Chapter 3
Simplification of Boolean Functions

Simplification of Boolean Functions

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3.1 The Map Method

- The map method provides a simple straightforward procedure for minimizing Boolean functions.
- The map method, first proposed by Veitch (1) and slightly modified by Karnaugh (2), is also known as the “Veitch diagram” or the “Karnaugh map.”
- The map is a diagram made up of squares.
- Each square represents one minterm

3.2 Two- and Three-variable Maps

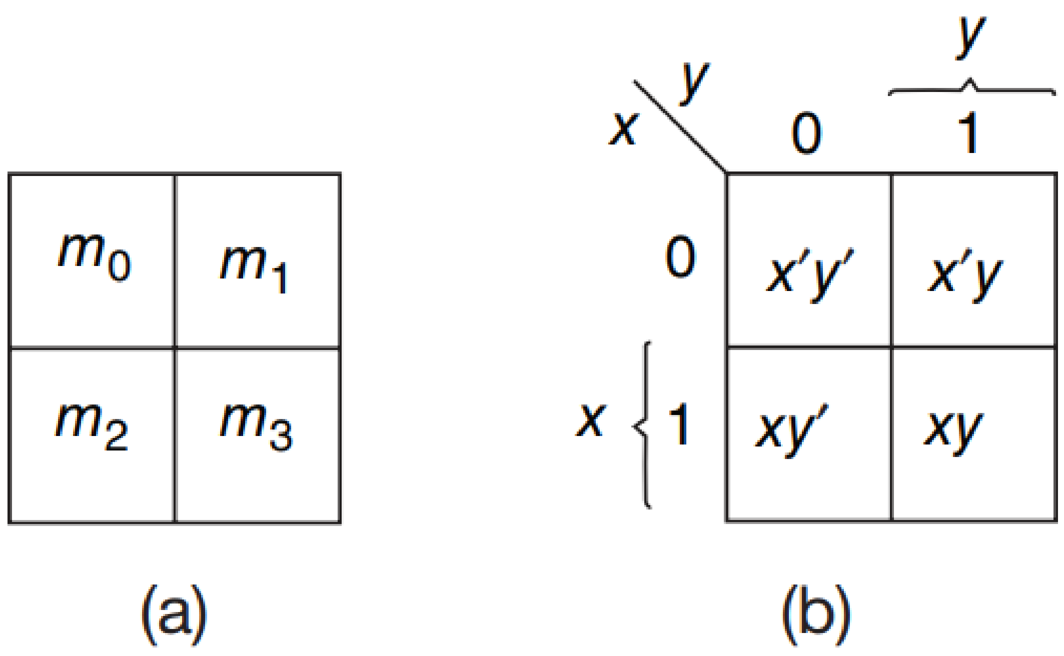


Figure 3.1 Two-variable map

3.2 Two- and Three-variable Maps

- These squares are found from the minterms of the function:
- $x + y = x'y + xy' + xy = m1 + m2 + m3$

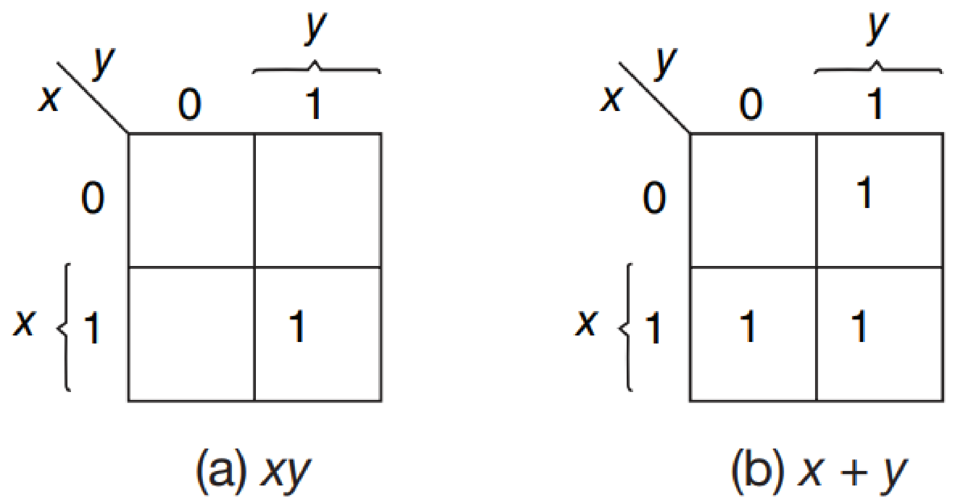


Figure 3-2 Representation of functions in the map

3.2 Two and Three-variable Maps

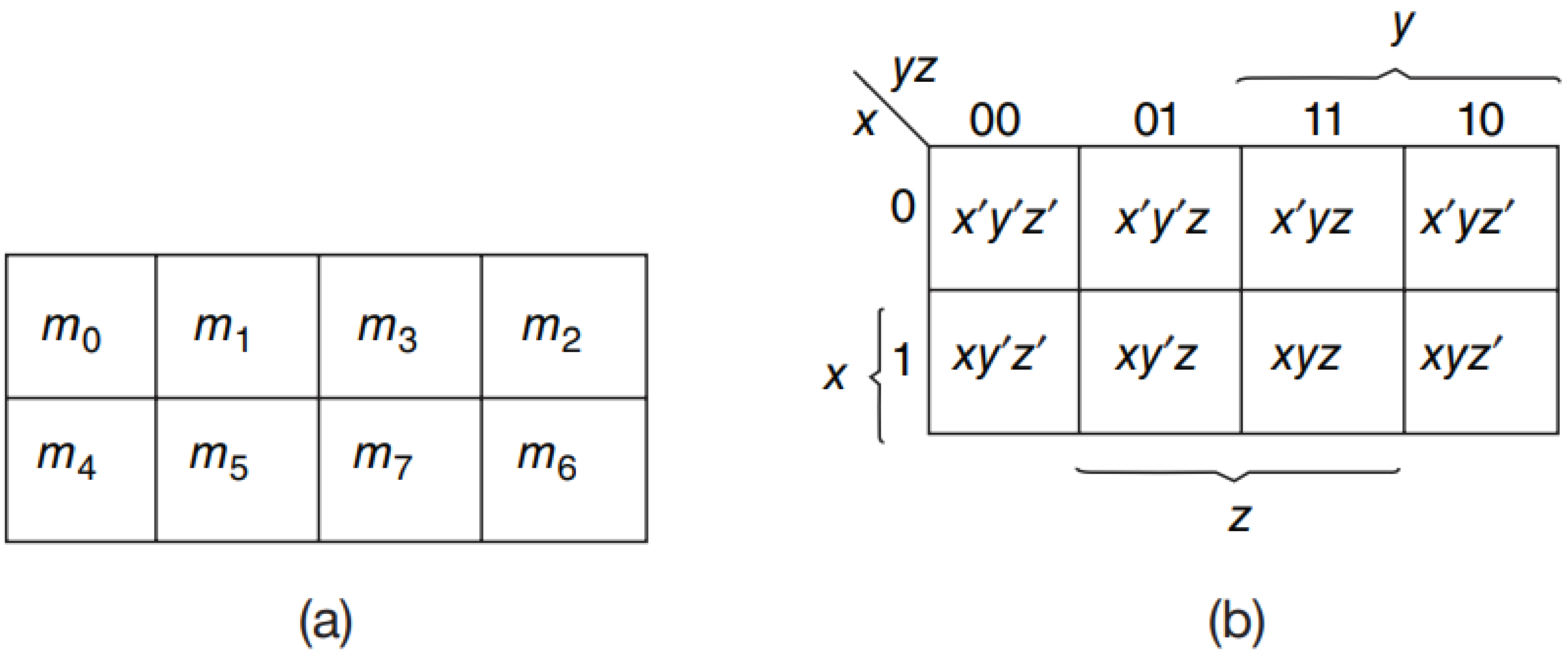


Figure 3-3 Three-variable map

3.2 Two and Three-variable Maps

EXAMPLE 3-1: Simplify the Boolean function:

$$F = x'yz + x'yz' + xy'z' + xy'z$$

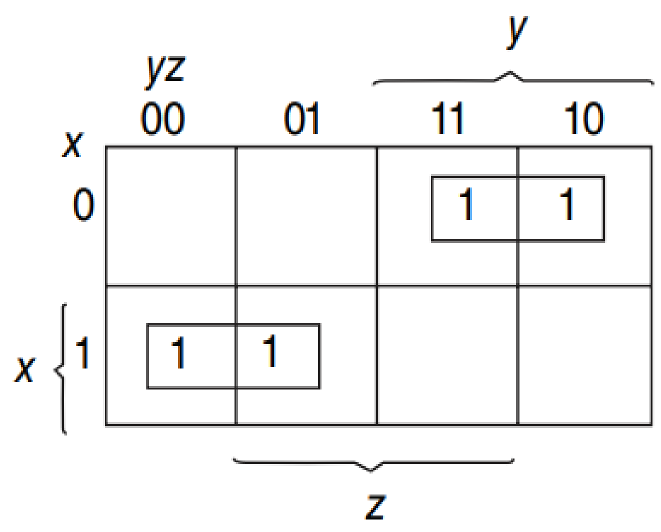
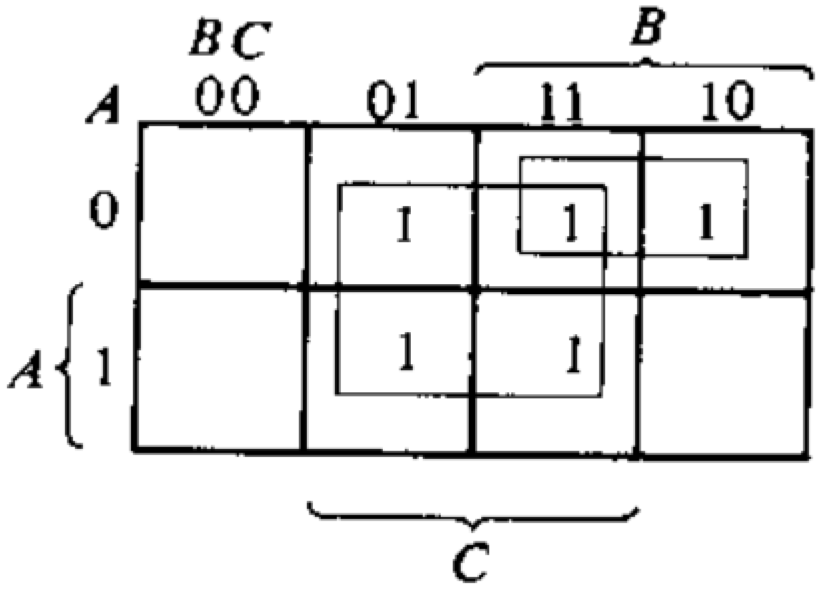


Figure 3.4 Map for Example 3-1; $x'yz + x'yz' + xy'z' + xy'z = x'y + xy'$

3.2 Two and Three-variable Maps

EXAMPLE 3-3: Simplify the Boolean function:

$$F = A'C + A'B + AB'C + BC$$



3.3 Four-variable Map

The map for Boolean functions of four binary variables are listed the 16 minterms and the squares assigned to each.

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6
m_{12}	m_{13}	m_{15}	m_{14}
m_8	m_9	m_{11}	m_{10}

(a)

$wx \backslash yz$	00	01	11	10
00	$w'x'y'z'$	$w'x'y'z$	$w'x'yz$	$w'x'yz'$
01	$w'xy'z'$	$w'xy'z$	$w'xyz$	$w'xyz'$
11	$wxy'z'$	$wxy'z$	$wxyz$	$wxyz'$
10	$wx'y'z'$	$wx'y'z$	$wx'yz$	$wx'yz'$

(b)

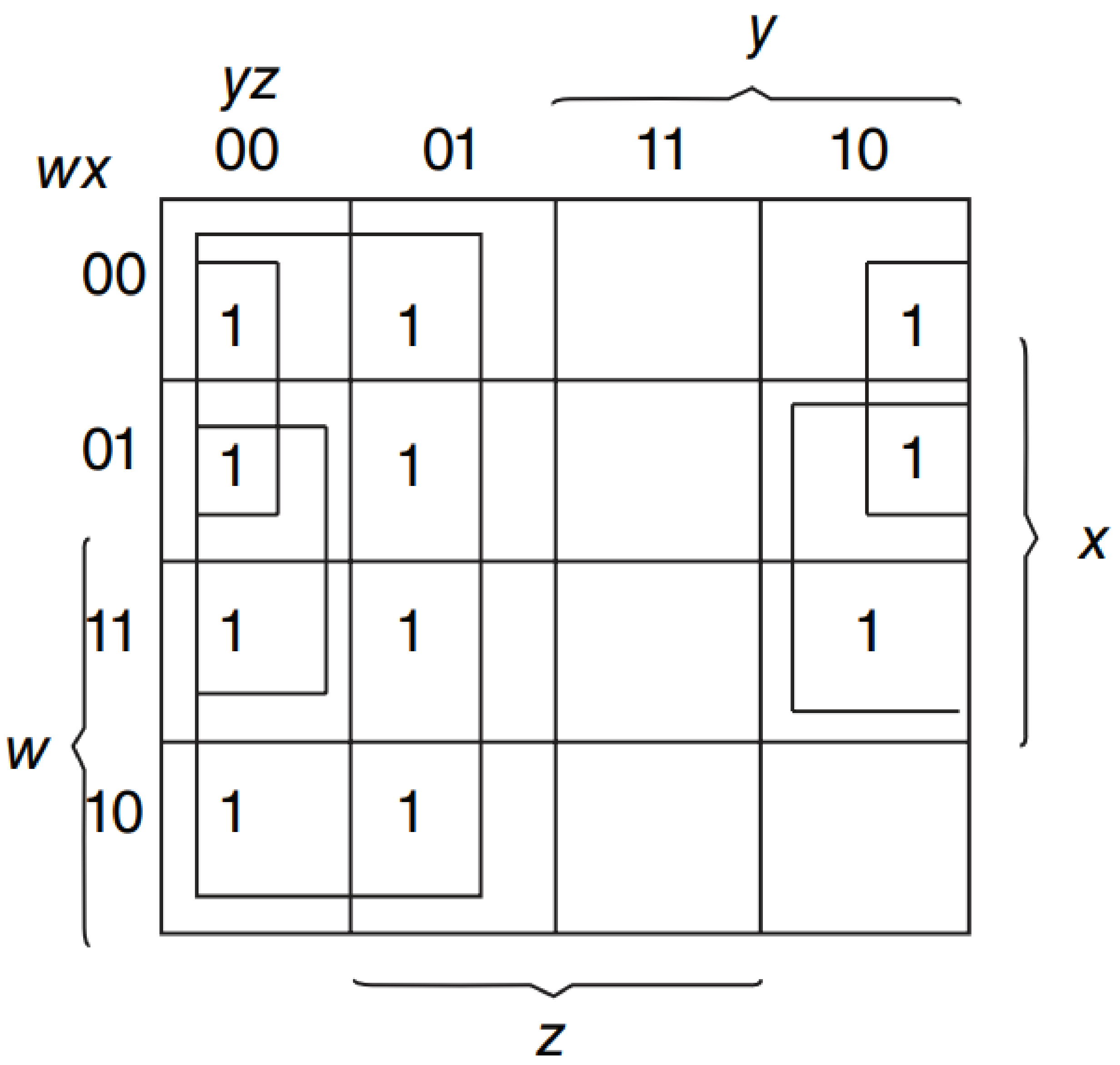
Figure 3.8 Four-variable map

3.3 Four-variable Map

EXAMPLE 3-5: Simplify the Boolean function;

$F(w, x, y, z) = \sum(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$

$y' + w'z' + xz'$



3.4 Five- and Six-Variable Maps

AB \ CDE	00	01	11	10	110	111	101	100
00	0	1	3	2	6	7	5	4
01	8	9	11	10	14	15	13	12
11	24	25	27	26	30	31	29	28
10	16	17	19	18	22	23	21	20

3.4 Five- and Six-Variable Maps

ABC \ DEF	000	001	011	010	110	111	101	100
000	0	1	3	2	6	7	5	4
001	8	9	11	10	14	15	13	12
011	24	25	27	26	30	31	29	28
010	16	17	19	18	22	23	21	20
110	48	49	51	50	54	55	53	52
111	56	57	59	58	62	63	61	60
101	40	41	43	42	46	47	45	44
100	32	33	35	34	38	39	37	36

3.5 Product of Sums Simplification

If we mark the empty squares by 0’s and combine them into valid adjacent squares, we obtain a simplified expression of the complement of the function, i.e., of F’.

The complement of F’ gives us back the function F.

Because of the generalized DeMorgan’s theorem, the function so obtained is automatically in the product of sums form.

3.5 Product of Sums Simplification

EXAMPLE 3-8: Simplify the following Boolean function in (a) sum of products and (b) product of sums. F (A, B, C, D) = ∑(0, 1, 2, 5, 8, 9, 10)

(a) F = B′D′ + B′C′ + A′C′D

(b) F = (A′ + B′)(C′ + D′)(B′ + D)