

Problem Sets due Jul 17, 2022 10:34 +03

Video



[Start of transcript. Skip to the end.](#)

- Hello again everyone.

Today we're going to talk about continuous distributions.

And when we move from discrete to continuous distribution

we'll observe that when we have discrete distributions

we're considering countable

At 18:00, for the expectation of triangle distribution, the term inside the integral is x^2x , not x .

You can also refer to the slides, which is correct.

9.1 Continuous Distributions

POLL

Which of the following is true about a continuous random variable on \mathbb{R} ?

RESULTS

- ☐ Its pdf must integrate to 1 on \mathbb{R} 69%
- ☒ Its cdf must integrate to 1 on \mathbb{R} 31%
- ☐ None of the above 0%

Submit

Results gathered from 42 respondents.

FEEDBACK

Its pdf must integrate to 1 on \mathbb{R} .

1 (Graded)

1/1 point (graded)

F is the cumulative distribution function for a continuous random variable. If

$F(b) - F(a) = 0.20$, then

- ☐ $[a, b]$ has length 0.20
- ☐ $P(X = b) - P(X = a) = 20\%$
- ☒ $P(X \in (a, b]) = 20\%$



Answer

Correct: Video: Continuous Distributions

Explanation

Recall that $F(b) = P(X \leq b)$, $F(a) = P(X \leq a)$. Hence

$P(a < X \leq b) = F(b) - F(a) = 0.2$.

Submit

You have used 2 of 2 attempts

2 (Graded)

2/2 points (graded)

Which of the following holds for all continuous probability distribution function $f(x)$ having support set \mathbb{R} ?

☒ $\forall x \in \mathbb{R}, \quad f(x) \geq 0$

☐ $\forall x \in \mathbb{R}, \quad f(x) \leq 1$

☒ $\exists x \in \mathbb{R}, \quad f(x) \leq 1$

☒ If the limits of $f(x)$ at positive and negative infinity exist, then
 $\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow -\infty} f(x) = 0$



Explanation

1. By definition, $f(x) \geq 0$.
2. Consider Gaussian $\mathcal{N}(0, 1/(8\pi))$. For this probability density function, $f(0) = 2 > 1$.
3. If $f > 1, \forall x \in \mathbb{R}, \int_{\mathbb{R}} f(z) dz = \infty$, but we require $\int_{\mathbb{R}} f(z) dz = 1$.
4. Suppose $\exists \epsilon, x_0 > 0$ such that $\forall x \geq x_0, f(x) > \epsilon$, then $\int_{\mathbb{R}} f(z) dz = \infty$. Thus there cannot exist such an $\epsilon, x_0 > 0$ and hence $\lim_{x \rightarrow \infty} f(x) = 0$. Similarly $\lim_{x \rightarrow -\infty} f(x) = 0$.

Submit

You have used 1 of 3 attempts

i Answers are displayed within the problem

3 - Power Law

0 points possible (ungraded)

Let X be a random variable with pdf $f_X(x) = Cx^{-\alpha}, x \geq 1$.

If $\alpha = 2$,

$C = ?$

If $\alpha = 3$,

$C = ?$

$E(X) = ?$

Submit

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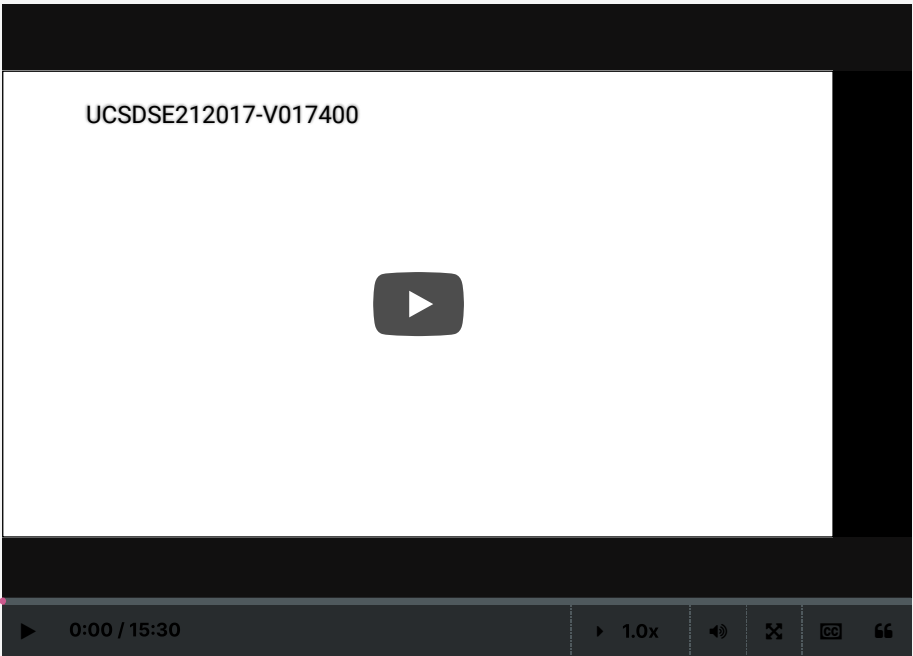
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? Is it just me that at this stage of the course I've started feeling a bit lost?

I feel that the first couple of chapters i.e. 1 to 5 were progressing more smoothly. Now I feel that I am h...

1

Video



[Start of transcript. Skip to the end.](#)

- Hello and welcome back.

In the last lecture, we talked about Random Variables,

and now we would like to start modifying them.

We'll talk about Functions of Random Variables, okay.

So just like we noticed,

observed for discrete random variables,

it's very useful to talk about

9.2 Functions of Random Variables

POLL

Let X be a continuous random variable. What type of function g will make the random variable $g(X)$ discrete?

RESULTS

<input type="radio"/>	increasing	12%
<input type="radio"/>	decreasing	7%
<input checked="" type="radio"/>	linear	10%
<input type="radio"/>	step	71%

Submit

Results gathered from 42 respondents.

FEEDBACK

A step function will make $g(X)$ discrete, as it will take only the y -values that correspond to the steps.

1 (Graded)

3/3 points (graded)

Let (X, Y) be distributed over $[0, 1] \times [0, 1]$ according to $f(x, y) = 6xy^2$. Find $P(XY^3 \leq 1/2)$.

3/4

✓ Answer: 0.75

$\frac{3}{4}$

Explanation

Let $Z = XY^3$.

For any $z \in (0, 1)$, $Z = XY^3 \leq z$ iff $Y \leq \min\{(z/X)^{1/3}, 1\}$. Therefore

$P(Z \leq z) = P(XY^3 \leq z) = \int_0^z \int_0^1 f(x, y) dy dx + \int_z^1 \int_0^{(z/x)^{1/3}} f(x, y) dy dx = \int_0^z \int_0^1 6xy^2 dy dx + \int_z^1 \int_0^{(z/x)^{1/3}} 6xy^2 dy dx = z^2 + \frac{1}{2}z^3$. Plugging in $z = 1/2$ gives the answer.

? **Hint (1 of 2):** Let $Z = XY^3$.

Next Hint

Hint (2 of 2): The cdf of Z is $F_Z(z) = P(Z \leq z) = P(XY^3 \leq z) = \int \int_{xy^3 \leq z} f(x, y) dx dy$.

Submit

You have used 1 of 4 attempts

Answers are displayed within the problem

2 (Graded)

4/4 points (graded)

A random variable X follows the distribution

$$f_X(x) = \begin{cases} Cx^2 & -1 \leq x \leq 2, \\ 0 & \text{otherwise,} \end{cases}$$

and $Y = X^2$. Calculate

• C

1/3

✓ Answer: 1/3

$\frac{1}{3}$

Explanation

Since $1 = \int_{-1}^2 f_X(x) dx = \int_{-1}^2 Cx^2 dx = 3C$, we must have $C = 1/3$.

• $P(X \geq 0)$

8/9

✓ Answer: 8/9

$\frac{8}{9}$

Explanation

$P(X \geq 0) = \int_0^2 f_X(x) dx = \int_0^2 \frac{1}{3}x^2 dx = \frac{1}{9} \cdot x^3 \Big|_0^2 = \frac{8}{9}$.

• $E[Y]$

33/15

✓ Answer: 11/5

$\frac{33}{15}$

Explanation

$$E(Y) = E(X^2) = \int_{-1}^2 x^2 f_X(x) dx = \int_{-1}^2 \frac{1}{3} \cdot x^4 dx = 33/15 = 11/5.$$

• $V(Y)$

✓ Answer: 228/175

1.3

Explanation

$$\text{First, } E(Y^2) = E(X^4) = \int_{-1}^2 x^4 f_X(x) dx = \int_{-1}^2 \frac{1}{3} \cdot x^6 dx = \frac{129}{21}.$$

$$\text{Hence } V(Y) = E(Y^2) - E(Y)^2 = \frac{129}{21} - \left(\frac{11}{5}\right)^2 = \frac{228}{175}.$$

Submit

You have used 1 of 4 attempts

❗ Answers are displayed within the problem

3

0 points possible (ungraded)

Let X be distributed according to $f(x) = ce^{-2x}$ over $x > 0$. Find $P(X > 2)$.

Submit

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Uniform Distribution

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Video

CDF

$$F(x) = \int_{-\infty}^x f(u)du = \begin{cases} \int_{-\infty}^x 0 \, du = 0 & x \leq a \\ \cancel{F(a)} + \int_a^x \frac{1}{b-a} du = \frac{x-a}{b-a} & a \leq x \leq b \\ 1 & x \geq b \end{cases}$$

$F(a) = 0$ $F(b) = 1$

0:00 / 0:00

1.0x

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[Start of transcript. Skip to the end.](#)

- Hello again everyone.

Today we'll start talking about continuous distributions, and the first distribution that we'll discuss will be the uniform distribution.

And to motivate a little bit, to explain it,

9.3 Uniform Distribution

POLL

Let X be a uniformly distributed continuous random variable, then which of the following is also uniform?

RESULTS

<input type="radio"/>	2X	8%
<input checked="" type="radio"/>	X+2	10%
<input type="radio"/>	Both	78%
<input type="radio"/>	Neither	5%

Submit

Results gathered from 40 respondents.

FEEDBACK

Both 2X and X+2 are also uniform.

1

0 points possible (ungraded)

The height of the probability density function of a uniformly distributed random variable is inversely proportional to the width of the interval it is distributed over.

☒ True

☐ False



Explanation

Recall that $\int_{-\infty}^{\infty} f_X(x) dx = 1$, which means the area under the pdf is one. For uniform distribution, it becomes

the area of a rectangle, which is 1. Hence the height is inversely proportional to the width.

Submit

You have used 1 of 1 attempt

Answers are displayed within the problem

2 (Graded)

1/1 point (graded)
The variance of a uniformly distributed random variable on $[a, b]$ is

- ☐ $(b - a) / 2$
- ☐ $(b - a) / 6$
- ☐ $(b - a)^2 / 6$
- ☒ $(b - a)^2 / 12$

✓
Answer
Correct: Video: Uniform Distribution

Explanation

The expectation of a uniformly distributed random variable X on $[a, b]$ is $E(X) = \frac{a+b}{2}$.

Its varaince is $V(X) = \int_a^b (x - \frac{a+b}{2})^2 \frac{1}{b-a} dx = (b - a)^2 / 12$

Submit

You have used 1 of 2 attempts

Answers are displayed within the problem

3 Max v. min (Graded)

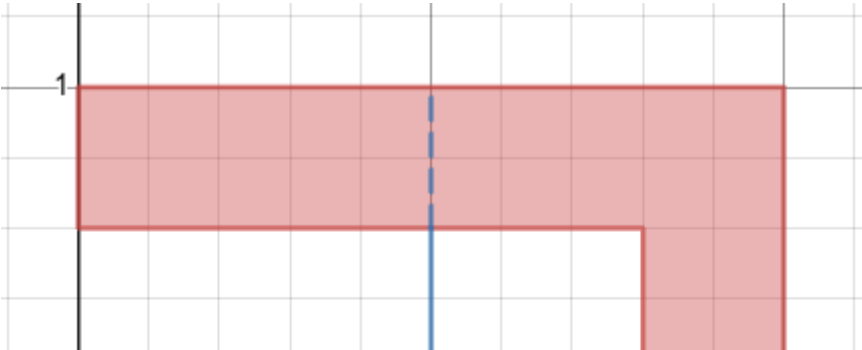
3/3 points (graded)
Let $X, Y \sim U_{[0,1]}$ independently. Find $P(\max(X, Y) \geq 0.8 \mid \min(X, Y) = 0.5)$.

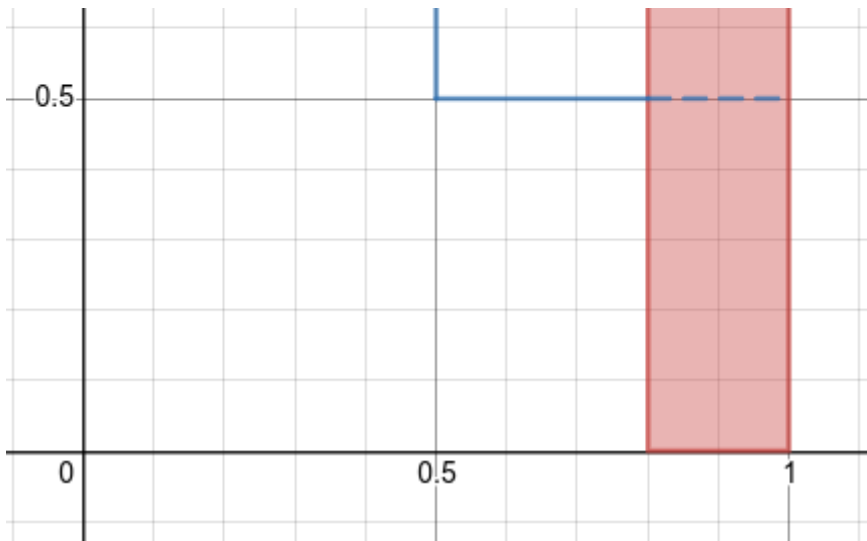
0.4

✓ Answer: 0.4

0.4

Explanation
Red area is the region that $\max(X, Y) \geq 0.8$.
Blue line (both solid and dash) is the region that $\min(X, Y) = 0.5$.
Blue dash line is the region that $\max(X, Y) \geq 0.8$ and $\min(X, Y) = 0.5$.
Notice that $P(\max(X, Y) \geq 0.8 \mid \min(X, Y) = 0.5) = \frac{P(\max(X, Y) \geq 0.8, \min(X, Y) = 0.5)}{P(\min(X, Y) = 0.5)}$.





Conditioning on $\min(X, Y) = 0.5$ restricts our focus to the L-shaped line from $(1, 0.5)$ to $(0.5, 0.5)$ to $(0.5, 1)$ whose total length is $0.5 + 0.5 = 1$, and the distribution over that line is uniform. Within this line, $\max(X, Y) \geq 0.8$ forms the segments from $(0.8, 0.5)$ to $(1, 0.5)$ and from $(0.5, 0.8)$ to $(0.5, 1)$ whose total length is $0.2 + 0.2 = 0.4$. The probability of falling within these segments given the L-shaped line is $0.4/1 = 0.4$.

? **Hint (1 of 1):** Think geometrically in \mathbb{R}^2 . What is the region $\min(X, Y) = 0.5$, and what fraction of this region intersects with $\max(X, Y) \geq 0.8$.

Next Hint

Submit

You have used 1 of 4 attempts

i Answers are displayed within the problem

4

0 points possible (ungraded)
Given $X \sim U_{[a,b]}$ with $E[X] = 2$ and $V(X) = 3$, find a and b .

• a

• b

Submit

You have used 0 of 4 attempts

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? Does Q 3 condition on a zero-probability event?
How can we get away with conditioning on the minimum of two continuous uniform RVs being equal to .5 when the probability of eit...

1

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Exponential Distribution

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Video

Variance

$$EX^2 = \int_0^\infty x^2 \lambda e^{-\lambda x} dx$$
$$u = x^2 \quad dv = \lambda e^{-\lambda x} dx$$
$$du = 2x dx \quad v = -e^{-\lambda x}$$
$$\int u dv = uv - \int v du$$
$$= -x^2 e^{-\lambda x} \Big|_0^\infty + \int_0^\infty 2x e^{-\lambda x} dx$$
$$= 0 + \frac{2}{\lambda} EX$$
$$EX = \int_0^\infty x \lambda e^{-\lambda x} dx = \frac{1}{\lambda}$$

[Start of transcript. Skip to the end.](#)

- Hello again, everyone.

In this video we're going to talk about the exponential distribution, and we're going to start.

So, the exponential distribution extends the geometric distribution to

9.4 Exponential Distribution

POLL

In terms of memorylessness, the exponential distribution is analogous to which discrete random variable distribution?

RESULTS

<input type="radio"/>	Bernoulli distribution	2%
<input type="radio"/>	Binomial distribution	2%
<input type="radio"/>	Poisson distribution	12%
<input checked="" type="radio"/>	Geometric distribution	84%

Submit

Results gathered from 43 respondents.

FEEDBACK

Geometric distributions are also memoryless.

1

0 points possible (ungraded)

The y-intercept of the pdf of an exponentially distribution with $\lambda = 2$ is

☐ 0

☐ 0.5

☐ 1

☐ 2

Submit

You have used 0 of 2 attempts

2 (Graded)

2/2 points (graded)

Assume the lifetimes of some kind of batteries follow exponential distribution with mean 1 year.

- What is the probability that one such batteries can be used for more than 1.5 years?

0.2231

✓ Answer: 0.22313

0.2231

Explanation

Let $X \sim \text{Exponential}(\lambda)$ denote the age of the battery. Since $1 = E(X) = 1/\lambda$, we have $\lambda = 1$. Further, for an exponential distribution, the CDF is given by $F_X(x) = P(X \leq x) = 1 - e^{-\lambda x}$, $x \geq 0$. Thus $P(X > 1.5) = 1 - P(X \leq 1.5) = 1 - F_X(1.5) = e^{-1.5} = 0.22313$.

- What is the probability that one such batteries can be used for more than 1.5 years **in total** if it has already been used for 0.5 year?

0.3679

✓ Answer: 0.367879

0.3679

Explanation

By the memoryless property of exponential distribution, $P(X > 1.5 | X > 0.5) = P(X > 1)$. Following the same steps as the previous part above, $P(X > 1) = e^{-1} = 0.367879$.

Submit

You have used 1 of 4 attempts

❗ Answers are displayed within the problem

3 (Graded)

3/3 points (graded)

Let X, Y be two independent exponential random variables with means **1** and **3**, respectively. Find $P(X > Y)$.

1/4

✓ Answer: 0.25

 $\frac{1}{4}$ **Explanation**

From the description we have $f_X(x) = e^{-x}$, $f_Y(y) = \frac{1}{3}e^{-y/3}$.

Hence $P(Y < y) = F_Y(y) = \int_{-\infty}^y f_Y(y') dy' = \int_0^y \frac{1}{3}e^{-y'/3} dy' = 1 - e^{-y/3}$.

$P(X > Y) = \int_0^\infty \int_0^t f_X(t) f_Y(y) dy dt = \int_0^\infty f_X(t) P(Y < t) dt = \int_0^\infty e^{-t} (1 - e^{-t/3}) dt = \int_0^\infty e^{-t} - e^{-4t/3} dt$

.

Submit

You have used 2 of 4 attempts

❗ Answers are displayed within the problem

4

0 points possible (ungraded)

In order to attend an important **8** A.M. lecture, you arrive at the shuttle stop at a time distributed uniformly between **7 : 20** A.M. and **7 : 30** A.M. The time between consecutive shuttle arrivals is known to be exponentially distributed with mean **15** minutes. If the journey takes **30** minutes, what is the probability that you arrive late to the lecture?

You have used 0 of 4 attempts

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Normal Distribution

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Video

UCSDSE212017-V017500

0:00 / 15:40

1.0x

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- Hello, and welcome back.
So next we would like to discuss Gaussian Distributions, or, commonly known as normal distributions.
You've noticed that some, many of the people we've talked about are quite famous, they've appeared on things like statues and stamps

9.5 Gaussian Distribution

POLL

If you fix the mean but increase the variance of a normal distribution, its pdf will

RESULTS

<input type="radio"/>	move to the left	5%
<input checked="" type="radio"/>	move to the right	7%
<input type="radio"/>	become taller and narrower	2%
<input type="radio"/>	become shorter and flatter	86%

Submit

Results gathered from 42 respondents.

FEEDBACK

The pdf will become shorter and flatter.

1 Highest probability

0 points possible (ungraded)

Let $X \sim \mathcal{N}(\mu, \sigma^2)$ be a normal random variable, then the maximum value of its pdf is

- ☐ 1
- ☐ $\frac{1}{\sqrt{2\pi}}$
- ☐ $\frac{1}{\sqrt{2\pi\sigma}}$

☐

$\frac{1}{\sqrt{2\pi\sigma^2}}$

Submit

You have used 0 of 2 attempts

2 Linear transformations

0 points possible (ungraded)
The linear transformation of a normal random variable is also a normal random variable.

☐ True

☐ False

Submit

You have used 0 of 1 attempt

3

0 points possible (ungraded)
If $\boldsymbol{X}, \boldsymbol{Y}$ are two independent random variable with $\boldsymbol{X} \sim \mathcal{N}(\boldsymbol{1}, \boldsymbol{16})$ and $\boldsymbol{Y} \sim \mathcal{N}(\boldsymbol{1}, \boldsymbol{9})$, then find $\mathbf{Var}(\boldsymbol{XY})$.

Submit

You have used 0 of 4 attempts

4

0 points possible (ungraded)
Suppose \boldsymbol{X} is a Gaussian random variable with mean $\boldsymbol{2}$ and variance $\boldsymbol{4}$. Find $\boldsymbol{E}\left(e^{\frac{\boldsymbol{X}}{2}}\right)$.

Submit

You have used 0 of 4 attempts

5

0 points possible (ungraded)
If $\boldsymbol{x} \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{1})$, find $\boldsymbol{E}\left(e^{-\boldsymbol{X}^2}\right)$.

Submit

You have used 0 of 4 attempts

6 (Graded)

3/3 points (graded)

Let X be distributed according to the pdf ke^{-x^2-7x} . Find $E(X^2)$.

12.75

✔ Answer: 51/4

12.75

Explanation

Notice that $f_X(x) = ke^{-x^2-7x} = (ke^{49/4}) \cdot e^{-\frac{(x+7/2)^2}{2 \times 0.5}} = c \cdot e^{-\frac{(x+7/2)^2}{2 \times 0.5}}$, where c is a constant. Therefore X is normally distributed with $\mu = -7/2$ and $\sigma^2 = 0.5$ and thus $E(X^2) = V(X) + E(X)^2 = 1/2 + 49/4 = 51/4$.

? Hint (1 of 1): Consider the pdf of Gaussian distribution.

Next Hint

Submit

You have used 1 of 4 attempts

ⓘ Answers are displayed within the problem

7 (Graded)

3/3 points (graded)

Let $X \sim N(0,9)$ have mean 0 and variance 9. Find the expected value of $X^2(X+1)$.

9

✔ Answer: 9

9

Explanation

Notice that since the Guassian distribution is symmetric around its mean 0, $E(X^3) = 0$. Further since $E(X^2) = V(X) + E(X)^2 = 9$, the answer follows.

Submit

You have used 1 of 4 attempts

ⓘ Answers are displayed within the problem

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Gaussian Probability

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Video

UCSDSE212017-V017800



0:00 / 27:00

▶ 1.0x

Start of transcript. Skip to the end.

- Hello and welcome back.
In the last lecture, we introduced the normal distribution,
and now we would like to calculate probabilities
when a random variable is distributed normally.
So let's see how we do that.
So we're going to talk about interval probabilities,

9.6 Gaussian Distribution Probabilities

POLL

Why z table only cover one half of the normal curve?

RESULTS

- | | | |
|----------------------------------|---|-----|
| <input type="radio"/> | The positive half is most frequently used. | 2% |
| <input type="radio"/> | The table will be too large to include the negative half. | 5% |
| <input checked="" type="radio"/> | The values of the negative half can be deduced from symmetry. | 93% |

Submit

Results gathered from 42 respondents.

FEEDBACK

The values of the negative half can be deduced from symmetry.

1

0 points possible (ungraded)

If X is a normal random variable with $\mu = -2$ and $\sigma = 3$, and has probability density function and cumulative density function $f_X(x)$, $F_X(x)$, calculate

- $P(-3 < X < 0)$

- $F(1/4)$

- $F^{-1}(1/4)$

Submit

You have used 0 of 4 attempts

2 (Graded)

2/2 points (graded)

Suppose X, Y are independent and $X \sim \mathcal{N}(1, 4)$ and $Y \sim \mathcal{N}(1, 9)$. If $P(2X + Y \leq a) = P(4X - 2Y \geq 4a)$, then find a .

4/3

✓ Answer: 4/3

$\frac{4}{3}$

Explanation

Notice that $2X + Y \sim \mathcal{N}(3, 25)$, $4X - 2Y \sim \mathcal{N}(2, 100)$. Then $P(2X + Y \leq a) = \Phi\left(\frac{a-3}{5}\right)$, and $P(4X - 2Y \geq 4a) = 1 - \Phi\left(\frac{4a-2}{10}\right) = \Phi\left(\frac{2-4a}{10}\right)$. By solving the equation $\frac{a-3}{5} = \frac{2-4a}{10}$, we have $a = \frac{4}{3}$.

Submit

You have used 1 of 4 attempts

❗ Answers are displayed within the problem

3

0 points possible (ungraded)

Let $X \sim B_{.36, 1600}$. Approximate $P(552 \leq X \leq 600)$.

Submit

You have used 0 of 4 attempts

4 (Graded)

6/6 points (graded)

Suppose a binary message is transmitted through a noisy channel. The transmitted signal S is equally likely to be 1 or -1 , the noise N follows a normal distribution $\mathcal{N}(0, 4)$, and the received signal is $R = S + N$. S and N are independent. The receiver concludes that the signal is 1 when $R \geq 0$ and -1 when $R < 0$.

- What is the error probability when one signal is transmitted?

0.308

✓ Answer: 0.308538

0.308

Explanation

An error occurs under either of these two events: $S = 1, Z < 0$ or if $S = -1, Z > 0$. Now

An error occurs under either of these two events, $S = 1, Z < 0$ or if $S = -1, Z \geq 0$. Now $P(S = 1, Z < 0) = P(S = 1) P(Z < 0 | S = 1) = 1/2 \cdot P(N < -1)$. Since $N/2 \sim \mathcal{N}(0, 1)$, $P(N < -1) = P(N/2 < -1/2) = \Phi(-1/2) = 0.308538$. Thus $P(S = 1, Z < 0) = 1/2 \cdot 0.308538$. Similarly, by symmetry it follows that $P(S = -1, Z \geq 0) = 1/2 \cdot 0.308538$. The probability of error thus is $2 \cdot 1/2 \cdot 0.308538 = 0.308538$.

- What is the error probability when one signal is transmitted if we triple the amplitude of the transmitted signal, namely, $S = 3$ or -3 with equal probability.

0.066

0.066

✓ Answer: 0.0668072

Explanation
Following the same analysis as above, the first error event $P(S = 3, Z < 0) = P(S = 3) P(Z < 0 | S = 3) = 1/2 \cdot P(N < -3)$. Since $N/2 \sim \mathcal{N}(0, 1)$, $P(N < -3) = P(N/2 < -3/2) = \Phi(-3/2) = 0.0668072$. Thus $P(S = 3, Z < 0) = 1/2 \cdot 0.0668072$. Similarly, by symmetry it follows that $P(S = -3, Z \geq 0) = 1/2 \cdot 0.0668072$. Thus the probability of error is $2 \cdot 1/2 \cdot 0.0668072 = 0.0668072$ and is drastically smaller than the previous scenario.

- What is the error probability if we send the origianl signal (with amplitude 1) three times, and take majority for conlusion? For example, if three received signal was concluded 1, -1, 1 by receiver, we determine the transmitted signal to be 1.

0.226

0.226

✓ Answer: 0.226844

Explanation
If we denote the error probability in the first part by $p (= 0.308538)$, on repeated transmission, an error occurs if and only if the incorrect signal is concluded in least 2 of the 3 transmissions. Since the trasmission are independent, the probability of error is thus $\binom{3}{2}p^2 \cdot (1 - p) + \binom{3}{3}p^3 = 0.226844$.

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Complexity of the problems

The complexity of the problems is way beyond the explanation or examples in the video.

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