Week 9

PROBABILITY THEORY

1.1 Random Experiment

An experiment is an operation whose output cannot be predicted with certainty. If in each trail of an experiment conducted under identical conditions, the outcome is not unique, but may be any one of the possible outcomes, then such an experiment is called Random Experiment.

1.2 Sample Space

A sample space can be defined as the set of all possible outcomes of an experiment and is denoted by S. The set S = {E1, E2, E3,..., En} is called a sample space of an experiment satisfying the following two conditions

(i) Each element of the set S denotes one of the possible outcomes

(ii) The outcome is one and only one element of the set S whenever the experiment is performed. For example, in a tossing a coin Sample space consists of head and tail S={H,T} and the two coins are tossed then the sample space S ={HH,HT,TH,TT}.

1.3 Trail and Events

Any particular performance of a random experiment is called trail and the outcome or combinations of outcomes are termed as event.

1.4 Exhaustive Events

The total number of possible outcome of a random experiment is known as the exhaustive events. For example, in a tossing a coin head and tail are the two exhaustive cases. In drawing two cards from a pack of cards, the exhaustive number of cases is $^{52}C_2$, since 2 cards can be drawn out of 52 cards in $^{52}C_2$ ways.

1.5 Favourable Events

The number of cases favourable to an event in a trail is the number of outcomes which entail the happening of the event. For example, in throwing of two dice, the number of cases favourable to getting the sum 5 is (2,3),(3,2),(1,4) and (4,1)

1.6 Mutually Exclusive Events

Events are said to be mutually exclusive or incompatible if the happening of any one of them precludes the happening of all the others, i.e., if no two or more of them can happen simultaneously in the same trail. For example, in tossing a coin, both head and tail cannot occur in a single trail.

1.7 Equally Likely Events

Outcomes of a trail are said to be equally likely if taking into consideration all the relevant evidences, there is no reason to expect one in preference to the others. For example, in tossing a coin, getting a head and tail are equally likely events.

1.8 Independent Events

Several events are said to be independent if the happening of an event is not affected by the supplementary knowledge concerning the occurrence of any number of the remaining events. For example, in tossing a unbiased coin, the event of getting a head in the first toss is independent of getting a head in the second, third and subsequent throws.

1.9 Algebraic Operations of Events

For events A, B, C, then

(i) $(A\cup B) = \{\omega\in S: \omega\in A \text{ or } \omega\in B\}$

(ii) $(A\cap B) = \{\omega\in S: \omega\in A \text{ and } \omega\in B\}$

(iii) Ac or A (A complement) = $\{\omega\in S: \omega\notin A\}$

(iv) $A-B= \{\omega\in S: \omega\in A \text{ but } \omega\notin B\}$

(v) $A \subset B \Rightarrow$ for every $\omega\in A, \omega\in B$

(vi) $B\supset A \Rightarrow A \subset B$

(vii) A=B if and only if A and B have same elements, *i.e.*, $A \subset B$ and $B \subset A$

(viii) $A\cup B$ can be denoted by A+B if A and B are disjoint.

(ix) A and B are disjoint (mutually exclusive) $\Rightarrow A\cap B =\phi$

Notes:**Algebra of Sets**

Commutative law $A\cup B = B\cup A,\ A\cap B = B\cap A$

Associative law $A\cup(B\cup C) = (A\cup B)\cup C$, $A\cap(B\cap C) = (A\cap B)\cap C$

Distributive Laws $A\cap(B\cup C) = (A\cap B)\cup(A\cap C)$,

$$A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$$

Complementary law $A\cup \bar{A} = S$, $A\cap \bar{A} = \phi$, $A\cup S=S$, $A\cap S= A$,

$$A\cup\phi = A,\ A\cap\phi = \phi,\ \bar{\bar{A}}=A,\ \bar{S} = \phi,\ \bar{\Phi} = S,$$

Difference law $A-B = A\cap \bar{B}$. $A-B = A-(A\cap B) =(A\cup B)-B$

$$A-(B-C) = (A-B)\cup(A-C) ,\ (A\cup B)-C = (A-C)\cup(B-C)$$

DeMorgan's Law

$(A\cup B)^c = A^c \cap B^c$, $(A\cap B)^c = A^c \cup B^c$

1.10 MATHEMATICAL (OR CLASSICAL OR PRIORI) PROBABILITY

If a random experiment or a trail results in‘n’ exhaustive, mutually exclusive and equally likely outcomes out of which ‘m’ are favourable to the occurrence of an event E, then the probability ‘P’ of occurrence of E , usually denoted by P(E), is given by

$$P(E) = \frac{\text{Number of favourable Cases}}{\text{Total number of exhaustive cases}} = \frac{m}{n}$$

Example: A bag contains 4 red and 6 green balls out of which 3 balls are drawn: Find the probability of drawing

i) 2 red and 1 green balls. ii) all red balls.

iii) one green ball. iv) no red ball.

Solution:

Red	Green	Total
4	6	10

Balls to be drawn = 3

$$\text{Sample space} = \binom{10}{3} = 120$$

i) Let A be the event of drawing 2 red and one green ball

$$n(A) = \binom{4}{2}\binom{6}{1} = 36$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{36}{120} = 0.30$$

ii) Let B be the event of drawing all red balls.

$$n(B) = \binom{4}{3}\binom{6}{0} = 4$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{4}{120} = 0.033$$

iii) Let C be the event of drawing one green ball

$$n(C) = \binom{4}{2}\binom{6}{1} = 36$$

$$P(C) = \frac{n(C)}{n(S)} = \frac{36}{120} = 0.30$$

iv) Let D be the event of drawing no red ball

$$n(D) = \binom{4}{0}\binom{6}{3} = 20$$

$$\therefore P(D) = \frac{n(D)}{n(S)} = \frac{20}{120} = 0.17$$

Example: If two fair dice are thrown, what is the probability of getting

i) a double six. ii) a sum of 8 or more dots.

Solution: Sample space is given by

$$S = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),$$
$$(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),$$
$$(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),$$
$$(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),$$
$$(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),$$
$$(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$$

$$\Rightarrow \quad n(S) = 36$$

i) Let A be the event that a double six occurs

$$A = \{(6,6)\} \Rightarrow n(A) = 1$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{36}$$

ii) Let B be the event that a sum 8 or more dots occurs.

$$B = \begin{matrix}\{(2,6),(3,5),(4,4),(5,3),(6,2),(3,6)(4,5)\ (4,6) \\ (5,4),(5,5),(5,6),(6,3),(6,4),(6,5),(6,6)\}\end{matrix}$$

$$\Rightarrow n(B) = 15$$

$$\therefore\ P(B) = \frac{n(B)}{n(S)} = \frac{15}{36} = \frac{5}{12}$$

Example 6.8: Six white balls and four black balls which are indistinguishable apart from colour, are placed in a bag. If six balls are taken from the bag, find the probability that their being three white and three black.

Solution:

White	Black	Total
6	4	10

$$\Rightarrow n(S) = \binom{10}{6} = 210$$

Let A be the event that three white and three black balls are taken

$$n(A) = \binom{6}{3}\binom{4}{3} = 80$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{80}{210} = \frac{8}{21}$$

Example: A fair die is tossed. Find the probability that the number on the uppermost face is not six.
Solution: Sample space is given by

$$S = \{1,2,3,4,5,6\} \Rightarrow n(S) = 6$$

Let A be the event that the uppermost face is 6 and $\bar{A}$ be the event that face is not 6. Then

$$A = \{6\} \Rightarrow n(A) = 1$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{6} \text{ and } P(\bar{A}) = 1 - P(A) = 1 - \frac{1}{6} = \frac{5}{6}$$

1.11 STATISTICAL (OR EMPIRICAL) PROBABILITY

If an experiment is performed repeatedly under essentially homogeneous and identical conditions, then the limiting value of the ratio of the number of times the event occurs to the numbers of the trails, as the number of trails becomes infinitely large, is called the probability of happening of the event, it begin assumed that the limit is finite and unique. Symbolically, if in N trails an event E happens M times, then the probability of the happening of E, denoted by P(E) is given by

$$P(E) = \lim_{N \to \infty} \frac{M}{N}$$

1.12 AXIOMS OF PROBABILITY

The axioms approach was given by A.N Kolmogrov. With each event E_i in a finite sample space S, associate a real number, say $P(E_i)$ called the probability of an event E_i satisfying the conditions:

(i) ***Nonnegative****:* $0 \leq P(E_i) \leq 1$.

This implies that the probability of an event is always non-negative and can never exceed. If P (A) = 1, the event A is certainly going to happen and if P (A) = 0, the event is certainly not going to happen (impossible event).

(ii) ***Certainty*** **:** The probability of the sample space is 1. P(S) =1,

(iii) **Union** : If $\{A_n\}$ is any finite or infinite sequence of disjoint events in B, then

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i) \qquad \text{(axioms of additivity)}$$

1.13 Theorems on Probability

Theorem 1.1 Probability of the impossible event is zero, *i.e.,* $P(\phi) = 0$

Proof:

Impossible event contains no sample point and hence the certain event S and the impossible event ϕ are mutually exclusive.

$$\therefore S \cup \phi = S \Rightarrow P(S \cup \phi) = P(S)$$

$$\Rightarrow P(S \cup \phi) = P(S) + P(\phi) \text{ using Axiom (iii) of probability}$$

$$P(S) = P(S) + P(\phi)$$

$$\Rightarrow P(\phi) = P(S) - P(S) = 0$$

$$\Rightarrow P(\phi) = 0$$

Theorem 1.2 Probability of the complementary event $\overline{A}$ of A is given by $P(\overline{A}) = 1 - P(A)$

Proof :

A and $\overline{A}$ are mutually disjoint events, so that $A \cup \overline{A} = S$

$$\Rightarrow P(A \cup \overline{A}) = P(S) \text{ from axioms (ii) and (iii)}$$

$$P(A) + P(\overline{A}) = P(S)$$

$$\Rightarrow P(A) + P(\overline{A}) = 1$$

$$\Rightarrow P(\overline{A}) = 1 - P(A)$$

$$P(\overline{A}) = 1 - P(A)$$

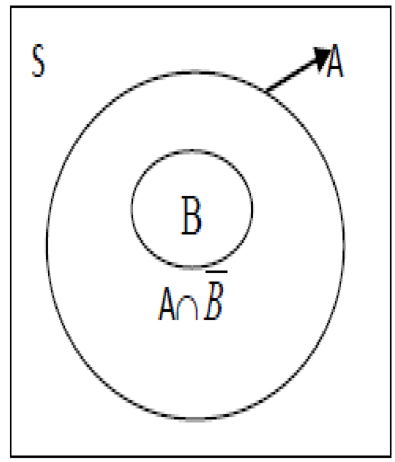
Theorem 1.3 If $B \subset A$, then

$$\text{(i) } P(A \cap \overline{B}) = P(A) - P(B) \quad \text{(ii) } P(B) \le P(A)$$

Proof

(i) When $B \subset A$, B and $A \cap \overline{B}$ are mutually exclusive

Events so that $A = B \cup (A \cap \overline{B})$



$\Rightarrow P(A) = P[B \cup (A \cap \overline{B})]$

$= P(B) + P(A \cap \overline{B})$ by axioms(iii)

$\Rightarrow P(A) - P(B) = P(A \cap \overline{B})$

$\therefore P(A \cap \overline{B}) = P(A) - P(B)$

(ii) $P(A \cap \overline{B}) \ge 0 \Rightarrow P(A) - P(B) \quad \ge 0 \Rightarrow P(A) \ge P(B)$

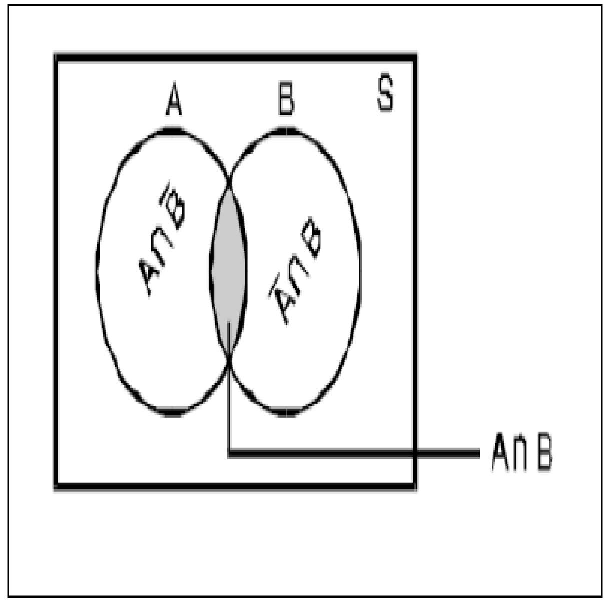
Hence $B \subset A \Rightarrow P(B) \le P(A)$

Theorem 1.4 If A and B are independent events, Prove that

(i) $\overline{A}$ and B are independent

(ii) A and $\overline{B}$ are independent and

(iii) $\overline{A}$ and $\overline{B}$ are independent



Proof : If A and B are independent events,

then $P(A\cap B) = P(A) .P(B)$

(i) From the diagram

$B = (A\cap B)\cup(\overline{A}\cap B)$

Also $(A\cap B)$ and $(\overline{A}\cap B)$ are mutually exclusive

$P(B) = P[(A\cap B)\cup(\overline{A}\cap B)]$

$= P (A\cap B) + P (\overline{A}\cap B)$

$P(B) - P (A\cap B) = P (\overline{A}\cap B)$

$P (\overline{A}\cap B) = P(B) - P (A\cap B) = P(B) - P (A) P(B)$

$= P(B) [1–P (A)] = P(B) P (\overline{A})$

$\therefore P (\overline{A}\cap B) = P(B) P (\overline{A})$ $\overline{A}$ and B are independent

(ii) From the diagram

$A = (A \cap \overline{B}) \cup (A \cup B)$

Also $(A \cap \overline{B})$ and $(A \cup B)$ are mutually exclusive

$P(A) = P[(A \cap \overline{B}) \cup (A \cap B)] = P(A \cap \overline{B}) + P(A \cap B)$

$P(A) - P(A \cap B) = P(A \cap \overline{B})$

$P(A \cap \overline{B}) = P(A) - P(A \cap B) = P(A) - P(A)P(B) = P(A)[1-P(B)]$

$$P(A \cap \overline{B}) = P(A)P(\overline{B})$$

$\therefore$ A and $\overline{B}$ are independent

(iii) $\overline{A}$ and $\overline{B}$ are independent

DeMorgan's Law, $\overline{A} \cap \overline{B} = \overline{A \cup B}$

$$\begin{aligned} P(\overline{A} \cap \overline{B}) = P(\overline{A \cup B}) = 1-P(A \cup B) &= 1-[P(A)+P(B)-P(A \cap B)] \\ &= 1-[P(A)+P(B)-P(A)P(B)] \\ &= 1-P(A)-P(B)+P(A)P(B) \\ &= 1-P(A)-P(B)[1-P(A)] \\ &= [1-P(A)][1-P(B)] = P(\overline{A})\, P(\overline{B}) \end{aligned}$$

$P(\overline{A} \cap \overline{B}) = P(\overline{A})\, P(\overline{B})$

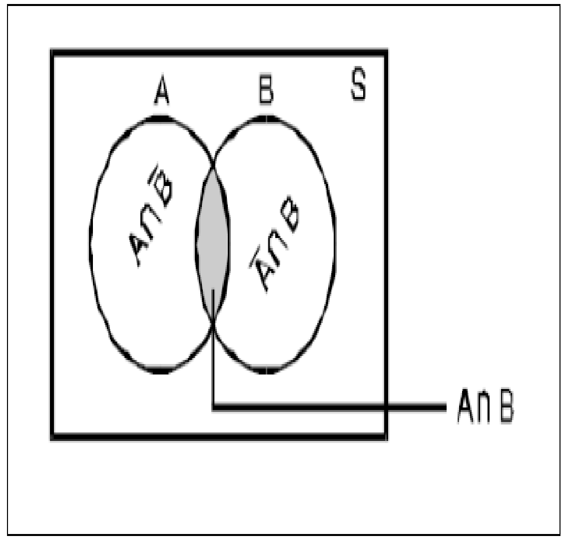
$\therefore$ $\overline{A}$ and $\overline{B}$ are independent

Addition Theorem For Not Mutually Exclusive Events

1.14 Addition Theorem of Probability

If A and B are any two events and are not disjoint, then $P(A\cup B) = P(A)+P(B)-P(A\cap B)$

Proof:



From the Venn diagram

Let $A = [(A\cap\bar{B})\cup(A\cap B)]$

$P(A) = P[(A\cap\bar{B})\cup(A\cap B)]$ using axiom (iii)

$\Rightarrow P(A) = P(A\cap\bar{B})+P(A\cap B)$ ....(1)

Let $B = [(\bar{A}\cap B)\cup(A\cap B)]$

$P(B) = P[(\bar{A}\cap B)\cup(A\cap B)]$ using axiom (iii)

$\Rightarrow P(B) = P(\bar{A}\cap B)+P(A\cap B)$ .....(2)

From (1)+(2), we get

$$\Rightarrow P(A)+P(B) = P(A\cap\bar{B})+P(A\cap B)+P(\bar{A}\cap B)+P(A\cap B)$$

$$=P(A\cup B)+P(A\cap B)$$

$$\Rightarrow P(A)+P(B) - P(A\cap B) = P(A\cup B)$$

$$\therefore \mathbf{P(A\cup B) = P(A)+P(B)-P(A\cap B)}$$

Similarly for the three events

$$P(A\cup B\cup C) = P(A)+P(B)+PC)-P(A\cap B)-P(A\cap C)-P(B\cap C)+P(A\cap B\cap C)$$

Example 6.10: A coin is tossed twice, points of the sample space are HH, HT, TH, TT and each sample point with probability $\frac{1}{4}$.

If A and B are the events that head at first coin and tail on second coin respectively. Then find $P(A \cup B)$.

Solution: Sample space is given by

$$S = \{HH, HT, TH, TT\} \Rightarrow n(S) = 4$$

$$\therefore \quad A = \{HH, HT\} \quad \Rightarrow n(A) = 2$$

$$\therefore \quad P(A) = \frac{n(A)}{n(S)} = \frac{2}{4}$$

$$B = \{TT, HT\} \Rightarrow n(B) = 2$$

$$\therefore \quad P(B) = \frac{n(B)}{n(S)} = \frac{2}{4}$$

$$A \cap B = \{HT\} \Rightarrow n(A \cap B) = 1$$

$$\therefore \quad P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{4}$$

$$\text{and } P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{2}{4} + \frac{2}{4} - \frac{1}{4} = \frac{3}{4}$$

Example 6.11: Two dice are rolled. If A and B are respectively the events that the sum of points is 8 and both dice should give odd numbers, Then find $P(A \cup B)$.

Solution: Sample space is shown in the example 6.7, where we see

$$n(S) = 36$$

$$A = \{(2,6), (3,5), (4,4), (5,3), (6,2)\} \Rightarrow n(A) = 5$$

$$\therefore \quad P(A) = \frac{n(A)}{n(S)} = \frac{5}{36}$$

$$\therefore B = \{(1,1), (1,3), (1,5), (3,1), (3,3), (3,5), (5,1), (5,3), (5,5)\} \Rightarrow n(B) = 9$$

$$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{9}{36}$$

$$A \cap B = \{(3,5), (5,3)\} \Rightarrow n(A \cap B) = 2$$

$$\therefore P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{2}{36}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{5}{36} + \frac{9}{36} - \frac{2}{36} = \frac{1}{3}$$

Addition Theorem for Mutually Exclusive Events

If A and B are two mutually exclusive events then the probability either of them happening is the sum of their respective probabilities i.e.,

$$\boxed{P(A \cup B) = P(A) + P(B)}$$

Example 6.12: A pair of dice are rolled. Find the probability that the sum of the uppermost dots is either 6 or 9.

Solution: Sample space is shown in the example 6.7, where we see

$$n(S) = 36$$

Let A be the event that the sum of the uppermost dots is 6, then

$$A = \{(1,5), (2,4), (3,3), (4,2), (5,1)\} \Rightarrow n(A) = 5$$

$$\therefore\ P(A) = \frac{n(A)}{n(S)} = \frac{5}{36}$$

Let B be the event that the sum of the uppermost dots is 9, then

$$B = \{(3,6), (4,5), (5,4), (6,3)\} \Rightarrow n(B) = 4$$

$$\therefore\ P(B) = \frac{n(B)}{n(S)} = \frac{4}{36} = \frac{1}{9}$$

Since A and B are mutually exclusive events.

$$\therefore P(A \cup B) = P(A) + P(B) = \frac{5}{36} + \frac{4}{36} = \frac{1}{4}$$

Example 6.13: A pair of dice is thrown. Find the probability of getting a total of either 5 or 11.

Solution: Sample space is shown in the example 6.7, where we see

$$n(S) = 36$$

Let A be the event that a total of 5 occurs, then

$$A = \{(1,4),(2,3),(3,2),(4,1)\} \Rightarrow n(A) = 4$$

$$\therefore\ P(A) = \frac{n(A)}{n(S)} = \frac{4}{36} = \frac{1}{9}$$

Let B be the event that a total of 11 occurs.

$$B = \{(5,6),(6,5)\} \Rightarrow n(B) = 2$$

$$\therefore\ P(B) = \frac{n(B)}{n(S)} = \frac{2}{36} = \frac{1}{18}$$

As events A and B are mutually exclusive, then

$$P(A \cup B) = P(A)+P(B) = \frac{1}{9}+\frac{1}{18} = \frac{1}{6}$$

Example 6.14: Three horses A, B and C are in a race. A is twice as likely to win as B and B is twice as likely to win as C, then

i) What are their respective chance of winning.

ii) What is the probability that B or C wins

Solution: Let $P(C) = P$

$$P(B) = 2P(C) = 2P$$
$$P(A) = 2P(B) = 2(2P) = 4P$$

Activate Window

i) Since A, B and C are mutually exclusive and collectively exhaustive events. Therefore, the probabilities must be equal to one i.e.,

$$P(A)+P(B)+P(C) = 1$$
$$\Rightarrow 4P+2P+P = 1$$
$$\Rightarrow 7P = 1$$
$$\Rightarrow P = \frac{1}{7}$$

$$\therefore P(A)=\frac{4}{7},\ P(B)=\frac{2}{7} \text{ and } P(C)=\frac{1}{7}$$

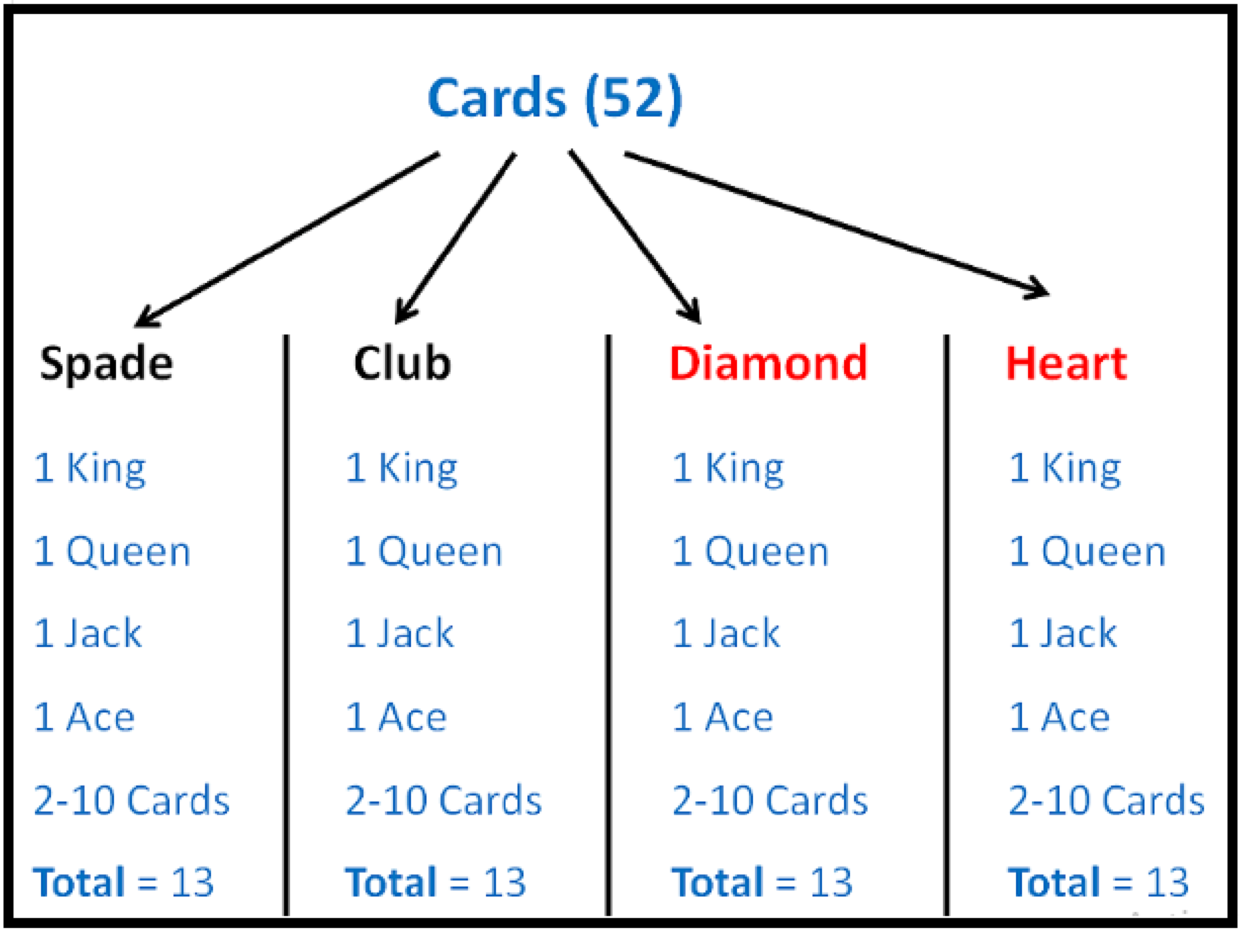
ii) As B and C are mutually exclusive events, so

$$P(B \cup C) = P(B)+P(C) = \frac{2}{7}+\frac{1}{7} = \frac{3}{7}$$

Example 15: A fair cion is tossed three times. What is the probability that atleast one head appears?

Ans:

P(A)=7/8



Deck of Cards Questions

- There are 52 cards in a standard deck of cards
- There are 4 of each card (4 Aces, 4 Kings, 4 Queens, etc.)
- There are 4 suits (Clubs, Hearts, Diamonds, and Spades) and there are 13 cards in each suit (Clubs/Spades are black,Hearts/Diamonds are red)
- Without replacement means the card IS NOT put back into the deck. With replacement means the card IS put back into the deck.

Example 16:

A card is drawn from a well shuffled pack of 52 cards. Find the probability of:

(i) ‘2’ of spades

(ii) a jack

(iii) a king of red colour

(iv) a card of diamond

(v) a king or a queen

(vi) a non-face card

(vii) a black face card

(viii) a black card

(ix) a non-ace

(x) non-face card of black colour

(xi) neither a spade nor a jack

(xii) neither a heart nor a red king

Solution:

In a playing card there are 52 cards.

Therefore the total number of possible outcomes = 52

(i) ‘2’ of spades:

Number of favourable outcomes i.e. ‘2’ of spades is 1 out of 52 cards.

Therefore, probability of getting ‘2’ of spade

$$P(A) = \frac{\textit{Number of favorable outcomes}}{\textit{Total number of possible outcome}}$$

= 1/52

(ii) a jack

Number of favourable outcomes i.e. ‘a jack’ is 4 out of 52 cards.

Therefore, probability of getting ‘a jack’

$$P(B) = \frac{\textit{Number of favorable outcomes}}{\textit{Total number of possible outcome}}$$

= 4/52
= 1/13

(iii) a king of red colour

Number of favourable outcomes i.e. ‘a king of red colour’ is 2 out of 52 cards.

Therefore, probability of getting ‘a king of red colour’

$$P(C) = \frac{\textit{Number of favorable outcomes}}{\textit{Total number of possible outcome}}$$

= 2/52
= 1/26

(iv) a card of diamond

Number of favourable outcomes i.e. ‘a card of diamond’ is 13 out of 52 cards.

Therefore, probability of getting ‘a card of diamond’

$$P(D) = \frac{\textit{Number of favorable outcomes}}{\textit{Total number of possible outcome}}$$

= 13/52

= 1/4

(v) a king or a queen

Total number of king is 4 out of 52 cards.

Total number of queen is 4 out of 52 cards

Number of favourable outcomes i.e. ‘a king or a queen’ is 4 + 4 = 8 out of 52 cards.

Therefore, probability of getting ‘a king or a queen’

$$P(E) = \frac{\textit{Number of favorable outcomes}}{\textit{Total number of possible outcome}}$$

= 8/52

= 2/13

(vi) a non-face card

Total number of face card out of 52 cards = 3 times 4 = 12

Total number of non-face card out of 52 cards = 52 - 12 = 40

Therefore, probability of getting ‘a non-face card’

$$P(F) = \frac{\textit{Number of favorable outcomes}}{\textit{Total number of possible outcome}}$$

= 40/52
= 10/13

(vii) a black face card:

Cards of Spades and Clubs are black cards.

Number of face card in spades (king, queen and jack or knaves) = 3

Number of face card in clubs (king, queen and jack or knaves) = 3

Therefore, total number of black face card out of 52 cards = 3 + 3 = 6

Therefore, probability of getting ‘a black face card’

$$P(G) = \frac{\textit{Number of favorable outcomes}}{\textit{Total number of possible outcome}}$$

= 6/52
= 3/26

(viii) a black card:

Cards of spades and clubs are black cards.

Number of spades = 13

Number of clubs = 13

Therefore, total number of black card out of 52 cards = 13 + 13 = 26

Therefore, probability of getting ‘a black card’

$$P(H) = \frac{\textit{Number of favorable outcomes}}{\textit{Total number of possible outcome}}$$

$= 26/52$
$= ½$

(ix) a non-ace:

Number of ace cards in each of four suits namely spades, hearts, diamonds and clubs = 1

Therefore, total number of ace cards out of 52 cards = 4

Thus, total number of non-ace cards out of 52 cards = 52 - 4

= 48

Therefore, probability of getting ‘a non-ace’

$$P(I) = \frac{\textit{Number of favorable outcomes}}{\textit{Total number of possible outcome}}$$

$= 48/52$
$= 12/13$

(x) non-face card of black colour:

Cards of spades and clubs are black cards.

Number of spades = 13

Number of clubs = 13

Therefore, total number of black card out of 52 cards = 13 + 13 = 26

Number of face cards in each suits namely spades and clubs = 3 + 3 = 6

Therefore, total number of non-face card of black colour out of 52 cards = 26 - 6 = 20

Therefore, probability of getting 'non-face card of black colour'

$$P(J) = \frac{\textit{Number of favorable outcomes}}{\textit{Total number of possible outcome}}$$

= 20/52
= 5/13

(xi) neither a spade nor a jack

Number of spades = 13

Total number of non-spades out of 52 cards = 52 - 13 = 39

Number of jack out of 52 cards = 4

Number of jack in each of three suits namely hearts, diamonds and clubs = 3

[Since, 1 jack is already included in the 13 spades so, here we will take number of jacks is 3]

Neither a spade nor a jack = 39 - 3 = 36

Therefore, probability of getting 'neither a spade nor a jack'

$$P(K) = \frac{\textit{Number of favorable outcomes}}{\textit{Total number of possible outcome}}$$

= 36/52
= 9/13

(xii) neither a heart nor a red king

Number of hearts = 13

Total number of non-hearts out of 52 cards = 52 - 13 = 39

Therefore, spades, clubs and diamonds are the 39 cards.

Cards of hearts and diamonds are red cards.

Number of red kings in red cards = 2

Therefore, neither a heart nor a red king = 39 - 1 = 38

[Since, 1 red king is already included in the 13 hearts so, here we will take number of red kings is 1]

Therefore, probability of getting ‘neither a heart nor a red king’

$$P(L) = \frac{\textit{Number of favorable outcomes}}{\textit{Total number of possible outcome}}$$

= 38/52
= 19/26

Example 17:

A card is drawn at random from a well-shuffled pack of cards numbered 1 to 20. Find the probability of

(i) getting a number less than 7

(ii) getting a number divisible by 3.

Solution:

(i) Total number of possible outcomes = 20 (since there are cards numbered 1, 2, 3, ..., 20).

Number of favourable outcomes for the event E

= number of cards showing less than 7 = 6 (namely 1, 2, 3, 4, 5, 6).

$$\text{So, } P(E) = \frac{\text{Number of Favourable Outcomes for the Event E}}{\text{Total Number of Possible Outcomes}}$$

=6/20

=3/10

(ii) Total number of possible outcomes = 20.

Number of favourable outcomes for the event F

= number of cards showing a number divisible by 3
= 6 (namely 3, 6, 9, 12, 15, 18).

So, P(F) = $\frac{\text{Number of Favourable Outcomes for the Event F}}{\text{Total Number of Possible Outcomes}}$

=6/20

=3/10

Example 18:

A card is drawn at random from a pack of 52 playing cards. Find the probability that the card drawn is

(i) a king

(ii) neither a queen nor a jack.

Solution:

Total number of possible outcomes = 52 (As there are 52 different cards).

(i) Number of favourable outcomes for the event E = number of kings in the pack = 4.

So, by definition, P(E) = 4/52
= 1/13

(ii) Number of favourable outcomes for the event F

= number of cards which are neither a queen nor a jack

= 52 - 4 - 4, [Since there are 4 queens and 4 jacks].

$= 44$

Therefore, by definition, $P(F) = 44/52$
$= 11/13$