KINEMATICS OF PARTICLES

Abdullah zaher

An Overview of Mechanics

Mechanics: The study of how bodies react to forces acting on them.

Statics: The study of bodies in equilibrium.

Dynamics:

1. **Kinematics** – concerned with the geometric aspects of motion
2. **Kinetics** - concerned with the forces causing the motion

Today’s Objectives:

- Students will be able to:

❖ Find the kinematic quantities (position, displacement, velocity, and acceleration) of a particle traveling along a straight path.

❖ Rectilinear means position given in Cartesian (x, y, and z) coordinates.

❖ We will stat with motion in a straight line

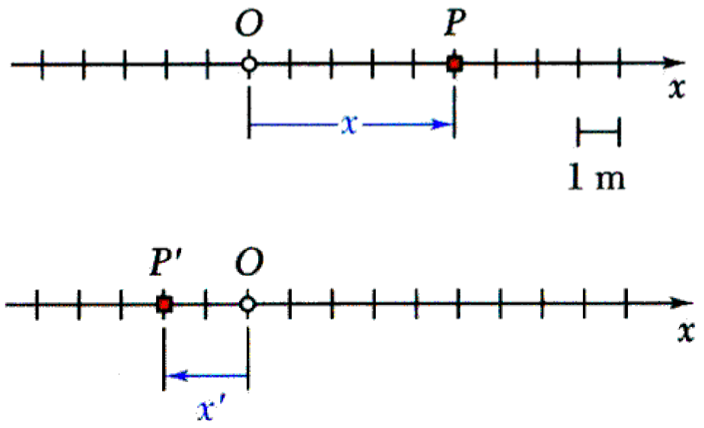
Introduction

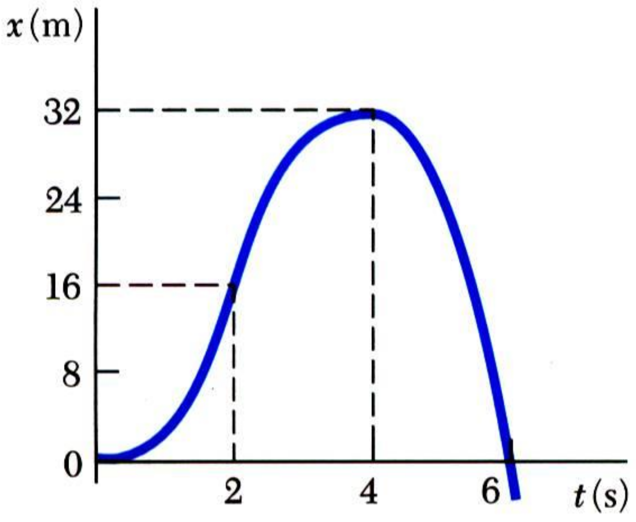
- **Particle kinetics includes:**

  - ***<u>Rectilinear motion</u>***: position, velocity, and acceleration of a particle as it moves along a straight line.

- ***<u>Curvilinear motion</u>***: position, velocity, and acceleration of a particle as it moves along a curved line in two or three dimensions.

Rectilinear Motion: Position, Velocity & Acceleration





- ***Rectilinear motion:*** particle moving along a straight line

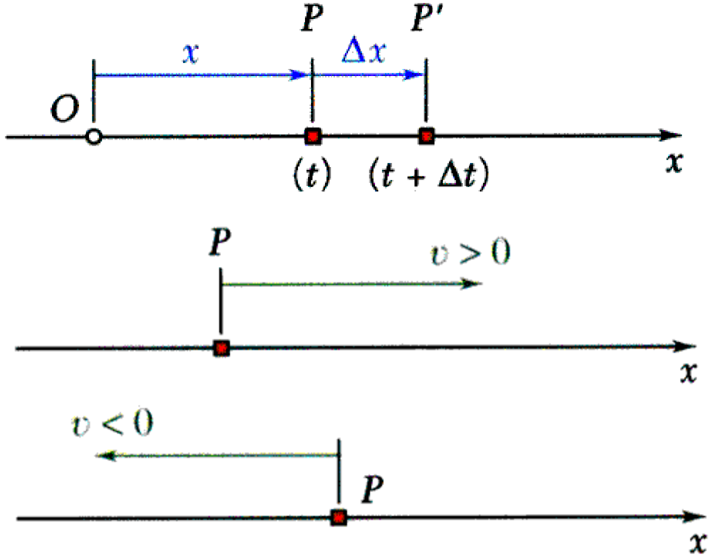
- ***Position coordinate:*** defined by positive or negative distance from a fixed origin on the line.

- The ***motion*** of a particle is known if the position coordinate for particle is known for every value of time *t*.

- May be expressed in the form of a function, e.g., $x = 6t^2 - t^3$

or in the form of a graph *x* vs. *t*.

Rectilinear Motion: Position, Velocity & Acceleration

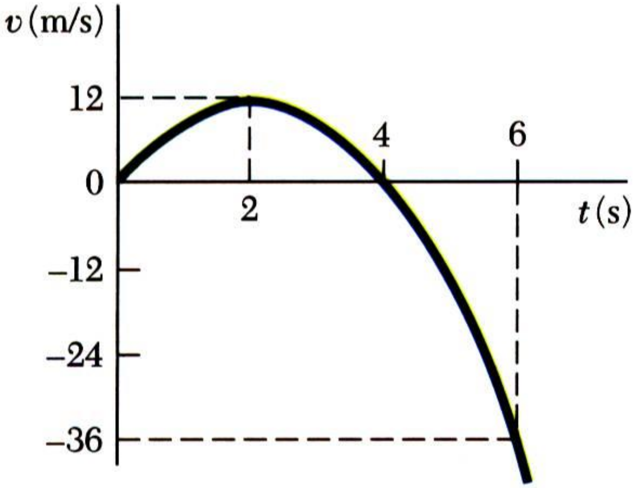


- Consider particle which occupies position P at time t and P' at $t+\Delta t$,

$$Average\ velocity = \frac{\Delta x}{\Delta t}$$

$$Instantaneous\ velocity = v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

- Instantaneous velocity may be positive or negative. Magnitude of velocity is referred to as *particle speed*.



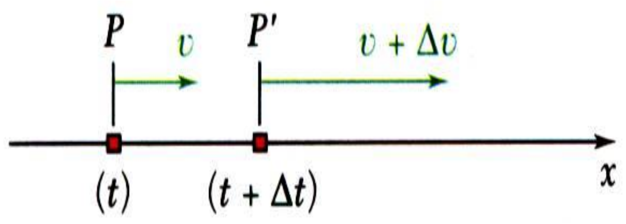
- From the definition of a derivative,

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

e.g., $x = 6t^2 - t^3$

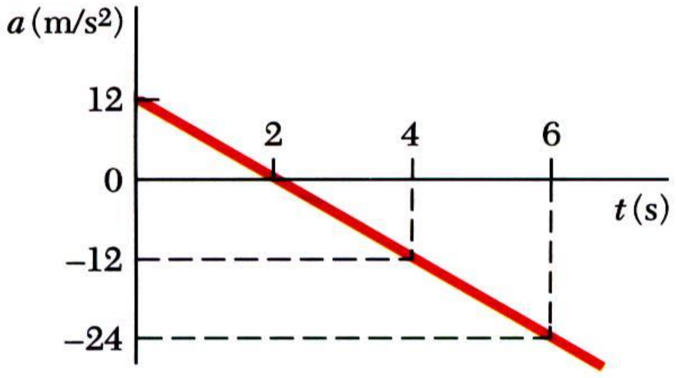
$$v = \frac{dx}{dt} = 12t - 3t^2$$

Rectilinear Motion: Position, Velocity & Acceleration



- Consider particle with velocity *v* at time *t* and *v'* at *t*+*Δt*,

$$\textit{Instantaneous acceleration} = a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$



- From the definition of a derivative,

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2 x}{dt^2}$$

e.g. $\quad v = 12t - 3t^2$

$$a = \frac{dv}{dt} = 12 - 6t$$

Determination of the Motion of a Particle

- **We often determine accelerations from the forces applied (kinetics will be covered later)**
- **Generally, have three classes of motion**
  - acceleration given as a function of *time*, $a = f(t)$
  - acceleration given as a function of *position*, $a = \mathrm{f}(x)$
  - acceleration given as a function of *velocity*, $a = \mathrm{f}(v)$

a spring

drag

SUMMARY OF KINEMATIC RELATIONS: RECTILINEAR MOTION

- Differentiate position to get velocity and acceleration.

$$v = ds/dt \; ; \qquad a = dv/dt \quad \text{or} \quad a = v\, dv/ds$$

- Integrate acceleration for velocity and position.

Velocity:

$$\int_{v_o}^{v} dv = \int_{0}^{t} a\, dt \text{ or } \int_{v_o}^{v} v\, dv = \int_{s_o}^{s} a\, ds$$

Position:

$$\int_{s_o}^{s} ds = \int_{0}^{t} v\, dt$$

- Note that s_o and v_o represent the initial position and velocity of the particle at $t = 0$.

CONSTANT ACCELERATION

The three kinematic equations can be integrated for the special case when acceleration is constant ($a = a_c$) to obtain very useful equations. A common example of constant acceleration is gravity; i.e., a body freely falling toward earth. In this case, $a_c = g = 9.81\ m/s^2 = 32.2\ ft/s^2$ downward. These equations are:

$$\int_{v_o}^{v} dv = \int_{o}^{t} a_c\, dt \quad \text{yields} \quad v = v_o + a_c t$$

$$\int_{s_o}^{s} ds = \int_{o}^{t} v\, dt \quad \text{yields} \quad s = s_o + v_o t + (1/2) a_c t^2$$

$$\int_{v_o}^{v} v\, dv = \int_{s_o}^{s} a_c\, ds \quad \text{yields} \quad v^2 = (v_o)^2 + 2a_c(s - s_o)$$

Acceleration as a function of time, position, or velocity

If….	Kinematic relationship	Integrate
$a = a(t)$	$\dfrac{dv}{dt} = a(t)$	$\displaystyle\int_{v_0}^{v} dv = \int_{0}^{t} a(t)\,dt$
$a = a(x)$	$dt = \dfrac{dx}{v}$ and $a = \dfrac{dv}{dt}$ $\downarrow$ $v\,dv = a(x)\,dx$	$\displaystyle\int_{v_0}^{v} v\,dv = \int_{x_0}^{x} a(x)\,dx$
$a = a(v)$	$\dfrac{dv}{dt} = a(v)$	$\displaystyle\int_{v_0}^{v} \frac{dv}{a(v)} = \int_{0}^{t} dt$
	$v\dfrac{dv}{dx} = a(v)$	$\displaystyle\int_{x_0}^{x} dx = \int_{v_0}^{v} \frac{v\,dv}{a(v)}$

Examples

- **<u>EX1:</u>** The car in Fig. 1.4 moves in a straight line such that for a short time its velocity is defined by $v = (3t^2 + 2t)$ ft/s, where t is in seconds. Determine its position and acceleration when $t = 3\ s$. Noting that $s = 0$ when $t = 0$
- SOLUTION
- $s = \int v\, dt$
- $s = \int_0^t (3t^2 + 2t)\, dt$
- $(S)_0^S = (t^3 + t^2)_0^t$
- S = $t^3 + t^2$
- = 36

$$\boldsymbol{a} = \frac{\boldsymbol{dv}}{\boldsymbol{dt}}$$

$$\boldsymbol{a} = \frac{\boldsymbol{d}}{\boldsymbol{dt}}(3t^2 + 2t)$$

$$= 6t + 2$$

$$= 6 * 3 + 2 = 20$$

- EX2- A particle begins its motion in a straight line such that its position relative to fixed point on that straight line is given by $s = t^3 - 9t^2 + 15t + 5\ m$. Find the position of the particle when the acceleration vanished
- $s = t^3 - 9t^2 + 15t + 5,$
- $v = \frac{dx}{dt} = 3t^2 - 18t + 15,$
- $a = \frac{dv}{dt} = 6\,t - 18,$
- when the acceleration vanished
- $a = 0$

$\longrightarrow \quad t = 3 \quad \longrightarrow \quad s = -4$

- **<u>Example 3:</u>** A particle moves in a straight line with acceleration inversely proportional with square distance from the origin. Find the velocity of the particle as a function of the distance knowing that the motion begins from rest when the particle is at a distance (a) from the origin.
- **<u>Solution</u>**
- We find that the motion is in straight line.
- **Velocity:** the acceleration is a function in x, then, the velocity can be obtained from a relation $a = \frac{vdv}{dx}$ by separating variables and integrating
- $a = v\frac{dv}{dx} = \frac{k}{x^2}$
- $\int_0^v vdv = \int_a^x \frac{k}{x^2}dx \quad , \frac{v^2}{2} = -\frac{k}{x} + \frac{k}{a} , \qquad v^2 = 2\left(\frac{k}{a} - \frac{k}{x}\right)$

EXAMPLE

Given: The acceleration of a body is $a = 5v$. At $t = 0$, $s = 0$, and $v_0 = 2$ m/s. (a) Find v(t). (b) Find v(s).

(a) Need the equation with a, v, and t.

$$a = \frac{dv}{dt}$$

Rearrange. Terms with t on one side and v on the other.

$$dt = \frac{dv}{a}$$

Create integrals

$$\int_0^t dt = \int_2^v \frac{dv}{5v}$$

EXAMPLE (continued)

Result is

$$t\Big|_0^t = \frac{1}{5}\ln(v)\Big|_2^v$$

$$t = \frac{1}{5}\ln\left(\frac{v}{2}\right)$$

$$v = 2e^{5t}$$

(b) Need the equation with a, v, and s.

$$a = v\frac{dv}{ds}$$

$$ds = v\frac{dv}{a}$$

EXAMPLE (continued)

Result is

$$\int_0^s ds = \int_2^v v\frac{dv}{5v}$$

$$s\Big|_0^s = \frac{1}{5}v\Big|_2^v$$

$$s = \frac{1}{5}(v - 2)$$

$$v = 5s + 2$$

Expression for the distance travelled in nth second

Let a body move with an initial velocity u and travel along a straight line with uniform acceleration a.
Distance travelled in the nth second of motion is,
sn = distance travelled during first n seconds – distance travelled during (n –1) seconds
Distance travelled during n seconds

$$Dn = un + 1/2an2$$

Distance travelled during (n -1) seconds

$$D_{(n-1)} = u(n-1) + \frac{1}{2} a(n-1)^2$$

. the distance travelled in the n^{th} second =

$$\text{(i.e)} \quad s_n = \left(un + \frac{1}{2}an^2\right) - \left[u(n-1) + \frac{1}{2}a(n-1)^2\right]$$

$$s_n = u + a\left(n - \frac{1}{2}\right)$$

$$s_n = u + \frac{1}{2}a(2n-1)$$

Special Cases

Case (i) : For downward motion
For a particle moving downwards, $a = g$, since the particle moves in the direction of gravity.

Case (ii) : For a freely falling body
For a freely falling body, $a = g$ and $u = 0$, since it starts from rest.

Case (iii) : For upward motion
For a particle moving upwards, $a = -g$, since the particle moves against the gravity

Coding using Python program

```python
import numpy as np
import matplotlib.pyplot as plt

finalTime=20 #Seconds
H=0

velocity=np.zeros(finalTime+1)#Initialize Velocity Array
height=np.zeros(finalTime+1)#Initialize Height Array

g=10 # m/s^2 you may change to 9.81 m/s^2
initialVelocity=100 #Initial Velocity in m/s

# calculate velocity and distance/height at each second
# Time=1; so not appearing in expressions v=u-gt and
H=ut-0.5*gt^2
for i in range (0,finalTime+1):
    if i==0:
        velocity[i]=initialVelocity
    else:
        velocity[i]= velocity[i-1]-g
        deltaH= velocity[i-1]-0.5*g
        H=H+deltaH
        height[i]=H
        # initialVelocity=velocity[i]
    #print(i, initialVelocity, H)
```

- ```
  timePoints=np.linspace(
  0,finalTime,
  finalTime+1)

  print("velocity",
  velocity)
  print("\n")
  print("height", height)
  ```

```
velocity [ 100.   90.   80.   70.   60.   50.   40.   30.
20.   10.    0.  -10.

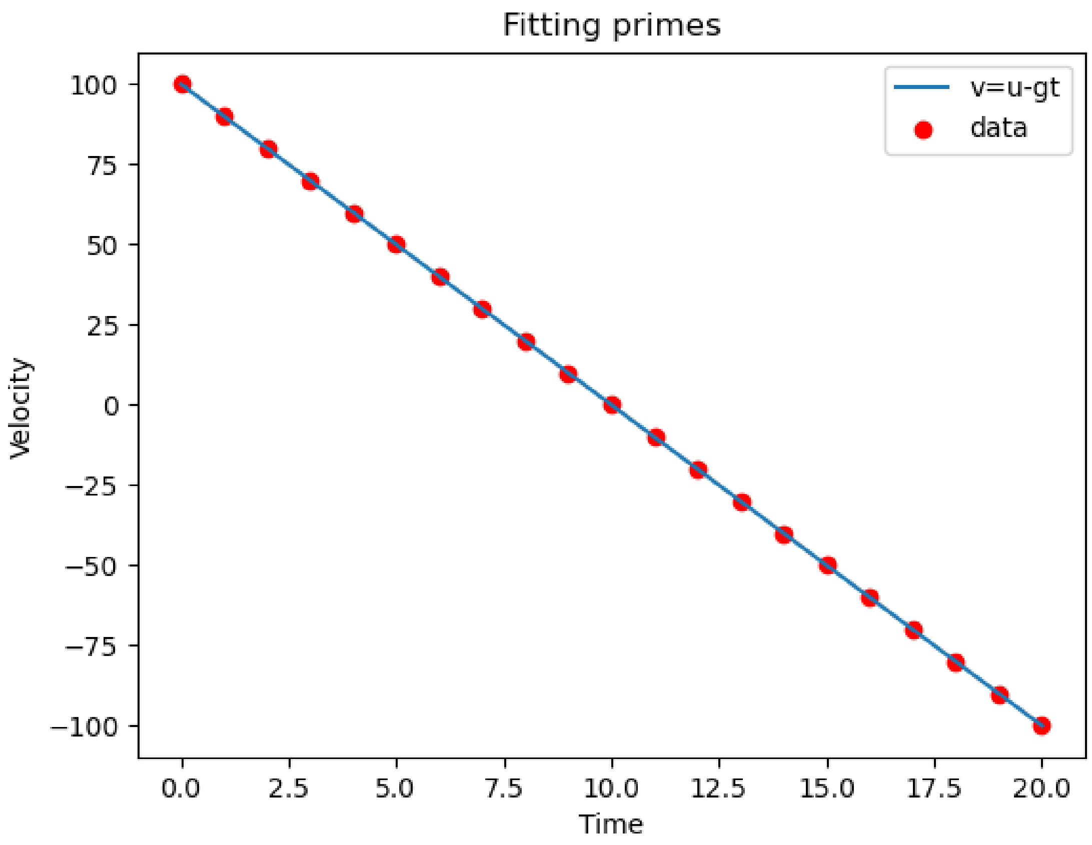
  -20.  -30.  -40.  -50.  -60.  -70.  -80.  -90. -100.]



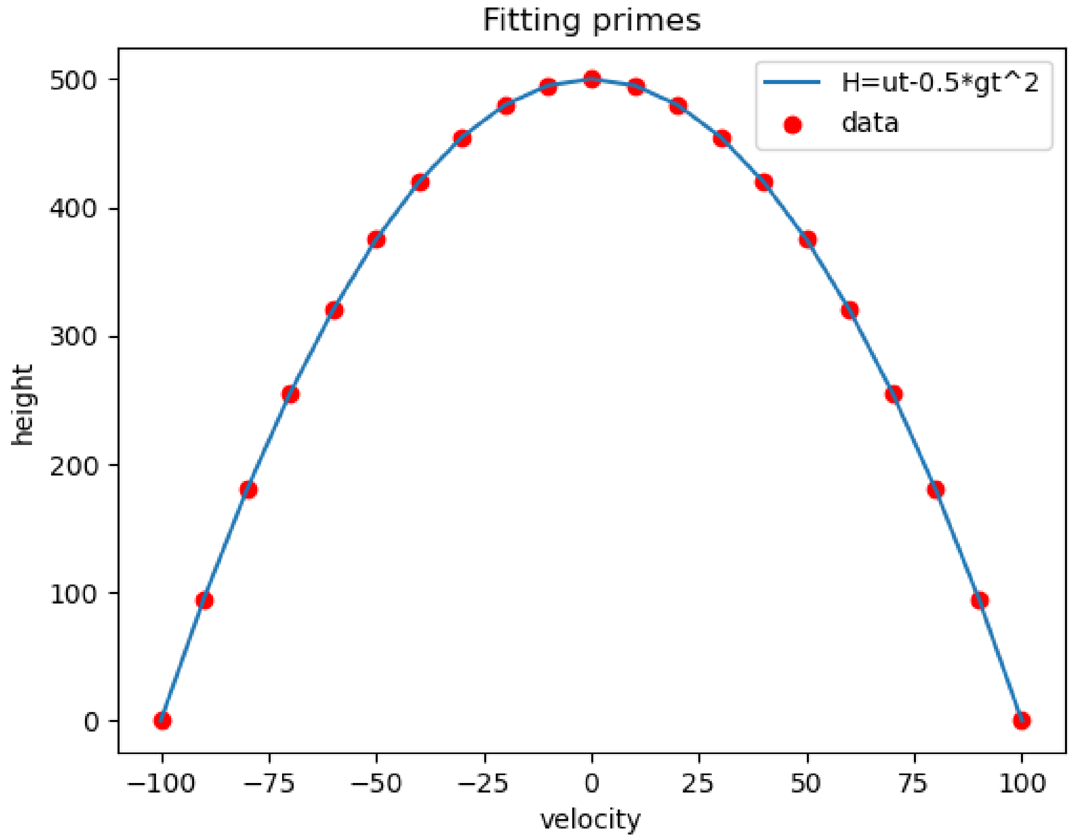
height [   0.   95. 180. 255. 320. 375. 420. 455. 480. 495.
500. 495. 480. 455.

 420. 375. 320. 255. 180.   95.    0.]
```
```

```
plt.scatter(timePoints, velocity, c='r', label='data')
plt.plot(timePoints, velocity, label='v=u-gt')
plt.xlabel('Time')
plt.ylabel('Velocity ')
plt.title('Fitting primes')
plt.legend()
plt.show()
```



- ```
  plt.scatter(velocity, height, c='r', label='data')
  plt.plot(velocity, height, label='H=ut-0.5*gt^2')
  plt.xlabel('velocity')
  plt.ylabel('height')
  plt.title('Fitting primes')
  plt.legend()
  plt.show()
  ```


  ```

Thank you