STUDY OF A MANUFACTURING FACILITY
SYSC 5001
Project Deliverable 2

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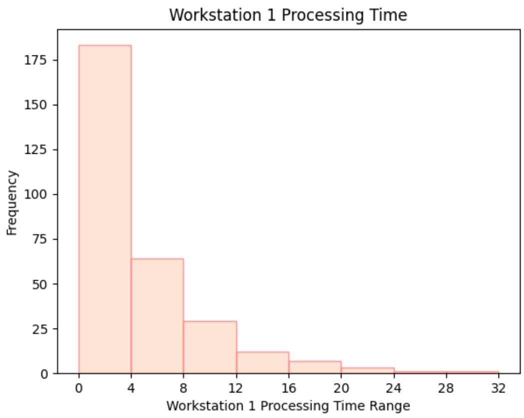
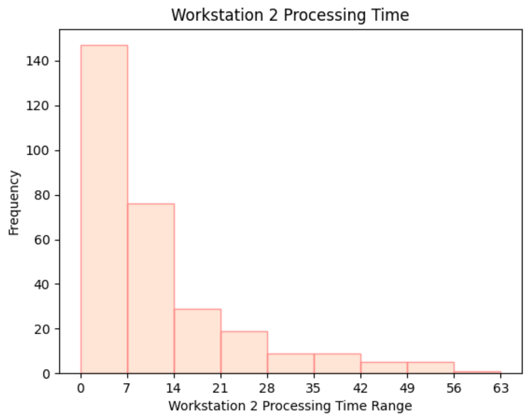
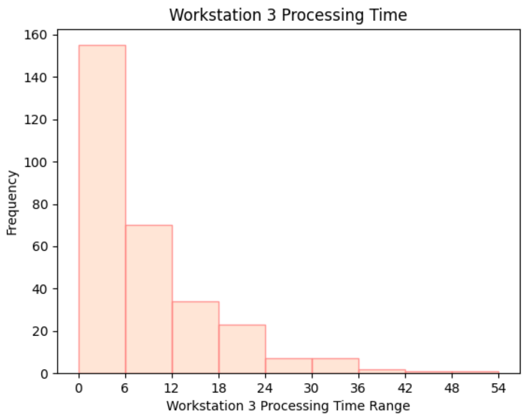
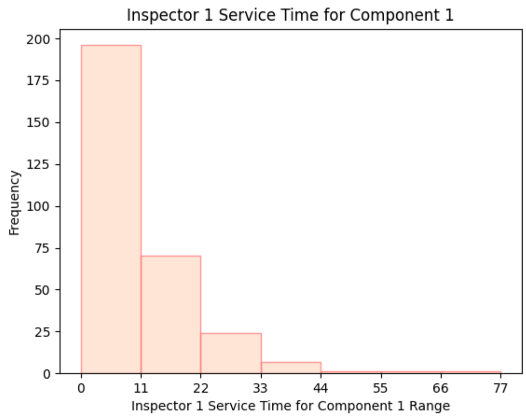
Identify Distribution (Histogram):

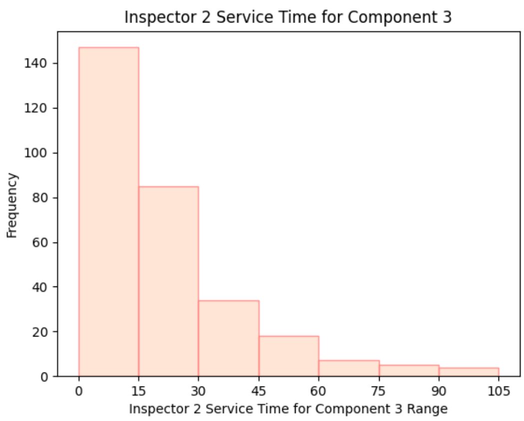
Below will be histograms for the data sets provided.

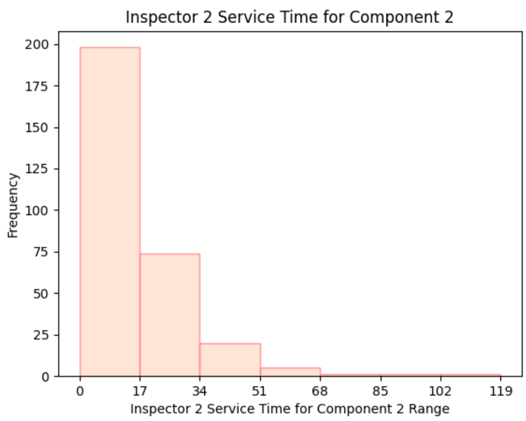












It can be seen form the distributions that all the datasets follow exponential distribution.

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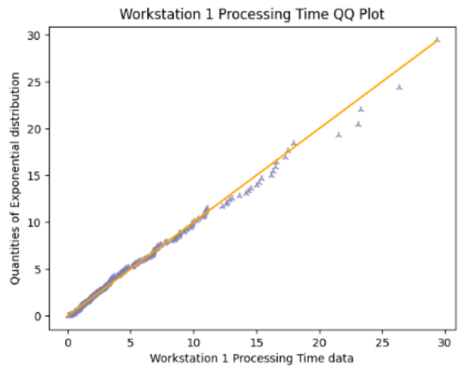
Evaluate Distributions (Q-Q Plots):

In this section the Q-Q plots will be provided based on which the evaluation of the data will be done.



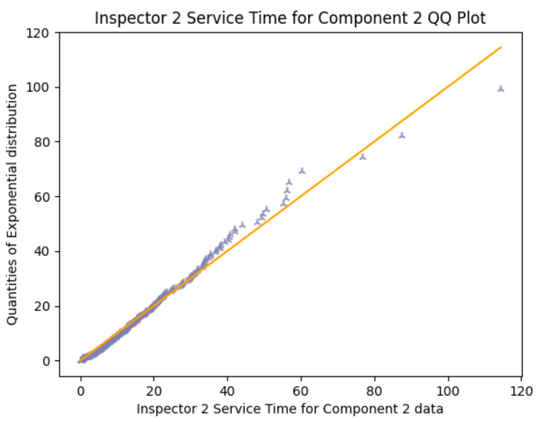
Based on the Q-Q Plot for the exponential distribution of the Workstation 1's Processing Time, we can say that "Exponential distribution" is approximately a good distribution as it mostly follows a straight line start to middle and start. Later for higher values at the ends it diverges.

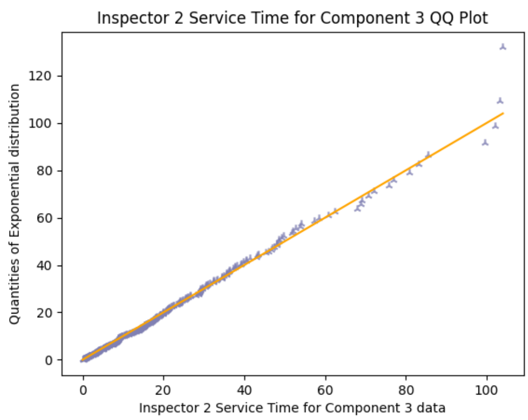
Based on the Q-Q Plot for the exponential distribution of the Workstation 2's Processing Time, we can say that "Exponential distribution" is not a very good distribution overall. This is because it only follows a straight line at the starting quantiles and later diverges from the straight line slightly in the middle and extremely towards the end. But still is a good fit.

Based on the Q-Q Plot for the exponential distribution of the Workstation 3's Processing Time, we can say that "Exponential distribution" is a very good distribution overall. It is a better fit that WS1. It follows a straight-line form start to end and barely deviates.

Based on the Q-Q Plot for Exponential distribution of Inspector 1's Service Time for Component 1 we can say it follows "Exponential distribution" as the points are mostly on straight line at the start, middle and end of the distribution.

Similarly for Inspectors 2's Service Times for Component 2 and 3, the distribution follows "Exponential Distribution" pattern as the points are mostly on a straight line at the start and middle of the distribution. For component 3 it is a much better fit as it is only slight deviating at the end while the rest is on a straight line.





Goodness of Fit (Chi-Square Test):

Workstation 1 Processing Time:

H_0: Random variable is exponential distribution.

H_1: Random variable is not exponential distribution.

	range	O	O (new)	E	E (new)	O-E	(O-E)^2/E
0	(0, 4]	183	183	174.155	174.155	8.845047	0.449226
1	(4, 8]	64	64	73.05513	73.05513	-9.05513	1.122376
2	(8, 12]	29	29	30.64542	30.64542	-1.64542	0.088346
3	(12, 16]	12	12	12.85525	12.85525	-0.85525	0.056899
4	(16, 20]	7	12	5.392564	9.00161	2.99838	0.998742
5	(20, 24]	3		2.262092			
6	(24, 28]	1		0.94891			
7	(28, 32]	1		0.398052			
						Sum:	2.715589

Degrees of freedom = k – s – 1 = 5 – 1 – 1 = 3

Level of significance = 0.05

Chi-Square (0.05,3) = 7.81

2.71 < 7.81

H_0 is accepted

Workstation 2 Processing Time:

H_0: Random variable is exponential distribution.

H_1: Random variable is not exponential distribution.

	range	O	O (New)	E	E(New)	O-E	(O-E)^2/E
0	(0, 7]	147	147	140.3903	140.3903	6.60971	0.311192
1	(7, 14]	76	76	74.69218	74.69218	1.307822	0.022899
2	(14, 21]	29	29	39.73866	39.73866	-10.7387	2.901929
3	(21, 28]	19	19	21.14225	21.14225	-2.14225	0.217065
4	(28, 35]	9	9	11.24836	11.24836	-2.24836	0.44941
5	(35, 42]	9	9	5.984493	5.984493	3.015507	1.519475
6	(42, 49]	5	11	3.183944	5.779147	5.220853	4.716492
7	(49, 56]	5		1.693961			
8	(56, 63]	1		0.901242			
						Sum:	10.138462

Degrees of freedom = k – s – 1 = 7 – 1 – 1 = 5

Level of significance = 0.05

Chi-Square (0.05,3) = 11.1

10.13 < 11.1

<u>H_0 is accepted</u>

Workstation 3 Processing Time:

H_0: Random variable is exponential distribution.

H_1: Random variable is not exponential distribution.

	range	O	O (New)	E	E (New)	O-E	(O-E)^2/E
0	(0, 6]	155	155	148.343	148.343	6.657041	0.298741
1	(6, 12]	70	70	74.99085	74.99085	-4.99085	0.332155
2	(12, 18]	34	34	37.90963	37.90963	-3.90963	0.403202
3	(18, 24]	23	23	19.16421	19.16421	3.835791	0.767748
4	(24, 30]	7	7	9.687958	9.687958	-2.68796	0.745783
5	(30, 36]	7	11	4.89749	9.257559	1.742441	0.327959
6	(36, 42]	2		2.475796			
7	(42, 48]	1		1.251573			
8	(48, 54]	1		0.6327			
						Sum:	2.875588

Degrees of freedom = k – s – 1 = 6 – 1 – 1 = 4

Level of significance = 0.05

Chi-Square (0.05,3) = 9.49

2.87< 9.49

<u>H_0 is accepted</u>

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Inspector 1 Service Time for Component 1:

H_0: Random variable is exponential distribution.

H_1: Random variable is not exponential distribution.

	range	O	O (New)	E	E (New)	O-E	(O-E)^2/E
0	(0, 11]	196	196	196.2699	196.2699	-0.26992	0.000371
1	(11, 22]	70	70	67.86365	67.86365	2.136351	0.067252
2	(22, 33]	24	24	23.46501	23.46501	0.534993	0.012198
3	(33, 44]	7	10	8.113424	12.22417	2.22417	0. 404684
4	(44, 55]	1		2.805354			
5	(55, 66]	1		0.969999			
6	(66, 77]	1		0.335393			
						Sum:	0.484505

Degrees of freedom = k – s – 1 = 4 – 1 – 1 = 2

Level of significance = 0.05

Chi-Square (0.05,3) = 5.99

0.48 < 5.99

<u>H_0 is accepted</u>

Inspector 2 Service Time for Component 2:

H_0: Random variable is exponential distribution.

H_1: Random variable is not exponential distribution.

	range	O	O (New)	E	E (New)	O-E	(O-E)^2/E
0	(0, 17]	198	198	199.5547	199.5547	-1.5547	0.012112
1	(17, 34]	74	74	66.81444	66.81444	7.18556	0.772771
2	(34, 51]	20	20	22.37066	22.37066	-2.37066	0.251222
3	(51, 68]	5	8	7.490091	11.1187	3.1187	0.874768
4	(68, 85]	1		2.507815			
5	(85, 102]	1		0.839661			
6	(102, 119]	1		0.281133			
						Sum:	1.910873

Degrees of freedom = k – s – 1 = 4 – 1 – 1 = 2

Level of significance = 0.05

Chi-Square (0.05,3) = 5.99

1.91 < 5.99

H_0 is accepted

Inspector 2 Service Time for Component 2:

H_0: Random variable is exponential distribution.

H_1: Random variable is not exponential distribution.

	range	O	O (New)	E	E (New)	O-E	(O-E)^2/E
0	(0, 15]	147	147	154.9928	154.9928	-7.99283	0.412183
1	(15, 30]	85	85	74.91691	74.91691	10.08309	1.357088
2	(30, 45]	34	34	36.21163	36.21163	-2.21163	0.135075
3	(45, 60]	18	18	17.50315	17.50315	0.496849	0.014104
4	(60, 75]	7	7	8.460275	8.460275	-1.46027	0.252049
5	(75, 90]	5	9	4.089335	6.065944	2.934056	1.419183
6	(90, 105]	4		1.976609			
						Sum:	3.589682

Degrees of freedom = k – s – 1 = 6 – 1 – 1 = 4

Level of significance = 0.05

Chi-Square (0.05,3) = 9.49

$3.58 < 9.49$

<u>H_0 is accepted</u>

Based on the Chi-Square Test conducted for all 6 Datasets we can say that all 6 follow Chi-Square Distribution.

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Generate Input Based on Model:

Model Identified: Exponential Distribution

Parameters need: λ

Parameter estimate: $\lambda = 1/\mu$

Procedure:

Step 1:

Generate uniform distribution of Sudo Random Number by Linear Congruential Method (Refer 7.3.1 in Textbook).

$X_{i+1} = (a X_i + c) \bmod m$ $\quad$ $i = 1, 2, 3, 4, \ldots$

Here follow best practices mentioned in the textbook.

m -> a power of 2, as large as possible

- ⇨ $m = 2^{48}$

c -> not equal to zero, relatively prime to m

- ⇨ $c = 2^7 - 1$

a -> 1 + 4k where k is integer

- ⇨ $a = 1 + 4 * 2 = 9$

X_0 any arbitrary value

Step 2:

Convert X_i to the range [0, 1] resulting in a uniform distribution.

$R_i = X_i / m$

Step 3:

Use this random uniform distribution of numbers to get exponential distribution with the following formula (detailed explanation in 8.1.1 Textbook):

$X_i = - \ln(R_i) / \lambda$

(or)

$X_i = - \ln(R_i) * \mu$

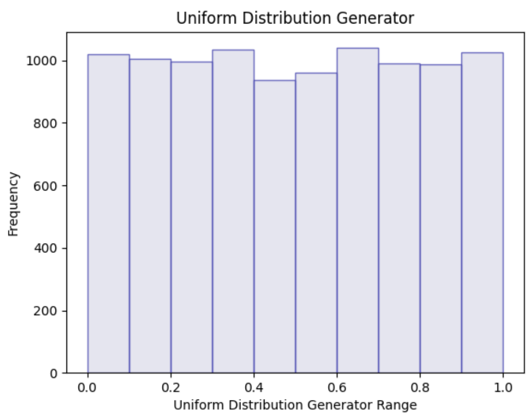
Step 4:

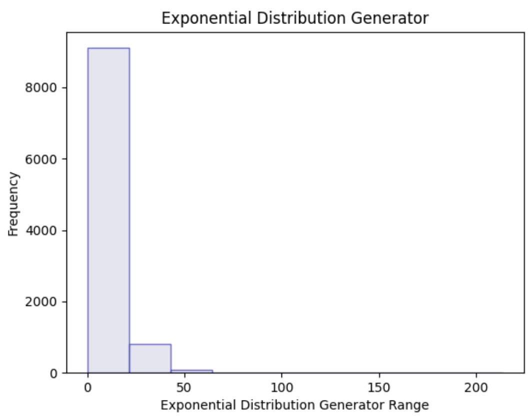
Generate the numbers

Resultant Distribution:

Based on the above-mentioned steps a generator was made.

The output of distribution for Step2, 3 are presented for the generator for the Mean = 8.45





Frequency Test (K-S Test):

H_0: Random variable is Uniform [0,1]

H_1: Random variable is not Uniform [0,1]

Range	O (Observed)	T (Theoretical)	F_O (X)	F_T(X)	| F_O (X) - F_T(X)|
(0.0, 0.1]	1018	1000	1018/10000	1000/10000	18/10000
(0.1, 0.2]	1006	1000	1006/10000	1000/10000	6/10000
(0.2, 0.3]	997	1000	997/10000	1000/10000	3/10000
(0.3, 0.4]	1035	1000	1035/10000	1000/10000	35/10000
(0.4, 0.5]	939	1000	939/10000	1000/10000	61/10000
(0.5, 0.6]	961	1000	961/10000	1000/10000	39/10000
(0.6, 0.7]	1040	1000	1040/10000	1000/10000	40/10000
(0.7, 0.8]	990	1000	990/10000	1000/10000	10/10000
(0.8, 0.9]	987	1000	987/10000	1000/10000	13/10000
(0.9, 1.0]	1027	1000	1027/10000	1000/10000	27/10000

N = 10000

α = 0.05

D = max (| F_O (X) - F_T(X)|)

- ⇨ D = 61/10000 = 0.0061

D_{10000} 0.05 = 1.36/Sqrt(10000) = 1.36/100 = 0.0136

0.0061 < 0.0136

<u>H_0 is accepted</u>

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Test for Autocorrelation:

First 10 random numbers are:

[0.33, 0.04, 0.40, 0.60, 0.44, 0.02, 0.22, 0.01, 0.12, 0.10]

H_0: $\rho_{im} = 0$ if numbers are independent

H_1: ρ_{im} != 0 if numbers are dependent

l = 1

α = 0.05

N = 10

i = 1 (start from the first number)

M = 8

- ⇨ i + (M +1) l <= N
- ⇨ 1 + (M + 1) 1 <= 10
- ⇨ M + 2 <= 10
- ⇨ M <= 8

ρ_{11} = 1/(M +1) [$Sum_{0, M}(R_{i+kl}*R_{i+(k+1)l})$] – 0.25

= 1/(8+1) [0.33*0.04 + 0.04*0.40 + 0.40*0.60 + 0.60*0.44 + 0.44*0.02 + 0.02*0.22 + 0.22*0.01 + 0.01*0.12] – 0.25

= 1/9 *[0.5498] – 0.25

= -0.188

$\sigma_{\rho 11}$ = Sqrt(13 * M + 7)/ (12 * (M + 1))

= Sqrt(13 * 8 + 7) / (12 * (8 + 1))

= Sqrt(111) / (108)

= 10.535 /108

= 0.097

Z_0 = - 0.188/0.097 = -1.947

$Z_{0.025}$ = 1.96

-1.947 < 1.96

H_0 is accepted

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Code:

```
class Random:

    __seed = None
    __multiplier = 9
    __increment = 2**7 - 1
    __modulus = 2**48
    __last_rand = None

    # Constructor
    def __init__(self, seed=0):
        self.seed = seed
        self.last_rand = seed

    # Returns uniform distribution of random number
    def random_probability(self):
        self.last_rand = (self.__multiplier * self.last_rand +
self.__increment) % self.__modulus
        return self.last_rand / self.__modulus

    # Returns randomly distributed exponential numbers
    def random_exponential(self, mean=None):
        if (mean is None) or (math.isnan(mean)):
            raise Exception("Mean Not Specified")
        uniform_rand = self.random_probability()
        return -1 * mean * math.log(uniform_rand, math.e)
```


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