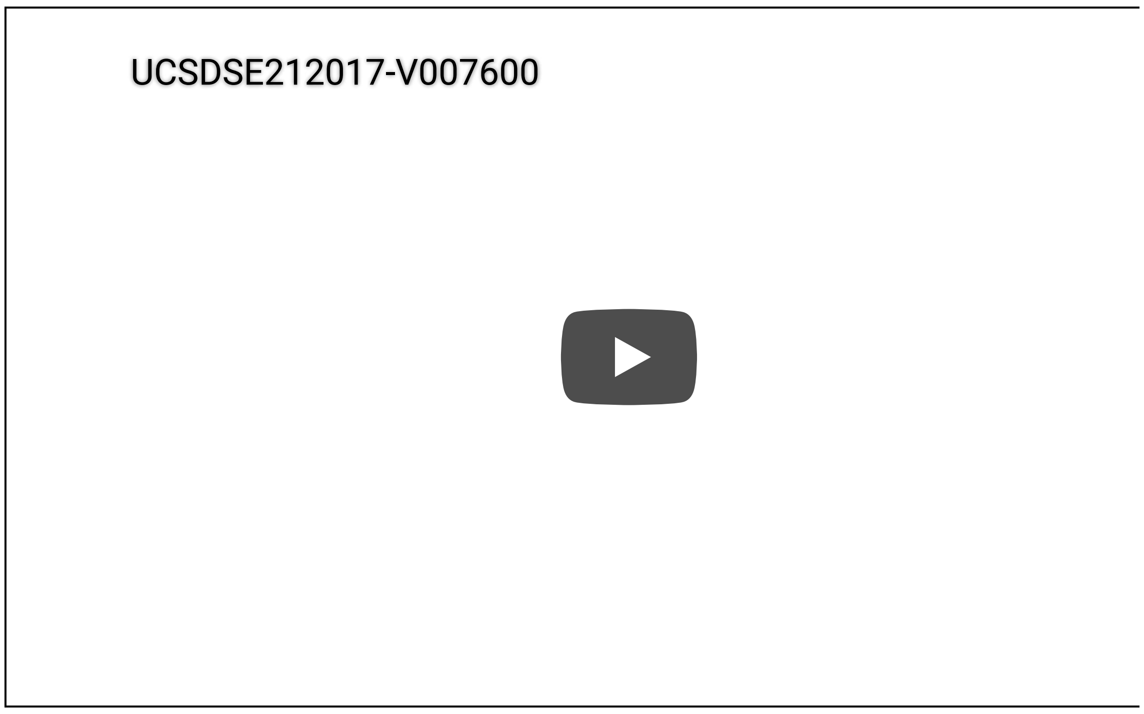
Partial Permutations

Problem Sets due May 4, 2022 18:05 +03 Completed

Video



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Start of transcript. Skip to the end.

- Welcome again everyone.

Last lecture, we talked about permutations

and now we want to talk about partial permutations.

These are permutations where you don't want to

arrange all the objects that you have,

but just some subset.

4.2_Partial_Permutations

POLL

How many 2-permutations do we have for set {1,2,3,4}?

RESULTS

8	0%
12	88%
16	12%

Submit

Results gathered from 24 respondents.

FEEDBACK

The answer is P(4 , 2) = 4 * 3 = 12.

1

0 points possible (ungraded)

In how many ways can 5 cars - a BMW, a Chevy, a Fiat, a Honda, and a Kia - park in 8 parking spots?

56*120 ✔ **Answer:** 6720

$56 \cdot 120$

Explanation

There are 8 locations for the BMW, the 7 for the Chevy, etc, so the total number of ways is $8^{\underline{5}} = 6720$.

? **Hint (1 of 1):** Note that both the order and locations of the cars matter.
So abbreviating the five models by their first letters and denoting an empty parking spot by X, the following three arrangments are considered different:
B C F H K X X X
X X X B C F H K
X X X K H F C B.

Next Hint

Submit You have used 2 of 4 attempts

Answers are displayed within the problem

2

0 points possible (ungraded)
In how many ways can 5 people sit in 8 numbered chairs?

8*7*6*5*4 ✔ **Answer:** 6720

$8 \cdot 7 \cdot 6 \cdot 5 \cdot 4$

Explanation
The first person can sit in any of the 8 chairs, the second in one of the remaining 7, etc. Hence $8^{\underline{5}} = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 = 6720$.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

3 (Graded)

6.0/6.0 points (graded)
Find the number of 7-character (capital letter or digit) license plates possible if no character can repeat and:

- there are no further restrictions,

36*35*34*33*32*31*30 ✔ **Answer:** 42072307200

$36 \cdot 35 \cdot 34 \cdot 33 \cdot 32 \cdot 31 \cdot 30$

Explanation
$36^{\underline{7}} = 42,072,307,200$.

- the first 3 characters are letters and the last 4 are numbers,

26*25*24*10*9*8*7 ✔ **Answer:** 78624000

$26 \cdot 25 \cdot 24 \cdot 10 \cdot 9 \cdot 8 \cdot 7$

Explanation
Choose 3 from capital letters, and 4 from digits, where the order matters. The result is $26^{\underline{3}} \cdot 10^{\underline{4}} = 78,624,000$.

- letters and numbers alternate, for example A3B9D7Q or 0Z3Q4A9.

(26*10*25*9*24*8*23)+(10*2 ✔ **Answer:** 336960000

$(26 \cdot 10 \cdot 25 \cdot 9 \cdot 24 \cdot 8 \cdot 23) + (10 \cdot 26 \cdot 9 \cdot 25 \cdot 8 \cdot 24 \cdot 7)$

Explanation

Explanation

Such plates contain either four letters and three digits, or the other way. The two sets are disjoint. Hence $26^{\underline{3}} \cdot 10^{\underline{4}} + 26^{\underline{4}} \cdot 10^{\underline{3}} = 336,960,000$.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

4 (Graded)

2.0/2.0 points (graded)

A derangement is a permutation of the elements such that none appear in its original position. For example, the only derangements of $\{1,2,3\}$ are $\{2,3,1\}$ and $\{3,1,2\}$. How many derangements does $\{1,2,3,4\}$ have?

9

Answer: 9

9

Explanation

Let F_1 be the set of permutations of $\{1,2,3,4\}$, where 1 is in location 1, for example 1324. Similarly let F_2 be the set of permutations where 2 is in location 2, for example 3214, etc.

Then $F_1 \cup F_2 \cup F_3 \cup F_4$ is the set of all 4-permutations where at least one element remains in its initial location. The set of permutations where no elements appears in its initial location is the complement of this set.

Note that $\sum_i |F_i| = 4^{\underline{3}}$ (1 location is fixed, so 3-permutation), $\sum_i \sum_j |F_i \cap F_j| = 4^{\underline{2}}$, $\sum_i \sum_j \sum_k |F_i \cap F_j \cap F_k| = 4^{\underline{1}}$, and $|F_1 \cap F_2 \cap F_3 \cap F_4| = 4^{\underline{0}}$.

Hence by inclusion exclusion, $|F_1 \cup F_2 \cup F_3 \cup F_4| = 4^{\underline{3}} - 4^{\underline{2}} + 4^{\underline{1}} - 4^{\underline{0}} = 24 - 12 + 4 - 1 = 15$.

It follows that the number of derangements is $4! - 15 = 9$.

Hint (1 of 1): Let F_1 be the set of permutations of $\{1,2,3,4\}$, where 1 is in location 1, for example 1324. Similarly let F_2 be the set of permutations where 2 is in location 2, for example 3214, etc. Use inclusion exclusion to calculate $F_1 \cup F_2 \cup F_3 \cup F_4$. Then observe that the the question asks for the complement of this set.

Next Hint

Submit You have used 2 of 4 attempts

Answers are displayed within the problem

5

0 points possible (ungraded)

Eight books are placed on a shelf. Three of them form a 3-volume series, two form a 2-volume series, and 3 stand on their own. In how many ways can the eight books be arranged so that the books in the 3-volume series are placed together according to their correct order, and so are the books in the 2-volume series? Noted that there is only one correct order for each series.

120

Answer: 120

120

Explanation

Since the 3-volume books must be placed in a unique order, we can view them as a just one "super book", similarly for the 2-volume books. We therefore have a total of 5 books that we can arrange freely, and we can do so in $5! = 120$ ways.

Submit You have used 1 of 4 attempts

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

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