Games of Chance

Problem Sets due Jun 1, 2022 17:13 +03

Video

So to summarize, we talked about the basics of roulette.

We calculated some simple probabilities.

We saw how much you expect to win

if you play a lot of times.

And what are we going to do next?

We're going to talk about dominoes.

See you then.

12:53 / 12:53 1.0x

End of transcript. Skip to the start.

5.5_Games_of_Chance_Roulette

Unless otherwise stated, in this and subsequent sections, a *die* refers to a fair six-sided die, and a *deck of cards* refers to a standard 52-card deck with four suits (Clubs, Diamonds, Hearts, and Spades) and thirteen ranks (2,..., 10, jack, Queen, King, and Ace).

POLL

What is the probability that two cards drawn from a standard deck without replacement have the same rank?

RESULTS

1/13	**5%**
1/17	**49%**
2/52	**14%**
None of the above	**32%**

Submit

Results gathered from 37 respondents.

FEEDBACK

There are 13 different ranks.

The number of ways to two cards from a specific rank is (4 choose 2).

Hence the probability is 13 * (4 choose 2) / (52 choose 2) = 3 / 51 = 1/17

1

0 points possible (ungraded)

What is the probability that a random four-card hand consists of a single suit?

What is the probability that a random four-card hand consists of a single suit?

- $\frac{4}{52}$
- $\frac{13}{52}$
- $\binom{13}{4}/\binom{52}{4}$
- $\binom{4}{1} \cdot \binom{13}{4}/\binom{52}{4}$

Explanation

There are $\binom{4}{1}$ ways to choose the suit (e.g. hearts) and $\binom{13}{4}$ ways to draw 4 cards from this suit.
Yet the total number of ways to draw 4 cards is $\binom{52}{4}$.
Hence, the probability is $4\binom{13}{4}/\binom{52}{4}$.

Submit You have used 1 of 2 attempts

Answers are displayed within the problem

2

0 points possible (ungraded)

Find the probability that a five-card hand contains:

- the ace of diamonds,

0.0962 **Answer:** 0.09615

0.0962

Explanation

The number of hands containing the ace of diamonds is $\binom{51}{4}$, corresponding to the choice of the remaining 4 cards from the other 51. Hence the probability is $\binom{51}{4}/\binom{52}{5} = 5/52$.

- at least an ace,

0.34 **Answer:** 0.3412

0.34

Explanation

The number of ways to draw 5 cards without any ace is $\binom{48}{5}$. By the complement rule, the answer is $1 - \binom{48}{5}/\binom{52}{5} = 0.3412$.

- at least a diamond.

0.77 **Answer:** 0.7785

0.77

Explanation

The number of ways to draw 5 cards without any diamond is $\binom{39}{5}$. $1 - \binom{39}{5}/\binom{52}{5} = 0.7785$.

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

3

0 points possible (ungraded)

Five cards are dealt from a poker deck. What is the probability of:

- three-of-a-kind (three cards of one rank and two cards of two other ranks),

0.0211 ✔ **Answer:** 0.0211

0.0211

Explanation

We deal with the 3 cards of the same rank first, and then the 2 remaining cards with different ranks.
The are 13 ranks. The number of ways to get 3 cards of a particular rank, (e.g. ace) is $\binom{4}{3} = 4$. In total the number of ways to get 3 cards of the same rank is $13 \cdot 4$.
The remaining 2 cards cannot have the same rank as the one we choose the first step, so there are 12 cards left. Since 4 suits can be chosen for each card, the number of ways in total is $\binom{12}{2} \cdot 4^2$.
The answer is $13 \cdot 4 \cdot \binom{12}{2} \cdot 4^2 / \binom{52}{5} = 0.0211$.

- two pairs (two pairs of same-rank cards),

0.0475 ✔ **Answer:** 0.0475

0.0475

Explanation

We first deal with the rank of the 2 pairs, and then the one left over.
There are $\binom{13}{2}$ ways to choose 2 ranks out of 13. The number of ways get 2 cards of a probaility of a particular rank, (e.g. ace), is $\binom{4}{2}$. We do the same for both the pair, so the total number of ways is $\binom{13}{2} \cdot \binom{4}{2}^2$,
For the one left over, there are 11 ranks left that can be chosen, and 4 suit can be chosen for each rank. The number of ways is $11 \cdot 4$.
The answer is $\binom{13}{2} \cdot \binom{4}{2}^2 \cdot 11 \cdot 4 / \binom{52}{5} = 0.0475$.

- one pair (a pair of same-rank cards, and the other three cards of three different ranks).

0.4226 ✔ **Answer:** 0.4226

0.4226

Explanation

We first deal with the rank of the pair, and then the three left over.
There are $\binom{13}{1} = 13$ ways to choose a ranks out of 13. The number of ways get 2 cards of a probaility of a particular rank, (e.g. ace), is $\binom{4}{2}$. The total number of ways is $13 \cdot \binom{4}{2}$,
For the one left over, there are 12 ranks left that can be chosen, and 4 suit can be chosen for each rank. The number of ways is $\binom{12}{3} \cdot 4^3$.
The answer is $13 \cdot \binom{4}{2} \cdot \binom{12}{3} \cdot 4^3 / \binom{52}{5} = 0.4226$.

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

4

0 points possible (ungraded)

A 52-card deck is randomly split into four 13-card hands. Find the probability that:

- each hand has an ace,

0.1

✔ **Answer:** 0.1055

0.1

Explanation
There are $4!$ ways to assign 4 aces to 4 hands. There are $\binom{48}{12,12,12,12}$ ways to assign the remaining 48 cards equally to 4 hands (12 for each). The answer is $4!\binom{48}{12,12,12,12}/\binom{52}{13,13,13,13} = 0.1055$.

- one hand has all four aces.

0.003

✘ **Answer:** 0.0106

0.003

Explanation
There are 4 ways to assign all 4 aces to one hand. There are $\binom{48}{9,13,13,13}$ ways to assign the remaining 48 cards to 4 hands (9 for the one which gets 4 aces, and 13 for the others). The answer is
$4\binom{48}{9,13,13,13}/\binom{52}{13,13,13,13} = 0.0106$.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

5 (Graded)

6/6 points (graded)
Assume that in blackjack, an ace is always worth 11, all face cards (Jack, Queen, King) are worth 10, and all number cards are woth the number they show. Given a shuffled deck of 52 cards:

- What is the probability that you draw 2 cards and they sum 21?

0.0483

✔ **Answer:** 0.0483

0.0483

Explanation
The possible combinations that sum to 21 are (A, 10), (A, J), (A, Q), (A, K). The number of them is $16 \cdot 4$. The answer is $16 \cdot 4/\binom{52}{2} = 0.0483$.

- What is the probability that you draw 2 cards and they sum 10?

0.0407

✔ **Answer:** 0.0407

0.0407

Explanation
The possible combinations that sum to 10 are (2, 8), (3, 7), (4, 6), (5, 5). The number of them is $3 \cdot 4^2 + \binom{4}{2}$. The answer is $\left(3 \cdot 4 \cdot 4 + \binom{4}{2}\right)/\binom{52}{2} = 0.0407$.

- Suppose you have drawn two cards: 10 of clubs and 4 of hearts. You now draw a third card from the remaining 50. What is the probability that the sum of all three cards is strictly larger than 21?

0.54

✔ **Answer:** 0.54

0.54

Explanation
To exceed 21, the third card belongs to { A, 8, 9, 10, J, Q, K }. As one 10 was drawn, $4 \cdot 7 - 1 = 27$ choices are left. The answer is $27/50 = 0.54$.

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

6

0 points possible (ungraded)
Three dice are rolled. What is the probability that the three outcomes

- contain at least a `1', e.g., 5,1,2,

- are all distinct, e.g., 3,2,5,

- in the order rolled, form an increasing consecutive sequence, e.g., 2,3,4.

- can be arranged to form a consecutive sequence, e.g., 3,2,4 that can form 2,3,4?

Submit You have used 0 of 4 attempts

7 (Graded)

4/4 points (graded)
An instructor assigns 10 problems and says that the final exam will consist of a random selection of 5 of them. If a student knows how to solve 7 of the problems, what is the probability that he or she will answer correctly

- all 5 problems,

0.0833 **Answer:** 0.083333

0.0833

Explanation
The student answers all 5 correctly in the event that all 5 questions appear from the 7 questions that he/show knows to solve. Thus the probability is $\binom{7}{5}/\binom{10}{5} = 0.083333$.

- at least 4 problems?

0.5 **Answer:** 0.50

0.5

Explanation

The student answers at least 4 correctly at least 4 questions appear from the 7 questions that he/show knows to solve. Thus the probability is $\binom{7}{4} \times \binom{3}{1} / \binom{10}{5} + \binom{7}{5} / \binom{10}{5} = 0.50$.

Submit You have used 1 of 4 attempts

Answers are displayed within the problem

8

0 points possible (ungraded)

Let X be the number of draws from a deck, without replacement, till an ace is observed. For example for draws Q, 2, A, $X = 3$. Find:

- $P(X = 10)$,

- $P(X = 50)$,

- $P(X < 10)$?

Submit You have used 0 of 4 attempts

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