

<u>Course</u> > <u>Week 9</u>... > <u>Proble</u>... > Proble...

Problem Set 9

Problems 1-6 correspond to "Linear Projections"

Problem 1

1/1 point (graded)

In \mathbb{R}^2 , what is the unit vector corresponding to the x_1 -direction?

 $\bigcirc (0,0)$

 \bigcirc (1,0)

 $\bigcirc (0,1)$

 \bigcirc (1,1)



Submit

1 Answers are displayed within the problem

Problem 2

1/1 point (graded)

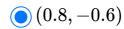
What is the unit vector in the same direction as (3, 2, 2, 2, 2)?

\bigcirc (1.5, 1, 1, 1, 1)	
\bigcirc (1, 0.67, 0.67, 0.67, 0.67)	
\bigcirc (0.6, 0.4, 0.4, 0.4, 0.4)	
$\bigcirc (0.5, 0.33, 0.33, 0.33, 0.33)$	
✓	
? Hint (1 of 1): To get a unit vector in the same direction as x , simply divide by $\ x\ $.	Next Hint
Submit	
Answers are displayed within the problem	
Problem 3	
1/1 point (graded) What is the projection of the vector $(3,5,-9)$ onto the direction $(0.6,-0.8,0)$	0)?
-2.2 ✓ Answer: -2.2	
-2.2	
? Hint (1 of 1): If u is a unit vector, then the projection of x onto direction u is simply $u \cdot x$.	Next Hint
Submit	

0	Answers are	displayed	within the	problem
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1/1 point (graded)

What is the (unit) direction along which the projection of (4,-3) is largest?



- \bigcirc (-0.6, -0.8)
- \bigcirc (-0.8, 0.6)
- $\bigcirc \, (0.8,0.6)$



Explanation

The projection of x=(4,-3) is going to be largest in the direction of x itself.

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1 Answers are displayed within the problem

Problem 5

1/1 point (graded)

What is the (unit) direction along which the projection of (4,-3) is smallest?

- \bigcirc (0.8, -0.6)
- $\bigcirc (-0.6, -0.8)$
- (-0.8, 0.6)

\bigcirc $(0.8,0.6)$
Explanation $ \text{The projection of } x=(4,-3) \text{ will be smallest in the direction opposite to } x \text{, that is, the direction of } -x. $
Answers are displayed within the problem
Problem 6
1/1 point (graded) The projection of vector \boldsymbol{x} onto direction \boldsymbol{u} is exactly zero. Which of the following statements is necessarily true? Select all that apply.
ightharpoonup u is orthogonal to x .
$oxedsymbol{oxed}u$ is in the opposite direction to $x.$
ightharpoonup u is at right angles to x .
It is not possible to have a projection of zero.
Submit
Answers are displayed within the problem
Problems 7-8 correspond to "Principal component analysis I: one-dimensional projection"

4 points possible (graded)

A three-dimensional data set has covariance matrix

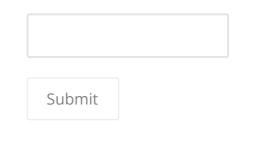
$$\Sigma = \left(egin{array}{ccc} 4 & 2 & -3 \ 2 & 9 & 0 \ -3 & 0 & 9 \end{array}
ight).$$

a) What is the variance of the data in the x_1 -direction?
b) What is the correlation between x_1 and x_3 ?
c) What is the variance in the direction $(0,-1,0)$?
d) What is the variance in the direction of $(1,1,0)$?
Submit

Pro	b	lem	8
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1 point possible (graded) Which of the following covariance matrices has the property that the variance is the same in any direction? Select all that apply.
The all-zeros matrix.
The all-ones matrix.
The identity matrix.
Any diagonal matrix.
Submit
Problems 9-11 correspond to "Principal component analysis II: the top k directions"
Problem 9
8 points possible (graded) Let $u_1,u_2\in\mathbb{R}^d$ be two vectors with $\ u_1\ =\ u_2\ =1$ and $u_1\cdot u_2=0$. Define U to be the matrix whose columns are u_1 and u_2 .
What are the dimensions of the following matrices?
a) $oldsymbol{U}$
of Rows =
of Columns =

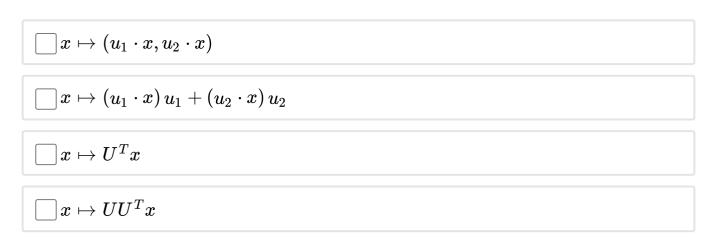
b) U^T	
# of Rows =	
# of Columns	
c) UU^T	
# of Rows =	
# of Columns =	
d) $u_1u_1^T$	
# of Rows =	
# of Columns =	



1 point possible (graded)

Continuing from the previous problem, let $u_1,u_2\in\mathbb{R}^d$ be two vectors with $\|u_1\|=\|u_2\|=1$ and $u_1\cdot u_2=0$, and define U to be the matrix whose columns are u_1 and u_2 .

Which of the following linear transformations sends points $x \in \mathbb{R}^d$ to their (two-dimensional) projections onto directions u_1 and u_2 ? Select all that apply.



Submit

Problem 11

2 points possible (graded)

For a particular four-dimensional data set, the top two eigenvectors of the covariance matrix are:

$$\frac{1}{2} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \quad \frac{1}{2} \begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \end{pmatrix}.$$

a) What is the PCA projection of point (2,4,2,6) into two dimensions? Write it in the form (a,b).

 \bigcirc (2,2)

 \bigcirc (2,3)

 \bigcirc (7,3)

 \bigcirc (4,6)

b) What is the reconstruction, from this projection, to a point in the original four-dimensional space? Write it in the form (a,b,c,d)

 \bigcirc (2, 5, 2, 5)

 \bigcirc (2,1,2,2)

 \bigcirc (4, 2, 2, 2)

 \bigcirc (2,6,2,4)

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Problems 12-14 correspond to "Linear algebra V: eigenvalues and eigenvectors"

2 points possible (graded)

Consider the 2 imes 2 matrix $M = \begin{pmatrix} 5 & 1 \\ 1 & 5 \end{pmatrix}$.

a) One of its eigenvectors is $\frac{1}{\sqrt{2}} \binom{1}{1}$. What is the corresponding eigenvalue?



b) Its other eigenvector is $\frac{1}{\sqrt{2}} \binom{1}{-1}$. What is the corresponding eigenvalue?



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Problem 13

6 points possible (graded)

A 2 imes 2 matrix M has eigenvalues 10 and 5 .

a) What are the eigenvalues of 2M (that is, each entry of M is multiplied by 2)?

Larger eigenvalue =



Smaller eigenvalue =

b) What are the eigenvalues of $M+3I$, where I is the $2 imes 2$ identity matrix?
Larger eigenvalue =
Smaller eigenvalue =
c) What are the eigenvalues of $M^2=MM$?
Larger eigenvalue =
Smaller eigenvalue =

7 points possible (graded)

A certain three-dimensional data set has covariance matrix

$$\begin{pmatrix} 5 & -3 & 0 \\ -3 & 5 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

.

a) Consider the direction $u=\left(1,1,1\right)/\sqrt{3}$. What is variance of the projection of the data onto direction u?

b) Which of the following are eigenvectors of the covariance matrix? Select all that apply.

/1
0
\setminus_0

$\int 0$
1
\setminus_0

$\int 0$
0
\setminus_1

$egin{array}{c} rac{1}{\sqrt{2}} egin{pmatrix} 0 \ 1 \ 1 \end{pmatrix}$
$egin{array}{c} igg rac{1}{\sqrt{2}} igg(rac{1}{-1} igg) \end{array}$
c) Find the eigenvalues of the covariance matrix. List them in decreasing order.
d) Suppose we used principal component analysis (PCA) to project points into two dimensions. What would be the resulting two-dimensional projection of the point $x=(\sqrt{2},-3\sqrt{2},2)$?
\bigcirc (1,0)
\bigcirc $(4,2)$
\bigcirc (1,4)

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/	_/	\ - 7	_	,

e) Now suppose we use the projection in (d) to reconstruct a point \widehat{x} in the original three-dimensional space. What is the Euclidean distance between x and \widehat{x} , that is, $||x-\widehat{x}||$?



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Problems 15-17 correspond to "Linear algebra VI: spectral decomposition"

Problem 15

1 point possible (graded)

M is a 2 imes 2 real-valued symmetric matrix with eigenvalues $\lambda_1 = 6, \lambda_2 = 1$ and corresponding eigenvectors

$$u_1=rac{1}{\sqrt{5}}inom{2}{1}, \ \ u_2=rac{1}{\sqrt{5}}inom{-1}{2}.$$

What is M?

- $\bigcirc \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$
- $\begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix}$

$igcirc igg(rac{3}{1} rac{1}{2} igg)$
$igcup_{\left(egin{array}{ccc} 5 & 2 \ 2 & 2 \end{array} ight)$
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Problem 16
1 point possible (graded) For a certain data set in d -dimensional space, the covariance matrix has the following interesting property: there are k positive eigenvalues and the rest are zero (where $k < d$). What can we conclude from this? Select all that apply.
$oxedsymbol{oxed}$ Each of the data points has at most k nonzero coordinates.
$oxedsymbol{ extstyle e$
$oxedsymbol{oxed}$ Each data point can be expressed as a linear combination of the top k eigenvectors.
It is possible to discard $d-k$ of the coordinates without losing any of the variance in the data.

1 point possible (graded)

A data set in \mathbb{R}^d has a covariance matrix with eigenvalues $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_d$. Under which of the following conditions is PCA most likely to be effective as a form of dimensionality reduction? Select all that apply.
$oxedsymbol{oxed}$ When the λ_i are approximately equal.
$oxedsymbol{oxed}$ When most of the λ_i are close to zero.
$oxedsymbol{oxed}$ When most of the λ_i are close to 1.
$oxedsymbol{oxed}$ When the sequence $\lambda_1,\lambda_2,\ldots$ is rapidly decreasing.
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