

# Algorithms

(& Big O Notation)

# What is an algorithm?

- AKA, Data Structures and Algorithms (sometimes referred to as DSA)
- A step-by-step procedure for solving a problem
- The procedures are reusable (functions/methods)
- Algorithms refer to virtually any computational task
- DSA knowledge can be applied to ANY language or framework
- This knowledge is often tested in technical interview - so worth understanding!

# Why do we care about Algorithms?

- A valid approach: ‘if my code works, it works’
- 🙌 often helpful to assess how your program is working
- “How good is my code?” “Is this the best solution?”
- How performant is the solution?
- Scalability
  - How can a start up serving a customer base of 10,000 move to a customer base of 2 million?
  - How many hits per day could the API handle?
- Assumed:
  - Thoughtful variable naming
  - Tidy code, formatted correctly, attention to spacing

# Comparing algorithms

- Possible to 'time' an algorithm...
- ...but there are problems with this, not least:
  - Tiny changes are hard to measure - so for comparing fast algorithms this is especially tricky
  - Different machines will record different times (for the same code)
  - Equally, the *same* machine will record different times!
- We need a way to *objectively* distinguish between one algorithm and another
- How can we analyse and compare their performance?
- Enter: **Big O Notation**
- (O = Order of)

# Big O Notation

- Every algorithm has an associated cost of execution
- Cost of execution can be expressed in:
  - TIME (how many operations are performed during execution)
  - SPACE (how much memory is utilised during execution)
- **Big O lets us express the efficiency of the algorithm in relation to the size of the input it takes**
- *How does the time/space complexity of this algorithm grow as the magnitude (size) of input increases?*
- *How does the performance of this algorithm scale as the magnitude (size) of input grows?*

# How does Big O help us?

- Good to have a vocabulary to talk about how our code is performing
- Which is better - this one or that one?
- If you encounter a problem - e.g. code performing very slowly - can help identify the parts of the code that are inefficient

# Big O (Time Complexity)

- We've covered some of the issues of using timers ❌
- Rather than counting seconds, we can count the number of simple operations the computer has to perform

*What is meant by 'simple operations'?*

- multiplication, division, assignments, comparators
- Counting the number of operations is tricky!
- Remember, with Big O we're focussing on the bigger picture, on the **general trend**

# Example 1a

A function/method to add all the numbers up to  $n$ . If  $n = 5 \rightarrow 15$ . If  $n = 3 \rightarrow 6$ .

Option 1: we could use a loop

```
public int addAllNumbersTo(int n) {  
    int total = 0;  
    for (int i = 0; i <= n; i++) {  
        total += i;  
    }  
    return total;  
}
```



# Counting Operations

```
public int addAllNumbersTo(int n) {  
    int total = 0;  
    for (int i = 0; i <= n; i++) {  
        total += i;  
    }  
    return total;  
}
```

assignment x 1

assignment x 1

increment x  $n$   
assignment x  $n$

comparison x  $n$

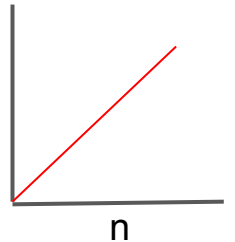
addition x  $n$   
assignment x  $n$

- We can conclude that the no. of operations is not static...
- ...it depends on  $n$ , as  $n$  increases so does the no. of operations
- It could be  $5n + 2$ , or  $5n$ , or  $2n$  (if we're only counting `total += i`)
- Doesn't matter, we just want the BIG PICTURE

# Counting Operations

```
public int addAllNumbersTo(int n) {  
    int total = 0;  
    for (int i = 0; i <= n; i++) {  
        total += i;  
    }  
    return total;  
}
```

- The general trend is:
- A function(method) with an input of  $n$ , has a runtime output of  $n$
- $f(n) = n$
- $O(n)$ , expressed as 'O of N', aka **Linear Time**



# Example 1b

Option 2: we could use a mathematical formula.

For now, just accept that  $n * (n + 1) / 2$  adds all numbers up to and including  $n$ .

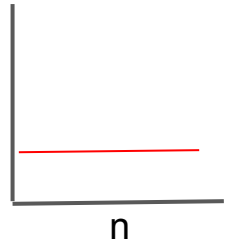
```
public int addAllNumbersTo(int n) {  
    return n * (n + 1) / 2;  
}
```

add x 1

multiply x 1

divide x 1

- Doesn't change if  $n$  increases
- Same no. of operations whether  $n = 5$ ,  $n = 50,000$ , etc
- $O(1)$ , expressed as 'O of 1', aka **Constant Time**



## Example 2

Suppose you have an array of elements and you want to check if there are any duplicates.

Option 1: loop through the entire array for each element, checking for a match each time.

```
public boolean hasDuplicates(int[] arr) {  
    // loop through array  
    for(int i = 0; i < arr.length; i++) {  
        // track the index from the end of the array  
        for(int j = arr.length - 1; j > 0; j--) {  
            // skip if index is same  
            if(i == j) continue;  
            else if(arr[i] == arr[j]) {  
                return true;  
            }  
        }  
    }  
  
    return false;  
}
```

value:	[ 10,	6,	7,	9,	5,	9 ]
index:	0	1	2	3	4	5

Iteration 1	i = 0	j = 5
no match	0 = 10	5 = 9
no match	0 = 10	4 = 5
no match	0 = 10	3 = 9
no match	0 = 10	2 = 7
no match	0 = 10	1 = 6
i == j so skipped	0 = 10	0 = 10

Iteration 2	i = 1	j = 5
no match	1 = 6	5 = 9
no match	1 = 6	4 = 5
no match	1 = 6	3 = 9
no match	1 = 6	2 = 7
i == j so skipped	1 = 6	1 = 6
no match	1 = 6	0 = 10

& so  
on...

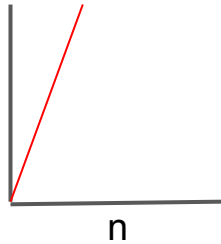
value:	[ 10,	6,	7,	9,	5,	9 ]
index:	0	1	2	3	4	5

Iteration 4	i = 3	j = 5
Match found!	3 = 9	5 = 9
	3 = 9	4 = 5
	3 = 9	3 = 9
	3 = 9	2 = 7
	3 = 9	1 = 6
	3 = 9	0 = 10

```
public boolean hasDuplicates(int[] arr) {  
    // loop through array  
    for(int i = 0; i < arr.length; i++) {  
        // track the index from the end of the array  
        for(int j = arr.length - 1; j >= 0; j--) {  
            // skip if index is same  
            if(i == j) continue;  
            else if(arr[i] == arr[j]) {  
                return true;  
            }  
        }  
    }  
    return false;  
}
```

$O(n)$

$O(n)$



- With Big O, always consider worst case scenario
- We could find the matching pair immediately, but could also work through entire array and find no matches
- With both loops, as  $n$  grows, so do the no. of operations
- $O(n^2)$ , expressed as 'O of N Squared', aka **Quadratic Time**

# Example 3

Logarithm is the inverse of exponentiation

roughly measures the number of times you can divide that number by 2 before you get to a value that's less than (or equal to) 1.

$$\log(8) = 3$$

- $8 / 2 = 4$  1
- $4 / 2 = 2$  2
- $2 / 2 = 1$  3



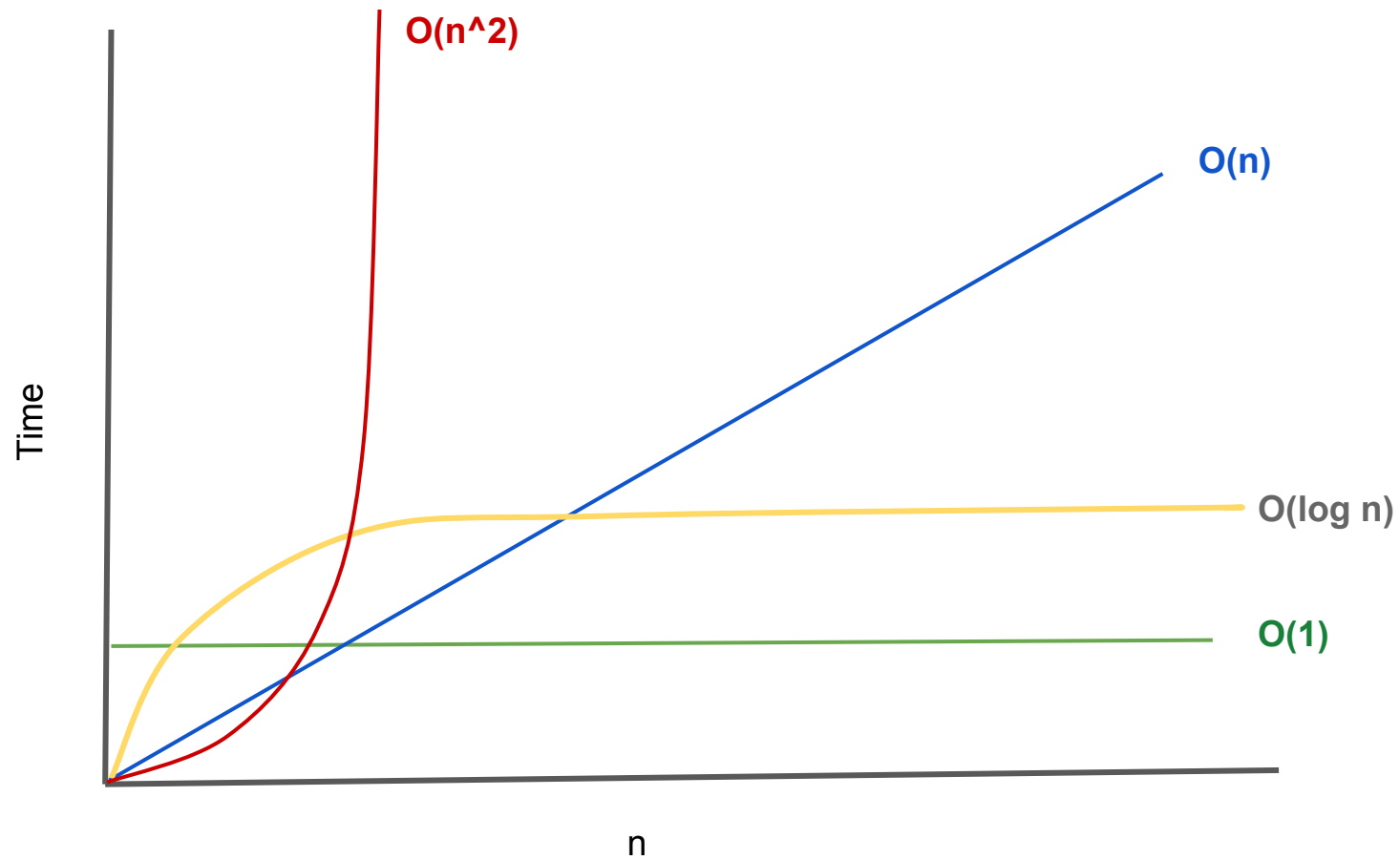
## Example 3 (cont'd)

```
public int logLookUp(int n) {  
    for (int i = 1; i < n; i = i * 2) {  
        System.out.println("looking up element at index " + i;  
    }  
}
```

- Loop is constructed in such a way that we effectively divide the number of remaining elements by 2 on each iteration, creating logarithmic behaviour
- Suppose  $n = 10$ 
  - Iteration 1 // "looking up element at index 1" ( $i = 2$ )
  - Iteration 2 // "looking up element at index 2" ( $i = 4$ )
  - Iteration 3 // "looking up element at index 4" ( $i = 8$ ) etc.
- No. of operations is related to magnitude of  $n$ , but some improvement on  $O(n)$
- $O(\log n)$ , expressed as 'O of Log N', aka **Logarithmic Time**
- Famous example: **Binary Search** (<https://www.youtube.com/watch?v=J3hM7xE9aFc>)

## A summary (in ascending order of runtime complexity)

- **$O(1)$** : the algorithm has the same (or, **constant**) complexity, no matter what input you provide.
- **$O(\log n)$** : having a **logarithmic** relationship with its input
- **$O(n)$** : a **linear** growth relationship between input and complexity. It says that an algorithm's efficiency is directly proportional to the magnitude of its input.
- **$O(n^2)$** : this notation depicts **quadratic** time. Every effort should be made to find a more efficient solution, as an algorithm with  $O(N^2)$  is inefficient, and not very scalable.
- **$O(n^3)$** : **cubic** time is extremely inefficient and not at all scalable



# Points to remember

- Big O == Big Picture
- You're trying to identify the overall trend
- Look for the worst case scenario
- In more complicated algorithms you may find multiple Big O terms, always go for the dominant term (weakest link)
- Practise! Codewars, Leetcode, Exercism
- <https://www.learnhowtoprogram.com/computer-science/big-o-notation-and-binary-trees/big-o-practice>
- It's not the be-all-and-end-all of what makes good code