Problem Sets due May 4, 2022 18:05 +03

Video

UCSDSE212017-V007800

13:50 / 21:04 1.0x HD

be 48.

And we can just look at the compliment of this set

relative to all possible permutations.

And then by the subtraction rule, we get

the total number of permutations or anagrams

where A and R are not adjacent is

five factorial, is the total number of permutations

minus 48, so 120 minus 48,

namely, 72.

Now let's look at more additional constrained permutations.

So how many ways can you

4.1_Combinatorics_Permutations

General comment

Unless other stated, in this and subsequent sections, the following are assumed to be different (distinguishable):

People (including, men, women, children, soccer players, etc.)

Orientations (left to right or right to left)

Rotations (around a circle)

POLL

How many permutations does the set {1,2,3,4} have?

RESULTS

○ 9	0%
○ 18	0%
◉ 24	92%
○ 36	8%

Submit

Results gathered from 13 respondents.

FEEDBACK

4! = 24

1

0 points possible (ungraded)

$0! =$

- ○ 0
- ◉ 1
- ○ ∞
- ○ undefined

✔

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

2

0 points possible (ungraded)

Which of the following are true for all $n, m \in \mathbb{N}$ and $n \geq 1$.

- [x] $n! = n \cdot (n-1)!$
- [ ] $(n \cdot m)! = n! \cdot m!$
- [ ] $(n+m)! = n! + m!$
- [ ] $(n^m)! = (n!)^m$

Submit

You have used 1 of 4 attempts

Answers are displayed within the problem

3

0 points possible (ungraded)

In how many ways can 11 soccer players form a line before a game?

- ( ) 11
- ( ) 11^2
- (•) $11!$
- ( ) None of the above

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

4 (Graded)

2/2 points (graded)
In how many ways can 8 identical rooks be placed on an 8×8 chessboard so that none can capture any other, namely no row and no column contains more than one rook?

40320

Answer: 40320

40320

Explanation
Since there are 8 rooks and 8 rows, each with at most one rook, each row must have exactly one rook. In the first row, there are 8 options for the location of the rook, and once that is chosen, there are 7 options for the second row, etc. Hence the number of ways to place the rooks is $8 \cdot 7 \cdot \ldots \cdot 2 \cdot 1 = 8! = 40,320$.

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

5

0 points possible (ungraded)
In how many ways can 8 distinguishable rooks be placed on an 8×8 chessboard so that none can capture any other, namely no row and no column contains more than one rook?

For example, in a 2×2 chessboard, you can place 2 rooks labled 'a' and 'b' in 4 ways. There are 4 locations to place 'a', and that location determines the location of 'b'.

1625702400

✔ **Answer:** 1625702400

1625702400

Explanation

You can eithr solve this based on the previous problem. There are $8!$ ways to place identical rooks. And once that is done, you can label them in $8!$ ways.
Alternatively, from scratch, there are 64 choices for the first rook, and once the first is placed, one row and column are ruled out for the second , resulting in 49 choices for the second, and so on. Therefore, number of ways is
$64 \cdot 49 \cdot \ldots \cdot 4 \cdot 1 = 8!^2 = 1625702400.$

Submit

You have used 2 of 4 attempts

Answers are displayed within the problem

6

0 points possible (ungraded)

In how many ways can 7 men and 7 women can sit around a table so that men and women alternate. Assume that all rotations of a configuration are identical hence counted as just one.

5040*720

✔ **Answer:** 3628800

$5040 \cdot 720$

Explanation

When rotations don't matter, there are 6! ways to seat the women. For each such configuration, there are 7! ways to seat the men. The total number of configurations is therefore 6!*7!=3,628,800.

Submit

You have used 3 of 4 attempts

Answers are displayed within the problem

7 (Graded)

2/4 points (graded)

In how many ways can three couples be seated in a row so that each couple sits stogether (namely next to each other):

- in a row,

48

✔ **Answer:** 48

48

Explanation

There are $3!$ ways to decide on the order of the couples, and then 2^3 ways to determine the order for each couple, hence a total of $3! \cdot 2^3 = 48$ ways.

- in a circle?

12

✘ **Answer:** 96

12

Explanation

Configuration where the mark is between two couples correspond to configurations in a row, hence there are $3! \cdot 2^3 = 48$ of them. Furthermore each circular shift of such a configuration results in one where the mark separates two members of the same couple. Hence there are also 48 such configurations, and the total number of configurations is $48 \cdot 2 = 96$.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

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Partial Permutations

Problem Sets due May 4, 2022 18:05 +03 Completed

Video

Start of transcript. Skip to the end.

UCSDSE212017-V007600

6:45 / 7:05 1.0x

- Welcome again everyone.

Last lecture, we talked about permutations

and now we want to talk about partial permutations.

These are permutations where you don't want to

arrange all the objects that you have,

but just some subset.

4.2_Partial_Permutations

POLL

How many 2-permutations do we have for set {1,2,3,4}?

RESULTS

8	**0%**
12	**88%**
16	**12%**

Submit

Results gathered from 24 respondents.

FEEDBACK

The answer is P(4 , 2) = 4 * 3 = 12.

1

0 points possible (ungraded)

In how many ways can 5 cars - a BMW, a Chevy, a Fiat, a Honda, and a Kia - park in 8 parking spots?

56*120 ✔ **Answer:** 6720

$56 \cdot 120$

Explanation

There are 8 locations for the BMW, the 7 for the Chevy, etc, so the total number of ways is $8^{\underline{5}} = 6720$.

? **Hint (1 of 1):** Note that both the order and locations of the cars matter. So abbreviating the five models by their first letters and denoting an empty parking spot by X, the following three arrangments are considered different:
B C F H K X X X
X X X B C F H K
X X X K H F C B.

Next Hint

Submit You have used 2 of 4 attempts

Answers are displayed within the problem

2

0 points possible (ungraded)
In how many ways can 5 people sit in 8 numbered chairs?

8*7*6*5*4 ✔ **Answer:** 6720

$8 \cdot 7 \cdot 6 \cdot 5 \cdot 4$

Explanation
The first person can sit in any of the 8 chairs, the second in one of the remaining 7, etc. Hence $8^{\underline{5}} = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 = 6720$.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

3 (Graded)

6.0/6.0 points (graded)
Find the number of 7-character (capital letter or digit) license plates possible if no character can repeat and:

- there are no further restrictions,

36*35*34*33*32*31*30 ✔ **Answer:** 42072307200

$36 \cdot 35 \cdot 34 \cdot 33 \cdot 32 \cdot 31 \cdot 30$

Explanation
$36^{\underline{7}} = 42,072,307,200.$

- the first 3 characters are letters and the last 4 are numbers,

26*25*24*10*9*8*7 ✔ **Answer:** 78624000

$26 \cdot 25 \cdot 24 \cdot 10 \cdot 9 \cdot 8 \cdot 7$

Explanation
Choose 3 from capital letters, and 4 from digits, where the order matters. The result is $26^{\underline{3}} \cdot 10^{\underline{4}} = 78,624,000.$

- letters and numbers alternate, for example A3B9D7Q or 0Z3Q4A9.

(26*10*25*9*24*8*23)+(10*2 ✔ **Answer:** 336960000

$(26 \cdot 10 \cdot 25 \cdot 9 \cdot 24 \cdot 8 \cdot 23) + (10 \cdot 26 \cdot 9 \cdot 25 \cdot 8 \cdot 24 \cdot 7)$

Explanation

Explanation
Such plates contain either four letters and three digits, or the other way. The two sets are disjoint. Hence $26^{\underline{3}} \cdot 10^{\underline{4}} + 26^{\underline{4}} \cdot 10^{\underline{3}} = 336,960,000$.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

4 (Graded)

2.0/2.0 points (graded)
A derangement is a permutation of the elements such that none appear in its original position. For example, the only derangements of $\{1,2,3\}$ are $\{2,3,1\}$ and $\{3,1,2\}$. How many derangements does $\{1,2,3,4\}$ have?

9

Answer: 9

9

Explanation
Let F_1 be the set of permutations of $\{1,2,3,4\}$, where 1 is in location 1, for example 1324. Similarly let F_2 be the set of permutations where 2 is in location 2, for example 3214, etc.
Then $F_1 \cup F_2 \cup F_3 \cup F_4$ is the set of all 4-permutations where at least one element remains in its initial location. The set of permutations where no elements appears in its initial location is the complement of this set.
Note that $\sum_i |F_i| = 4^{\underline{3}}$ (1 location is fixed, so 3-permutation), $\sum_i \sum_j |F_i \cap F_j| = 4^{\underline{2}}$,
$\sum_i \sum_j \sum_k |F_i \cap F_j \cap F_k| = 4^{\underline{1}}$, and $|F_1 \cap F_2 \cap F_3 \cap F_4| = 4^{\underline{0}}$.
Hence by inclusion exclusion, $|F_1 \cup F_2 \cup F_3 \cup F_4| = 4^{\underline{3}} - 4^{\underline{2}} + 4^{\underline{1}} - 4^{\underline{0}} = 24 - 12 + 4 - 1 = 15$.
It follows that the number of derangements is $4! - 15 = 9$.

? Hint (1 of 1): Let F_1 be the set of permutations of $\{1,2,3,4\}$, where 1 is in location 1, for example 1324. Similarly let F_2 be the set of permutations where 2 is in location 2, for example 3214, etc. Use inclusion exclusion to calculate $F_1 \cup F_2 \cup F_3 \cup F_4$. Then observe that the the question asks for the complement of this set.

Next Hint

Submit You have used 2 of 4 attempts

Answers are displayed within the problem

5

0 points possible (ungraded)
Eight books are placed on a shelf. Three of them form a 3-volume series, two form a 2-volume series, and 3 stand on their own. In how many ways can the eight books be arranged so that the books in the 3-volume series are placed together according to their correct order, and so are the books in the 2-volume series? Noted that there is only one correct order for each series.

120

Answer: 120

120

Explanation
Since the 3-volume books must be placed in a unique order, we can view them as a just one "super book", similarly for the 2-volume books. We therefore have a total of 5 books that we can arrange freely, and we can do so in $5! = 120$ ways.

Submit You have used 1 of 4 attempts

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

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Combinations

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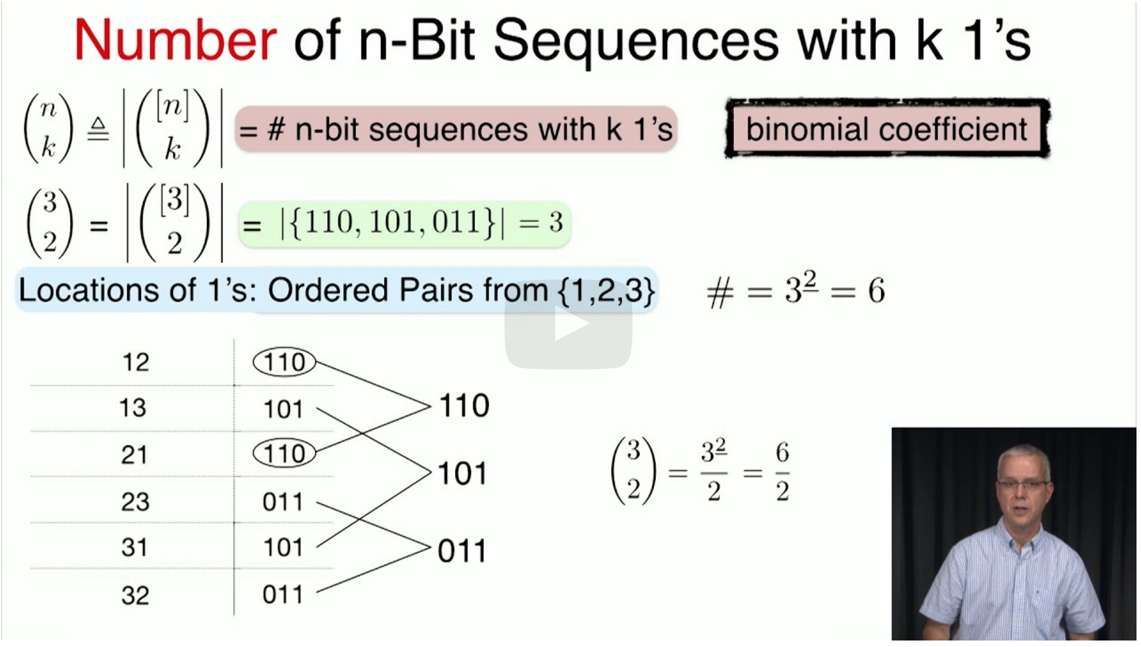

0:00 / 0:00 1.0x

- Hello again, everyone.

Last time we talked about permutations,

and in this lecture we'll discuss combinations.

So what are they?

So first we're going to look at subsets of a set.

And so a subset of size k is called a k-subset.

4.3_Combinations

POLL

Which of the following is larger for k≤n?

RESULTS

The number of k-permutations of an n-set	**90%**
The number of k-subsets of an n-set	**10%**

Submit

Results gathered from 21 respondents.

FEEDBACK

The number of k-permutations is larger.

In selecting subsets, the order doesn't matter, hence the number of k-subsets is the number of k-permutations divided by k!

1

0 points possible (ungraded)

In how many ways can a basketball coach select 5 starting players form a team of 15?

- (selected) $\frac{15!}{5!10!}$
- $\frac{15!}{10!}$
- $\frac{15!}{5!}$
- None of the above

Explanation

It can be deducted from partial permutation, but the order does not matter. It is $\binom{15}{5} = \frac{15^5}{5!} = \frac{15!}{5!10!}$.

Submit

You have used 1 of 2 attempts

Answers are displayed within the problem

2

0 points possible (ungraded)

- In how many ways can you select a group of 2 people out of 5?

- [x] 10
- [ ] 25
- [ ] 125
- [ ] None of the above

Explantion

$\binom{5}{2} = 10.$

- In how many ways can you select a group of 3 people out of 5?

- [x] 10
- [ ] 25
- [ ] 125
- [ ] None of the above

Explantion

$\binom{5}{3} = 10.$

- In how many ways can you divide 5 people into two groups, where the first group has 2 people and the second has 3?

- [x] 10
- [ ] 25
- [ ] 125
- [ ] None of the above

Explantion

After we determine the group of 2, the group of 3 is determined as well, hence the answer is $\binom{5}{2} = 10$.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

3

0 points possible (ungraded)

Ten points are placed on a plane, with no three on the same line. Find the number of:

- lines connecting two of the points,

45 ✔ **Answer:** 45

45

Explanation

Choosing any 2 points out of the 10 points can make a line: $\binom{10}{2}$

- these lines that do not pass through two specific points (say A or B),

28 ✔ **Answer:** 28

28

Explanation

Choosing any 2 points out of the remaining 8 points (except A, B): $\binom{8}{2}$

- triangles formed by three of the points,

120 ✔ **Answer:** 120

120

Explanation

As no three on the same line, choosing any 3 points out of the 10 points make a triangle: $\binom{10}{3}$

- these triangles that contain a given point (say point A),

(9*8*7)/6 ✖ **Answer:** 36

$\frac{9 \cdot 8 \cdot 7}{6}$

Explanation

With point A fixed, choosing any 2 points out of the remaining 9 points make a triangle: $\binom{9}{2}$

- these triangles contain the side AB.

8 ✔ **Answer:** 8

8

Explanation

With point A and B fixed, choosing any 1 point out of the remaining 8 points make a triangle: $\binom{8}{1}$

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

4

0 points possible (ungraded)
The set $\{1, 2, 3\}$ contains 6 nonempty intervals: $\{1\}$, $\{2\}$, $\{3\}$, $\{1, 2\}$, $\{2, 3\}$, and $\{1, 2, 3\}$.

How many nonempty intervals does $\{1, 2, \ldots, 10\}$ contain?

(2^10)

✖ **Answer:** 55

(2^{10})

Explanation
$\{1, 2, \ldots, n\}$ contains $\binom{n}{1}$ singleton intervals and $\binom{n}{2}$ intervals of 2 or more elements. Hence the total number of intervals is $\binom{n}{2} + \binom{n}{1}$. By Pascal's identity $\binom{n}{2} + \binom{n}{1} = \binom{n+1}{2}$. This can also be seen by considering the $n + 1$ midpoints $\{0.5, 1.5, \ldots n + 0.5\}$. Any pair of these points defines an interval in $\{1, 2, \cdots n\}$.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

5

0 points possible (ungraded)
A rectangle in an $m \times n$ chessboard is a cartesian product $S \times T$, where S and T are nonempty intervals in $\{1, \ldots, m\}$ and $\{1, 2, \ldots, n\}$ respectively. How many rectangles does the 3×6 chessboard have?

6*21

✔ **Answer:** 126

$6 \cdot 21$

Explanation
Repeating the same analysis as the above question, but for two different intervals, we have $\binom{4}{2} \cdot \binom{7}{2} = 126.$

? **Hint (1 of 1):** For example, the 2×2 chessboard has $3 \cdot 3 = 9$ rectangles. Next Hint

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

6 (Graded)

8.0/8.0 points (graded)
A standard 52-card deck consists of 4 suits and 13 ranks. Find the number of 5-card hands where:

- any hand is allowed (namely the number of different hands),

2598960

✔ **Answer:** 2598960

2598960

2598960

Explanation
This is simply $\binom{52}{5}$.

- all five cards are of same suit,

4*1287 ✔ **Answer:** 5148

$4 \cdot 1287$

Explanation
There are 4 suits in total and 13 cards in each suit, hence $4 \cdot \binom{13}{5}$ hands.

- all four suits are present,

685464 ✔ **Answer:** 685464

685464

Explanation
One of the 4 suits will appear twice, hence $4 \cdot \binom{13}{2} \cdot 13^3$ hands.

- all cards are of distinct ranks.

1317888 ✔ **Answer:** 1317888

1317888

Explanation
First pick 5 out of 13 ranks, then choose their suits. Therefore there are $\binom{13}{5} \cdot 4^5$ hands.

? **Hint (1 of 1):** For example, for hands where all cards are of the same suit, count the number of hands with 5 clubs, or with 5 diamonds, etc. Next Hint

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

7 (Graded)

2.0/2.0 points (graded)
A company employs 4 men and 3 women. How many teams of three employees have at most one woman?

- ○ 21
- ◉ 22
- ○ 23
- ○ 24

✔

Explanation
There are $\binom{4}{3} = 4$ teams with 0 women and $\binom{3}{1} \times \binom{4}{2} = 3 \times 6 = 18$ teams with 1 woman, for a total of 22.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

8 (Graded)

5.0/5.0 points (graded)

A (tiny) library has 5 history texts, 3 sociology texts, 6 anthropology texts and 4 psychology texts. Find the number of ways a student can choose:

- one of the texts,

18 ✔ **Answer:** 18

18

Explanation

- two of the texts,

153 ✔ **Answer:** 153

153

Explanation

- one history text and one other type of text,

65 ✔ **Answer:** 65

65

Explanation

The student can choose 5 different history texts, and $3+6+4=13$ other texts, by the product rule there are $5 \cdot 13 = 65$ ways of doing that.

- one of each type of text,

360 ✔ **Answer:** 360

360

Explanation

The student selects one text of each type, by the product rule this can be done in $5 \cdot 3 \cdot 6 \cdot 4 = 360$ ways.

- two of the texts with different types.

119 ✔ **Answer:** 119

119

Explanation

There are $5 \cdot 3 = 15$ ways to choose one history and one sociology text, $5 \cdot 6 = 30$ ways to choose one history and one anthropology text, etc. In total there are $5 \cdot 3 + 5 \cdot 6 + 5 \cdot 4 + 3 \cdot 6 + 3 \cdot 4 + 6 \cdot 4 = 119$ ways.

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

9

0 points possible (ungraded)

In how many ways can 7 distinct red balls and 5 distinct blue balls be placed in a row such that

- all red balls are adjacent,

3628800

✔ **Answer:** 3628800

3628800

Explanation

There are 6 ways to place 7 red balls adjacent. Hence the number of ways is $6 \times 7! \times 5! = 3628800$.

- all blue balls are adjacent,

4838400

✔ **Answer:** 4838400

4838400

Explanation

There are 8 ways to place 5 red balls adjacent. Hence the number of ways is $8 \times 7! \times 5! = 4838400$.

- no two blue balls are adjacent.

✖ **Answer:** 33868800

Explanation

First, decide on the locations of the red and blue balls. Arrange all 7 red balls in a line, we can then choose 5 out of the 8 gaps (including those at the beginning and end) to place the blue balls. Since the balls are distinct we can permute the blue balls, and the red balls, for a total of $\binom{8}{5}7!5!$ arrangements.

Submit

You have used 4 of 4 attempts

Answers are displayed within the problem

10

0 points possible (ungraded)

For the set $\{1, 2, 3, 4, 5, 6, 7\}$ find the number of:

- subsets,

2^7

✔ **Answer:** 2^7

2^7

Explanation

There are 7 elements in the set. The number of subsets is 2^7.

- 3-subsets,

✖ **Answer:** 35

Explanation

Choose 3 elements out of 7. The number of ways is $\binom{7}{3} = 35$.

- 3-subsets containing the number 1,

✖ **Answer:** 15

Explanation
1 is fixed.
Choose 2 elements out of 6. The number of ways is $\binom{6}{2} = 15$.

- 3-subsets not containing the number 1.

✖ **Answer:** 20

Explanation
Choose 3 elements out of 6 (excluding 1). The number of ways is $\binom{6}{3} = 20$.

? **Hint (1 of 1):** A 3-subset is a subset with 3 elements. Next Hint

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

11 Functions.

0 points possible (ungraded)
A function $f : X \to Y$ is *injective* or *one-to-one* if different elements in X map to different elements in Y, namely,

$$\forall x \neq x' \in X, \quad f(x) \neq f(x').$$

A function $f : X \to Y$ is *surjective* or *onto* if all elements in Y are images of at least one element of X, namely,

$$\forall y \in Y \quad \exists x \in X, \quad f(x) = y.$$

For sets $A = \{1, 2, 3\}$ and $B = \{a, b, c, d\}$, find the number of

- functions from A to B,

- functions from B to A,

- one-to-one functions from A to B,

- onto functions from B to A.

Submit You have used 0 of 4 attempts

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Binomial Coefficients

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Video

UCSDSE212017-V007500

5:31 / 5:57 1.0x

formula which is

n factorial divided k factorial

and divide it again by n minus k factorial

and what we want to do next is find some properties of the

Binomial Coefficients.

So that's what we're going to do next.

See you then.

End of transcript. Skip to the start.

4.4_Applications_of_Binomial_Coefficients

POLL

You school offers 6 science classes and 5 art classes. How many schedules can you form with 2 science and 2 art classes if order doesn't matter.

RESULTS

25	**12%**
55	**6%**
60	**0%**
150	**82%**

Submit

Results gathered from 17 respondents.

FEEDBACK

The answer is (6 choose 2) * (5 choose 2) = 150.

1 (Graded)

2.0/2.0 points (graded)

How many ordered pairs (A, B), where A, B are subsets of $\{1, 2, 3, 4, 5\}$, are there if:

- $|A| + |B| = 4$

210

Answer: 210

210

Explanation

Number of ways is $\binom{5}{0}\binom{5}{4}+\binom{5}{1}\binom{5}{3}+\cdots+\binom{5}{4}\binom{5}{0}$.

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

2 (Graded)

0.0/2.0 points (graded)
In the video (slide 6, minute 4:04), we discussed the number of non-decreasing grid-paths from $(0,0)$ to $(6,4)$.

How many of these paths go through the point $(2,2)$?

7

✖ **Answer:** 90

7

Explanation
From $(0,0)$ to $(2,2)$, there are $\binom{4}{2}=6$ paths.
From $(2,2)$ to $(6,4)$, there are $\binom{6}{2}=15$ paths.
The total number of paths is $6\times 15=90$.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

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STAFF: Please check grader on question 2
The question, as phrased, seems to suggest an answer that is marked incorrect by the grader. 1

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Video

UCSDSE212017-V007700

7:00 / 11:14 1.0x HD

And we're going to show it's 2 to the n,

and again you could, so here is an example.

3 choose 0, plus 3 choose 1, plus 3 choose 2,

plus 3 choose 3, is what?

3 choose 0 is 1, 3 choose 1 is 3,

3 choose 2 is 3, 3 choose 3 is 1,

so we have 1 plus 3 plus 3 plus 1,

which gives us 8, which is 2 cubed.

And this is (speaks softly) summation

of n choose I is 2 to the n,

and the question is why?

And again you could give an

4.5_Properties_of_Binomial_Coefficient

POLL

For a positive integer n, n choose (n-1) equals to

RESULTS

1	**8%**
n-1	**15%**
n	**69%**

n+1 8%

Submit

Results gathered from 13 respondents.

FEEDBACK
The answer is n.

1 (Graded)

1/1 point (graded)
A deck $n \geq 5$ cards has as many 5-card hands as 2-card hands. What is n?

7 **Answer:** 7

7

Explanation
From the information given, we have $\binom{n}{5} = \binom{n}{2}$ which clearly holds for $n = 7$ since $\binom{n}{5} = \binom{n}{n-5}$.

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

2 (Graded)

1/1 point (graded)
If $\binom{n+2}{5} = 12\binom{n}{3}$, find n.

14 **Answer:** 14

14

Explanation

As $\binom{n+2}{5} = \frac{(n+2)(n+1)}{5 \cdot 4}\binom{n}{3}$, $\frac{(n+2)(n+1)}{5 \cdot 4} = 12$. Hence $n = 14$.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

3

0 points possible (ungraded)

Which of the following is the expansion of $(x+y)^3$?

- $x^3 + y^3$
- $x^3 + x^2y + xy^2 + y^3$
- $x^3 + 6xy + y^3$
- $x^3 + 3x^2y + 3xy^2 + y^3$

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

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The binomial Theorem

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18:45 / 21:28 1.0x HD

of the binomial coefficients.

In other words, we talked about binary sequences

or choosing a set, subset from a set of size k,

and what we want to do next is

talk about larger alphabet sequence,

also for larger alphabets,

And that's what we'll do in the next lecture. See you then.

End of transcript. Skip to the start.

4.6_Binomial_Theorem

POLL

What is the coefficient of x^2 in the expansion of (x+2)^4?

- ○ **12**
- ○ **24**
- ○ **48**
- ◉ **None of the above**

Submit

FEEDBACK

The answer is (4 choose 2) * 2^2 = 24.

1 (Graded)

2/2 points (graded)

- What is the coefficient of x^4 in the expansion of $(2x-1)^7$?

-560 ✔ **Answer:** -560

-560

Explanation

By binomial theorem, the number of terms that contain $(2x)^4$ is $\binom{7}{4}$. Hence, the coefficient of x^4 is $2^4 \times (-1)^3 \times \binom{7}{4} = -560$.

- What is the constant term in the expansion of $\left(x-\frac{2}{}\right)^{6}$?

-160

✔ **Answer:** -160

-160

Explanation

$\left(x-\frac{2}{x}\right)^6=\left(x^2-2\right)^6\left(\frac{1}{x}\right)^6$. To find the constant term, we just need to find the coefficient of x^6 in $\left(x^2-2\right)^6$. The number of terms that contain x^6 is $\binom{6}{3}$, so the coefficient is $1^3\times(-2)^3\times\binom{6}{3}=-160$

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

2 (Graded)

2/2 points (graded)

What is the coefficient of x^2 in the expansion of $(x+2)^4(x+3)^5$?

23112

✔ **Answer:** 23112

23112

Explanation

Consider $(x+2)^4(x+3)^5$ as the product of $(x+2)^4$ and $(x+3)^5$, there are 3 ways to get x^2: (1) multiply the x^2 term in $(x+2)^4$ and the constant term in $(x+3)^5$, (2) multiply the x term in $(x+2)^4$ and the x term in $(x+3)^5$, (3) multiply the constant term in $(x+2)^4$ and the x^2 term in $(x+3)^5$.

Hence the result is the sum of these 3 ways $\binom{5}{2}2^4 3^3+\binom{4}{1}\binom{5}{1}2^3 3^4+\binom{4}{2}3^5 2^2=23112$.

Submit You have used 3 of 4 attempts

Answers are displayed within the problem

3

0 points possible (ungraded)

In an earlier section, we solved this question by mapping the sets A and B to ternary sequences. In this section, we ask you to solve it using the binomial theorem.

How many ordered pairs (A, B), where A, B are subsets of $\{1,2,3,4,5\}$ have:

- $A\cap B=\emptyset$

- $A\cup B=\{1,2,3,4,5\}$

Submit You have used 0 of 4 attempts

4

0 points possible (ungraded)

Which of the followings are equal?

- [ ] $\binom{10}{4}$
- [ ] $\binom{10}{5}$
- [ ] $\binom{10}{6}$
- [ ] $\binom{9}{5} + \binom{9}{6}$

Submit You have used 0 of 3 attempts

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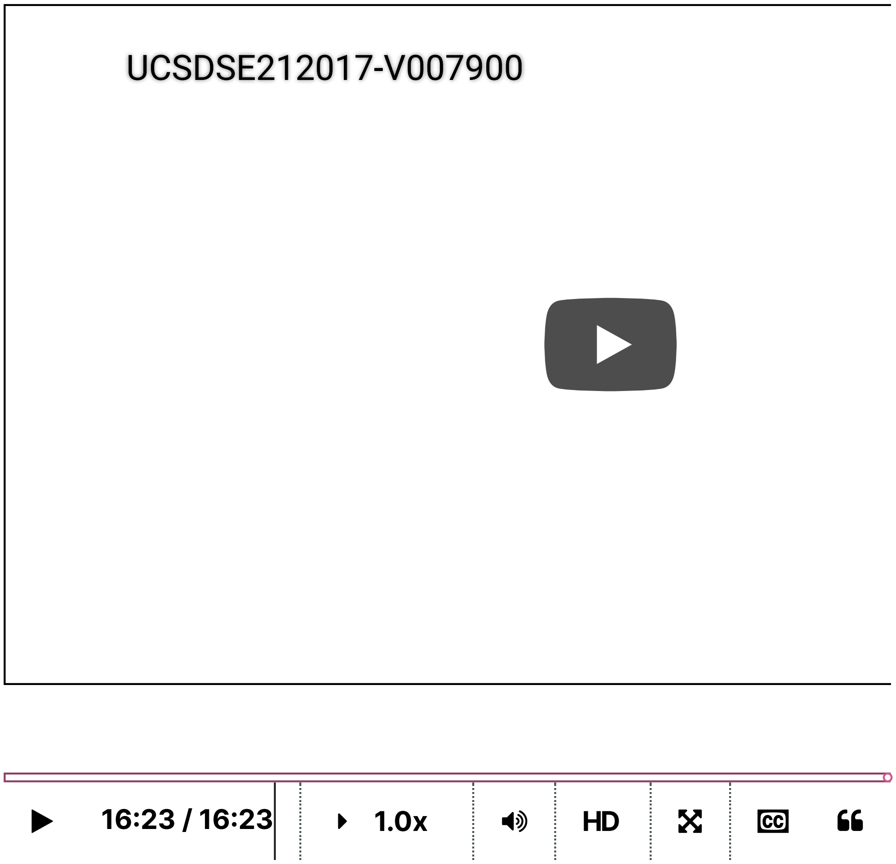

a simple extension of the binomial coefficients

to multinomial coefficients and the binomial theorem

to the multinomial theorem and what we're going to do next

is look at another application

of binomial and related coefficients.

End of transcript. Skip to the start.

4.7_Multinomials

POLL

What is the coefficient of xy in the expansion of (x+y+2)^4?

RESULTS

◉ **12**	**8%**
○ **24**	**21%**
○ **48**	**67%**

None of the above 4%

Submit

Results gathered from 24 respondents.

FEEDBACK
The answer is 48. The number of ways to have 2"2"s, 1 "x", 1 "y" is 12 (using multinomial coefficient). Then we multiply it with 2^2 = 4 and get the answer.

1 (Graded)

3/3 points (graded)
In how many ways can you give three baseball tickets, three soccer tickets, and three opera tickets, all general admission, to nine friend so each gets one ticket?

1680

Answer: 1680

1680

Explanation
Using the multinomial coefficient, we get the answer $\binom{9}{3,3,3} = 1680$.

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

2

0 points possible (ungraded)
How many ways can we divide 12 people into:

- three labeled groups evenly

- three unlabeled groups evenly

- three labeled groups with 3, 4 and 5 people

- three unlabeled groups with 3, 4 and 5 people

- three unlabeled groups with 3, 3 and 6 people

Submit

You have used 0 of 4 attempts

---

3 (Graded)

4/4 points (graded)

- What is the coefficient of x^3y^2 in expansion of $(x+2y+1)^{10}$?

10080

Answer: 10080

10080

Explanation

$(x+2y+1)^{10} = \underbrace{(x+2y+1)\cdots(x+2y+1)}_{10\ (x+2y+1)s}.$

To form x^3y^2, we need to pick three x's, two $2y$'s, and five 1's. The number of ways is $\binom{10}{3,2,5}$.

The resulting term of x^3y^2 is $\binom{10}{3,2,5}\left(x^3(2y)^2 1^5\right)$. Hence the coefficient is $\binom{10}{3,2,5}2^2 = 10080$.

- What is the coefficient of x^3 in expansion of $(x^2-x+2)^{10}$

-38400

✔ **Answer:** -38400

-38400

Explanation

$(x^2-x+2)^{10} = \underbrace{(x^2-x+2)\cdots(x^2-x+2)}_{10\ (x^2-x+2)s}.$

To form x^3, we can pick one x^2's, one $-x$'s, and eight 2's. The number of ways is $\binom{10}{1,1,8}$.
Or we can pick zero x^2's, three $-x$'s, and seven 2's. The number of ways is $\binom{10}{0,3,7}$.
The resulting term of x^3 is $\binom{10}{1,1,8}\left(x^2(-x)2^8\right) + \binom{10}{0,3,7}\left((x^2)^0(-x)^3 2^7\right)$. Hence the coefficient is $\binom{10}{1,1,8}(-1)2^8 + \binom{10}{0,3,7}(-1)^3 2^7 = -38400$.

Submit

You have used 2 of 4 attempts

Answers are displayed within the problem

4

0 points possible (ungraded)

How many terms are there in the expansion of $(x+y+z)^{10} + (x-y+z)^{10}$?

Submit

You have used 0 of 4 attempts

5

0 points possible (ungraded)

How many anagrams, with or without meaning, does "REFEREE" have such that:

- there is no constraint

- two "R"'s are separated

- it contains subword “EE”

- it begins with letter “R”

Submit

You have used 0 of 4 attempts

6

0 points possible (ungraded)

How many anagrams, with or without meaning, do the following words have?

- CHAIR

- INDIA

- SWIMMING

Submit You have used 0 of 4 attempts

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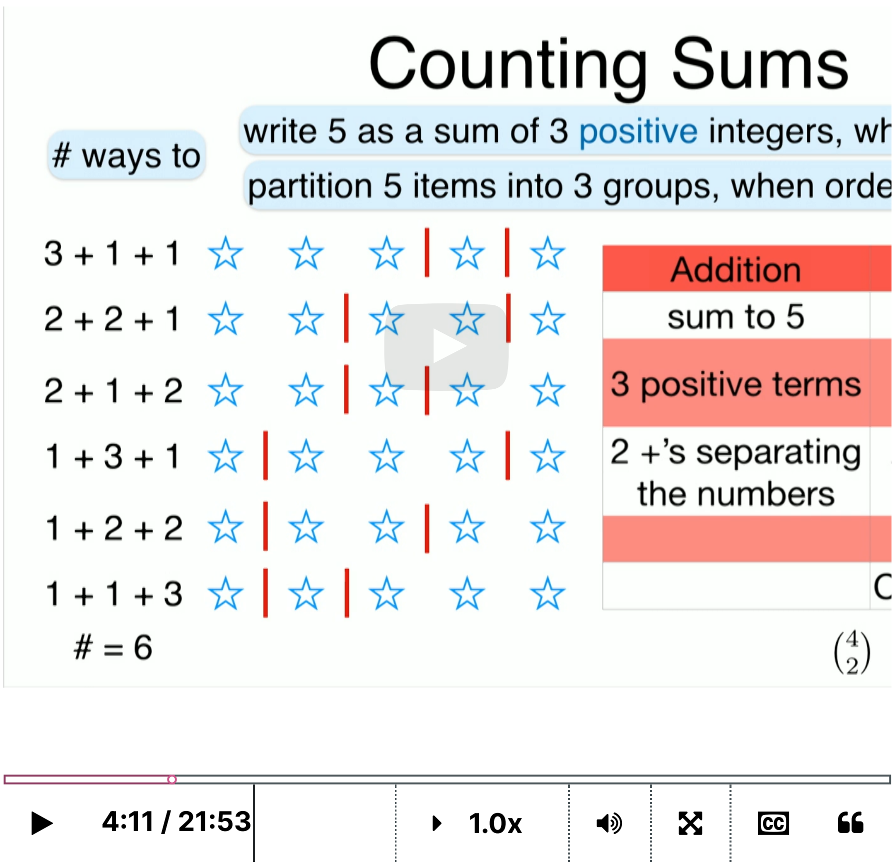

which as we know is six.

So we see that we could get this number

by just looking at the number of spaces

or gaps that we have which was five minus one, four.

And the number of bars that we have

which was three minus one, or two.

And we have four choose two which is six

which is the number that we got here.

Now this of course generalizes,

and if we want to have K terms that will add to N,

then we would like to find

[4.8_Stars_and_Bars]()

POLL

In how many different ways can you write 11 as a sum of 3 **positive** integers if order matters?

RESULTS

- ○ **28** **0%**
- ◉ **36** **9%**

- 45 70%
- None of the above 22%

Submit

Results gathered from 23 respondents.

FEEDBACK

The answer is 45. Following the equation mentioned in the video, it is "10 choose 2".

1

0 points possible (ungraded)

If $a+b+c+d=10$, how many ordered integer solutions (a,b,c,d) are there, when all elements are

- non-negative,

- positive?

Submit You have used 0 of 4 attempts

2 (Graded)

3/3 points (graded)

In how many ways can we place 10 idential red balls and 10 identical blue balls into 4 distinct urns if:

- there are no constraints,

81796

✔ **Answer:** 81796

81796

Explanation

$\binom{13}{3} \cdot \binom{13}{3}$, by combining stars and bars for both balls evaluated separately.

- the first urn has at least 1 red ball and at least 2 blue balls,

36300

✔ **Answer:** 36300

36300

Explanation

First place 1 red ball and 2 blue balls in the first urn, and then repeat the above part with 9 red balls and 8 blue balls, resulting in $\binom{12}{3} \cdot \binom{11}{3}$.

- each urn has at least 1 ball?

65094

✔ **Answer:** 65094

65094

Explanation

There are $\binom{12}{2}^2$ ways to place the balls so that urn 1 is empty, $\binom{11}{1}^2$ ways so that urns 1 and 2 are empty and $\binom{10}{0}^2 = 1$ so that urns 1 2 and 3 are empty. By inclusion exclusion, there are $\binom{4}{1}\binom{12}{2}^2 - \binom{4}{2}\binom{11}{1}^2 + \binom{4}{3}\binom{10}{0}^2$ placements where at least one urn is empty. And by the complement rule, the answer is $\binom{13}{3}^2 - \binom{4}{1}\binom{12}{2}^2 + \binom{4}{2}\binom{11}{1}^2 - \binom{4}{3}\binom{10}{0}^2 = 65,094$.

? **Hint (1 of 2):** Let (a, b, c, d) be the number of balls you put into the 4 urns respectively. Then $(4, 3, 2, 1)$ and $(1, 2, 3, 4)$ are different.

Hint (2 of 2): (Part 3) Use complement and inclusion exclusion

Next Hint

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

3 (Graded)

4/4 points (graded)

How many 6-digit sequences are:

- strictly ascending, as 024579 or 135789, but not 011234,

210

Answer: 210

210

Explanation

Every six-digit strictly asending sequence corresponts to 6 distinct digits. There are $\binom{10}{6} = 210$ ways to choose them.

- ascending (not necessarily strictly), as 023689, 033588, or 222222.

5005

Answer: 5005

5005

Explanation

Every six-digit (not necessarily striclty) assending sequence corresponds to a collection of 6 digits, possibly with repetition. Let x_i denote the number of times digit i is included in the number. Using stars and bars, the number of ways of assigning $x_0 + x_1 + \cdots + x_9 = 6$ is $\binom{6+10-1}{6} = 5005.$

? **Hint (1 of 2):** For the first part, the number of striclty assending 6-digit sequences is the number of waays to choose 6 digits without repetition.

Hint (2 of 2): For the second part, the number of (not-necessaariy-strictly) assending 6-digit sequences is the number of ways to choose 6 digits, with possible repetion.

Next Hint

Submit You have used 3 of 4 attempts

Answers are displayed within the problem

4

0 points possible (ungraded)

How many terms are there in the expansion of $(x+y+z)^{10}$?

Submit You have used 0 of 4 attempts

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? How to understand the solution to question 2 part 3?
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