Conditional Probability

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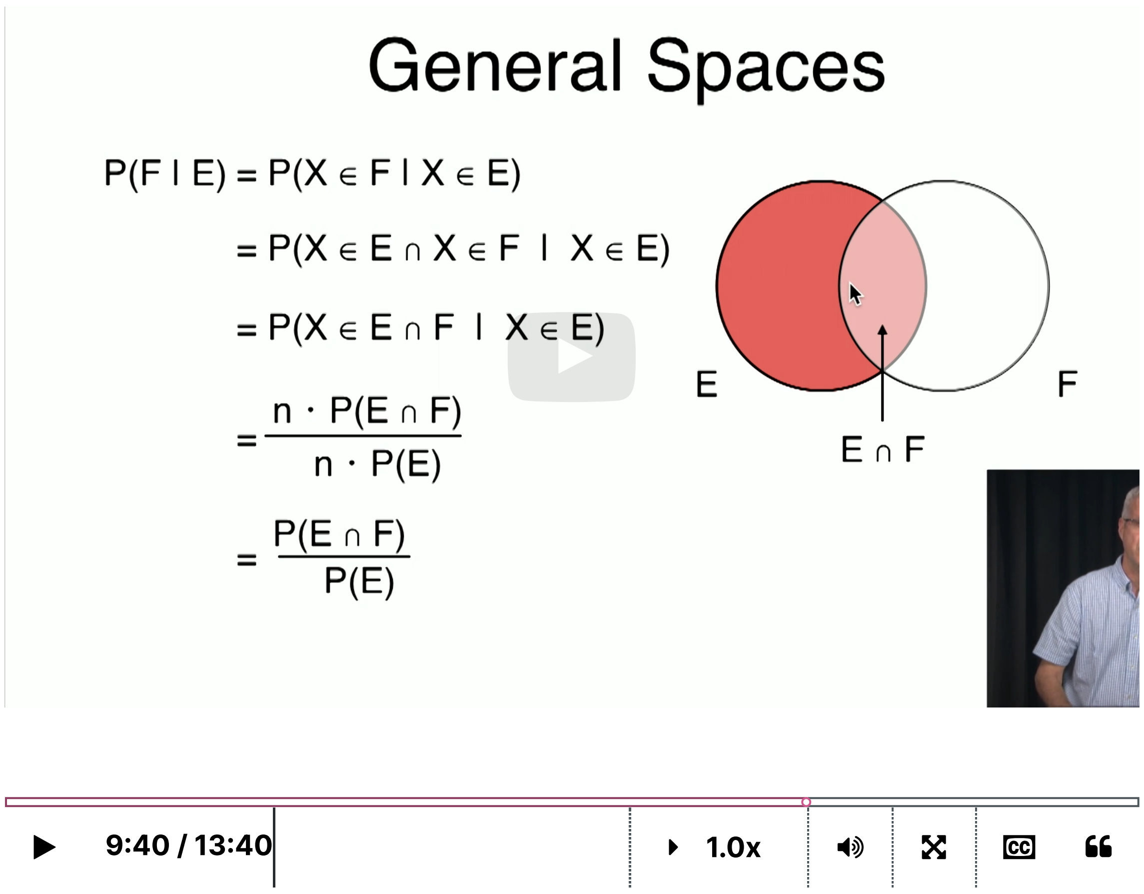
Problem Sets due Jun 6, 2022 02:13 +03

Video



General Spaces

$$P(F \mid E) = P(X \in F \mid X \in E)$$
$$= P(X \in E \cap X \in F \mid X \in E)$$
$$= P(X \in E \cap F \mid X \in E)$$
$$= \frac{n \cdot P(E \cap F)}{n \cdot P(E)}$$
$$= \frac{P(E \cap F)}{P(E)}$$

So, let R1 be the event that the first ball is red

and R2 be the event that the second ball is red.

So, the probability that both are red

is the probability of R1 intersection with R2,

when both happen, and that's going to be equal

to the probability of R1 times the probability

of R2 given R1.

Now, what is the probability of R1 is two thirds, right?

Because we have two red balls and one blue,

so if we take one at random,

the probability that it will be red will be two thirds

and once we do that, then, if R1

6.1_Conditional_Probability

POLL

Let A and B be two positive-probability events. Does P(A|B)>P(A) imply P(B|A)>P(B)?

RESULTS

○ **Yes**	**33%**
● **Not necessarily**	**67%**

Submit

Results gathered from 46 respondents.

FEEDBACK

Yes.
P(A|B)=P(A,B) / P(B) and P(B|A)=P(A,B) / P(A).
Hence, P(A|B)>P(A) iff P(A,B)>P(A) * P(B) iff P(B|A)>P(B).

1

0 points possible (ungraded)

Suppose $P(A) > 0$. Find $P(B|A)$ when:

- $B = A$,

1 ✓ **Answer:** 1

1

Explanation

Given that A happens, B must happens. Hence $P(B|A) = 1$.

- $B \supseteq A$,

1

✓ **Answer:** 1

1

Explanation
Same as above.

- $B = \Omega$,

1

✓ **Answer:** 1

1

Explanation
Same as above.

- $B = A^c$,

0

✓ **Answer:** 0

0

Explanation
Given that A happens, B can never happens. Hence $P\left(B|A\right) = 0$.

- $A \cap B = \emptyset$,

0

✓ **Answer:** 0

0

Explanation
Same as above.

- $B = \emptyset$.

0

✓ **Answer:** 0

0

Explanation
Same as above.

Submit You have used 3 of 4 attempts

🛈 Answers are displayed within the problem

2

0 points possible (ungraded)
If A and B are disjoint positive-probablity events, then $P\left(A|B\right)$=

☐ $P\left(A\right)$,

☑ $P\left(B|A\right)$,
✓

☐ $P\left(A \cup B\right)$,

☐ $P(A \cap B)$.
✓

✗

Explanation
Since A and B are disjoint, $P(A|B) = 0$.
$P(A \cap B) = P(B|A) = 0$, while $P(A)$ and $P(A \cup B)$ are positive as A and B are positive-probablity events.

Submit You have used 2 of 2 attempts

ℹ Answers are displayed within the problem

3 (Graded)

4/4 points (graded)
Given events A, B with $P(A) = 0.5$, $P(B) = 0.7$, and $P(A \cap B) = 0.3$, find:

- $P(A|B)$,

0.3/0.7 ✓ **Answer:** 3/7

$\frac{0.3}{0.7}$

Explanation
$P(A|B) = P(A \cap B)/P(B) = 0.3/0.7 = 3/7$.

- $P(B|A)$,

0.6 ✓ **Answer:** 3/5

0.6

Explanation
$P(B|A) = P(B \cap A)/P(A) = 0.3/0.5 = 3/5$.

- $P(A^c|B^c)$,

(1-(0.5+0.7-0.3))/(1-0.7) ✓ **Answer:** 1/3

$\frac{1-(0.5+0.7-0.3)}{1-0.7}$

Explanation
$P(A^c|B^c) = P(A^c \cap B^c)/P(B^c) = 0.1/0.3 = 1/3$.

- $P(B^c|A^c)$.

0.2 ✓ **Answer:** 1/5

0.2

Explanation
$P(B^c|A^c) = P(B^c \cap A^c)/P(A^c) = 0.1/0.5 = 1/5$.

Submit You have used 2 of 4 attempts

ℹ Answers are displayed within the problem

4

0 points possible (ungraded)

Find the probability that the outcome of a fair-die roll is at least 5, given that it is at least 4.

● $\frac{2}{3}$

○ $\frac{2}{4}$

○ $\frac{1}{3}$

○ $\frac{1}{2}$

✓

Explanation

$$P\left(\text{at least } 5 \middle| \text{at least } 4\right) = \frac{P(\text{at least } 5 \cap \text{ at least } 4)}{P(\text{at least } 4)} = \frac{P(\text{at least } 5)}{P(\text{at least } 4)} = \frac{2}{3}.$$

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ℹ Answers are displayed within the problem

5

0 points possible (ungraded)

Two balls are painted red or blue uniformly and independently. Find the probability that both balls are red if:

- at least one is red,

1/3 ✓ **Answer:** 1/3

$\frac{1}{3}$

Explanation

$$P\left(2R \middle| \text{at least } 1R\right) = \frac{P(2R \cap \text{at least } 1R)}{P(\text{at least } 1R)} = \frac{P(2R)}{P(\text{at least } 1R)} = \frac{1/4}{3/4} = \frac{1}{3}.$$

- a ball is picked at random and it is pained red.

1/2 ✓ **Answer:** 1/2

$\frac{1}{2}$

Explanation

$$P\left(2R \middle| \text{random ball is R}\right) = \frac{P(2R \wedge \text{random ball is R})}{P(\text{random ball is R})} = \frac{P(2R)}{P(\text{random ball is R})} = \frac{1/4}{1/2} = \frac{1}{2}.$$

Submit You have used 2 of 4 attempts

ℹ Answers are displayed within the problem

6 (Graded)

3/3 points (graded)

Three fair coins are sequentialy tossed. Find the probability that all are heads if:

- the first is tails,

0

✓ **Answer:** 0

0

Explanation

If the fisrt coin is tails, it's impossible for all coins to be heads, hence the probability is 0.

More formally, $P\left(X_1 \cap X_2 \cap X_3 | \overline{X_3}\right) = \frac{P(X_1 \cap X_2 \cap X_3 \cap \overline{X_3})}{P(\overline{X_3})} = \frac{P(\emptyset)}{P(\overline{X_3})} = \frac{0}{1/2} = 0.$

- the first is heads,

1/4

✓ **Answer:** 1/4

$\frac{1}{4}$

Explanation

First intuitively, if the first coin is heads, then all are heads iff the second and third coins are heads, which by independence of coin flips happens with probability $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$.

A bit more formally, let X_1, X_2, X_3 be the events that the first, second, and third coin is heads. Then $P\left(X_1 \cap X_2 \cap X_3 | X_1\right) = \frac{P(X_1 \cap X_2 \cap X_3 \cap X_1)}{P(X_1)} = \frac{P(X_1 \cap X_2 \cap X_3)}{P(X_1)} = \frac{1/8}{1/2} = \frac{1}{4}.$

- at least one is heads.

1/7

✓ **Answer:** 1/7

$\frac{1}{7}$

Explanation

First intuitively, there are seven possible outcome triples where at least one of the coins is heads, and only one of them has all heads. Hence the probability of all heads given that one is heads is $1/7$.

More formally,

$P\left(X_1 \cap X_2 \cap X_3 | X_1 \cup X_2 \cup X_3\right) = \frac{P((X_1 \cap X_2 \cap X_3) \cap (X_1 \cup X_2 \cup X_3))}{P(X_1 \cup X_2 \cup X_3)} = \frac{P(X_1 \cap X_2 \cap X_3)}{P(X_1 \cup X_2 \cup X_3)} = \frac{1/8}{7/8} = \frac{1}{7}.$

Submit

You have used 3 of 4 attempts

ℹ Answers are displayed within the problem

7

0 points possible (ungraded)

A 5-card poker hand is drwan randomly from a standard 52-card deck. Find the probability that:

- all cards in the hand are ≥ 7 (7, 8,..., K, Ace), given that the hand contains at least one face card (J, Q, or K),

- there are exactly two suits given that the hand contains exactly one queen.

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