\mathbf{1}} + \boldsymbol{p_2}\Delta_{\mathbf{2}}}{\Delta}$$

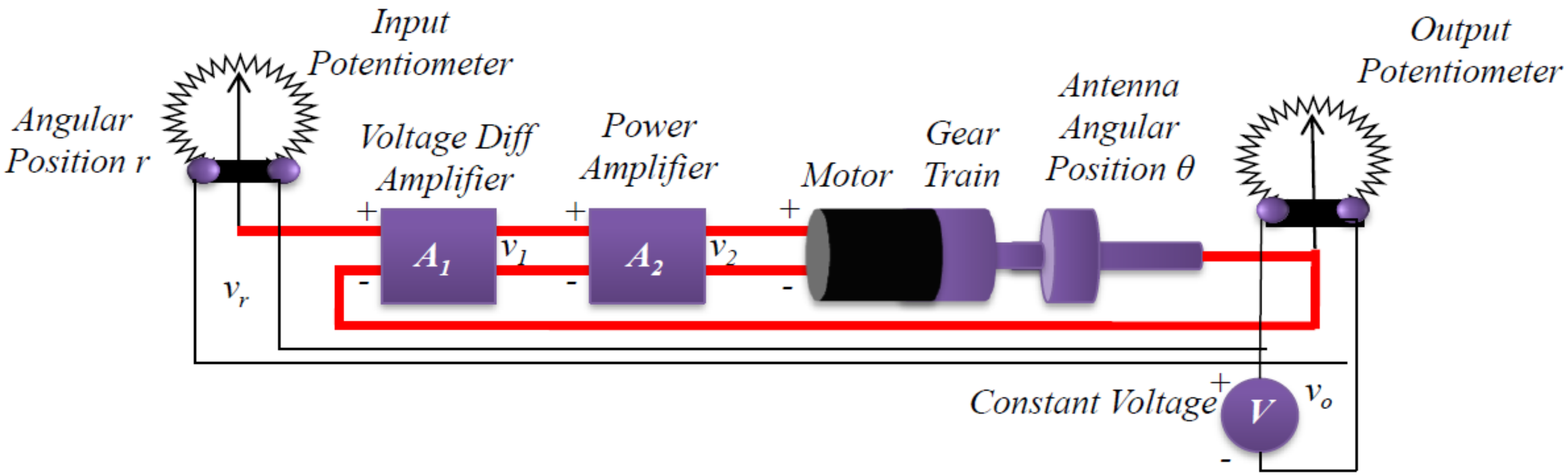
$$= \frac{G_1G_2G_3G_4G_5 + G_6(1 + G_2H_1 + G_4H_2 + G_2G_4H_1H_2)}{\mathbf{1} + (G_2H_1 + G_4H_2 + G_1G_2G_3G_4G_5H_3 + G_6H_3)}$$

$$+(G_2G_4H_1H_2 + G_2G_6H_1H_3 + G_4G_6H_2H_3)$$

$$+(G_2G_4G_6H_1H_2H_3)$$

Electromechanical Systems

Example – A Position Servo a large video satellite antenna

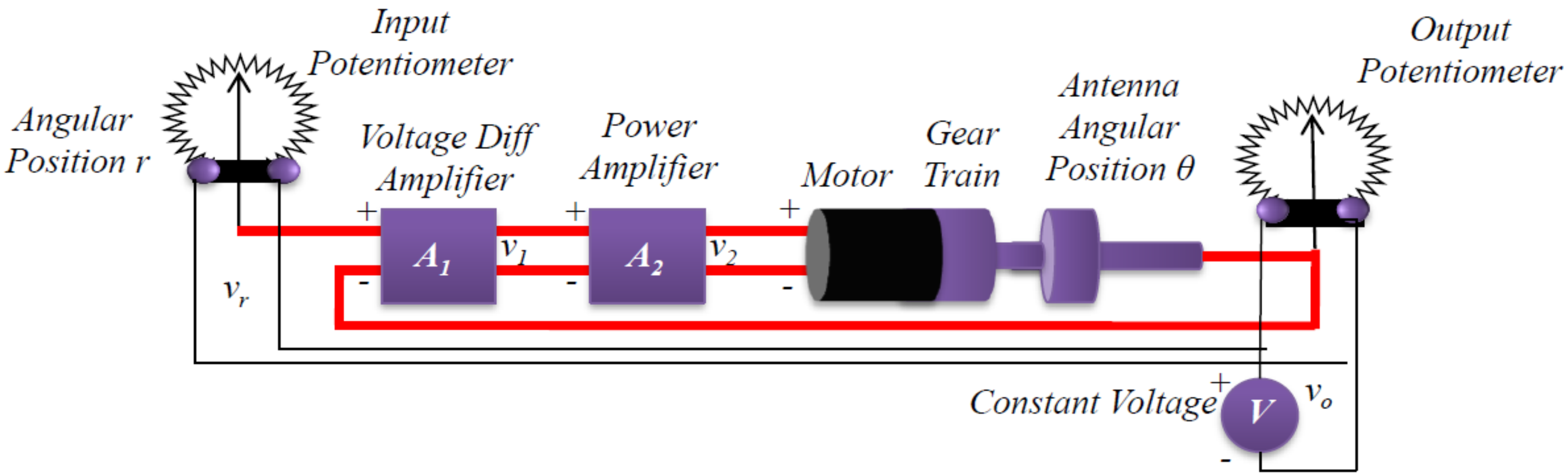


- Output potentiometer measures the output shaft position and converts it to a potential voltage.
  $v_o = K_p \theta$
  $\theta = output\ shaft\ angle$
  $K_p = proportionality\ const. = \dfrac{V}{\theta_{max}}$ volts/radian

- *The input potentiometer slider position is converted to a voltage in a similar manner:*
  $v_r = K_p r$

Electromechanical Systems

Example – A Position Servo a large video satellite antenna



- Difference between the two potentiometer signals is then amplified with gain A_1

$v_1 = A_1(v_r - v_0) = A_1 K_p(r - \theta)$ *where,* $v_1 = error\ voltage\ output$

- This voltage is then further amplified with gain A_2 and applied to the motor terminals:

$v_2 = A_2 v_1 = A_2 A_1 K_p(r - \theta)$ *where,* $v_2 = motor\ drive\ voltage$

- *The second amplifier is the power amplifier capable of providing the electrical power needed to drive the motor.*
- *The motor is coupled to the antenna with a gear train ratio of*

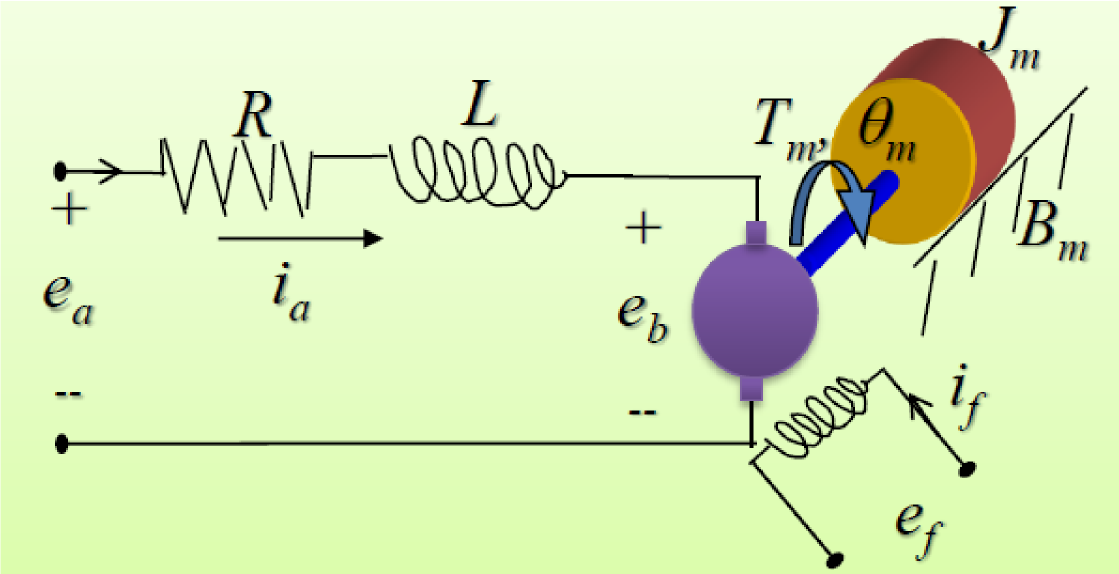
$\theta = \frac{N_1}{N_2}\theta_m$ *where* θ_m *is the motor shaft angle.*

Electromechanical Systems

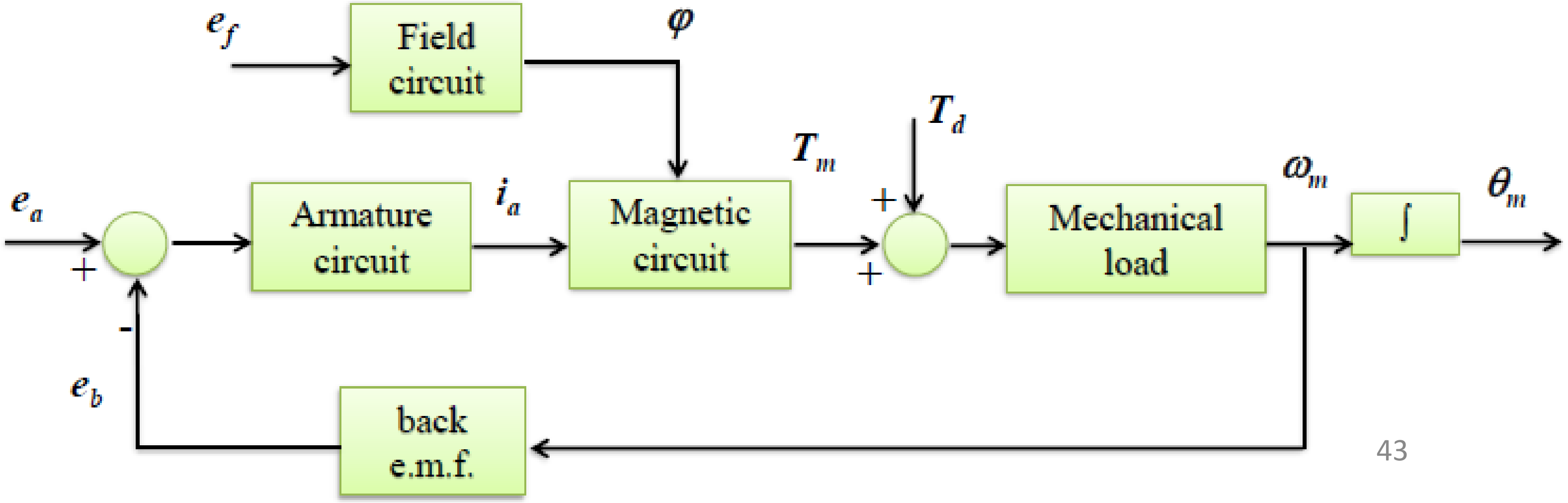
Example – A Position Servo a large video satellite antenna

DC Servo Motor – Modeling

- The **armature current** depends upon the applied voltage and the back emf.
- The **electromagnetic torque** is produced by the interaction of the armature current and the field current.
- The **electromagnetic torque** minus disturbance or **load torque** drives the inertial load.



- The functional block diagram of a DC motor.

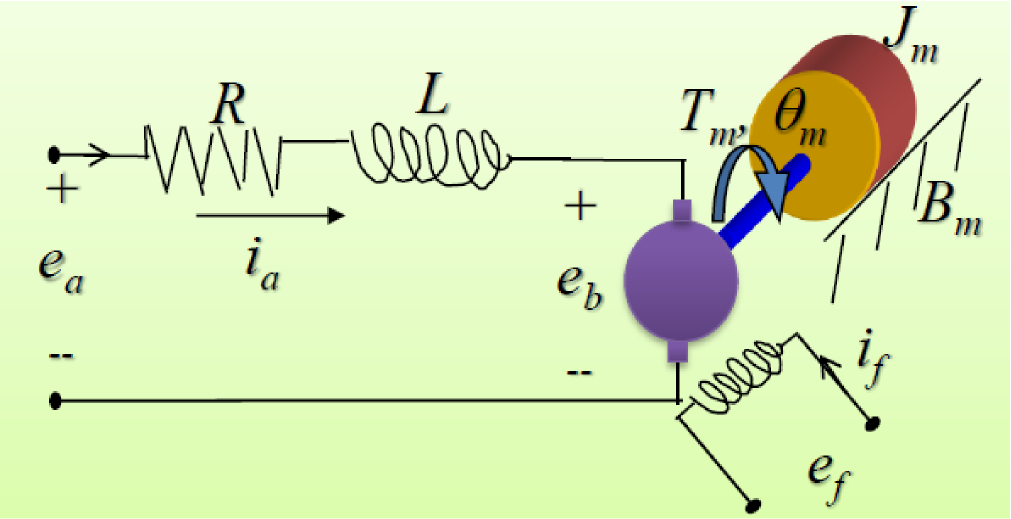


$$TF = \frac{\theta_m(s)}{E_a(s)}$$

Electromechanical Systems

Example – A Position Servo a large video satellite antenna

DC Servo Motor – Modeling



- The relationship between the armature current $i_a(t)$, the applied armature voltage $e_a(t)$, and the back emf $\boldsymbol{e_b(t)}$, is found by applying KVL and then taking Laplace transform:

$$E_a(s) = R_a I_a(s) + L_a s I_a(s) + E_b\ (s) \ldots\ldots (a)$$

- The **back emf $\boldsymbol{e_b(t)}$** is directly proportional to the speed of the motor and it can be written in Laplace domain as:

$$E_b(s) = K_b s \theta_m(s) \ \ldots\ldots\ldots (b)$$

- The **torque developed by the motor (armature torque)** is proportional to the armature current; thus,

$$T_m(s) = K_i I_a(s) \ \rightarrow \ I_a(s) = \frac{T_m}{K_i} \ldots\ldots\ldots (c)$$

✓ The **armature torque** is directly proportional to the product of the **flux** and the **armature current**:

$$T_m \propto \emptyset I_a \ \rightarrow \ T_m = K_m \emptyset I_a$$

$$\emptyset = K_f i_f \rightarrow \ \textit{field flux}$$

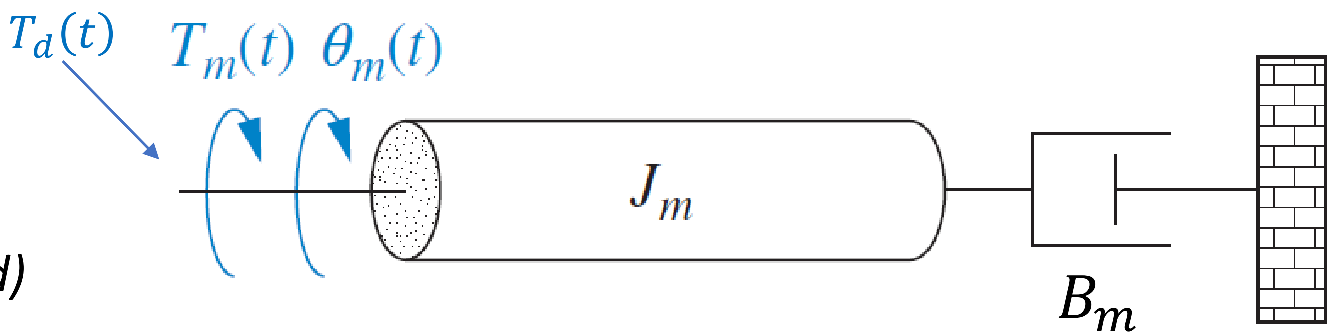
$$T_m = K_m K_f I_f i_a = K_i i_a \rightarrow \textit{for const. field current}$$

$$K_i = \textit{motor torque constant}$$

Electromechanical Systems

Example – A Position Servo a large video satellite antenna

DC Servo Motor – Modeling



- Substituting ***(b)*** *and* ***(c)*** in ***(a)*;**

$$E_a(s) = (R_a + L_a s)\frac{T_m}{K_i} + K_b s\theta_m(s) \text{....... (d)}$$

- The **mechanical load** on the motor can be modeled in Laplace domain as:

$$T_m(s) + T_d(s) = (J_m s^2 + B_m s)\theta_m(s) \text{..........(e)}$$

$$\textit{Substituting } \textbf{(e)} \textit{ in } \textbf{(d)} \textit{ and setting } T_d(s) = 0$$

Recalling the transfer function: $\boldsymbol{TF} = \frac{\boldsymbol{\theta_m(s)}}{\boldsymbol{E_a(s)}}$

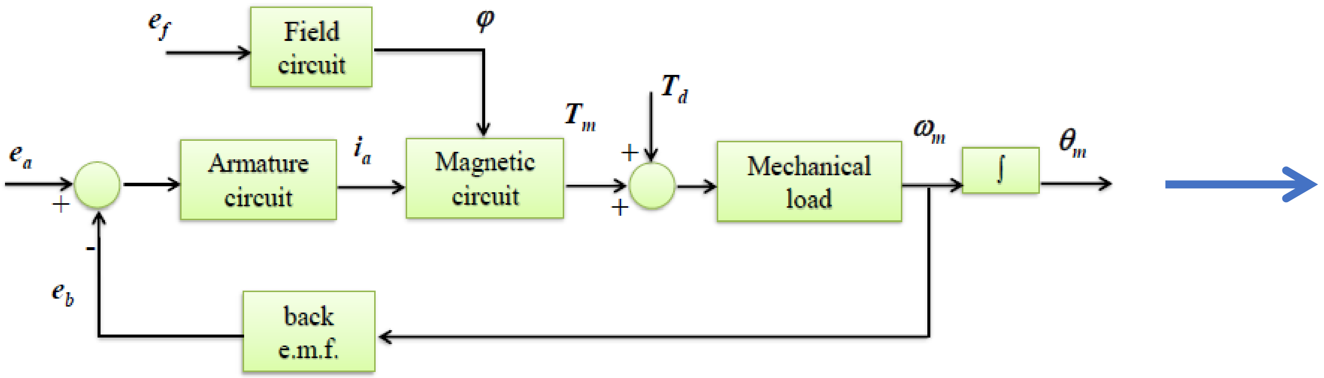
$$E_a(s) = \frac{(R_a + L_a s)(J_m s^2 + B_m s)\theta_m(s)}{K_i} + K_b s\theta_m(s)$$

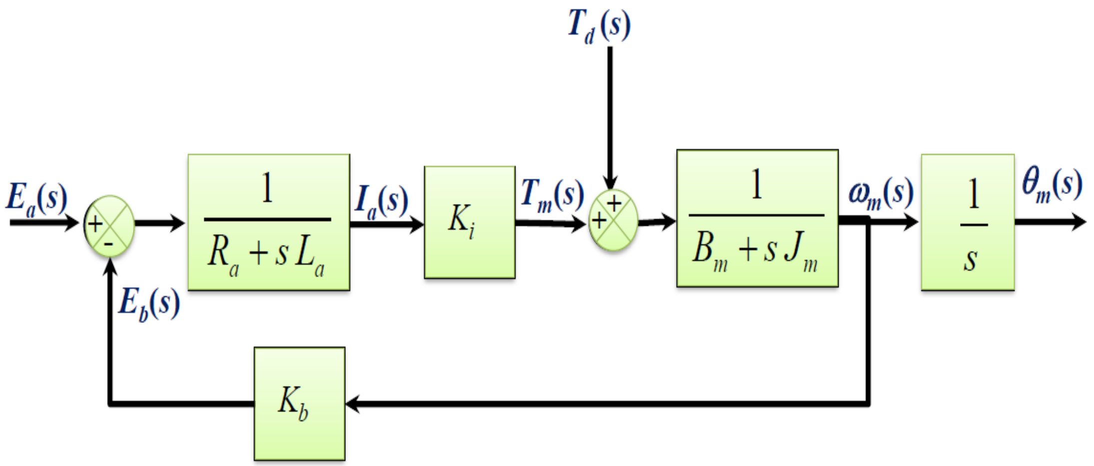
$$\boldsymbol{TF} = \frac{\boldsymbol{\theta_m(s)}}{\boldsymbol{E_a(s)}} = \underbrace{\frac{K_i}{s[L_a J_m s^2 + (R_a J_m + L_a B_m)s + (R_a B_m + K_b K_i)]}}_{G(s)}$$

Electromechanical Systems

Example – A Position Servo a large video satellite antenna

DC Servo Motor – Modeling





In the text book $K_i = K_T$, $K_b = K_v$, and $E_a = V_2$ which means:

$$\frac{\boldsymbol{\theta_m(s)}}{\boldsymbol{V_2(s)}} = \frac{K_T}{s[L_aJ_ms^2 + (R_aJ_m + L_aB_m)s + (R_aB_m + K_vK_T)]}$$

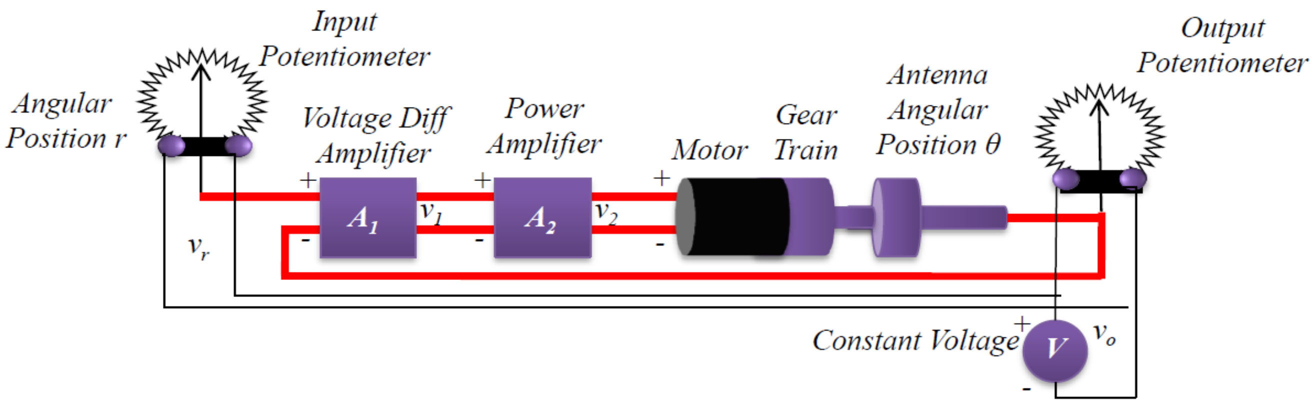
Neglecting the terms L_a, B_m and dividing by K_vK_T

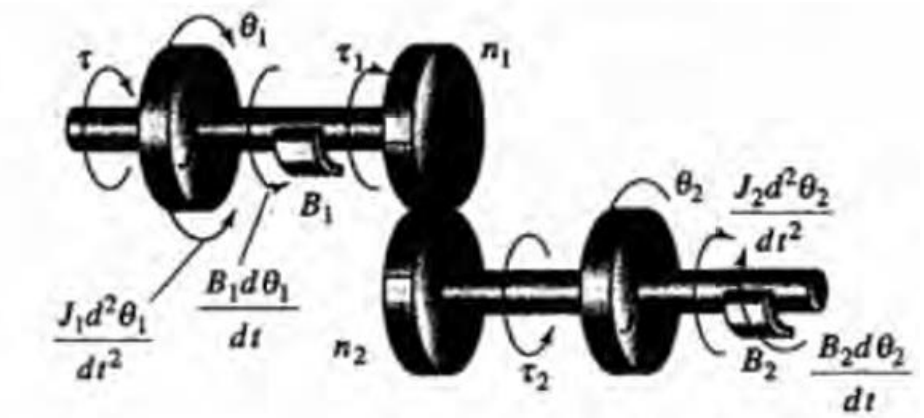
$$\frac{\boldsymbol{\theta_m(s)}}{\boldsymbol{V_2(s)}} = \frac{1/K_v}{s[1 + (\frac{R_aJ_m}{K_vK_T})]}$$

- $T_m = K_m \emptyset I_a$
  $K_m =$ *motor torque constant*
- $v_m = K_v \omega_m$
  $K_v =$ *motor voltage constant*
- $K_m = 1/K_v$

Electromechanical Systems

Example – A Position Servo a large video satellite antenna





- **"θ"** is the angular position of the antenna with the moment of inertia **"J"**. N1 <<< N2, since the high-speed shaft of the motor must drive the antenna at low speed and high torque.
- $J_1 s^2\theta_1(s) + B_1 s\theta_1(s) + \left(\frac{n_1}{n_2}\right)^2 J_2 s^2\theta_1(s) + \left(\frac{n_1}{n_2}\right)^2 B_1 s\theta_1(s)$ ←

$$\boxed{\begin{aligned} &\tau = \tau_1 + J_1\frac{d^2\theta_1}{dt^2} + B_1\frac{d\theta_2}{dt} \\ &\tau_2 = J_2\frac{d^2\theta_2}{dt^2} + B_2\frac{d\theta_2}{dt} \\ &\tau_2 = \frac{n_2}{n_1}\tau_1 \qquad \theta_2 = \frac{n_1}{n_2}\theta_1 \end{aligned}}$$

- $\frac{\boldsymbol{\theta}\,(\boldsymbol{s})}{\boldsymbol{V_2}(\boldsymbol{s})} = \frac{K_m\,(\frac{N_1}{N_2})}{s(1 + (R_a/K_v K_T[J_m + \left(\frac{N_1}{N_2}\right)^2 J_L]\,)s)}$

$$\boxed{\begin{aligned} J_1 &= J_m \\ J_2 &= J_L \end{aligned}}$$

- Taking Laplace of v_2 gives: **(ref: slide 42)**
- $V_2(s) = A_1 A_2 K_p(R(s) - \theta(s))$

❖ $\theta(s) = \theta_m\left[\frac{N_1}{N_2}\right]$

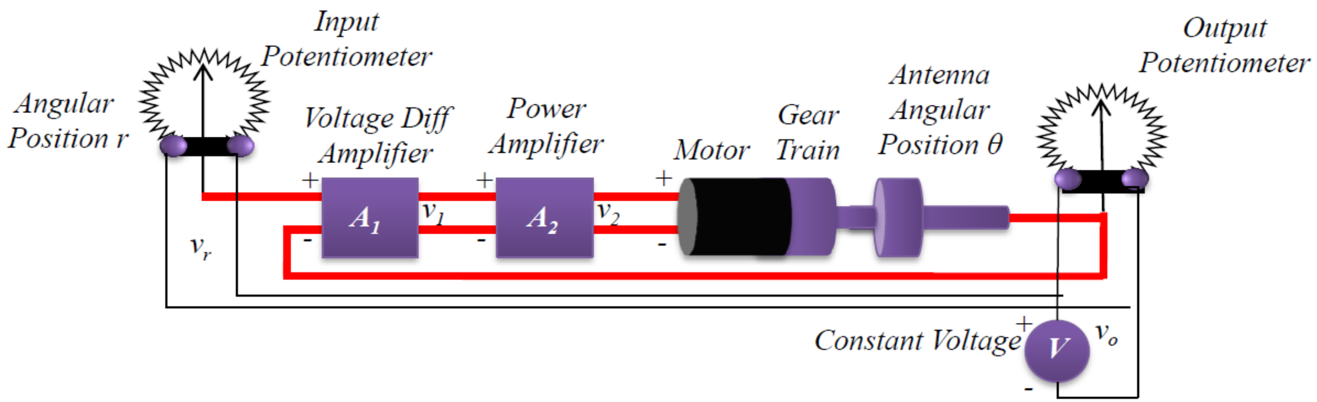
Electromechanical Systems

Example – A Position Servo a large video satellite antenna

substituting to find value of $\theta(s)$ we get

$$\boldsymbol{\theta(s) = \frac{K_m\,(\frac{N_1}{N_2})A_1A_2K_p(R(s) - \theta(s))}{s(1\ + \tau_L s)}}$$

Where, $\tau_L = (R_a/K_v K_T[J_m + \left(\frac{N_1}{N_2}\right)^2 J_L\ ]$



$$\boldsymbol{\theta(s) = \frac{K_m\,(\frac{N_1}{N_2})A_1A_2K_p(R(s) - \theta(s))}{s(1\ + \tau_L s)}}$$

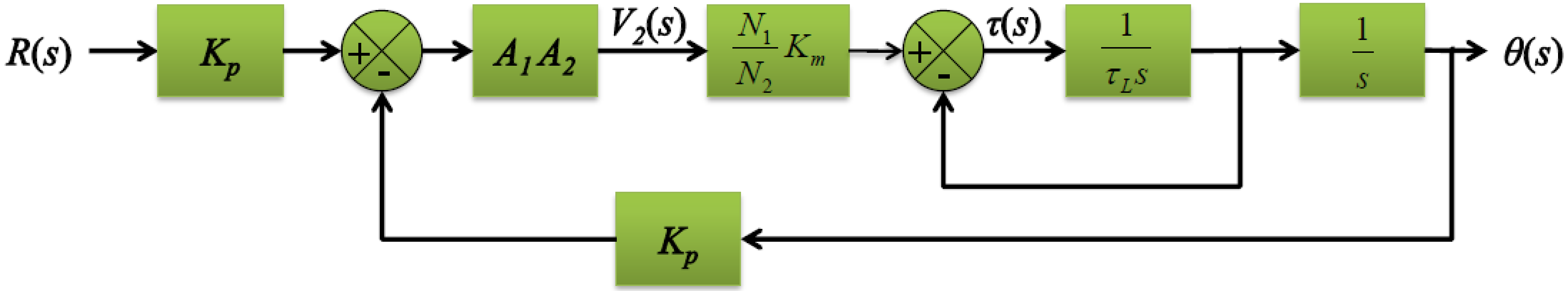
- Some of the coefficients, and thus some of the system properties can be selected by the designer by appropriately choosing the control components.
- However, the moment of inertia of the load J cannot be changed.
- The transfer function relating the input position R(s) to the output position Θ(s) is given by:

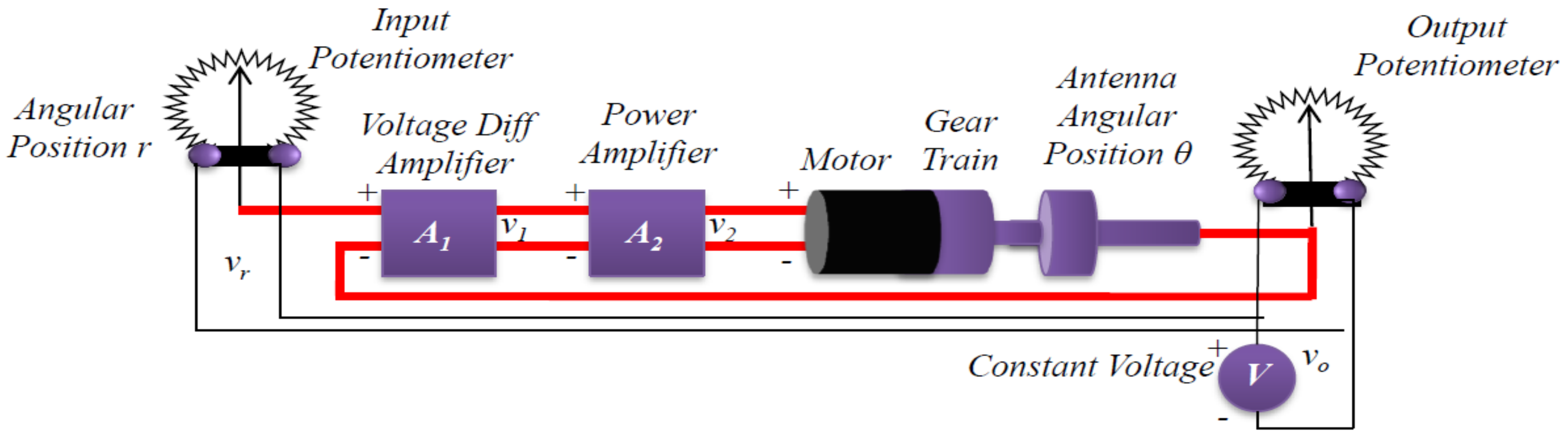
$$\boldsymbol{T(s) = \frac{\theta(s)}{R\ (s)} = \frac{(\frac{N_1}{N_2})A_1A_2K_m\,K_p}{\tau_L s^2 + s + (\frac{N_1}{N_2})A_1A_2K_m\,K_p}}$$

Electromechanical Systems

Example – A Position Servo a large video satellite antenna
DC Servo Motor – Block Diagram

- The transfer function can also be derived using a block diagram as shown below:





Electromechanical Systems

Example – A Position Servo a large video satellite antenna
DC Servo Motor – Block Diagram – Signal Flow Graph

- It can also be obtained by reducing the equivalent signal flow graph shown on this slide
- In this graph for **one forward path** Mason's rule gives.

$$P_1 = K_P A_1 A_2 \frac{N_1}{N_2} \frac{1}{\tau_L s} \frac{1}{s} K_m \ , \qquad \Delta_1 = 1$$

$$L_1 = A_1 A_2 \frac{N_1}{N_2} K_m \frac{1}{\tau_L s} \frac{1}{s} (-K_P), \qquad L_2 = \frac{-1}{\tau_L s}$$

$$= \frac{-A_1 A_2 (\frac{N_1}{N_2}) K_m K_p}{\tau_L s^2}$$

$$T(s) = \frac{K_P A_1 A_2 (\frac{N_1}{N_2}) K_m}{\tau_L s^2 + s + (\frac{N_1}{N_2}) K_P A_1 A_2 K_m}$$