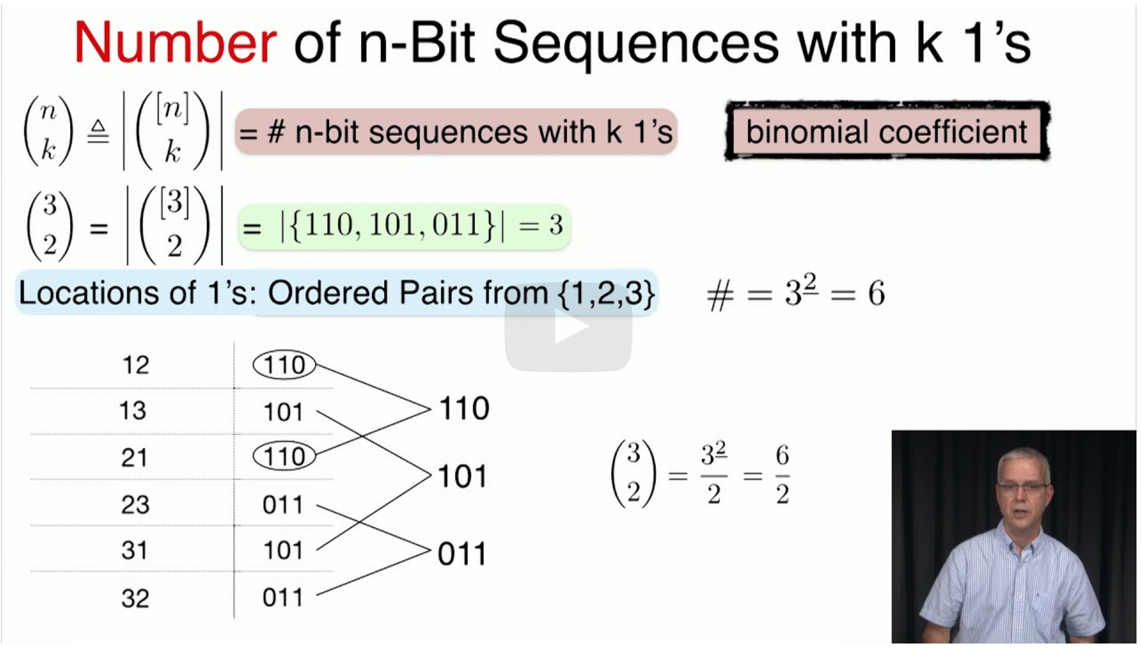
Combinations

Problem Sets due May 4, 2022 18:05 +03 Completed

Video

Start of transcript. Skip to the end.



0:00 / 0:00 1.0x

- Hello again, everyone.

Last time we talked about permutations,

and in this lecture we'll discuss combinations.

So what are they?

So first we're going to look at subsets of a set.

And so a subset of size k is called a k-subset.

4.3_Combinations

POLL

Which of the following is larger for k≤n?

RESULTS

The number of k-permutations of an n-set	**90%**
The number of k-subsets of an n-set	**10%**

Submit

Results gathered from 21 respondents.

FEEDBACK

The number of k-permutations is larger.

In selecting subsets, the order doesn't matter, hence the number of k-subsets is the number of k-permutations divided by k!

1

0 points possible (ungraded)

In how many ways can a basketball coach select 5 starting players form a team of 15?

- (selected) $\frac{15!}{5!10!}$
- $\frac{15!}{10!}$
- $\frac{15!}{5!}$
- None of the above

Explanation

It can be deducted from partial permutation, but the order does not matter. It is $\binom{15}{5} = \frac{15^5}{5!} = \frac{15!}{5!10!}$.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

2

0 points possible (ungraded)

- In how many ways can you select a group of 2 people out of 5?

- (x) 10
- ( ) 25
- ( ) 125
- ( ) None of the above

Explantion

$\binom{5}{2} = 10.$

- In how many ways can you select a group of 3 people out of 5?

- (x) 10
- ( ) 25
- ( ) 125
- ( ) None of the above

Explantion

$\binom{5}{3} = 10.$

- In how many ways can you divide 5 people into two groups, where the first group has 2 people and the second has 3?

- (x) 10
- ( ) 25
- ( ) 125
- ( ) None of the above

Explantion

After we determine the group of 2, the group of 3 is determined as well, hence the answer is $\binom{5}{2} = 10$.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

3

0 points possible (ungraded)

Ten points are placed on a plane, with no three on the same line. Find the number of:

- lines connecting two of the points,

45 ✔ **Answer:** 45

45

Explanation

Choosing any 2 points out of the 10 points can make a line: $\binom{10}{2}$

- these lines that do not pass through two specific points (say A or B),

28 ✔ **Answer:** 28

28

Explanation

Choosing any 2 points out of the remaining 8 points (except A, B): $\binom{8}{2}$

- triangles formed by three of the points,

120 ✔ **Answer:** 120

120

Explanation

As no three on the same line, choosing any 3 points out of the 10 points make a triangle: $\binom{10}{3}$

- these triangles that contain a given point (say point A),

(9*8*7)/6 ✘ **Answer:** 36

$\frac{9 \cdot 8 \cdot 7}{6}$

Explanation

With point A fixed, choosing any 2 points out of the remaining 9 points make a triangle: $\binom{9}{2}$

- these triangles contain the side AB.

8 ✔ **Answer:** 8

8

Explanation

With point A and B fixed, choosing any 1 point out of the remaining 8 points make a triangle: $\binom{8}{1}$

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

4

0 points possible (ungraded)
The set $\{1,2,3\}$ contains 6 nonempty intervals: $\{1\}$, $\{2\}$, $\{3\}$, $\{1,2\}$, $\{2,3\}$, and $\{1,2,3\}$.

How many nonempty intervals does $\{1,2,\ldots,10\}$ contain?

(2^10) ✖ **Answer:** 55

(2^{10})

Explanation
$\{1,2,\ldots,n\}$ contains $\binom{n}{1}$ singleton intervals and $\binom{n}{2}$ intervals of 2 or more elements. Hence the total number of intervals is $\binom{n}{2}+\binom{n}{1}$. By Pascal's identity $\binom{n}{2}+\binom{n}{1}=\binom{n+1}{2}$. This can also be seen by considering the $n+1$ midpoints $\{0.5,1.5,\ldots n+0.5\}$. Any pair of these points defines an interval in $\{1,2,\cdots n\}$.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

5

0 points possible (ungraded)
A rectangle in an $m\times n$ chessboard is a cartesian product $S\times T$, where S and T are nonempty intervals in $\{1,\ldots,m\}$ and $\{1,2,\ldots,n\}$ respectively. How many rectangles does the 3×6 chessboard have?

6*21 ✔ **Answer:** 126

$6\cdot 21$

Explanation
Repeating the same analysis as the above question, but for two different intervals, we have $\binom{4}{2}\cdot\binom{7}{2}=126.$

? **Hint (1 of 1):** For example, the 2×2 chessboard has $3\cdot 3=9$ rectangles. Next Hint

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

6 (Graded)

8.0/8.0 points (graded)
A standard 52-card deck consists of 4 suits and 13 ranks. Find the number of 5-card hands where:

- any hand is allowed (namely the number of different hands),

2598960 ✔ **Answer:** 2598960

2598960

2598960

Explanation
This is simply $\binom{52}{5}$.

- all five cards are of same suit,

4*1287 ✔ **Answer:** 5148

$4 \cdot 1287$

Explanation
There are 4 suits in total and 13 cards in each suit, hence $4 \cdot \binom{13}{5}$ hands.

- all four suits are present,

685464 ✔ **Answer:** 685464

685464

Explanation
One of the 4 suits will appear twice, hence $4 \cdot \binom{13}{2} \cdot 13^3$ hands.

- all cards are of distinct ranks.

1317888 ✔ **Answer:** 1317888

1317888

Explanation
First pick 5 out of 13 ranks, then choose their suits. Therefore there are $\binom{13}{5} \cdot 4^5$ hands.

? **Hint (1 of 1):** For example, for hands where all cards are of the same suit, count the number of hands with 5 clubs, or with 5 diamonds, etc. Next Hint

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

7 (Graded)

2.0/2.0 points (graded)
A company employs 4 men and 3 women. How many teams of three employees have at most one woman?

- ( ) 21
- (x) 22
- ( ) 23
- ( ) 24

✔

Explanation
There are $\binom{4}{3} = 4$ teams with 0 women and $\binom{3}{1} \times \binom{4}{2} = 3 \times 6 = 18$ teams with 1 woman, for a total of 22.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

8 (Graded)

5.0/5.0 points (graded)

A (tiny) library has 5 history texts, 3 sociology texts, 6 anthropology texts and 4 psychology texts. Find the number of ways a student can choose:

- one of the texts,

18 **Answer:** 18

18

Explanation

- two of the texts,

153 **Answer:** 153

153

Explanation

- one history text and one other type of text,

65 **Answer:** 65

65

Explanation

The student can choose 5 different history texts, and $3+6+4=13$ other texts, by the product rule there are $5 \cdot 13 = 65$ ways of doing that.

- one of each type of text,

360 **Answer:** 360

360

Explanation

The student selects one text of each type, by the product rule this can be done in $5 \cdot 3 \cdot 6 \cdot 4 = 360$ ways.

- two of the texts with different types.

119 **Answer:** 119

119

Explanation

There are $5 \cdot 3 = 15$ ways to choose one history and one sociology text, $5 \cdot 6 = 30$ ways to choose one history and one anthropology text, etc. In total there are $5 \cdot 3 + 5 \cdot 6 + 5 \cdot 4 + 3 \cdot 6 + 3 \cdot 4 + 6 \cdot 4 = 119$ ways.

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

9

0 points possible (ungraded)

In how many ways can 7 distinct red balls and 5 distinct blue balls be placed in a row such that

- all red balls are adjacent,

3628800

Answer: 3628800

3628800

Explanation

There are 6 ways to place 7 red balls adjacent. Hence the number of ways is $6 \times 7! \times 5! = 3628800$.

- all blue balls are adjacent,

4838400

Answer: 4838400

4838400

Explanation

There are 8 ways to place 5 red balls adjacent. Hence the number of ways is $8 \times 7! \times 5! = 4838400$.

- no two blue balls are adjacent.

Answer: 33868800

Explanation

First, decide on the locations of the red and blue balls. Arrange all 7 red balls in a line, we can then choose 5 out of the 8 gaps (including those at the beginning and end) to place the blue balls. Since the balls are distinct we can permute the blue balls, and the red balls, for a total of $\binom{8}{5}7!5!$ arrangements.

Submit

You have used 4 of 4 attempts

Answers are displayed within the problem

10

0 points possible (ungraded)

For the set $\{1, 2, 3, 4, 5, 6, 7\}$ find the number of:

- subsets,

2^7

Answer: 2^7

2^7

Explanation

There are 7 elements in the set. The number of subsets is 2^7.

- 3-subsets,

Answer: 35

Explanation

Choose 3 elements out of 7. The number of ways is $\binom{7}{3} = 35$.

- 3-subsets containing the number 1,

✖ **Answer:** 15

Explanation
1 is fixed.
Choose 2 elements out of 6. The number of ways is $\binom{6}{2} = 15$.

- 3-subsets not containing the number 1.

✖ **Answer:** 20

Explanation
Choose 3 elements out of 6 (excluding 1). The number of ways is $\binom{6}{3} = 20$.

? Hint (1 of 1): A 3-subset is a subset with 3 elements. Next Hint

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

11 Functions.

0 points possible (ungraded)
A function $f : X \to Y$ is *injective* or *one-to-one* if different elements in X map to different elements in Y, namely,

$$\forall x \neq x' \in X, \quad f(x) \neq f(x').$$

A function $f : X \to Y$ is *surjective* or *onto* if all elements in Y are images of at least one element of X, namely,

$$\forall y \in Y \quad \exists x \in X, \quad f(x) = y.$$

For sets $A = \{1, 2, 3\}$ and $B = \{a, b, c, d\}$, find the number of

- functions from A to B,

- functions from B to A,

- one-to-one functions from A to B,

- onto functions from B to A.

Submit You have used 0 of 4 attempts

Discussion

Topic: Topic 4 / Combinations

Hide Discussion

Add a Post

Show all posts by recent activity

There are no posts in this topic yet.

×

< Previous Next >

edX

About
Affiliates
edX for Business
Open edX
Careers
News

Legal

Terms of Service & Honor Code
Privacy Policy
Accessibility Policy
Trademark Policy
Sitemap

Connect

Blog