Random Variables

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Problem Sets due Jul 8, 2022 16:34 +03

Video

what the pmf, probability mass function

for random variable is, we saw how to visualize

using histogram, using plot and using stem plot

and next time we're going to talk about

cumulative distribution functions.

See you then.

End of transcript. Skip to the start.

7.1_Random_Variables

POLL

Which of the following statements is correct?

RESULTS

○	**Random variables are mappings between outcomes and real numbers.**	**67%**
●	**Random variables are mappings between events and real numbers.**	**28%**
○	**Neither**	**5%**

Submit

Results gathered from 43 respondents.

FEEDBACK

Random variables are mappings between outcomes and real numbers.

---

1 (Graded)

0/3 points (graded)

For which value of α is the function $p_i = \frac{(\alpha+1)(i-\alpha)+2}{120}$ over $\{1, 2, \cdots, 10\}$ a p.m.f.?

7

✖ **Answer:** 1.5

7

Explanation

The p.m.f should add up to 1, hence,

$$\sum_{i=1}^{10} p_i = \sum_{i=1}^{10} \frac{(\alpha+1)(i-\alpha)+2}{120} = \sum_{i=1}^{10} \frac{-\alpha^2+(i-1)\alpha+i+2}{120} = 1$$

.

This reduces to the quadratic equation $2\alpha^2 - 9\alpha + 9 = 0$ with two solutions $\alpha = \frac{3}{2}$ and $\alpha = 3$. Recall that $0 \le p_i \le 1$, the solution $\alpha = 3$ is discarded as some p_i's are negative, and we are left eith $\alpha = \frac{3}{2}$.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

2

0 points possible (ungraded)
Which of the following are true for random variables?

☐ A random variable X defines an event.

☑ For a random variable X and a fixed real number a, "$X \le a$" defines an event.

☐ Random variables for the same sample space must be same.

☑ For a random variable X, possible values for $P(X = x)$ include $0, 0.5$ and 1.

✓

Explanation
Recall either the informal definition of a random variable as a real-valued random experiment, or the more formal one as a function that maps the sample set Ω to real numbers $\mathbb{R}$. Therefore:
- False. A random variable does not define an event.
- True. "$X \le a$" is the set of outcomes that are at most a.
- False. A fair coin and a biased coin are two different variables with the same sample space \(\{h,t\}).
- True. $0 \le P(X = x) \le 1$, hence both $0, 0.5$ and 1 are possible.

Submit You have used 2 of 4 attempts

Answers are displayed within the problem

3 (Graded)

3/3 points (graded)
An urn contains 20 balls numbered 1 through 20. Three of the balls are selected from the run randomly without replacement, and X denotes the largest number selected.

- How many values can X take?

18 ✓ **Answer:** 18

18

Explanation
1 and 2 are impossible, the remaining 18 outcomes can occur.

- What is $P(X = 18)$?

34/285 ✓ **Answer:** 0.119

$\frac{34}{285}$

Explanation

18 is fixed, while the other 2 balls should selected from 1 to 17. $P(X=18)=\binom{17}{2}/\binom{20}{3}=0.119$.

- What is $P(X\geq 17)$?

0.5

✓ **Answer:** 0.508

0.5

Explanation

$$P(X\geq 17)=P(X=17)+P(X=18)+P(X=19)+P(X=20)=\frac{\binom{16}{2}+\binom{17}{2}+\binom{18}{2}+\binom{19}{2}}{\binom{20}{3}}=0.508$$

.

Submit

You have used 1 of 4 attempts

Answers are displayed within the problem

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Video

So to summarize, we talked about

cumulative distribution functions,

we defined them, we saw some of the properties

and we saw that they are useful

to calculate interval probabilities

and next time, we're going to talk about expectations.

See you then.

End of transcript. Skip to the start.

7.2_Cumulative_Distribution_Function

POLL

All cumulative distribution functions are:

RESULTS

○	**Continuous.**	**16%**
○	**Left continuous.**	**7%**
●	**Right continuous.**	**70%**
○	**None of the above.**	**7%**

Submit

Results gathered from 43 respondents.

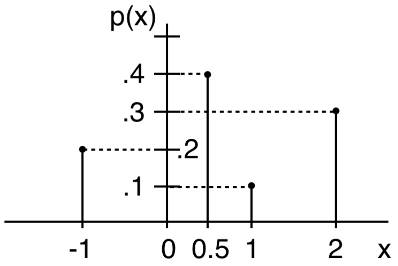
FEEDBACK

All cdf's are right continuous.

1 (Graded)

3/3 points (graded)

For the probability mass function



Find:

- $P(X = 1)$,

0.1

✓ **Answer:** 0.1

0.1

Explanation
$P(X = 1) = 0.1$ from the figure.

- $P(X \geq 1)$,

0.4

✓ **Answer:** 0.4

0.4

Explanation
$P(X \geq 1) = P(X = 1) + P(X = 2) = 0.4$.

- $P(X \in \mathbb{Z})$.

0.6

✓ **Answer:** 0.6

0.6

Explanation
$P(X \in \mathbb{Z}) = P(X = -1) + P(X = 0) + P(X = 1) + P(X = 2) = 0.6$.

Submit

You have used 1 of 4 attempts

Answers are displayed within the problem

2 (Graded)

4/4 points (graded)
Recall that the "floor" of a real number x, denoted $\lfloor x \rfloor$, is the largest interger $\leq x$.

$F(x) = \begin{cases} k - \frac{1}{\lfloor x \rfloor}, x \geq 1, \\ 0, x < 1, \end{cases}$ is a cummulative distribution function (cdf) for some fixed number k. Find:

- k,

1

✓ **Answer:** 1

1

Explanation
Recall that $F(\infty) = 1$. Here $F(\infty) = k$, hence $k = 1$.

- x_{min} (the smallest number with non-zero probability), 2

✓ **Answer:** 2

2

Explanation
Observe that $F(x) = 0$ for $x < 1$, and since $k = 1$, also $F(1) = 0$, hence the smallest number with non-zero probability is 2.

- $P(X = 4)$,

1/12

✓ **Answer:** 1/12

$\frac{1}{12}$

Explanation
$P(X = 4) = F(4) - F(3) = \frac{3}{4} - \frac{2}{3} = \frac{1}{12}$.

- $P(2 < X \leq 5)$.

3/10

✓ **Answer:** 3/10

$\frac{3}{10}$

Explanation
$P(2 < X \leq 5) = F(5) - F(2) = \frac{4}{5} - \frac{1}{2} = \frac{3}{10}$.

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

3

0 points possible (ungraded)
Flip a coin with heads probability 0.6 repeatedly till it lands on tails, and let X be the total number of flips, for example, for h, h, t, $X = 3$. Find:

- $P(X \leq 3)$,

0.784

✓ **Answer:** 0.784

0.784

Explanation
$P(X \leq 3) = P(X = 1) + P(X = 2) + P(X = 3) = 0.4 + 0.6 \times 0.4 + 0.6 \times 0.6 \times 0.4 = 0.784$.

- $P(X \geq 5)$.

0.1296

✓ **Answer:** 0.1296

0.1296

Explanation
$P(X \geq 5) = 1 - P(X < 5) = 1 - P(X \leq 4) = 1 - (P(X \leq 3) + P(X = 4)) = 1 - (P(X \leq 3) + 0.6 \times 0.6 \times 0.6 \times 0.4$
.

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

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Expectation

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Video

Start of transcript. Skip to the end.

- Hello and welcome back.

In the last lecture we talked

about the cumulative distribution function

and now we would like to move on

and calculate expectations.

This picture and images that we'll get to

later on are taken from the Daily Mirror.

7.3_Expectation

POLL

The expectation of a random variable X must be a number X can take.

RESULTS

○	**True**	**20%**
●	**Not true**	**80%**

Submit

Results gathered from 50 respondents.

FEEDBACK

The expectation of a die roll is 3.5.

1

0 points possible (ungraded)

Which 2 of the following are true about the expectation of a random variable?

- ☑ Not random
- ☐ Random value
- ☑ Property of the distribution
- ☐ Independent of the distribution

✓

Answer

Correct:
Video: Expectation
Video: Expectation
Video: Expectation
Video: Expectation

Explanation
An expectation of a distribution is a constant, which can be deducted by the distribution.

Submit You have used 3 of 4 attempts

ℹ Answers are displayed within the problem

2 (Graded)

2.0/2.0 points (graded)
A quiz-show contestant is presented with two questions, question 1 and question 2, and she can choose which question to answer first. If her initial answer is incorrect, she is not allowed to answer the other question. If the rewards for correctly answering question 1 and 2 are $200 and $100 respectively, and the contestant is 60% and 80% certain of answering question 1 and 2, which question should she answer first as to maximize the expected reward?

Question 2 ✓ **Answer:** Question 2

Explanation
The expected reward if Question 1 is answered first is given by
$300 \times 0.6 \times 0.8 + 200 \times 0.6 \times 0.2 + 0 = 168$,
and if Question 2 is chosen to be answered first,
$300 \times 0.8 \times 0.6 + 100 \times 0.8 \times 0.4 + 0 = 176$.
Thus she should choose to answer Question 2 first.

Submit You have used 1 of 1 attempt

ℹ Answers are displayed within the problem

3

0 points possible (ungraded)
If we draw cards from a 52-deck with replacement 100 times, how many times can we expect to draw a black king?

● 3.846

○ 1.923

○ 0.038

○ 7.692

✓

Answer
Correct: Video: Expectation

Explanation
Create 100 random variables $X_1, X_2, \cdots, X_{100}$, each of which is a binary number, with 1 denotes we get a black king and 0 otherwise. It is easy to show that $E\left[X_i\right] = \frac{2}{52}$.
The times we expect to draw a black king can be calculated using
$E\left[X_1 + X_2 + \cdots + X_{100}\right] = E\left[X_1\right] + E\left[X_2\right] + \cdots + E\left[X_{100}\right] = \frac{200}{52} = 3.846$.

Submit You have used 2 of 2 attempts

ℹ Answers are displayed within the problem

4 (Graded)

2.0/2.0 points (graded)

Each time you play a die rolling game you must pay $1. If you roll an even number, you win $2. If you roll an odd number, you lose additional $1. What is the expected value of your winnings?

● $-\$0.50$

○ $+\$0.50$

○ $+\$0.00$

○ $+\$1.00$

○ $-\$1.00$

✓

Answer

Correct: Video: Expectation

Explanation

Since each time you need to pay $1 for the game, the quesion is equivalent to "If you roll an even number, you win $1. If you roll an odd number, you lose $2."

With $P(\text{even}) = P(\text{odd}) = \frac{1}{2}$, the expetation is $1 \times \frac{1}{2} + (-2) \times \frac{1}{2} = -0.5$.

Submit You have used 1 of 2 attempts

ℹ Answers are displayed within the problem

5

0 points possible (ungraded)

Choose a random subset of $\{2^1, 2^2, \cdots, 2^{10}\}$ by selecting each of the 10 elements independently with probability $1/2$. Find the expected value of the smallest element in the subset (e.g. the subset can be $\{2^1, 2^3, 2^4, 2^7\}$. The smallest element is 2^1).

10 ✓ **Answer:** 10

10

Explanation

An element 2^j, $(j \in \{1, \cdots, 10\})$ is the smallest if and only if all elements less than it have not been chosen and j is chosen. The probability of this happening is $1/2^j$. Therefore the expectation is $\sum_{j=1}^{10} 1/2^j \cdot 2^j = 10$.

Submit You have used 1 of 4 attempts

ℹ Answers are displayed within the problem

6

0 points possible (ungraded)
An edX assignment has 50 multiple-choice questions, each with four choices of which one is correct. A student gets 3 points for solving a question correctly, and loses a point for an incorrect answer. What is the expected score of a student who answers all questions uniformly at random?

Submit You have used 0 of 4 attempts

7

0 points possible (ungraded)
Which of the following statements are true for a random variable X?

- ☐ $E(X)$ must be in the range $(0,1)$
- ☐ $E(X)$ can take a value that X does not take
- ☐ $P(X \leq E(X)) = 1/2$
- ☐ $E(X) = \frac{1}{2}(x_{\max} + x_{\min})$

Submit You have used 0 of 4 attempts

8

0 points possible (ungraded)
A bag contains five balls numbered 1 to 5. Randomly draw two balls from the bag and let X denote the sum of the numbers.

- What is $P(X \leq 5)$?

- What is $E(X)$?

Submit You have used 0 of 4 attempts

9

0 points possible (ungraded)
A player flips two fair coins. The player wins $\$3$ if 2 heads occur and $\$1$ if 1 head occurs. How much money (in $\$$) should the player lose when no heads occur for the game to be fair (expected gain is 0)?

Submit You have used 0 of 4 attempts

10

0 points possible (ungraded)

There are 3 classes with 20, 22 and 25 students in each class for a total of 67 students. Choose one out of the 67 students uniformly at random, and let X denote the number of students in his or her class. What is $E\left(X\right)$?

Submit You have used 0 of 4 attempts

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Video

which is what we expected, okay,

so this is called a linearity of expectation,

the expected value of X plus b

is a times the expected value of X plus b, alright.

So in this, we discussed the expectation

of functions of variables

and next, we want to talk about variance.

See you then.

End of transcript. Skip to the start.

7.5_Expectation_of_Modified_Variables

POLL

Which of the following does not hold for all random variables?

RESULTS

○	**E[X+2]=E[X]+2**	**2%**
●	**E[2X]=2E[X]**	**8%**
○	**E[X^2]=E[X]^2**	**80%**
○	**All of them hold**	**10%**

Submit

Results gathered from 49 respondents.

FEEDBACK

E[X^2]=E[X]^2 doesn't hold.

For example, if X is equally likely to be -1 or 1, then E(X)=0 so E(X)^2=0^2=0

But X^2=1, so E(X^2)=E(1)=1

1

0 points possible (ungraded)

Let X be distributed over the set $\mathbb{N}$ of non-negative integers, with pmf

$$P(X=i)=\frac{\alpha}{2^i}$$

- α

1/2

$\frac{1}{2}$

✓ **Answer:** 1/2

Explanation

Since the total probability must sum to 1, We must have $1 = \sum_{i=0}^{\infty} P(X = i) = \sum_{i=0}^{\infty} \frac{\alpha}{2^i} = \alpha \cdot \sum_{i=0}^{\infty} \frac{1}{2^i} = \alpha \cdot 2$. Thus $\alpha = 1/2$.

- $E[X]$

1

1

✓ **Answer:** 1

Explanation

By definition, $E(X) = \sum_{i=0}^{\infty} i \cdot P(X = i) = \sum_{i=1}^{\infty} i.\, \frac{\alpha}{2^i}$. This may be re-written to give $2E(X) = \sum_{i=0}^{\infty} (i+1)\, .\, \frac{\alpha}{2^i}$. Subtracting the former from the latter we have $E(X) = \sum_{i=0}^{\infty} \frac{\alpha}{2^i} = 2 \cdot \alpha = 1$.

For $Y = X \mod 3$, find

- $P(Y = 1)$

2/7

$\frac{2}{7}$

✓ **Answer:** 2/7

Explanation

Here $P(Y = 1) = \sum_{j=0}^{\infty} P(X = 3j + 1) = \sum_{j=0}^{\infty} \frac{\alpha}{2^{3j+1}} = 1/2 \cdot \sum_{j=0}^{\infty} \frac{\alpha}{8^j} = 1/2 \cdot \alpha \cdot 8/7 = 2/7$.

- $E[Y]$

4/7

$\frac{4}{7}$

✓ **Answer:** 4/7

Explanation

First note that $P(Y = 2) = \sum_{j=0}^{\infty} P(X = 3j + 2) = \sum_{j=0}^{\infty} \frac{\alpha}{2^{3j+2}} = 1/4 \cdot \sum_{j=0}^{\infty} \frac{\alpha}{8^j} = 1/4 \cdot \alpha \cdot 8/7 = 1/7$. Now $E(Y) = 1 \cdot P(Y = 1) + 2 \cdot P(Y = 2) = 2/7 + 2 \cdot 1/7 = 4/7$.

? **Hint (1 of 2):** What is $2E(X)$? Next Hint

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

2 (Graded)

0/2 points (graded)

Which of the following statements hold for all finite-expectation random variables X, Y and all fixed numbers $a, b \in \mathbb{R}$?

☑ $E[X+a] = E[X+b] \quad \Rightarrow \quad a = b$
✓

☐ $E[aX] = E[bX] \quad \Rightarrow \quad a = b$

☑ $E[X] \neq E[Y] \quad \Rightarrow \quad E[aX+b] \neq E[aY+b]$ for $a \neq 0$
✓

☑ $E[X] \neq E[Y] \quad \Rightarrow \quad E[X^2] \neq E[Y^2]$

☐ $E[X^2] \neq E[Y^2] \quad \Rightarrow \quad E[X] \neq E[Y]$

✗

Explanation

- Ture. $E[X+a] = E[X+b] \Rightarrow E[X]+a = E[X]+b \Rightarrow a = b$.
- False. $E[aX] = E[bX] \Rightarrow aE[X] = bE[X]$. If $E[X] = 0$, it does not guarantee $a = b$.
- True. If $a \neq 0$ and $E[X] \neq E[Y]$, $aE[X] \neq aE[Y] \Rightarrow aE[X]+b \neq aE[Y]+b \Rightarrow E[aX+b] \neq E[aY+b]$.
- False. Suppose X is uniformly distributed over $\{-1,0\}$, Y is uniformly distributed over $\{0,1\}$. Then $E[X] = -\frac{1}{2}$, $E[Y] = \frac{1}{2}$. However, $E[X^2] = E[Y^2] = \frac{1}{2}$.
- False. Suppose X is uniformly distributed over $\{-1,0,1\}$, Y is uniformly distributed over $\{-2,0,2\}$. Then $E[X^2] = \frac{2}{3}$, $E[Y^2] = \frac{8}{3}$. However, $E[X] = E[Y] = 0$.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

3

0 points possible (ungraded)

Every morning, the campus coffeeshop orderes the day's croissant supply. The coffeeshop buys each croissant for $1, and sells it for $4. Experience has shown that the number of croissants customers wish to buy on any given day is distirbuted uniformly between 0 and 49. Once the coffeeshop runs out of crossants, they cannot sell any more, while on the other hand, all croissants left at the end of the day are given to charity for free.

How many croissants should the coffeeshop order to maximize their expected profit?

For example, if the coffeeshop orders 1 croissant, then they spend $1 to buy it, and then with probability 0.02 the don't sell any croissants and with probability 0.98 they sell it and bring in $4, hence their expected $ profit is $0.02 \cdot 0 + 0.98 \cdot 4 - 1 = 2.92$. If the coffeeshop orders 2 croissants, then they spend $2 to buy them, and then with probability 0.02 they sell nothing, with probability 0.02 they sell 1, and with probability 0.96 they sell both, hence their expected $ profit is
$0.02 \cdot 0 + 0.02 \cdot 4 + 0.96 \cdot 8 - 2 = 7.76 - 2 = 5.76$.

37 ✓

37

? Hint (1 of 2): Calculate the expected profit as a function of the number r of croissants the coffeeshop orders, then maximize over r.

Next Hint

Submit You have used 1 of 4 attempts

✓ Correct

4 (Graded)

2/2 points (graded)

Let X follows a distribution P over Ω. The indicator function of an event $A \subseteq \Omega$, is the 0-1 function $I_A(x) = \begin{cases} 1 & \text{if } x \in A, \\ 0 & \text{if } x \notin A. \end{cases}$.

Observe that $I_A(X)$ is a random variable whose value is 1 if A occurred, and 0 if A did not occur.

$E[I_A(X)]$ is:

○ always 0 or 1,

○ $E(X)$,

● $P(A)$.

✓

Explanation

$E(I_A(X)) = \sum_{x \in \Omega} I_A(X = x) P(X = x) = \sum_{x \in A} I_A(X = x) P(X = x) + \sum_{x \notin A} I_A(X = x) P(X = x) = \sum_{x \in A} P(X =$

? Hint (1 of 1): For example, for $\Omega = \{1, 2, \dots, 10\}$, $I_{\{2,4\}}(2) = I_{\{2,4\}}(4) = 1$, while $I_{\{2,4\}}(x) = 0$ for all $x \neq 2, 4$.
Then $I_{\{2,4\}}(X)$ is the random variable that is 1 if 2 or 4 occur and is 0 if any other number occurs.

Next Hint

Submit You have used 1 of 1 attempt

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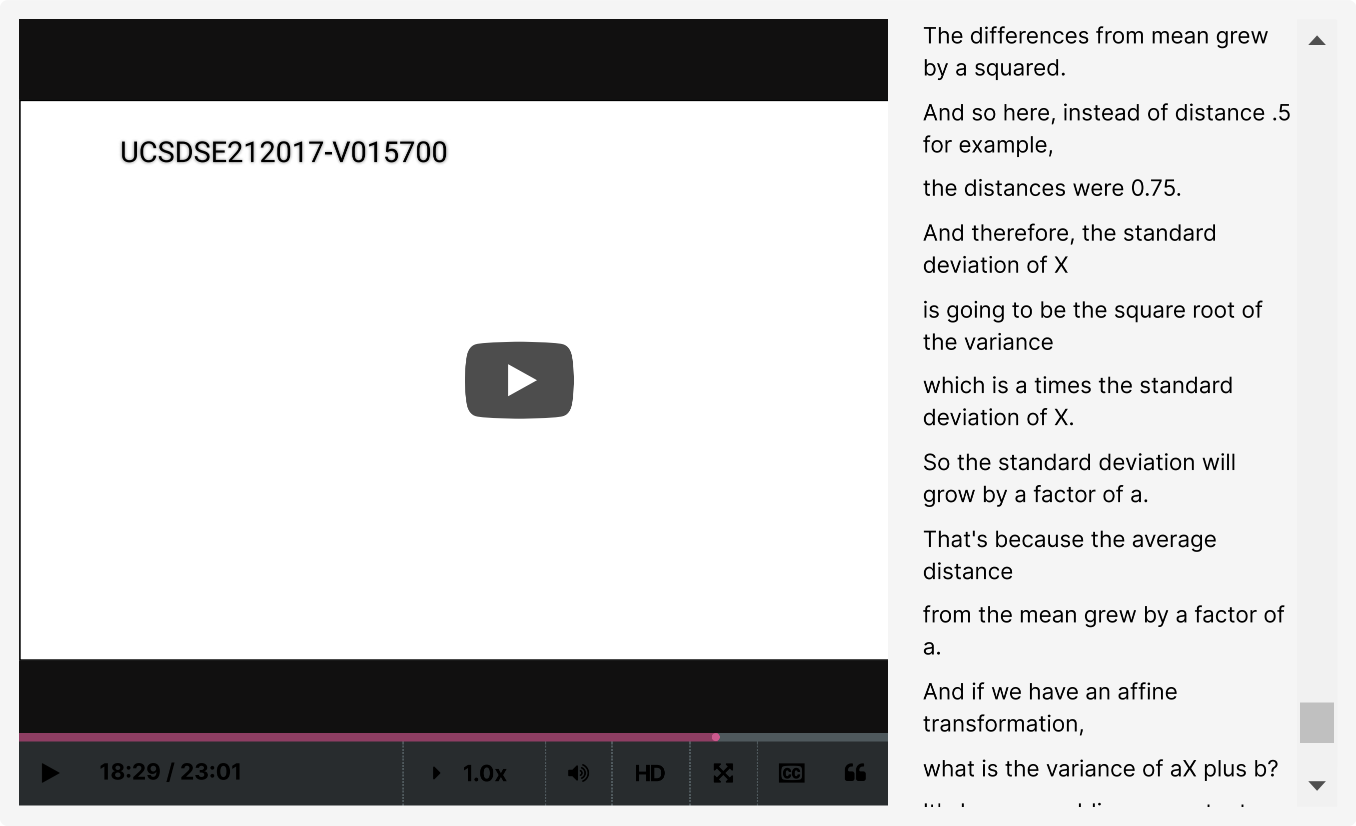


The differences from mean grew by a squared.

And so here, instead of distance .5 for example,

the distances were 0.75.

And therefore, the standard deviation of X

is going to be the square root of the variance

which is a times the standard deviation of X.

So the standard deviation will grow by a factor of a.

That's because the average distance

from the mean grew by a factor of a.

And if we have an affine transformation,

what is the variance of aX plus b?

7.6_Variance

POLL

Which of the following is greater (≥) for a random variable X?

RESULTS

○ **E[X^2]** **36%**
● **E[X]^2** **18%**
○ **Depends on X** **47%**

Submit

Results gathered from 45 respondents.

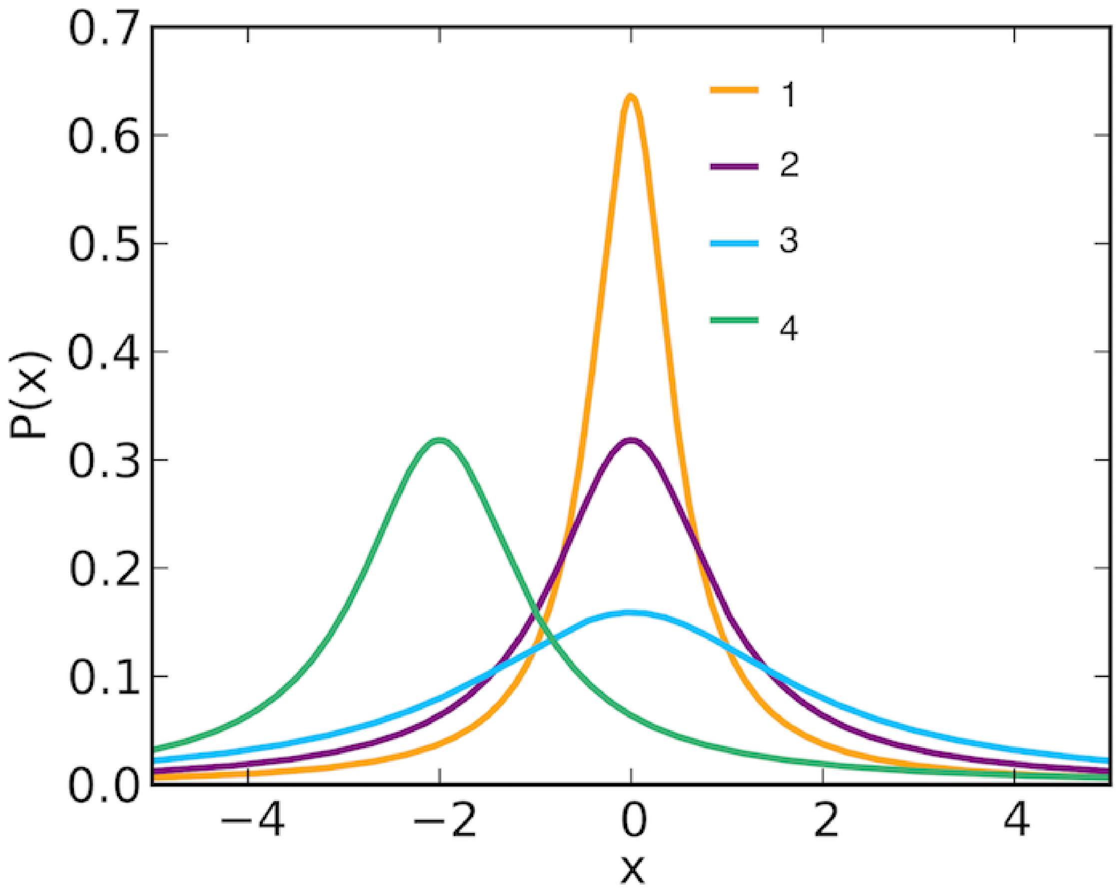
FEEDBACK

E[X^2] will be greater. Since V(X)=E[X^2]-E[X]^2, and V(X) is always non-negative.

1

0 points possible (ungraded)

Given 4 probability density functions, which one shows the greatest variance?



● 1

○ 2

○ 3 ✓

○ 4

✗

Answer
Incorrect: Video: Variance

Explanation
Variance measures how far a set of (random) numbers are spread out from their average value. 3 is the brodest one.

Submit You have used 2 of 2 attempts

Answers are displayed within the problem

2

0 points possible (ungraded)

A random variable X is distributed over $\{-1, 0, 1\}$ according to the p.m.f. $P\left(X = x\right) = \frac{|x|+1}{5}$.

Find its expectation $E(X)$

0

✓ **Answer:** 0

0

Explanation
The pmf is symmetric around 0, hence the mean is 0.

and variance $V(X)$

4/5

✓ **Answer:** 4/5

$\frac{4}{5}$

Explanation
By definition, Var$(X) = \frac{2}{5} \times (-1-0)^2 + \frac{1}{5} \times (0-0)^2 + \frac{2}{5} \times (1-0)^2 = \frac{2}{5} + 0 + \frac{2}{5} = \frac{4}{5}$
Or, Var$(X) = \mathbb{E}\left(X^2\right) - \mathbb{E}(X)^2 = 4/5 - 0 = 4/5$

Submit You have used 1 of 4 attempts

ℹ Answers are displayed within the problem

3 (Graded)

4/4 points (graded)
Let random variable X be distributed according to the p.m.f

x	1	2	3
$P(x)$	0.3	0.5	0.2

- If $Y = 2^X$, what are

$E[Y]$

4.2

✓ **Answer:** 4.2

4.2

Explanation
$E(Y) = E\left(2^X\right) = 2 \times 0.3 + 4 \times 0.5 + 8 \times 0.2 = 4.2.$

$\mathrm{Var}(Y)$

22-(4.2^2)

✓ **Answer:** 4.36

$22 - \left(4.2^2\right)$

Explanation
For any random variable Z, $V\left(Z\right)=E\left(Z^2\right)-E(Z)^2$. Here $E\left(Y^2\right)=E\left(2^{2X}\right)=4\times 0.3+16\times 0.5+64\times 0.2=22$. Thus $V\left(Y\right)=22-4.2^2=4.36$.

- If $Z=aX+b$ has $E\left[Z\right]=0$ and $\mathrm{Var}\left(Z\right)=1$, what are:

$|a|$

10/7

✓ **Answer:** 1.42857

$\frac{10}{7}$

$|b|$

2.714

✓ **Answer:** 2.714285

2.714

Explanation
First, $E\left(X\right)=0.3\times 1+0.5\times 2+0.2\times 3=1.9$, $E\left(X^2\right)=0.3\times 1+0.5\times 4+0.2\times 9=4.1$ and thus $\mathrm{Var}\left(X\right)=E\left(X^2\right)-E(X)^2=4.1-1.9^2=0.49$.
Now, by linearity of expectation, $0=E\left(Z\right)=aE\left(X\right)+b=1.9\cdot a+b$. Further, we know $1=\mathrm{Var}\left(Z\right)=\mathrm{Var}\left(aX+b\right)=a^2{\cdot}\mathrm{Var}\left(X\right)=a^2\cdot 0.49$. Solving these two equations gives $|a|=1.42857$, $|b|=2.71485$.

Submit

You have used 2 of 4 attempts

ℹ Answers are displayed within the problem

---

4 (Graded)

5/5 points (graded)
Consider two games. One with a guaranteed payout $P_1=90$, and the other whose payout P_2 is equally likely to be 80 or 120. Find:

- $E\left(P_1\right)$

90

✓ **Answer:** 90

90

Explanation
The distribution of P_1 is $P\left(P_1=90\right)=1$. Hence, $E\left(P_1\right)=1\times 90=90$.

- $E\left(P_2\right)$

(80 + 120)/2

✓ **Answer:** 100

$\frac{80+120}{2}$

Explanation
The distribution of P_2 is $P\left(P_2=80\right)=P\left(P_2=120\right)=\frac{1}{2}$. Hence, $E\left(P_2\right)=\frac{1}{2}\times 80+\frac{1}{2}\times 120=100$.

- $\operatorname{Var}\left(P_1\right)$

0

✓ **Answer:** 0

0

Explanation
By definition, $\operatorname{Var}\left(P_1\right)=1\times(90-90)^2=0$.

- $\operatorname{Var}\left(P_2\right)$

400

✓ **Answer:** 400

400

Explanation
By definition, $\operatorname{Var}\left(P_2\right)=\frac{1}{2}\times(80-100)^2+\frac{1}{2}\times(120-100)^2=400$.

- Which of games 1 and 2 maximizes the `risk-adjusted reward' $E\left(P_i\right)-\sqrt{\operatorname{Var}\left(P_i\right)}$?

● 1

○ 2

✓

Explanation
By definition, $E\left(P_1\right)-\sqrt{\operatorname{Var}\left(P_1\right)}=90$, $E\left(P_2\right)-\sqrt{\operatorname{Var}\left(P_2\right)}=80$.

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

5 (Graded)

2/2 points (graded)
Which of the following are always true for random variables X, Y and real numbers a, b?

☑ The variance of X is always non-negative.

☑ The standard deviation of X is always non-negative.

☑ If $V(X) = V(Y)$, then $V(X+a) = V(Y+b)$.

☐ If $V(aX) = V(bX)$ for $a \neq 0$ and $b \neq 0$, then $a = b$.

☐ If $E[X] = E[Y]$ and $V(X) = V(Y)$, then $X = Y$.

☑ If $E[X] = E[Y]$ and $V(X) = V(Y)$, then $E[X^2] = E[Y^2]$.

✓

Explanation

- True.
- True. Standard deviation is defined by $\sqrt{V(X)}$, which is also non-negative.
- True. Adding a constant a to random varianle X will not affect its varaince.

$$V(X+a) = E\left((X+a-E(X+a))^2\right) = E\left((X+a-E(X)-a)^2\right) = E\left((X-E(X))^2\right) = V(X)$$

.

- False. When $V(X) = 0$, this does not hold.
- False. Consider two random variables X, Y with pmf, $P(X=x) = \begin{cases} \frac{1}{2}, x = -1, \\ \frac{1}{2}, x = 1 \end{cases}$ and

$P(Y=y) = \begin{cases} \frac{1}{8}, y = -2 \\ \frac{3}{4}, y = 0 \\ \frac{1}{8}, y = 2 \end{cases}$. Now $E(X) = E(Y) = 0, V(X) = V(Y) = 1$. However, $X \neq Y$.

- True. As $E(X^2) = V(X) + E^2[X]$, if $E(X) = E(Y)$ and $V(X) = V(Y)$, then $E(X^2) = E(Y^2)$.

Submit

You have used 1 of 4 attempts

Answers are displayed within the problem

6

0 points possible (ungraded)

We say X_A is an indicator variable for event $A: X_A = 1 \text{ if A occurs, } X_A = 0 \text{ if A does not occur.}$

If $P(A) = 0.35$, what is:

- $E(X_A)$?

- $\text{Var}(X_A)$?

Submit You have used 0 of 4 attempts

7

0 points possible (ungraded)

Let X denote the number when rolling a fair six-sided die, then what is:

- $\mathrm{Var}\left(X\right)$?

- σ_X?

Submit You have used 0 of 4 attempts

8

0 points possible (ungraded)

Let X and Y be independent random variables with expectations 1 and 2, and variances 3 and 4, respectively. Find the variance of $V\left(XY\right)$.

Submit You have used 0 of 4 attempts

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? Proof of E(XY) = E(X)*E(Y)

How can we prove that E(XY) = E(X)*E(Y) in general?

2

Two Variables

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Problem Sets due Jul 8, 2022 16:34 +03

Video

they're not proportional to each other.

So with this we've introduced pairs of random variables

and in the next presentation,

we're going to start talking about expectation

of different functions of pairs of random variables.

See you then.

End of transcript. Skip to the start.

7.7a_Two_variables

POLL

If X has three different outcomes and Y has four different outcomes, how many outcomes does the joint random variable (X,Y) have?

RESULTS

○	**4**	**0%**
○	**7**	**4%**
●	**12**	**93%**
○	**None of the above**	**2%**

Submit

Results gathered from 45 respondents.

FEEDBACK

The answer is 3×4=12.

1

0 points possible (ungraded)

Which of the following hold for all **Independent** random variables, X and Y?

☑ $P\left(X=x|Y=y\right)=P\left(X=x\right)$

☐ $P\left(X=x|Y=y\right)=P\left(Y=y|X=x\right)$

✓

Explanation

If two random variables are independent, by definition, $P(X=x, Y=y) = P(X=x)\,P(Y=y)$. Since $P(X=x, Y=y) = P(X=x|Y=y)\,P(Y=y)$, we have $P(X=x|Y=y) = P(X=x)$.

Submit You have used 2 of 3 attempts

Answers are displayed within the problem

2 (Graded)

3/3 points (graded)

A joint probabilty mass table is given as follows:

$X\backslash Y$	0	1
0	0.15	0.25
1	0.45	0.15

1) Choose the correct marginal PMFs for X and Y.

○

x, y	$P(x)$	$P(y)$
0	0.15	0.45
1	0.25	0.5

●

x, y	$P(x)$	$P(y)$
0	0.4	0.6
1	0.6	0.4

○

x, y	$P(x)$	$P(y)$
0	0.6	0.4
1	0.4	0.6

✓

Answer

Correct: Video: Two Variables

x, y	$P(x)$	$P(y)$
0	0.4	0.6
1	0.6	0.4

Explanaton

$$P(X=0) = P(X=0, Y=0) + P(X=0, Y=1) = 0.15 + 0.25 = 0.4$$
$$P(X=1) = P(X=1, Y=0) + P(X=1, Y=1) = 0.45 + 0.15 = 0.6$$
$$P(Y=0) = P(Y=0, X=0) + P(Y=0, X=1) = 0.15 + 0.45 = 0.6$$
$$P(Y=1) = P(Y=1, X=0) + P(Y=1, X=1) = 0.25 + 0.15 = 0.4$$

2) Find $P(X=0|Y=0)$.

● 0.250

○ 0.375

○ 0.667

○ 1

✓

Answer

Answer

Correct: Video: Two Variables

Explanaton

$P\left(X=0|Y=0\right)=\frac{P(X=0,Y=0)}{P(Y=0)}=\frac{0.15}{0.6}=0.25$

3) Find $P\left(Y=1|X=0\right)$.

○ 0.375

○ 0.417

● 0.625

○ 0.750

✓

Answer

Correct: Video: Two Variables

Explanaton

$P\left(Y=1|X=0\right)=\frac{P(X=0,Y=1)}{P(X=0)}=\frac{0.25}{0.4}=0.625$

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

3

0 points possible (ungraded)

Given independent random variables X and Y with the following joint distribution. Find

$X\setminus Y$	0	1	sum
0	b	$?$	0.7
1	$?$	0.18	$?$
sum	a	$?$	

- a

0.4 ✓ **Answer:** 0.4

0.4

Explanation

$P\left(X=1\right)=1-P\left(X=0\right)=0.3$, $P\left(Y=1\right)=1-P\left(Y=0\right)=1-a$. By independence of X and Y, $P\left(X=1,Y=1\right)=0.18=P\left(X=1\right)\cdot P\left(Y=1\right)=0.3\cdot\left(1-a\right)$. Thus $a=0.4$.

- b

0.28 ✓ **Answer:** 0.28

0.28

Explanation

$b=P\left(X=0,Y=0\right)=P\left(X=0\right)\cdot P\left(Y=0\right)=P\left(X=0\right)\cdot a=0.7\times 0.4=0.28$.

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

4

0 points possible (ungraded)

Which equation accurately describes the marginal PMFs for the random variables, X and Y?

○ $P(X=x)=\sum_x p(X=x,Y=y), \quad P(Y=y)=\sum_y p(X=x,Y=y)$

● $P(X=x)=\sum_y p(X=x,Y=y), \quad P(Y=y)=\sum_x p(X=x,Y=y)$

○ $P(X=x)=\sum_x p(Y=y), \quad P(Y=y)=\sum_y p(X=x)$

○ $P(X=x)=\sum_y p(X=x), \quad P(Y=y)=\sum_x p(Y=y)$

✓

Answer

Correct: Video: Two Variables

Explanation

Refer to the video and slides.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

5 (Graded)

8/8 points (graded)

Roll two fair six-sided dice, and let X, Y denote the first and the second numbers.

If $Z=\max\{X,Y\}$, find

- $E(Z)$

161/36 ✓ **Answer:** 4.4722

$\frac{161}{36}$

Explanation

The distribution of Z is

$P(Z=1)=\frac{1}{36}, P(Z=2)=\frac{3}{36}, P(Z=3)=\frac{5}{36}, P(Z=4)=\frac{7}{36}, P(Z=5)=\frac{9}{36}, P(Z=6)=\frac{11}{36}$

The expectation of Z is $E(Z)=\sum_{i=1}^{6} i \cdot P(Z=i)=\frac{161}{36}=4.472$

- $V(Z)$

1.972 ✓ **Answer:** 1.9715

1.972

Explanation

$E(Z^2)=\sum_{i=1}^{6} i^2 \cdot P(Z=i)=\frac{791}{36}$

The variance of Z is $V(Z)=E(Z^2)-E^2(Z)=1.9715$

If $Z=|X-Y|$, find

- $E(Z)$

70/36

✓ **Answer:** 1.9444

$\frac{70}{36}$

Explanation
The distribution of Z is
$P(Z=0)=\frac{6}{36}, P(Z=1)=\frac{10}{36}, P(Z=2)=\frac{8}{36}, P(Z=3)=\frac{6}{36}, P(Z=4)=\frac{4}{36}, P(Z=5)=\frac{2}{36}$
The expectation of Z is $E(Z)=\sum_{i=0}^{5} i\cdot P(Z=i)=\frac{35}{18}=1.9444$

- $V(Z)$

2.0525

✓ **Answer:** 2.0525

2.0525

Explanation
$E(Z^2)=\sum_{i=0}^{5} i^2\cdot P(Z=i)=\frac{35}{6}$
The variance of Z is $V(Z)=E(Z^2)-E^2(Z)=2.0525$

? **Hint (1 of 1):** What is the PMF of Z (i.e. $P_Z(z)$)? Next Hint

Submit You have used 1 of 4 attempts

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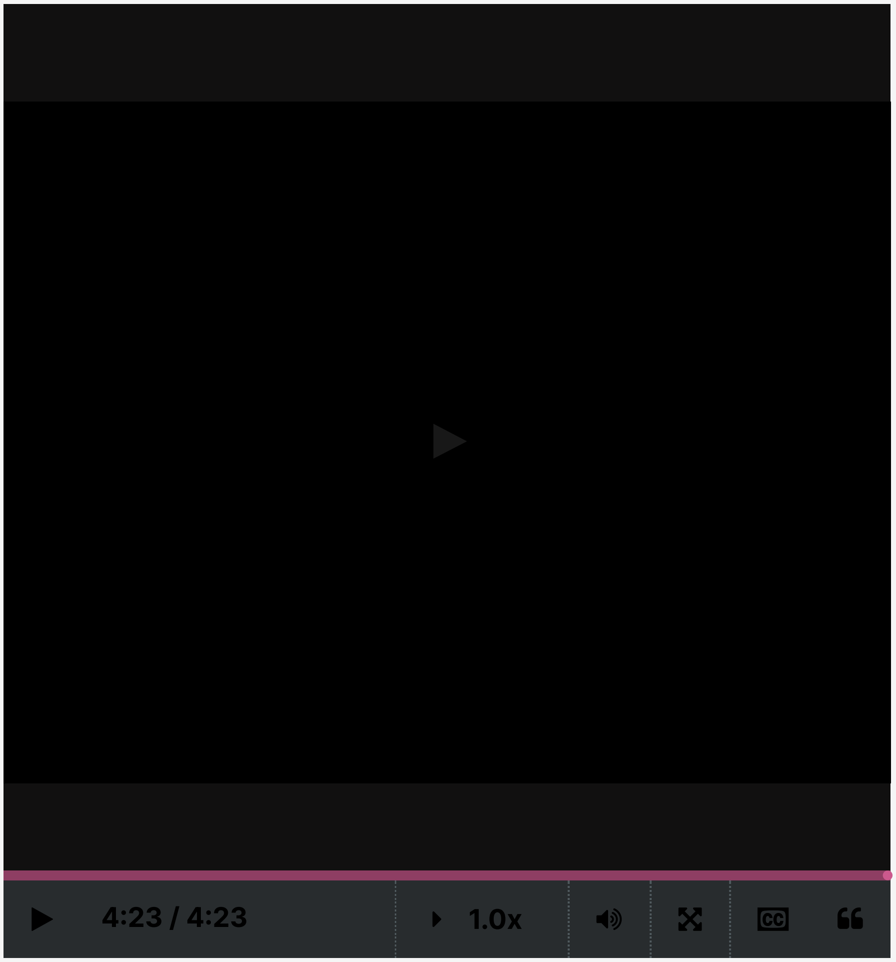

than whether the variances add,

and that's what we want to look at next.

So, this is what we're going to do in a separate video

because it would take us some time to discuss this.

See you then.

End of transcript. Skip to the start.

7.8_Linearity_of_Expectation

POLL

Which of the following always holds?

RESULTS

○	**E[X+Y]=E[X]+E[Y]**	**16%**
○	**E[X-Y]=E[X]-E[Y]**	**2%**
●	**Both**	**80%**

○ **None** **2%**

Submit

Results gathered from 45 respondents.

FEEDBACK

Both of them hold.

1

0 points possible (ungraded)

Let X be number of heads you get by flipping a fair coin 100 times. Then what is $E\left(X\right)$?

○ $E\left[X\right] = 25$

● $E\left[X\right] = 50$

○ $E\left[X\right] = 75$

○ None of the above

✓

Explanation

Let X_i be the random variable for the i-th flip, with 1 representing heads and 0 representing tails. Then $E\left(X_i\right) = \frac{1}{2}$.

It is obvious that $X = \sum_{i=1}^{100} X_i$. Its expectation

$$E\left(X\right) = E\left(\sum_{i=1}^{100} X_i\right) = \sum_{i=1}^{100} E\left(X_i\right) = 100 \times \frac{1}{2} = 50.$$

? **Hint (1 of 1):** Expectation is linear. Next Hint

Submit You have used 1 of 2 attempts

ℹ Answers are displayed within the problem

---

2 (Graded)

3/3 points (graded)

Starting with 10 blue balls, in each of 10 sequential rounds, we remove a random ball and replace it with a new red ball. For example, after the first round we have 9 blue balls and one red ball, after the second round, with probability $9/10$ we have 8 blue balls and 2 red balls, and with probability $1/10$ we have 9 blue balls and one red ball, etc.

What is the probability that the ball we remove at the 11th round is blue?

0.34

✓ **Answer:** 0.349

0.34

Explanation

Imagine that the balls are placed in 10 locations 1 to 10. Let B_i be the event that at the final $(11$th) round, the ball in location i is blue. B_i occurs iff the ball in location i was not discarded in any of the previous 10 rounds, hence $P(B_i) = (1 - 1/10)^{10} = (9/10)^{10}$. Let B be the event that the final ball, picked at the 11th round, is blue. By the rule of total probability, $P(B) = \sum_{i=1}^{10} \frac{1}{10} P(B_i) = 10 \cdot \frac{1}{10} \left(\frac{9}{10}\right)^{10} = \left(\frac{9}{10}\right)^{10} = 0.3486$.

? **Hint (1 of 1):** Imagine that the balls are placed in 10 distinct locations, and first find the probability that at the end of the 10th round, the ball in a given location is still blue.

Next Hint

Submit

You have used 2 of 4 attempts

ℹ Answers are displayed within the problem

---

3 (Graded)

2/2 points (graded)

$\mathbb{E}(X) = 2$ and $\mathbb{E}(X(X-1)) = 5$. Find $V(X)$.

3

✓ **Answer:** 3

3

Explanation

$5 = \mathbb{E}(X(X-1))$
$= \mathbb{E}(X^2 - X)$
$= \mathbb{E}(X^2) - \mathbb{E}(X)$
$= \mathbb{E}(X^2) - 2$
$\rightarrow \mathbb{E}(X^2) = 5 + 2 = 7$
$V(X) = \mathbb{E}(X^2) - \mathbb{E}(X)^2 = 7 - 4 = 3$

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