



Assignment 4

CSI2120 Programming Paradigms

Winter 2017

Due on March 22nd, 2017 before 11:00 pm in Virtual Campus

[5 marks in total]

Question 1. Happy Numbers Series [5 marks]

Consider the digits $d_k, d_{k-1}, \dots, d_1, d_0$ of a positive integer number. The squares of the digits are then $d_k^2, d_{k-1}^2, \dots, d_1^2, d_0^2$ and we name the sum of these squares s . We can create a recursive series of the squares of the digits of positive integers. This series will be $s_0, s_1, s_2, s_3, \dots$ where s_0 is the sum of squares of the original number, s_1 the sum of the squares of the digits of s_0 , s_2 the sum of the squares of the digits of s_1 , and so on. For example:

$$120 \rightarrow 1^2 + 2^2 = 5 \rightarrow 5^2 = 25 \rightarrow 2^2 + 5^2 = 29 \rightarrow \dots$$

a) Create a function `sosd` that calculates the sum of square digits. Example:

```
(sosd 130)  
⇒ 10
```

It has been shown that for any starting number, the series described will always reach one of the following numbers: 0, 1, 4, 16, 20, 37, 42, 58, 89, 145 (OEIS A039943; Porges 1945). In the following I call these numbers stop numbers. If the series reaches the number 1 for a starting number H , then the number H is called a 'happy number'.

b) Write a function `stop?` that is true if the argument is one of the stop numbers in the above list.

```
(stop? 42)  
⇒ #t  
  
(stop? 31)  
⇒ #f
```

c) Create a function `sspd_series` that returns a list containing all the sum of square digits calculated until (and including) a stop number is reached.

```
(sspd_series 120)  
⇒ (5 25 29 85 89)
```

d) Create a function `happy?` that returns true if the function `ssod_series` ends in a 1. Example:

```
(happy? 44)
```

```
⇒ #t
```

```
(happy? 120)
```

```
⇒ #f
```