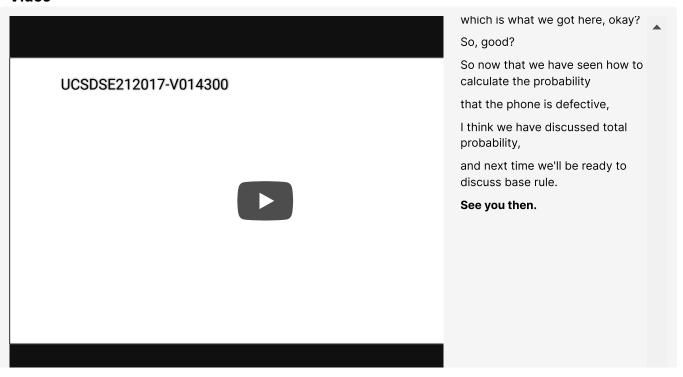
## Problem Sets due Jul 4, 2022 07:34 +03

### Video



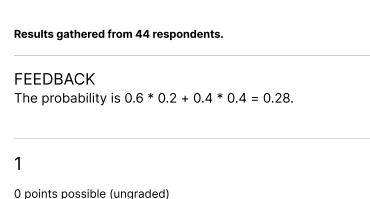
## 6.4\_Total\_Probability

## **POLL**

60% of our students are American (born), and 40% are foreign (born). 20% of the Americans and 40% of the foreigners speak two languages. What is the probability that a random student speaks two languages?

## **RESULTS**





Three 100-marble bags are placed on a table. One bag has 60 red and 40 blue marbles, one as 75 red and 25 blue marbles, and one has 45 red and 55 blue marbles.

You select one bag at random and then choose a marble at random. What is the probability that the marble is red?

 ○ 0.2025

 ○ 0.33

 ○ 0.50



0.60



#### **Answer**

Correct: Video: Total Probability

## **Explanation**

The probability is  $\frac{1}{3}(0.6 + 0.75 + 0.45) = 0.60$ .

Submit

You have used 2 of 2 attempts

**1** Answers are displayed within the problem

# 2 (Graded)

2/2 points (graded)

Each of Alice and Bob has an identical bag containing 6 balls numbered 1, 2, 3, 4, 5, and 6. Alice randomly selects one ball from her bag and places it in Bob's bag, then Bob randomly select one ball from his bag and places it in Alice's bag. What is the probability that after this process the content in two bags remain unchanged?

2/7

**✓ Answer:** 2/7

1

### **Explanation**

The two bags will remain unchanged if the ball Bob picks has the same number as the one Alice placed there. Once Alice puts a ball numbered n in Bob's bag, the probability the Bob picks a ball numbered n is  $\frac{2}{7}$ . The total probability is  $\sum_{n=1}^{6} \frac{1}{6} \cdot \frac{2}{7} = \frac{2}{7}$ .

Submit

You have used 1 of 4 attempts

**1** Answers are displayed within the problem

# 3. Two subsets

0 points possible (ungraded)

Let A and B be two random subsets of  $\{1,2,3,4\}$ . What is the probability that  $A\subseteq B$ ?

? Hint (1 of 1): A random subset of  $\{1,2,3,4\}$  is one of the  $2^4=16$  subsets, selected uniformly at random. An equivalent way of selecting a random subset is to include each of the four elements in the subset independently with probability 1/2.

Next Hint

Submit

You have used 0 of 4 attempts

# 4 (Graded)

2/2 points (graded)

Eight equal-strength players, including Alice and Bob, are randomly split into  $\bf 4$  pairs, and each pair plays a game (i.e. 4 games in total), resulting in four winners. What is the probability that exactly one of Alice and Bob will be among the four winners?

0.57

**✓ Answer:** 4/7

0.57

### **Explanation**

Here are two ways of solving the problem. One using total probability, the other by symmetry.

Total Probability.

Let E be the desired event that exactly one of Alice or Bob is a winner. We divide the sample space into two disjoint events,  $E_1$ ,  $E_2$ .  $E_1$  is the event that Alice and Bob play against each other and  $E_2$  is the complimentary event that Alice and Bob play against other players. Since Alice is equally likely to play with any of the seven other players,  $P(E_1)=1/7$ , hence  $P(E_2)=6/7$ . Now  $P(E|E_1)=1$ , while  $P(E|E_2)=1/2$  since Alice and Bob each play an independent game where the probability of winning is 1/2. Therefore  $P(E)=P(E_1)\cdot P(E|E_1)+P(E_2)\cdot P(E|E_2)=1/7\cdot 1+6/7\cdot 1/2=4/7$ .

Symmetry.

In the end, 4 of the 8 players will be declared winners. There are  $\binom{8}{4}$  such 4-winner "quartets", all equally likely.

The number of "quartets" that contain exactly one of Alice and Bob is  $\binom{2}{1} \cdot \binom{6}{3}$ .

Hence the probability that this occurs is  $\frac{\binom{2}{1}\cdot\binom{6}{3}}{\binom{8}{4}}=\frac{4}{7}$ .



You have used 1 of 4 attempts

• Answers are displayed within the problem

# Discussion

Topic: Topic 6 / Total Probability

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