

# EE-379 Linear Control Systems

## Week No. 2: Continuous Time System Description

- The Concept of Stability in Control Systems
- Block Diagrams
- Signal Flow Graphs
- Modeling Example: A Position Servo

# System Description

## Stability

- Stability is usually evaluated from two viewpoints **internal** and **external**.

## Internal Stability

- For **internal asymptotic** stability, the **zero-input (natural)** response decays to zero as the time approaches  $\infty$ , for **all possible initial conditions**.
- The **characteristic polynomial roots** influence the response.
- Ensured if **all roots are located in the LHP**

# System Description

## Stability

- Stability is usually evaluated from two viewpoints **internal** and **external**.

## External Stability

- For **external** (bounded input bounded output or BIBO) **stability**, the **zero-state response** is bounded as the time approaches  $\infty$ , for all bounded inputs.
- For a **bounded input** we can expect the **forced response to be bounded**.
- For the system to be BIBO stable, **the natural response of the output** should also be bounded.
- **If natural response decays to zero** as time approaches  $\infty$  system is BIBO.
- It is ensured if the **zero-state impulse response** of the output decays to zero.

# System Description

## Stability – Evaluation Criterion

- If all characteristic polynomial roots are in the **LHP** then the natural response decays to zero and the system is **asymptotically and BIBO stable**.
- Another possibility for **BIBO** exists wherein suppose the characteristic polynomial contains some **RHP** roots (*poles*).
- **RHP** zeros of the  $T(s)$  cancel all the **RHP** poles from the **TF** and from the zero-state impulse response.
- At least **one term** in the **zero-input response** will go to  $\infty$ , while the **zero-state impulse response** decays to zero. So, the system will be **asymptotically unstable but BIBO stable**.

# System Description

## Stability – Evaluation Criterion

- Thus, the two types of stability differ when all **RHP poles** are canceled by **RHP zeros**.
- We **evaluate BIBO stability** by examining the **zero-state impulse response** which allows for poles zero cancellation.
- We **evaluate asymptotic stability** by examining the **zero-input response of the system**.

# System Description

## Stability – Evaluation Criterion (simple words)

- A system is stable if the natural response approaches zero as time approaches infinity.
- A system is unstable if the natural response approaches infinity as time approaches infinity.
- A system is marginally stable if the natural response neither decays nor grows but remains constant or oscillates.
- A system is stable if every bounded input yields a bounded output.
- A system is unstable if any bounded input yields an unbounded output.

# System Description

## Stability – Example

- Consider the following three **TFs**.
- The **first system** will have **zero input response**

$$y_{zi1} = K_1 e^{-3t} + K_2 e^{-2t}$$

- $K_1$  and  $K_2$  are determined by initial conditions
- Zero-state impulse response** when input  $r(t)$  is a unit impulse function is;

$$y_{zs,impulse1}(t) = 2e^{-3t} - e^{-2t}$$

$$T_1(s) = \frac{(s+1)}{(s+2)(s+3)} = \frac{2}{(s+3)} - \frac{1}{(s+2)}$$

$$T_2(s) = \frac{(s-7)}{(s-2)(s+3)} = \frac{2}{(s+3)} - \frac{1}{(s-2)}$$

$$T_3(s) = \frac{2(s-2)}{(s-2)(s+3)} = \frac{2}{(s+3)} + \frac{0}{(s-2)}$$

# System Description

## Stability – Example

- Consider the following three *TFs*.
- For the **second system**

$$y_{zi2} = K_1 e^{-3t} + K_2 e^{2t}$$

$$y_{zs,impulse2}(t) = 2e^{-3t} - e^{2t}$$

- This is asymptotically unstable because  $e^{2t}$  will approach infinity and it is *BIBO* unstable due to the same term.

$$T_1(s) = \frac{(s+1)}{(s+2)(s+3)} = \frac{2}{(s+3)} - \frac{1}{(s+2)}$$

$$T_2(s) = \frac{(s-7)}{(s-2)(s+3)} = \frac{2}{(s+3)} - \frac{1}{(s-2)}$$

$$T_3(s) = \frac{2(s-2)}{(s-2)(s+3)} = \frac{2}{(s+3)} - \frac{0}{(s-2)}$$



# System Description

## Stability – Example

- Consider the following three *TFs*.
- For the **third system**

$$y_{zi3} = K_1 e^{-3t} + K_2 e^{2t}$$

$$y_{zs,impulse3}(t) = 2e^{-3t}$$

$$T_1(s) = \frac{(s+1)}{(s+2)(s+3)} = \frac{2}{(s+3)} - \frac{1}{(s+2)}$$

$$T_2(s) = \frac{(s-7)}{(s-2)(s+3)} = \frac{2}{(s+3)} - \frac{1}{(s-2)}$$

$$T_3(s) = \frac{2(s-2)}{(s-2)(s+3)} = \frac{2}{(s+3)} - \frac{0}{(s-2)}$$

- This is asymptotically unstable because  $e^{2t}$  term, but *BIBO* stable due to the cancellation of *RHP* zero also located at +2

# System Description

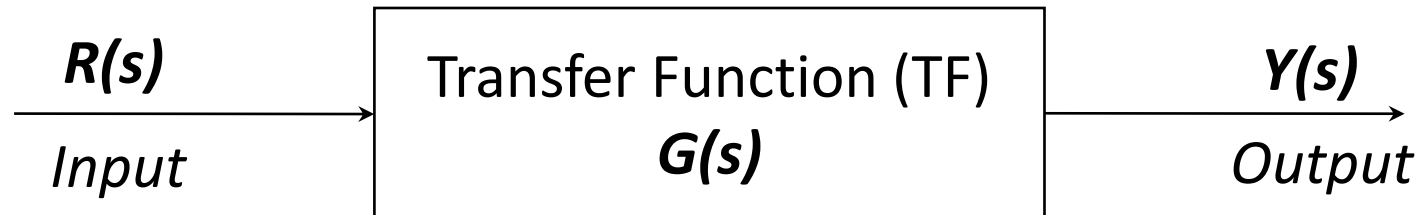
## Stability Description – Example

- The most conservative approach is to demand **asymptotic stability**.
- Suppose the **zero-state response** of each of the three systems provides the **external position of an aircraft** while the zero-input response provides the **response of some internal device**.
- The first system has an acceptable response for both the aircraft and the device.
- While the second system has an unacceptable response for both the aircraft and the device.
- Although the zero-state response of the third system is stable, the electrical device is probably destroyed.
- This is unacceptable since this device may play a vital role in the future activity of the aircraft.
- Only **asymptotic stability** is **accepted**

# System Description

## Block Diagram - Uses

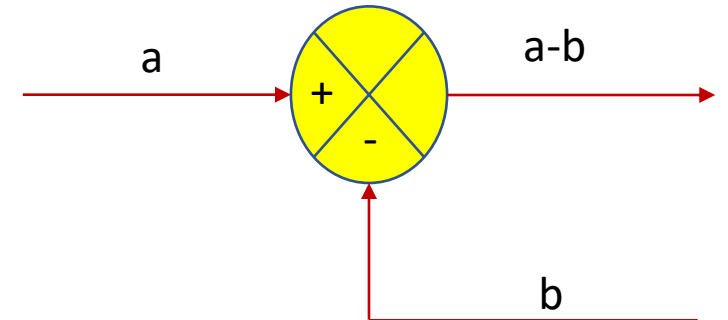
- Block diagrams are used to model all types of systems because of their simplicity and versatility.
- A block diagram can be used to describe the **composition** and **interconnection** of a system.
- Also, it can be used together with transfer functions, to describe the **cause** and **effect** relationships throughout the system.
- A block diagram consists of unidirectional, operational blocks that represent the **TF**.



# System Description

## Block Diagram - Uses

- Many systems are composed of **multiple subsystems**.
- When multiple subsystems are interconnected, a few more **schematic elements** must be added to the block diagram. Like **summing junctions** and **pickoff points**.
- **Summing Point:** A **circle with a cross** indicates a summing operation. The plus + and minus - sign at each arrowhead indicates whether the signal is to be added or to be subtracted.
- **Junction/Pick-off Point:** A point from which the signal from a block goes **concurrently** to other blocks or summing points.

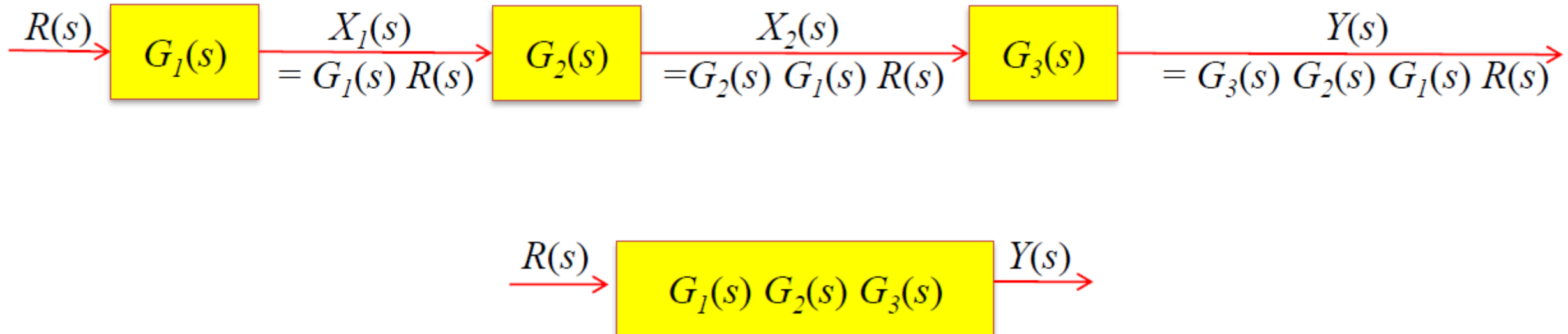


# System Description

## Block Diagram – Common Forms

- There are three basic forms, by which the subsystems are connected together.
  - **Cascade form**
  - Parallel form
  - Feedback form

### Cascade Form

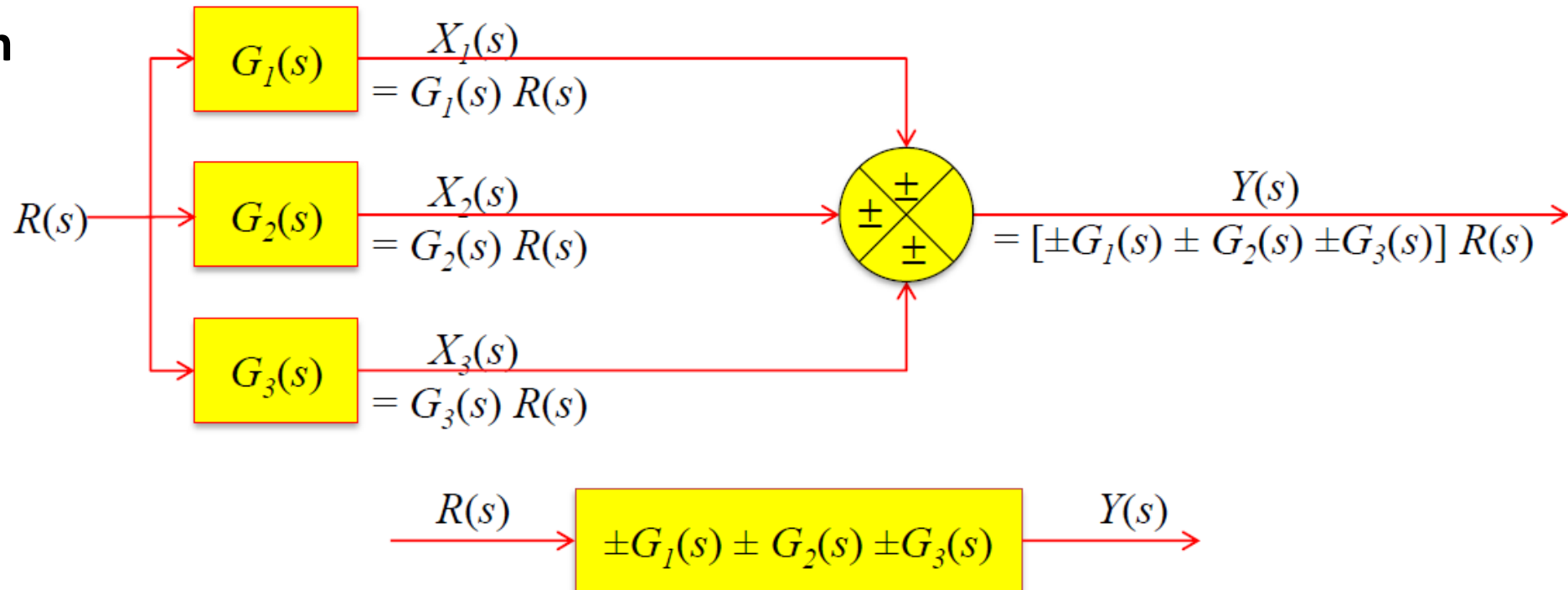


# System Description

## Block Diagram – Common Forms

- There are three basic common forms, by which the subsystems are connected together.
  - Cascade form
  - **Parallel form**
  - Feedback form

### Parallel Form

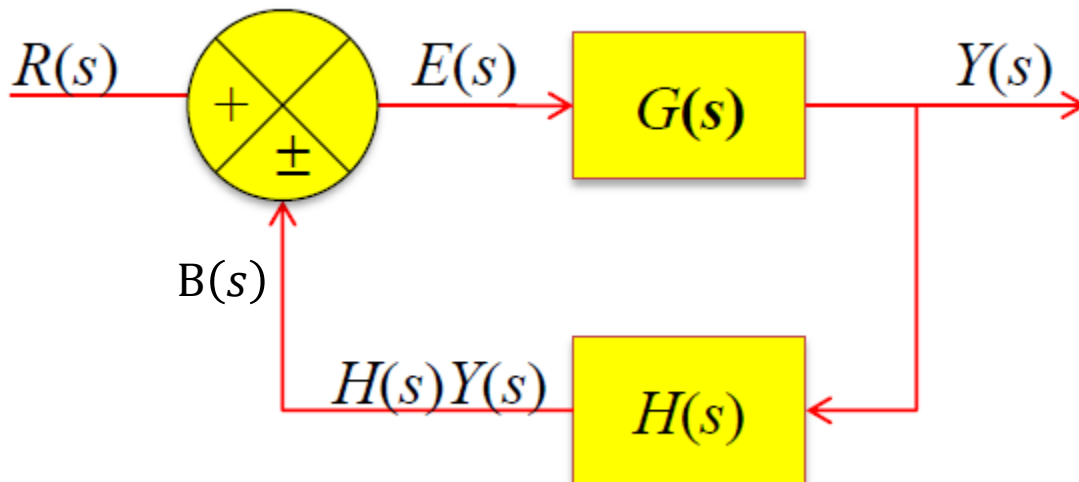


# System Description

## Block Diagram – Common Forms

- There are three basic common forms, by which the subsystems are connected together.
  - Cascade form
  - Parallel form
  - **Feedback form**

### Feedback Form



### Open-Loop Transfer Function

- The ratio of the **feedback signal**  $H(s)Y(s)$  to the **actuating error signal**  $E(s)$  is called the open-loop transfer function.

$$T(s) = \frac{B(s)}{E(s)} = G(s)H(s)$$

### Feed-forward Transfer Function

- The ratio of the **output**  $Y(s)$  to the **actuating error signal**  $E(s)$  is called the feedforward transfer function.

$$T(s) = \frac{Y(s)}{E(s)} = G(s)$$

- If the **feedback transfer function**  $H(s)$  is **unity**, then the **open-loop transfer function** and the **feedforward transfer function** are the **same**.

# System Description

## Block Diagram – Common Forms – Feedback

- The relationship between the signals is:

$$Y(s) = G(s)E(s)$$
$$E(s) = R(s) \pm H(s)Y(s)$$

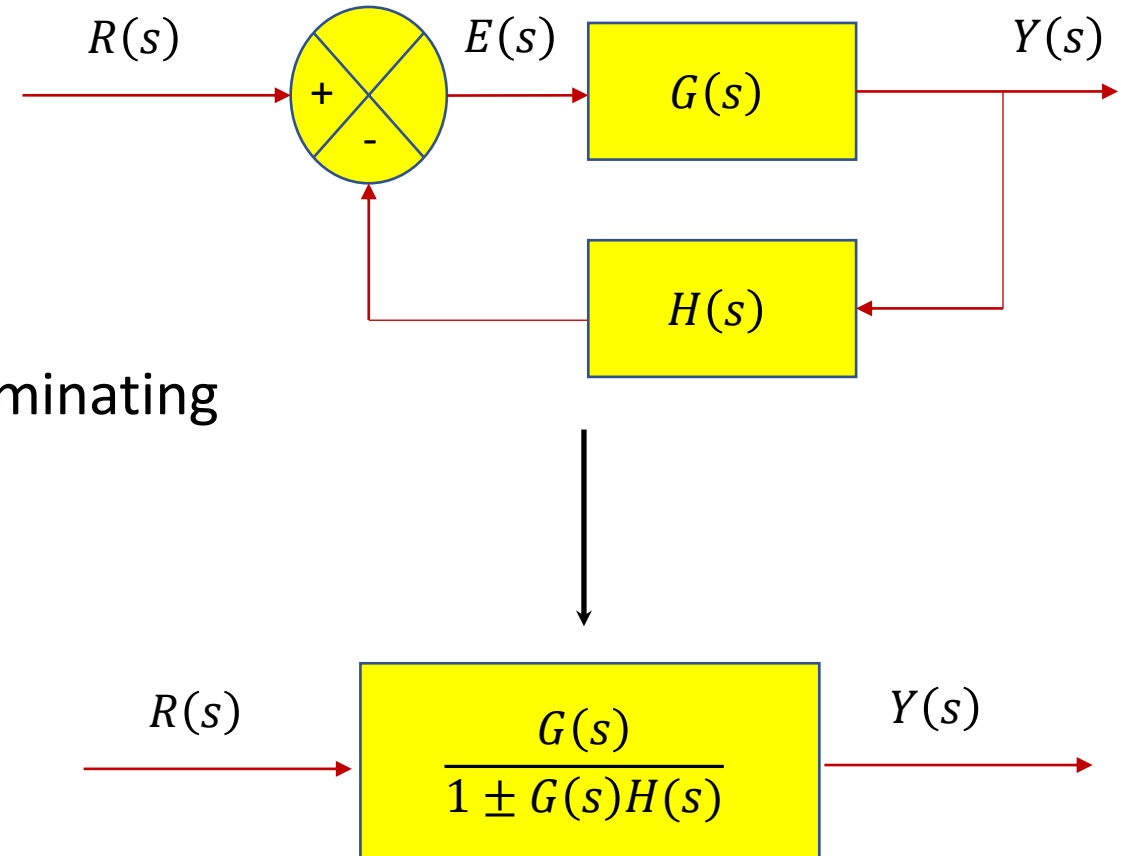
Solving for  $Y(s)$  in terms of  $R(s)$  and eliminating  $E(s)$ , we get.

$$Y(s) = G(s)[R(s) \pm H(s)Y(s)]$$
$$Y(s) = G(s)R(s) \pm G(s)H(s)Y(s)$$

which results in

$$T(s) = \frac{Y(s)}{R(s)} = \frac{G(s)}{1 \pm G(s)H(s)}$$

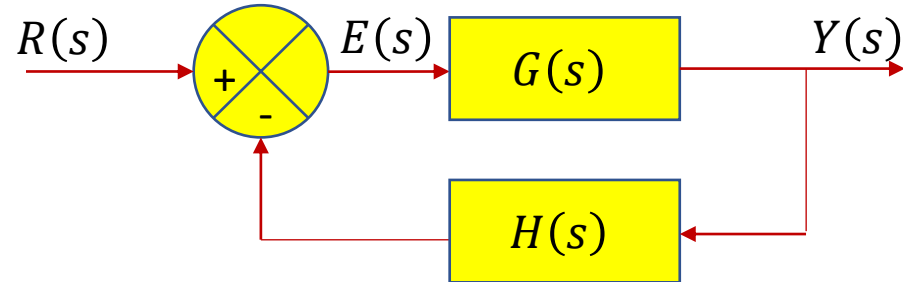
**Closed-loop** transfer function





# System Description

## Block Diagram – Common Forms – Feedback



- Fundamental to control engineering as it reveals the **effect** of applying **feedback** to a system.
- Practical meaning of two possible summation (positive and negative):

Lets imagine,

- $G(s)$  – the combination of cruise controller and dynamics of a vehicle.
- $H(s)$  – velocity measuring device.
- $R(s)$  – desired velocity
- $Y(s)$  – actual velocity
- Say, desired velocity = 65 km/h
- Actual velocity = 55 km/h

### Using negative summation:

- Error =  $(65-55)\text{km/h} = 10 \text{ km/h}$
- This would speed up the car by 10 km/h and would be a logical choice.

### Using positive summation

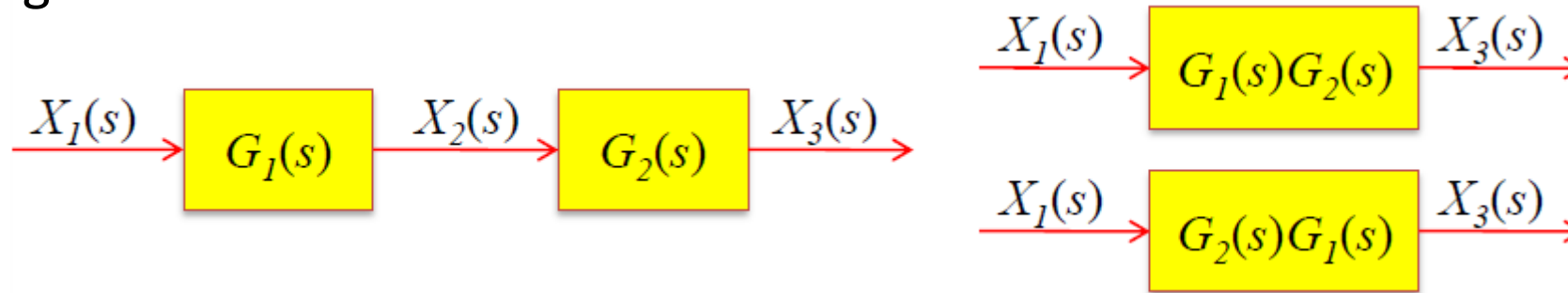
- Error =  $(65+55) = 110 \text{ km/h}$
- +’ve sign on summer introduces negative co-eff and results in RHP roots.
- This causes instability.

Therefore, negative feedback should be used.

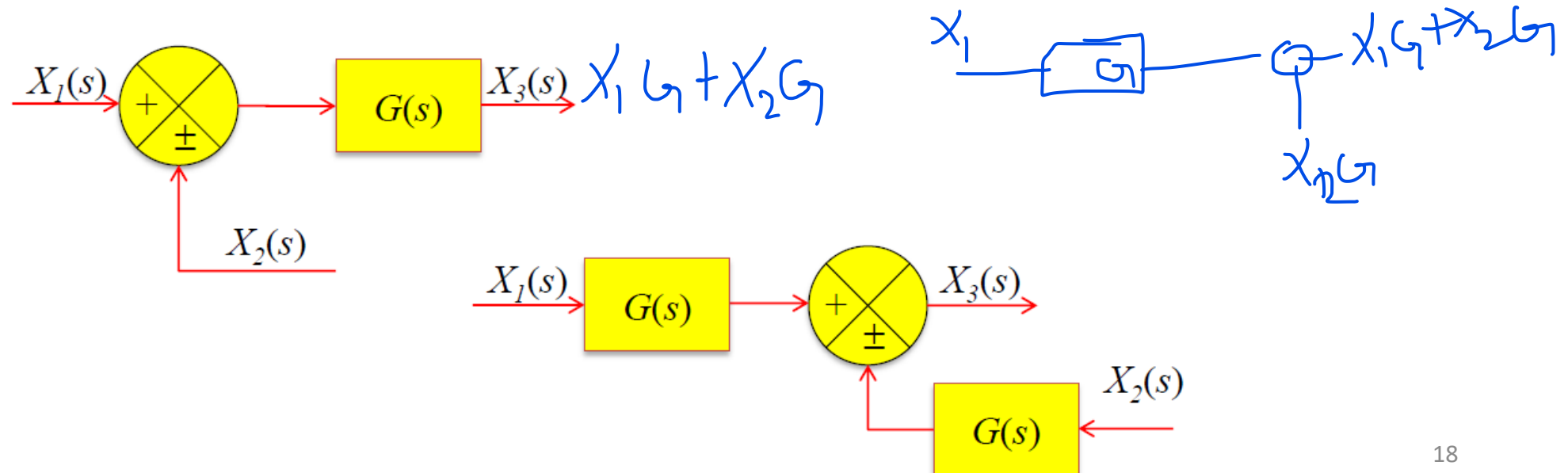
# System Description

## Block Diagram – Reductions (6)

1) Combining blocks in cascade



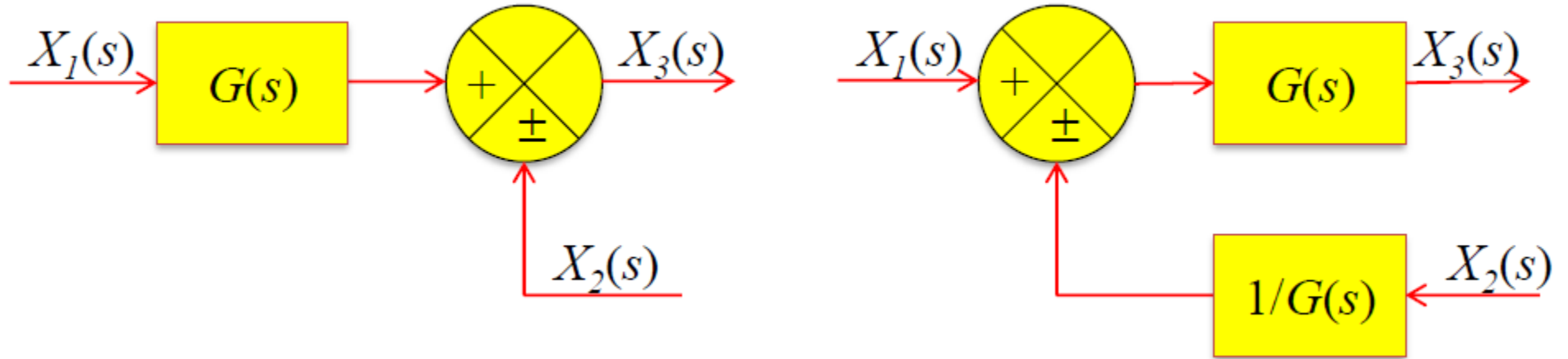
2) Moving a summing point forward



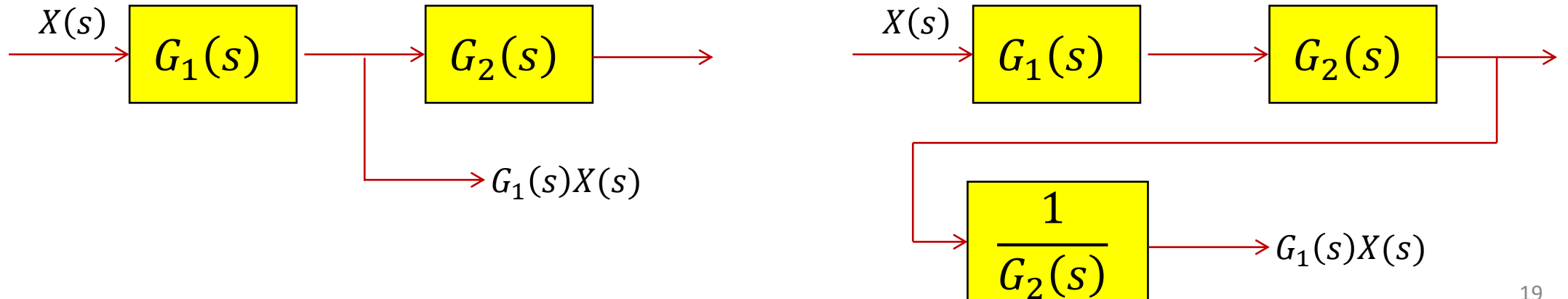
# System Description

## Block Diagram – Reductions (6)

3) Moving a summing point back



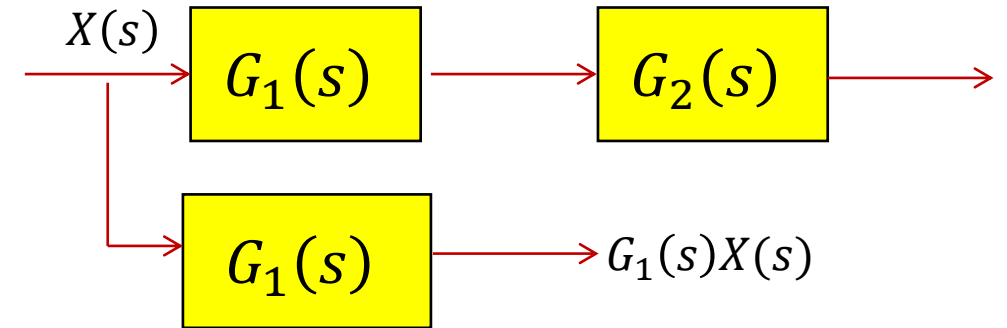
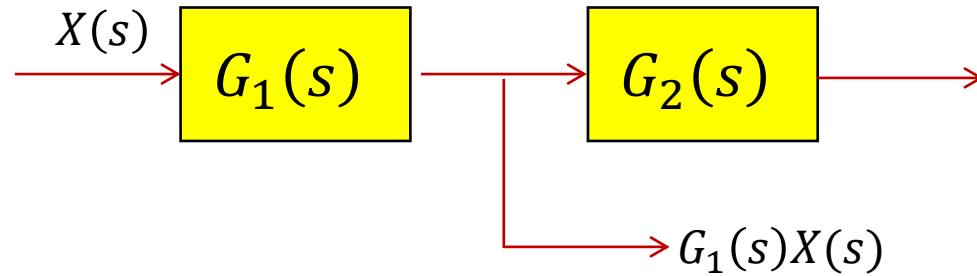
4) Moving a pickoff point forward



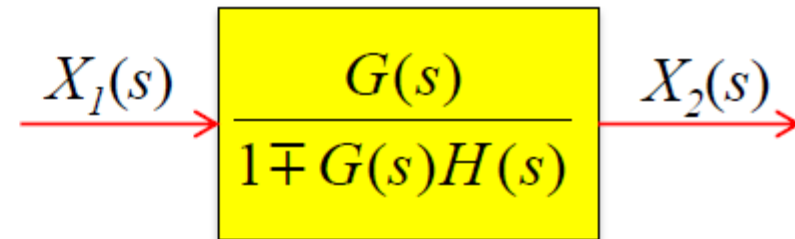
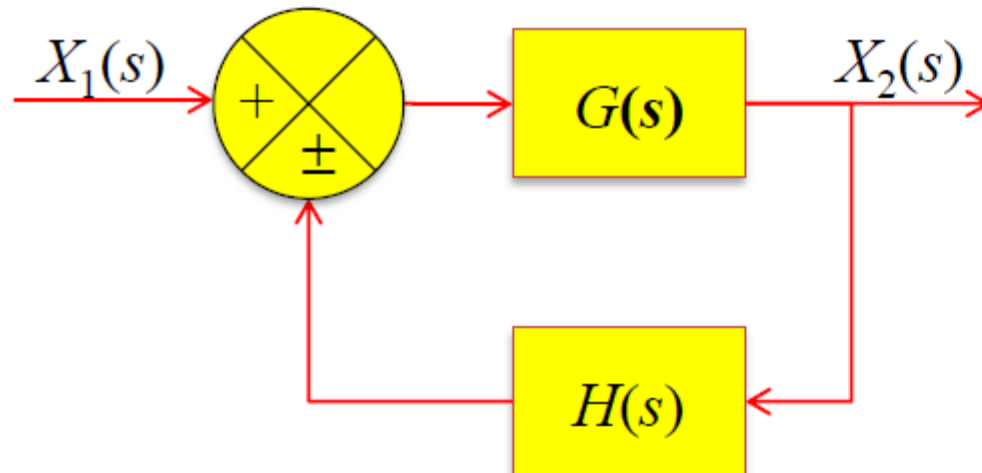
# System Description

## Block Diagram – Reductions (6)

5) Moving a pickoff point back

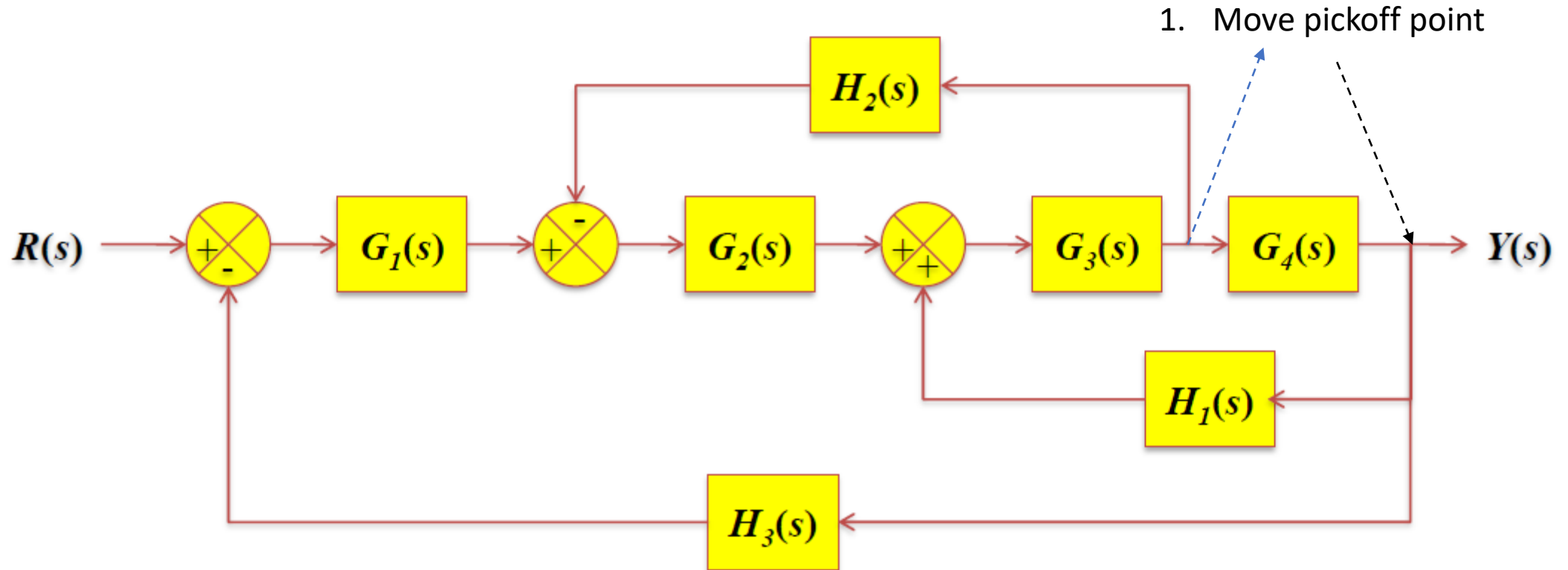


6) Eliminating a feedback loop



# System Description

## Block Diagram Reduction – Example

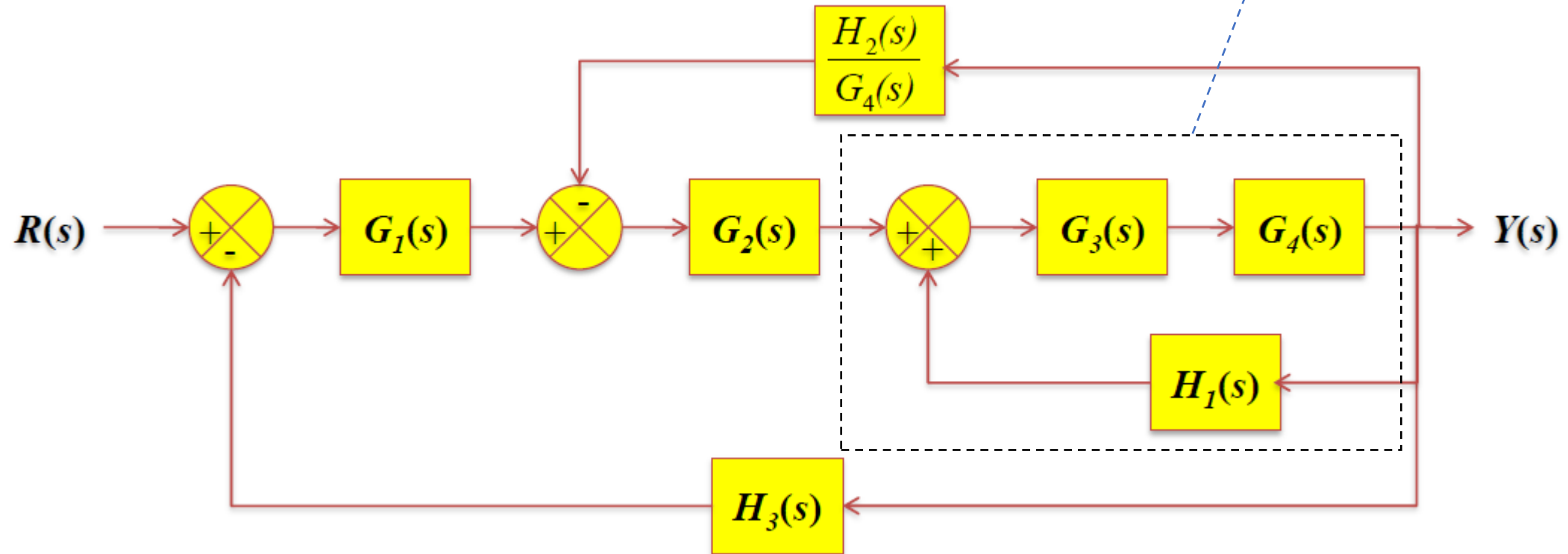


# System Description

## Block Diagram Reduction – Example

- Step-by-step solution

2. Calculate closed loop transfer function

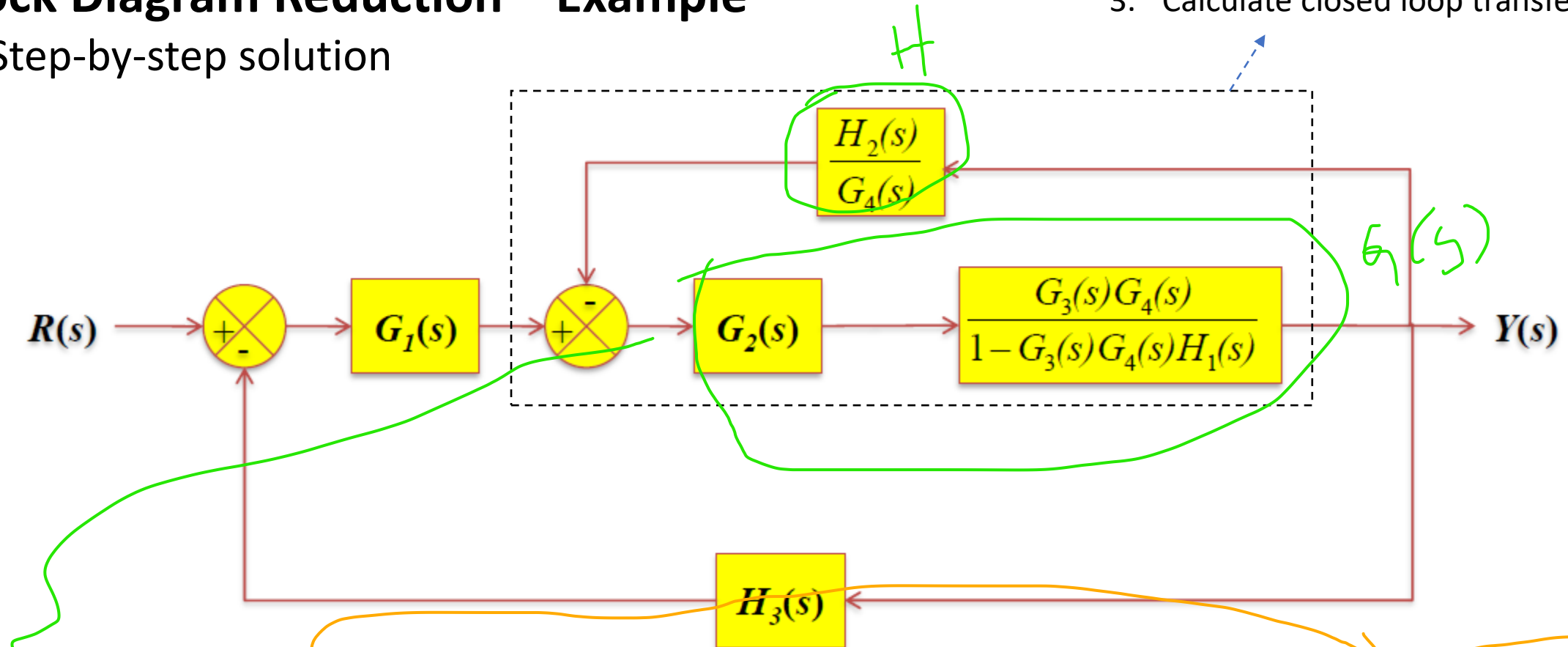


# System Description

## Block Diagram Reduction – Example

- Step-by-step solution

3. Calculate closed loop transfer function



Handwritten green note:

$$\frac{G}{1 - G H}$$

Handwritten orange note showing the derivation of the closed-loop transfer function:

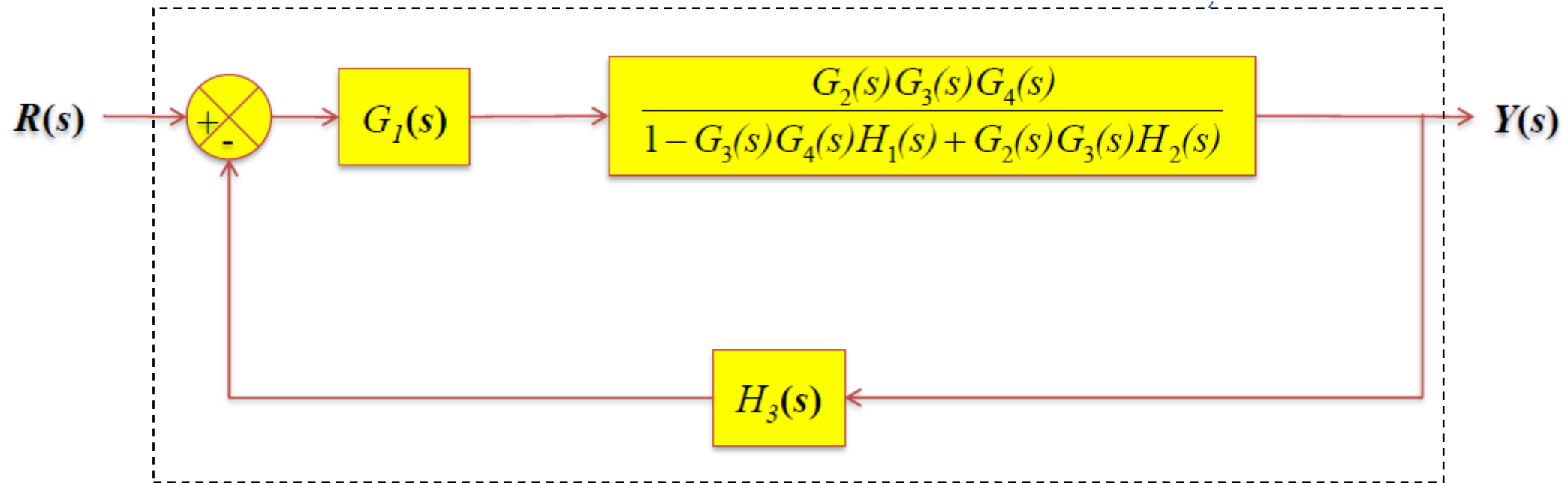
$$\frac{G_{1234}}{1 - G_{134}H_1} \times \frac{H_2}{G_4} \Rightarrow 1 - G_1 H = 1 - \frac{G_{123}H_2}{1 - G_{134}H_1} \rightarrow \frac{1 - G_{134}H_1 - G_{123}H_2}{1 - G_{134}H_1}$$

# System Description

## Block Diagram Reduction – Example

- Step-by-step solution

3. Calculate closed loop transfer function

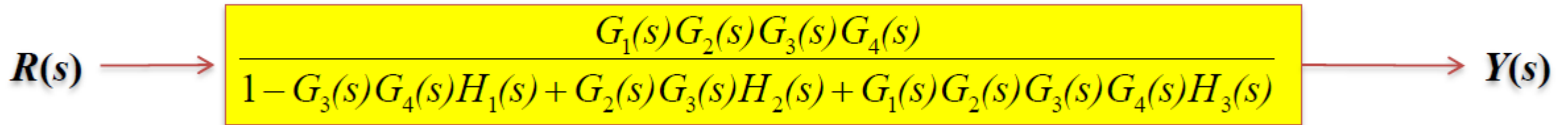




# System Description

## Block Diagram Reduction – Example

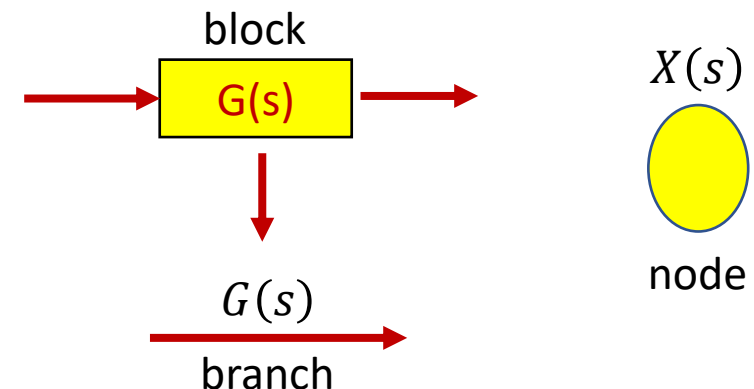
- Step-by-step solution
- **Answer**



# Signal Flow Graphs

## What is it?

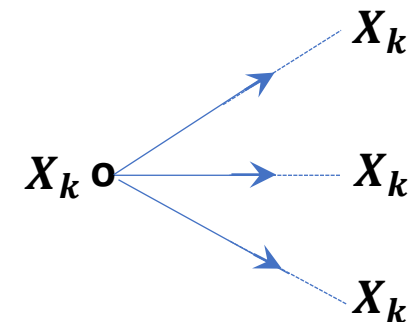
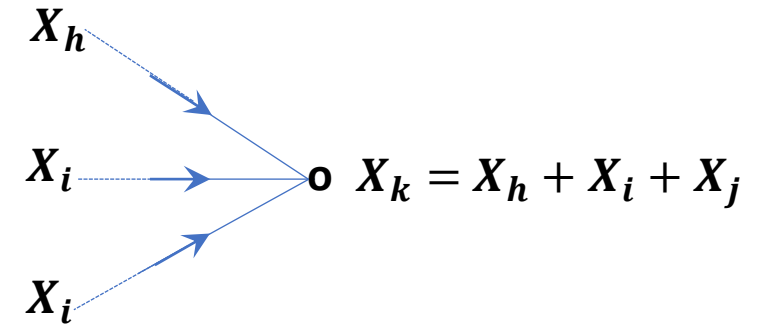
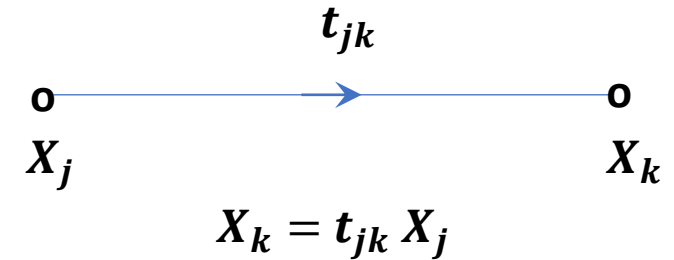
- A Signal Flow Graph SFG ) is a **special type of block diagram** and directed graph solution.
- It consists of **nodes** and **branches**. Its nodes are the **variables** of a set of linear algebraic relations.
- SFG can only represent **multiplications** and **additions**.
  - **Multiplications** are represented by the **weights of the branches**.
  - **Additions** are represented by **multiple branches going into one node**.
  - It has a one-to-one relationship with a system of linear equations and can also be used to represent the signal flow in a physical system; i.e., it can represent relations of **cause and effect**.
- **Consists of branches** (**represent system**)
- **Nodes** (**represents signals**)



# Signal Flow Graphs

## Basic Properties

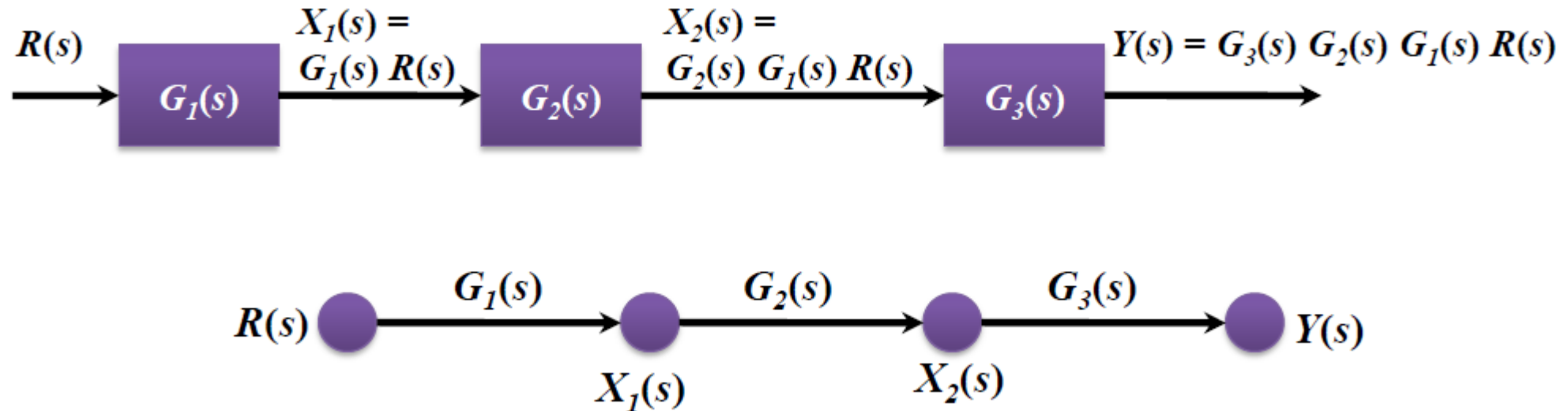
- A signal flows along a branch only in the direction defined by the arrow and is multiplied by the transmittance of that branch.
- A node signal is equal to the algebraic sum of all signals entering the pertinent node via the incoming branches.
- The signal at a node is applied to each outgoing branch that originates from that node.



# Signal Flow Graphs

## Conversion between Block Diagrams and SFG

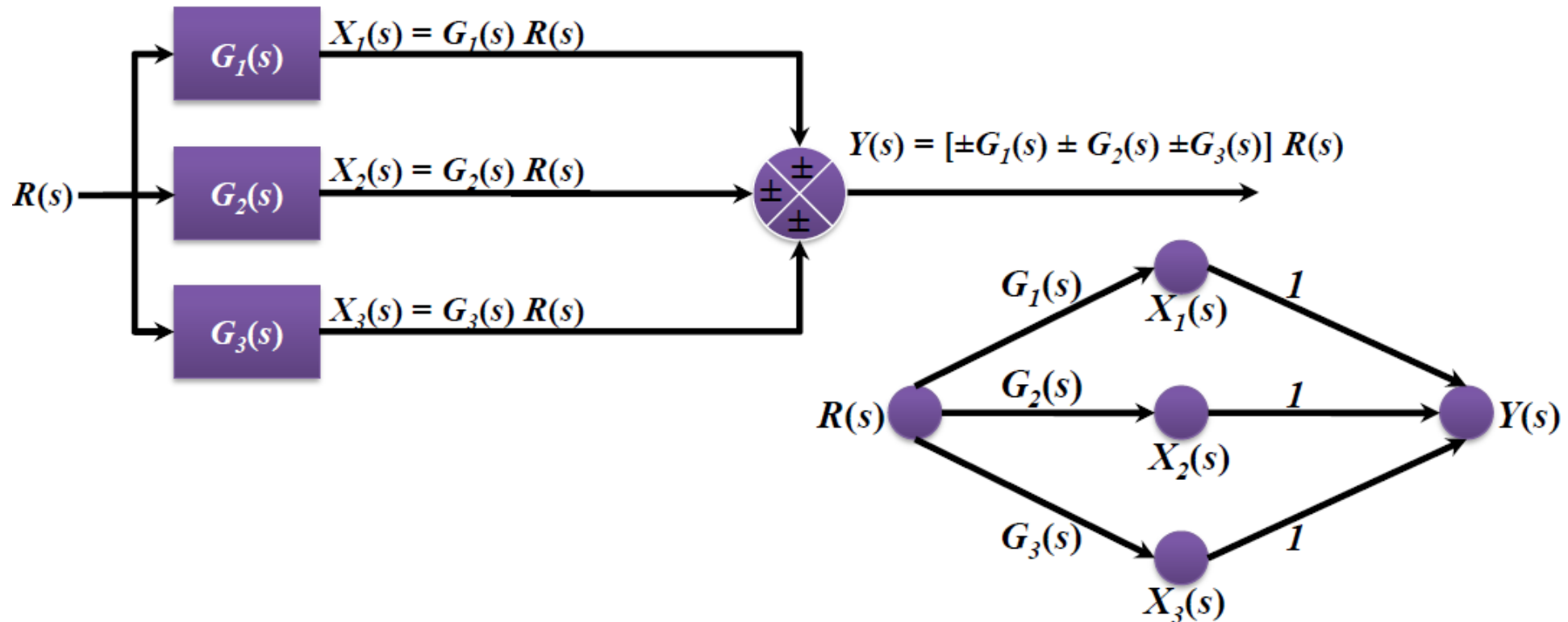
- We can convert the block diagrams in cascade, parallel, and feedback forms into signal flow diagrams.
  - **Cascade Form**



# Signal Flow Graphs

## Conversion between Block Diagrams and SFG

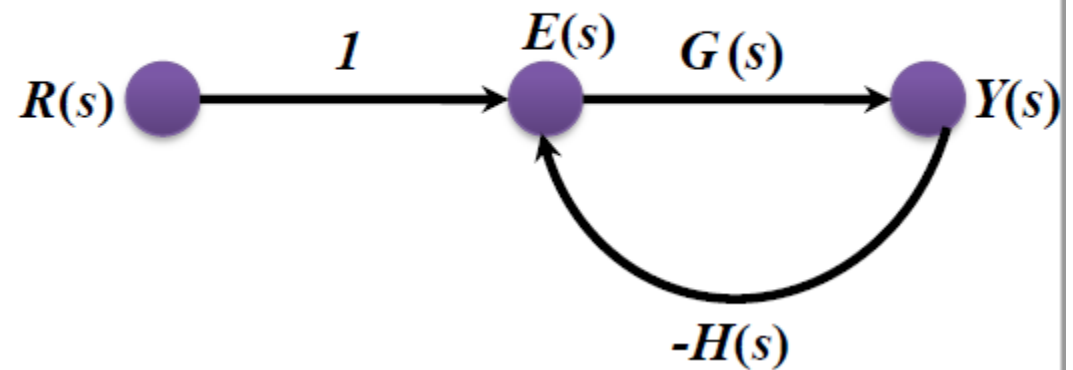
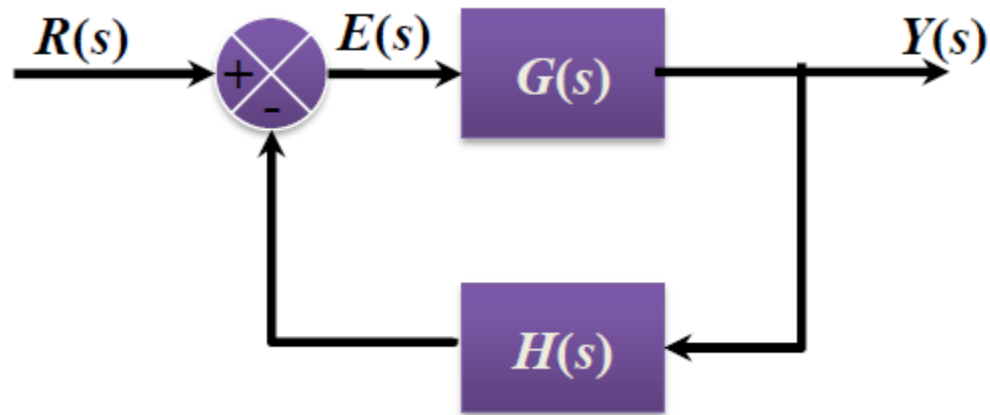
- We can convert the block diagrams in cascade, parallel, and feedback forms into signal flow diagrams.
  - Cascade Form
  - **Parallel Form**



# Signal Flow Graphs

## Conversion between Block Diagrams and SFG

- We can convert the block diagrams in cascade, parallel, and feedback forms into signal flow diagrams.
  - Cascade Form
  - Parallel Form
  - **Feedback Form**



# Signal Flow Graphs

## Conversion: Example

- Replace every **block** with a **branch**.
- Replace each combination of **summer** and **pick-off points** with a **node** in the signal flow graph (all sums are assumed to be +ve. For -ve sums add a -ve sign)
- Replace each solitary **pick-off point** (not connected to a summer) with a **label of the variable** assigned to the pick-off point.
- For each input show a node labeled with the variable assigned to the input.
- Add unity branches as needed or for clarity.

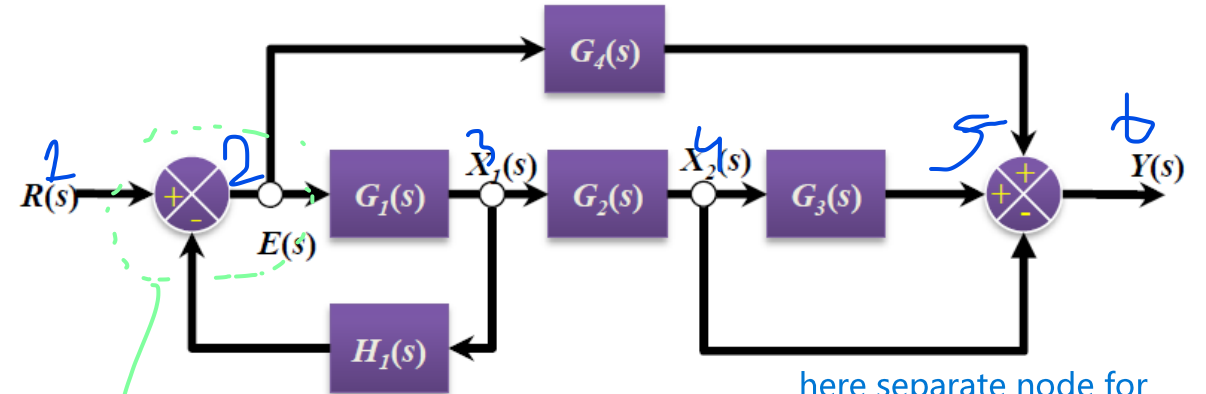
make nodes in place of

1. inputs and outputs

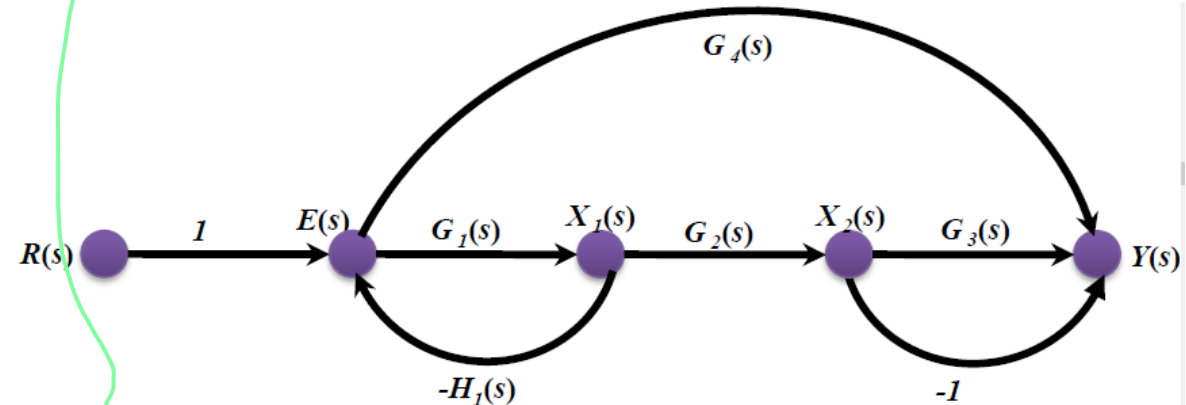
2. summing points (if branching is from the output of summer --this combined would be considered as a single node

3. branch points

4. in between cascaded blocks



here separate node for output isn't required as no feedback/branching is originating from it



# Signal Flow Graphs

## Mason's Gain Formula

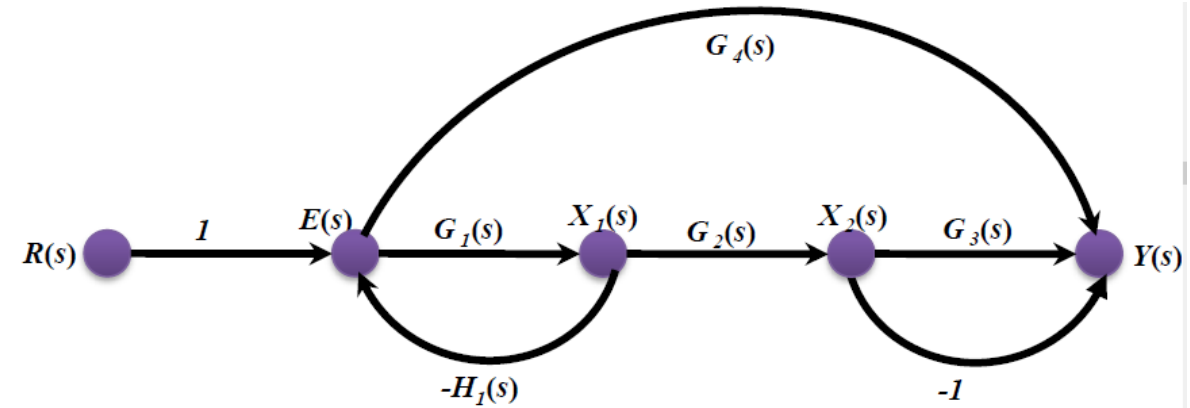
$$G(s) = \frac{Y(s)}{R(s)} = \frac{\sum_i^N p_i \Delta_i}{\Delta}$$

$N$  = total number of forward paths

$p_i$  = gain of the  $i$ th forward path

$\Delta = 1 - (\sum \text{all individual feedback loop gains including self-loops}) + (\sum \text{gain product of all possible combinations of two nontouching loops}) - (\sum \text{gain product of all possible combinations of three nontouching loops}) + \dots$

$\Delta_i$  = value of  $\Delta$  after eliminating all loops that touch its  $i$ th forward path

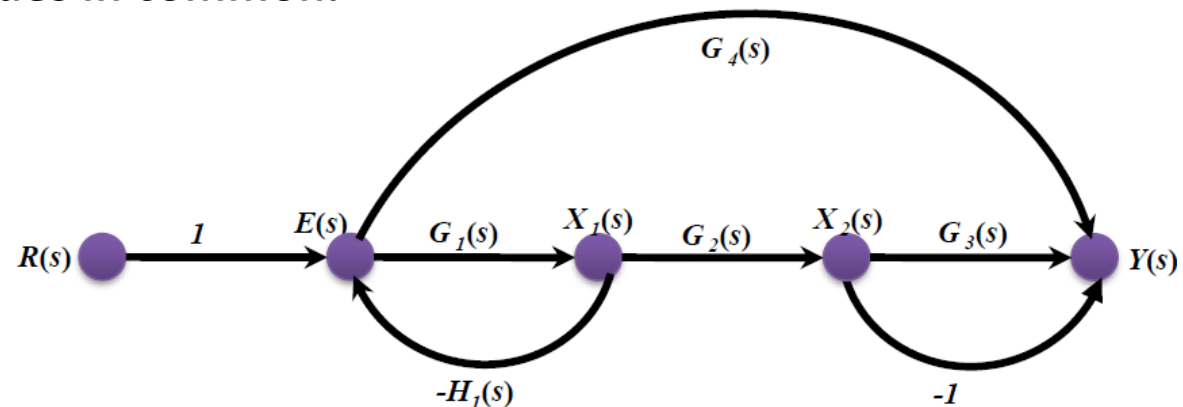


1. Path =  $p_i$
2. Loop
3. Touching loops
4. Determinant =  $\Delta$
5. Cofactor =  $\Delta_i$



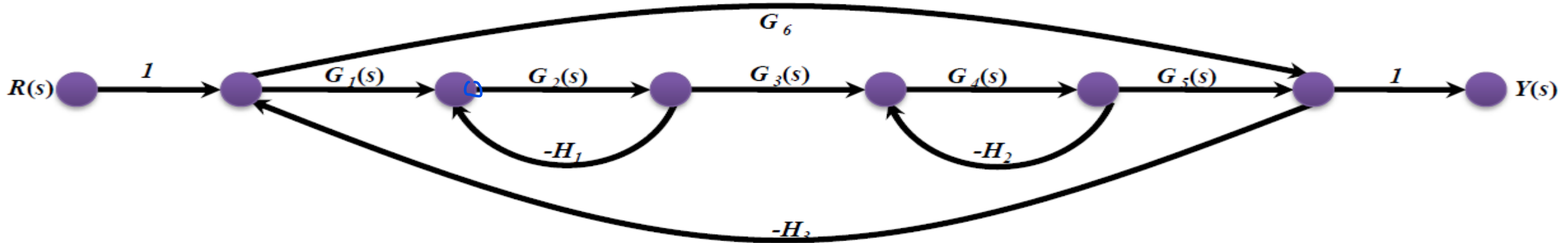
# Signal Flow Graphs

1. **Path** =  $p_i$  = A succession of branches, from input to output in the direction of arrows, that does not pass any node more than once.
2. **Path gain** = **Product of the transmittances** of the branches of the path. For the  $i$ th path, the path gain is denoted by  $p_i$ .
3. **Loop** = **A closed succession of branches**, in the direction of the arrows, , that does not pass any node more than once.
4. **Loop gain** = **Product of the transmittances** of the branches of the loop.
5. **Touching loops** = Loops with one or more **nodes in common**.
6. **Determinant** =  $\Delta$
4. **Cofactor** =  $\Delta_i$

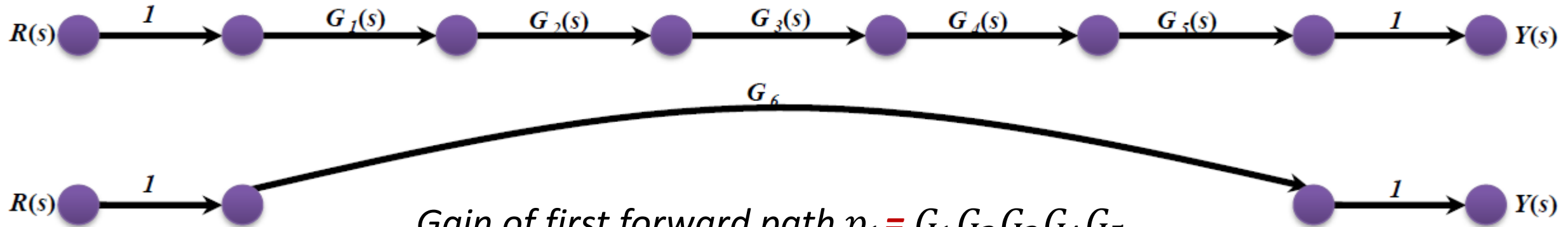


# Signal Flow Graphs

## Mason's Gain Formula: Example



**Step 1:** There are two forward paths as below so  **$N=2$**

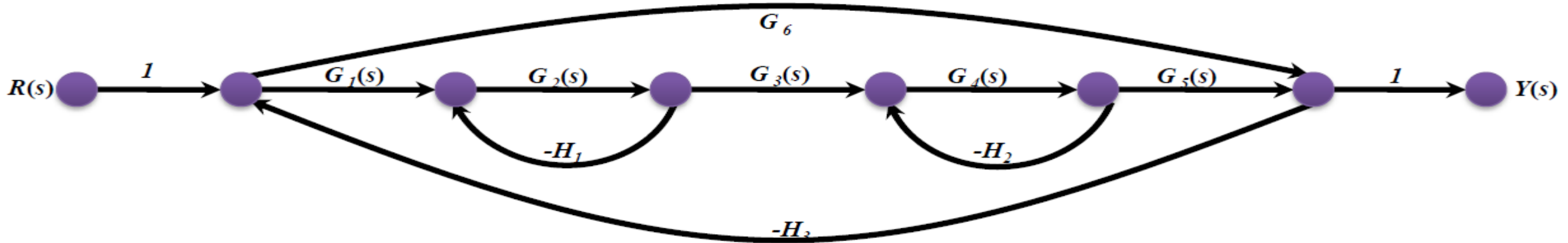


Gain of first forward path  $p_1 = G_1 G_2 G_3 G_4 G_5$

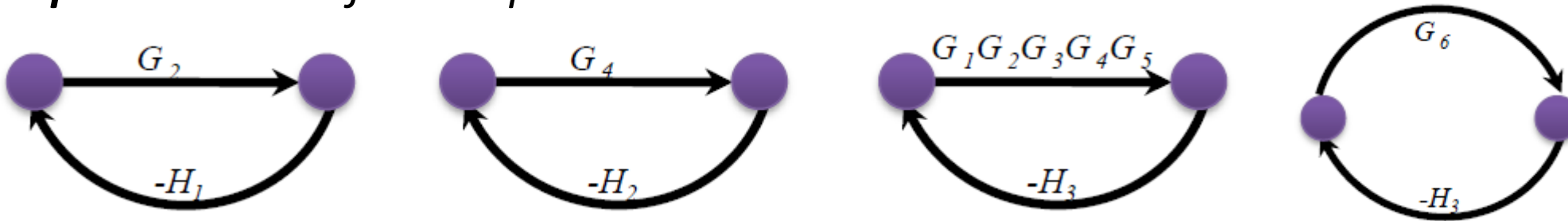
Gain of second forward path  $p_2 = G_6$

# Signal Flow Graphs

## Mason's Gain Formula: Example



**Step 2:** There are four loops



Loop gain of first loop ( $L_1$ ) =  $-G_2H_1$

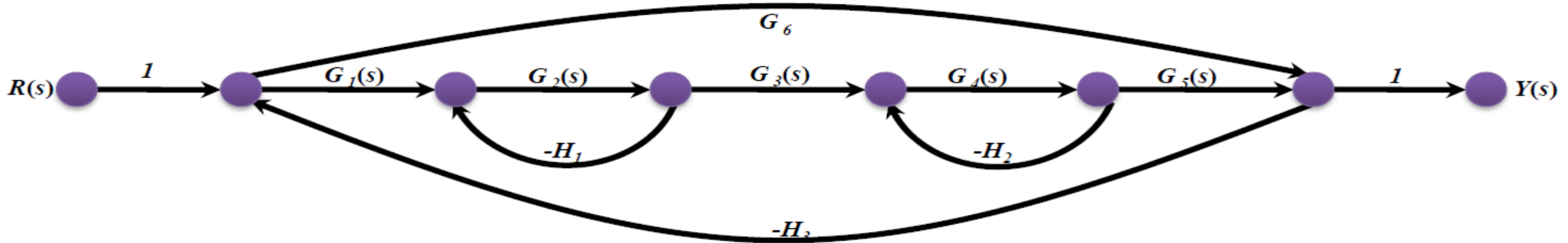
Loop gain of second loop ( $L_2$ ) =  $-G_4H_2$

Loop gain of third loop ( $L_3$ ) =  $-G_1G_2G_3G_4G_5H_3$

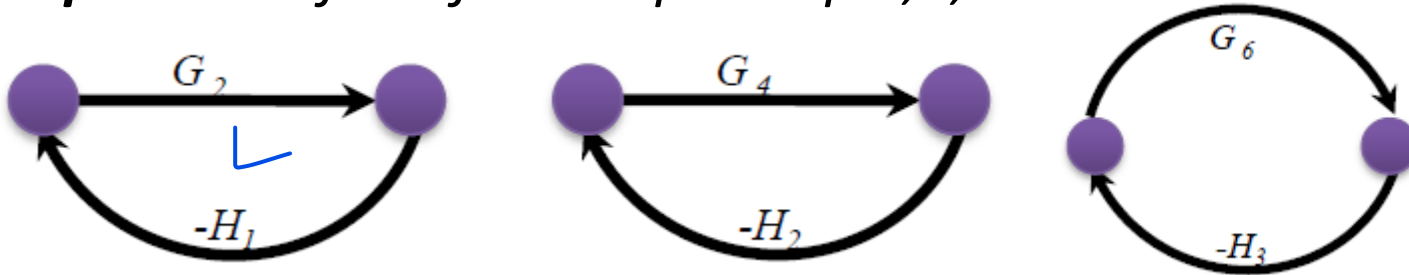
Loop gain of fourth loop ( $L_4$ ) =  $-G_6H_3$

# Signal Flow Graphs

## Mason's Gain Formula: Example



**Step 3:** Out of the four loops: loop 1, 2, and 4 are non touching



Combinations of two non touching loops are:

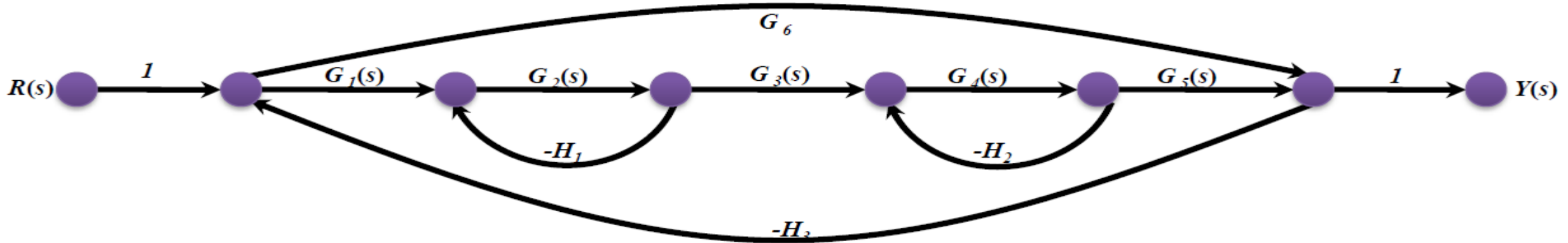
**Loop 1, Loop 2:** Loop gain ( $L_{12}$ ) =  $G_2G_4H_1H_2$

**Loop 1, Loop 4:** Loop gain ( $L_{14}$ ) =  $G_2G_6H_1H_3$

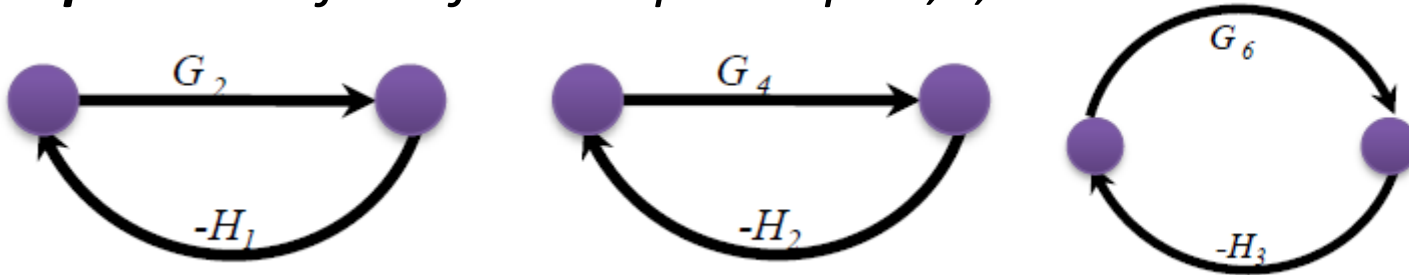
**Loop 2, Loop 4:** Loop gain ( $L_{24}$ ) =  $G_4G_6H_2H_3$

# Signal Flow Graphs

## Mason's Gain Formula: Example



**Step 4:** Out of the four loops: loops 1, 2, and 4 are non-touching



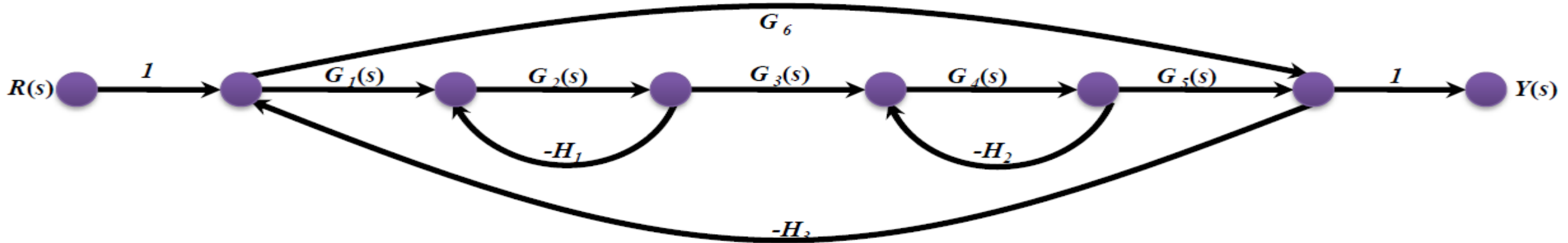
Combinations of three non touching loops are:

**Loop 1, Loop 2, Loop 4:** Loop gain ( $L_{124}$ ) =  $-G_2G_4G_6H_1H_2H_3$

**Step 5:** There are no higher order non-touching loops.

# Signal Flow Graphs

## Mason's Gain Formula: Example

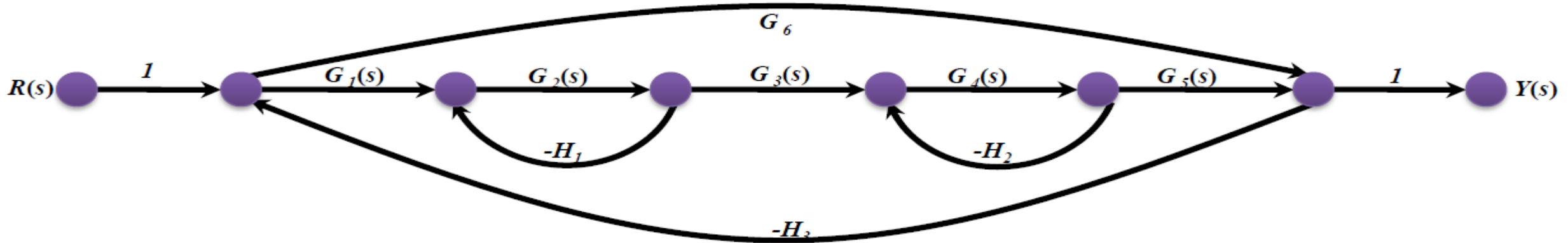


**Step 6:** Calculate  $\Delta$

$$\begin{aligned}
 \Delta &= \left[ 1 - (L_1 + L_2 + L_3 + L_4) \right] + \left[ (L_{12} + L_{14} + L_{24}) - (L_{124}) \right] \\
 &= 1 + (G_2H_1 + G_4H_2 + G_1G_2G_3G_4G_5H_3 + G_6H_3) \\
 &\quad + (G_2G_4H_1H_2 + G_2G_6H_1H_3 + G_4G_6H_2H_3) \\
 &\quad + (G_2G_4G_6H_1H_2H_3)
 \end{aligned}$$

# Signal Flow Graphs

## Mason's Gain Formula: Example



**Step 7:** Calculate  $\Delta_i$

We know that:  $\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + (L_{12} + L_{14} + L_{24}) - (L_{124})$

Considering path  $p_1$ , loops 1,2,3,4 touch it: eliminating all these from  $\Delta$

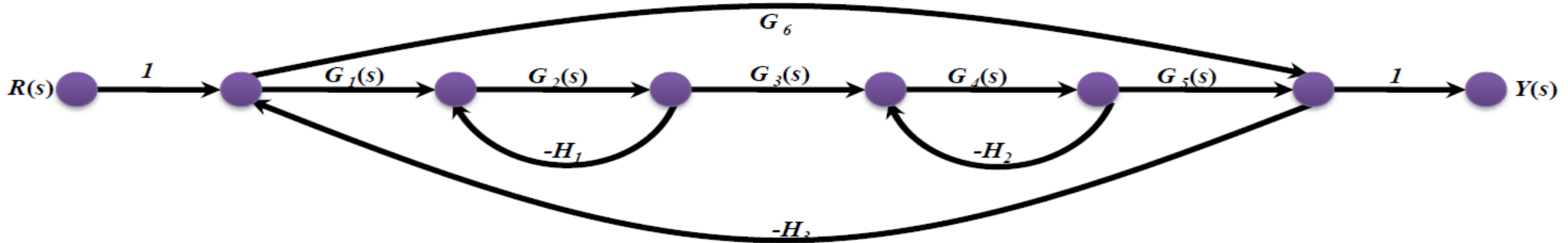
$$\Delta_1 = 1 - (0) = 1 \quad \text{1- (loops which aren't touching first forward path)}$$

Considering path  $p_2$ , loops 3,4 touch it: eliminating loops 3, 4 from  $\Delta$

$$\begin{aligned} \Delta_2 &= 1 - (L_1 + L_2) + (L_{12}) = 1 - (-G_2H_1 - G_4H_2) + G_2G_4H_1H_2 \\ &= 1 + G_2H_1 + G_4H_2 + G_2G_4H_1H_2 \end{aligned}$$

# Signal Flow Graphs

## Mason's Gain Formula: Example



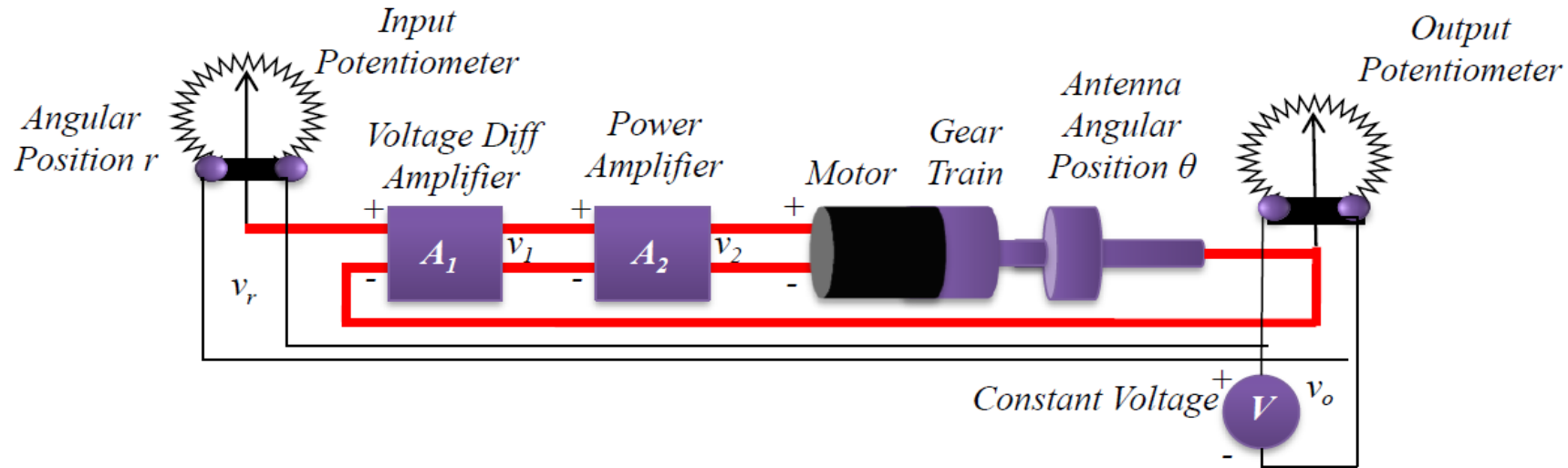
**Step 8: Transfer Function**

$$\begin{aligned}
 G(s) &= \frac{p_1 \Delta_1 + p_2 \Delta_2}{\Delta} \\
 &= \frac{G_1 G_2 G_3 G_4 G_5 + G_6 (1 + G_2 H_1 + G_4 H_2 + G_2 G_4 H_1 H_2)}{1 + (G_2 H_1 + G_4 H_2 + G_1 G_2 G_3 G_4 G_5 H_3 + G_6 H_3) \\
 &\quad + (G_2 G_4 H_1 H_2 + G_2 G_6 H_1 H_3 + G_4 G_6 H_2 H_3) \\
 &\quad + (G_2 G_4 G_6 H_1 H_2 H_3)}
 \end{aligned}$$



# Electromechanical Systems

## Example – A Position Servo a large video satellite antenna



- Output potentiometer measures the output shaft position and converts it to a potential voltage.

$$v_o = K_p \theta$$

$\theta = \text{output shaft angle}$

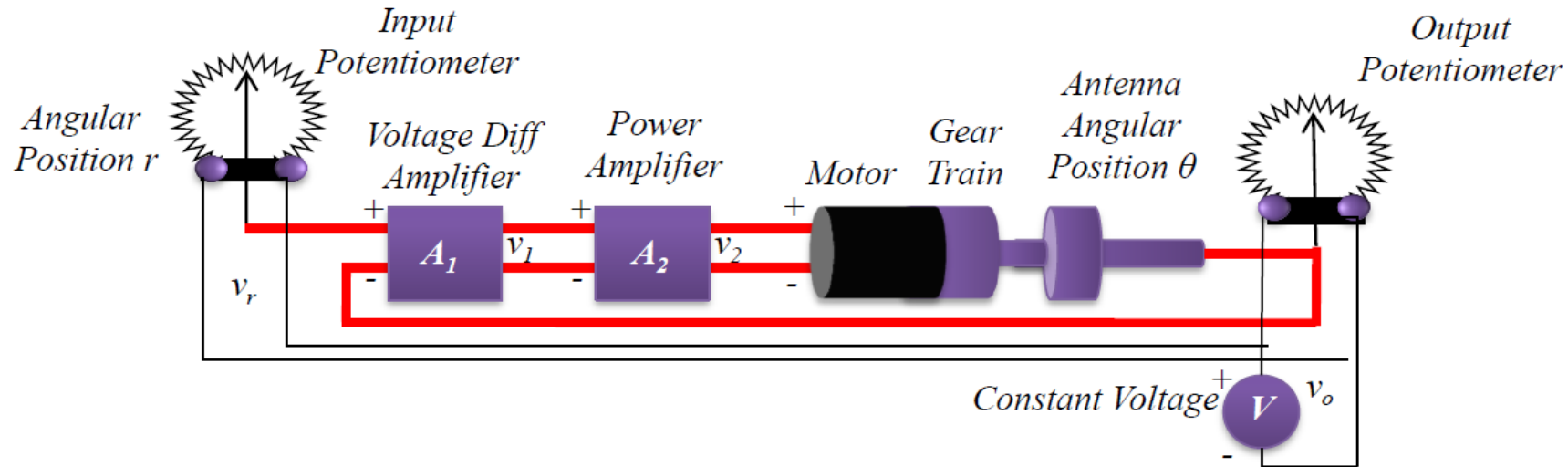
$$K_p = \text{proportionality const.} = \frac{V}{\theta_{\max}} \text{ volts/radian}$$

- The input potentiometer slider position is converted to a voltage in a similar manner:

$$v_r = K_p r$$

# Electromechanical Systems

## Example – A Position Servo a large video satellite antenna



- Difference between the two potentiometer signals is then amplified with gain  $A_1$   

$$v_1 = A_1(v_r - v_0) = A_1K_p(r - \theta) \quad \text{where, } v_1 = \text{error voltage output}$$
- This voltage is then further amplified with gain  $A_2$  and applied to the motor terminals:  

$$v_2 = A_2v_1 = A_2A_1K_p(r - \theta) \quad \text{where, } v_2 = \text{motor drive voltage}$$
- The second amplifier is the power amplifier capable of providing the electrical power needed to drive the motor.
- The motor is coupled to the antenna with a gear train ratio of  

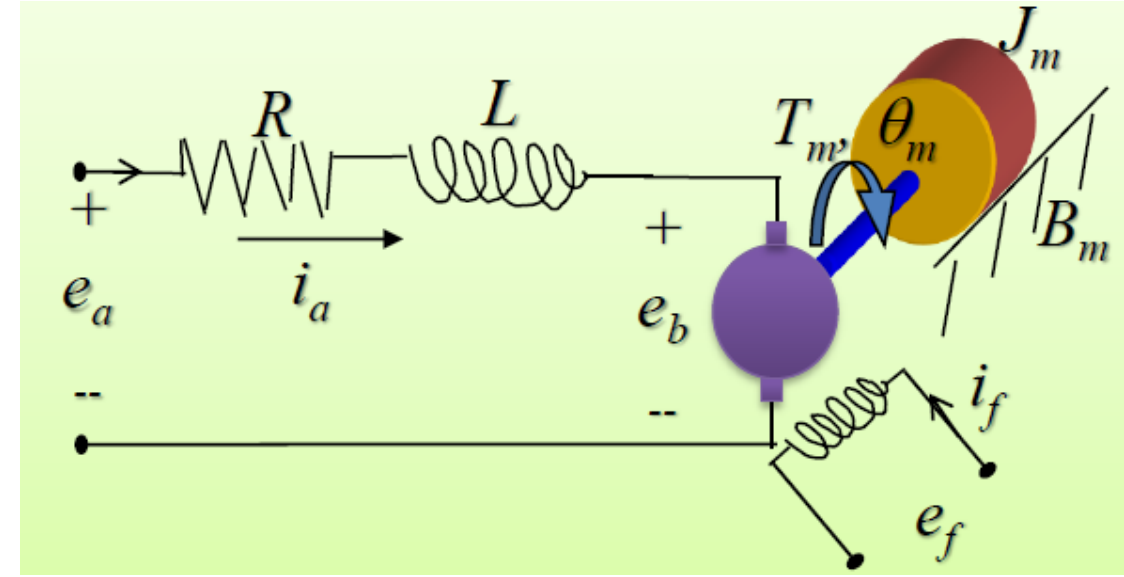
$$\theta = \frac{N_1}{N_2} \theta_m \quad \text{where } \theta_m \text{ is the motor shaft angle.}$$

# Electromechanical Systems

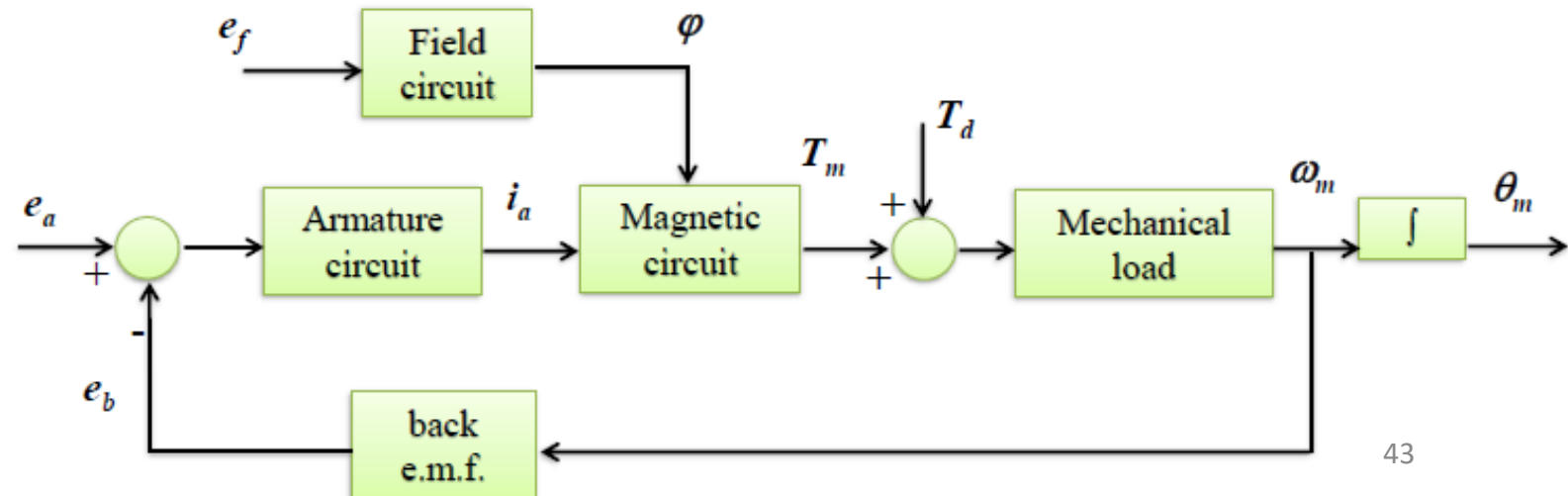
## Example – A Position Servo a large video satellite antenna

### DC Servo Motor – Modeling

- The **armature current** depends upon the applied voltage and the back emf.
- The **electromagnetic torque** is produced by the interaction of the armature current and the field current.
- The **electromagnetic torque** minus disturbance or **load torque** drives the inertial load.
- The functional block diagram of a DC motor.



$$TF = \frac{\theta_m(s)}{E_a(s)}$$



# Electromechanical Systems

## Example – A Position Servo a large video satellite antenna

### DC Servo Motor – Modeling

- The relationship between the armature current  $i_a(t)$ , the applied armature voltage  $e_a(t)$ , and the back emf  $e_b(t)$ , is found by applying KVL and then taking Laplace transform:

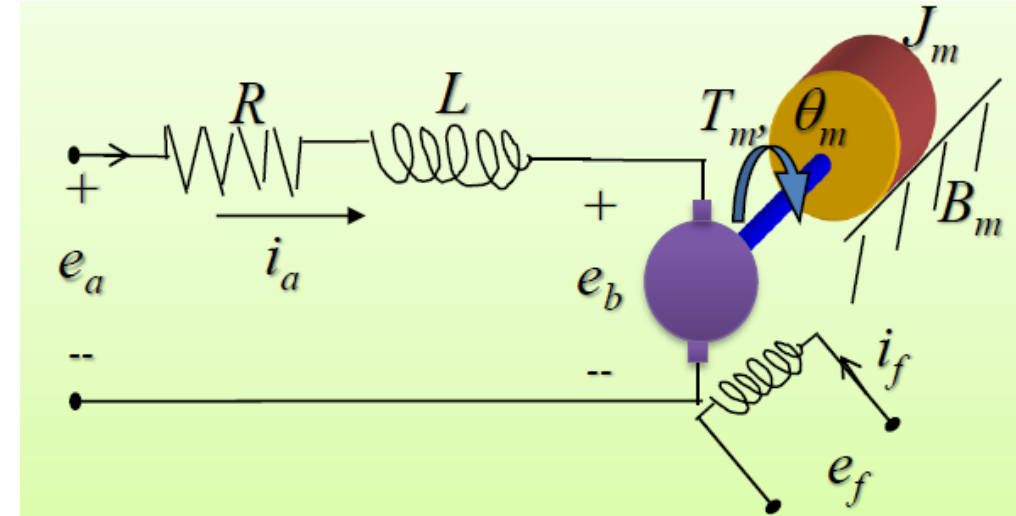
$$E_a(s) = R_a I_a(s) + L_a s I_a(s) + E_b(s) \dots\dots (a)$$

- The **back emf**  $e_b(t)$  is directly proportional to the speed of the motor and it can be written in Laplace domain as:

$$E_b(s) = K_b s \theta_m(s) \dots\dots\dots (b)$$

- The **torque developed by the motor (armature torque)** is proportional to the armature current; thus,

$$T_m(s) = K_i I_a(s) \rightarrow I_a(s) = \frac{T_m}{K_i} \dots\dots\dots (c) \quad \longrightarrow$$



- ✓ The **armature torque** is directly proportional to the product of the **flux** and the **armature current**:

$$T_m \propto \Phi I_a \rightarrow T_m = K_m \Phi I_a$$

$$\Phi = K_f i_f \rightarrow \text{field flux}$$

$$T_m = K_m K_f I_f i_a = K_i i_a \rightarrow \text{for const. field current}$$

$$K_i = \text{motor torque constant}$$

# Electromechanical Systems

## Example – A Position Servo a large video satellite antenna DC Servo Motor – Modeling

- Substituting *(b)* and *(c)* in *(a)*;

$$E_a(s) = (R_a + L_a s) \frac{T_m}{K_i} + K_b s \theta_m(s) \dots\dots (d)$$

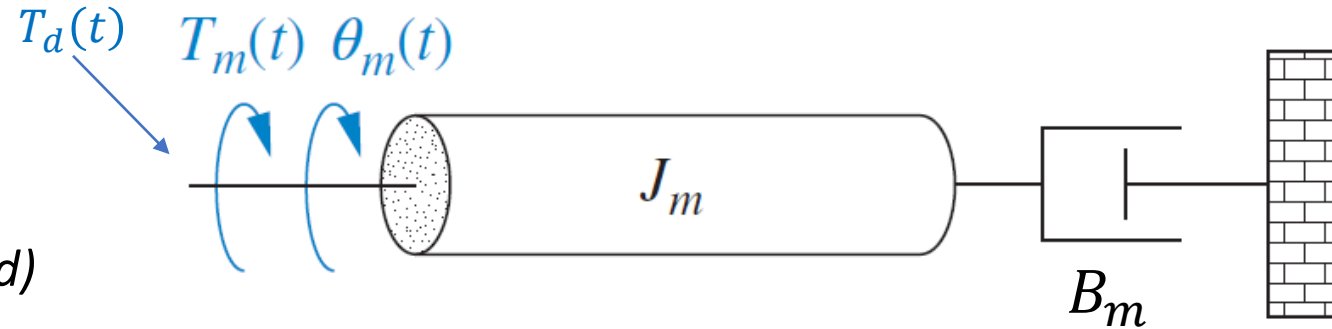
- The **mechanical load** on the motor can be modeled in Laplace domain as:

$$T_m(s) + T_d(s) = (J_m s^2 + B_m s) \theta_m(s) \dots\dots\dots (e)$$

*Substituting (e) in (d) and setting  $T_d(s) = 0$*

$$E_a(s) = \frac{(R_a + L_a s)(J_m s^2 + B_m s) \theta_m(s)}{K_i} + K_b s \theta_m(s)$$

$$TF = \frac{\theta_m(s)}{E_a(s)} = \frac{K_i}{\underbrace{s[L_a J_m s^2 + (R_a J_m + L_a B_m)s + (R_a B_m + K_b K_i)]}_{G(s)}}$$

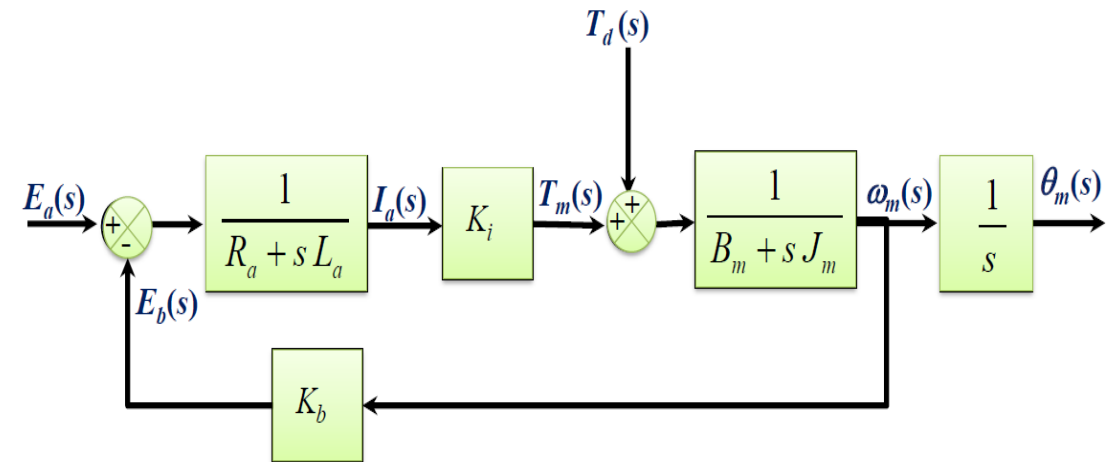
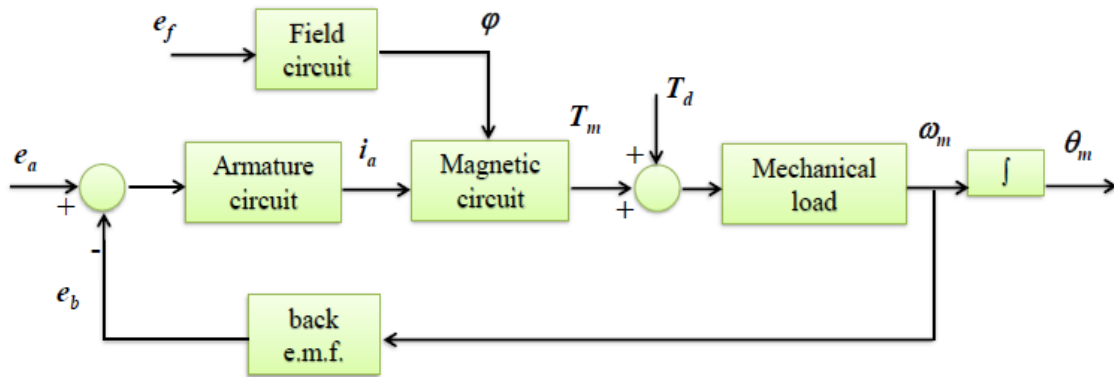


Recalling the transfer function:  $TF = \frac{\theta_m(s)}{E_a(s)}$



# Electromechanical Systems

## Example – A Position Servo a large video satellite antenna DC Servo Motor – Modeling



In the text book  $K_i = K_T$ ,  $K_b = K_v$ , and  $E_a = V_2$  which means:

$$\frac{\theta_m(s)}{V_2(s)} = \frac{K_T}{s[L_a J_m s^2 + (R_a J_m + L_a B_m)s + (R_a B_m + K_v K_T)]}$$

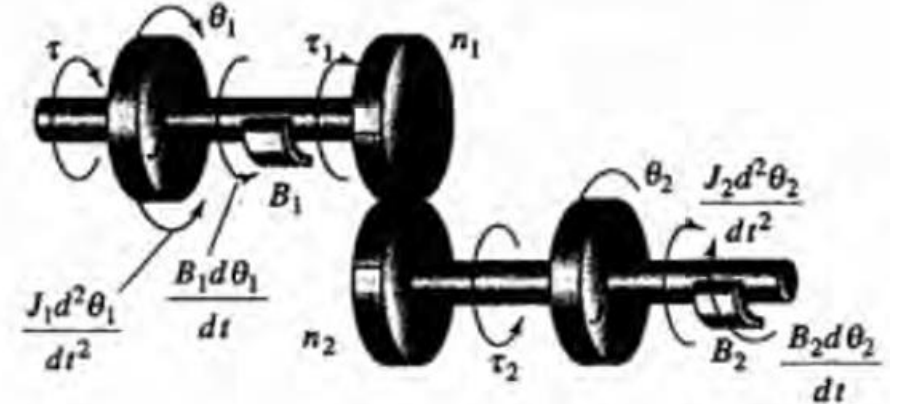
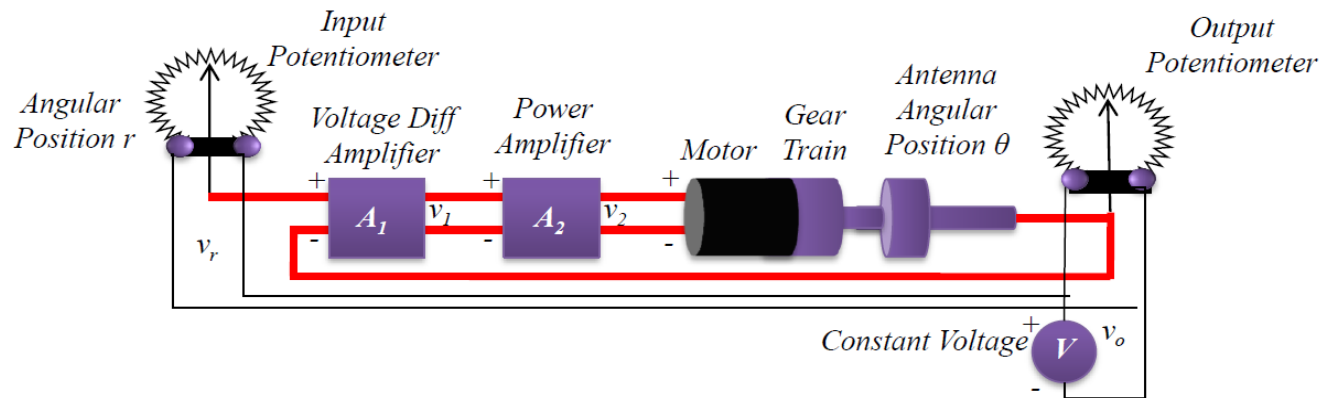
Neglecting the terms  $L_a$ ,  $B_m$  and dividing by  $K_v K_T$

$$\frac{\theta_m(s)}{V_2(s)} = \frac{1/K_v}{s[1 + (\frac{R_a J_m}{K_v K_T})]}$$

- ❖  $T_m = K_m \phi I_a$   
 $K_m = \text{motor torque constant}$
- ❖  $v_m = K_v \omega_m$   
 $K_v = \text{motor voltage constant}$
- ❖  $K_m = 1/K_v$

# Electromechanical Systems

## Example – A Position Servo a large video satellite antenna



- “ $\theta$ ” is the angular position of the antenna with the moment of inertia “ $J$ ”.  $N_1 \ll N_2$ , since the high-speed shaft of the motor must drive the antenna at low speed and high torque.

- $J_1 s^2 \theta_1(s) + B_1 s \theta_1(s) + \left(\frac{n_1}{n_2}\right)^2 J_2 s^2 \theta_1(s) + \left(\frac{n_1}{n_2}\right)^2 B_1 s \theta_1(s)$  ←

- $$\frac{\theta(s)}{V_2(s)} = \frac{K_m \left(\frac{N_1}{N_2}\right)}{s(1 + (R_a/K_v K_T [J_m + \left(\frac{N_1}{N_2}\right)^2 J_L]) s)}$$

$$\begin{aligned} J_1 &= J_m \\ J_2 &= J_L \end{aligned}$$

$$\begin{aligned} \tau &= \tau_1 + J_1 \frac{d^2 \theta_1}{dt^2} + B_1 \frac{d\theta_1}{dt} \\ \tau_2 &= J_2 \frac{d^2 \theta_2}{dt^2} + B_2 \frac{d\theta_2}{dt} \\ \tau_2 &= \frac{n_2}{n_1} \tau_1 & \theta_2 &= \frac{n_1}{n_2} \theta_1 \end{aligned}$$

- Taking Laplace of  $v_2$  gives: (ref: slide 42)
- $V_2(s) = A_1 A_2 K_p (R(s) - \theta(s))$

$$\diamond \theta(s) = \theta_m \left[ \frac{N_1}{N_2} \right]$$

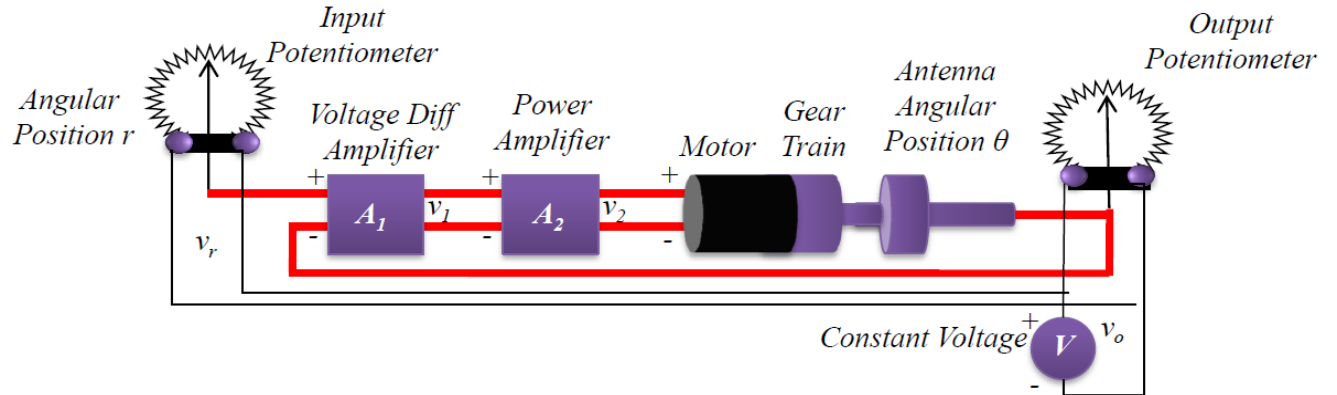
# Electromechanical Systems

## Example – A Position Servo a large video satellite antenna

substituting to find value of  $\theta(s)$  we get

$$\theta(s) = \frac{K_m \left(\frac{N_1}{N_2}\right) A_1 A_2 K_p (R(s) - \theta(s))}{s(1 + \tau_L s)}$$

Where,  $\tau_L = (R_a / K_v K_T [J_m + \left(\frac{N_1}{N_2}\right)^2 J_L])$



$$\theta(s) = \frac{K_m \left(\frac{N_1}{N_2}\right) A_1 A_2 K_p (R(s) - \theta(s))}{s(1 + \tau_L s)}$$

- Some of the coefficients, and thus some of the system properties can be selected by the designer by appropriately choosing the control components.
- However, the moment of inertia of the load J cannot be changed.
- The transfer function relating the input position  $R(s)$  to the output position  $\Theta(s)$  is given by:

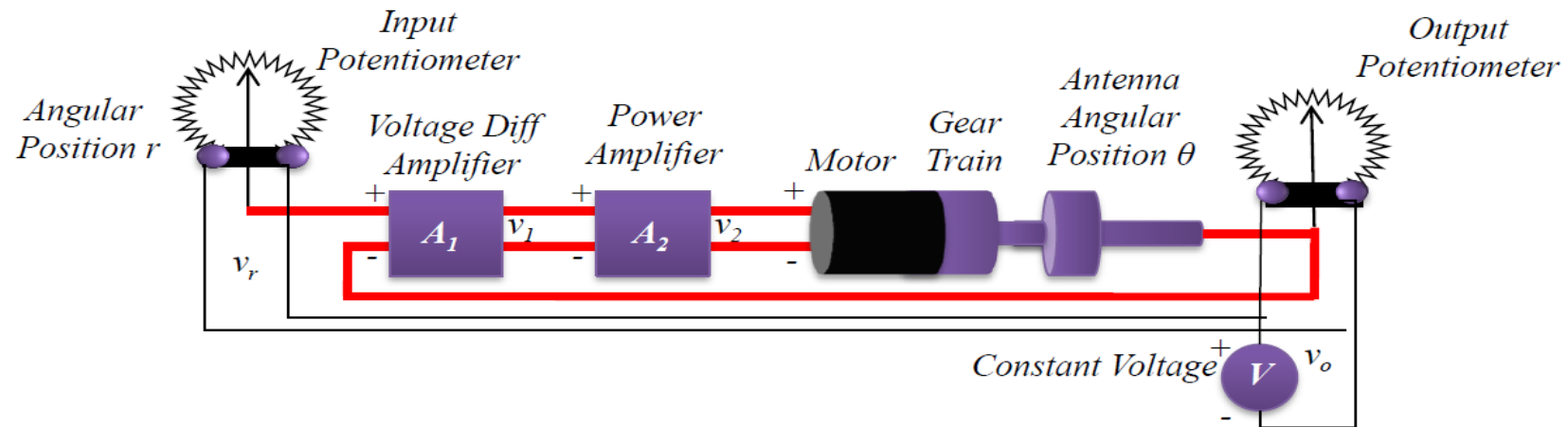
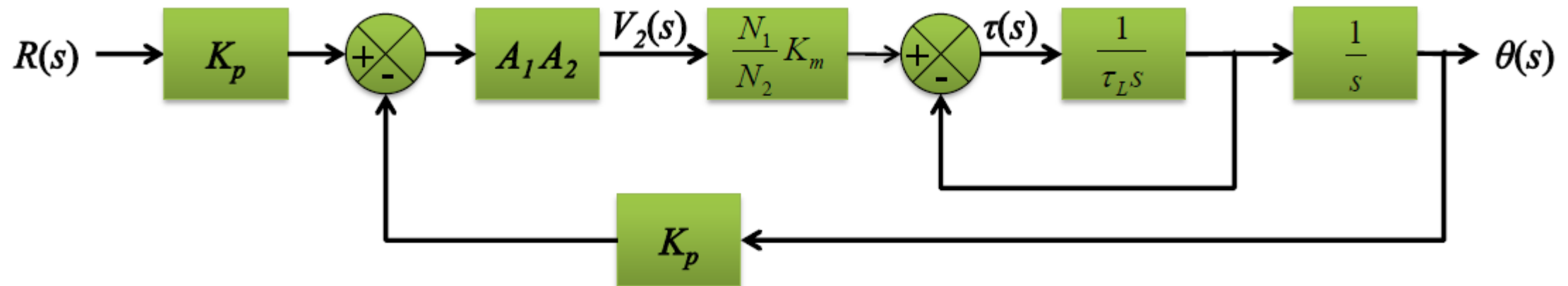
$$T(s) = \frac{\theta(s)}{R(s)} = \frac{\left(\frac{N_1}{N_2}\right) A_1 A_2 K_m K_p}{\tau_L s^2 + s + \left(\frac{N_1}{N_2}\right) A_1 A_2 K_m K_p}$$



# Electromechanical Systems

## Example – A Position Servo a large video satellite antenna DC Servo Motor – Block Diagram

- The transfer function can also be derived using a block diagram as shown below:



# Electromechanical Systems

## Example – A Position Servo a large video satellite antenna DC Servo Motor – Block Diagram – Signal Flow Graph

- It can also be obtained by reducing the equivalent signal flow graph shown on this slide
- In this graph for **one forward path** Mason's rule gives.

$$P_1 = K_p A_1 A_2 \frac{N_1}{N_2} \frac{1}{\tau_L s} \frac{1}{s} K_m, \quad \Delta_1 = 1$$

$$L_1 = A_1 A_2 \frac{N_1}{N_2} K_m \frac{1}{\tau_L s} \frac{1}{s} (-K_p), \quad L_2 = \frac{-1}{\tau_L s}$$

$$= \frac{-A_1 A_2 \left(\frac{N_1}{N_2}\right) K_m K_p}{\tau_L s^2}$$

$$T(s) = \frac{K_p A_1 A_2 \left(\frac{N_1}{N_2}\right) K_m}{\tau_L s^2 + s + \left(\frac{N_1}{N_2}\right) K_p A_1 A_2 K_m}$$

