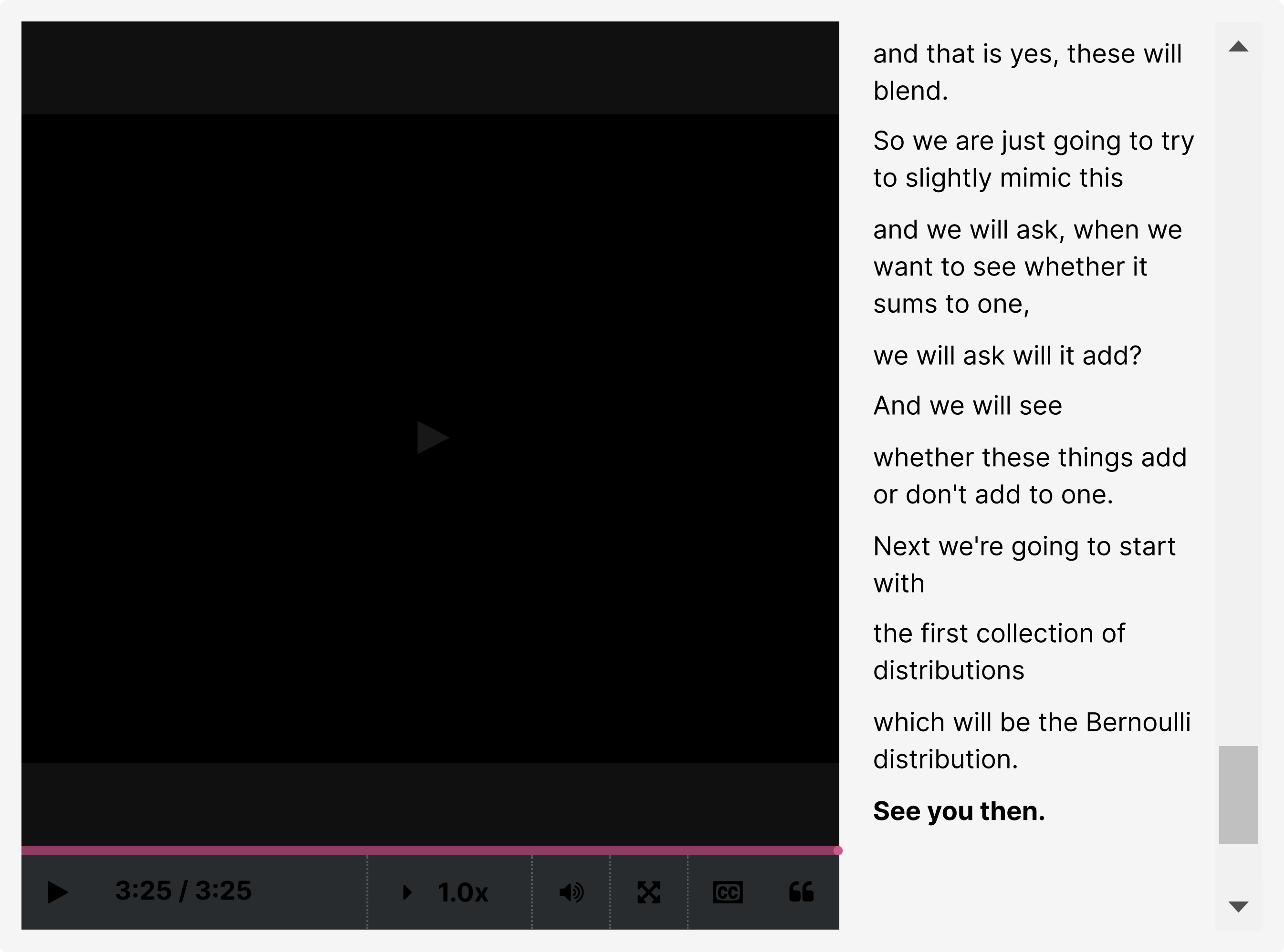
Problem Sets due Jul 13, 2022 01:34 +03

Video



and that is yes, these will blend.

So we are just going to try to slightly mimic this

and we will ask, when we want to see whether it sums to one,

we will ask will it add?

And we will see

whether these things add or don't add to one.

Next we're going to start with

the first collection of distributions

which will be the Bernoulli distribution.

See you then.

8.0_Distributions_Introduction

---

1 (Graded)

2/2 points (graded)

For which value of the parameter α is the function $f\,(x) = \frac{2(10-x)+\alpha}{100}$ over $\{1, 2, \cdots, 10\}$ a p.m.f.?

○ -1

○ 0

● 1

○ 2

✓

Explanation

Following $\sum_{x=1}^{10} f\left(x\right)=1$, we have $\alpha=1$.

Submit You have used 2 of 2 attempts

Answers are displayed within the problem

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Bernoulli

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Video

Start of transcript. Skip to the end.

- Hello and welcome back.

So in this lecture, we're going to start talking

about the first of the families

of distribution that we're going to present,

and we'll talk about the Bernoulli Distributions.

And what we're going to show is we're going

8.1_Bernoulli

POLL

Every random variable distributed over {0, 1} is Bernoulli.

RESULTS

●	**Yes**	**63%**
○	**Not necessarily**	**37%**

Submit

Results gathered from 49 respondents.

FEEDBACK

Every random variable over {0,1} attains the value 1 with some probability (p) and 0 with the remaining probability (1-p), hence is (B_p). So the answer is Yes.

---

1 (Graded)

1/1 point (graded)

$X \sim B_p$ with $p > 0.5$ and $V(X) = 0.24$. Find

- p,

0.6 ✓ **Answer:** 0.6

0.6

Explanation

For a Bernoulli distribution, $E(X^2) = E(X) = p$. Thus $V(X) = E(X^2) - E(X)^2 = p - p^2 = p(1-p)$. Since $0.24 = V(X) = p(1-p)$ and $p \geq 0.5$, we must have $p = 0.6$.

- $E[X]$.

0.6 ✓ **Answer:** 0.6

0.6

Explanation

$E(X) = p = 0.6$.

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

2

0 points possible (ungraded)

Which of the following hold for two Bernoulli variables?

☐ Independent implies uncorrelated, ✓

☑ Uncorrelated implies independent. ✓

✗

Explanation

- True. It is trivial.

- True. Let $X \sim B_{p_x}, Y \sim B_{p_y}$.

If X and Y are uncorrelated, $\text{Cov}(X,Y) = E(XY) - E(X)E(Y)$.

$$\begin{aligned} &= \textstyle\sum_{x=0}^{1}\sum_{y=0}^{1} xyP(X=x,Y=y) - p_x p_y \\ &= P(X=1,Y=1) - p_x p_y \\ &= P(X=1|Y=1)P(Y=1) - p_x p_y \\ &= (P(X=1|Y=1) - p_x)p_y \\ &= 0 \end{aligned}$$

Hence, $P(X=1|Y=1) = p_x = P(X=1)$ and similarly $P(Y=1|X=1) = p_y = P(Y=1)$. From that, we have

$P(X=0|Y=1) = \frac{P(Y=1|X=0)P(X=0)}{P(Y=1)} = 1 - p_x = P(X=0) \Rightarrow P(Y=1|X=0) = p_y = P(Y=1)$, and similarly $P(X=1|Y=0) = p_x = P(X=1)$. Thus, X and Y are independent.

? Hint (1 of 1): One part may require some thought. Next Hint

Submit You have used 2 of 2 attempts

Answers are displayed within the problem

3 (Graded)

1/1 point (graded)

Consider ten independent $B_{0.3}$ trials. Which of the following is the most probable?

● 0000000000

○ 1111111111

○ 1111111111

○ 1110000000

○ 0001111111

✓

Explanation

Under $B_{0.3}$, the probability of sequence with w ones and $n-w$ zeros is $0.3^w \cdot 0.7^{(n-w)} = 0.7^n \cdot (3/7)^w$, which decreases with w.

Hence 0000000000 is the most likely sequence with probability 0.7^{10}, while 1111111111 is least likely with probability 0.3^{10}. This is also logical as under $B_{0.3}$, every bit is more likely to be a 0 than a 1.

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

4

0 points possible (ungraded)

Consider ten independent $B_{0.3}$ trials. Which of the following is the most probable?

Try to reconcile with the previous question.

○ 10 zeros

○ 10 ones

● 3 ones and 7 zeros

○ 3 zeros and 7 ones.

✓

Explanation

First, inutitively, for $B_{0.3}$ we expect to see roughly 30% 1's.

Slightly more rigorously, while individually, a sequence with 10 zeros is the most likely among all sequences, there is only one such sequence. When you balance the probability of each sequence with the number of such sequence, you see that observing a sequence with 3 ones and 7 zeros is most likely.

We will do this calculation formally when we study binomial distributions in the next section.

Submit You have used 3 of 4 attempts

Answers are displayed within the problem

5 Bernoulli modifications

0 points possible (ungraded)

Let $X \sim B_{0.2}$. Find the Bernoulli parameter for the following random variables. Write -1 if they are not Bernoulli.

- X^2,

0.2 ✓ **Answer:** 0.2

0.2

Explanation

Since $X \in \{0, 1\}$, we have $X^2 = X$.

- $+\sqrt{X}$,

0.2

✓ **Answer:** 0.2

0.2

Explanation
Since $X \in \{0, 1\}$, we have $+\sqrt{X} = X$.

- $1 - X$,

0.8

✓ **Answer:** 0.8

0.8

Explanation
$1 - X$ takes values in $\{0, 1\}$, hence is Bernoulli, and $1 - X = 1$ iff $X = 0$, which happens with probability 0.8.

- $-X$.

-1

✓ **Answer:** -1

-1

Explanation
$-X$ takes values in $\{0, -1\}$, hence is not Bernoulli.

Submit

You have used 1 of 4 attempts

Answers are displayed within the problem

6 Bernoulli pairs

0 points possible (ungraded)
Let $X \sim B_{0.4}$, $Y \sim B_{0.2}$, and they are independent. Find the Bernoulli parameter for the following random variables. Write -1 if they are not Bernoulli.

- $X \cdot Y$,

0.08

✓ **Answer:** 0.08

0.08

Explanation
$X \cdot Y$ takes values in $\{0, 1\}$, hence is Bernoulli. It is 1 iff $X = Y = 1$ which happens with probability $0.4 \cdot 0.2 = 0.08$.

- X^Y, recall that $0^0 = 1$,

0.88

✓ **Answer:** 0.88

0.88

Explanation
X^Y takes values in $\{0, 1\}$, hence is Bernoulli. It is 0 iff $X = 0$ and $Y = 1$, which happens with probability $0.6 \cdot 0.2 = 0.12$, hence it is 1 with probability 0.88.

- $|X - Y|$,

0.44 ✓ **Answer:** 0.44

0.44

Explanation
$|X - Y|$ takes values in $\{0, 1\}$, hence is Bernoulli. It is 1 iff $X \neq Y$, which happens with probability $0.6 \cdot 0.2 + 0.4 \cdot 0.8 = 0.44$.

- $X + Y$.

-1 ✓ **Answer:** -1

-1

Explanation
$X + Y$ takes values in $\{0, 1, 2\}$, hence is not Bernoulli.

Submit You have used 1 of 4 attempts

ℹ Answers are displayed within the problem

7 Bernoulli sum

0 points possible (ungraded)
$X = U + V$, where U and V are independent Bernoulli variables with different expectations but the same variance 0.21. Find:

- $E(X)$,

- $V(X)$,

- σ_X.

Submit You have used 0 of 4 attempts

8 (Graded)

3/3 points (graded)
Let X be the number of heads when flipping four coins with heads probabilities 0.3, 0.4, 0.7, and 0.8. Find:

- $P(X = 1)$,

0.187 ✓ **Answer:** 0.1872

0.187

Explanation
$P(X=1) = 0.3 \cdot 0.6 \cdot 0.3 \cdot 0.2 + 0.7 \cdot 0.4 \cdot 0.3 \cdot 0.2 + 0.7 \cdot 0.6 \cdot 0.7 \cdot 0.2 + 0.7 \cdot 0.6 \cdot 0.3 \cdot 0.8 = 0.1872$
.

- $E(X)$,

2.2 ✓ **Answer:** 2.2

2.2

Explanation
$E(X) = 0.3 + 0.4 + 0.7 + 0.8 = 2.2$.

- $V(X)$.

0.82 ✓ **Answer:** 0.82

0.82

Explanation
$V(X) = 0.21 + 0.24 + 0.21 + 0.16 = 0.82$.

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

9 Light bulbs

0 points possible (ungraded)
Every light bulb is defective with 2% probability. What is the probabity that a package of 8 bulbs will not suffice for a project requiring 7?

0.0103 ✓ **Answer:** 0.0103

0.0103

Explanation
Let $X \sim B(0.02, 8)$ be the number of defective bulbs in a package.
The box will not suffice if there are 2 or more defective bulbs, which happens with probability.
$P(X > 1) = 1 - P(X=0) - P(X=1) = 1 - \binom{8}{0} \cdot (1-0.02)^8 - \binom{8}{1} \cdot (1-0.02)^7 \cdot 0.02 = 0.0103.$

Submit You have used 3 of 4 attempts

Answers are displayed within the problem

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? Question 3 and 4
Both are asking for the most probable, I do not understand the difference. How are these questions different? Could someone pleas... 1

Binomial Distribution

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Video

Start of transcript. Skip to the end.

- Hello, and welcome back.

In the last lecture,

we talked about the Bernoulli distribution,

and we said that it forms the foundation

of many other families of distribution,

and in this lecture we're going to start with the first one,

8.2_Binomial_Distribution

POLL

If you flip a fair coin 10 times and let X be the total number of heads, then V(X) is

○ **1.5**

○ **2.5**

○ **3.5**

○ **None of the above**

Submit

---

1 (Graded)

3/3 points (graded)

There are 5 traffic signals between your home and work. Each is red with probability 0.35, independently of all others. Find:

a) the probability of encountering no red lights,

○ 2.26%

○ 5.2%

● 11.6%

○ 17.5%

✓

Answer

Correct: Video: Binomial Distribution

Correct: Video: Binomial Distribution

Explanation
$(1 - 0.35)^5 = 0.116$

b) the probability of encountaring only red lights,

○ 0.03%

● 0.52%

○ 1.16%

○ 16.4%

✓

Answer
Correct: Video: Binomial Distribution

Explanation
$0.35^5 = 0.0052$

c) the expected number of red lights you will encounter?

○ 0.75

○ 1.42

● 1.75

○ 2.25

✓

Answer
Correct: Video: Binomial Distribution

Explanation
The expection of the sum is the sum of the expections. $0.35 + 0.35 + 0.35 + 0.35 + 0.35 = 1.75$.

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

2 (Graded)

2/2 points (graded)
If every student is independently late with probability 10%, find the probability that in a class of 30 students:

a) nobody is late,

● 4.2%

○ 8.0%

○ 17.4%

○ 33.3%

✓

Answer
Correct: Video: Binomial Distribution

Explanation
$(1-0.1)^{30} = 0.042$

b) exactly 1 student is late.

○ 3.33%

○ 5.25%

○ 7.75%

● 14.1%

✓

Answer
Correct: Video: Binomial Distribution

Explanation
$(1-0.1)^{29} \times 0.1 \times \binom{30}{1} = 0.141$

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

3

0 points possible (ungraded)
A coin with heads probability 0.6 is tossed 6 times, calculate the probability of observing:

- exactly two heads,

- at most one tails,

- an even number of heads.

Submit You have used 0 of 4 attempts

4 (Graded)

3/3 points (graded)
A Binomial distribution $B_{p,n}$, where $p \neq 0$, has the same mean and standard deviation, namely $\mu = \sigma$.

Find the mean of $B_{p,n+1}$.

1 ✓ **Answer:** 1

1

Explanation
Since $B_{p,n}$ has $\sigma = \mu$, we have $np = \sqrt{npq}$, hence $1 - p = q = np$, or $p \cdot (n+1) = 1$.

? Hint (1 of 1): Express the mean and variance of $B_{p,n}$ in terms of n and p. Next Hint

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

5

0 points possible (ungraded)
For $X \sim B_{0.7,10}$, find:

- $E(X)$,

- $V(X)$,

- σ_X,

- The most likely outcome of X.

Submit You have used 0 of 4 attempts

---

6. Balls in urns

0 points possible (ungraded)
Ten balls are randomly dropped into four urns. Let X be the number of balls dropped into one preselected urn. Find:

- $P(X = 0)$,

0.056 ✓ **Answer:** 0.056

0.056

Explanation

Cleraly X is distributed $B_{1/4,10}$. Hence $P(X=0)=\binom{10}{0}\cdot(1/4)^0\cdot(3/4)^{10}=(3/4)^{10}=0.056$.

- $P(X=1)$,

0.187 ✓ **Answer:** 0.188

0.187

Explanation

$P(X=1)=\binom{10}{1}\cdot(1/4)^1\cdot(3/4)^9=0.188$.

- $E(X)$,

2.5 ✓ **Answer:** 5/2

2.5

Explanation

$E(X)=np=10\cdot\frac{1}{4}=\frac{5}{2}$.

- $V(X)$.

1.87 ✓ **Answer:** 15/8

1.87

Explanation

$V(X)=np(1-p)=10\cdot\frac{1}{4}\frac{3}{4}=\frac{30}{16}=\frac{15}{8}$.

? **Hint (1 of 2):** Why is this question in this particular section? Next Hint

Hint (2 of 2): X is distributed binomially.

Submit You have used 2 of 4 attempts

Answers are displayed within the problem

7

0 points possible (ungraded)

Our TA owns four Porsches, each works 80% of the time, and two Ferraris, each works 60% of the time. What is the probability that on a given day, at least half of the Porsches and at least half of the Ferraris work?

0.817 ✓ **Answer:** 0.817152

0.817

Explanation

Let Q be the number of Porshes that work and F be the number of Ferraris that work.
$P(Q\geq 2)=1-P(Q=0)-P(Q=1)=1-\binom{4}{0}0.2^4-\binom{4}{1}0.8^1\cdot 0.2^3=0.9728$. Similarly
$P(F\geq 1)=1-P(F=0)=1-\binom{2}{0}0.4^2=0.84$. Therefore the required probability is
$P(Q\geq 2)\cdot P(F\geq 1)=0.817152$.

Submit You have used 2 of 4 attempts

Answers are displayed within the problem

9

0 points possible (ungraded)

Alice solves every puzzle with probability 0.6, and Bob, with probability 0.5. They are given 7 puzzle and each chooses 5 out of the 7 puzzles randomly and solves them independently. A puzzle is considered solved if at least one of them solves it. What is the probability that all the 7 puzzles are solved?

0.02 ✓ **Answer:** 0.021

0.02

Explanation

The probability that all the 7 puzzles are chosen is the probability that Bob chooses the two puzzles Alice did not pick, namely, $\frac{\binom{5}{3}\binom{2}{2}}{\binom{7}{5}} = \frac{10}{21}$. Every puzzle they both attempt, they both fail with probability $0.4 \cdot 0.5 = 0.2$, hence at least one solves with probability $1 - 0.2 = 0.8$. It follows that all puzzles are solved with probability $\frac{10}{21} \cdot 0.6^2 \cdot 0.5^2 \cdot 0.8^3 = 0.0219$.

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

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Poisson

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Video

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- Hello and welcome back.

In the last lecture we discussed

the binomial distribution which as you remember

was an extension of the Bernoulli distribution

and now we want to list of course

an extension of the binomial distribution

which is called the Poisson

8.3_Poisson_Distribution

POLL

The mean and the variance of a Poisson distribution is the same.

RESULTS

●	**True**	**100%**
○	**False**	**0%**

Submit

Results gathered from 45 respondents.

FEEDBACK

It's true.

1

0 points possible (ungraded)

Assume a telemarketer's successful sales per hour is a Poisson random variable with $\lambda = 2$. What is the probability that the telemarketer makes no sales in 1 hour?

● 13.5%

○ 22.5%

○ 27.7%

○ 31.2%

✓

Answer
Correct: Video: Poisson Distribution

Explanation
$P\left(X=k\right)=\frac{\lambda^k}{k!}e^{-\lambda}$. With $\lambda=2$, $P\left(X=0\right)=e^{-2}=0.135$

Submit You have used 1 of 2 attempts

🛈 Answers are displayed within the problem

2

0 points possible (ungraded)
The expectation of a Poisson random variable and its variance are

○ equal
✓

● not equal

✗

Explanation
$E\left(X\right)=V\left(X\right)=\lambda.$

Submit You have used 1 of 1 attempt

🛈 Answers are displayed within the problem

---

3 (Graded)

2.0/2.0 points (graded)
Random variable X is distributed Poisson, and $P\left(X=2\right)=P\left(X=4\right)$. Find $P\left(X=3\right)$.

0.2169 ✓ **Answer:** 0.2169

0.2169

Explanation
$P\left(X=2\right)=P\left(X=4\right)$ implies $\lambda=2\sqrt{3}$. Hence $P\left(X=3\right)=4\sqrt{3}\cdot e^{-2\sqrt{3}}$.

Submit You have used 1 of 4 attempts

🛈 Answers are displayed within the problem

4

0 points possible (ungraded)
Let X be distributed Poisson with parameter 1. Find $P\left(X\geq 2\mid X\leq 4\right)$.

Submit You have used 0 of 4 attempts

5

0 points possible (ungraded)
Assume the number of typo errors on a single page of a book follows Poisson distribution with parameter 1/3. Calculate the probability that on one page there are

- no typo,

- exactly two typos,

- more than one typo?

Submit You have used 0 of 4 attempts

6

0 points possible (ungraded)
If a random variable X follows Poisson distribution with $\lambda = 2.5$, calculate

- $E[X]$

- $E[X^2]$

- $V(X)$

Submit You have used 0 of 4 attempts

7

0 points possible (ungraded)
Assume the number of tropical storms making landfall in the Philippines each year follows Poisson distribution with parameter 9. What is the probability that there are less than 6 tropical storms making landfall in Philippines in one year?

Submit You have used 0 of 3 attempts

8 (Graded)

3/3 points (graded)

A computer manufacturing company produce chips with defect probability 0.001. In a package of 2000 chips, denote the number of defective chips by X. Use Poisson distribution for approximation:

- The Poisson parameter for X is:

2

✓ **Answer:** 2

2

Explanation

Poisson approximation yields $\lambda = np = 2000 \cdot 0.001 = 2$.

- $P(X > 1) =?$

0.5940

✓ **Answer:** 0.5940

0.5940

Explanation

Evaluate Poission probability of $P(X > 1)$ with $\lambda = 2$.

- $P(X \leq 3) =?$

0.85

✓ **Answer:** 0.8571

0.85

Explanation

Evaluate Poission probability of $P(X \leq 3)$ with $\lambda = 2$.

Submit You have used 1 of 4 attempts

ℹ Answers are displayed within the problem

9

0 points possible (ungraded)

A vendor sells merchandise through Amazon and Ebay. On Ebay she sells an average rate of 2 items per day, while on Amzaon the daily average is 3. Both sales follow a Poisson distribution and are independent of each other. What is the probability that she sells 5 items on a given day?

Submit You have used 0 of 4 attempts

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Geometric

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Video

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- Hello, and welcome back.

In the previous lecture,

we talked about the Poisson distribution,

which was one form of derivative of binomial

and therefore Bernoulli distribution.

Now we're going to talk about yet another derivative

of Bernoulli distributions

8.4_Geometric_Distribution

POLL

Which of the following distributions is memoryless?

RESULTS

○	**Poisson**	**7%**
●	**Geometric**	**88%**
○	**Both**	**5%**
○	**Neither**	**0%**

Submit

Results gathered from 43 respondents.

FEEDBACK

Only the geometric distribution.

Several of the following questions ask about the number of experiments performed till a certain outcome is observed. Unless otherwise stated, include the final experiment (where the outcome is observed) in the count. For example, the number of coin tosses till observing a heads in the sequence t, t, h, is 3.

1

0 points possible (ungraded)

A die is rolled until the number 1 turns up. The expected number of rolls is

○ 2,

○ 4,

○ 6,

○ 8.

Submit You have used 0 of 2 attempts

2

0 points possible (ungraded)

A pair of dice are repeatedly rolled till the two sum to ≥ 10. For example (6,3), (2,4), (5,5), stopping after three pair rolls. The expected number of times the pair is rolled is:

○ 2,

○ 4,

○ 6,

○ 8.

Submit You have used 0 of 2 attempts

---

3 (Graded)

3.0/3.0 points (graded)

A G_p random variable is odd with probability

○ $\frac{1-p}{2-p}$,

○ $\frac{p}{2-p}$,

● $\frac{1}{2-p}$,

○ $p + (1-p)^2 \cdot p$.

✓

Explanation

There are two natural ways to find the probability that $X \sim G_p$ is odd.

The first is "brute force".

Recall that $1 + q + q^2 + \ldots = \frac{1}{1-q}$.

Hence,

$P\left(X \text{ is odd}\right) = P\left(X = 1\right) + P\left(X = 3\right) + \ldots = p + \bar{p}^2 \cdot p + \bar{p}^4 \cdot p + \ldots = \frac{p}{1-\bar{p}^2} = \frac{p}{1-(1-p)^2} = \frac{p}{2p-p^2} = \frac{1}{2-p}$

.

The second method is by relating $\left(P\left(X \text{ is even}\right)\right.$ to $P\left(X \text{ is odd}\right)$.

$P\left(X \text{ is even}\right) = P\left(X \text{ is even} \cap X > 1\right) = P\left(X > 1\right) \cdot P\left(X \text{ is even} | X > 1\right) = P\left(X > 1\right) \cdot P\left(X \text{ is odd}\right)$

.

X is even or odd, hence $1 = P\left(X \text{ is odd}\right) + (1-p) \cdot P\left(X \text{ is odd}\right) = (2-p) \cdot P\left(X \text{ is odd}\right)$.

Hence $P\left(X \text{ is odd}\right) = \frac{1}{2-p}$.

? **Hint (1 of 2):** There are two natural ways to find the probability that $X \sim G_p$ is odd. You may want to try both.
One is "brute force", by adding the probabilities that $X = 1, 3, 5, \ldots$.
The other is by relating $P(X \text{ is even})$ to $P(X \text{ is odd})$.

Hint (2 of 2): For the brute-force way, recall that $1 + q + q^2 + \ldots = \frac{1}{1-q}$.

If you use the second method, first show that
$P(X \text{ is even}) = (1-p) \cdot P(X \text{ is odd})$.

Next Hint

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

4

0 points possible (ungraded)

Find the expected number of coin tosses till the third heads appears, (e.g., for h, t, h, t, h, five coins were tossed).

? **Hint (1 of 1):** You may want to let X_i, for $1 \le i \le 3$, be the number of tosses between the $i-1$th and ith heads.
For example, for t,h,t,t,h,h, then $X_1 = 2$, $X_2 = 3$, and $X_3 = 1$.

Next Hint

Submit You have used 0 of 4 attempts

5

0 points possible (ungraded)

X is the random number of times a coin with heads probability $1/4$ is tossed till the first heads appears, find:

- $E(X)$,
- $E(X^2)$,
- $V(X)$,
- σ_X,

- $P(X \leq 10)$,

- $P(X > 5)$.

Submit You have used 0 of 4 attempts

---

6 (Graded)

9.0/9.0 points (graded)

Two coins with heads probabilities $1/3$ and $1/4$ are alternately tossed, starting with the $1/3$ coin, until one of them turns up heads. Let X denote the total number of tosses, including the last. Find:

- $P(X = 5)$,

0.08 ✓ **Answer:** 1/12

0.08

Explanation

$P(X = 5) = \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{1}{3} = \frac{1}{12}$.

- $P(X \text{ is odd})$,

2/3 ✓ **Answer:** 2/3

$\frac{2}{3}$

Explanation

Similar to Problem 3, this can be done in two ways. Brute force or relating two probabilities.

For the brute force, $P(X \text{ is odd}) = P(X = 1) + P(X = 3) + \ldots = \frac{1}{3} + \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{1}{3} + \ldots$
$= \frac{1}{3} + \frac{1}{2} \cdot \frac{1}{3} + \left(\frac{1}{2}\right)^2 \cdot \frac{1}{3} + \ldots = \frac{1}{3} \cdot \left(1 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \ldots\right) = \frac{1}{3} \cdot \frac{1}{1 - \frac{1}{2}} = \frac{2}{3}$.

Alternatively,

$P(X \text{ is odd}) = P(X = 1) + P(X \text{ is odd} \cap X \geq 3) = P(X = 1) + P(X \geq 3) \cdot P(X \text{ is odd} | X \geq 3)$
$= P(X = 1) + P(X \geq 3) \cdot P(X \text{ is odd}) = \frac{1}{3} + \frac{2}{3} \cdot \frac{3}{4} \cdot P(X \text{ is odd}) = \frac{1}{3} + \frac{1}{2} \cdot P(X \text{ is odd})$.

Hence $\frac{1}{2} \cdot P(X \text{ is odd}) = \frac{1}{3}$, or $P(X \text{ is odd}) = \frac{2}{3}$,

- $E(X)$.

10/3 ✓ **Answer:** 10/3

$\frac{10}{3}$

Explanation

$E(X) = \frac{1}{3} \cdot 1 + \frac{2}{3} \cdot \frac{1}{4} \cdot 2 + \sum_{i=3}^{\infty} i \cdot P(X = i)$.

$= \frac{1}{3} \cdot 1 + \frac{2}{3} \cdot \frac{1}{4} \cdot 2 + \sum_{i=1}^{\infty} (i + 2) \cdot P(X = i + 2, X > 2)$

$= \frac{1}{3} \cdot 1 + \frac{2}{3} \cdot \frac{1}{4} \cdot 2 + \sum_{i=1}^{\infty} (i + 2) \cdot P(X = i + 2 | X > 2) \cdot P(X > 2)$

$= \frac{1}{3}\cdot 1+\frac{2}{3}\cdot\frac{1}{4}\cdot 2+\frac{2}{3}\cdot\frac{3}{4}\cdot\sum_{i=1}^{\infty}(i+2)\cdot P(X=i)$
$= \frac{1}{3}\cdot 1+\frac{2}{3}\cdot\frac{1}{4}\cdot 2+\frac{2}{3}\cdot\frac{3}{4}\cdot(E(X)+2)$

Hence $E(X)\cdot(1-\frac{1}{2})=\frac{1}{3}+\frac{1}{3}+1=\frac{5}{3}$.
And therefore $E(X)=\frac{10}{3}$.

? **Hint (1 of 2):** The first part is straight-forward.
As in problem 3, the second and third parts can be done via a brute force, or using a more clever calculation (The geometric distribution is memoryless).

Next Hint

Hint (2 of 2): For the second part, relate $P(X \text{ is odd})$ to $P(X \text{ is odd}|X\geq 3)$. Similarly for the third part.

Submit

You have used 3 of 4 attempts

Answers are displayed within the problem

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Missing argument in the "Will it add ? " proof
At minute 8:06 it might be clearer to add, (1+q+q^2+q^3+ ... + q^(n-1))*(1-q) = 1 -q^n which tends to 1 when n tends to infinity iff |q|...
1

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Geometric Distribution Examples

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Problem Sets due Jul 13, 2022 01:34 +03 Completed

Video

Start of transcript. Skip to the end.

- Hello, and welcome back.

In the last lecture, we talked about

geometric distributions, and now we would like

to discuss a couple of example related to them.

So let's start with some fake statistics about startups.

Let's assume that a startup succeeds

8.5_Geometric_Examples

POLL

If X and Y are two independent geometric random variables, then X+Y also is also geometric.

RESULTS

○	**True**	**68%**
●	**Not true**	**32%**

Submit

Results gathered from 41 respondents.

FEEDBACK

False. For example, if (X) and (Y) have the same success probability, (X+Y) will follow a negative-binomial distribution.

1

0 points possible (ungraded)

In a basketball shooting workout, a player keeps shooting until she makes 10 baskets. Suppose the probability that she makes any given shot is 0.7, and let X be the total number of shots she takes. Calculate:

- $E[X]$,

100/7 ✓ **Answer:** 100/7

$\frac{100}{7}$

Explanation

Let X_i be the random variable indicating the number of shots between the $(i-1)^{th}$ and i^{th} shots

$(i \in \{1, \ldots, 10\})$. Then, the total number of shots, $T = \sum_{i=1} X_i$. Using the fact that here each of the random variables $X_i \sim \text{Geometric}(0.7)$, and that for a geometric distribution with parameter p, $E(X_i) = 1/p\, E(X_i) = 1/0.7 = 10/7$. Further, by linearity of expecatation $E(T) = \sum_{i=1}^{10} E(X_i) = 100/7$.

- $V(X)$.

300/49 ✓ **Answer:** 300/49

$\frac{300}{49}$

Explanation

Using that $X_i \sim \text{Geometric}(0.7)$, and that for a geometric distribution with parameter p, $V(X_i) = (1-p)/p^2\, V(X_i) = 0.3/0.49 = 30/49$. Here, each of the X_i, $(i \in \{1, \ldots, 10\})$, are also independent. Thus $V(T) = \sum_{i=1}^{10} V(X_i) = 300/49$.

Submit You have used 2 of 4 attempts

Answers are displayed within the problem

2 (Graded)

2.0/2.0 points (graded)

A production line has a 5% defective rate, and its products are inspected one-by-one until the first defect is found. Given that the first 10 inspections do not find any defect, what is the probability that the number of inspections is at most 20?

0.401 ✓ **Answer:** 1-0.95^10

0.401

Explanation

Let D be the event of interest here. Further let E denote the event that any 10 consecutive inspections find a defect. Since the inspections here are independent, the required probability $P(D) = P(E)$. Now if $\bar{E}$ denotes the compliment of event E, $P(\bar{E}) = 1 - P(E) = (1-0.05)^{10}$ since in $\bar{E}$ we require that no defective item be discovered in each of the 10 inspections. Thus $P(D) = P(E) = 1 - 0.95^{10}$.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

3 (Graded)

2.0/2.0 points (graded)

A bag contains K blue balls and $N-K$ red balls. Find the expected number of blue balls observed when n balls are randomly drawn.

- ● $n\frac{K}{N}$
- ○ $(n-1)\frac{K}{N}$
- ○ $(n-1)\frac{K-1}{N-1}$
- ○ $(n)\frac{K-1}{N-1}$

✓

Explanation

Without replacement, the expecatation is

$$\sum_{k=0}^{n} k\frac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}} = \frac{K}{\binom{N}{n}}\sum_{k=1}^{n}\binom{K-1}{k-1}\binom{N-K}{n-k} = \frac{K}{\binom{N}{n}}\underbrace{\sum_{k=0}^{n-1}\binom{K-1}{k}\binom{N-K}{n-1-k}}_{\substack{\#\text{ of ways}\\ \text{to choose } n-1 \text{ balls}\\ \text{out of } N-1 \text{ balls}}} = \frac{K}{\binom{N}{n}}\binom{N-1}{n-1} = n\frac{K}{N}.$$

With replacement, the expecatation is trivial, which is $n\frac{K}{N}$.

Hence, the answer does not depend on whether the selection is with or without replacement.

? **Hint (1 of 1):** Does the answer depend on whether the selection is with or without replacement?

Next Hint

Submit You have used 1 of 2 attempts

ⓘ Answers are displayed within the problem

4

0 points possible (ungraded)

A bag contains 6 blue balls and 9 red balls, if 5 balls are randomly picked from the bag with replacement, what is the most likely number of blue balls that will be picked?

2 ✓ **Answer:** 2

2

Explanation

Intuitively, it is most likely to get 2 blue balls and 3 red balls.

Let X be the number of blue balls. $P(X=k)=\frac{\binom{6}{k}\binom{9}{5-k}}{\binom{15}{5}}$, and we can show that it reaches its maximum when $k=2$.

Submit You have used 1 of 4 attempts

ⓘ Answers are displayed within the problem

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? confused between p(success with uncle if dad's fund failed) and p(success with uncle) 1

I am confused and wondering as to what's so much different between these questions: p(success with uncle if dad's fund failed) : U

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