Independence

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Problem Sets due Jun 6, 2022 02:13 +03

Video

and seeing whether the probability change or not

and that's our measure of when

probabilities are dependent or independent.

With that, we're going to finish this slide

and we talked about independence

and next time we'll talk about base rule.

▶ 10:36 / 10:36 ▸ 1.0x

End of transcript. Skip to the start.

6.2_Independence

POLL

Two disjoint events cannot be independent.

RESULTS

● **Yes**	**32%**
○ **Not exactly**	**68%**

Submit

Results gathered from 40 respondents.

FEEDBACK

Not exactly.
If the two disjoint events have positive probability, they are dependent.
But if one of the two events has zero probability, they are independent .

1

0 points possible (ungraded)

Two dice are rolled. The event that the first die is 1 and the event that two dice sum up to be 7 are

● Independent

○ Dependent

✓

Explanation

Let X be the outcome of the first die and Y be the outcome of the second die.
$P(X=1|X+Y=7)=\frac{1}{6}=P(X=1)$. Hence, they are independent.

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

2

0 points possible (ungraded)

Of 10 students, 4 take only history, 3 take only math, and 3 take both history and math. If you select a student at random, the event that the student takes history and the event that the student takes math are:

○ Independent

● Dependent

✓

Explanation

Let H be the event that the student takes history, and M the event that the student takes math. Then $P\left(H\right) = \frac{7}{10}$, $P\left(M\right) = \frac{6}{10}$, and $P\left(H,M\right) = \frac{3}{10}$. Since $P\left(H\right)P\left(M\right) \neq P\left(H,M\right)$, the two events are dependent.

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

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3 (Graded)

2/2 points (graded)

4 freshman boys, 6 freshman girls, and 6 sophomore boys go on a trip. How many sophomore girls must join them if a student's gender and class are to be independent when a student is selected at random?

9 ✓ **Answer:** 9

9

Explanation

First, let's do it the formal but hard way. Let SG denote the number of sophomore girls. Then the total number of students is $4 + 6 + 6 + SG = 16 + SG$.

If a student is selected at randdom, the probabilty that the student is a freshman is $\frac{4+6}{16+SG}$, the probability that a random student is a boy is $\frac{4+6}{16+SG}$, and the probability that the student is both a freshman and boy is $\frac{4}{16+SG}$. If the student's gender and class are independent, then by the product rule, the probability of the intersection is the profuct of the probabilities, hence

$\frac{4}{16+SG} = \frac{4+6}{16+SG} \cdot \frac{4+6}{16+SG}$, hence $100 = 4 \cdot (16 + SG)$, or $SG = 9$.

Another way to see this is to observe that if the gender and class are independent, then the fraction of girls that are freshmen, namely $\frac{6}{6+SG}$ should be the same as the fraction of boys that are freshmen, namely $\frac{4}{4+6} = \frac{2}{5}$. Therefore $\frac{6}{6+SG} = \frac{2}{5}$, or $SG = 9$.

Submit You have used 2 of 4 attempts

Answers are displayed within the problem

4

0 points possible (ungraded)
Every event A is independent of:

☑ $\emptyset$,

☑ Ω,

☐ A itself,

☐ A^c.

✓

Explanation
Intuitively:
A is indpendent of the null event because occurance of A doesn't change the 0 probability of the null event.
Similarly A is independent of Ω because occurance of A does not change the probability 1 of Ω.
If A has probability strictly between 0 and 1, then its occurance changes the probability of both itself and A^c, implying dependence.
Mathematically:
- True. $P(\emptyset|A) = 0 = P(\emptyset)$.
- True. $P(A|\Omega) = \frac{P(A\cap\Omega)}{P(\Omega)} = \frac{P(A)}{P(\Omega)} = P(A)$.
- False.
- False.

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

5

0 points possible (ungraded)
Which of the following ensure that events A and B are independent:

☑ A and B^c are independent,
✓

☑ $A \cap B = \emptyset$,

☑ $A \subseteq B$,

☐ at least one of A or B is $\emptyset$ or Ω?
✓

✗

Explanation
- True. If A and B^c are independent, $1 - P(B|A) = P(B^c|A) = P(B^c) = 1 - P(B)$, which implies $P(B|A) = P(B)$.
- False.
- False.
- True. For $\emptyset$, $P(\emptyset|A) = 0 = P(\emptyset)$. For Ω, $P(A|\Omega) = \frac{P(A\cap\Omega)}{P(\Omega)} = \frac{P(A)}{P(\Omega)} = P(A)$. $\emptyset$ and Ω are indepedent with any sets.

Submit You have used 4 of 4 attempts

ℹ Answers are displayed within the problem

6 (Graded)

2/2 points (graded)

When rolling two dice, which of the following events are independent of the event that the first die is 4:

☑ the second is 2,

☐ the sum is 6,

☑ the sum is 7,

☑ the sum is even.

✓

Explanation

Let X be the outcome of the first dice, and Y be the second one.

- True. $P(X=4|Y=2)=P(X=4)=\frac{1}{6}$.
- False. $P(X+Y=6|X=4)=\frac{1}{6}$. $P(X+Y=6)=\frac{5}{36}$. $P(X+Y=6)\neq P(X+Y=6|Y=4)$.
- True. $P(X+Y=6|X=4)=\frac{1}{6}=P(X+Y=7)$.
- True. $P(X+Y\text{ is even }|X=4)=P(Y\text{ is even })=\frac{1}{2}=P(X+Y\text{ is even })$.

Submit You have used 1 of 4 attempts

ℹ Answers are displayed within the problem

7

0 points possible (ungraded)

Roll two dice, and let F_e be the event that the first die is even, S_4 the event that the second die is 4, and Σ_o the event that the sum of the two dice is odd. Which of the following events are independent:

☑ F_e and S_4, ✓

☑ F_e and Σ_o, ✓

☐ S_4 and Σ_o, ✓

☐ F_e, S_4, and Σ_o (mutually independent)?

✗

Explanation

- True. $P(F_e,S_4)=\frac{1}{12}, P(F_e)=\frac{1}{2}, P(S_4)=\frac{1}{6}$. As $P(F_e,S_4)=P(F_e)P(S_4)$, F_e and S_4 are independent.
- True. $P(F_e,\Sigma_o)=\frac{1}{4}, P(F_e)=\frac{1}{2}, P(\Sigma_o)=\frac{1}{2}$. As $P(F_e,\Sigma_o)=P(F_e)P(\Sigma_o)$, F_e and Σ_o are independent.
- True. $P(S_4,\Sigma_o)=\frac{1}{12}, P(S_4)=\frac{1}{6}, P(\Sigma_o)=\frac{1}{2}$. As $P(S_4,\Sigma_o)=P(S_4)P(\Sigma_o)$, S_4 and Σ_o are independent.
- False. $P(F_e,S_4,\Sigma_o)=0\neq P(F_e)P(S_4)P(\Sigma_o)$.

Submit You have used 4 of 4 attempts

Submit You have used 4 of 4 attempts

Answers are displayed within the problem

8

0 points possible (ungraded)
Two dice are rolled. Let F_3 be the event that the first die is 3, S_4 the event that the second die is 4, and Σ_7 the event that the sum is 7. Which of the following are independent:

☑ F_3 and S_4,

☑ F_3 and Σ_7,

☑ S_4 and Σ_7,

☐ F_3, S_4, and Σ_7 (mutually independent)?

✓

Explanation

- True. $P(F_3, S_4) = \frac{1}{36}, P(F_3) = \frac{1}{6}, P(S_4) = \frac{1}{6}$. As $P(F_3, S_4) = P(F_3)\, P(S_4)$, F_3 and S_4 are independent.
- True. $P(F_3, \Sigma_7) = \frac{1}{36}, P(F_3) = \frac{1}{6}, P(\Sigma_7) = \frac{1}{6}$. As $P(F_3, \Sigma_7) = P(F_3)\, P(\Sigma_7)$, F_3 and Σ_7 are independent.
- True. $P(S_4, \Sigma_7) = \frac{1}{36}, P(S_4) = \frac{1}{6}, P(\Sigma_7) = \frac{1}{6}$. As $P(S_4, \Sigma_7) = P(S_4)\, P(\Sigma_7)$, S_4 and Σ_7 are independent.
- False. $P(F_3, S_4, \Sigma_7) = \frac{1}{36} \neq P(F_3)\, P(S_4)\, P(\Sigma_7) = \frac{1}{6}\frac{1}{6}\frac{1}{6} = \frac{1}{216}$.

Submit You have used 2 of 4 attempts

Answers are displayed within the problem

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