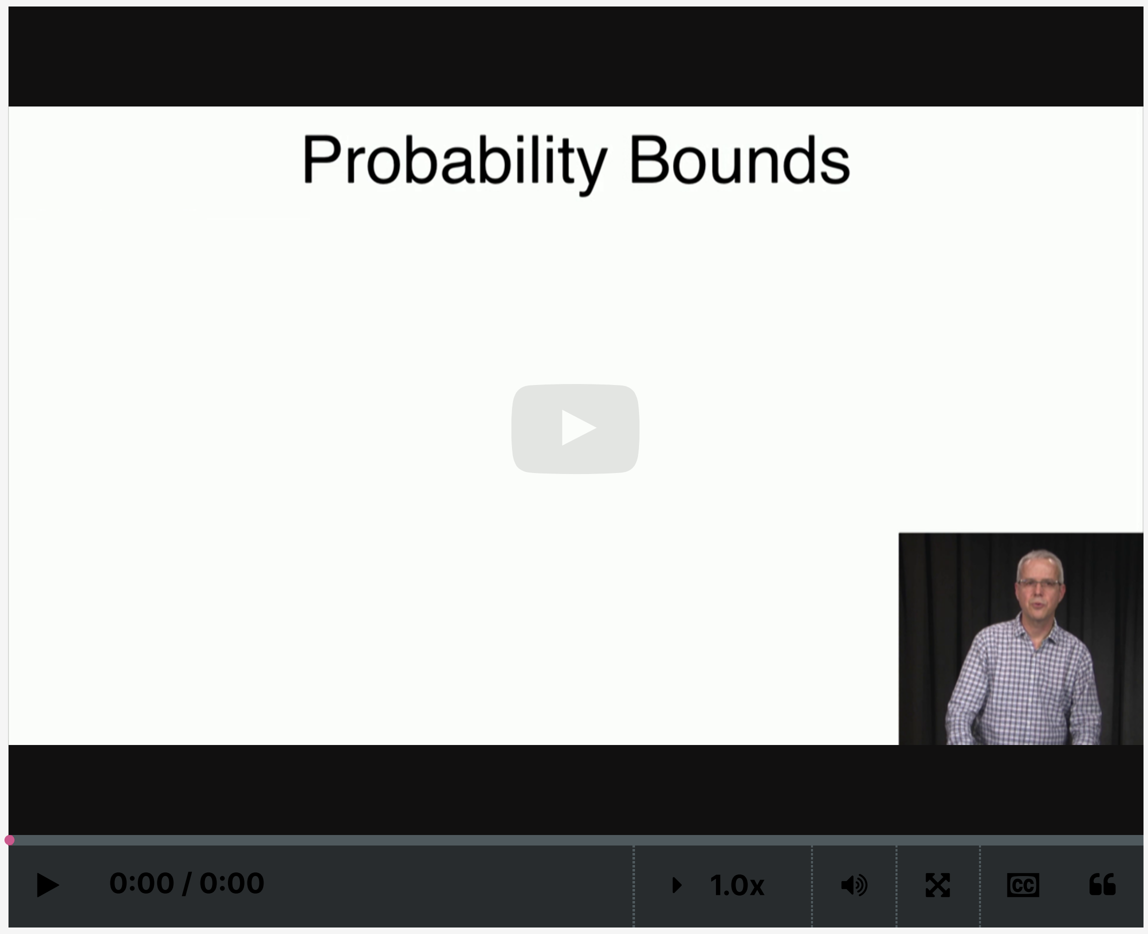
Markov's Inequality

Problem Sets due Jul 21, 2022 19:34 +03

Video



Start of transcript. Skip to the end.

- Hello, and welcome back.

In this set of lectures,

we're going to talk about inequalities

that are related to probability distributions.

We'll start with the,

you might want to think of it

as the mother of all such

10.1_Markov_compressed

POLL

A mob of 30 meerkats has an average height of 10”, and 10 of them are 30” tall. According to Markov's Inequality this is:

RESULTS

◉	**Possible**	**70%**
○	**Impossible**	**30%**

Submit

Results gathered from 37 respondents.

FEEDBACK

Impossible. For the average to be 10, the remaining 20 meerkats would need to have height zero.

1

0 points possible (ungraded)

Which of the following are correct versions of Markov’s Inequality for a nonnegative random variable X:

- [ ] $P(X \geq \alpha\mu) \leq \frac{1}{\alpha}$
- [ ] $P(X \geq \alpha\mu) \leq \mu\alpha$
- [ ] $P(X \geq \mu) \leq \frac{1}{\alpha}$
- [ ] $P(X \geq \alpha) \leq \frac{\mu}{\alpha}$

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2 - Markov variations (Graded)

2/2 points (graded)
Upper bound $P(X \geq 3)$ when $X \geq 2$ and $E[X] = 2.5$.

1/2 ✔ **Answer:** 1/2

$\frac{1}{2}$

Explanation
Let $Y = X - 2$. Then $Y \geq 0$ and $E(Y) = E(X) - 2 = 0.5$. By Markov's inequality,
$P(X \geq 3) = P(Y \geq 1) \leq \frac{E(Y)}{1} = 0.5$.

? **Hint (1 of 1):** Modify X and apply Markov's inequality. Next Hint

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Answers are displayed within the problem

3 (Graded)

4/4 points (graded)

- In a town of 30 families, the average annual family income is $80,000. What is the largest number of families that can have income at least $100,000 according to Markov's Inequality?

Note: The annual family income can be any **non-negative** number.

24 ✔ **Answer:** 24

24

Explanation
This question can be answered using the Meerkat paradigm, or we can convert it to a probability question and use Markov's Inequality. Imagine that you pick one of the 30 families uniformly at random. The expected income is the average over all families, $80,000. The probability that the random family has income at least $100,000 is the number of families with such income, normalized by 30. By Markov's Inequality, this probability is at most $80000/100000 = 0.8$. Hence the number of families with such income is at most $30 \cdot 0.8 = 24$.

- In the same town of 30 families, the average household size is 2.5. What is the largest number of families that can have at least 4 members according to Markov's Inequality?

Note the household size can be any **postive** integer.

15 ✔ **Answer:** 15

15

Explanation
Let X be the size of a family picked uniformly at random. Then $X \geq 1$ and $E(X) = 2.5$. Define $Y = X - 1$. Then $Y \geq 0$ and $E(Y) = E(X) - 1 = 1.5$. By Markov's Inequality $P(X \geq 4) = P(Y \geq 3) \leq \frac{1.5}{3} = \frac{1}{2}$. Hence the fraction of families with at least 4 members is at most $\frac{1}{2} \cdot 30 = 15$.

? **Hint (1 of 1):** Pay attention to the range of X. Next Hint

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Answers are displayed within the problem

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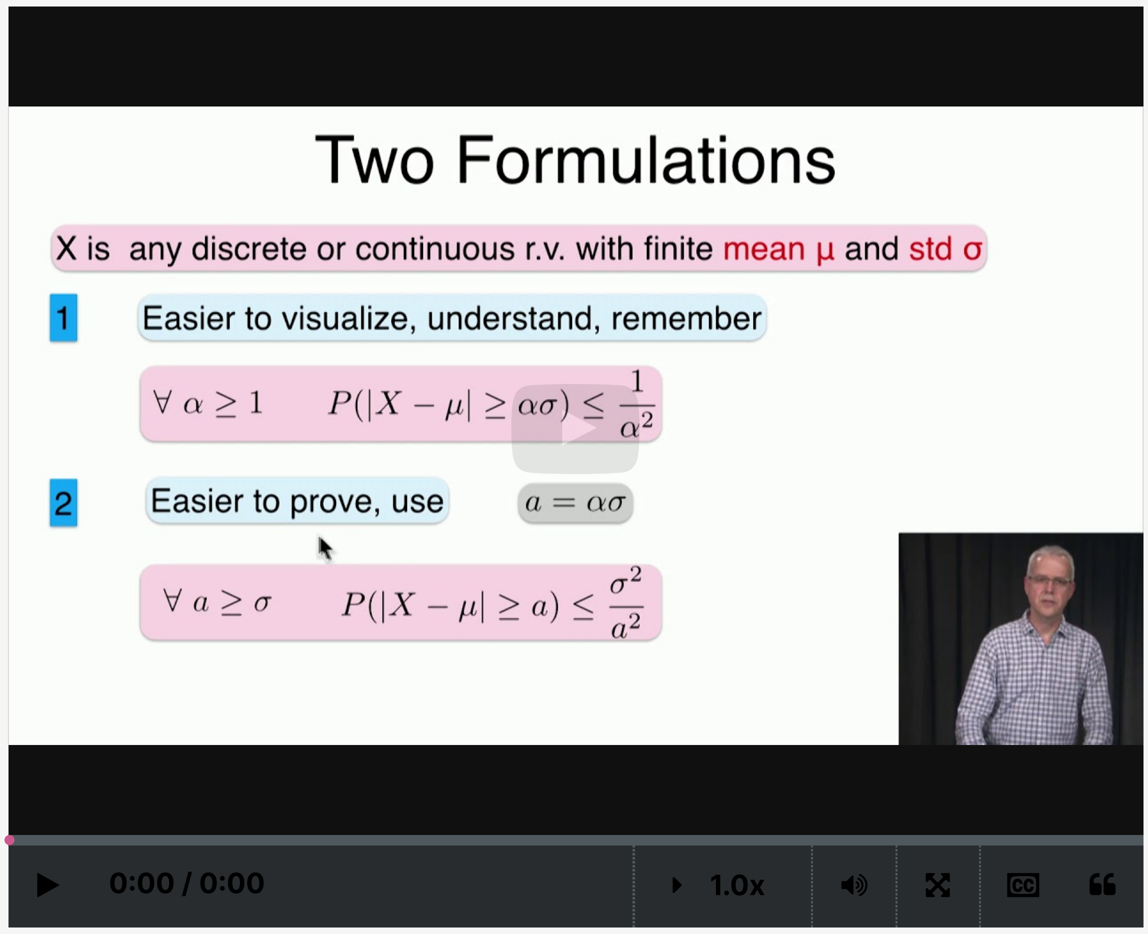

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- Hello and welcome back.

In the last lecture we

talked about Markov's inequality, and as we said then,

it's the basis of many other inequalities

and today we're going to talk about one of these,

which is Chebyshev's inequality.

Chebyshev was actually Markov's

10.2_Chebyshev

POLL

Which of the following is correct about Chebyshev's inequality?

- ○ **It only applys to non-negative distribution**
- ○ **It only applys to discrete distribution**
- ○ **It only applys to continuous distribution**
- ○ **None of the above**

Submit

1

0 points possible (ungraded)

Apply Chebyshev's Inequality to lower bound $P(0 < X < 4)$ when $E(X) = 2$ and $E(X^2) = 5$.

Submit You have used 0 of 4 attempts

2

0 points possible (ungraded)

The average number of spelling errors on a page is 5 and the standard deviation is 2. What is the probability of more than 20 mistakes on a page?

no greater than 1%

- [x] no greater than 2%
- [ ] no greater than 5%
- [ ] no greater than 10%

Explanation
Using Chebyshev's inequality, we have
$P(X > 20) < P(X \geq 20) = P(X - 5 \geq 15) = P(|X - 5| \geq 15) \leq (\frac{2}{15})^2 \leq \frac{1}{50} = 2\%$.
P.S. Since we cannot get negative number of mistakes, $P(X - 5 \leq -15) = 0$. Hence,
$P(|X - 5| \geq 15) = P(X - 5 \geq 15) + P(X - 5 \leq -15) = P(X - 5 \geq 15)$

Submit You have used 2 of 2 attempts

Answers are displayed within the problem

3 (Graded)

6/6 points (graded)
Let $X \sim \text{Exponential}(1)$. For $P(X \geq 4)$, evaluate:

- Markov's inequality,

1/4 **Answer:** 0.25

$\frac{1}{4}$

Explanation
$E(X) = \frac{1}{\lambda} = 1$.
$P(X \geq 4) \leq \frac{E(X)}{4} = \frac{1}{4}$.

- Chebyshev's inequality,

0.111 **Answer:** 0.1111

0.111

Explanation
$E(X) = \frac{1}{\lambda} = 1, V(X) = \frac{1}{\lambda^2} = 1$.
$P(X \geq 4) = P(|X - 1| \geq 3) \leq \frac{V(X)}{9} = \frac{1}{9}$.

- the exact value.

0.018 **Answer:** 0.0183

0.018

Explanation
$P(X \geq 4) = \int_4^\infty e^{-x} dx = e^{-4} = 0.0183$.

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

4

0 points possible (ungraded)

A gardener has new tomato plants sprouting up in her garden. Their expected height is 8”, with standard deviation of 1". Which of the following lower bounds the probability that a plant will be between 6" and 10" tall?

- [ ] 10% ✓
- [ ] 25% ✓
- [x] 50% ✓
- [x] 75% ✓

Explanation

By Chebyshev's Inequality, $P\left(|X-8| \geq 2\right) \leq \frac{V(X)}{4} = \frac{1}{4}$. Hence $P\left(6 \leq X \leq 10\right) = 1 - P\left(|X-8| \geq 2\right) \geq 1 - \frac{1}{4} = \frac{3}{4} = 75\%$.

Since the probability is at least 75%, it is also at least 50%, etc.

? **Hint (1 of 1):** More than one answer may hold. Next Hint

Submit You have used 2 of 2 attempts

Answers are displayed within the problem

5 (Graded)

2/2 points (graded)

If $E\left(X\right) = 15$, $P\left(X \leq 11\right) = 0.2$, and $P\left(X \geq 19\right) = 0.3$, which of the following is $\textit{impossible}$?

- (x) $V\left(X\right) \leq 7$
- ( ) $V\left(X\right) \leq 8$
- ( ) $V\left(X\right) > 8$
- ( ) $V\left(X\right) > 7$

✓

Explanation

According to Chebyshev's inequality, $P\left(|X-15| \geq 4\right) \leq \frac{V(X)}{16}$. As $P\left(|X-15| \geq 4\right) = P\left(X \leq 11\right) + P\left(X \geq 19\right) = 0.5$, we have $V\left(X\right) \geq 8$.

Submit You have used 2 of 2 attempts

Answers are displayed within the problem

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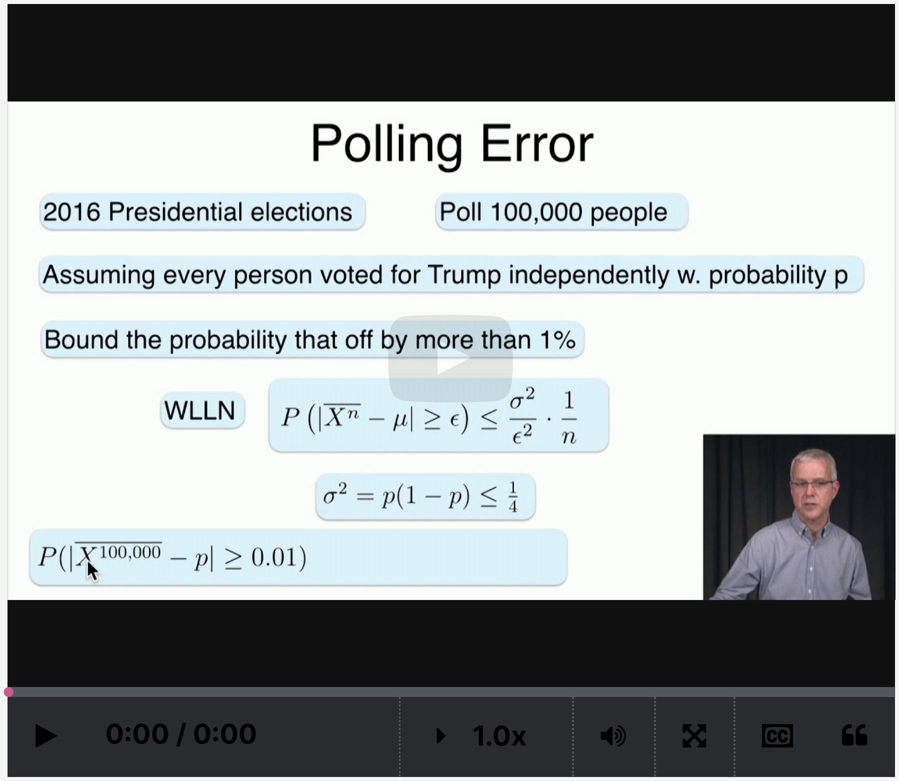

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- Hello and welcome back.

In the previous lectures,

we talked about Markov and Chebyshev's inequality

and now we would like to apply them

to get the Weak Law of

10.3_Law_of_Large_Numbers

POLL

You have two fair coins, and you toss the pair 10,000 times (so you get 10,000 outcome pairs). Roughly how many pairs will not show any tails?

RESULTS

0	3%
1250	11%
2500	61%
5000	25%

Submit

Results gathered from 36 respondents.

FEEDBACK

The probability of not getting any tails is 1/4. According to the weak law of large number, when the number of experiments grows, the sample mean gets closer to the true mean, which is 1/4 in this case. Hence, the answer is 10000 * 1/4 = 2500.

1 (Graded)

1/1 point (graded)

In plain terms, the Weak Law of Large Numbers states that as the number of experiments approaches infinity, the difference between the sample mean and the distribution mean can be as small as possible.

- [x] True
- [ ] False

✔

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

2

0 points possible (ungraded)

Given n iid random varibles $X_1, X_2, \cdots, X_n$ with mean μ, standard deviation $\sigma < \infty$, and the sample mean $S_n = \frac{1}{n}\sum_{i=1}^{n} X_i$, is it true that $\lim_{n\to\infty} E\left((S_n - \mu)^2\right) = 0$?

- [ ] True
- [ ] False

Submit You have used 0 of 1 attempt

3 (Graded)

3/3 points (graded)

The height of a person is a random variable with variance ≤ 5 inches^2. According to Mr. Chebyshev, how many people do we need to sample to ensure that the sample mean is at most 1 inch away from the distribution mean with probability $\geq 95\%$?

100 ✔ **Answer:** 100

100

Explanation

Recall from the proof of the weak law of large numbers that if $X_1, \ldots X_n$ are iid samples each with variance σ^2, then the variance of the sample mean $\overline{X^n}$ is σ^2/n. Therefore, if we sample n people, the sample mean of their heights will have a variance $\leq 5/n$ inches^2. By Chebyshev's Inequality, the probability that the sample mean will be at least 1 inch away from the mean is at most $\frac{5/n}{1^2} = \frac{5}{n}$, hence the probability that the sample mean will be at most 1 inch away is at least $1 - \frac{5}{n}$. We would like to have $1 - \frac{5}{n} \geq 0.95$, hence $\frac{5}{n} \leq 0.05$, or $n \geq 100$.

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

4

0 points possible (ungraded)

For $i = 1, 2, \cdots, n$, let $X_i \sim \mathcal{U}(0, 4)$, $Y_i \sim \mathcal{N}(2, 4)$, and they are independent. Calculate,

$E(X_i)$

$V(X_i)$

$E(Y_i)$

$V(Y_i)$

Find the limit in probability of when $n \to \infty$

$\frac{1}{n}\sum_{i=1}^{n}(X_i + Y_i)$

$\frac{1}{n}\sum_{i=1}^{n}(X_i Y_i)$

Submit

You have used 0 of 4 attempts

5

0 points possible (ungraded)

Flip a fair coin n times and let X_n be the number of heads. Is it true that $P(|X_n - \frac{n}{2}| > 1000) < 0.99$?

- True
- False

Does the result above contradict with the WLLW?

- Yes
- No

Submit You have used 0 of 4 attempts

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Moment Generating

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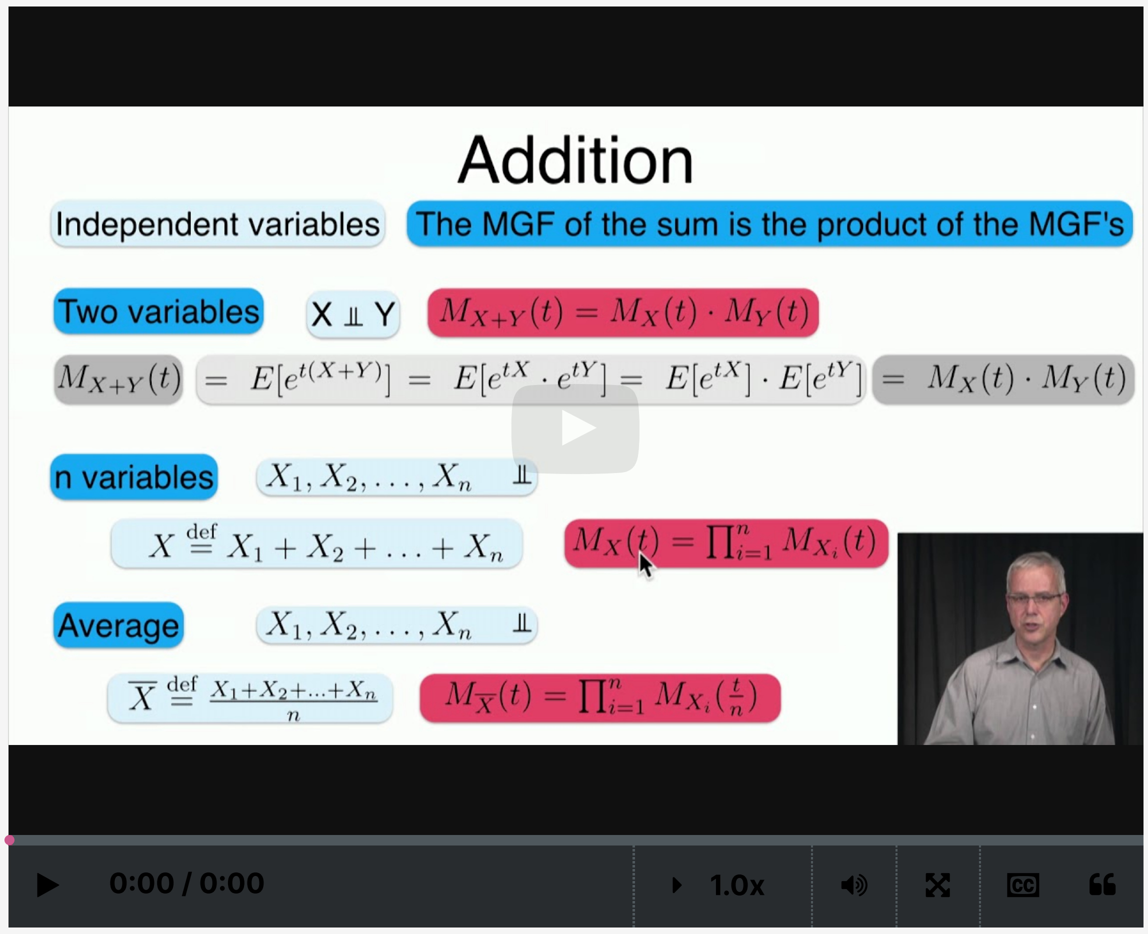

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- Hello and welcome back.

We have so far talked about Markov's

and Chebyshev's inequality and we want to move

to more sophisticated and stronger inequalities.

But before we do that I want to

10.4a_Moment_Generating_Functions

10.4b_Moment_Generating_Functions_Examples

POLL

If M(t) is a moment generating function, then what is M(0)?

RESULTS

0	**10%**
1	**71%**
infinity	**0%**
depends on the distribution	**19%**

Submit

Results gathered from 31 respondents.

FEEDBACK

M(0)=E[e^0]=1

1

0 points possible (ungraded)

If X has moment generating function $M_X(t) = (1-3t)^{-1}$, what is $V(X)$?

- 6
- 9
- 12

Submit You have used 0 of 1 attempt

2

0 points possible (ungraded)
Let $M_X(t)$ be the MGF of X. Which of the following hold for all X and Y?

- [ ] $M_X(0) = 1$
- [ ] $M_X(t) \geq 0$ for all t
- [ ] $M_{3X+2}(t) = e^{2t} \cdot M_X(3t)$
- [ ] $M_{X+Y}(t) = M_X(t) M_Y(t)$

Submit You have used 0 of 3 attempts

3

0 points possible (ungraded)
If X is a non-negative continuous random variable with moment generating function

$$M_X(t) = \frac{1}{(1-2t)^2}, \quad t < \frac{1}{2}$$

Calculate

- $E[X]$

- $V(X)$

Submit You have used 0 of 4 attempts

4

0 points possible (ungraded)
Let $X_1, X_2, \ldots$ be independent $B_{1/2}$ random variables, and let $M \sim P_4$, namely Poisson with mean 4. Which of the following is the MGF of $X_1 + X_2 + \ldots + X_M$?

- ( ) $e^{2(1+e^t)}e^{-4}$
- ( ) $e^{1+e^t}e^{-2}$
- ( ) $\frac{1+e^t}{2}$

2

$\frac{1+e^{2t}}{2}$

Submit You have used 0 of 2 attempts

5 (Graded)

3/3 points (graded)

Let X be a random variable with MGF $M_X(t) = \frac{1}{3}e^{-t} + \frac{1}{6} + \frac{1}{2}e^{2t}$. What is $P(X \leq 1)$?

1/2 ✔ **Answer:** 0.5

$\frac{1}{2}$

Explanation

The pmf of X is $P(X = x) = \begin{cases} \frac{1}{2}, x = 2 \\ \frac{1}{6}, x = 0 \\ \frac{1}{3}, x = -1 \end{cases}$.

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

6

0 points possible (ungraded)

Let $M_X(t)$ be an MGF, which of the following are valid MGF's?

- [x] $M_X(2t)\, M_X(7t)$
- [x] $e^{-5t} M_X(t)$
- [ ] $3M_X(t)$

✔

Explanation

- True.
- True. $e^{-5t} M(t) = E\left(e^{t(X-5)}\right)$.
- False. $3M(0) = 3 \neq 1$.

Submit You have used 2 of 2 attempts

Answers are displayed within the problem

7

0 points possible (ungraded)

If $M_X(t) = e^{-5(1-e^t)}$, find $V(X)$.

$P(X = 3)$.

Submit You have used 0 of 4 attempts

8 (Graded)

3/3 points (graded)

Find the MGF of $(X_1 + X_2 + X_3 + X_4)/3$ where each X_i is an independent $B_{1/2}$ random variable?

- [x] $((1 + e^{t/3})/2)^4$
- [ ] $((1 + e^t)/2)^4$
- [ ] $((2/3 + e^t/3))^4$
- [ ] $((2/3 + e^{t/3}/3))^4$

Explanation

$E(e^{\frac{tX_1}{3}}) = \frac{(1+e^{\frac{t}{3}})}{2}$.

$M_X(t) = E(e^{\frac{tX_1}{3}})E(e^{\frac{tX_2}{3}})E(e^{\frac{tX_3}{3}})E(e^{\frac{tX_4}{3}}) = (\frac{1+e^{\frac{t}{3}}}{2})^4$

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Chernoff

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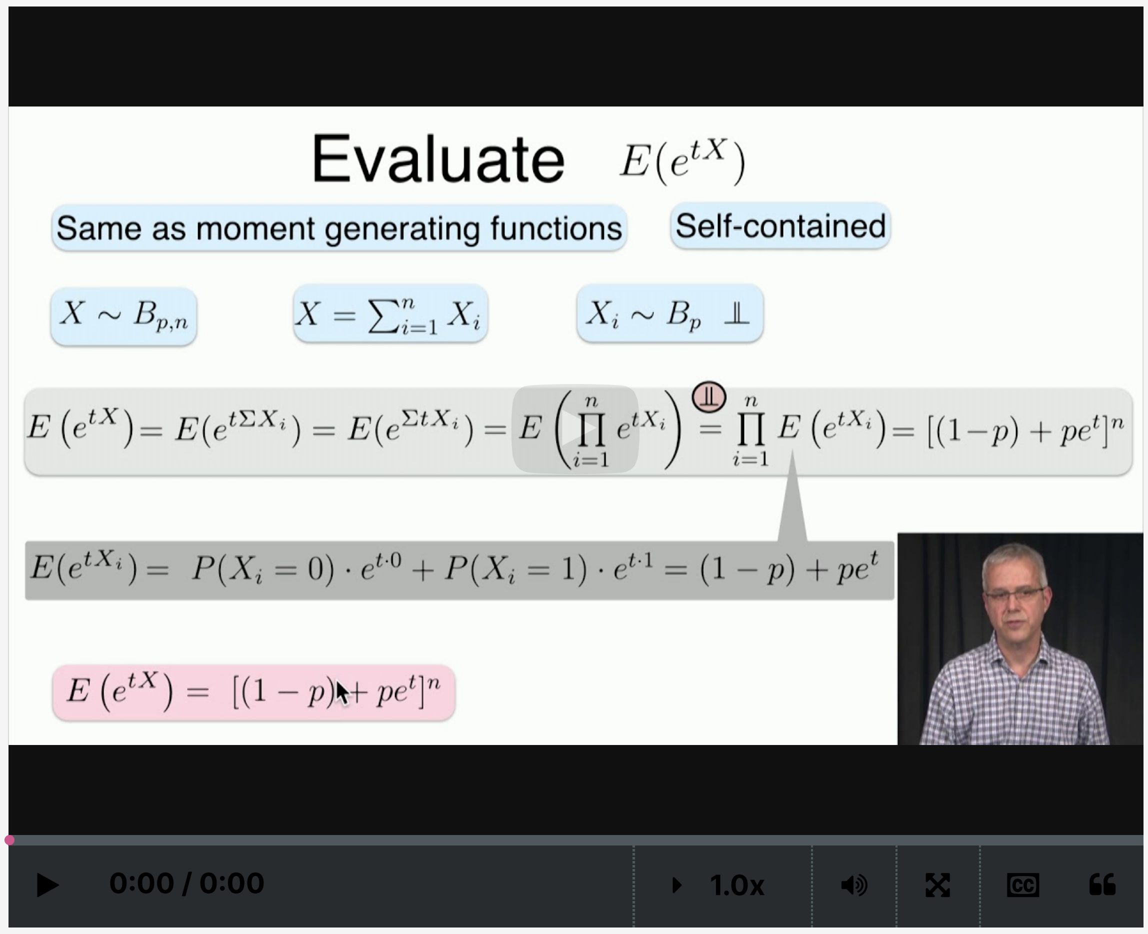

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- Hello and welcome back.

In previous lectures, we talked about

the Markov inequality and the Chebyshev inequalities,

both of them bound the probability

that an element is far away from its mean

and what we want to do in this lecture

is talk about a significantly stronger

10.5_Chernoff_Bound

POLL

If we want to apply Chernoff bound to other distributions, the formulas are going to be different from Chernoff bound on binomial distributions. Because different distributions have the different moment generating functions.

RESULTS

True	88%
False	12%

Submit

Results gathered from 26 respondents.

FEEDBACK

True

1 (Graded)

3.0/3.0 points (graded)

You toss a fair coin 1000 times and take a step forward if the coin lands head and a step backward if it lands tail. Upper bound the probability that you end up ≥ 100 steps from your starting point (in either direction) using Chernoff bound (after the final simplification as in the slides).

0.184 ✔ **Answer:** 0.1745

0.184

Explanation

The expected number of heads is $\mu = 500$ and you will be off by ≥ 100 if and only if the humber of heads is ≥ 550 or ≤ 450. For both the upper and lower bounds, $\delta = 0.1$, and according to the Chernoff bound, the probability is $< e^{-\frac{\delta^2}{2+\delta}\mu} + e^{-\frac{\delta^2}{2}\mu} \approx 0.1745$.

? **Hint (1 of 1):** If you get 650 heads and 350 tails, you are $650 - 350 = 300$ steps aways from the origin.

Next Hint

Submit You have used 3 of 4 attempts

Answers are displayed within the problem

2 (Graded)

0.0/3.0 points (graded)

A coin is equally likely to be either $B_{1/3}$ or $B_{2/3}$. To figure out the bias, we toss the coin 99 times and declare $B_{1/3}$ if the number of heads is less than 49.5 and $B_{2/3}$ otherwise. Bound the error probability using the Chernoff bound derived in lecture video (in its final form, after simplifcation).

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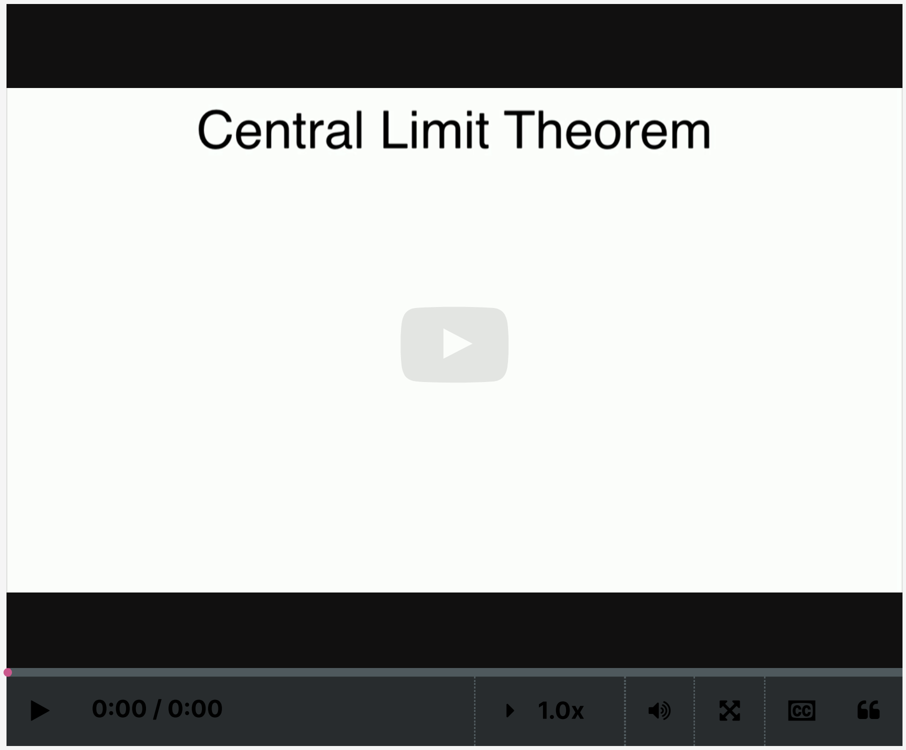

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- Hello and welcome back.

Now that we have talked about several bounds on probability,

we're ready to talk about one of the most important results

in probability and statistics, the central limit theorem.

So just a little bit of an overview.

The central limit theorem, which you'll abbreviate

10.6_Central_Limit_Theorem

POLL

Let X be a random variable with μ = 10 and σ = 4. If X is sampled 100 times, what is the approximate probability that the sample mean of these 100 observations is less than 9?

RESULTS

0.002	**18%**
0.004	**14%**
0.006	**64%**
None of the above	**4%**

Submit

Results gathered from 28 respondents.

FEEDBACK

The answer is 0.006.

1 (Graded)

2/2 points (graded)

For $i \geq 1$, let $X_i \sim G_{1/2}$ be distributed Geometrically with parameter $1/2$.

Define

$$Y_n = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} (X_i - 2)$$

Approximate $P(-1 \leq Y_n \leq 2)$ with large enough n.

0.6818 ✔ **Answer:** 0.681600335381381

0.6818

Explanation

Recall that the Geometric Distribution G_p has mean $\frac{1}{p}$ and standard deviation $\frac{\sqrt{(1-p)}}{p}$.

Since the $X_i \sim G_{1/2}$, their mean is 2 and their standard deviation is $\frac{\sqrt{1/2}}{1/2} = \sqrt{2}$.

Let $Z_n = \frac{Y_n}{\sqrt{2}}$. Then by the central limit theorem, for sufficiently large n, $Z_n \sim N(0, 1)$.

Hence

$P(-1 \leq Y_n \leq 2) = P(-1/\sqrt{2} \leq Z_n \leq \sqrt{2}) = \Phi(\sqrt{2}) - \Phi(-1/\sqrt{2}) = 0.9214 - 0.2398 = 0.6816$

.

? **Hint (1 of 1):** Note that Y_n is not "properly" normalized. Next Hint

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

2 (Graded)

3/3 points (graded)

A class has 100 students. Each student's score is a random variable with mean 85 and standard deviation 40. Use the CLT to approximate the proability that the class average score is below 80.

0.1056

Answer: 0.1056

0.1056

Explanation

The class average score $\frac{1}{100}\sum_{i=1}^{100} X_i$ has mean 85 and standard deviation $\frac{40}{\sqrt{100}} = 4$. The probability can be calculated using $\Phi\left(\frac{80-85}{4}\right) = \Phi(-1.25) = 0.1056$.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

3

0 points possible (ungraded)

The time between consecutive shuttle arrivals is known to be exponentially distributed with mean 10 minutes.

You arrive at the shuttle stop at a uniformly-distributed time.

What is the probability that you wait for less than 9 minutes?

Assume that you took the shuttle once a day during the past 30 days. What is the approximate probability, according to the CLT, that your average wait time was less than 9 muinutes?

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