Conditional Probability

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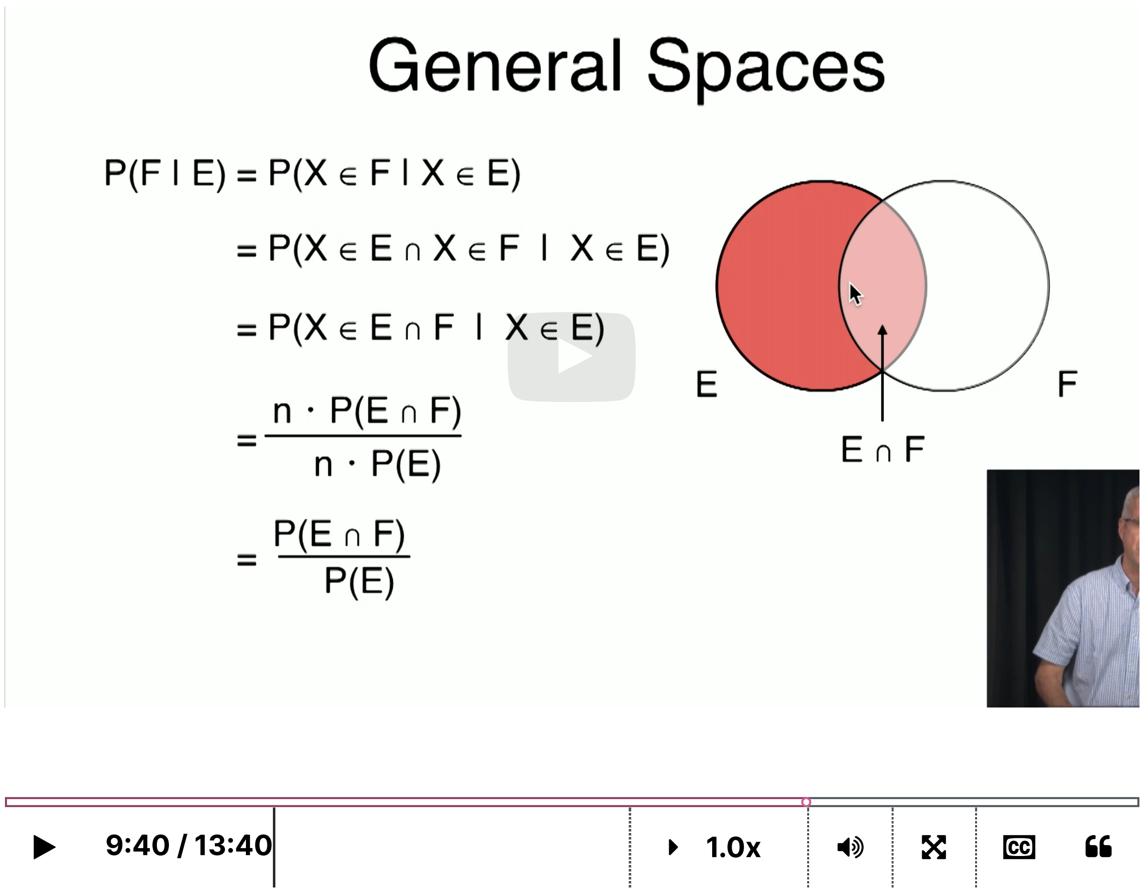
Problem Sets due Jun 6, 2022 02:13 +03

Video



General Spaces

$$P(F \mid E) = P(X \in F \mid X \in E)$$
$$= P(X \in E \cap X \in F \mid X \in E)$$
$$= P(X \in E \cap F \mid X \in E)$$
$$= \frac{n \cdot P(E \cap F)}{n \cdot P(E)}$$
$$= \frac{P(E \cap F)}{P(E)}$$

So, let R1 be the event that the first ball is red

and R2 be the event that the second ball is red.

So, the probability that both are red

is the probability of R1 intersection with R2,

when both happen, and that's going to be equal

to the probability of R1 times the probability

of R2 given R1.

Now, what is the probability of R1 is two thirds, right?

Because we have two red balls and one blue,

so if we take one at random,

the probability that it will be red will be two thirds

and once we do that, then, if R1

6.1_Conditional_Probability

POLL

Let A and B be two positive-probability events. Does P(A|B)>P(A) imply P(B|A)>P(B)?

RESULTS

○ **Yes**	**33%**
● **Not necessarily**	**67%**

Submit

Results gathered from 46 respondents.

FEEDBACK

Yes.

P(A|B)=P(A,B) / P(B) and P(B|A)=P(A,B) / P(A).

Hence, P(A|B)>P(A) iff P(A,B)>P(A) * P(B) iff P(B|A)>P(B).

1

0 points possible (ungraded)

Suppose $P(A) > 0$. Find $P(B|A)$ when:

- $B = A$,

1 ✓ **Answer:** 1

1

Explanation

Given that A happens, B must happens. Hence $P(B|A) = 1$.

- $B \supseteq A$,

1

✓ **Answer:** 1

1

Explanation
Same as above.

- $B = \Omega$,

1

✓ **Answer:** 1

1

Explanation
Same as above.

- $B = A^c$,

0

✓ **Answer:** 0

0

Explanation
Given that A happens, B can never happens. Hence $P(B|A) = 0$.

- $A \cap B = \emptyset$,

0

✓ **Answer:** 0

0

Explanation
Same as above.

- $B = \emptyset$.

0

✓ **Answer:** 0

0

Explanation
Same as above.

Submit You have used 3 of 4 attempts

Answers are displayed within the problem

2

0 points possible (ungraded)
If A and B are disjoint positive-probablity events, then $P(A|B)$=

☐ $P(A)$,

☑ $P(B|A)$,
✓

☐ $P(A \cup B)$,

☐ $P(A \cap B)$.
✓

✗

Explanation
Since A and B are disjoint, $P(A|B) = 0$.
$P(A \cap B) = P(B|A) = 0$, while $P(A)$ and $P(A \cup B)$ are positive as A and B are positive-probablity events.

Submit You have used 2 of 2 attempts

🛈 Answers are displayed within the problem

3 (Graded)

4/4 points (graded)
Given events A, B with $P(A) = 0.5$, $P(B) = 0.7$, and $P(A \cap B) = 0.3$, find:

- $P(A|B)$,

0.3/0.7 ✓ **Answer:** 3/7

$\frac{0.3}{0.7}$

Explanation
$P(A|B) = P(A \cap B) / P(B) = 0.3/0.7 = 3/7$.

- $P(B|A)$,

0.6 ✓ **Answer:** 3/5

0.6

Explanation
$P(B|A) = P(B \cap A) / P(A) = 0.3/0.5 = 3/5$.

- $P(A^c|B^c)$,

(1-(0.5+0.7-0.3))/(1-0.7) ✓ **Answer:** 1/3

$\frac{1-(0.5+0.7-0.3)}{1-0.7}$

Explanation
$P(A^c|B^c) = P(A^c \cap B^c) / P(B^c) = 0.1/0.3 = 1/3$.

- $P(B^c|A^c)$.

0.2 ✓ **Answer:** 1/5

0.2

Explanation
$P(B^c|A^c) = P(B^c \cap A^c) / P(A^c) = 0.1/0.5 = 1/5$.

Submit You have used 2 of 4 attempts

Answers are displayed within the problem

4

0 points possible (ungraded)

Find the probability that the outcome of a fair-die roll is at least 5, given that it is at least 4.

● $\frac{2}{3}$

○ $\frac{2}{4}$

○ $\frac{1}{3}$

○ $\frac{1}{2}$

✓

Explanation

$$P\left(\text{at least } 5 | \text{at least } 4\right) = \frac{P(\text{at least } 5 \cap \text{at least } 4)}{P(\text{at least } 4)} = \frac{P(\text{at least } 5)}{P(\text{at least } 4)} = \frac{2}{3}.$$

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

5

0 points possible (ungraded)

Two balls are painted red or blue uniformly and independently. Find the probability that both balls are red if:

- at least one is red,

1/3 ✓ **Answer:** 1/3

$\frac{1}{3}$

Explanation

$$P\left(2R | \text{at least } 1R\right) = \frac{P(2R \cap \text{at least } 1R)}{P(\text{at least } 1R)} = \frac{P(2R)}{P(\text{at least } 1R)} = \frac{1/4}{3/4} = \frac{1}{3}.$$

- a ball is picked at random and it is pained red.

1/2 ✓ **Answer:** 1/2

$\frac{1}{2}$

Explanation

$$P\left(2R | \text{random ball is R}\right) = \frac{P(2R \wedge \text{random ball is R})}{P(\text{random ball is R})} = \frac{P(2R)}{P(\text{random ball is R})} = \frac{1/4}{1/2} = \frac{1}{2}.$$

Submit You have used 2 of 4 attempts

Answers are displayed within the problem

6 (Graded)

6 (Graded)

3/3 points (graded)

Three fair coins are sequentialy tossed. Find the probability that all are heads if:

- the first is tails,

0

✓ **Answer:** 0

0

Explanation

If the fisrt coin is tails, it's impossible for all coins to be heads, hence the probability is 0.

More formally, $P\left(X_1 \cap X_2 \cap X_3 | \overline{X_3}\right) = \frac{P(X_1 \cap X_2 \cap X_3 \cap \overline{X_3})}{P(\overline{X_3})} = \frac{P(\emptyset)}{P(\overline{X_3})} = \frac{0}{1/2} = 0.$

- the first is heads,

1/4

✓ **Answer:** 1/4

$\frac{1}{4}$

Explanation

First intuitively, if the first coin is heads, then all are heads iff the second and third coins are heads, which by independence of coin flips happens with probability $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$.

A bit more formally, let X_1, X_2, X_3 be the events that the first, second, and third coin is heads. Then

$P\left(X_1 \cap X_2 \cap X_3 | X_1\right) = \frac{P(X_1 \cap X_2 \cap X_3 \cap X_1)}{P(X_1)} = \frac{P(X_1 \cap X_2 \cap X_3)}{P(X_1)} = \frac{1/8}{1/2} = \frac{1}{4}.$

- at least one is heads.

1/7

✓ **Answer:** 1/7

$\frac{1}{7}$

Explanation

First intuitively, there are seven possible outcome triples where at least one of the coins is heads, and only one of them has all heads. Hence the probability of all heads given that one is heads is $1/7$.

More formally,

$P\left(X_1 \cap X_2 \cap X_3 | X_1 \cup X_2 \cup X_3\right) = \frac{P((X_1 \cap X_2 \cap X_3) \cap (X_1 \cup X_2 \cup X_3))}{P(X_1 \cup X_2 \cup X_3)} = \frac{P(X_1 \cap X_2 \cap X_3)}{P(X_1 \cup X_2 \cup X_3)} = \frac{1/8}{7/8} = \frac{1}{7}.$

Submit

You have used 3 of 4 attempts

ℹ Answers are displayed within the problem

7

0 points possible (ungraded)

A 5-card poker hand is drwan randomly from a standard 52-card deck. Find the probability that:

- all cards in the hand are ≥ 7 (7, 8,..., K, Ace), given that the hand contains at least one face card (J, Q, or K),

- there are exactly two suits given that the hand contains exactly one queen.

Submit You have used 0 of 4 attempts

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Video

10:36 / 10:36

1.0x

and seeing whether the probability change or not

and that's our measure of when

probabilities are dependent or independent.

With that, we're going to finish this slide

and we talked about independence

and next time we'll talk about base rule.

End of transcript. Skip to the start.

6.2_Independence

POLL

Two disjoint events cannot be independent.

RESULTS

●	**Yes**	**32%**
○	**Not exactly**	**68%**

Submit

Results gathered from 40 respondents.

FEEDBACK

Not exactly.

If the two disjoint events have positive probability, they are dependent.

But if one of the two events has zero probability, they are independent .

1

0 points possible (ungraded)

Two dice are rolled. The event that the first die is 1 and the event that two dice sum up to be 7 are

● Independent

○ Dependent

✓

Explanation

Let X be the outcome of the first die and Y be the outcome of the second die.

$P(X=1|X+Y=7)=\frac{1}{6}=P(X=1)$. Hence, they are independent.

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

2

0 points possible (ungraded)

Of 10 students, 4 take only history, 3 take only math, and 3 take both history and math. If you select a student at random, the event that the student takes history and the event that the student takes math are:

○ Independent

● Dependent

✓

Explanation

Let H be the event that the student takes history, and M the event that the student takes math. Then $P(H) = \frac{7}{10}$, $P(M) = \frac{6}{10}$, and $P(H, M) = \frac{3}{10}$. Since $P(H)\,P(M) \neq P(H, M)$, the two events are dependent.

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

3 (Graded)

2/2 points (graded)

4 freshman boys, 6 freshman girls, and 6 sophomore boys go on a trip. How many sophomore girls must join them if a student's gender and class are to be independent when a student is selected at random?

9 ✓ **Answer:** 9

9

Explanation

First, let's do it the formal but hard way. Let SG denote the number of sophomore girls. Then the total number of students is $4 + 6 + 6 + SG = 16 + SG$.

If a student is selected at randdom, the probabilty that the student is a freshman is $\frac{4+6}{16+SG}$, the probability that a random student is a boy is $\frac{4+6}{16+SG}$, and the probability that the student is both a freshman and boy is $\frac{4}{16+SG}$. If the student's gender and class are independent, then by the product rule, the probability of the intersection is the profuct of the probabilities, hence $\frac{4}{16+SG} = \frac{4+6}{16+SG} \cdot \frac{4+6}{16+SG}$, hence $100 = 4 \cdot (16 + SG)$, or $SG = 9$.

Another way to see this is to observe that if the gender and class are independent, then the fraction of girls that are freshmen, namely $\frac{6}{6+SG}$ should be the same as the fraction of boys that are freshmen, namely $\frac{4}{4+6} = \frac{2}{5}$. Therefore $\frac{6}{6+SG} = \frac{2}{5}$, or $SG = 9$.

Submit You have used 2 of 4 attempts

Answers are displayed within the problem

4

0 points possible (ungraded)

Every event A is independent of:

☑ $\emptyset$,

☑ Ω,

☐ A itself,

☐ A^c.

✓

Explanation

Intuitively:

A is indpendent of the null event because occurance of A doesn't change the 0 probability of the null event.

Similarly A is independent of Ω because occurance of A does not change the probability 1 of Ω.

If A has probability strictly between 0 and 1, then its occurance changes the probability of both itself and A^c, implying dependence.

Mathematically:

- True. $P(\emptyset|A) = 0 = P(\emptyset)$.
- True. $P(A|\Omega) = \frac{P(A\cap\Omega)}{P(\Omega)} = \frac{P(A)}{P(\Omega)} = P(A)$.
- False.
- False.

Submit You have used 4 of 4 attempts

ℹ Answers are displayed within the problem

5

0 points possible (ungraded)

Which of the following ensure that events A and B are independent:

☑ A and B^c are independent,
✓

☑ $A \cap B = \emptyset$,

☑ $A \subseteq B$,

☐ at least one of A or B is $\emptyset$ or Ω?
✓

✗

Explanation

- True. If A and B^c are independent, $1 - P(B|A) = P(B^c|A) = P(B^c) = 1 - P(B)$, which implies $P(B|A) = P(B)$.
- False.
- False.
- True. For $\emptyset$, $P(\emptyset|A) = 0 = P(\emptyset)$. For Ω, $P(A|\Omega) = \frac{P(A\cap\Omega)}{P(\Omega)} = \frac{P(A)}{P(\Omega)} = P(A)$. $\emptyset$ and Ω are indepedent with any sets.

Submit You have used 4 of 4 attempts

🛈 Answers are displayed within the problem

6 (Graded)

2/2 points (graded)

When rolling two dice, which of the following events are independent of the event that the first die is 4:

☑ the second is 2,

☐ the sum is 6,

☑ the sum is 7,

☑ the sum is even.

✓

Explanation

Let X be the outcome of the first dice, and Y be the second one.

- True. $P(X=4|Y=2)=P(X=4)=\frac{1}{6}$.
- False. $P(X+Y=6|X=4)=\frac{1}{6}$. $P(X+Y=6)=\frac{5}{36}$. $P(X+Y=6)\neq P(X+Y=6|Y=4)$.
- True. $P(X+Y=6|X=4)=\frac{1}{6}=P(X+Y=7)$.
- True. $P(X+Y \text{ is even } |X=4)=P(Y \text{ is even })=\frac{1}{2}=P(X+Y \text{ is even })$.

Submit You have used 1 of 4 attempts

🛈 Answers are displayed within the problem

7

0 points possible (ungraded)

Roll two dice, and let F_e be the event that the first die is even, S_4 the event that the second die is 4, and Σ_o the event that the sum of the two dice is odd. Which of the following events are independent:

☑ F_e and S_4,
✓

☑ F_e and Σ_o,
✓

☐ S_4 and Σ_o,
✓

☐ F_e, S_4, and Σ_o (mutually independent)?

✗

Explanation

- True. $P(F_e,S_4)=\frac{1}{12}, P(F_e)=\frac{1}{2}, P(S_4)=\frac{1}{6}$. As $P(F_e,S_4)=P(F_e)P(S_4)$, F_e and S_4 are independent.
- True. $P(F_e,\Sigma_o)=\frac{1}{4}, P(F_e)=\frac{1}{2}, P(\Sigma_o)=\frac{1}{2}$. As $P(F_e,\Sigma_o)=P(F_e)P(\Sigma_o)$, F_e and Σ_o are independent.
- True. $P(S_4,\Sigma_o)=\frac{1}{12}, P(S_4)=\frac{1}{6}, P(\Sigma_o)=\frac{1}{2}$. As $P(S_4,\Sigma_o)=P(S_4)P(\Sigma_o)$, S_4 and Σ_o are independent.
- False. $P(F_e,S_4,\Sigma_o)=0\neq P(F_e)P(S_4)P(\Sigma_o)$.

Submit You have used 4 of 4 attempts

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

8

0 points possible (ungraded)
Two dice are rolled. Let F_3 be the event that the first die is 3, S_4 the event that the second die is 4, and Σ_7 the event that the sum is 7. Which of the following are independent:

☑ F_3 and S_4,

☑ F_3 and Σ_7,

☑ S_4 and Σ_7,

☐ F_3, S_4, and Σ_7 (mutually independent)?

✓

Explanation
- True. $P(F_3, S_4) = \frac{1}{36}, P(F_3) = \frac{1}{6}, P(S_4) = \frac{1}{6}$. As $P(F_3, S_4) = P(F_3) P(S_4)$, F_3 and S_4 are independent.
- True. $P(F_3, \Sigma_7) = \frac{1}{36}, P(F_3) = \frac{1}{6}, P(\Sigma_7) = \frac{1}{6}$. As $P(F_3, \Sigma_7) = P(F_3) P(\Sigma_7)$, F_3 and Σ_7 are independent.
- True. $P(S_4, \Sigma_7) = \frac{1}{36}, P(S_4) = \frac{1}{6}, P(\Sigma_7) = \frac{1}{6}$. As $P(S_4, \Sigma_7) = P(S_4) P(\Sigma_7)$, S_4 and Σ_7 are independent.
- False. $P(F_3, S_4, \Sigma_7) = \frac{1}{36} \neq P(F_3) P(S_4) P(\Sigma_7) = \frac{1}{6}\frac{1}{6}\frac{1}{6} = \frac{1}{216}$.

Submit You have used 2 of 4 attempts

Answers are displayed within the problem

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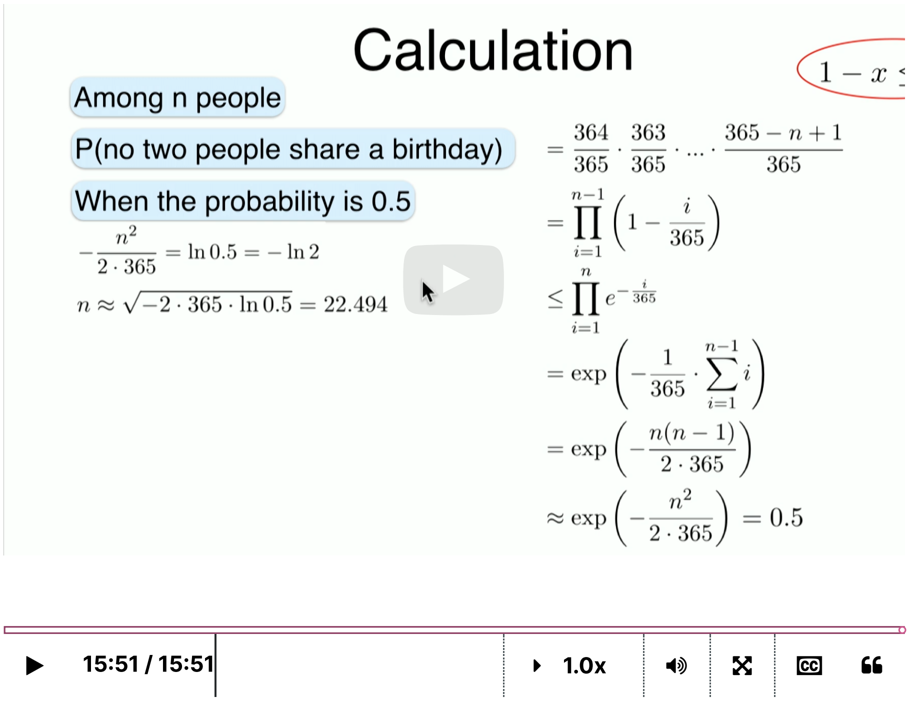

Calculation

Among n people

P(no two people share a birthday)

When the probability is 0.5

$$-\frac{n^2}{2\cdot 365} = \ln 0.5 = -\ln 2$$

$$n \approx \sqrt{-2\cdot 365\cdot \ln 0.5} = 22.494$$

$$1 - x$$

$$\begin{aligned}
&= \frac{364}{365}\cdot\frac{363}{365}\cdot\ldots\cdot\frac{365-n+1}{365}\\
&= \prod_{i=1}^{n-1}\left(1-\frac{i}{365}\right)\\
&\leq \prod_{i=1}^{n} e^{-\frac{i}{365}}\\
&= \exp\left(-\frac{1}{365}\cdot\sum_{i=1}^{n-1} i\right)\\
&= \exp\left(-\frac{n(n-1)}{2\cdot 365}\right)\\
&\approx \exp\left(-\frac{n^2}{2\cdot 365}\right) = 0.5
\end{aligned}$$

you can see how to calculate these probabilities

for any given number of n.

Alright, so in this (mumbles) lecture

we talked about Sequential Probability

and next we will talk about Total Probability.

See you then.

End of transcript. Skip to the start.

6.3_Sequential_Probability

POLL

The equality P(A∩B)=P(A)P(B) holds whenever the events A and B are

RESULTS

● **independent**	**83%**
○ **disjoint**	**14%**
○ **intersecting**	**2%**

Submit

Results gathered from 42 respondents.

FEEDBACK

Independent. In fact, that's the definition of independence.

1

0 points possible (ungraded)

An urn contains b black balls and w white balls. Sequentially remove a random ball from the urn, till none is left.

Which of the following observed color sequences would you think is more likely: first all white balls then all black ones (e.g. wwbbb), or alternating white (first) and black, till one color is exhausted, then the other color till it is exhausted (e.g. wbwbb)?

For $b = 4$ and $w = 2$, calculate the probability of:

white, white, black black, black black,

(2/6)*(1/5) * 1

✓ **Answer:** 0.0666

$\left(\frac{2}{6}\right) \cdot \left(\frac{1}{5}\right) \cdot 1$

white, black, white, black, black, black,

(2/6)*(4/5)*(1/4) * 1

✓ **Answer:** 0.0666

$\left(\frac{2}{6}\right) \cdot \left(\frac{4}{5}\right) \cdot \left(\frac{1}{4}\right) \cdot 1$

Try to understand the observed outcome.

Explanation

By sequential probability, it is easy to see that the for any order of the colors, the denominator will be $(b + w)!$ while the numerator will be $b! \cdot w!$.

This can also be seen by symmetry. Imagine that the balls are colored from 1 to $b + w$. Then each of the $(b + w)!$ permutations of the balls is equally likely to be observed, hence will happen with probability $1/\,(b + w)!$, and $b! \cdot w!$ of them will correspnd to each specified order of the colors.

Submit

You have used 1 of 4 attempts

Answers are displayed within the problem

2 (Graded)

6/6 points (graded)

An urn contains one red and one black ball. Each time, a ball is drawn independently at random from the urn, and then returned to the urn along with another ball of the same color. For example, if the first ball drawn is red, the urn will subsequently contain two red balls and one black ball.

What is the probability of observing the sequence r,b,b,r,r?

1/60

✓ **Answer:** 0.016

$\frac{1}{60}$

Explanation

$P\left(r,b,b,r,r\right) = P\left(r\right)\cdot P\left(b|r\right)\cdot P\left(b|r,b\right)\cdot P\left(r|r,b,b\right)\cdot P\left(r|r,b,b,r\right) = \frac{1}{2}\cdot\frac{1}{3}\cdot\frac{2}{4}\cdot\frac{2}{5}\cdot\frac{3}{6} = \frac{1}{60} = 0.01666..$

.

What is the probability of observing 3 red and 2 black balls?

1/6 ✓ **Answer:** 1/6

$\frac{1}{6}$

What is the probability of observing 7 red and 9 black balls?

1/17 ✓ **Answer:** 1/17

$\frac{1}{17}$

Explanation

It can be verified that for any sequence with n_r red balls and n_b black balls, the probability $p = n_r!\cdot n_b!/\left(n_r+n_b+1\right)!$.
Hence the probability of observing n_r red balls and n_b black balls is $n_r!\cdot n_b!/\left(n_r+n_b+1\right)!\binom{n_r+n_b}{n_b} = \frac{1}{n_r+n_b+1}$.

? Hint (1 of 1): (Part 2) Note that any sequence with 3 red and 2 black balls, e.g. r,r,r,b,b is observed with the same probability.

Next Hint

Submit

You have used 1 of 4 attempts

ⓘ Answers are displayed within the problem

3

0 points possible (ungraded)

A box has seven tennis balls. Five are brand new, and the remaining two had been previously used. Two of the balls are randomly chosen, played with, and then returned to the box. Later, two balls are again randomly chosen from the seven and played with. What is the probability that all four balls picked were brand new.

Submit

You have used 0 of 4 attempts

4

0 points possible (ungraded)

A box contains six tennis balls. Peter picks two of the balls at random, plays with them, and returns them to the box. Next, Paul picks two balls at random from the box (they can be the same or different from Peter's balls), plays with them, and returns them to the box. Finally, Mary picks two balls at random and plays with them. What is the probability that each of the six balls in the box was played with exactly once?

2/75 ✓ **Answer:** 2/75

$\frac{2}{75}$

Explanation

The probability that every ball picked was played with exactly once is the probability that the 2 balls Paul picks differ from the 2 Peter picked, and that the 2 balls Mary picks differ from the 4 Peter or Paul picked. This probability is

$$\frac{\binom{6-2}{2}}{\binom{6}{2}}\cdot\frac{\binom{6-2-2}{2}}{\binom{6}{2}}=\frac{\binom{4}{2}}{\binom{6}{2}}\cdot\frac{\binom{2}{2}}{\binom{6}{2}}=\frac{6}{15}\cdot\frac{1}{15}=\frac{2}{75}.$$

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

5 (Graded)

2/2 points (graded)

A bag contains 4 white and 3 blue balls. Remove a random ball and put it aside. Then remove another random ball from the bag. What is the probability that the second ball is white?

○ 3/6

○ 4/6

○ 3/7

● 4/7

✓

Explanation

This can be done in two simple ways.

First, by symmetry. There are 4 white balls and 3 blue balls. The second ball picked is equally likely to be any of the 7 balls, hence the probability that it is white is 4/7.

Second, by total probability. The probability that the second ball is white is the probability that the first is white and the second is white namely $\frac{4}{7} \cdot \frac{3}{6}$, plus the probability that the first is blue and the second is white, namely $\frac{3}{7} \cdot \frac{4}{6}$, and $\frac{4}{7} \cdot \frac{3}{6} + \frac{3}{7} \cdot \frac{4}{6} = \frac{4}{7}$.
Note that the first, symmetry, argument is easier to extend to the third ball picked etc. But both derivation are of interest, and you may want to use the total-variation for a general case with W white balls and R red balls.

? **Hint (1 of 1):** This problem can be solved using basic symmetry agruments, or using total probability discussed in the next section. Next Hint

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

6

0 points possible (ungraded)
An urn contains 15 white and 20 black balls. The balls are withdrawn randomly, one at a time, until all remaining balls have the same color. Find the probability that:

- all remaining balls are white (if needed, see hints below),

- there are 5 remaining balls.

Submit You have used 0 of 4 attempts

7 Tennis matches

0 points possible (ungraded)
Eight equal-strength players, including Alice and Bob, are randomly split into 4 pairs, and each pair plays a game, resulting in four winners. Find the probability that:

both Alice and Bob will be among the four winners, neither Alice and Bob will be among the four winners.

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Problem Sets due Jul 4, 2022 07:34 +03

Video

which is what we got here, okay?

So, good?

So now that we have seen how to calculate the probability

that the phone is defective,

I think we have discussed total probability,

and next time we'll be ready to discuss base rule.

See you then.

End of transcript. Skip to the start.

6.4_Total_Probability

POLL

60% of our students are American (born), and 40% are foreign (born). 20% of the Americans and 40% of the foreigners speak two languages. What is the probability that a random student speaks two languages?

RESULTS

○	**0.18**	**2%**
●	**0.28**	**89%**
○	**0.34**	**5%**
○	**0.45**	**5%**

Submit

Results gathered from 44 respondents.

FEEDBACK

The probability is 0.6 * 0.2 + 0.4 * 0.4 = 0.28.

1

0 points possible (ungraded)

Three 100-marble bags are placed on a table. One bag has 60 red and 40 blue marbles, one as 75 red and 25 blue marbles, and one has 45 red and 55 blue marbles.

You select one bag at random and then choose a marble at random. What is the probability that the marble is red?

○ 0.2025

○ 0.33

○ 0.50

● 0.60

✓

Answer

Correct: Video: Total Probability

Explanation

The probability is $\frac{1}{3}(0.6+0.75+0.45)=0.60$.

Submit

You have used 2 of 2 attempts

ℹ Answers are displayed within the problem

2 (Graded)

2/2 points (graded)

Each of Alice and Bob has an identical bag containing 6 balls numbered 1, 2, 3, 4, 5, and 6. Alice randomly selects one ball from her bag and places it in Bob’s bag, then Bob randomly select one ball from his bag and places it in Alice’s bag. What is the probability that after this process the content in two bags remain unchanged?

2/7

✓ **Answer:** 2/7

$\frac{2}{7}$

Explanation

The two bags will remain unchanged if the ball Bob picks has the same number as the one Alice placed there. Once Alice puts a ball numbered n in Bob's bag, the probability the Bob picks a ball numbered n is $\frac{2}{7}$. The total probability is $\sum_{n=1}^{6} \frac{1}{6} \cdot \frac{2}{7} = \frac{2}{7}$.

Submit

You have used 1 of 4 attempts

Answers are displayed within the problem

3. Two subsets

0 points possible (ungraded)

Let A and B be two random subsets of $\{1, 2, 3, 4\}$. What is the probability that $A \subseteq B$?

? **Hint (1 of 1):** A random subset of $\{1, 2, 3, 4\}$ is one of the $2^4 = 16$ subsets, selected uniformly at random. An equivalent way of selecting a random subset is to include each of the four elements in the subset independently with probability $1/2$.

Next Hint

Submit

You have used 0 of 4 attempts

---

4 (Graded)

2/2 points (graded)

Eight equal-strength players, including Alice and Bob, are randomly split into 4 pairs, and each pair plays a game (i.e. 4 games in total), resulting in four winners. What is the probability that exactly one of Alice and Bob will be among the four winners?

0.57

✓ **Answer:** 4/7

0.57

Explanation

Here are two ways of solving the problem. One using total probability, the other by symmetry.

Total Probability.

Let E be the desired event that exactly one of Alice or Bob is a winner. We divide the sample space into two disjoint events, E_1, E_2. E_1 is the event that Alice and Bob play against each other and E_2 is the complimentary event that Alice and Bob play against other players. Since Alice is equally likely to play with any of the seven other players, $P\left(E_1\right) = 1/7$, hence $P\left(E_2\right) = 6/7$. Now $P\left(E|E_1\right) = 1$, while $P\left(E|E_2\right) = 1/2$ since Alice and Bob each play an independent game where the probability of winning is $1/2$. Therefore $P\left(E\right) = P\left(E_1\right) \cdot P\left(E|E_1\right) + P\left(E_2\right) \cdot P\left(E|E_2\right) = 1/7 \cdot 1 + 6/7 \cdot 1/2 = 4/7$.

Symmetry.

In the end, 4 of the 8 players will be declared winners. There are $\binom{8}{4}$ such 4-winner "quartets", all equally likely.

The number of "quartets" that contain exactly one of Alice and Bob is $\binom{2}{1} \cdot \binom{6}{3}$.

Hence the probability that this occurs is $\frac{\binom{2}{1} \cdot \binom{6}{3}}{\binom{8}{4}} = \frac{4}{7}$.

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

Discussion

Topic: Topic 6 / Total Probability

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Offensive abbreviation in poll

"60% of our students are American (born), and 40% are foreign (born). " Using "foreign" as an abbreviation for "foreign bor...

2

Bayes' Rule

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Video

to even find the probability

that a defective phone was made in a different

a different iPhone was made in different places.

I think we have exhausted, for now, Bayes' rule,

and next time we're going to talk about random variables.

See you then.

End of transcript. Skip to the start.

6.5_Bayes_Rule

POLL

Monty Hall Problem:

Suppose you're on a game show, and you're given the choice of three doors. Behind one door is a car and behind the others are goats. You pick a door, say door 1. The host knows what is behind each door. He opens another door, say door 3, which has a goat. He then says to you, "Do you want to change your selection to door 2?" Is it to your advantage to switch your choice?

RESULTS

○ **It is better to keep my choice of door 1.**	**12%**
● **It is better to switch to door 2.**	**68%**
○ **There is no difference.**	**20%**

Submit

Results gathered from 41 respondents.

FEEDBACK

It is better to switch.

See the explanation to the Monte Hall problem here.

1

0 points possible (ungraded)

A rare disease occurs randomly in one out of 10,000 people, and a test for the disease is accurate 99% of the time, both for those who have and don't have the disease. You take the test and the result is postive. The chances you actually have the disease are approximately:

○ 10%

○ 1%
✓

● 0.1%

○ 0.01%

✗

Explanation
Let H and $D)$ be the events that you Have and Don't have the disease, respectively, and let S be the event that the result is poSitive.

By the streamlined version of Bayes' Rule, $P(H|S) = \frac{P(H,S)}{P(S)} = \frac{P(H,S)}{P(H,S)+P(D,S)}$.

Now, $P(H,S) = P(H) \cdot P(S|H) = 0.0001 \cdot 0.99 \approx 0.0001$, and
$P(D,S) = P(D) \cdot P(S|D) = 0.9999 \cdot 0.01 \approx 0.01$.

Hence $P(H|S) = \frac{0.0001}{0.0001+0.01} \approx 0.01$.

Submit You have used 2 of 2 attempts

Answers are displayed within the problem

2

0 points possible (ungraded)
A car manufacturer has three factories producing 21%, 35%, and 44% of its cars, respectively. Of these cars, 7%, 6%, and 2%, respectively, are defective. A car is chosen at random from the manufacturer's supply.

- What is the probability that the car is defective?

0.044 ✓ **Answer:** 0.0445

0.044

Explanation
Let F_1, F_2, F_3 be the events that the care is made by the first, second, and third factory, respectively, and let D be the event that the car is defective. By the law of total probability,
$$P(D) = P(F_1) \cdot P(D|F_1) + P(F_2) \cdot P(D|F_2) + P(F_3) \cdot P(D|F_3) = 0.21 \cdot 0.07 + 0.35 \cdot 0.06 + 0.44 \cdot 0.0$$
.

- Given that the car is defective, what is the probability that was produced by the first factory?

0.33 ✓ **Answer:** 0.3303

0.33

Explanation
By Bayes' Rule and using $P(D)$ from above, $P(F_1|D) = \frac{P(F_1) \cdot P(D|F_1)}{P(D)} = \frac{0.21 \cdot 0.07}{0.0445} = 0.3303$.

Submit You have used 3 of 4 attempts

Answers are displayed within the problem

---

3 (Graded)

2/2 points (graded)

A college graduate is applying for a job and has 3 interviews. She passes the first, second, and third interviews with probabilities 0.9, 0.8, and 0.7, respectively. If she fails any interview, she cannot proceed with subsequent interview(s) and will not get the job. If she didn't get the job, what is the probability that she failed the second interview?

0.36

✓ **Answer:** 45/124

0.36

Explanation

Let F, S, and T denote the events that the applicant passed the first, second, and third interviews, respectively. The probability that she failed the second interview given that she didn't get the job is $P\left(\overline{S}|\overline{FST}\right) = P\left(F\overline{S}|\overline{FST}\right) = \frac{P(F\overline{S}\wedge\overline{FST})}{P(\overline{FST})} = \frac{P(F\overline{S})}{P(\overline{FST})} = \frac{0.9\cdot0.2}{1-0.9\cdot0.8\cdot0.7}$, where the first equality follows as the applicant fails the second interview iff she passes the first interview and fails the second.

Submit You have used 1 of 4 attempts

ℹ Answers are displayed within the problem

4

0 points possible (ungraded)

An ectopic pregnancy is twice as likely to develop when a pregnant woman is a smoker than when she is a nonsmoker. If 32% of women of childbearing age are smokers, what fraction of women having ectopic pregnancies are smokers?

Submit You have used 0 of 4 attempts

5 (Graded)

3/3 points (graded)

Each of Alice, Bob, and Chuck shoots at a target once, and hits it independently with probabilities 1/6, 1/4, and 1/3, respectively. If only one shot hit the target, what is the probability that Alice's shot hit the target?

○ 31/72

● 6/31

○ 10/31

○ 15/31

✓

Explanation

Let A, B, and C, be the events that Alice, Bob, and Chuck hit the target, respectively, and let $E = A\overline{BC} \cup \overline{A}B\overline{C} \cup \overline{AB}C$ be the event that only one shot hit the target.

Then $P\left(E\right) = \frac{1}{6}\cdot\frac{3}{4}\cdot\frac{2}{3} + \frac{5}{6}\cdot\frac{1}{4}\cdot\frac{2}{3} + \frac{5}{6}\cdot\frac{3}{4}\cdot\frac{1}{3} = \frac{31}{72}$.

By Bayes' Rule, $P\left(A|E\right) = \frac{P(AE)}{P(E)} = \frac{P(A\overline{BC})}{P(E)} = \frac{6/72}{31/72} = \frac{6}{31}$.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

6

0 points possible (ungraded)

Jack has two coins in his pocket, one fair, and one "rigged" with heads on both sides. Jack randomly picks one of the two coins, flips it, and observes heads. What is the probability that he picked the fair coin?

○ 3/4

○ 2/3

○ 1/3

○ 1/4

Submit You have used 0 of 2 attempts

7

0 points possible (ungraded)

It rains in Seattle one out of three days, and the weather forecast is correct two thirds of the time (for both sunny and rainy days). You take an umbrella if and only if rain is forecasted.

- What is the probability that you are caught in the rain without an umbrella?

- What is the probability that you carry an umbrella and it does not rain?

Submit You have used 0 of 4 attempts

8

0 points possible (ungraded)

On any night, there is a 92% chance that an burglary attempt will trigger the alarm, and a 1% chance of a false alarm, namely that the alarm will go off when there is no burglary. The chance that a house will be burglarized on a given night is $1/1000$. What is the chance of a burglary attempt if you wake up at night to the sound of your alarm?

Submit You have used 0 of 4 attempts

9

0 points possible (ungraded)
An urn labeled "heads" has 5 white and 7 black balls, and an urn labeled "tails" has 3 white and 12 black balls. Flip a fair coin, and randomly select on ball from the "heads" or "tails" urn according to the coin outcome. Suppose a white ball is selected, what is the probability that the coin landed tails?

Submit You have used 0 of 4 attempts

10

0 points possible (ungraded)
A car manufacturer receives its air conditioning units from 3 suppliers. 20% of the unitws come from supplier A, 30% from supplier B, and 50% from supplier C. 10% of the units from supplier A are defective, 8% of units from supplier B are defective, and 5% of units from supplier C are defective. If a unit is selected at random and is found to be defective.

What is the probability that a unit came from supplier A if it is:

defective,

non-defective,

Submit You have used 0 of 4 attempts

11

0 points possible (ungraded)
Suppose that 15% of the population have cancer, 50% of the population smokes, and 75% of those with cancer smoke. What fraction of smokers have cancer?

○ 0.05625

○ 0.225

○ 0.25

○ 0.75

Submit You have used 0 of 2 attempts

12

0 points possible (ungraded)

A fair coin with $P(heads) = 0.5$ and a biased coin with $P(heads) = 0.75$ are placed in an urn. One of the two coins is picked at random and tossed twice. Find the probability:

of observing two heads,

that the biased coin was picked if two heads are observed.

Submit You have used 0 of 4 attempts

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Topic: Topic 6 / Bayes's Rule

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Why switching?

I guess you are more likely to win in this situation if you switch doors, but only because the host knows what is behind them? So swi... 1

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