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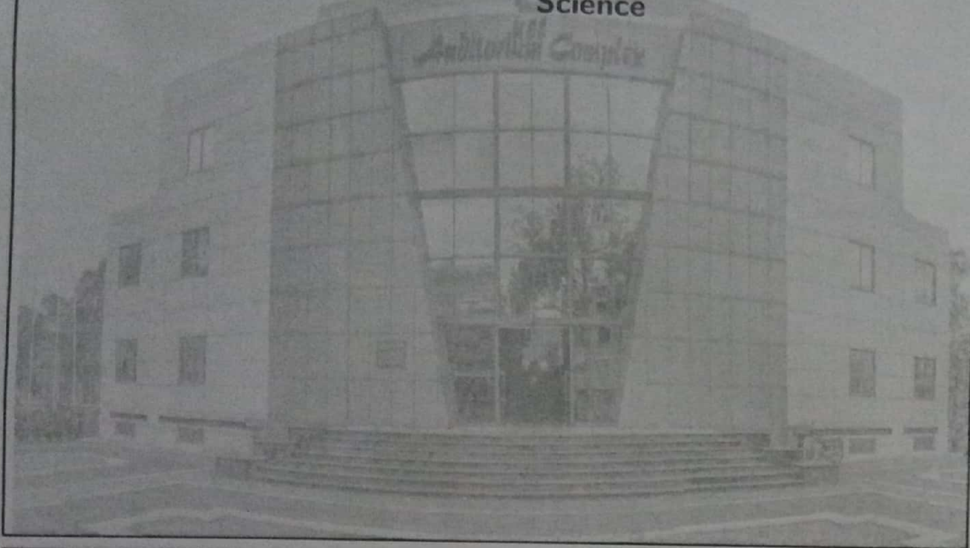
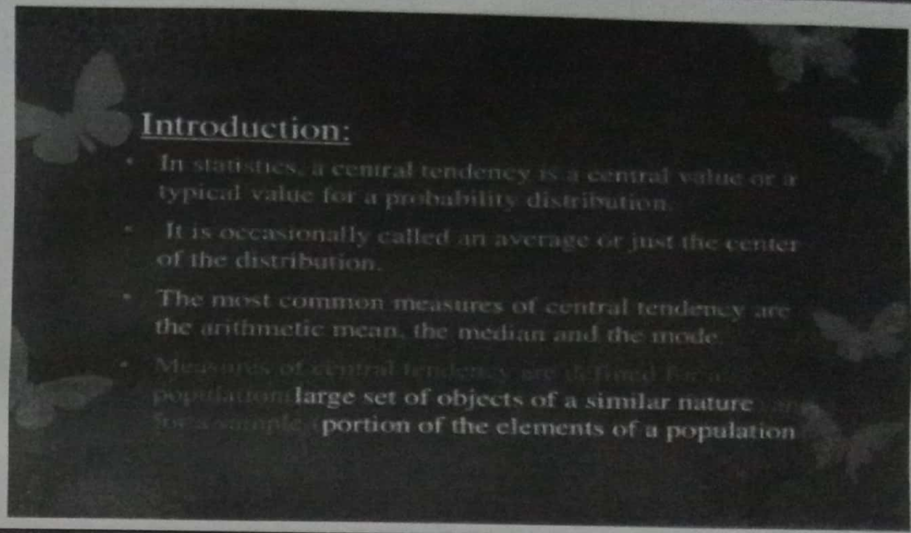


Introduction of Measures of Central Tendency OR Averages



Introduction:

- In statistics, a central tendency is a central value or a typical value for a probability distribution.
- It is occasionally called an average or just the center of the distribution.
- The most common measures of central tendency are the arithmetic mean, the median and the mode.
- Measures of central tendency are defined for a population large set of objects of a similar nature and for a sample portion of the elements of a population

Measure of Central Tendency OR Averages

Types of Measure of Central Tendency

- Arithmetic Mean
- Geometric Mean
- Harmonic Mean
- Mode
- Median

Arithmetic Mean or Simply Mean:

A value obtained by dividing the sum of all the observations by the number of observation is called arithmetic Mean.

$$\text{Mean} = \frac{\textit{Sum Mean of All observation}}{\textit{Number of observation}}$$

Formula Chart

Methods	Ungrouped data	Grouped data
Direct Method	$\bar{x} = \frac{\sum x_i}{n}$	$\bar{x} = \frac{\sum fx}{n}$; Here $n = \sum f$
Short cut Method	$\bar{x} = A + \frac{\sum D}{n}$	$\bar{x} = A + \frac{\sum fD}{n}$; Here $n = \sum f$
	Where $D = X - A$ and A is the provisional or assumed mean.	
Step deviation Method	$\bar{x} = A + \frac{\sum u}{n} \times h$	$\bar{x} = A + \frac{\sum fu}{n} \times h$; Here $n = \sum f$
	Where $u = \frac{X - A}{h}$ and h is the common width of the class intervals	

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Solution: Using formula of arithmetic mean for ungrouped data:

Example

Calculate the arithmetic mean for the following the marks obtained by 9 students are given below:

x_i
45
32
37
46
39
36
41
48
36
$\sum_{i=1}^{n} x_i = 360$

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$n = 9$$

$$\bar{x} = \frac{360}{9} = 40 \ marks$$

Formula of direct method of arithmetic mean for grouped data:

$$\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}, \quad n = \sum_{i=1}^{n} f_i$$

Example:Using formula of direct method of arithmetic mean for grouped data:

Calculate the arithmetic mean for the following data given below:

The weights recorded to nearest grams of 60 apples picked out at random,

106	111	100	98	148	123	107	92
186	110	90	115	76	86	84	78
107	90	82	70	99	185	181	158
109	126	113	162	131	118	107	68
204	178	75	125	115	130	111	140
184	128	187	139	173	110	194	95
119	123	146	80	82	93	129	141
152	104	115	136				

Direct Method

Weight (grams)	Frequency
65----84	09
85----104	10
105----124	17
125----144	10
145----164	05
165----184	04
185----204	05

Solution:

Weight (grams)	Midpoints (x_i)	Frequency (f_i)	$f_i x_i$
65----84	(65+84)/2 = 74.5	09	9×74.5=670.5
85----104	94.5	10	945.0
105----124	114.5	17	1946.5
125----144	134.5	10	1345.0
145----164	154.5	05	772.5
165----184	174.5	04	698.0
185----204	194.5	05	972.5
		$\sum_{i=1}^{n} f_i = 60$	$\sum_{i=1}^{n} f_i x_i = 7350.0$

$$\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$$

$$= \frac{7350.0}{60}$$

$= 122.5$ grams

(Answer).

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Example

Example:

Find mean days of confinement after delivery in the following series:-

Day of confinement	No. of patients
6	5
7	4
8	4
9	3
10	2

Solution:-

Day of confinement (x)	No. of patients (f)	x*f
6	5	30
7	4	28
8	4	32
9	3	27
10	2	20
Total	18	137

$$\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{n} = 137/18 = 7.61$$

Using formula of **short cut method of arithmetic mean** for grouped data:

$$\bar{x} = A + \frac{\sum_{i=1}^{n} f_i D_i}{\sum_{i=1}^{n} f_i}, \quad n = \sum_{i=1}^{n} f_i$$

Where $D_i = X_i - A$ and

A is the provisional or assumed mean

Short cut method of arithmetic mean

Weight (grams)	Midpoints (x_i)	Frequency (f_i)	$D_i = X_i - A$, $A = 114.5$	$f_i D_i$
65----84	$(65+84)/2 = 74.5$	09	-40	-360
85----104	94.5	10	-20	-200
105----124	114.5	17	0	0
125----144	134.5	10	20	200
145----164	154.5	05	40	200
165----184	174.5	04	60	240
185----204	194.5	05	80	400
		$\sum_{i=1}^{n} f_i = 60$		$\sum_{i=1}^{n} f_i D_i = 480$

$$\bar{x} = A + \frac{\sum_{i=1}^{n} f_i D_i}{\sum_{i=1}^{n} f_i} = 114.5 + \frac{480}{60} = 122.5 \text{ grams} \quad \text{(Answer).}$$

Using formula of **step deviation method** of arithmetic mean for grouped data

$\bar{x} = A + \frac{\sum_{i=1}^{n} f_i u_i}{\sum_{i=1}^{n} f_i} \times h \; u_i = \frac{x_i - A}{h}$

$$\bar{x} = A + \frac{\sum_{i=1}^{n} f_i u_i}{\sum_{i=1}^{n} f_i} \times h, \quad u_i = \frac{x_i - A}{h}$$

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Step deviation method

Weight (grams)	Midpoints (x_i)	Frequency (f_i)	$u_i = \frac{X_i - A}{h}$ $A = 114.5$, h=20	$f_i u_i$
65----84	$(65+84)/2 = 74.5$	09	-2	-18
85----104	94.5	10	-1	-10
105----124	114.5	17	0	0
125----144	134.5	10	1	10
145----164	154.5	05	2	10
165----184	174.5	04	3	12
185----204	194.5	05	4	20
		$\sum_{i=1}^{n} f_i = 60$		$\sum_{i=1}^{n} f_i u_i = 24$

$$\bar{x} = A + \frac{\sum_{i=1}^{n} f_i u_i}{\sum_{i=1}^{n} f_i} \times h \quad = 114.5 + \frac{24}{60} \times 20 \quad = 114.5 + 08 \quad = 122.5 \text{ grams} \quad \text{(Answer)}$$

Properties of Arithmetic Mean

Property 1:

★ The mean of a constant is that constant.

Proof: By definition of arithmetic mean: $\bar{x} = \frac{\sum xi}{n}$

If "c" is any constant, then $\bar{x} = \frac{\sum c}{n}$

$$\Rightarrow \bar{x} = \frac{nc}{n} \qquad (\because \sum c = nc)$$

$$\Rightarrow \bar{x} = c$$

Properties of Arithmetic Mean

Property 2:

★ The sum of deviations from mean is equal to zero. i.e. $\sum (xi - \bar{x}) = 0$

Proof: Sum of Deviation $= \sum (xi - \bar{x})$

$$= \sum xi - \sum \bar{x}$$

$$= \sum xi - n\bar{x} \qquad (\because \bar{x} \text{ is constant})$$

$$= \sum xi - n\left(\frac{\sum xi}{n}\right) \qquad \left(\because \bar{x} = \frac{\sum xi}{n}\right)$$

$$= \sum xi - \sum xi$$

$$= 0$$

Properties of Arithmetic Mean

Property 3

★ The sum of squared deviations from the mean is smaller than the sum of squared deviations from any arbitrary value or provisional mean. i.e. $\sum (xi - \bar{x})^2 < \sum (xi - A)^2$

Proof: Taking $\sum (xi - A)^2 = \sum (xi - A + \bar{x} - \bar{x})^2$

$$= \sum [(xi - \bar{x}) + (\bar{x} - A)]^2$$

$$= \sum [(xi - \bar{x})^2 + (\bar{x} - A)^2 + 2(xi - \bar{x})(\bar{x} - A)]$$

$$= \sum (xi - \bar{x})^2 + \sum (\bar{x} - A)^2 + 2\sum (xi - \bar{x})(\bar{x} - A)$$

$$= \sum (xi - \bar{x})^2 + n(\bar{x} - A)^2 + 2(\bar{x} - A)\sum (xi - \bar{x})$$

$$= \sum (xi - \bar{x})^2 + n(\bar{x} - A)^2 \qquad [\because \sum (xi - \bar{x}) = 0]$$

$$\Rightarrow \sum (xi - A)^2 < \sum (xi - \bar{x})^2 \qquad [\because n(\bar{x} - A)^2 > 0]$$

Note: If $A = \bar{x}$ Then $\sum (xi - A)^2 = \sum (xi - \bar{x})^2$

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Properties of Arithmetic Mean

Property 4:

The arithmetic mean is affected by the change of origin and scale i.e., when a constant is added to or subtracted from each value of a variable or if each value of a variable is multiplied or divided by a constant, then arithmetic mean is affected by these changes.

Variable	Mean
X_i	$\overline{X}$
$X_i \pm a$	$\overline{X} \pm a$
$a X_i$	$a \overline{X}$

Geometric Mean

The nth root of the product of "n" positive values is called geometric mean"

Ungrouped data	Grouped data
$G = Antilog\left(\frac{\sum \log x}{n}\right)$	$G = Antilog\left(\frac{\sum f \log x}{n}\right)$;Here $n = \sum f$

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