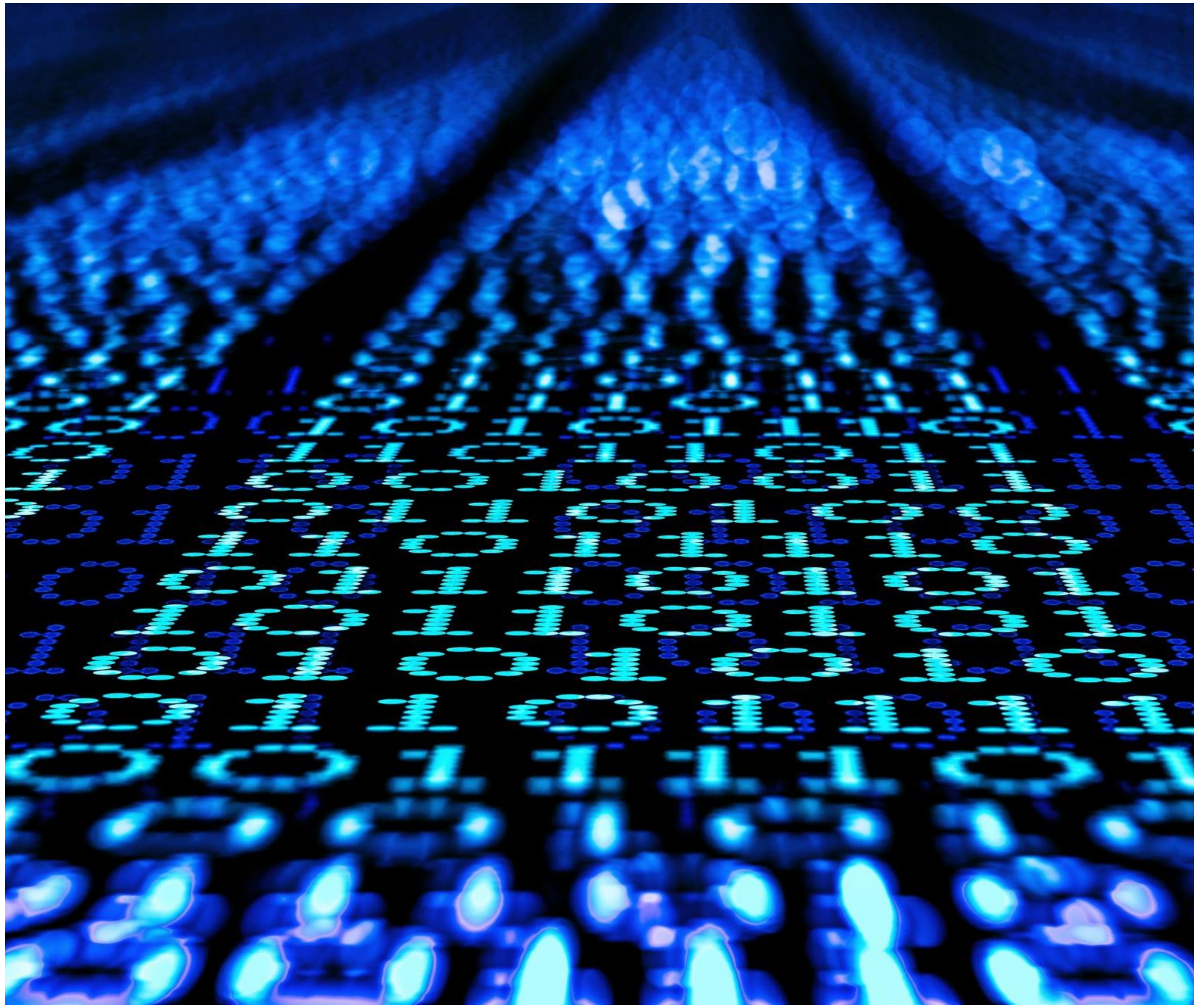
Chapter 1
Binary Systems



Binary Systems

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1.2 Binary Numbers

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1.1 Digital Computers and Digital Systems

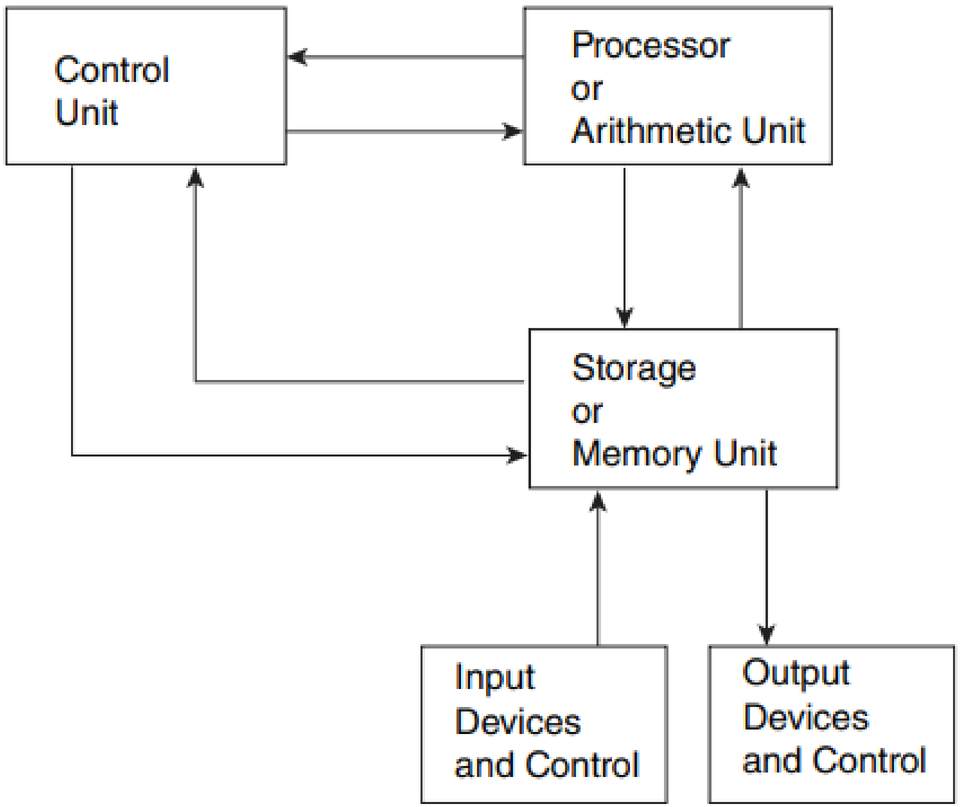


Figure 1.1 Block diagram of a digital computer

1.2 Binary Numbers

Table 1-1 Numbers with different bases

Decimal (base 10)	Binary (base 2)	Octal (base 8)	Hexadecimal (base 16)
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

1.3 Number Base Conversions

EXAMPLE 1-1: Convert decimal 41 to binary. First, 41 is divided by 2 to give an integer quotient of 20 and a remainder of $\frac{1}{2}$. The quotient is again divided by 2 to give a new quotient and remainder. This process is continued until the integer quotient becomes 0. The *coefficients* of the desired binary number are obtained from the *remainders* as follows:

integer quotient		*remainder*	*coefficient*
$\frac{41}{2} = 20$	+	$\frac{1}{2}$	$a_0 = 1$
$\frac{20}{2} = 10$	+	0	$a_1 = 0$
$\frac{10}{2} = 5$	+	0	$a_2 = 0$
$\frac{5}{2} = 2$	+	$\frac{1}{2}$	$a_3 = 1$
$\frac{2}{2} = 1$	+	0	$a_4 = 0$
$\frac{1}{2} = 0$	+	$\frac{1}{2}$	$a_5 = 1$

answer: $(41)_{10} = (a_5a_4a_3a_2a_1a_0)_2 = (101001)_2$

1.3 Number Base Conversions

The arithmetic process can be manipulated more conveniently as follows:

integer	*remainder*
41	
20	1
10	0
5	0
2	1
1	0
0	1

101001 = answer

1.3 Number Base Conversions

EXAMPLE 1-4: Convert $(0.513)_{10}$ to octal.

$$0.513 \times 8 = 4.104$$
$$0.104 \times 8 = 0.832$$
$$0.832 \times 8 = 6.656$$
$$0.656 \times 8 = 5{,}248$$
$$0.248 \times 8 = 1.984$$
$$0.984 \times 8 = 7.872$$

The answer, to seven significant figures, is obtained from the integer part of the products:

$$(0.513)_{10} = (0.406517 \ldots)_8$$

1.4 Octal and Hexadecimal Numbers

$$(\underbrace{10}_{2}\ \underbrace{110}_{6}\ \underbrace{001}_{1}\ \underbrace{101}_{5}\ \underbrace{011}_{3} \cdot \underbrace{111}_{7}\ \underbrace{100}_{4}\ \underbrace{000}_{0}\ \underbrace{110}_{6})_2 = (26153.7406)_8$$

Conversion from binary to hexadecimal is similar, except that the binary number is divided into groups of four digits:

$$(\underbrace{10}_{2}\ \underbrace{1100}_{C}\ \underbrace{0110}_{6}\ \underbrace{1011}_{B} \cdot \underbrace{1111}_{F}\ \underbrace{0010}_{2})_2 = (2C6B.F2)_{16}$$

1.4 Octal and Hexadecimal Numbers

Conversion from octal or hexadecimal to binary is done by a procedure reverse to the above. Each octal digit is converted to its three-digit binary equivalent. Similarly, each hexadecimal digit is converted to its four-digit binary equivalent. This is illustrated in the following examples:

$$(673.124)_8 = (\underbrace{110}_{6}\ \underbrace{111}_{7}\ \underbrace{011}_{3} \cdot \underbrace{001}_{1}\ \underbrace{010}_{2}\ \underbrace{100}_{4})_2$$

$$(306.\ D)_{16} = (\underbrace{0011}_{3}\ \underbrace{0000}_{0}\ \underbrace{0110}_{6} \cdot \underbrace{1101}_{D})_2$$

1.5 Complements

1.5.1 The r's Complement

1.5.2 The (r – 1)'s Complement

1.5.3 Subtraction with r's Complements

1.5.4 Subtraction with (r – 1)'s Complement

1.5.5 Comparison between 1's and 2's Complements

1.5.1 The r's Complement

Given a positive number N in base r with an integer part of n digits, the r's complement of N is defined as $r^n - N$ for $N \neq 0$ and 0 for $N = 0$. The following numerical example will help clarify the definition.

The 10's complement of $(52520)_{10}$ is $10^5 - 52520 = 47480$.

The number of digits in the number is $n = 5$.

The 10's complement or $(0.3267)_{10}$ is $1 - 0.3267 = 0.6733$.

1.5.2 The (r – 1)'s Complement

The 9's complement of $(52520)_{10}$ is $(10^5 - 1 - 52520) = 99999 - 52520 = 47479$.

No fraction part, so $10^{-m} = 10^0 = 1$.

The 9's complement of $(0.3267)_{10}$ is $(1 - 10^{-4}\ 0.3267) = 0.9999 - 0.3267 = 0.6732$.

No integer part, so $10^n = 10^0 - 1$.

The 9's complement of $(25.639)_{10}$ is $(10^2 - 10^{-3} - 25.639) = 99.999 - 25.639 = 74.360$.

The l's complement of $(101100)_2$ is $(2^6 - 1) - (101100) = (111111 - 101100)_2 = 010011$.

The l's complement of $(0.0110)_2$ is $(1 - 2^{-4})_{10} - (0.0110)_2 = (0.1111 - 0.0110)_2 = 0.1001$.

The 7's complement of $(76)_8$ is $(8^2 - 8^0)_{10} - 76_8 = (63)_{10} - 76_8 = (77 - 76)_8 = 1_8$

1.5.3 Subtraction with r's Complements

EXAMPLE 1-5: Using 10's complement, subtract 72532 – 3250.

$$
\begin{array}{rr}
M = 72532 & 72532 \\
N = 03250 & \\
 & + \\
\text{10's complement of } N = 96750 & 96750 \\
\hline
\text{end carry} \rightarrow 1 & 69282
\end{array}
$$

answer: 69282

1.5.3 Subtraction with r's Complements

EXAMPLE 1-6: Subtract: $(3250 - 72532)_{10}$.

$$M = 03250 \qquad\qquad 03250$$
$$N = 72532$$
$$+$$
$$\text{10's complement of } N = 27468 \qquad\qquad 27468$$
$$\text{no carry} \qquad\qquad 30718$$

answer: – 69282 = – (10**'s** complement of 30718)

1.5.3 Subtraction with r's Complements

EXAMPLE 1-7: Use 2's complement to perform $M - N$ with the given binary numbers.

(a) $M = 1010100$
$N = 1000100$

2's complement of $N = 0111100$

$$\begin{array}{r} 1010100 \\ +\ 0111100 \\ \hline \text{end carry} \rightarrow 1\ \ 0010000 \end{array}$$

answer: 10000

(b) $M = 1000100$
$N = 1010100$

2's complement of $N = 0101100$

$$\begin{array}{r} 1000100 \\ +\ 0101100 \\ \hline \text{no carry}\ \ 1110000 \end{array}$$

answer: – 10000 = – (2's complement of 1110000)

1.5.4 Subtraction with (r – 1)'s Complement

EXAMPLE 1-8: Repeat Examples 1-5 and 1-6 using 9's complements.

(a)

$$M = 72532$$
$$N = 03250$$
$$\text{9's complement of } N = 96749$$

$$\begin{array}{r} 72532 \\ +\ 96749 \\ \hline 1\ 69281 \\ +\ 1 \\ \hline 69282 \end{array}$$

end-around carry

answer: 69282

(b)

$$M = 03250$$
$$N = 72532$$
$$\text{9's complement of } N = 27467$$

$$\begin{array}{r} 03250 \\ +\ 27467 \\ \hline 30717 \end{array}$$

no carry

answer: – 69282 = – (9's complement of 30717)

1.5.4 Subtraction with (r – 1)'s Complement

EXAMPLE 1-9: Repeat Example 1 = 7 using l's complement.

(a)	$M = 1010100$	1010100
	$N = 1000100$	
	l's complement of $N = 0111011$	+ 0111011
	end-around carry	1 0001111
		+ 1
		0010000

answer: 10000

(b)	$M = 1000100$	1000100
	$N = 1010100$	+
	l's complement of $N = 0101011$	0101011
	no carry	1101111

answer: – 10000 = – (l's complement of 1101111)

1.5.5 Comparison between 1's and 2's Complements

Using l's complement:

$$\begin{array}{r} 1100 \\ + \quad 0011 \\ \hline + \quad 1111 \end{array}$$

Complement again to obtain −0000.

Using 2's complement:

$$\begin{array}{r} 1100 \\ + \quad 0100 \\ \hline + \quad 0000 \end{array}$$

While the 2's complement has only one arithmetic zero, the l's complement zero can be positive or negative, which may complicate matters.

1.6 Binary Codes

Table 1-2 Binary codes for the decimal digits

Decimal digit	(BCD) 8421	Excess-3	84-2-1	2421	(Biquinary) 5043210
0	0000	0011	0000	0000	0100001
1	0001	0100	0111	0001	0100010
2	0010	0101	0110	0010	0100100
3	0011	0110	0101	0011	0101000
4	0100	0111	0100	0100	0110000
5	0101	1000	1011	1011	1000001
6	0110	1001	1010	1100	1000010
7	0111	1010	1001	1101	1000100
8	1000	1011	1000	1110	1001000
9	1001	1100	1111	1111	1010000

1.6.1 Decimal Codes

- Numbers are represented in digital computers either in binary or in decimal through a binary code.
- For example, the decimal number 395, when converted to binary, is equal to 110001011 and consists of nine binary digits.
- The same number, when represented internally in the BCD code, occupies four bits for each decimal digit, for a total of 12 bits: 001110010101. The first four bits represent a 3, the next four a 9, and the last four a 5

1.6.2 Error-Detection Codes

Table 1-3 Parity-bit generation

(a) Message	P(odd)	(b) Message	P (even)
0000	1	0000	0
0001	0	0001	1
0010	0	0010	1
0011	1	0011	0
0100	0	0100	1
0101	1	0101	0
0110	1	0110	0
0111	0	0111	1
1000	0	1000	1
1001	1	1001	0
1010	1	1010	0
1011	0	1011	1
1100	1	1100	0
1101	0	1101	1
1110	0	1110	1
1111	1	1111	0

1.6.3 The Reflected Code

Table 1-4 Four-bit reflected code

Reflected code	Decimal equivalent
0000	0
0001	1
0011	2
0010	3
0110	4
0111	5
0101	6
0100	7
1100	8
1101	9
1111	10
1110	11
1010	12
1011	13
1001	14
1000	15

1.6.4 Alphanumeric Codes

Table 1-5 Alphanumeric character codes

Character	6-Bit internal code		7-Bit ASCII code		8-Bit EBCDIC code		12-Bit card code
A	010	001	100	0001	1100	0001	12,1
B	010	010	100	0010	1100	0010	12,2
C	010	011	100	0011	1100	0011	12,3
D	010	100	100	0100	1100	0100	12,4
E	010	101	100	0101	1100	0101	12,5
F	010	110	100	0110	1100	0110	12,6
G	010	111	100	0111	1100	0111	12,7
H	011	000	100	1000	1100	1000	12,8
I	011	001	100	1001	1100	1001	12,9
J	100	001	100	1010	1101	0001	11,1
K	100	010	100	1011	1101	0010	11,2
L	100	011	100	1100	1101	0011	11.3
M	100	100	100	1101	1101	0100	11,4

1.7 Binary Storage and Registers

1.7.1 Registers

1.7.2 Register Transfer

1.7.1 Registers

A *register* is a group of binary cells. Since a cell stores one bit of information, it follows that a register with n cells can store any discrete quantity of information that contains n bits. The *state* of a register is an n-tuple number of 1's and 0's, with each bit designating the state of one cell in the register. The *content* of a register is a function of the interpretation given to the information stored in it. Consider, for example, the following 16-cell register:

1	1	0	0	0	0	1	1	1	1	0	0	1	0	0	1
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

1.7.2 Register Transfer(a)

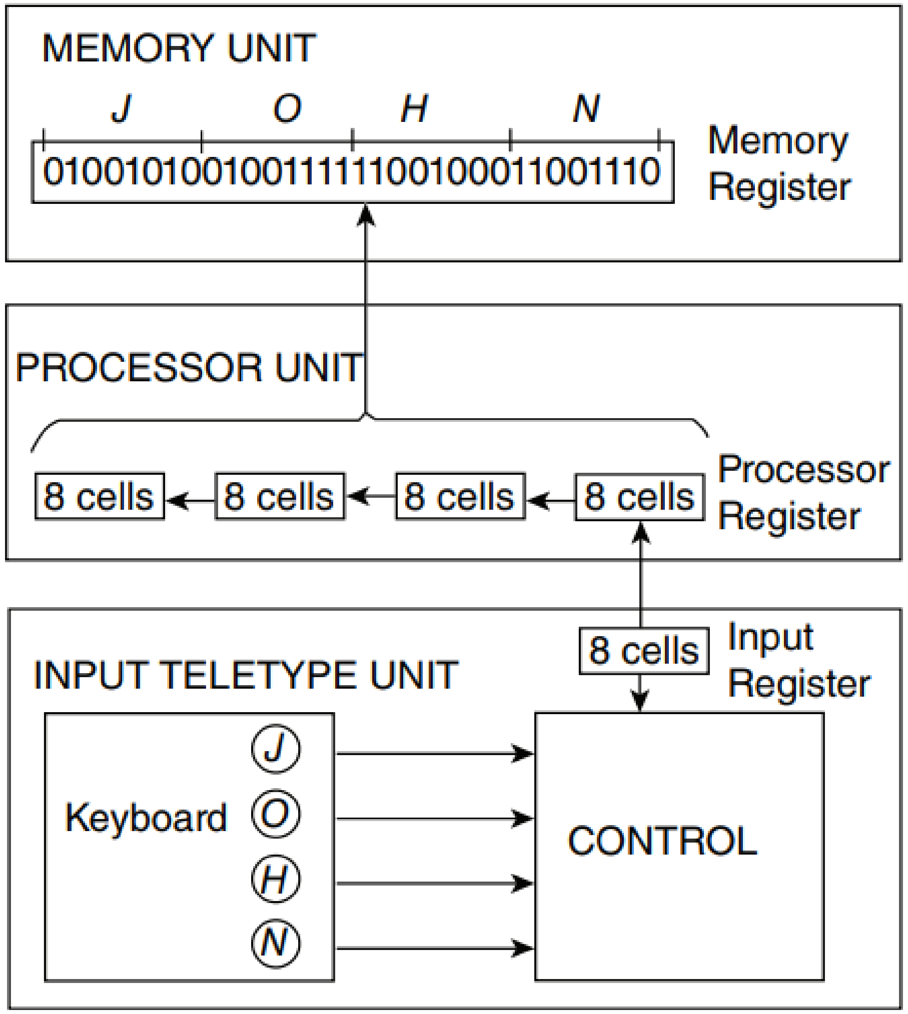


Figure 1.2 Transfer of information with registers

1.7.2 Register Transfer(b)

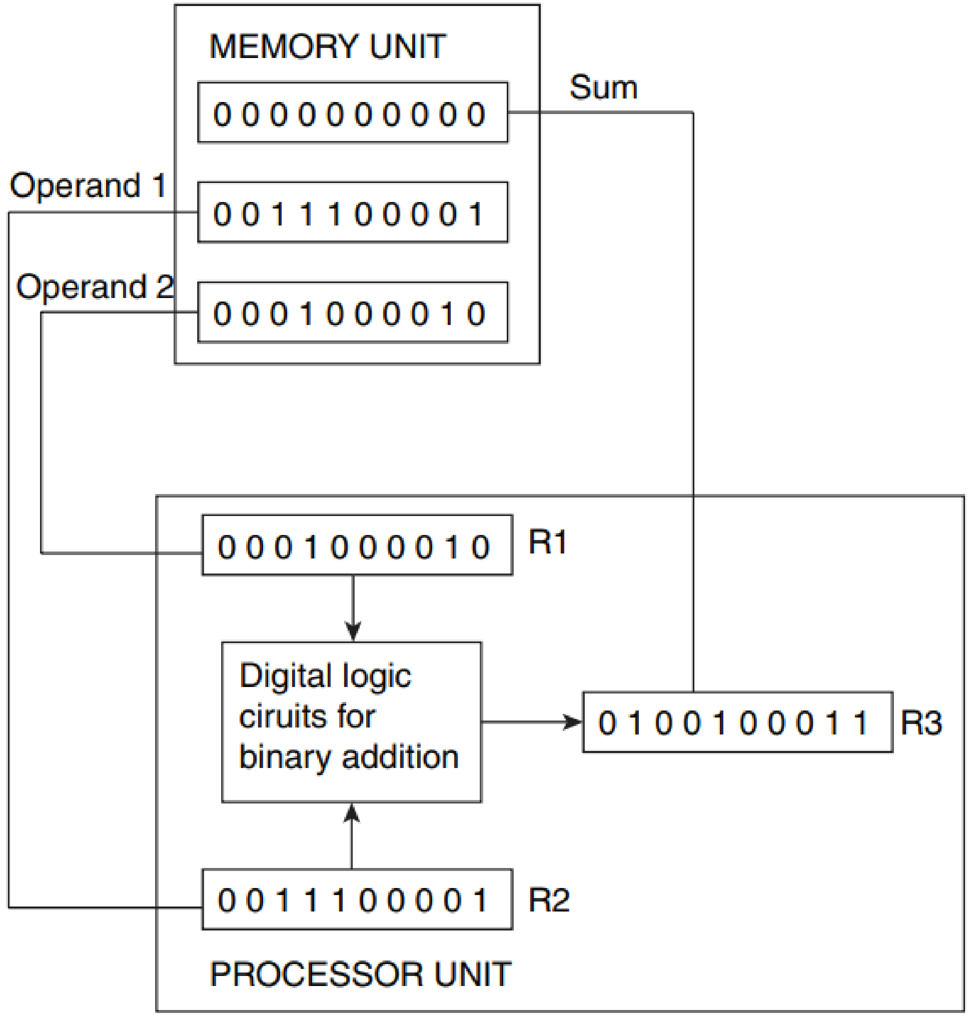


Figure 1.3 Example of binary information processing

1.8 Binary Logics

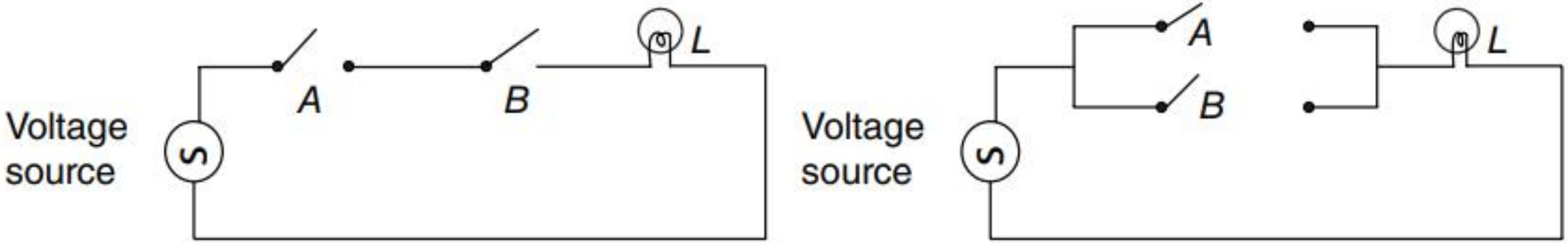
- 1.8.1 Definition of Binary Logic
- 1.8.2 Switching Circuits and Binary Signals
- 1.8.3 Logic Gates

1.8.1 Definition of Binary Logic

Table 1-6 Truth tables of logical operations

AND			OR			NOT	
x	y	$x \cdot y$	x	y	$x + y$	x	x'
0	0	0	0	0	0	0	1
0	1	0	0	1	1	1	0
1	0	0	1	0	1		
1	1	1	1	1	1		

1.8.2 Switching Circuits and Binary Signals



(a) Switches in series – logic AND (b) Switches in parallel – Logic OR

Figure 1.4 Switching circuits that demonstrate binary logic

1.8.3 Logic Gates

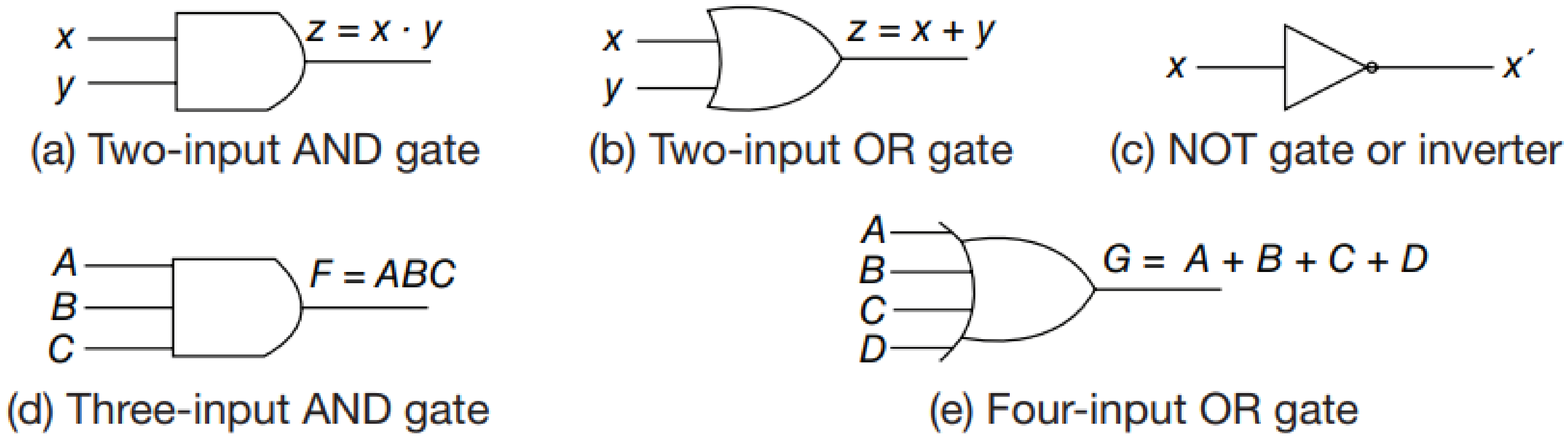


Figure 1.6 Symbols for digital logic circuits

1.9 Integrated Circuits(a)

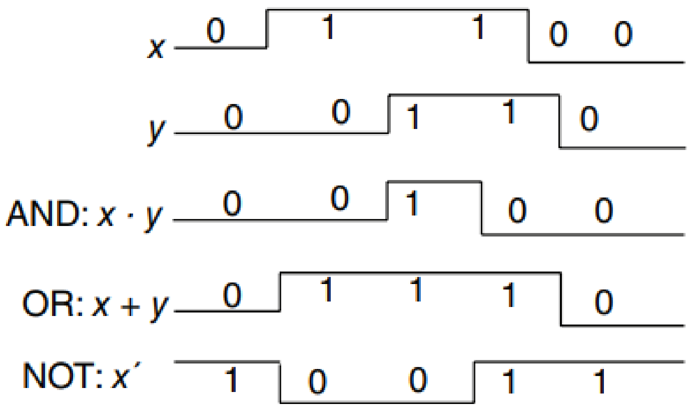


Figure 1.7 Input-output signals for gates (a), (b), and (c) of Fig. 1.6

1.9 Integrated Circuits(b)

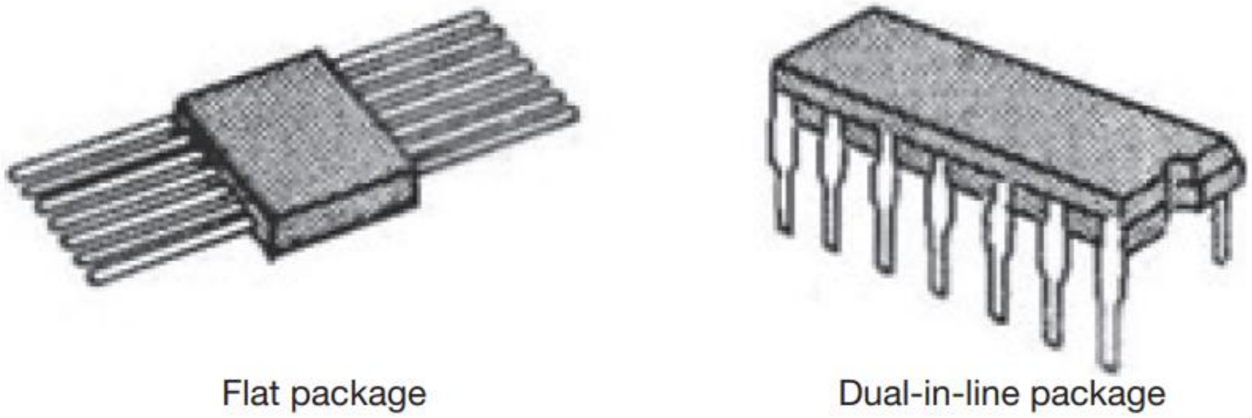


Figure 1.8 Integrated-circuit packages

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