

Sir Syed University of Engineering & Technology

ANSWER SCRIPT

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Section:	A
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$$a = 0$$

$$b = 2$$

$$c = 2$$

Qno. 1 (i)

$$f(x, y) = (x + y)$$

diff w.r.t x

$$\frac{df}{dx} = 1 + 0 = 1$$

diff w.r.t y

$$\frac{df}{dy} = 0 + 1 = 1$$

Qno 1: (ii)

$$w = x^2 + y^2, \quad x = \cos t, \quad y = \sin t$$

$$\text{at } t = \pi$$

sol

$$\frac{dw}{dx} = 2x, \quad \frac{dw}{dy} = 2y$$

$$\frac{dx}{dt} = -\sin t, \quad \frac{dy}{dt} = \cos t$$

$$\Rightarrow \frac{dw}{dt} = \frac{dw}{dx} \cdot \frac{dx}{dt} + \frac{dw}{dy} \cdot \frac{dy}{dt}$$

$$\frac{dw}{dt} = (2x)(-\sin t) + (2y)(\cos t)$$

$$\begin{cases} x = \cos t \\ y = \sin t \end{cases}$$

$$\frac{dw}{dt} = -2\cos t \sin t + 2\sin t \cos t$$

$$\frac{dw}{dt} = 0$$

Ques: 2 (i)

$$\lim_{x \rightarrow 0} \frac{d}{(c+1)x^2} = (2+1)$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{3x^2}$$

Apply lin.

$$\frac{\sin 0}{3(0)^2}$$

$$\frac{0}{0} \text{ undefined.}$$

Apply L'Hopital Rule

$$\lim_{x \rightarrow 0} \frac{\cos x}{6x}$$

$$\lim_{x \rightarrow 0} \frac{-\sin x}{6} = \frac{0}{6}$$

Qno: 2 (ii)

$$f(x) = (b+1)(a) \pi x$$

$$f(x) = (2+1)(\sin) \pi x$$

$$= 3 \sin \pi x$$

$$f'(x) = 3\pi \cos \pi x$$

$$f''(x) = -3\pi^2 \sin \pi x$$

$$f(0) = \sin(\pi(0)) = 0$$

$$f'(0) = 3\pi \cos(\pi(0)) = 3\pi$$

$$f''(0) = -3\pi^2 \sin(\pi(0)) = 0$$

$$f(x) = f(0) + f'(0)x + \frac{x^2}{2!} f''(0)$$

$$f(x) = 0 + 3\pi x + \frac{x^2}{2!} (0)$$

$$= 3\pi x$$

Q5 (i)

$$P(0, 0, 0)$$

$$Q(1, 2, 3)$$

$$R(0, 2, 2)$$

2)

$$\overrightarrow{PQ} = (1-0)\hat{i} + (2-0)\hat{j} + (3-0)\hat{k}$$

$$\overrightarrow{PQ} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\begin{aligned}\overrightarrow{PR} &= (0-0)\hat{i} + (2-0)\hat{j} + (2-0)\hat{k} \\ &= 0\hat{i} + 2\hat{j} + 2\hat{k}\end{aligned}$$

The area of parallelogram is determined by \overrightarrow{PQ} and \overrightarrow{PR} is.

$$|\overrightarrow{PQ} \times \overrightarrow{PR}|: \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 0 & 2 & 2 \end{vmatrix}$$

$$\begin{aligned}&= \hat{i}(2 \cdot 2 - 3 \cdot 2) - \hat{j}(1 \cdot 2 - 3 \cdot 0) + \hat{k}(1 \cdot 2 - 2 \cdot 0) \\ &= \hat{i}(4 - 6) - \hat{j}(2 - 0) + \hat{k}(2 - 0) = \{-2; -2; 2\}\end{aligned}$$

$$|\vec{c}| = \sqrt{(-2)^2 + (-2)^2 + (2)^2} = \sqrt{4 + 4 + 4}$$

$$= \sqrt{12} = 2\sqrt{3}$$

Triangle Area.

$$A = \frac{1}{2} 2\sqrt{3} = \sqrt{3} \approx 1.732$$

Q 5 (ii)

~~Data:~~

~~$P = (0, 2, 2)$~~

~~$V = (0, 0, -2)$~~

Data.

$$P = (0, 2, 2)$$

$$V = (0, 0, -2)$$

Sol

$$\begin{aligned} \vec{PV} &= (0-0)\hat{i} + (0-2)\hat{j} + (-2-2)\hat{k} \\ &= 0\hat{i} + 0\hat{j} - 4\hat{k} \end{aligned}$$

is parallel to the line with

$$(x, y, z) = (0, 2, 2) \text{ given.}$$

$$x = 0 + 0t, \quad y = 2 - 2t, \quad z = 2 - 4t$$

Q4 (i)

Here, $a = 0$, $d = \cos x$

$$= \int_0^{\pi} \int_0^{\cos x} dy \, dx$$

$$= \int_0^{\pi} |y|_0^{\cos x} dx$$

$$= \int_0^{\pi} [\cos x - 0] dx = \int_0^{\pi} (0) x \, dx$$

$$= \sin(\pi) - 0$$

$$= \sin 0 - 0$$

$$= 0$$

Q4 (ii)

$$\vec{A} = a\mathbf{i} + b\mathbf{j}, \quad \vec{B} = 4\mathbf{j} + 3\mathbf{k}$$

$$a = 0$$

$$b = 2$$

$$\vec{A} = 0\mathbf{i} + 2\mathbf{j}, \quad \vec{B} = 4\mathbf{j} + 3\mathbf{k}$$

$$\text{Formula: } \theta = \cos^{-1} \left(\frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}| |\mathbf{v}|} \right)$$

$$\mathbf{A} \cdot \mathbf{B} = (0)(2) + (2)(4) + (0)(3)$$
$$0 + 8 + 0$$

Date _____

$$|A| = \sqrt{(2)^2} = \sqrt{4} = 2$$

$$|B| = \sqrt{(4)^2 + (3)^2} = \sqrt{25} = 5$$

$$\theta = \cos^{-1} \left(\frac{8}{4 \times 5} \right)$$

$$\theta = \cos^{-1} \left(\frac{8}{20} \right) = \frac{4}{5}$$

$$\theta = 36.86^\circ$$

Qno 3 (i) $a=0$ $b=2$ $c=2$.

$$\int_0^{2\pi} \cos^{-1} y \, dy.$$

$$I = \left[\cos^{-1} y \int 1 \, dy - \int \frac{1}{\sqrt{1-y^2}} \, dy \right]_0^3$$

$$= \left[y \cos^{-1} y - \sqrt{1-y^2} \right]_0^3$$

$$3 \cos^{-1}(3) - \sqrt{1-(3)^2} - 0 - \sqrt{1-(0)^2}$$

$$3 \cos^{-1}(3) - \sqrt{-8} - \sqrt{1}$$

$$3 \cos^{-1}(3) - \sqrt{-8} - 1 + C \quad A$$

Q. No 3. (ii)

$$a = 0$$

$$b = 2$$

$$c = 2$$

$$d = e^x$$

$$\Rightarrow \int_0^{\pi} (0 + 2 + 2) x^2 (e^x) dx$$

$$\Rightarrow \int_0^{\pi} 4x^2 e^x dx$$

$$\Rightarrow 4 \int_0^{\pi} x^2 e^x dx$$

$$\Rightarrow 4 [x^2 e^x - \int 2x e^x dx]$$

$$\Rightarrow 4 [x^2 e^x - 2 [x e^x - \int e^x dx]]$$

$$\Rightarrow 4 [x^2 e^x - 2 [x e^x - e^x]]$$

$$\Rightarrow 4 [x^2 e^x - 2x e^x + 2e^x]$$

Applying limit

$$\Rightarrow 4 [[\pi^2 e^{\pi} - 2\pi e^{\pi} + 2e^{\pi}]]$$

$$\Rightarrow 4 [[\pi e^{\pi} - 2\pi e^{\pi} + 2e^{\pi}] - 2]$$

$$\Rightarrow 127.27(4)$$

$$\Rightarrow 509.080 \text{ A}$$