Combinations

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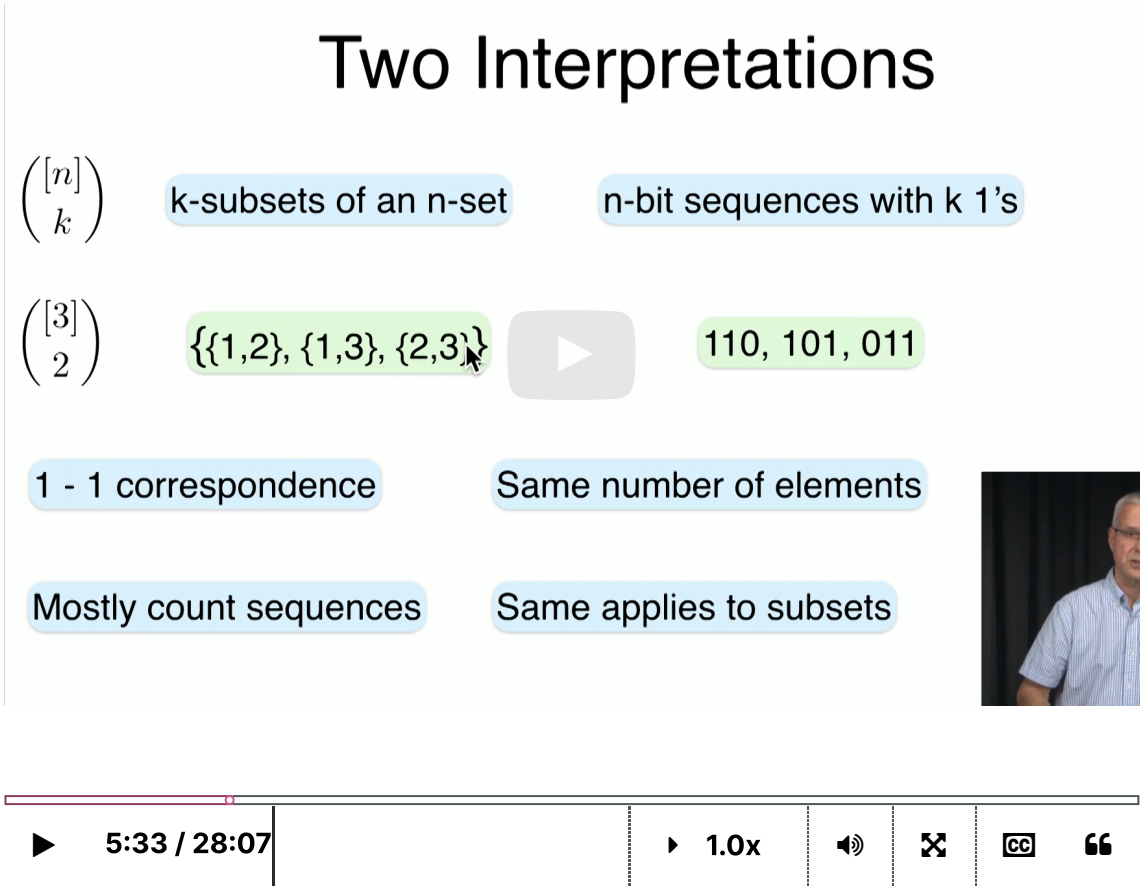
Problem Sets due May 4, 2022 18:05 +03

Video



there are three elements,

therefore the binomial coefficient three choose two

is equal to three.

And if you want to see where this number comes from,

why it has to be three,

then one way to do it is to specify

the locations of the ones.

So we can do that by specifying

ordered pairs from one up to three.

For example, we can specify that we put one in location one

and then a one in location two.

That will correspond to the sequence 110.

Or that we put a one in location one

and then a one in location three,

and that will correspond to the sequence 101.

4.3_Combinations

POLL

Which of the following is larger for k≤n?

- **The number of k-permutations of an n-set**
- **The number of k-subsets of an n-set**

Submit

1

0 points possible (ungraded)

In how many ways can a basketball coach select 5 starting players form a team of 15?

- $\frac{15!}{5!10!}$
- $\frac{15!}{10!}$
- $\frac{15!}{5!}$
- None of the above

Explanation

It can be deducted from partial permutation, but the order does not matter. It is $\binom{15}{5} = \frac{15^5}{5!} = \frac{15!}{5!10!}$.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

2

0 points possible (ungraded)

- In how many ways can you select a group of 2 people out of 5?

- 10
- 25
- 125
- None of the above

Explantion

$\binom{5}{2} = 10$.

- In how many ways can you select a group of 3 people out of 5?

- 10
- 25
- 125
- None of the above

Explantion

$\binom{5}{3} = 10$.

- In how many ways can you divide 5 people into two groups, where the first group has 2 people and the second has 3?

- 10
- 25
- 125
- None of the above

Explantion

After we determine the group of 2, the group of 3 is determined as well, hence the answer is $\binom{5}{2} = 10$.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

3

0 points possible (ungraded)

Ten points are placed on a plane, with no three on the same line. Find the number of:

- lines connecting two of the points,

45

45

- these lines that do not pass through two specific points (say A or B),

- triangles formed by three of the points,

- these triangles that contain a given point (say point A),

- these triangles contain the side AB.

Submit You have used 0 of 4 attempts

4

0 points possible (ungraded)

The set $\{1, 2, 3\}$ contains 6 nonempty intervals: $\{1\}$, $\{2\}$, $\{3\}$, $\{1, 2\}$, $\{2, 3\}$, and $\{1, 2, 3\}$.

How many nonempty intervals does $\{1, 2, \ldots, 10\}$ contain?

Submit You have used 1 of 4 attempts

5

0 points possible (ungraded)

A rectangle in an $m \times n$ chessboard is a cartesian product $S \times T$, where S and T are nonempty intervals in $\{1, \ldots, m\}$ and $\{1, 2, \ldots, n\}$ respectively. How many rectangles does the 3×6 chessboard have?

Submit You have used 0 of 4 attempts

6 (Graded)

8/8 points (graded)

A standard 52-card deck consists of 4 suits and 13 ranks. Find the number of 5-card hands where:

- any hand is allowed (namely the number of different hands),

2598960 ✔ **Answer:** 2598960

2598960

Explanation

This is simply $\binom{52}{5}$.

- all five cards are of same suit,

4*1287 ✔ **Answer:** 5148

$4 \cdot 1287$

Explanation

There are 4 suits in total and 13 cards in each suit, hence $4 \cdot \binom{13}{5}$ hands.

- all four suits are present,

685464 ✔ **Answer:** 685464

685464

Explanation

One of the 4 suits will appear twice, hence $4 \cdot \binom{13}{2} \cdot 13^3$ hands.

- all cards are of distinct ranks.

1317888 ✔ **Answer:** 1317888

1317888

Explanation

First pick 5 out of 13 ranks, then choose their suits. Therefore there are $\binom{13}{5} \cdot 4^5$ hands.

? **Hint (1 of 1):** For example, for hands where all cards are of the same suit, count the number of hands with 5 clubs, or with 5 diamonds, etc. Next Hint

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

7 (Graded)

2/2 points (graded)

A company employs 4 men and 3 women. How many teams of three employees have at most one woman?

- 21
- 22
- 23
- 24

✔

Explanation

There are $\binom{4}{3} = 4$ teams with 0 women and $\binom{3}{1} \times \binom{4}{2} = 3 \times 6 = 18$ teams with 1 woman, for a total of 22.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

8 (Graded)

5/5 points (graded)

A (tiny) library has 5 history texts, 3 sociology texts, 6 anthropology texts and 4 psychology texts. Find the number of ways a student can choose:

- one of the texts,

18 ✔ **Answer:** 18

18

Explanation

- two of the texts,

153 ✔ **Answer:** 153

153

Explanation

- one history text and one other type of text,

65 ✔ **Answer:** 65

65

Explanation

The student can choose 5 different history texts, and $3 + 6 + 4 = 13$ other texts, by the product rule there are $5 \cdot 13 = 65$ ways of doing that.

- one of each type of text,

360 ✔ **Answer:** 360

360

Explanation

The student selects one text of each type, by the product rule this can be done in $5 \cdot 3 \cdot 6 \cdot 4 = 360$ ways.

- two of the texts with different types.

119 ✔ **Answer:** 119

119

Explanation
There are $5 \cdot 3 = 15$ ways to choose one history and one sociology text, $5 \cdot 6 = 30$ ways to choose one history and one anthropology text, etc. In total there are $5 \cdot 3 + 5 \cdot 6 + 5 \cdot 4 + 3 \cdot 6 + 3 \cdot 4 + 6 \cdot 4 = 119$ ways.

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

9

0 points possible (ungraded)
In how many ways can 7 distinct red balls and 5 distinct blue balls be placed in a row such that

- all red balls are adjacent,

- all blue balls are adjacent,

- no two blue balls are adjacent.

Submit You have used 0 of 4 attempts

10

0 points possible (ungraded)
For the set $\{1, 2, 3, 4, 5, 6, 7\}$ find the number of:

- subsets,

- 3-subsets,

- 3-subsets containing the number 1,

- 3-subsets not containing the number 1.

Submit You have used 0 of 4 attempts

11 Functions.

0 points possible (ungraded)

A function $f : X \to Y$ is *injective* or *one-to-one* if different elements in X map to different elements in Y, namely,

$$\forall x \neq x' \in X, \quad f(x) \neq f(x').$$

A function $f : X \to Y$ is *surjective* or *onto* if all elements in Y are images of at least one element of X, namely,

$$\forall y \in Y \quad \exists x \in X, \quad f(x) = y.$$

For sets $A = \{1, 2, 3\}$ and $B = \{a, b, c, d\}$, find the number of

- functions from A to B,
- functions from B to A,
- one-to-one functions from A to B,
- onto functions from B to A.

Submit You have used 0 of 4 attempts

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