Set Size

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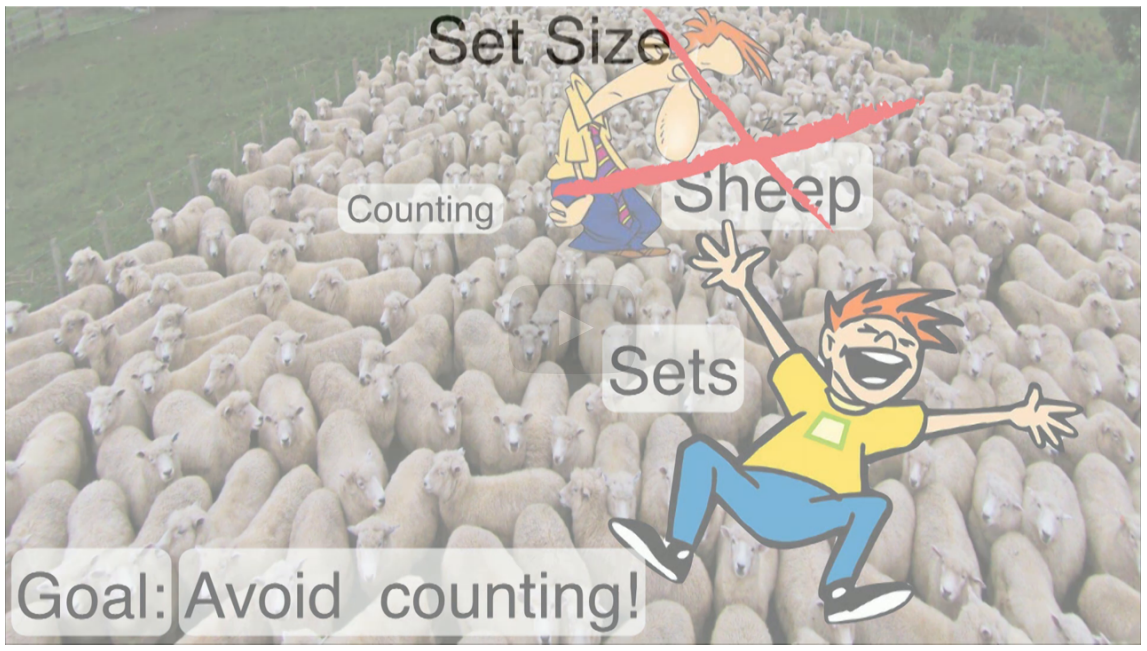
Problem Sets due Apr 30, 2022 09:05 +03 Completed

Video



0:00 / 0:00 1.0x

Start of transcript. Skip to the end.

Hello, and welcome back.

So we're done talking about just sets on their own,

and we're going to start talking about counting,

and don't worry, we won't be counting sheep

and fall asleep because we'll be counting sets,

and in fact, our goal in these next few lectures

3.1_Counting

POLL

The Python definition `A = set(range(1,10))` implies that A has size

RESULTS

2	0%
9	86%
10	14%
11	0%

Submit

Results gathered from 28 respondents.

FEEDBACK

A has size 9 as the elements are 1 to 9.

1 Perfect squares (Graded)

1.0/1.0 point (graded)

A square of an integer, for example, 0, 1, 4 and 9, is called a *perfect square*. How many perfect squares are ≤ 100?

11

Answer: 11

11

Explanation

The perfect squares ≤ 100 are 0^2, 1^2, 2^2, ..., 10^2. Hence there are 11.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

2

0 points possible (ungraded)

Which of the following sets are finite?

- [x] Weeks in a year
- [x] Students at UCSD
- [ ] Odd primes
- [x] Positive integer divisors of 30

Explanation

- True.
- True. Despite appearances, luckily, UCSD has only a finite number of students.
- False.
- True. It is $\{1, 2, 3, 5, 6, 10, 15, 30\}$.

Submit You have used 2 of 4 attempts

Answers are displayed within the problem

3 (Graded)

3.0/3.0 points (graded)

Which of the following sets are finite?

- [x] $\{x \in Z | x^2 \leq 10\}$
- [ ] $\{x \in Z | x^3 \leq 10\}$
- [x] $\{x \in N | x^3 \leq 10\}$
- [ ] $\{x \in R | x^2 \leq 10\}$
- [x] $\{x \in R | x^3 = 10\}$

Explanation

- True. It is $\{-3, -2, \cdots, 3\}$.
- False. It is $\{x \in Z | x \leq 2\}$.
- True. It is $\{0, 1, 2\}$.
- False. It is $\{x \in R | -\sqrt{10} \leq x \leq \sqrt{10}\}$.
- True. It is $\{10^{\frac{1}{3}}\}$.

Submit

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Disjoint Unions

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Video

Start of transcript. Skip to the end.

0:00 / 0:00 1.0x

Hello and welcome back.

Now that we've got basic counting under our belt,

let's talk about Disjoint Unions.

So we're going to discuss the addition rule,

subtraction rule, how it applies to pets

and also a hairy problem, okay.

3.2_Disjoint_Unions

POLL

We saw that the size of a union of two disjoint sets is the sum of their sizes.

If the two sets are not necessarily disjoint, then the size of their union is:

RESULTS

At least the sum of the set sizes	**0%**
At most the sum of the set sizes	**85%**
Could be smaller, same, or larger than the sum of the set sizes.	**15%**

Submit

Results gathered from 27 respondents.

FEEDBACK

at most the sum of the sizes as some elements may be in both sets, and adding the sizes counts these elements twice.

1

0 points possible (ungraded)

Which of the following are finite for every finite set A and an infinite set B?

- [x] $A \cap B$
- [ ] $A \cup B$
- [x] $A - B$
- [ ] $B - A$

- [ ] $A \Delta B$

✔

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Answers are displayed within the problem

2

1/1 point (graded)

Which of the following pairs A and B satisfy $|A \cup B| = |A| + |B|$?

- [x] $\{1, 2\}$ and $\{0, 5\}$
- [ ] $\{1, 2\}$ and $\{2, 3\}$
- [ ] $\{i \in \mathbb{Z} : |i| \leq 3\}$ and $\{i \in \mathbb{Z} : 2 \leq |i| \leq 5\}$
- [ ] $\{\text{English words starting with the letter 'a'}\}$ and $\{\text{English words ending with the letter 'a'}\}$

✔

Explanation

$|A \cup B| = |A| + |B|$ holds when A and B are disjoint.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

3

2.0/2.0 points (graded)

$|A \cup B \cup C| = |A| + |B| + |C|$ whenever:

A and B are disjoint and B and C are disjoint,

- ( ) True
- (•) False

✔

Explanation

False. Let $A = C = \{1\}$ and $B = \{2\}$. Then A and B are disjoint, B and C are disjoint. But $|A \cup B \cup C| = 2$, while $|A| + |B| + |C| = 3$.

A and B are disjoint, B and C are disjoint, and A and C are disjoint.

- (•) True
- ( ) False

✔

Explanation

True. Since A and C are disjoint, and B and C are disjoint, we must have that $A \cup B$ and C are disjoint. Hence $|A \cup B \cup C| = |A \cup B| + |C|$.
Since A and B are disjoint, we have $|A \cup B| = |A| + |B|$.
Hence $|A \cup B \cup C| = |A| + |B| + |C|$.

Submit You have used 2 of 4 attempts

Answers are displayed within the problem

4. Non perfect-squares

0 points possible (ungraded)
Recall that a square of an integer, for example, 1, 4 and 9, is called a *perfect square*. How many integers between 1, and 100, inclusive, are *not* perfect squares?

3

✖ **Answer:** 90

3

Explanation
The perfect squares between 1 and 100 are 1^2, 2^2, ..., 10^2. Hence there are 10. By the complement rule, 100-10=90 integers between 1 and 100 are not perfect squares.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

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wording confusion
In exercises like number 4. the word 'between' is confusing. Typically I would think that means x is greater than 1 and less than 100 b... 3

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General Unions

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Video

Start of transcript. Skip to the end.

Hello and welcome back.

In the previous lecture,

we talked about disjoint unions,

and now we would like to find the size of general units.

I will show that it's actually doing that is a piece of PIE.

And so we'll talk about

0:00 / 0:00 1.0x

3.3_General_Unions

POLL

When does |AUB|=|A|+|B|?

RESULTS

When at least one of A and B is empty	0%
When A and B are disjoint	35%
Both of the above.	65%

Submit

Results gathered from 26 respondents.

FEEDBACK

Both of the above. If A and B are disjoint then either by the disjoint union rule or by inclusion exclusion, the size of the union is the sum of the sizes. That implies that if one of the sets is empty the same holds.

1

0 points possible (ungraded)

In a high school graduation exam, 80% of examinees passed the English exam, 85% passed the math exam, and 75% passed both. If 40 examinees failed both subjects, what what the total number of examinees?

70

✖ **Answer:** 400

70

Explanation

By inclusion exclusion (applied to fractions), $80 + 85 - 75 = 90$% of the students passed at least one topic. Therefore 10% failed both topics. It follows that 40 students correspond to 10%, hence 400 students took the exam.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

2

0 points possible (ungraded)

How many integers in $\{1, 2, \ldots, 100\}$ do not contain the digit 6?

100-(10+9)

Answer: 81

$100 - (10 + 9)$

Explanation

Let $U_6 = \{6, 16, \ldots, 96\}$ be the sets of integers between 1 and 100 whose units digit is 6, and let $T_6 = \{60, 61, \ldots, 69\}$ be the corresponding set for the tens digit.

The set of integers between 1 and 100 containing 6 is $U_6 \cup T_6$, and by inclusion-exclusion, its size is $|U_6 \cup T_6|$= $|U_6| + |T_6| - |U_6 \cap T_6| = 10 + 10 - 1 = 19$.

Hence 100-19=81 integers between 1 and 100 do not contain the digit 6.

Submit You have used 2 of 4 attempts

Answers are displayed within the problem

3 (Graded)

6.0/6.0 points (graded)

Of 100 foreign journalists who can speak Chinese, English or French at a press conference:

60 speak Chinese.

65 speak English.

60 speak French.

35 speak both Chinese and English.

25 speak both Chinese and French.

35 speak both English and French.

How many journalists speak **exactly**

- one language,

25

Answer: 25

25

- two languages,

65

Answer: 65

65

- three languages?

10

Answer: 10

10

Explanation
By the Principle of Inclusion and Exclusion for three sets,
$100 = |A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C| = 90 + |A \cap B \cap C|$
.

Submit You have used 3 of 4 attempts

Answers are displayed within the problem

4 (Graded)

1.0/1.0 point (graded)
$|A \cup B| = |A| + |B|$ when

- [x] A and B are disjoint,
- [x] A is the complement of B,
- [x] A and B do not intersect,
- [x] At least one of A and B is empty.

Explanation
It holds whenever A and B are disjoint.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

5

0 points possible (ungraded)
The following equation is incorrect. What needs to be added to make it correct?

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C|$

- ( ) $-|A \cap B \cap C|$
- (•) $+|A \cap B \cap C|$
- ( ) $+3|A \cap B \cap C|$

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

6

0 points possible (ungraded)

In a highschool graduation exam 70% of examinees passed the English exam, 76% passed the math exam, and 66% passed both. If 40 examinees failed in both subjects, what is the total number of examinees?

200

✔ **Answer:** 200

200

Explanation

By the inclusion-exclusion principle, the percentage of students who passed at least one of the two exams is 70+76-66=80. Therefore 20% have failed both subjects. Hence the total number of examinees is 40 / 20% = 200.

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Cartesian Products

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Video

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Hello, and welcome back.

So, in the last video we talked about

the size of general unions and now

we would like to discuss Cartesian Products.

So,

if you have a set, for example, {a,b}

and another set, {1,2,3},

the set {a,b} is size two.

0:00 / 0:00 1.0x

3.4_Cartesian_Products_

POLL

Do AxB and BxA have the same size for any sets A and B?

RESULTS

Yes	**96%**
No	**4%**

Submit

Results gathered from 23 respondents.

FEEDBACK

Yes, just like a rectangle has the same area if you rotate it 90 degrees.

1. Finite X infinite

0 points possible (ungraded)

If A is finite and B is infinite then $A \times B$ can be:

- [x] empty
- [ ] nonempty finite
- [x] infinite

Explanation

If A is empty then $A \times \backslash B$, otherwise it is infinite.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

2. Order matters (Graded)

1/1 point (graded)

Which of the following ensures that $A \times B = B \times A$?

- [x] $A = B$
- [x] $A = \emptyset$
- [x] $B = \emptyset$
- [ ] $A \cap B = \emptyset$
- [ ] $|A| = |B|$

Explanation

- True.
- True. $A \times B = B \times A = \emptyset$.
- True. Same as above.
- False. Let $A = \{1\}, B = \{2\}$. Then $A \times B \neq B \times A$.
- False. Same as above.

Submit You have used 3 of 4 attempts

Answers are displayed within the problem

3. Cartesian product shape

0 points possible (ungraded)

Taking the geometric view of Cartesian products, if A and B are real intervals of positive length in $\mathbb{R}$, then $A \times B$ is a:

- ( ) line,
- (•) rectangle,
- ( ) circle,
- ( ) triangle,
- ( ) none of above.

Explanation

Let $A = [a, b)\,, B = [c, d)$. Then $A \times B = \{(x, y)\,|a \leq x < b, c \leq y < d, x \in \mathbb{R}, y \in \mathbb{R}\}$, which is a rectangle.

Submit You have used 2 of 2 attempts

Answers are displayed within the problem

4. Divisors (Graded)

2.0/2.0 points (graded)

How many positive divisors does 2016 have?

36

Answer: 36

36

Explanation

$2016 = 2^5 \cdot 3^2 \cdot 7$.

Hence any positive divisor of 2016 can factored as $2^x \cdot 3^y \cdot 7^z$, where $x \in \{0, 1, \ldots, 5\}$, $y \in \{0, 1, 2\}$ and $z \in \{0, 1\}$. By the product rule, there are $6 \times 3 \times 2 = 36$ divisors.

Submit

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Cartesian Powers

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Video

Start of transcript. Skip to the end.

Hello, and welcome back.

In the last lecture, we talked about Cartesian products

and today we want to talk about Cartesian Powers.

So we have shown an analogy between number operations

and set operations.

For example, we said that number addition corresponds to

0:00 / 0:00 1.0x

3.5_Cartesian_Powers

POLL

The set {000, 001, ...,111} of all 3-bit strings has the following number of subsets:

RESULTS

2^3	59%
2^6	0%
2^8	32%
2^9	9%

Submit

Results gathered from 22 respondents.

FEEDBACK

2^8. The set has (2^3=8) strings, and a set of 8 elements has 2^8 subsets.

1

0 points possible (ungraded)

Rank the functions n^2, 2^n, n from slow to fast in terms of their growth rate as n increases.

- $n, 2^n, n^2$
- $n, n^2, 2^n$
- $2^n, n, n^2$
- $2^n, n^2, n$

Submit You have used 2 of 2 attempts

Answers are displayed within the problem

2 (Graded)

6.0/6.0 points (graded)

Find the number of 7-character (capital letter or digit) license plates possible if:

- there are no further restrictions,

36^7 **Answer:** 78364164096

36^7

Explanation

In each position there are $26 + 10 = 36$ choices (26 letters and 10 digits).

The number of ways is $(26 + 10)^7 = 36^7 = 78,364,164,096$.

- the first 3 characters are letters and the last 4 are numbers,

(26^3)*(10^4) **Answer:** 175760000

$(26^3) \cdot (10^4)$

Explanation

For the first 3 positions, the number of ways is 26^3 (letters only).

For the last 4 positions, the number of ways is 10^4 (digits only).

The total number of ways is $26^3 \cdot 10^4 = 175,760,000$.

- letters and numbers alternate, for example A3B5A7Q or 0Z3Q4Q9.

(26^4)*(10^3)+(26^3)*(10^4) **Answer:** 632736000

$(26^4) \cdot (10^3) + (26^3) \cdot (10^4)$

Explanation

If there are 4 digits and 3 letters, the number of ways is $26^3 \cdot 10^4$.

If there are 4 letters and 3 digits, the number of ways is $26^4 \cdot 10^3$.

The total number of ways is $26^3 \cdot 10^4 + 26^4 \cdot 10^3 = 175,760,000 + 456,976,000 = 632,736,000$.

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

3

0 points possible (ungraded)

If P and Q are sets, then $|P|^{|Q|}$ is the number of functions

- ( ) from P to Q,
- (•) from Q to P.

Explanation
The number of functions from Q to P as there are $|P|$ possible images for every element of Q.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

4 (Graded)

6.0/6.0 points (graded)
Recall that the power set $\mathcal{P}(S)$ of a set S is the collection of all subsets of S.

For A = $\{1, 2, 3\}$ and $B = \{x, y\}$, calculate the following cardinalities.

- $|\mathcal{P}(A)|$

2^3

✔ **Answer:** 8

2^3

- $|\mathcal{P}(B)|$

2^2

✔ **Answer:** 4

2^2

- $|A \times B^2|$

3*(2^2)

✔ **Answer:** 12

$3 \cdot (2^2)$

- $|\mathcal{P}(A \times B)|$

2^(2*3)

✔ **Answer:** 64

$2^{2 \cdot 3}$

- $|\mathcal{P}(A) \times B|$

(2^3)*2

✔ **Answer:** 16

$(2^3) \cdot 2$

- $|\mathcal{P}(\mathcal{P}(A))|$

2^((2^3))

✔ **Answer:** 256

$2^{(2^3)}$

Submit You have used 3 of 4 attempts

Answers are displayed within the problem

5

0 points possible (ungraded)

Let $G = \{0, 2, 4, 6, 8\}$. What is $|G^4|$?

- (selected) 5^4
- 4^5
- $5 \times 4 \times 3 \times 2 \times 1$
- $0 + 2 + 4 + 6 + 8$

✔

Submit You have used 2 of 2 attempts

Answers are displayed within the problem

6

0 points possible (ungraded)

Let A be a set with size 5. How many **strict** subsets does A have?

(2^5)-1 ✔ **Answer:** 31

$(2^5) - 1$

Explanation

A itself is not included. $2^5 - 1 = 31$.

Submit You have used 2 of 3 attempts

Answers are displayed within the problem

7

0 points possible (ungraded)

How many subsets of $\{1, 2, \ldots, 9\}$

- are there,

2^9 ✔ **Answer:** 512

2^9

Explanation

Every element of $\{1, 2, \ldots, 9\}$ is an either in the subset of not, hence there are $2^9 = 512$ subsets.

- contain '1',

2^8 ✔ **Answer:** 256

2^8

Explanation

Ever set cotaining '1' corresponds to an abritrary subset of $\{2, 3, \cdots, 9\}$, and there are $2^8 = 256$ such subsets.

- have > 5 elements

- have ≥ 5 elements,

2^5

✖ **Answer:** 256

2^5

Explanation
Every subset of size ≥ 5 corresponds to its complement that has size ≤ 4. For example, $\{1, 2, 3, 4, 5\}$ has 5 elements, while its complement $\{6, 7, 8, 9\}$ has 4 elements.
Hence the number of subsets of size ≥ 5 is the same as the number of subsets of size ≤ 4 and together they comprise all 2^9 subsets. Hence the number of subsets of size ≥ 5 is $\frac{2^9}{2} = 2^8 = 256$.

- have no odd elements,

2^5

✖ **Answer:** 16

2^5

Explanation
Any subset with no odd elements is a subset of $\{2, 4, 6, 8\}$ and there are $2^4 = 16$ such subsets.

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Counting Variations

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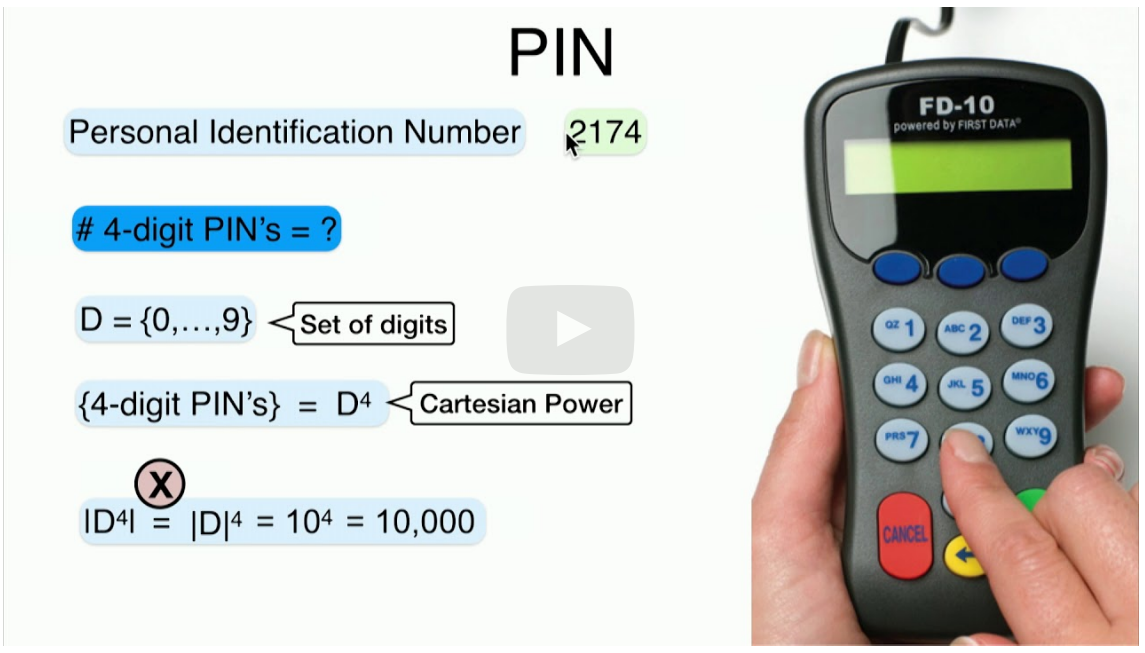

0:00 / 0:00 1.0x

Hello, and welcome back.

So far, we talked about counting different types of sets

like unions, Cartesian products, Cartesian powers,

and what we want to do today

is discuss several variations of these sets.

But before we do that, let's consider the question

3.6_Variations

POLL

Sam tries to find how many 2-letter sequences do not include the letter Q, for example BR or ZZ, but not BQ, or QQ.

First, Sam calculates the total number of 2-letter sequences, allowing all letters, as 26*26=676.

Then Sam calculates the number of disallowed sequences:

There are 26 sequences where a Q is the first letter (e.g. QS), and 26 where Q is the second letter (e.g. BQ).

The total number of disallowed sequences is therefore 26+26=52. Subtracting 52 from 676, Sam deduces that there are 624 two-letter sequences without Q.

Where did Sam go wrong?

RESULTS

Made a multiplication error.	**7%**
Made a subtraction error	**14%**
Subtracted the sequence QQ from the total twice.	**79%**
Nothing, Sam is correct.	**0%**

Submit

Results gathered from 14 respondents.

FEEDBACK

Sam subtracted the sequence QQ from the total twice.

The problems in this section are either variations or combinations of techniques we have learned.

1 (Graded)

3.0/3.0 points (graded)

An n-variable Boolean function maps $\{0,1\}^n$ to $\{0,1\}$. How many 4-variable Boolean functions are there?

- ○ 16
- ○ 256
- ◉ $65,536$

✔

Explanation

From the video, there are 2^{2^n} Boolean functions of n-variables, and $2^{2^4} = 2^{16} = 65,436$.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

2 (Graded)

6.0/6.0 points (graded)

A word that reads the same from left to right and right to left is a *palindrome*. For example, "I", "noon" and "racecar" are palindromes. In this question, we consider *palindromic integers* such as 101 and 66. Note that the first digit of an integer **cannot** be 0.

How many positive 5-digit integer palindromes

- are there?

9*10*10*1*1 ✔ **Answer:** 900

$9 \cdot 10 \cdot 10 \cdot 1 \cdot 1$

Explanation

Since the number is palindromic, it is determined by its first three digits. The first digit must be 1,2,...,9, while the second and third are unconstrained. Hence the total number is $9 \cdot 10 \cdot 10 = 900$.

- are even, for example 29192?

4*10*10*1*1 ✔ **Answer:** 400

$4 \cdot 10 \cdot 10 \cdot 1 \cdot 1$

Explanation

The first digit can be only 2,4,6,8, hence $4 \cdot 10 \cdot 10 = 400$.

- contain 7 or 8, for example 27172 or 38783?

452 ✔ **Answer:** 452

452

Explanation

$7 \cdot 8 \cdot 8 = 448$ palindromic integers do not contain 7 or 8. By the complement rule, 900-448=452 contain 7 or 8.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

3

3

0 points possible (ungraded)

How many 5-digit ternary strings are there without 4 consecutive 0s, 1s or 2s?

For example, 01210 and 11211 are counted, but 20000, 11112, and 22222 are excluded.

(3^5)-(4*3)

✖ **Answer:** 228

$(3^5) - (4 \cdot 3)$

Explanation

First count the number of excluded sequences.
A 5-digit string contains 4 consecutive 0's if it starts with 0000, or ends with 0000, or both.
The number of 5-digit ternary strings that start with 0000 is 3, the number that end with 0000 is 3, and the number that both starts and ends with 0000 is 1 (the sequence 00000). By inclusion exclusion, there are 3+3-1=5 sequences containing 0000.
Similarly there are 5 sequencess with 1111 and 5 sequences with 2222.
These sequences are disjoint, hence by the sum rule, the total number of forbidden sequences is 5+5+5=15.
The total number of 5-digit ternary strings is $3^5 = 243$.
By the subtraction rule, the number of allowed sequences is $243 - 15 = 228$.

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Answers are displayed within the problem

4

0 points possible (ungraded)

A password consists of four or five characters, each an upper-case letter, a lower-case letter, or a digit. How many passwords are there if each of the three character types must appear at least once?

((62^4) - ((26^4)+(26^4) + (10

✖ **Answer:** 443888640

$((62^4) - ((26^4) + (26^4) + (10^4))) + ((62^5) - ((26^5) + (26^5) + (10^5)))$

Explanation

For each of length 4 and 5, we use inclusion-exclusion to count the number of password that do not contain at least one of the three character types, then use the complement rule.
Let U, L, and D denote the sets of passwords that do not contain upper-case letters, lower-case letters, and digits, respectively, and let n be the password length. Then $|U| = |L| = 36^n$, and $|D| = 52^n$. Furthermore, $|L \cap U| = 10^n$, while $|L \cap D| = |U \cap D| = 26^n$, and $U \cap L \cap D = \emptyset$. Finally, the number of length-n character sequences is 62^n.
$|U \cup L \cup D|$ is the number of sequences that have at least one character type missed.
By the 3-set inclusion-exclusion principle,
$|U \cup L \cup D| = |U| + |L| + |D| - |L \cap U| - |L \cap D| - |U \cap D| + |U \cap L \cap D|$. Hence, there are
$\left(62^4 - (36^4 + 36^4 + 52^4 - 26^4 - 26^4 - 10^4 + 0)\right) + \left(62^5 - (36^5 + 36^5 + 52^5 - 26^5 - 26^5 - 10^5 + 0)\right) = 4$
valid passwords.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

5

0 points possible (ungraded)

In the US, telephone numbers are 7-digit long. While real phone numbers have some restrictions, for example,

they cannot start with 0 or 1, here we assume that all 7-digit sequences are possible, even 0000000.

How many phone numbers:

- start or end with two identical digits, for example 0012345, 1234511, or 2222222,

- contain a substring of 5 consecutive digits. For example 0034567, 2567892, or 0123456.

Submit You have used 0 of 4 attempts

6 (Graded)

6.0/6.0 points (graded)

How many ordered pairs (A, B), where A, B are subsets of $\{1, 2, 3, 4, 5\}$ have:

- $A \cap B = \emptyset$

243 ✔ **Answer:** 243

243

Explanation

Represent each pair (A, B) of disjoint subssets of $\{1, 2, 3, 4, 5\}$ as a ternary sequence of length 5, where 1 in location i indicaes that $i \in A$, 2 in location i indicates that $i \in B$, and 0 in location i indicates that i is in neither A nor B. For example, 10201 corresponds to $A = \{1, 5\}$ and $B = \{3\}$. The number of disjoint subset pairs is therefore the same as the number of ternary sequences of length 5, namely 3^5.

- $A \cap B = \{1\}$

81 ✔ **Answer:** 81

81

Explanation

Include 1 in both A, B and then repeat above part with the set $\{2, 3, 4, 5\}$.

- $|A \cap B| = 1$

405 ✔ **Answer:** 405

405

Explanation

Same as the above part, except the element in common can be any from $\{1, 2, \ldots, 5\}$ and therefore $5 \cdot 3^4 = 405.$

? **Hint (1 of 3):** Each element in the set has 4 states: (1) In A; (2) In B; (3) In A and B; (4) Neither in A nor B.

Hint (2 of 3): If $A \cap B = \emptyset$, what are the possible states for the elements?

Hint (3 of 3): For example, if $A = \{1, 2\}, B = \{3, 4\}$, the states of the five elements are 11224.

Next Hint

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

7

0 points possible (ungraded)

How many ordered pairs (A, B), where A, B are subsets of $\{1, 2, 3, 4, 5\}$, are there if:

- $A \cup B = \{1, 2, 3, 4, 5\}$

- $|A \cup B| = 4$

Submit You have used 0 of 4 attempts

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typo in #1 explanation
There's a typo in the explanation for #1 1

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