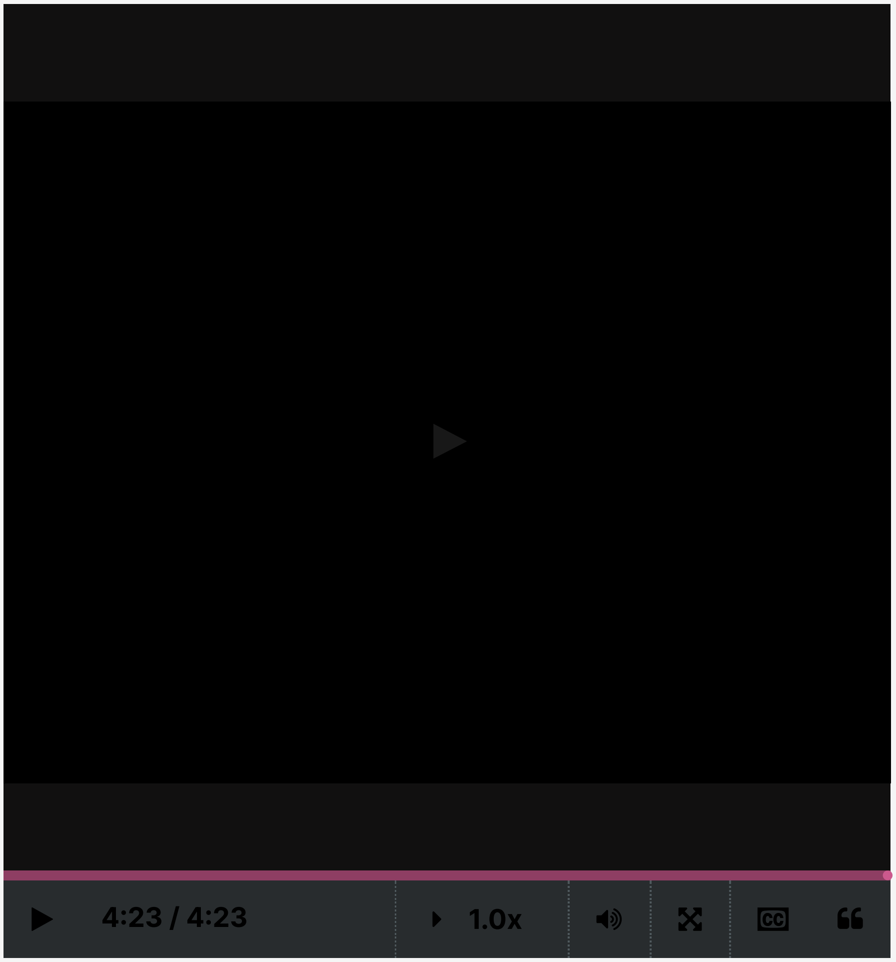
Problem Sets due Jul 8, 2022 16:34 +03

Video



than whether the variances add,

and that's what we want to look at next.

So, this is what we're going to do in a separate video

because it would take us some time to discuss this.

See you then.

End of transcript. Skip to the start.

7.8_Linearity_of_Expectation

POLL

Which of the following always holds?

RESULTS

E[X+Y]=E[X]+E[Y]	16%
E[X-Y]=E[X]-E[Y]	2%
Both	80%

None 2%

Submit

Results gathered from 45 respondents.

FEEDBACK

Both of them hold.

1

0 points possible (ungraded)

Let X be number of heads you get by flipping a fair coin 100 times. Then what is $E(X)$?

- $E[X] = 25$
- $E[X] = 50$
- $E[X] = 75$
- None of the above

Explanation

Let X_i be the random variable for the i-th flip, with 1 representing heads and 0 representing tails. Then $E(X_i) = \frac{1}{2}$.

It is obvious that $X = \sum_{i=1}^{100} X_i$. Its expectation

$E(X) = E\left(\sum_{i=1}^{100} X_i\right) = \sum_{i=1}^{100} E(X_i) = 100 \times \frac{1}{2} = 50.$

? **Hint (1 of 1):** Expectation is linear. Next Hint

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

2 (Graded)

3/3 points (graded)

Starting with 10 blue balls, in each of 10 sequential rounds, we remove a random ball and replace it with a new red ball. For example, after the first round we have 9 blue balls and one red ball, after the second round, with probability $9/10$ we have 8 blue balls and 2 red balls, and with probability $1/10$ we have 9 blue balls and one red ball, etc.

What is the probability that the ball we remove at the 11th round is blue?

0.34

Answer: 0.349

0.34

Explanation

Imagine that the balls are placed in 10 locations 1 to 10. Let B_i be the event that at the final (11th) round, the ball in location i is blue. B_i occurs iff the ball in location i was not discarded in any of the previous 10 rounds, hence $P\left(B_i\right) = \left(1 - 1/10\right)^{10} = \left(9/10\right)^{10}$. Let B be the event that the final ball, picked at the 11th round, is blue. By the rule of total probability, $P\left(B\right) = \sum_{i=1}^{10} \frac{1}{10} P\left(B_i\right) = 10 \cdot \frac{1}{10} \left(\frac{9}{10}\right)^{10} = \left(\frac{9}{10}\right)^{10} = 0.3486$.

? **Hint (1 of 1):** Imagine that the balls are placed in 10 distinct locations, and first find the probability that at the end of the 10th round, the ball in a given location is still blue.

Next Hint

Submit

You have used 2 of 4 attempts

Answers are displayed within the problem

3 (Graded)

2/2 points (graded)

$\mathbb{E}(X) = 2$ and $\mathbb{E}(X(X-1)) = 5$. Find $V(X)$.

3

✔ **Answer:** 3

3

Explanation

$5 = \mathbb{E}(X(X-1))$

$= \mathbb{E}(X^2 - X)$

$= \mathbb{E}(X^2) - \mathbb{E}(X)$

$= \mathbb{E}(X^2) - 2$

$\rightarrow \mathbb{E}(X^2) = 5 + 2 = 7$

$V(X) = \mathbb{E}(X^2) - \mathbb{E}(X)^2 = 7 - 4 = 3$

Submit

You have used 1 of 4 attempts

Answers are displayed within the problem

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