Name: ____________

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Roll No: ___________

Section: ___________

UET Lahore
Department of Computer Sciences

Mid Examination

Date & Time: 15-11-2021, 12:00PM
Instructor Name: Mr. Nazeef Ul Haq
Program/Semester: BSCS-Section A, B, C
Course Code: CSC-270
Course Title: Discrete Mathematics
Time Allowed: 60 Minutes
Max Marks: 30

Attempt All Questions. If anyone is found in cheating case, his/her exam will be cancelled and awarded zero marks. Also, marks will be deducted from class participation.

Question 1: Prove either argument is true or not.
If at least one of these two numbers is divisible by 6, then the product of these two numbers is divisible by 6. Neither of these two numbers is divisible by 6. Therefore the product of these two numbers is not divisible by 6.
Solution: Let d = at least one of these two numbers is divisible by 6.
p = product of these two numbers is divisible by 6.
Then the argument become in these symbols
$d \to p$
$\sim d$
$\therefore \sim p$
We will made the truth table for premises and conclusion as given below

d	p	$d \to p$	$\sim d$	$\sim p$
T	T	T	F	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

The Argument is invalid.

Question 2: In a school, 100 students have access to three software packages, A, B and C
28 did not use any software
8 used only packages A
26 used only packages B
7 used only packages C
10 used all three packages
13 used both A and B
Draw a Venn diagram with all sets enumerated as for as possible.
Label the two subsets which cannot be enumerated as x and y, in any order.

(ii) If twice as many students used package B as package A, write down a pair of simultaneous equations in x and y.
(iii) Solve these equations to find x and y.
(iv) How many students used package C?

Solution:

SOLUTION:

We are given
students using package B = 2 (# students using package A)
Now the number of students which used package B and A are clear from the diagrams given below. So we have the following equation

$$\Rightarrow 3 + 10 + 26 + y = 2\,(8 + 3 + 10 + x)$$
$$\Rightarrow 39 + y = 42 + 2x$$
$$\text{or} \quad y = 2x + 3 \quad \text{............(1)}$$

Also, total number of students = 100.
Hence, $8 + 3 + 26 + 10 + 7 + 28 + x + y = 100$
or $82 + x + y = 100$
or $x + y = 18$ ....(2)

(iii)Solving simultaneous equations for x and y.

SOLUTION:

$y = 2x + 3$ ....(1)
$x + y = 18$ ....(2)

Using (1) in (2), we get,

$$x + (2\,x + 3) = 18$$
$$\text{or} \quad 3x + 3 = 18$$
$$\text{or} \quad 3x = 15$$
$$\Rightarrow x = 5$$

Consequently $y = 13$

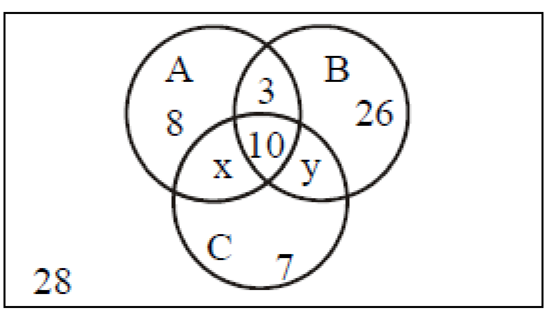
How many students used package C?

SOLUTION:

No. of students using package C
$= x + y + 10 + 7$
$= 5 + 13 + 10 + 7$
$= 35$

SOLUTION(i)

Venn Diagram with all sets enumerated.



(ii) If twice as many students used package B as package A, write down a pair of simultaneous equations in x and y.

Question 3: Write inclusion-exclusion principle for three sets A, B, and C.
Solution: The inclusion-exclusion principle for three sets will be following.

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|.$$

Question 04: Question 04: Suppose there are *n* people in a group, each aware of a scandal no one else in the group knows about. These people communicate by telephone; when two people in the group talk, they share information about all scandals each knows about. For example, on the first call, two people share information, so by the end of the call, each of these people knows about two scandals. The **gossip problem** asks for *G(n)*, the minimum number of telephone calls that are needed for all *n* people to learn about all the scandals.

i. Find G(1), G(2), G(3), and G(4).
ii. Use mathematical induction to prove that $G(n) \leq 2n - 4$ for $n \geq 4$. [*Hint:* In the inductive step, have a new person call a particular person at the start and at the end.]

Solution:

i. G(1) = 0, G(2) = 1, G(3)= 3, and G(4) = 4
ii. The base step is given in the previous question: $G(4) \leq 4 = 2 .4 - 4$.
For the inductive step we use as hypothesis that $G(n) \leq 2n - 4$. That is, that a group of n persons can share all their gossips with at most 2n - 4 calls. Suppose now, that there are n+1, v1, v2, …, vn, vn+1, each with one gossip to share. One way to spread all rumors is the following: First, let vn+1 call v1. Now, v1 knows two gossips. Second, let v1, …, vn communicate with G(n) calls and share all gossips. Now, each vi, $1 \leq i \leq n$ knows all gossips, including the one from vn+1. Last, let vn+1 call v1 (or any other person) for it to learn the n - 2 missing gossips. This means that $G(n + 1) \leq G(n) + 2 = 2(n + 1) - 4$: The proof by induction is complete.

Question 05: Some airline tickets have a 15-digit identification number $a1a2 \ldots a15$ where $a15$ is a check digit that equals $a1a2 \ldots a14$ **mod** 7. Determine whether this given 15-digit numbers 101333341789013 is a valid airline ticket identification number.

Solution:

10133334178901 = 7 · 1447619168414 + 3. Therefore 10133334178901 **mod** 7 = 3 = a 15 , so this is a valid airline ticket number.