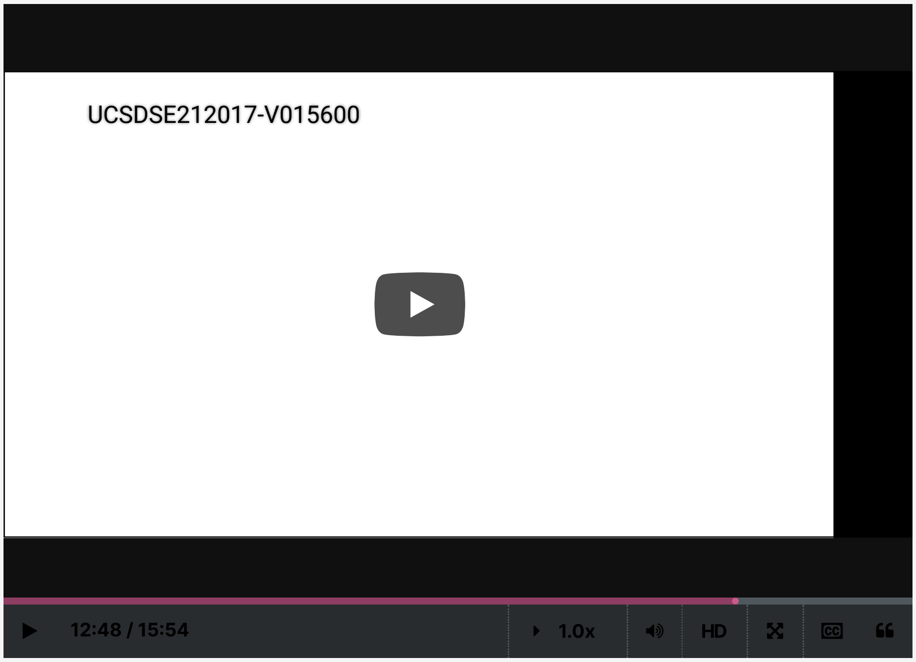
Problem Sets due Jul 8, 2022 16:34 +03

Video



which is what we expected, okay,

so this is called a linearity of expectation,

the expected value of X plus b

is a times the expected value of X plus b, alright.

So in this, we discussed the expectation

of functions of variables

and next, we want to talk about variance.

See you then.

End of transcript. Skip to the start.

7.5_Expectation_of_Modified_Variables

POLL

Which of the following does not hold for all random variables?

RESULTS

- E[X+2]=E[X]+2 — 2%
- E[2X]=2E[X] — 8%
- E[X^2]=E[X]^2 — 80%
- All of them hold — 10%

Submit

Results gathered from 49 respondents.

FEEDBACK

E[X^2]=E[X]^2 doesn't hold.

For example, if X is equally likely to be -1 or 1, then E(X)=0 so E(X)^2=0^2=0

But X^2=1, so E(X^2)=E(1)=1

1

0 points possible (ungraded)

Let X be distributed over the set $\mathbb{N}$ of non-negative integers, with pmf

$$P(X=i)=\frac{\alpha}{2^i}$$

- α

1/2 ✔ **Answer:** 1/2

$\frac{1}{2}$

Explanation
Since the total probability must sum to 1, We must have $1 = \sum_{i=0}^{\infty} P(X = i) = \sum_{i=0}^{\infty} \frac{\alpha}{2^i} = \alpha \cdot \sum_{i=0}^{\infty} \frac{1}{2^i} = \alpha \cdot 2$. Thus $\alpha = 1/2$.

- $E[X]$

1 ✔ **Answer:** 1

1

Explanation
By definition, $E(X) = \sum_{i=0}^{\infty} i \cdot P(X = i) = \sum_{i=1}^{\infty} i. \frac{\alpha}{2^i}$. This may be re-written to give $2E(X) = \sum_{i=0}^{\infty} (i+1) . \frac{\alpha}{2^i}$. Subtracting the former from the latter we have $E(X) = \sum_{i=0}^{\infty} \frac{\alpha}{2^i} = 2 \cdot \alpha = 1$.

For $Y = X \mod 3$, find

- $P(Y = 1)$

2/7 ✔ **Answer:** 2/7

$\frac{2}{7}$

Explanation
Here $P(Y = 1) = \sum_{j=0}^{\infty} P(X = 3j + 1) = \sum_{j=0}^{\infty} \frac{\alpha}{2^{3j+1}} = 1/2 \cdot \sum_{j=0}^{\infty} \frac{\alpha}{8^j} = 1/2 \cdot \alpha \cdot 8/7 = 2/7$.

- $E[Y]$

4/7 ✔ **Answer:** 4/7

$\frac{4}{7}$

Explanation
First note that $P(Y = 2) = \sum_{j=0}^{\infty} P(X = 3j + 2) = \sum_{j=0}^{\infty} \frac{\alpha}{2^{3j+2}} = 1/4 \cdot \sum_{j=0}^{\infty} \frac{\alpha}{8^j} = 1/4 \cdot \alpha \cdot 8/7 = 1/7$. Now
$E(Y) = 1 \cdot P(Y = 1) + 2 \cdot P(Y = 2) = 2/7 + 2 \cdot 1/7 = 4/7$.

? Hint (1 of 2): What is $2E(X)$? Next Hint

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

2 (Graded)

0/2 points (graded)
Which of the following statements hold for all finite-expectation random variables X, Y and all fixed numbers $a, b \in \mathbb{R}$?

- [x] $E[X+a]=E[X+b] \quad \Rightarrow \quad a=b$
- [ ] $E[aX]=E[bX] \quad \Rightarrow \quad a=b$
- [x] $E[X]\neq E[Y] \quad \Rightarrow \quad E[aX+b]\neq E[aY+b]$ for $a\neq 0$
- [x] $E[X]\neq E[Y] \quad \Rightarrow \quad E[X^2]\neq E[Y^2]$
- [ ] $E[X^2]\neq E[Y^2] \quad \Rightarrow \quad E[X]\neq E[Y]$

Explanation

- Ture. $E[X+a]=E[X+b]\Rightarrow E[X]+a=E[X]+b\Rightarrow a=b$.
- False. $E[aX]=E[bX]\Rightarrow aE[X]=bE[X]$. If $E[X]=0$, it does not guarantee $a=b$.
- True. If $a\neq 0$ and $E[X]\neq E[Y]$, $aE[X]\neq aE[Y]\Rightarrow aE[X]+b\neq aE[Y]+b\Rightarrow E[aX+b]\neq E[aY+b]$.
- False. Suppose X is uniformly distributed over $\{-1,0\}$, Y is uniformly distributed over $\{0,1\}$. Then $E[X]=-\frac{1}{2}$, $E[Y]=\frac{1}{2}$. However, $E[X^2]=E[Y^2]=\frac{1}{2}$.
- False. Suppose X is uniformly distributed over $\{-1,0,1\}$, Y is uniformly distributed over $\{-2,0,2\}$. Then $E[X^2]=\frac{2}{3}$, $E[Y^2]=\frac{8}{3}$. However, $E[X]=E[Y]=0$.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

3

0 points possible (ungraded)

Every morning, the campus coffeeshop orderes the day's croissant supply. The coffeeshop buys each croissant for $1, and sells it for $4. Experience has shown that the number of croissants customers wish to buy on any given day is distirbuted uniformly between 0 and 49. Once the coffeeshop runs out of crossants, they cannot sell any more, while on the other hand, all croissants left at the end of the day are given to charity for free.

How many croissants should the coffeeshop order to maximize their expected profit?

For example, if the coffeeshop orders 1 croissant, then they spend $1 to buy it, and then with probability 0.02 the don't sell any croissants and with probability 0.98 they sell it and bring in $4, hence their expected $ profit is $0.02\cdot 0+0.98\cdot 4-1=2.92$. If the coffeeshop orders 2 croissants, then they spend $2 to buy them, and then with probability 0.02 they sell nothing, with probability 0.02 they sell 1, and with probability 0.96 they sell both, hence their expected $ profit is
$0.02\cdot 0+0.02\cdot 4+0.96\cdot 8-2=7.76-2=5.76$.

37

37

? Hint (1 of 2): Calculate the expected profit as a function of the number r of croissants the coffeeshop orders, then maximize over r. Next Hint

Submit You have used 1 of 4 attempts

✔ Correct

4 (Graded)

2/2 points (graded)

Let X follows a distribution P over Ω. The indicator function of an event $A \subseteq \Omega$, is the 0-1 function $I_A(x) = \begin{cases} 1 & \text{if } x \in A, \\ 0 & \text{if } x \notin A. \end{cases}$.

Observe that $I_A(X)$ is a random variable whose value is 1 if A occurred, and 0 if A did not occur.

$E[I_A(X)]$ is:

- ◯ always 0 or 1,
- ◯ $E(X)$,
- ◉ $P(A)$.

✔

Explanation

$E(I_A(X)) = \sum_{x\in\Omega} I_A(X=x) P(X=x) = \sum_{x\in A} I_A(X=x) P(X=x) + \sum_{x\notin A} I_A(X=x) P(X=x) = \sum_{x\in A} P(X =$

? **Hint (1 of 1):** For example, for $\Omega = \{1, 2, \ldots, 10\}$,
$I_{\{2,4\}}(2) = I_{\{2,4\}}(4) = 1$, while $I_{\{2,4\}}(x) = 0$ for all $x \neq 2, 4$.
Then $I_{\{2,4\}}(X)$ is the random variable that is 1 if 2 or 4 occur and is 0 if any other number occurs.

Next Hint

Submit You have used 1 of 1 attempt

ℹ Answers are displayed within the problem

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