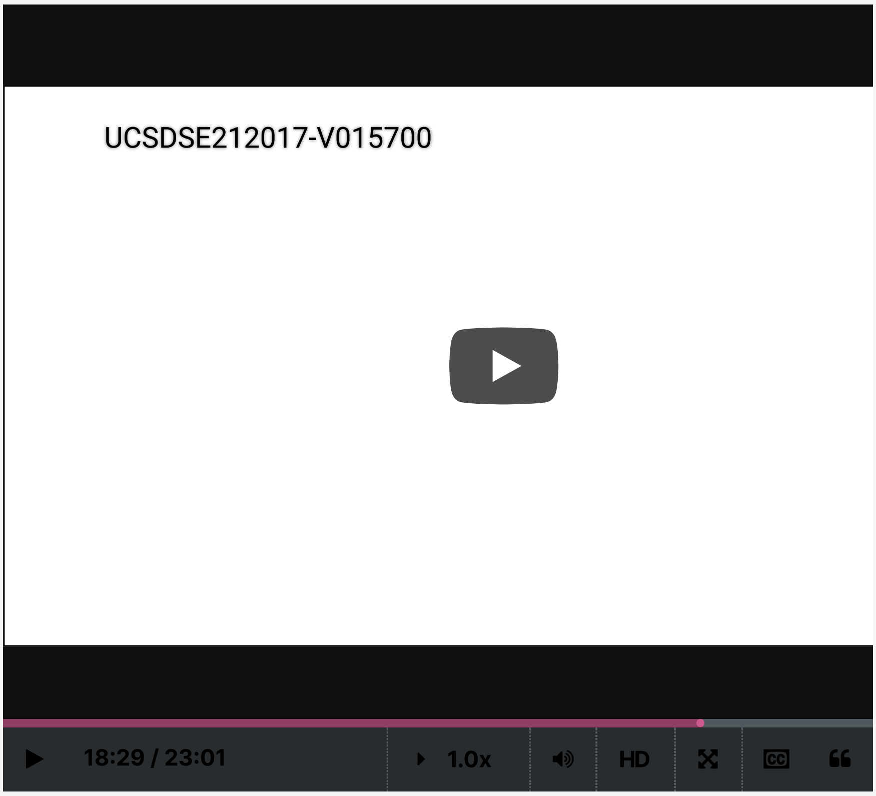
Problem Sets due Jul 8, 2022 16:34 +03

Video



The differences from mean grew by a squared.

And so here, instead of distance .5 for example,

the distances were 0.75.

And therefore, the standard deviation of X

is going to be the square root of the variance

which is a times the standard deviation of X.

So the standard deviation will grow by a factor of a.

That's because the average distance

from the mean grew by a factor of a.

And if we have an affine transformation,

what is the variance of aX plus b?

7.6_Variance

POLL

Which of the following is greater (≥) for a random variable X?

RESULTS

- ○ **E[X^2]** **36%**
- ◉ **E[X]^2** **18%**
- ○ **Depends on X** **47%**

Submit

Results gathered from 45 respondents.

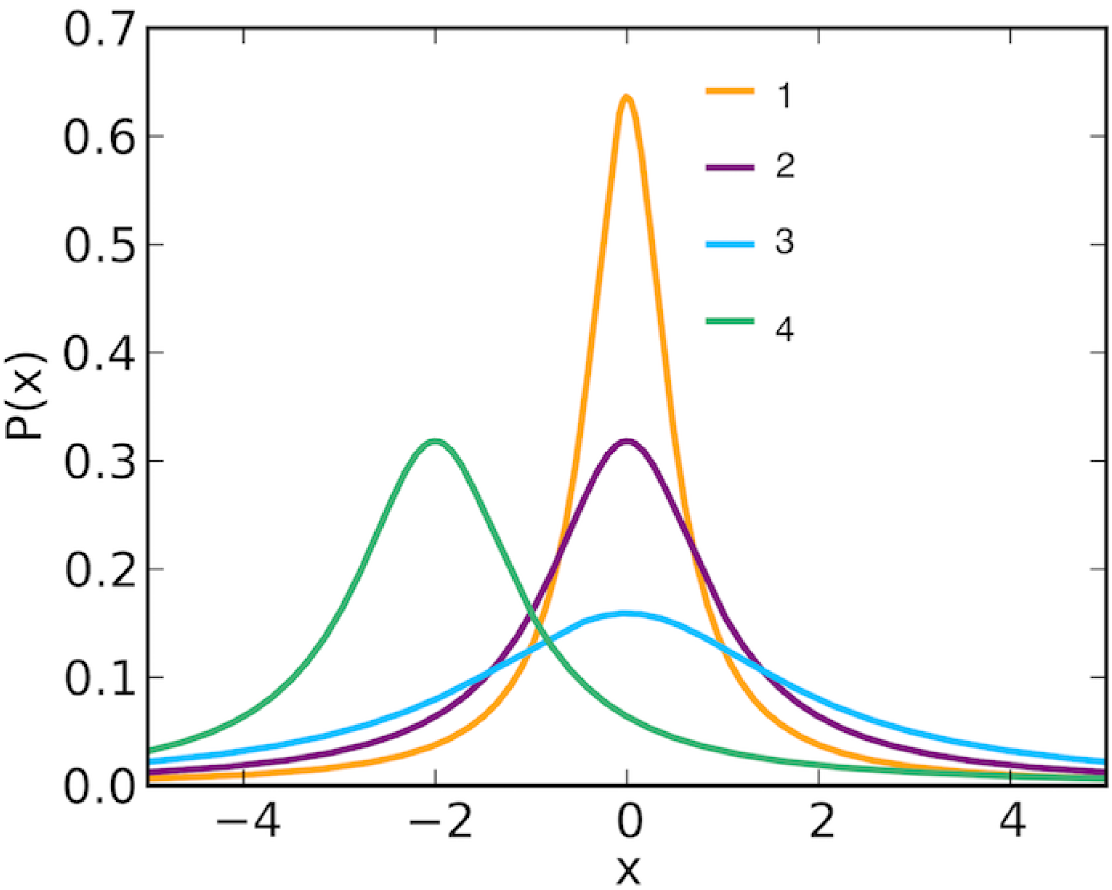
FEEDBACK

E[X^2] will be greater. Since V(X)=E[X^2]-E[X]^2, and V(X) is always non-negative.

1

0 points possible (ungraded)

Given 4 probability density functions, which one shows the greatest variance?



- ◉ 1
- ○ 2
- ○ 3 ✔
- ○ 4

✖

Answer
Incorrect: Video: Variance

Explanation
Variance measures how far a set of (random) numbers are spread out from their average value. 3 is the brodest one.

Submit You have used 2 of 2 attempts

ℹ Answers are displayed within the problem

2

0 points possible (ungraded)

A random variable X is distributed over $\{-1, 0, 1\}$ according to the p.m.f. $P(X = x) = \frac{|x|+1}{5}$.

Find its expectation $E\left(X\right)$

0 **Answer:** 0

0

Explanation
The pmf is symmetric around 0, hence the mean is 0.

and variance $V\left(X\right)$

4/5 **Answer:** 4/5

$\frac{4}{5}$

Explanation
By definition, Var$(X) = \frac{2}{5} \times (-1-0)^2 + \frac{1}{5} \times (0-0)^2 + \frac{2}{5} \times (1-0)^2 = \frac{2}{5} + 0 + \frac{2}{5} = \frac{4}{5}$
Or, Var$(X) = \mathbb{E}\left(X^2\right) - \mathbb{E}(X)^2 = 4/5 - 0 = 4/5$

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

3 (Graded)

4/4 points (graded)
Let random variable X be distributed according to the p.m.f

x	1	2	3
$P\left(x\right)$	0.3	0.5	0.2

- If $Y = 2^X$, what are

$E\left[Y\right]$

4.2 **Answer:** 4.2

4.2

Explanation
$E\left(Y\right) = E\left(2^X\right) = 2 \times 0.3 + 4 \times 0.5 + 8 \times 0.2 = 4.2.$

$\text{Var}\left(Y\right)$

22-(4.2^2) **Answer:** 4.36

$22 - \left(4.2^2\right)$

Explanation

For any random variable Z, $V(Z) = E(Z^2) - E(Z)^2$. Here $E(Y^2) = E(2^{2X}) = 4 \times 0.3 + 16 \times 0.5 + 64 \times 0.2 = 22$. Thus $V(Y) = 22 - 4.2^2 = 4.36$.

- If $Z = aX + b$ has $E[Z] = 0$ and $\mathrm{Var}(Z) = 1$, what are:

$|a|$

10/7

✔ **Answer:** 1.42857

$\frac{10}{7}$

$|b|$

2.714

✔ **Answer:** 2.714285

2.714

Explanation

First, $E(X) = 0.3 \times 1 + 0.5 \times 2 + 0.2 \times 3 = 1.9$, $E(X^2) = 0.3 \times 1 + 0.5 \times 4 + 0.2 \times 9 = 4.1$ and thus $\mathrm{Var}(X) = E(X^2) - E(X)^2 = 4.1 - 1.9^2 = 0.49$.

Now, by linearity of expectation, $0 = E(Z) = aE(X) + b = 1.9 \cdot a + b$. Further, we know $1 = \mathrm{Var}(Z) = \mathrm{Var}(aX + b) = a^2 \cdot \mathrm{Var}(X) = a^2 \cdot 0.49$. Solving these two equations gives $|a| = 1.42857$, $|b| = 2.71485$.

Submit You have used 2 of 4 attempts

Answers are displayed within the problem

4 (Graded)

5/5 points (graded)

Consider two games. One with a guaranteed payout $P_1 = 90$, and the other whose payout P_2 is equally likely to be 80 or 120. Find:

- $E(P_1)$

90

✔ **Answer:** 90

90

Explanation

The distribution of P_1 is $P(P_1 = 90) = 1$. Hence, $E(P_1) = 1 \times 90 = 90$.

- $E(P_2)$

(80 + 120)/2

✔ **Answer:** 100

$\frac{80+120}{2}$

Explanation
The distribution of P_2 is $P(P_2 = 80) = P(P_2 = 120) = \frac{1}{2}$. Hence, $E(P_2) = \frac{1}{2} \times 80 + \frac{1}{2} \times 120 = 100$.

- $\mathrm{Var}(P_1)$

0 ✔ **Answer:** 0

0

Explanation
By definition, $\mathrm{Var}(P_1) = 1 \times (90 - 90)^2 = 0$.

- $\mathrm{Var}(P_2)$

400 ✔ **Answer:** 400

400

Explanation
By definition, $\mathrm{Var}(P_2) = \frac{1}{2} \times (80 - 100)^2 + \frac{1}{2} \times (120 - 100)^2 = 400$.

- Which of games 1 and 2 maximizes the `risk-adjusted reward' $E(P_i) - \sqrt{\mathrm{Var}(P_i)}$?

- [x] 1
- [ ] 2

✔

Explanation
By definition, $E(P_1) - \sqrt{\mathrm{Var}(P_1)} = 90$, $E(P_2) - \sqrt{\mathrm{Var}(P_2)} = 80$.

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

5 (Graded)

2/2 points (graded)
Which of the following are always true for random variables X, Y and real numbers a, b?

- [x] The variance of X is always non-negative.

- [x] The standard deviation of X is always non-negative.
- [x] If $V(X) = V(Y)$, then $V(X+a) = V(Y+b)$.
- [ ] If $V(aX) = V(bX)$ for $a \neq 0$ and $b \neq 0$, then $a = b$.
- [ ] If $E[X] = E[Y]$ and $V(X) = V(Y)$, then $X = Y$.
- [x] If $E[X] = E[Y]$ and $V(X) = V(Y)$, then $E[X^2] = E[Y^2]$.

✓

Explanation

- True.
- True. Standard deviation is defined by $\sqrt{V(X)}$, which is also non-negative.
- True. Adding a constant a to random varianle X will not affect its varaince.

$V(X+a) = E((X+a-E(X+a))^2) = E((X+a-E(X)-a)^2) = E((X-E(X))^2) = V(X)$

.
- False. When $V(X) = 0$, this does not hold.
- False. Consider two random variables X, Y with pmf, $P(X=x) = \begin{cases} \frac{1}{2}, x = -1, \\ \frac{1}{2}, x = 1 \end{cases}$ and

$P(Y=y) = \begin{cases} \frac{1}{8}, y = -2 \\ \frac{3}{4}, y = 0 \\ \frac{1}{8}, y = 2 \end{cases}$. Now $E(X) = E(Y) = 0, V(X) = V(Y) = 1$. However, $X \neq Y$.
- True. As $E(X^2) = V(X) + E^2[X]$, if $E(X) = E(Y)$ and $V(X) = V(Y)$, then $E(X^2) = E(Y^2)$.

Submit

You have used 1 of 4 attempts

Answers are displayed within the problem

6

0 points possible (ungraded)

We say X_A is an indicator variable for event $A: X_A = 1 \text{ if A occurs, } X_A = 0 \text{ if A does not occur.}$

If $P(A) = 0.35$, what is:

- $E(X_A)$?

- $\text{Var}(X_A)$?

Submit You have used 0 of 4 attempts

7

0 points possible (ungraded)

Let X denote the number when rolling a fair six-sided die, then what is:

- $\mathrm{Var}\left(X\right)$?

- σ_X?

Submit You have used 0 of 4 attempts

8

0 points possible (ungraded)

Let X and Y be independent random variables with expectations 1 and 2, and variances 3 and 4, respectively. Find the variance of $V\left(XY\right)$.

Submit You have used 0 of 4 attempts

Discussion

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Topic: Topic 7 / Variance

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? Proof of E(XY) = E(X)*E(Y)

How can we prove that E(XY) = E(X)*E(Y) in general?

2