EE-379 Linear Control Systems

Week No. 2: Continuous Time System Description

- The Concept of Stability in Control Systems
- Block Diagrams
- Signal Flow Graphs
- Modeling Example: A Position Servo

System Description

Stability

- Stability is usually evaluated from two viewpoints internal and external.

Internal Stability

- For internal asymptotic stability, the **zero-input (natural)** response decays to zero as the time approaches ∞, for **all possible initial conditions**.
- The **characteristic polynomial roots** influence the response.
- Ensured if **all roots are located in the LHP**

System Description

Stability

- Stability is usually evaluated from two viewpoints internal and external.

External Stability

- For external (bounded input bounded output or BIBO) stability, the **zero-state response** is bounded as the time approaches ∞, for all bounded inputs.

- For **a bounded input** we can expect the **forced response to be bounded**.
- For the system to be BIBO stable, **the natural response of the output** should also be bounded.
- **If natural response decays to zero** as time approaches ∞ system is BIBO.
- It is ensured if the **zero-state impulse response** of the output decays to zero.

System Description

Stability – Evaluation Criterion

- If all characteristic polynomial roots are in the **LHP** then the natural response decays to zero and the system is asymptotically and BIBO stable.
- Another possibility for BIBO exists wherein suppose the characteristic polynomial contains some RHP roots (*poles*).
- RHP zeros of the *T(s)* cancel all the RHP poles from the TF and from the zero-state impulse response.
- At least **one term** in the **zero-input response** will go to ∞ , while the **zero-state impulse response decays to zero**. So, the system will be **asymptotically unstable but BIBO stable**.

System Description

Stability – Evaluation Criterion

- Thus, the two types of stability differ when all RHP poles are canceled by RHP zeros.
- We **evaluate BIBO stability** by examining the **zero-state impulse response** which allows for poles zero cancellation.
- We **evaluate asymptotic stability** by examining the **zero-input response of the system.**

System Description

Stability – Evaluation Criterion (simple words)

- **A system is stable** if the **natural response approaches zero** as time approaches infinity.
- **A system is unstable** if the **natural response approaches infinity** as time approaches infinity.
- **A system is marginally stable** if the **natural response neither decays nor grows** but **remains constant or oscillates**.
- **A system is stable** if every bounded input yields a bounded output.
- **A system is unstable** if any bounded input yields an unbounded output.

System Description

Stability – Example

- Consider the following three *TFs.*

$$T_1(s) = \frac{(s+1)}{(s+2)(s+3)} = \frac{2}{(s+3)} - \frac{1}{(s+2)}$$

$$T_2(s) = \frac{(s-7)}{(s-2)(s+3)} = \frac{2}{(s+3)} - \frac{1}{(s-2)}$$

$$T_3(s) = \frac{2(s-2)}{(s-2)(s+3)} = \frac{2}{(s+3)} + \frac{0}{(s-2)}$$

- The **first system** will have **zero input response**

$$y_{zi1} = K_1 e^{-3t} + K_2 e^{-2t}$$

- K_1 and K_2 are determined by initial conditions
- **Zero-state impulse response** when input *r(t)* is a unit impulse function is;

$$y_{zs,impulse1}(t) = 2e^{-3t} - e^{-2t}$$

- Which decays to zero, so the system is both asymptotically and BIBO stable

System Description

Stability – Example

- Consider the following three *TFs.*

$$T_1(s) = \frac{(s+1)}{(s+2)(s+3)} = \frac{2}{(s+3)} - \frac{1}{(s+2)}$$

$$T_2(s) = \frac{(s-7)}{(s-2)(s+3)} = \frac{2}{(s+3)} - \frac{1}{(s-2)}$$

$$T_3(s) = \frac{2(s-2)}{(s-2)(s+3)} = \frac{2}{(s+3)} - \frac{0}{(s-2)}$$

- For the **second system**

$$y_{zi2} = K_1 e^{-3t} + K_2 e^{2t}$$

$$y_{zs,impulse2}(t) = 2e^{-3t} - e^{2t}$$

- This is asymptotically unstable because e^{2t} will approach infinity and it is *BIBO* unstable due to the same term.

System Description

Stability – Example

- Consider the following three *TFs.*
- For the **third system**

$$T_1(s) = \frac{(s+1)}{(s+2)(s+3)} = \frac{2}{(s+3)} - \frac{1}{(s+2)}$$

$$T_2(s) = \frac{(s-7)}{(s-2)(s+3)} = \frac{2}{(s+3)} - \frac{1}{(s-2)}$$

$$T_3(s) = \frac{2(s-2)}{(s-2)(s+3)} = \frac{2}{(s+3)} - \frac{0}{(s-2)}$$

$$y_{zi3} = K_1 e^{-3t} + K_2 e^{2t}$$

$$y_{zs,impulse3}(t) = 2e^{-3t}$$

- This is asymptotically unstable because e^{2t} term, but *BIBO* stable due to the cancellation of *RHP zero also located at +2*

System Description

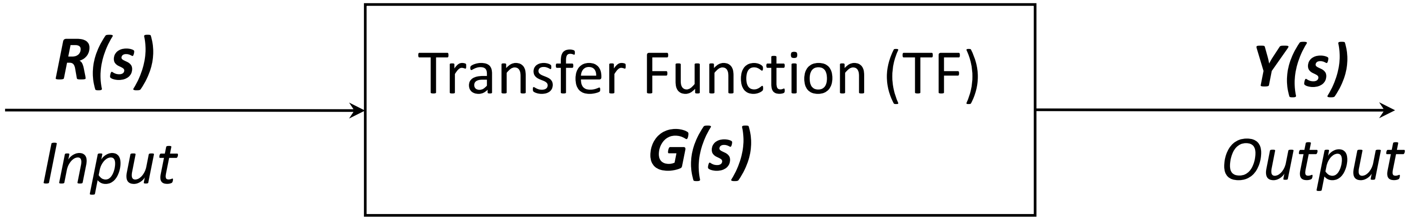
Stability Description – Example

- The most conservative approach is to demand **asymptotic stability**.
- Suppose the **zero-state response** of each of the three systems provides the **external position of an aircraft** while the zero-input response provides the **response of some internal device**.
- The first system has an acceptable response for both the aircraft and the device.
- While the second system has an unacceptable response for both the aircraft and the device.
- Although the zero-state response of the third system is stable, the electrical device is probably destroyed.
- This is unacceptable since this device may play a vital role in the future activity of the aircraft.
- Only **asymptotic stability is accepted**

System Description

Block Diagram - Uses

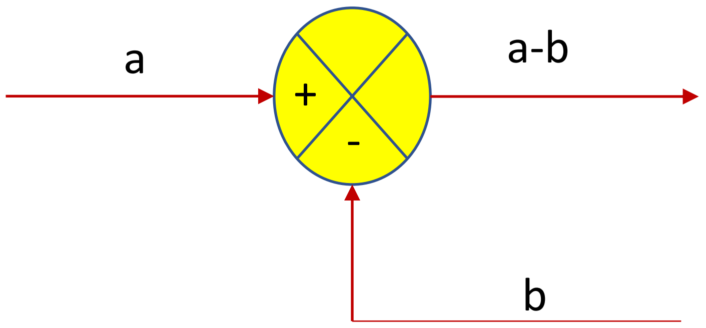
- Block diagrams are used to model all types of systems because of their simplicity and versatility.
- A block diagram can be used to describe the **composition** and **interconnection** of a system.
- Also, it can be used together with transfer functions, to describe the **cause** and **effect** relationships throughout the system.
- A block diagram consists of unidirectional, operational blocks that represent the *TF*.



System Description

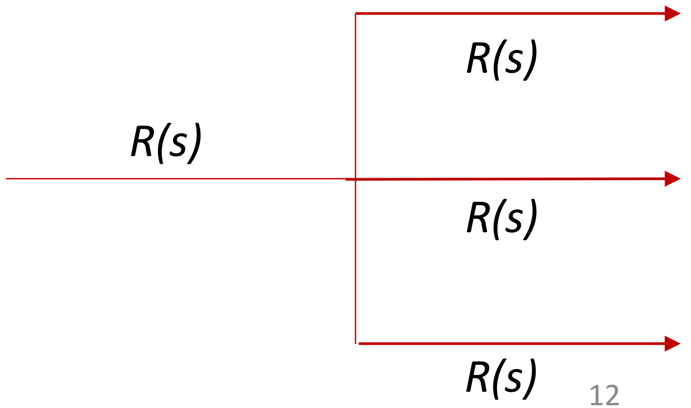
Block Diagram - Uses

- Many systems are composed of **multiple subsystems**.
- When multiple subsystems are interconnected, a few more **schematic elements** must be added to the block diagram. Like **summing junctions** and **pickoff points**.



- Summing Point: A **circle with a cross** indicates a summing operation. The plus + and minus - sign at each arrowhead indicates whether the signal is to be added or to be subtracted.

- Junction/Pick-off Point: A point from which the signal from a block goes **concurrently** to other blocks or summing points.

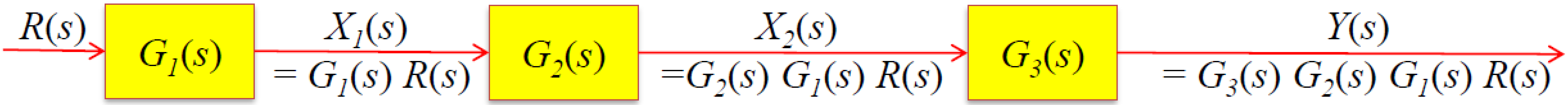


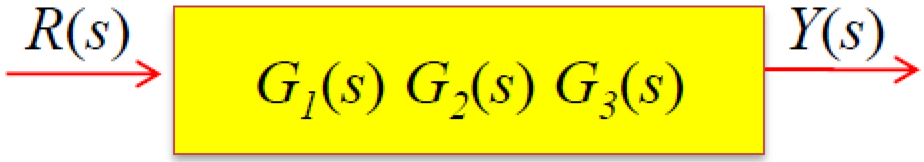
System Description

Block Diagram – Common Forms

- There are three basic forms, by which the subsystems are connected together.
  - **Cascade form**
  - Parallel form
  - Feedback form

Cascade Form



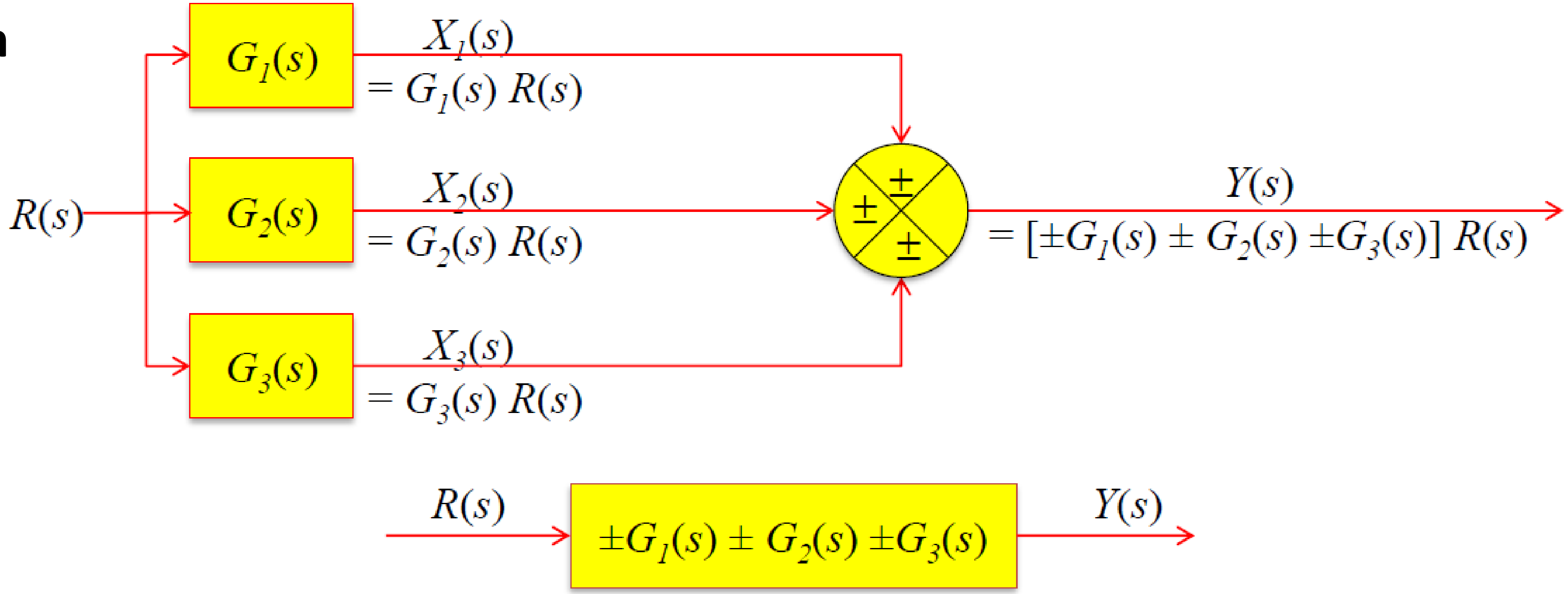


System Description

Block Diagram – Common Forms

- There are three basic common forms, by which the subsystems are connected together.
  - Cascade form
  - **Parallel form**
  - Feedback form

Parallel Form

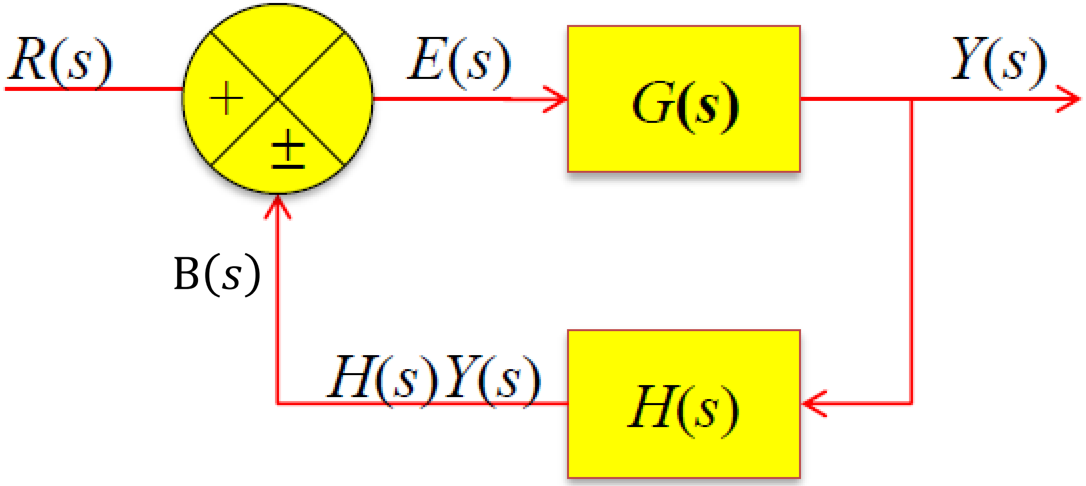


System Description

Block Diagram – Common Forms

- There are three basic common forms, by which the subsystems are connected together.
  - Cascade form
  - Parallel form
  - **Feedback form**

Feedback Form



Open-Loop Transfer Function

- The ratio of the **feedback signal** $\mathbf{H}(\boldsymbol{s})\boldsymbol{Y}(\boldsymbol{s})$ to the **actuating error signal** $\mathbf{E}(\boldsymbol{s})$ is called the open-loop transfer function.

$$T(s) = \frac{B(s)}{E(s)} = \mathrm{G}(s)H(s)$$

Feed-forward Transfer Function

- The ratio of the **output** $\boldsymbol{Y}(\boldsymbol{s})$ to the **actuating error signal** $\mathbf{E}(\boldsymbol{s})$ is called the feedforward transfer function.

$$T(s) = \frac{Y(s)}{E(s)} = \mathrm{G}(s)$$

- If the **feedback transfer function** $\mathbf{H}(\boldsymbol{s})$ is **unity**, then the **open-loop transfer function** and the **feedforward transfer function** are the **same**.

System Description

Block Diagram – Common Forms – Feedback

- The relationship between the signals is:

$$Y(s) = G(s)E(s)$$
$$E(s) = R(s) \pm H(s)Y(s)$$

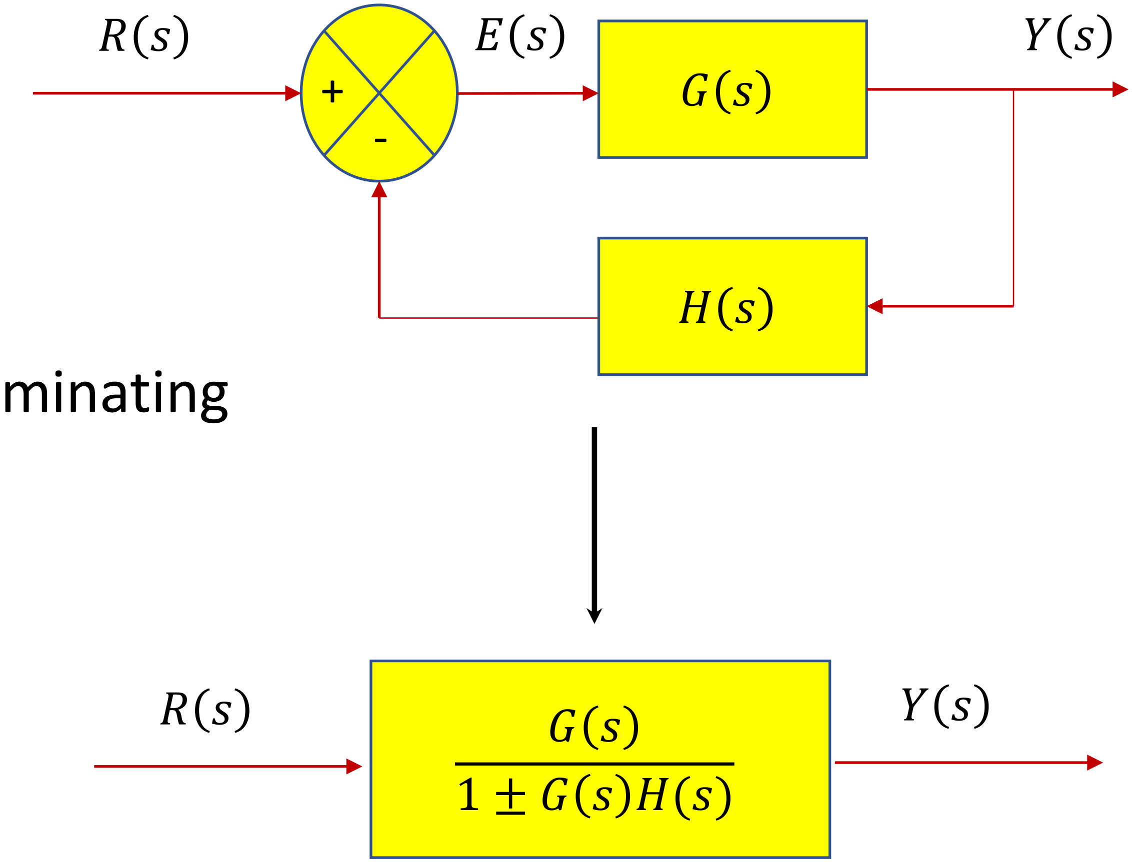
Solving for $Y(s)$ in terms of $\mathrm{R}(s)$ and eliminating $\mathrm{E}(s)$, we get.

$$Y(s) = G(s)[R(s) \pm H(s)Y(s)]$$
$$Y(s) = G(s)R(s) \pm G(s)H(s)Y(s)$$

which results in

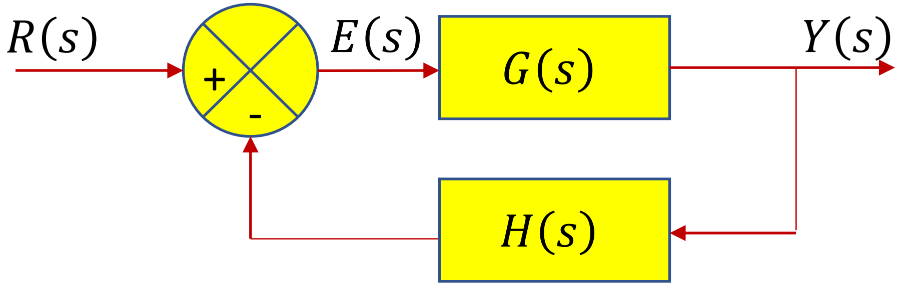
$$T(s) = \frac{Y(s)}{R(s)} = \frac{G(s)}{1 \pm G(s)H(s)}$$

Closed-loop transfer function



System Description

Block Diagram – Common Forms – Feedback



- Fundamental to control engineering as it reveals the **effect** of applying **feedback** to a system.
- Practical meaning of two possible summation (positive and negative):

  Lets imagine,
  - $G(s)$ – the combination of cruise controller and dynamics of a vehicle.
  - $H(s)$ – velocity measuring device.
  - $R(s)$ – desired velocity
  - $Y(s)$ – actual velocity
  - Say, desired velocity = 65 km/h
  - Actual velocity = 55 km/h

Using negative summation:

- Error = (65-55)km/h = 10 km/h
- This would speed up the car by 10 km/h and would be a logical choice.

Using positive summation

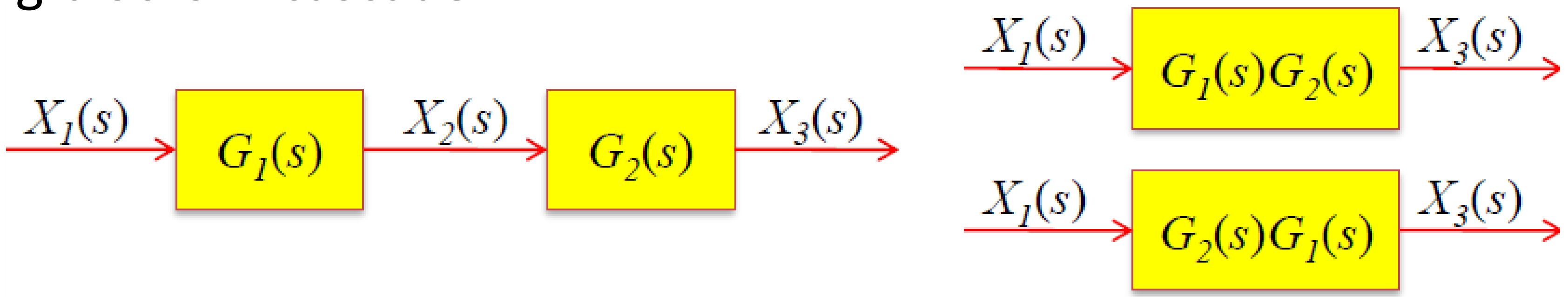
- Error = (65+55) = 110 km/h
- +’ve sign on summer introduces negative co-eff and results in RHP roots.
- This causes instability.

Therefore, negative feedback should be used.

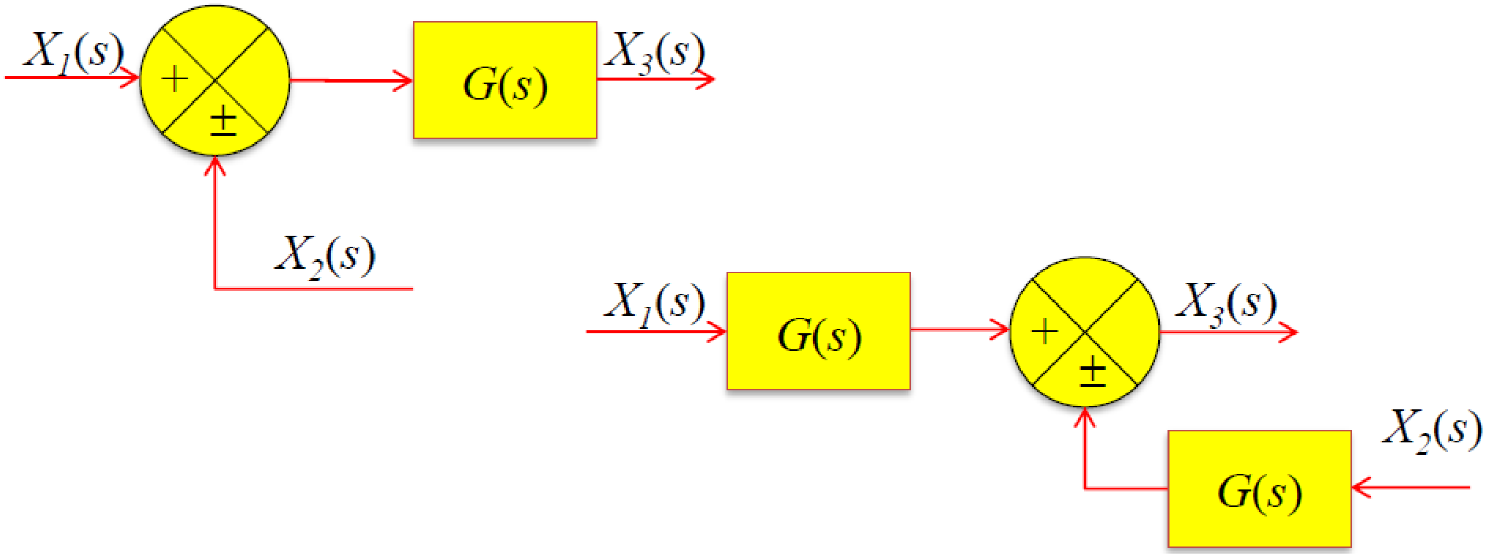
System Description

Block Diagram – Reductions (6)

1) Combining blocks in cascade



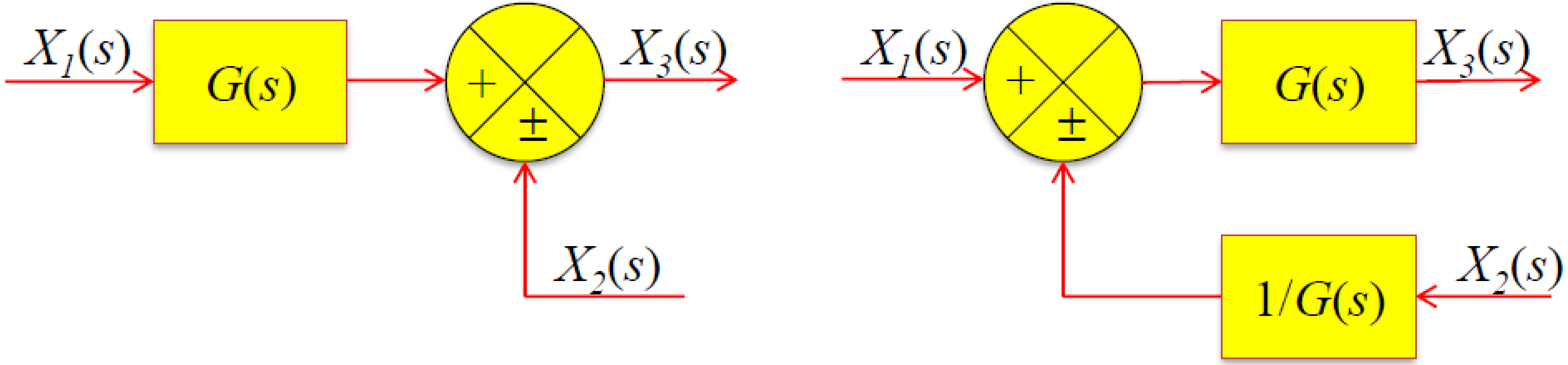
2) Moving a summing point forward



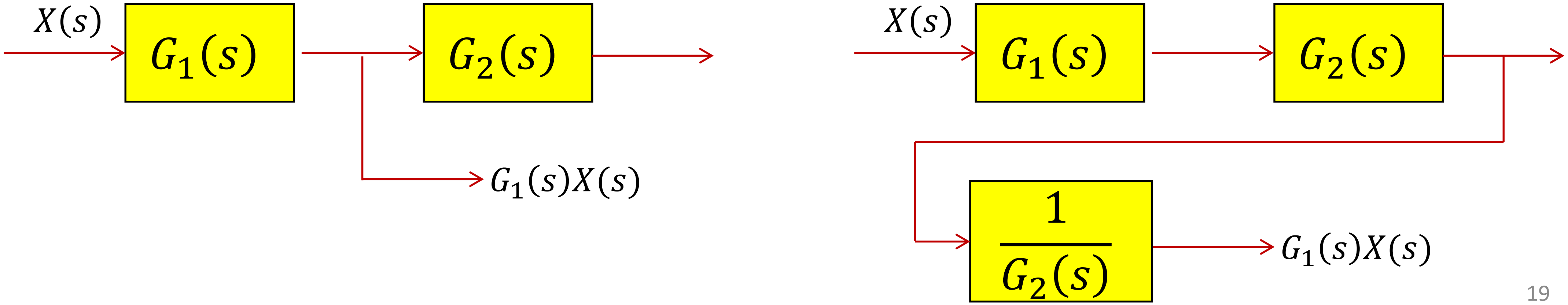
System Description

Block Diagram – Reductions (6)

3) Moving a summing point back



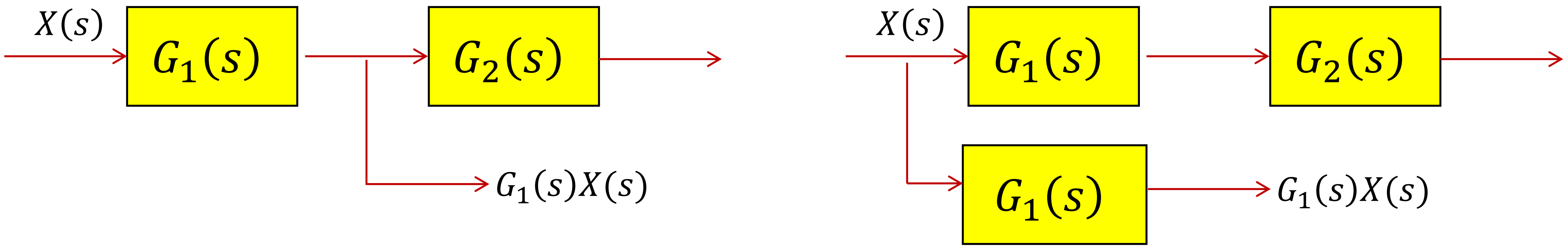
4) Moving a pickoff point forward



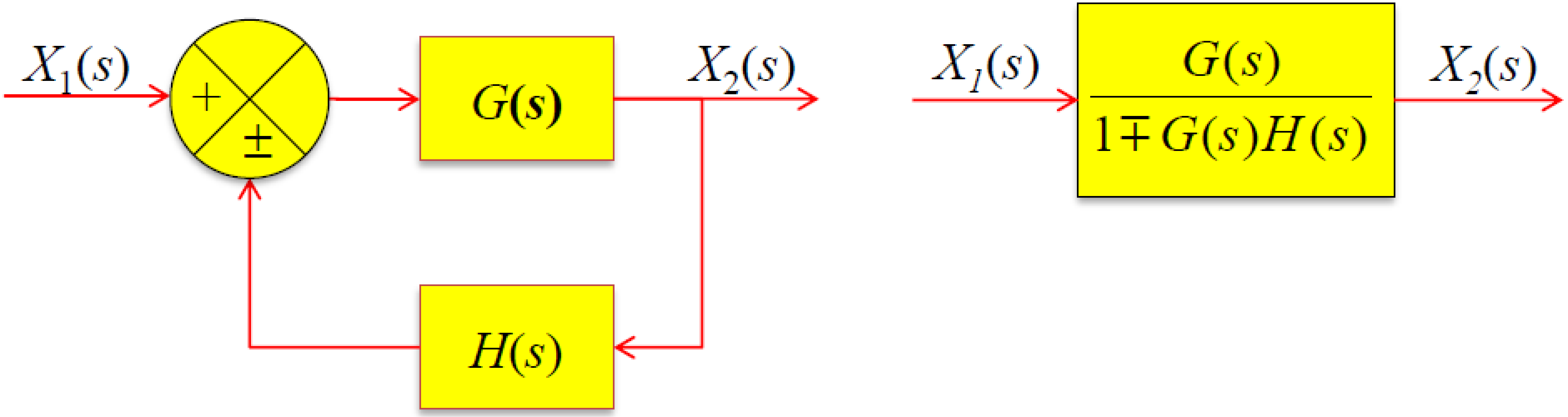
System Description

Block Diagram – Reductions (6)

5) Moving a pickoff point back



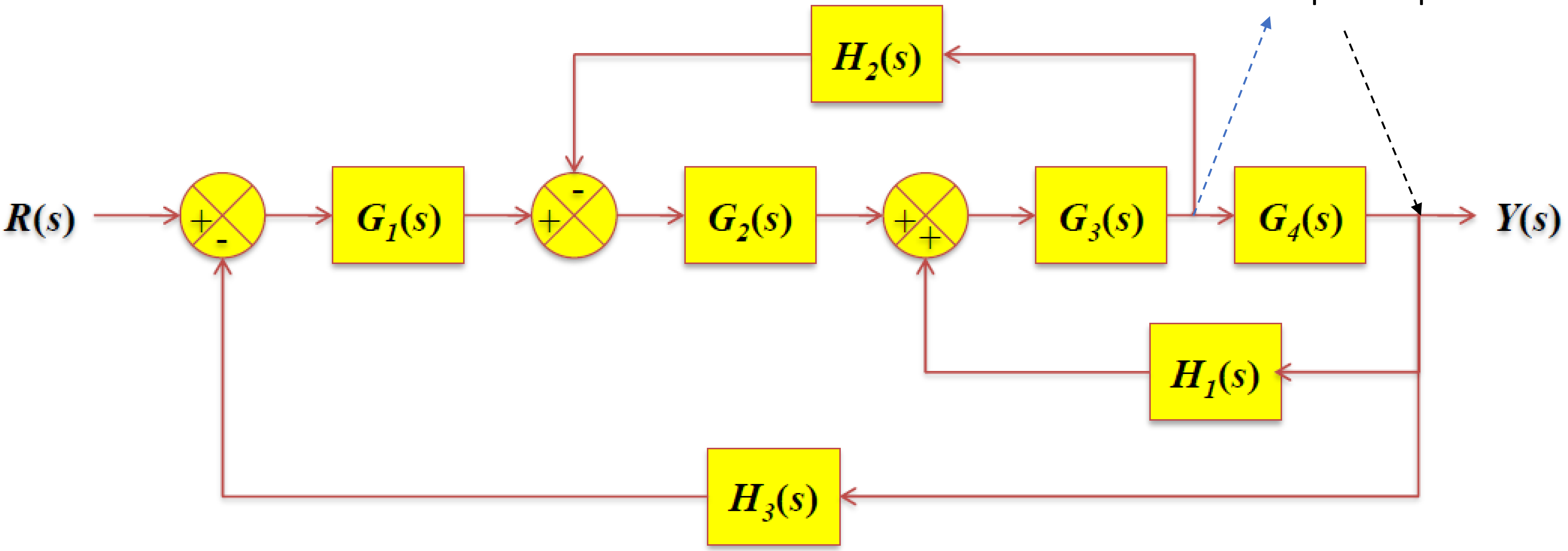
6) Eliminating a feedback loop



System Description

Block Diagram Reduction – Example

1. Move pickoff point

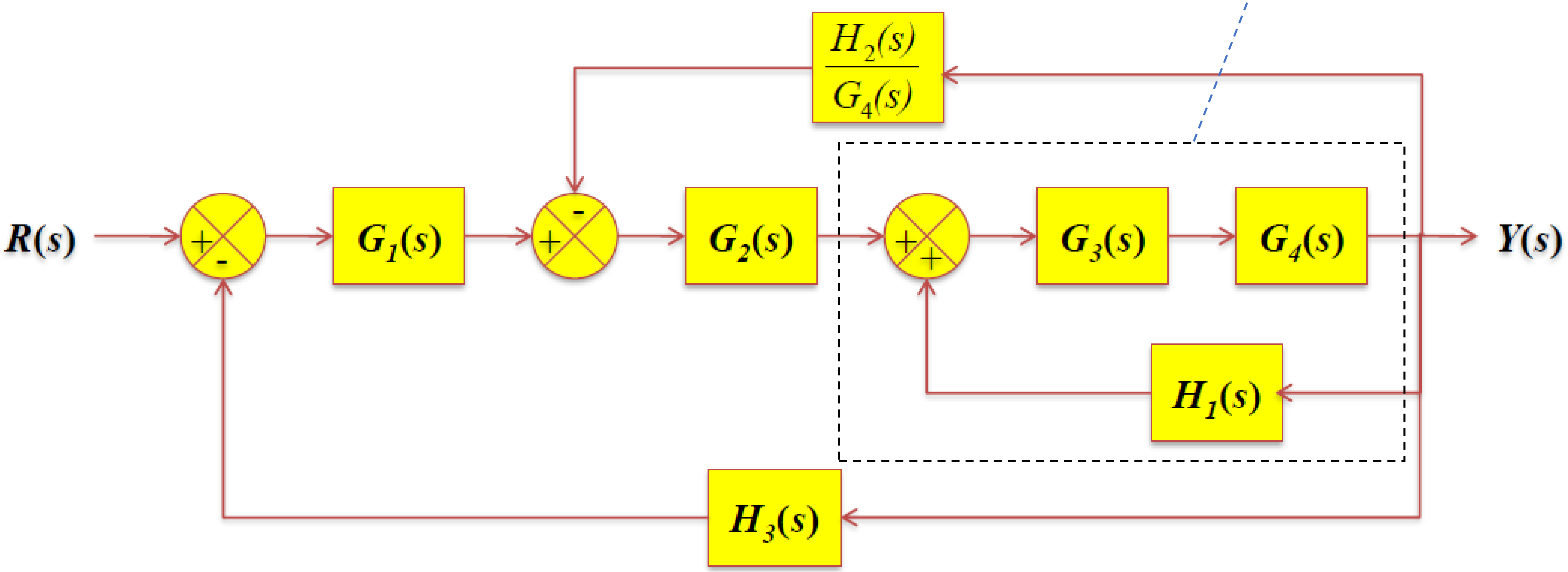


System Description

Block Diagram Reduction – Example

- Step-by-step solution

2. Calculate closed loop transfer function

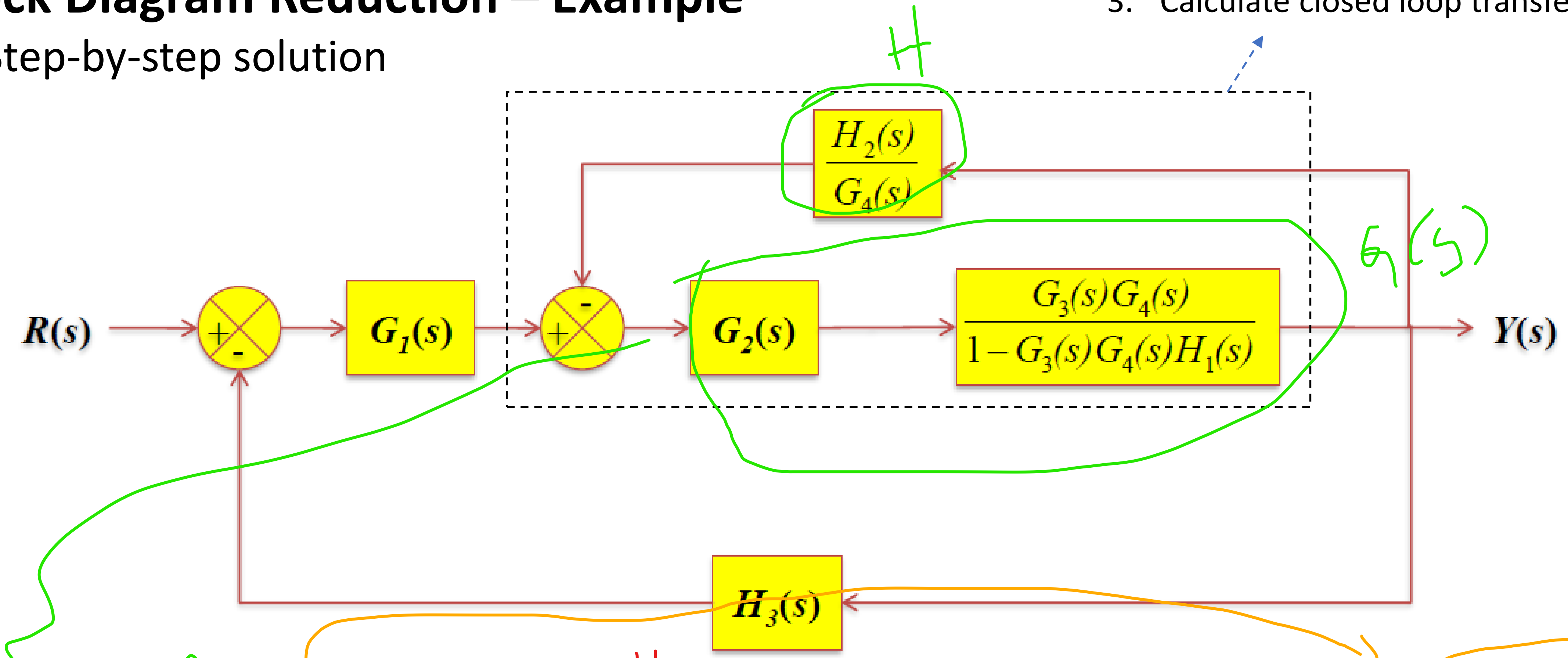


System Description

Block Diagram Reduction – Example

- Step-by-step solution

3. Calculate closed loop transfer function



$$\frac{G}{1-GH}$$

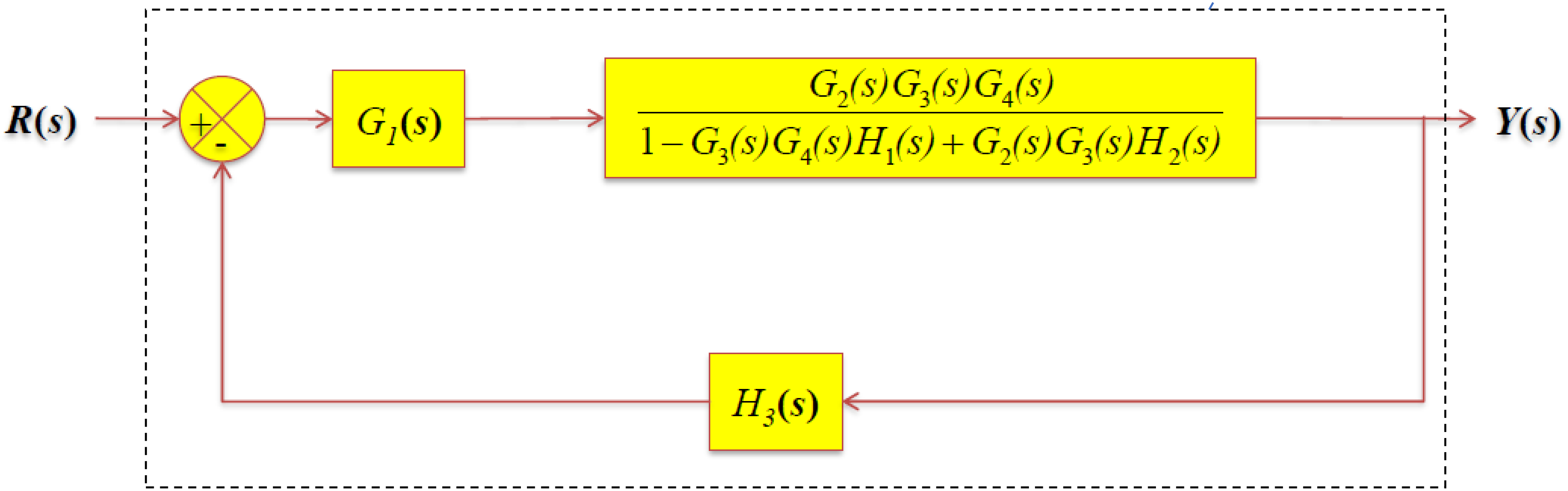
$$\frac{G_{23\cancel{4}}}{1-G_{34}H_1} \times \frac{H_2}{\cancel{G_4}} \Rightarrow 1-GH = 1-\frac{G_{23}H_2}{1-G_{34}H_1} \rightarrow \frac{1-G_{34}H_1-G_{23}H_2}{1-G_{34}H_1}$$

System Description

Block Diagram Reduction – Example

- Step-by-step solution

3. Calculate closed loop transfer function



System Description

Block Diagram Reduction – Example

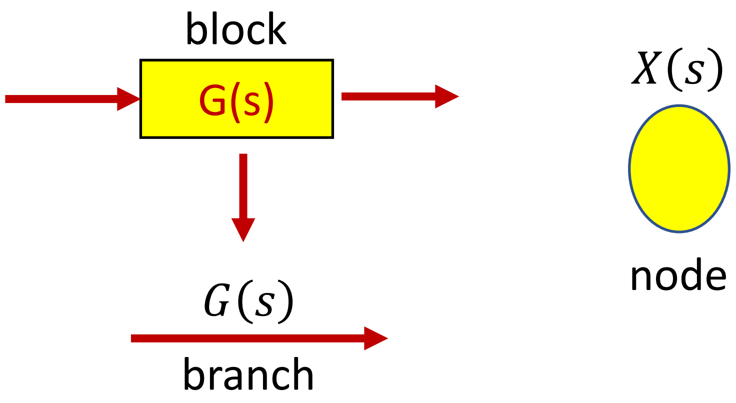
- Step-by-step solution
- **Answer**

$$R(s) \longrightarrow \boxed{\dfrac{G_1(s)G_2(s)G_3(s)G_4(s)}{1-G_3(s)G_4(s)H_1(s)+G_2(s)G_3(s)H_2(s)+G_1(s)G_2(s)G_3(s)G_4(s)H_3(s)}} \longrightarrow Y(s)$$

Signal Flow Graphs

What is it?

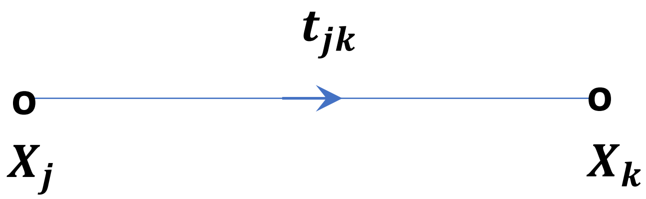
- A Signal Flow Graph SFG) is a **special type of block diagram** and directed graph solution.
- It consists of **nodes** and **branches**. Its nodes are the **variables** of a set of linear algebraic relations.
- SFG can only represent **multiplications** and **additions**.
    - **Multiplications** are represented by the **weights of the branches**.
    - **Additions** are represented by **multiple branches going into one node**.
    - It has a one-to-one relationship with a system of linear equations and can also be used to represent the signal flow in a physical system; i.e., it can represent relations of **cause and effect**.
    - **Consists of branches** (**represent system**)
    - **Nodes** (**represents signals**)



Signal Flow Graphs

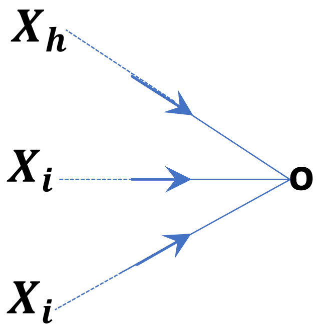
Basic Properties

- A signal flows along a branch only in the direction defined by the arrow and is multiplied by the transmittance of that branch.



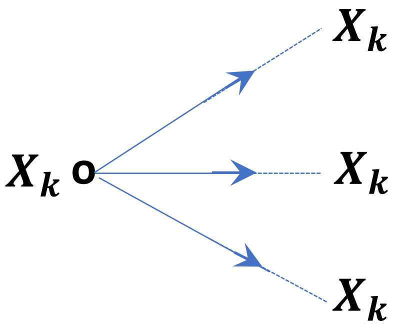
$$X_k = t_{jk}\, X_j$$

- A node signal is equal to the algebraic sum of all signals entering the pertinent node via the incoming branches.



$$X_k = X_h + X_i + X_j$$

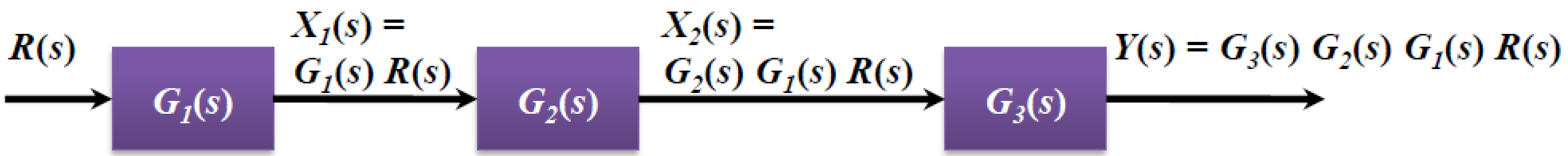
- The signal at a node is applied to each outgoing branch that originates from that node.

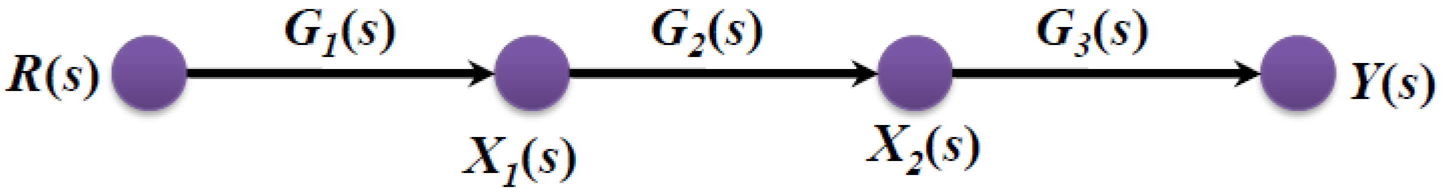


Signal Flow Graphs

Conversion between Block Diagrams and SFG

- We can convert the block diagrams in cascade, parallel, and feedback forms into signal flow diagrams.
  - **Cascade Form**

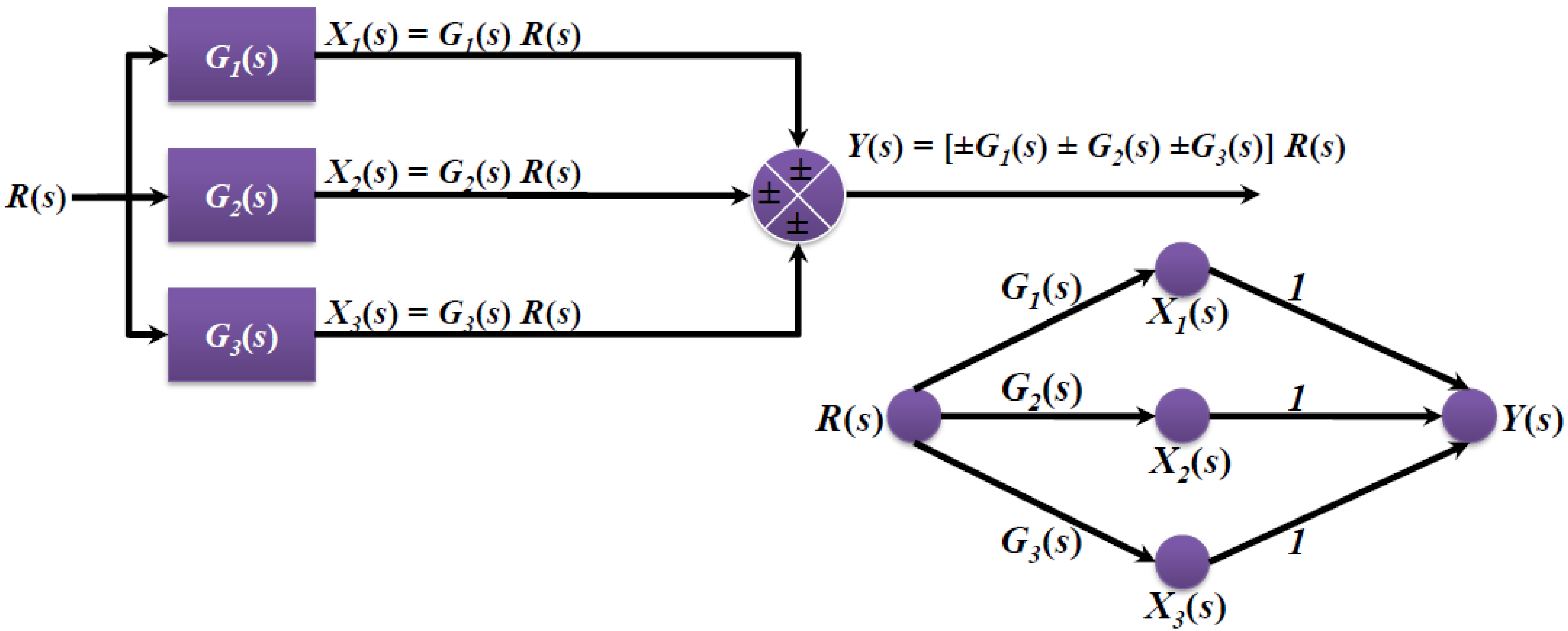




Signal Flow Graphs

Conversion between Block Diagrams and SFG

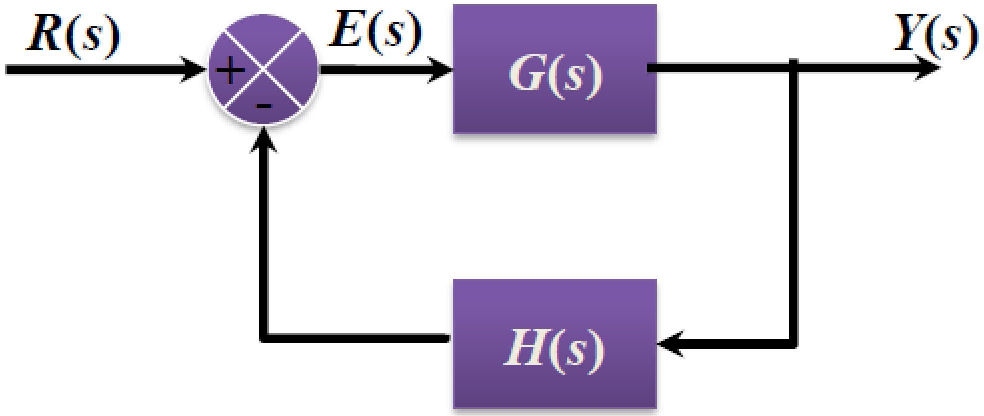
- We can convert the block diagrams in cascade, parallel, and feedback forms into signal flow diagrams.
  - Cascade Form
  - **Parallel Form**

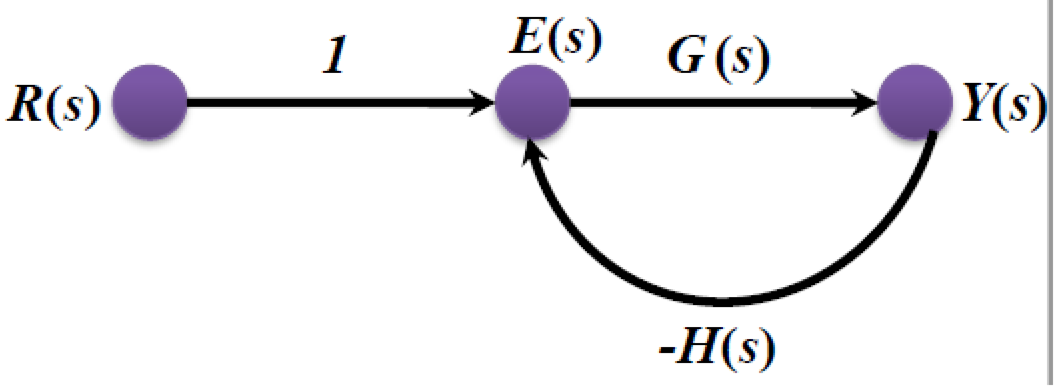


Signal Flow Graphs

Conversion between Block Diagrams and SFG

- We can convert the block diagrams in cascade, parallel, and feedback forms into signal flow diagrams.
  - Cascade Form
  - Parallel Form
  - **Feedback Form**

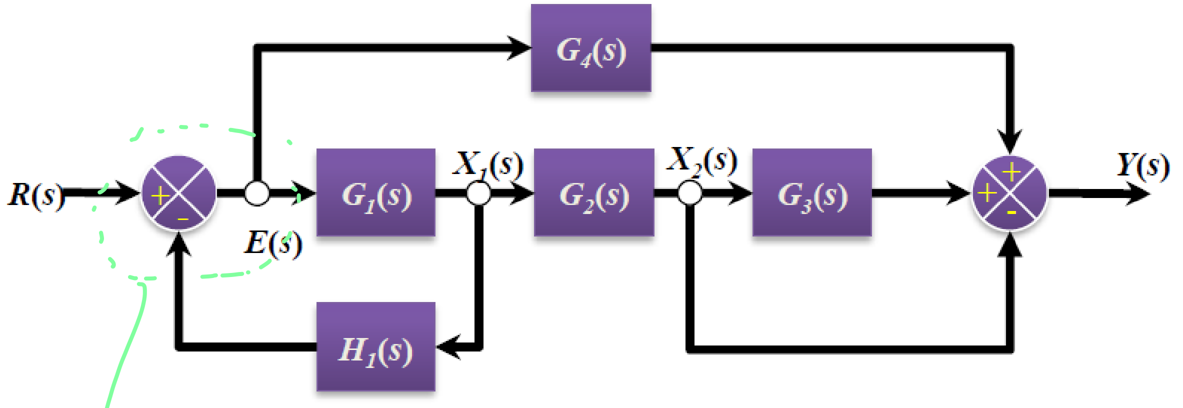


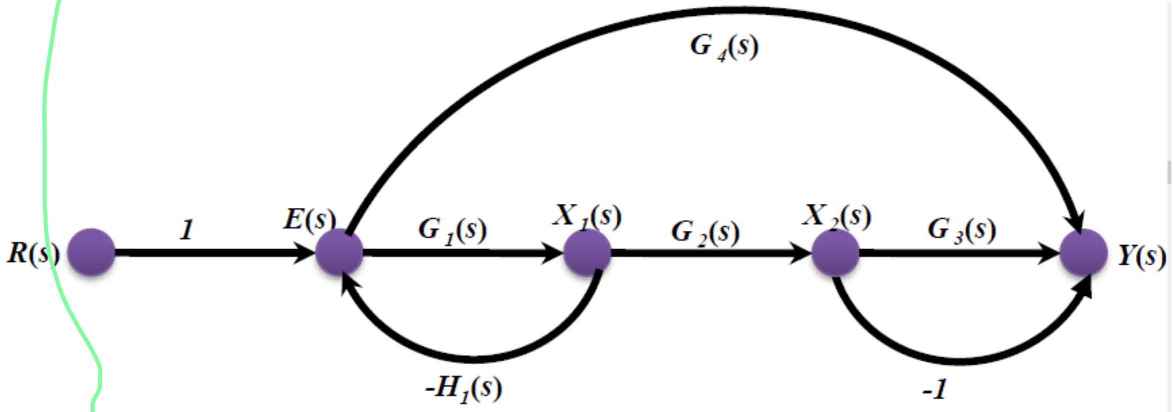


Signal Flow Graphs

Conversion: Example

- Replace every **block** with a **branch**.
- Replace each combination of **summer** and **pick-off points** with a **node** in the signal flow graph (all sums are assumed to be +ve. For -ve sums add a -ve sign
- Replace each solitary **pick-off point** (not connected to a summer) with a **label of the variable** assigned to the pick-off point.
- For each input show a node labeled with the variable assigned to the input.
- Add unity branches as needed or for clarity.



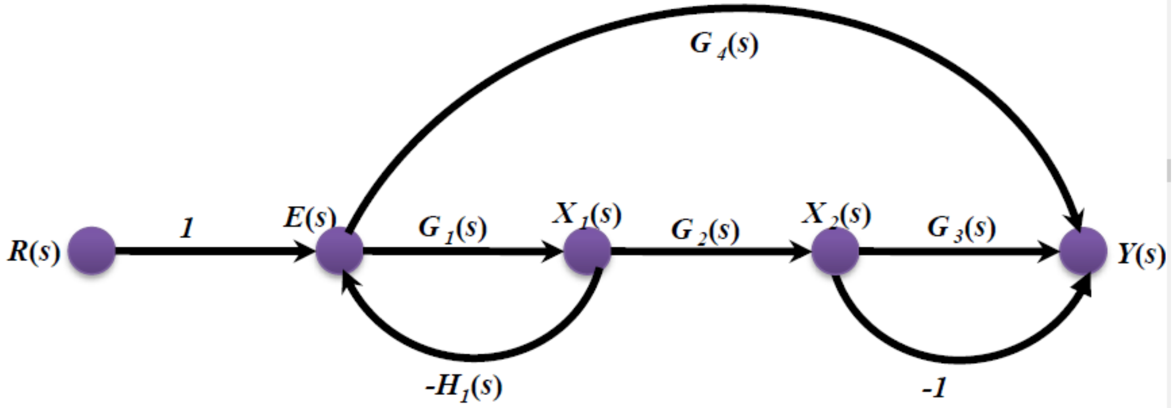


make nodes in place of
1. inputs and outputs
2. summing points (if branching is from the output of summer --this combined would be considered as a single node
3. branch points
4. in between cascaded blocks

Signal Flow Graphs

Mason's Gain Formula

$$G(s) = \frac{Y(s)}{R(s)} = \frac{\sum_i^N p_i \Delta_i}{\Delta}$$



N = total number of forward paths

p_i*= gain of the ith forward path*

Δ = 1- (Σ all individual feedback loop gains including self-loops) + (Σ gain product of all possible combinations of two nontouching loops) - (Σ gain product of all possible combinations of three non-touching loops)+…….
Δ_i *= value of Δ after eliminating all loops that touch its ith forward path*

1. *Path =* p_i
2. *Loop*
3. *Touching loops*
4. *Determinant* $= \Delta$
5. *Cofactor* $= \Delta_i$

Signal Flow Graphs

1. ***Path*** = $p_i =$ *A succession of branches, from input to output in the direction of arrows, that does not pass any node more than once.*

2. ***Path gain*** = ***Product of the transmittances*** *of the branches of the path. For the ith path, the path gain is denoted by* p_i.

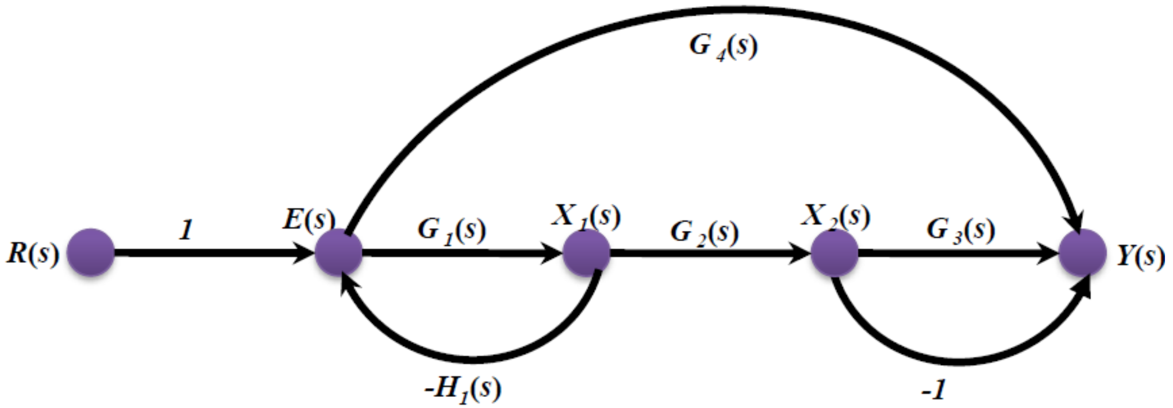
3. ***Loop*** = ***A closed succession of branches****, in the direction of the arrows, , that does not pass any node more than once.*

4. ***Loop gain*** = ***Product of the transmittances*** *of the branches of the loop.*

5. ***Touching loops*** = *Loops with one or more* **nodes in common.**

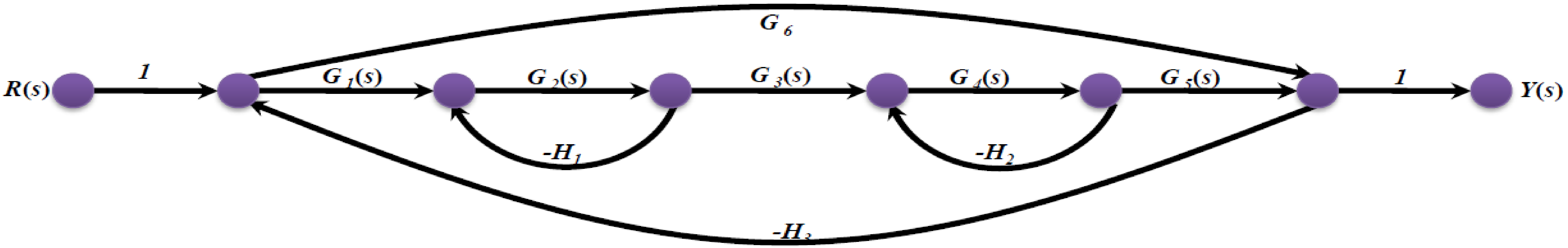
6. ***Determinant*** $= \Delta$

4. ***Cofactor*** $= \Delta_i$

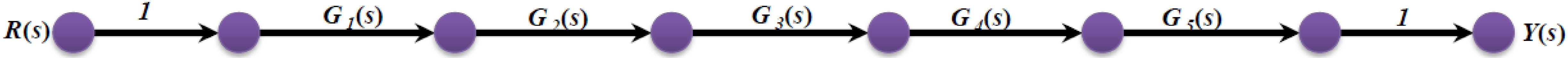


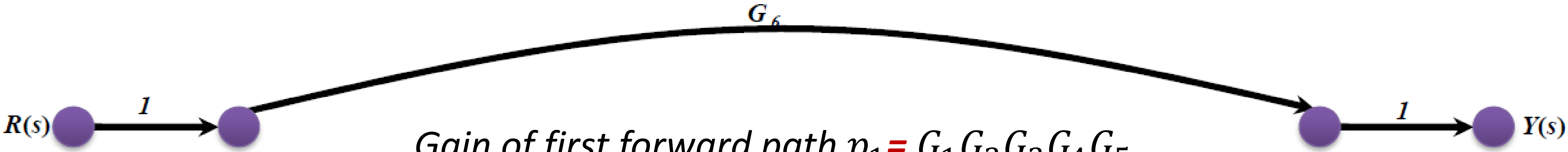
Signal Flow Graphs

Mason’s Gain Formula: Example



Step 1: *There are two forward paths as below so* **N=2**



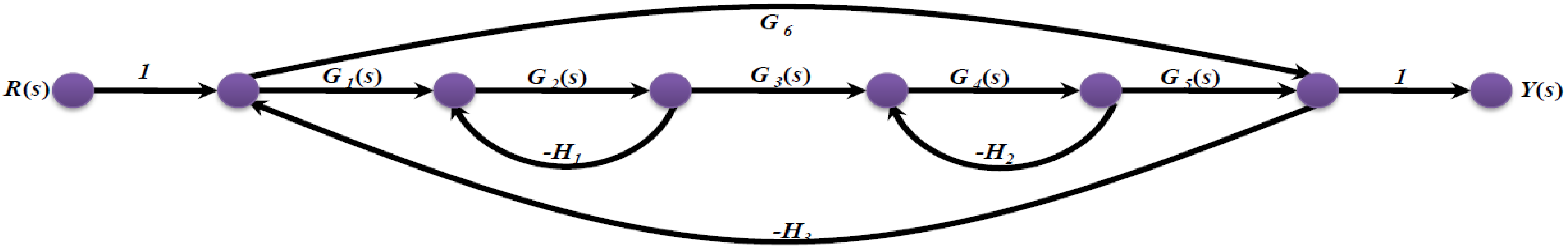


Gain of first forward path $p_1 = G_1G_2G_3G_4G_5$

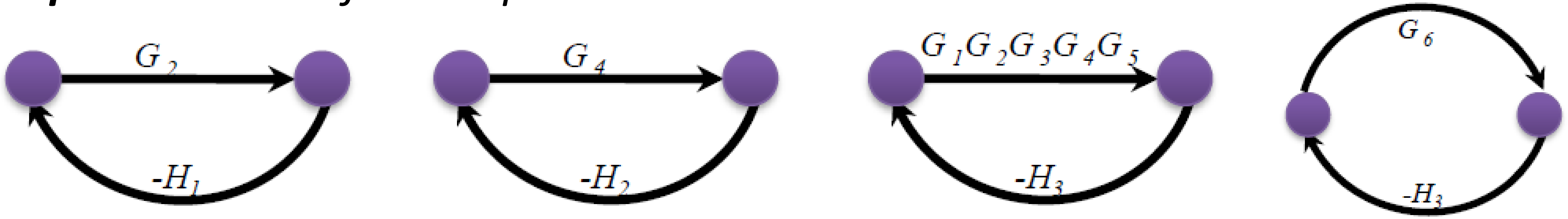
Gain of second forward path $p_2 = G_6$

Signal Flow Graphs

Mason's Gain Formula: Example



Step 2: *There are four loops*



Loop gain of first loop (L_1) = $-G_2H_1$

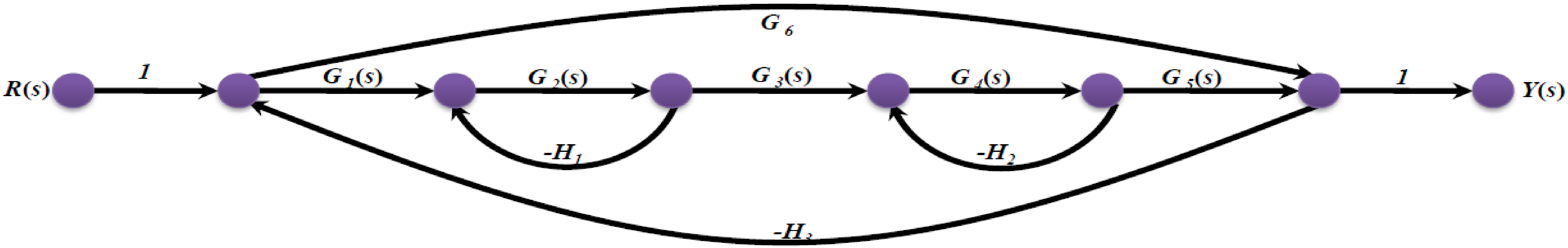
Loop gain of second loop (L_2) = $-G_4H_2$

Loop gain of third loop (L_3) = $-G_1G_2G_3G_4G_5H_3$

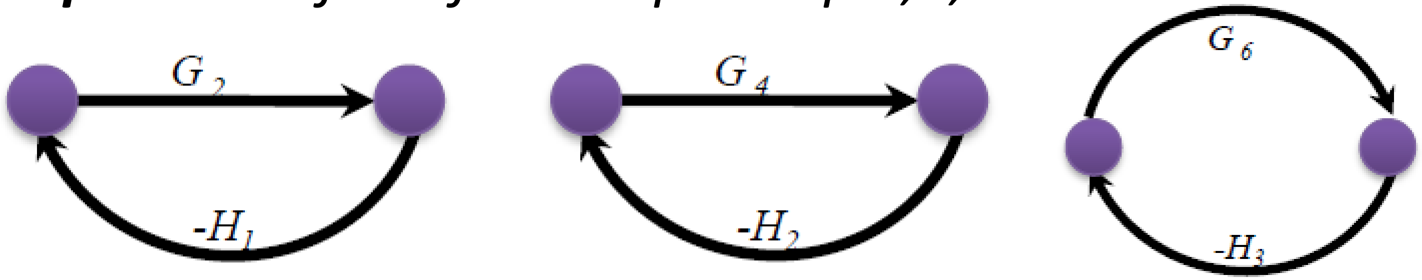
Loop gain of fourth loop (L_4) = $-G_6H_3$

Signal Flow Graphs

Mason's Gain Formula: Example



Step 3: *Out of the four loops: loop 1,2, and 4 are non touching*



Combinations of two non touching loops are:

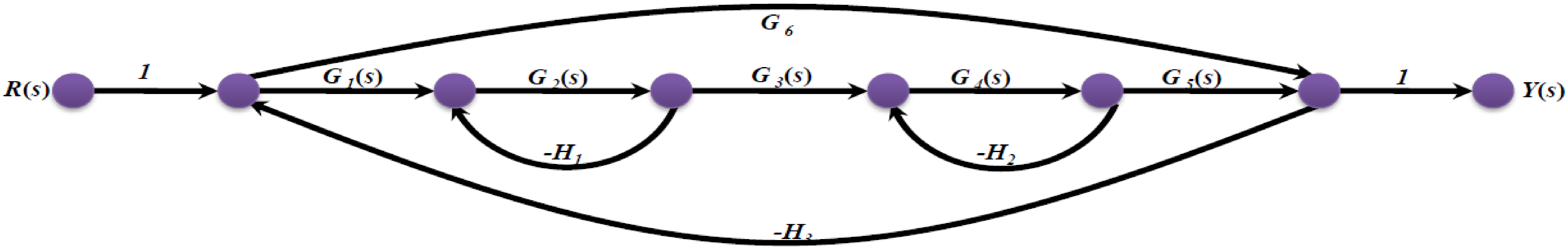
Loop 1, Loop 2: *Loop gain* (L_{12}) = $G_2G_4H_1H_2$

Loop 1, Loop 4: *Loop gain* (L_{14}) = $G_2G_6H_1H_3$

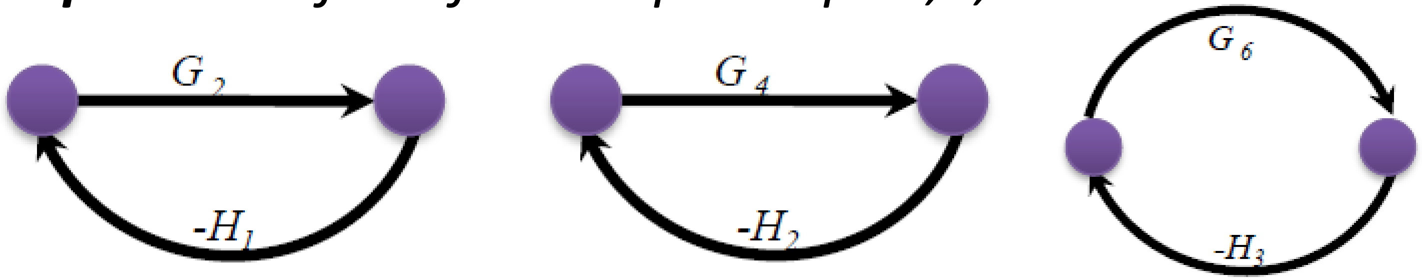
Loop 2, Loop 4: *Loop gain* (L_{24}) = $G_4G_6H_2H_3$

Signal Flow Graphs

Mason's Gain Formula: Example



Step 4: *Out of the four loops: loops 1,2, and 4 are non-touching*



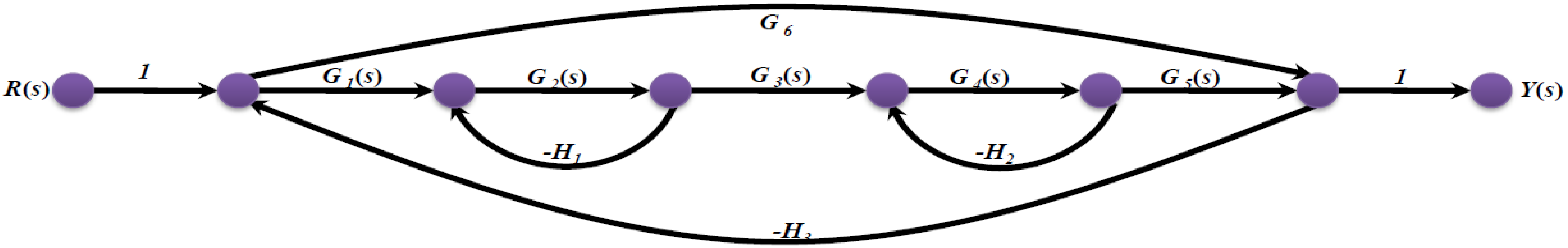
Combinations of three non touching loops are:

Loop 1, Loop 2, Loop 4: *Loop gain* (L_{124}) **=** $-G_2G_4G_6H_1H_2H_3$

Step 5: *There are no higher order non-touching loops.*

Signal Flow Graphs

Mason's Gain Formula: Example



Step 6: *Calculate* Δ

$$\Delta = \mathbf{1} - (\boldsymbol{L_1} + \boldsymbol{L_2} + \boldsymbol{L_3} + \boldsymbol{L_4}) + (\boldsymbol{L_{12}} + \boldsymbol{L_{14}} + \boldsymbol{L_{24}}) - (\boldsymbol{L_{124}})$$

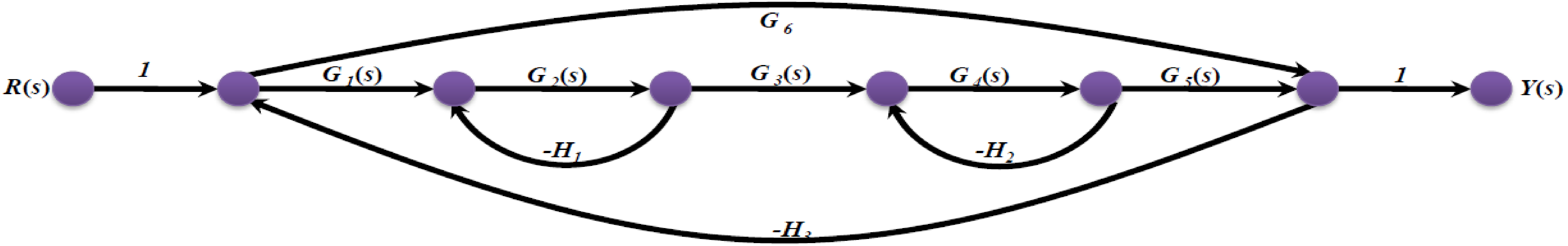
$$= \mathbf{1} + (G_2H_1 + G_4H_2 + G_1G_2G_3G_4G_5H_3 + G_6H_3)$$

$$+(G_2G_4H_1H_2 + G_2G_6H_1H_3 + G_4G_6H_2H_3)$$

$$+(G_2G_4G_6H_1H_2H_3)$$

Signal Flow Graphs

Mason's Gain Formula: Example



Step 7: *Calculate* Δ_i

We know that: $\Delta = \mathbf{1} - (\boldsymbol{L_1} + \boldsymbol{L_2} + \boldsymbol{L_3} + \boldsymbol{L_4}) + (\boldsymbol{L_{12}} + \boldsymbol{L_{14}} + \boldsymbol{L_{24}}) - (\boldsymbol{L_{124}})$

Considering path $\boldsymbol{p_1}$, $loops$ 1,2,3,4 touch it: eliminating all these from Δ

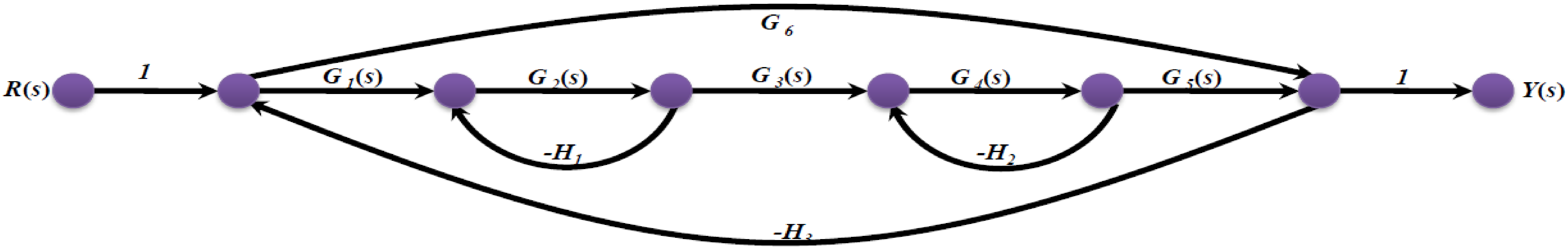
$\Delta_{\mathbf{1}} = 1 - (0) = 1$

Considering path $\boldsymbol{p_2}$, $loops$ 3,4 touch it: eliminating loops 3, 4 from Δ

$$\Delta_{\mathbf{2}} = 1 - (L_1 + L_2) + (L_{12}) = 1 - (-G_2H_1 - G_4H_2) + G_2G_4H_1H_2$$
$$= 1 + G_2H_1 + G_4H_2 + G_2G_4H_1H_2$$

Signal Flow Graphs

Mason's Gain Formula: Example



Step 8: *Transfer Function*

$$\boldsymbol{G}(\boldsymbol{s}) = \frac{\boldsymbol{p_1}\Delta_1 + \boldsymbol{p_2}\Delta_2}{\Delta}$$

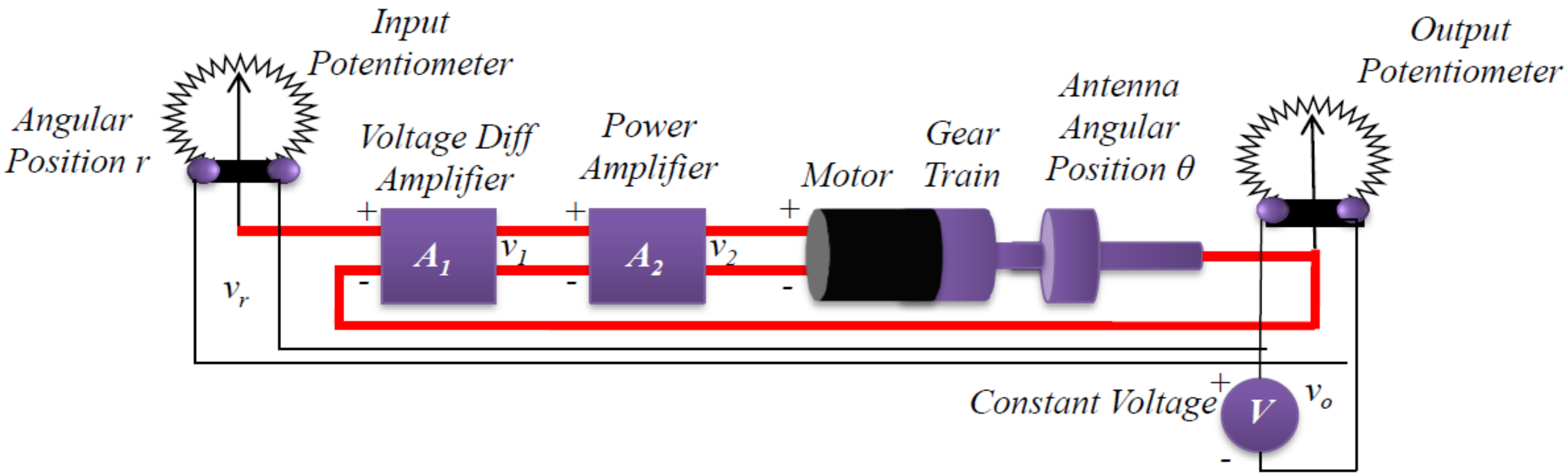
$$= \frac{G_1G_2G_3G_4G_5 + G_6(1 + G_2H_1 + G_4H_2 + G_2G_4H_1H_2)}{\mathbf{1} + (G_2H_1 + G_4H_2 + G_1G_2G_3G_4G_5H_3 + G_6H_3)}$$

$$+(G_2G_4H_1H_2 + G_2G_6H_1H_3 + G_4G_6H_2H_3)$$

$$+(G_2G_4G_6H_1H_2H_3)$$

Electromechanical Systems

Example – A Position Servo a large video satellite antenna



- Output potentiometer measures the output shaft position and converts it to a potential voltage.

  $v_o = K_p\theta$

  $\theta = output\ shaft\ angle$

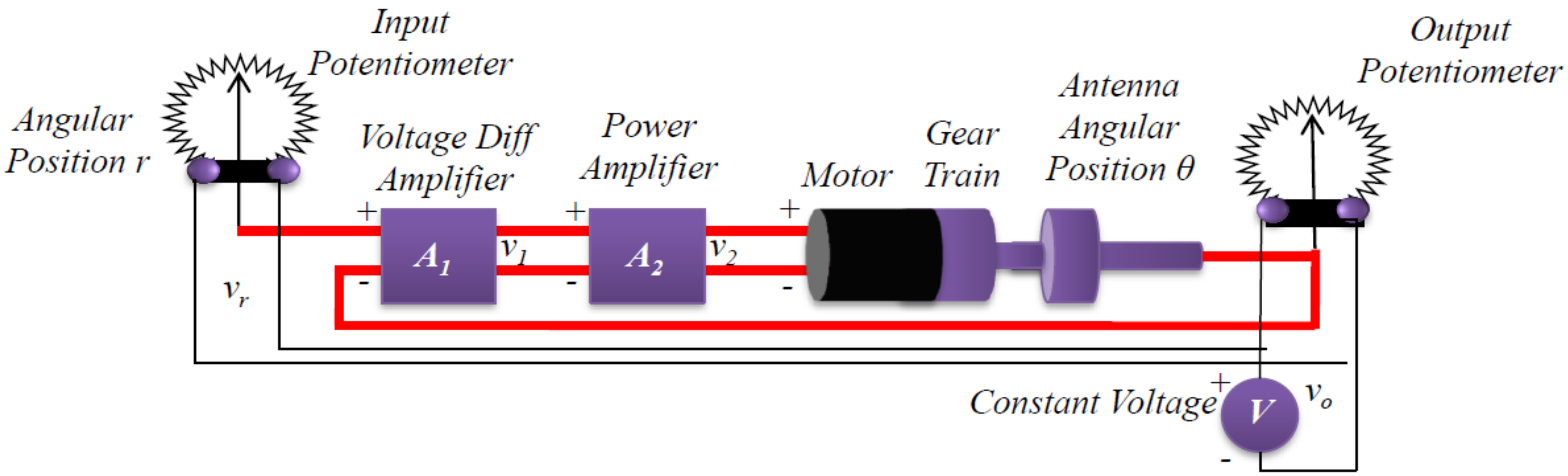
  $K_p = proportionality\ const. = \dfrac{V}{\theta_{max}}$ volts/radian

- *The input potentiometer slider position is converted to a voltage in a similar manner:*

  $v_r = K_p r$

Electromechanical Systems

Example – A Position Servo a large video satellite antenna



- Difference between the two potentiometer signals is then amplified with gain A_1

$v_1 = A_1(v_r - v_0) = A_1K_p(r - \theta)$ *where,* $v_1 = error\ voltage\ output$

- This voltage is then further amplified with gain A_2 and applied to the motor terminals:

$v_2 = A_2v_1 = A_2A_1K_p(r - \theta)$ *where,* $v_2 = motor\ drive\ voltage$

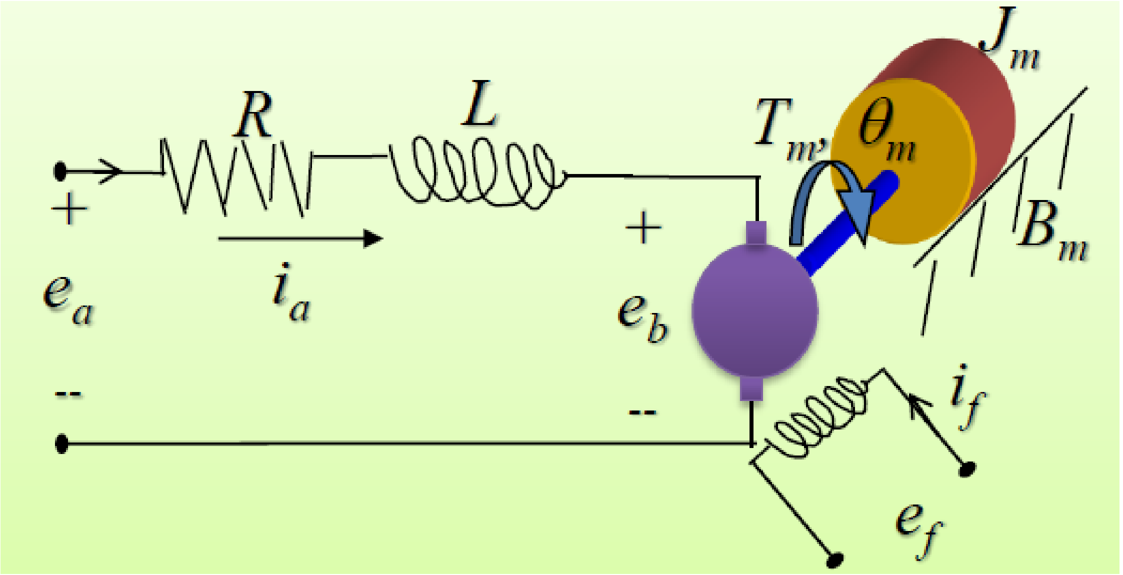
- *The second amplifier is the power amplifier capable of providing the electrical power needed to drive the motor.*
- *The motor is coupled to the antenna with a gear train ratio of*

$\theta = \frac{N_1}{N_2}\theta_m$ *where* θ_m *is the motor shaft angle.*

Electromechanical Systems

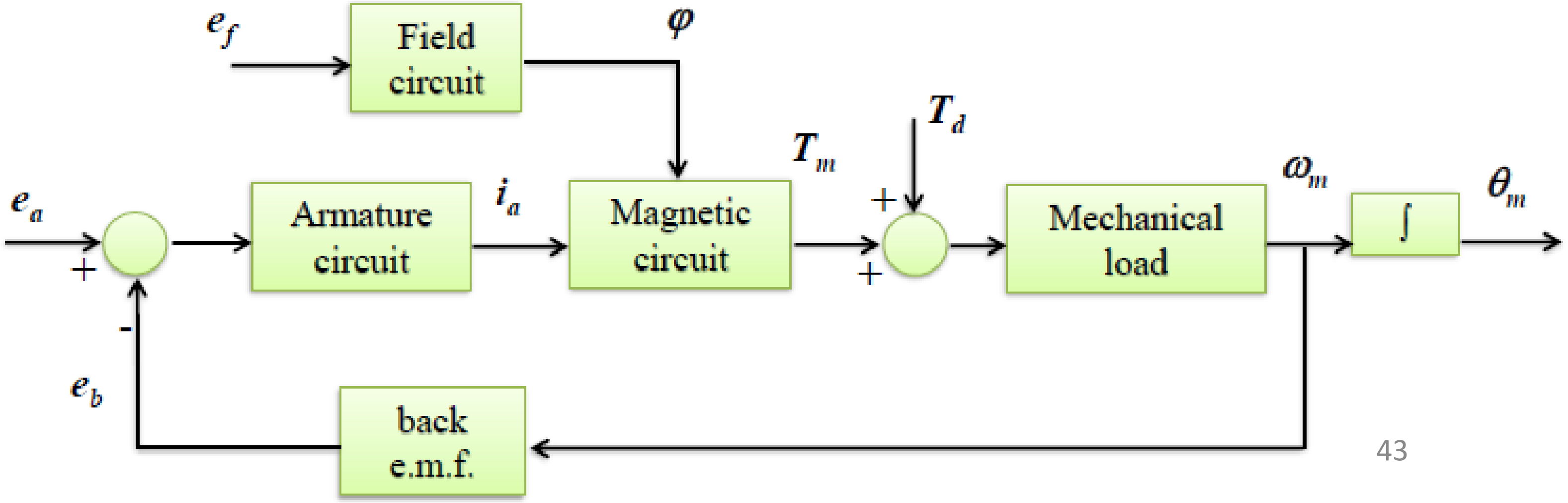
Example – A Position Servo a large video satellite antenna

DC Servo Motor – Modeling



- The **armature current** depends upon the applied voltage and the back emf.
- The **electromagnetic torque** is produced by the interaction of the armature current and the field current.
- The **electromagnetic torque** minus disturbance or **load torque** drives the inertial load.

- The functional block diagram of a DC motor.

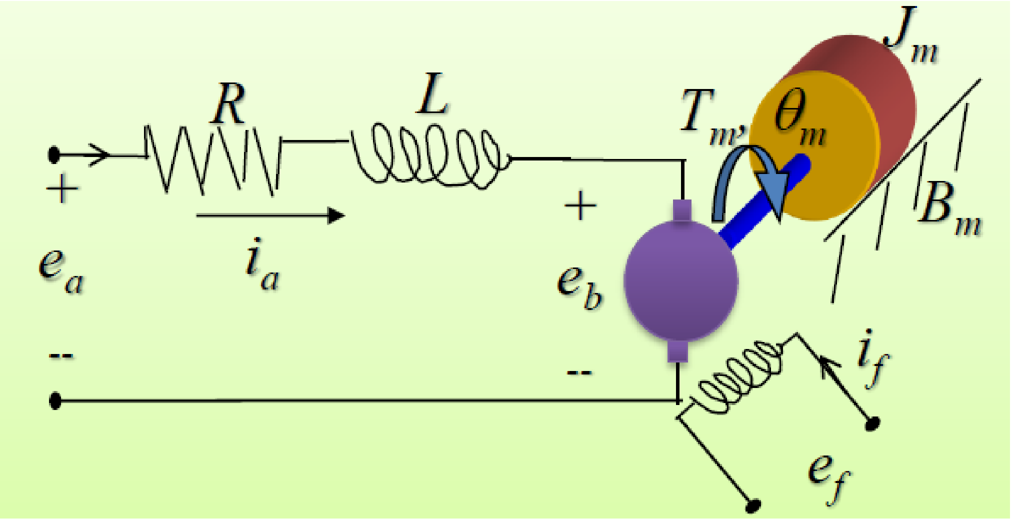


$$TF = \frac{\theta_m(s)}{E_a(s)}$$

Electromechanical Systems

Example – A Position Servo a large video satellite antenna

DC Servo Motor – Modeling



- The relationship between the armature current $i_a(t)$, the applied armature voltage $e_a(t)$, and the back emf $\boldsymbol{e_b(t)}$, is found by applying KVL and then taking Laplace transform:

$$E_a(s) = R_a I_a(s) + L_a s I_a(s) + E_b\,(s) \ldots\ldots (a)$$

- The **back emf $\boldsymbol{e_b(t)}$** is directly proportional to the speed of the motor and it can be written in Laplace domain as:

$$E_b(s) = K_b s\theta_m(s) \ldots\ldots\ldots (b)$$

- The **torque developed by the motor (armature torque)** is proportional to the armature current; thus,

$$T_m(s) = K_i I_a(s) \;\rightarrow\; I_a(s) = \frac{T_m}{K_i} \ldots\ldots\ldots (c)$$

✓ The **armature torque** is directly proportional to the product of the **flux** and the **armature current**:

$$T_m \propto \emptyset I_a \;\rightarrow\; T_m = K_m \emptyset I_a$$

$$\emptyset = K_f i_f \rightarrow \textit{field flux}$$

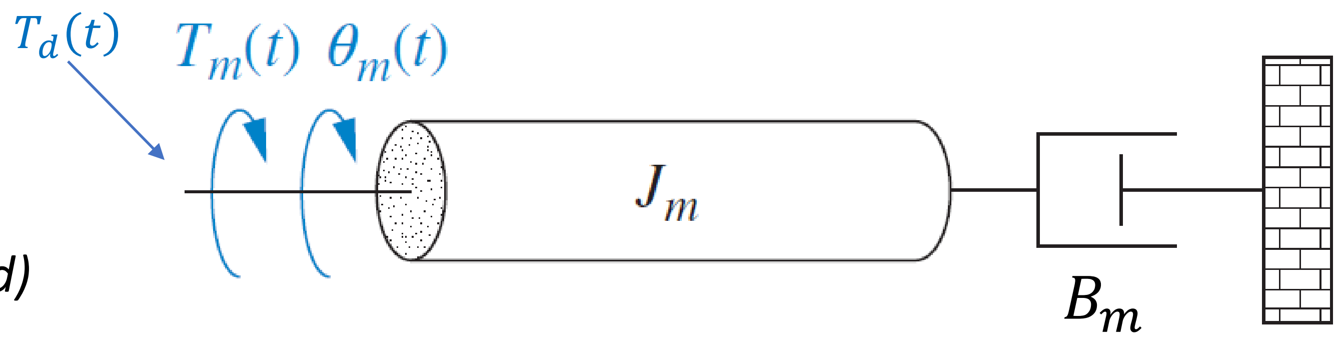
$$T_m = K_m K_f I_f i_a = K_i i_a \rightarrow \textit{for const. field current}$$

$$K_i = \textit{motor torque constant}$$

Electromechanical Systems

Example – A Position Servo a large video satellite antenna

DC Servo Motor – Modeling



- Substituting ***(b)*** *and* ***(c)*** in ***(a);***

$$E_a(s) = (R_a+L_a s)\frac{T_m}{K_i} + K_b s\theta_m(s) \ldots\ldots\ (d)$$

- The **mechanical load** on the motor can be modeled in Laplace domain as:

$$T_m(s) + T_d(s) = (J_m s^2 + B_m s)\theta_m(s) \ldots\ldots\ldots (e)$$

$$\mathit{Substituting}\ \boldsymbol{(e)}\ \mathit{in}\ \boldsymbol{(d)}\ \mathit{and\ setting}\ T_d(s) = 0$$

Recalling the transfer function: $\boldsymbol{TF} = \dfrac{\boldsymbol{\theta_m(s)}}{\boldsymbol{E_a(s)}}$

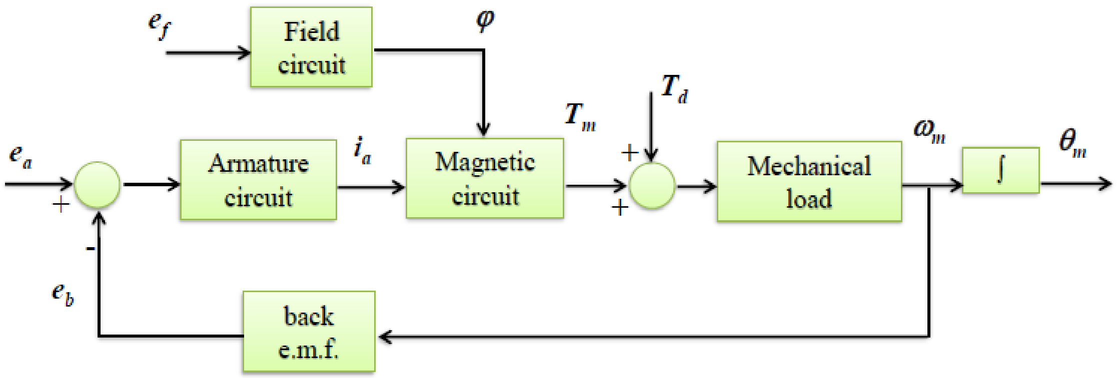
$$E_a(s) = \frac{(R_a+L_a s)(J_m s^2+B_m s)\theta_m(s)}{K_i} + K_b s\theta_m(s)$$

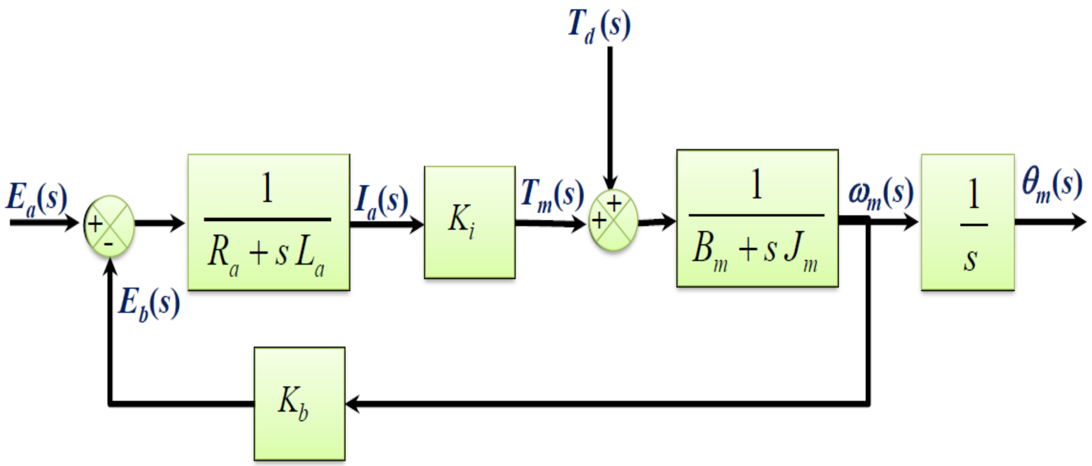
$$\boldsymbol{TF} = \frac{\boldsymbol{\theta_m(s)}}{\boldsymbol{E_a(s)}} = \frac{K_i}{\underbrace{s[L_a J_m s^2+(R_a J_m+L_a B_m)s+(R_a B_m+K_b K_i)]}_{G(s)}}$$

Electromechanical Systems

Example – A Position Servo a large video satellite antenna

DC Servo Motor – Modeling





In the text book $K_i = K_T$, $K_b = K_v$, and $E_a = V_2$ which means:

$$\frac{\boldsymbol{\theta_m(s)}}{\boldsymbol{V_2(s)}} = \frac{K_T}{s[L_aJ_ms^2 + (R_aJ_m + L_aB_m)s + (R_aB_m + K_vK_T)]}$$

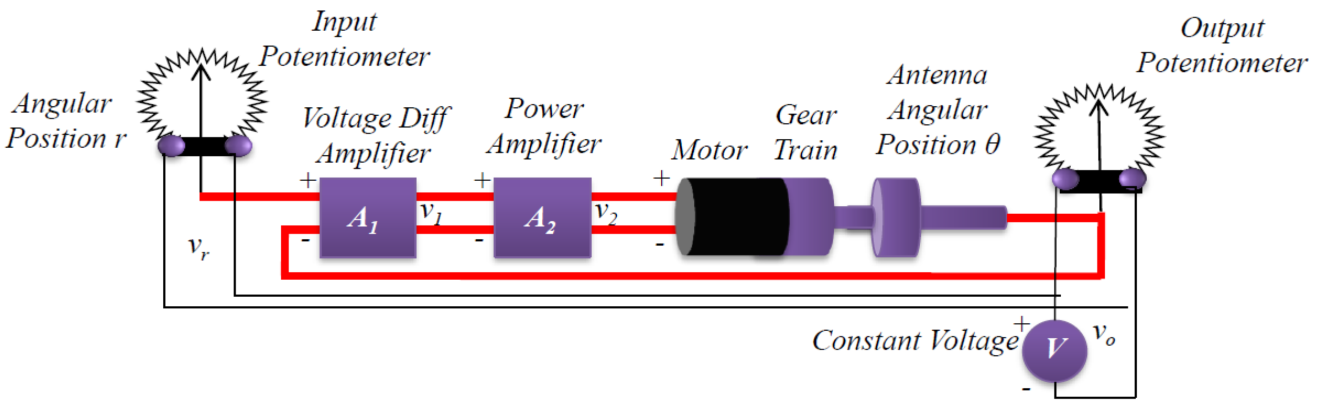
Neglecting the terms L_a, B_m and dividing by K_vK_T

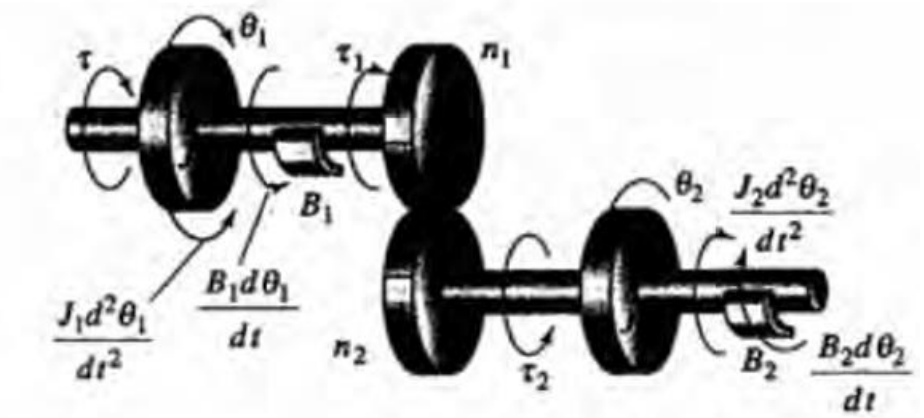
$$\frac{\boldsymbol{\theta_m(s)}}{\boldsymbol{V_2(s)}} = \frac{1/K_v}{s[1 + (\frac{R_aJ_m}{K_vK_T})]}$$

- $T_m = K_m \emptyset I_a$
  $K_m =$ *motor torque constant*
- $v_m = K_v \omega_m$
  $K_v =$ *motor voltage constant*
- $K_m = 1/K_v$

Electromechanical Systems

Example – A Position Servo a large video satellite antenna





- **"θ"** is the angular position of the antenna with the moment of inertia **"J"**. N1 <<< N2, since the high-speed shaft of the motor must drive the antenna at low speed and high torque.
- $J_1 s^2\theta_1(s) + B_1 s\theta_1(s) + \left(\frac{n_1}{n_2}\right)^2 J_2 s^2\theta_1(s) + (\frac{n_1}{n_2})^2 B_1 s\theta_1(s)$
- $\frac{\boldsymbol{\theta}\,(\boldsymbol{s})}{\boldsymbol{V_2}(\boldsymbol{s})} = \frac{K_m\,(\frac{N_1}{N_2})}{s(1 + (R_a/K_v K_T[J_m + \left(\frac{N_1}{N_2}\right)^2 J_L]\,)s)}$
- Taking Laplace of v_2 gives: **(ref: slide 42)**
- $V_2(s) = A_1 A_2 K_p(R(s) - \theta(s))$

$$\boxed{\begin{aligned} J_1 &= J_m \\ J_2 &= J_L \end{aligned}}$$

$$\boxed{\begin{gathered} \tau = \tau_1 + J_1\frac{d^2\theta_1}{dt^2} + B_1\frac{d\theta_2}{dt} \\ \tau_2 = J_2\frac{d^2\theta_2}{dt^2} + B_2\frac{d\theta_2}{dt} \\ \tau_2 = \frac{n_2}{n_1}\tau_1 \qquad \theta_2 = \frac{n_1}{n_2}\theta_1 \end{gathered}}$$

❖ $\theta(s) = \theta_m\left[\frac{N_1}{N_2}\right]$

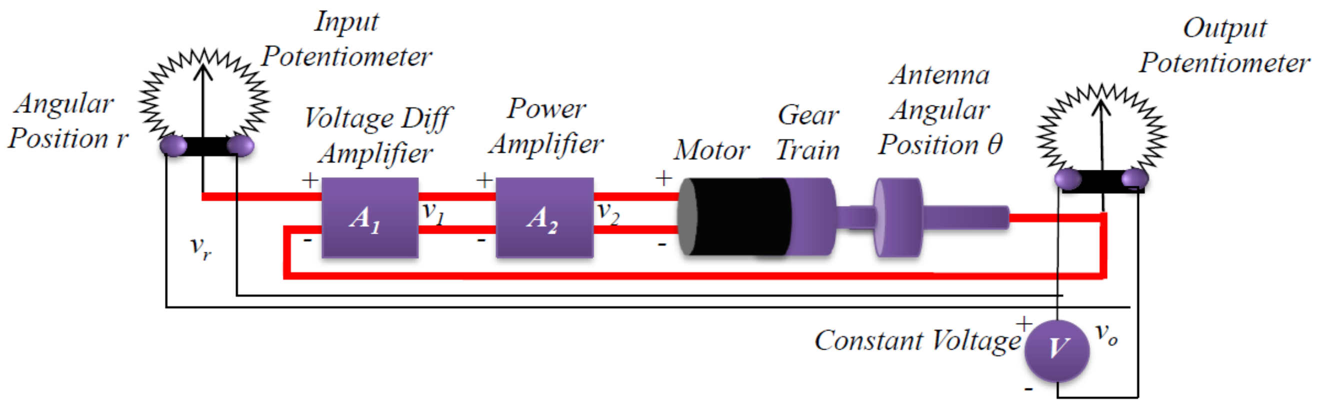
Electromechanical Systems

Example – A Position Servo a large video satellite antenna

substituting to find value of $\theta(s)$ we get

$$\boldsymbol{\theta(s) = \frac{K_m\,(\frac{N_1}{N_2})A_1A_2K_p(R(s)-\theta(s))}{s(1\ +\tau_L s)}}$$

Where, $\tau_L = (R_a/K_vK_T[J_m + \left(\frac{N_1}{N_2}\right)^2 J_L\,]$



$$\boldsymbol{\theta(s) = \frac{K_m\,(\frac{N_1}{N_2})A_1A_2K_p(R(s)-\theta(s))}{s(1\ +\tau_L s)}}$$

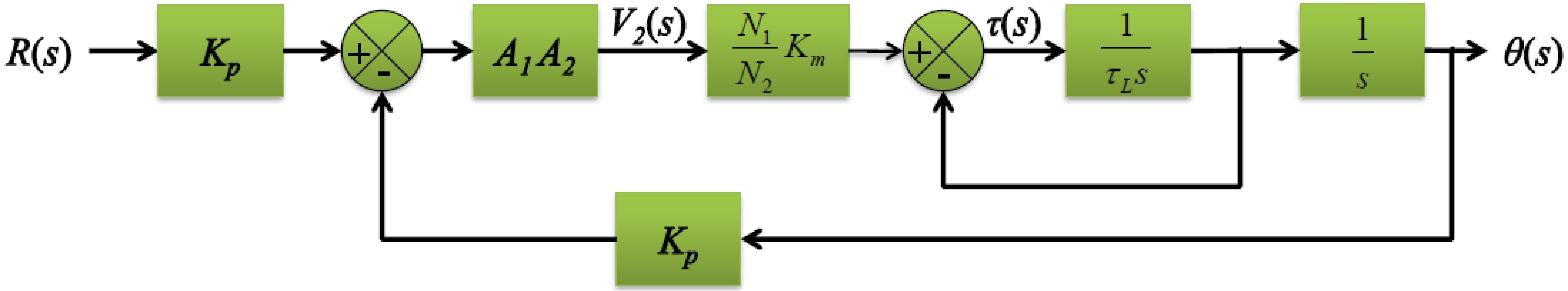
- Some of the coefficients, and thus some of the system properties can be selected by the designer by appropriately choosing the control components.
- However, the moment of inertia of the load J cannot be changed.
- The transfer function relating the input position R(s) to the output position Θ(s) is given by:

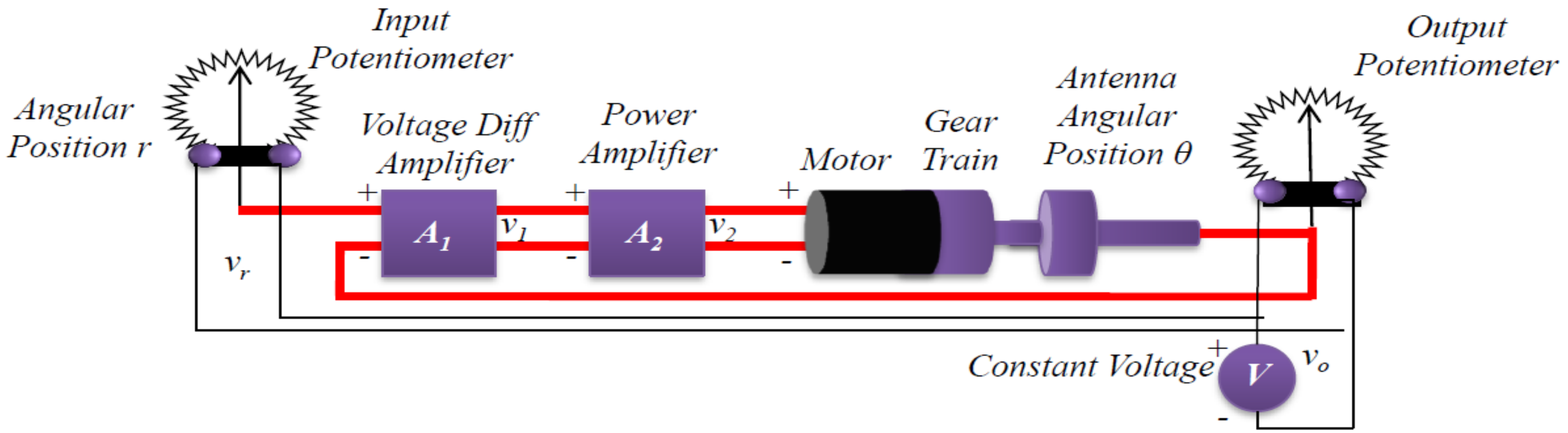
$$\boldsymbol{T(s) = \frac{\theta(s)}{R\ (s)} = \frac{(\frac{N_1}{N_2})A_1A_2K_m\,K_p}{\tau_L s^2 + s + (\frac{N_1}{N_2})A_1A_2K_m\,K_p}}$$

Electromechanical Systems

Example – A Position Servo a large video satellite antenna
DC Servo Motor – Block Diagram

- The transfer function can also be derived using a block diagram as shown below:





Electromechanical Systems

Example – A Position Servo a large video satellite antenna
DC Servo Motor – Block Diagram – Signal Flow Graph

- It can also be obtained by reducing the equivalent signal flow graph shown on this slide
- In this graph for **one forward path** Mason's rule gives.

$$P_1 = K_P A_1 A_2 \frac{N_1}{N_2} \frac{1}{\tau_L s} \frac{1}{s} K_m \ , \qquad \Delta_1 = 1$$

$$L_1 = A_1 A_2 \frac{N_1}{N_2} K_m \frac{1}{\tau_L s} \frac{1}{s} (-K_P), \qquad L_2 = \frac{-1}{\tau_L s}$$

$$= \frac{-A_1 A_2 (\frac{N_1}{N_2}) K_m K_p}{\tau_L s^2}$$

$$T(s) = \frac{K_P A_1 A_2 (\frac{N_1}{N_2}) K_m}{\tau_L s^2 + s + (\frac{N_1}{N_2}) K_P A_1 A_2 K_m}$$