Chapter 1 :

Solve linear system by matrix

Section ②

Three possibilities of linear system of equations

$$\left.\begin{aligned} &1)\ \textit{Has exactly one solution} \\ &2)\ \textit{Has infinitly many solutions} \end{aligned}\right\}\ \text{Consistent System}$$

$$3)\ \textit{Has no solution} \quad \}\ \text{Inconsistent System}$$

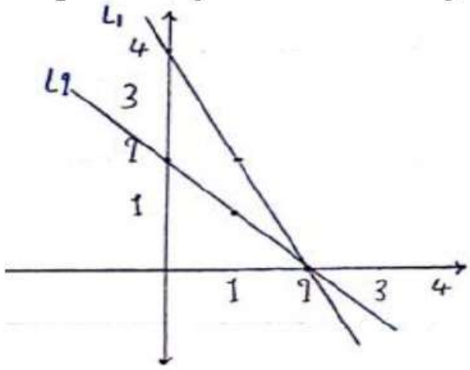
Consistent System: System of equations that has at least one solution .

Inconsistent System: System of equations that has no solution .

① The line L_1 intersect line L_2 at only one point. (One Solution)

$$\left.\begin{aligned} 2x + y &= 4 \\ x + y &= 2 \end{aligned}\right\}$$

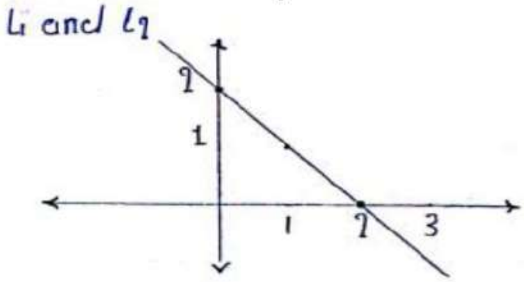
Solution: $\boxed{X = 2}$ $\boxed{y = 0}$



② The line L_1 coincide line L_2 . (Infinite Solutions)

$$\left.\begin{aligned} 2x + 2y &= 4 \\ x + y &= 2 \end{aligned}\right\}$$

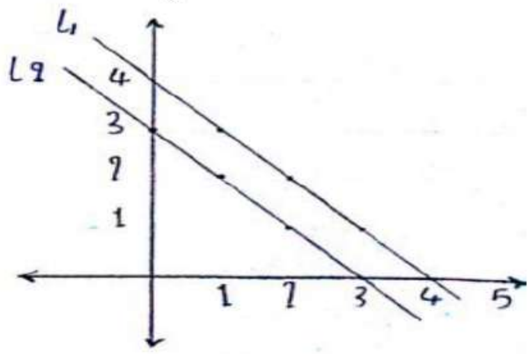
Solution: Infinitely many solutions



③ The lines L_1, L_2 are parallel. (No Solution)

$$\left.\begin{aligned} x + y &= 4 \\ 2x + 2y &= 6 \end{aligned}\right\}$$

No Solution



How to determine if a system has no solution, or infinite solutions from augmented matrix?

✓ A system has no solution

$$\left[\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & 0 & 1 \end{array}\right]$$

✓ A system infinite solutions

$$\left[\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right]$$

Example 1 : Solve by Gaussian Elimination

$$\left.\begin{array}{r} x_1 - 2x_2 - 6x_3 = 12 \\ 2x_1 + 4x_2 + 12x_3 = -17 \\ x_1 - 4x_2 - 12x_3 = 22 \end{array}\right\}$$

----------------------------**Solution**----------------------------------

$$\left[\begin{array}{ccc|c} 1 & -2 & -6 & 12 \\ 2 & 4 & 12 & -17 \\ 1 & -4 & -12 & 22 \end{array}\right]$$

$$\xrightarrow[-\boldsymbol{R_1} + \boldsymbol{R_3} \to \boldsymbol{R_3}]{-2\boldsymbol{R_1} + \boldsymbol{R_2} \to \boldsymbol{R_2}} \begin{bmatrix} 1 & -2 & -6 & 12 \\ 0 & 8 & 24 & -41 \\ 0 & -2 & -6 & 10 \end{bmatrix}$$

$$\xrightarrow{\boldsymbol{R_2}/8 \to \boldsymbol{R_2}} \begin{bmatrix} 1 & -2 & -6 & 12 \\ 0 & 1 & 3 & -41/8 \\ 0 & -2 & -6 & 10 \end{bmatrix}$$

$$\xrightarrow{2\boldsymbol{R_2} + \boldsymbol{R_3} \to \boldsymbol{R_3}} \begin{bmatrix} 1 & -2 & -6 & 12 \\ 0 & 1 & 3 & -41/8 \\ 0 & 0 & 0 & -1/4 \end{bmatrix}$$

The System has no solution

Example 2 **: Solve by Gauss-Jordan**

$$\left.\begin{aligned} x + 2y - 3z + w &= -2 \\ 3x - y - 2z - 4w &= 1 \\ 2x + 3y - 5z + w &= -3 \end{aligned}\right\}$$

------------------------------Solution------------------------------

$$\left[\begin{array}{cccc:c} 1 & 2 & -3 & 1 & -2 \\ 3 & -1 & -2 & -4 & 1 \\ 2 & 3 & -5 & 1 & -3 \end{array}\right]$$

$$\xrightarrow[-2R_1 + R_3 \to R_3]{-3R_1 + R_2 \to R_2} \begin{bmatrix} 1 & 2 & -3 & 1 & -2 \\ 0 & -7 & 7 & -7 & 7 \\ 0 & -1 & 1 & -1 & 1 \end{bmatrix}$$

$$\xrightarrow{R_2/-7 \to R_2} \begin{bmatrix} 1 & 2 & -3 & 1 & -2 \\ 0 & 1 & -1 & 1 & -1 \\ 0 & -1 & 1 & -1 & 1 \end{bmatrix}$$

$$\xrightarrow[R_2 + R_3 \to R_3]{-2R_2 + R_1 \to R_1} \begin{bmatrix} 1 & 0 & -1 & -1 & 0 \\ 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The System has infinitely many solutions

X , y are ***leading*** *variables*
Z , w are ***free*** *variables*

X – z – w = 0 ⟹ x = z + w

Y – z + w = -1 ⟹ y = z – w – 1

Solution 1	Solution 2	Solution 3	………………	Solution n
X = 0	X = 2			
Y = -1	Y = -1			
Z = 0	Z = 1			
W = 0	W = 1			

***Example $\underline{\underline{3}}$* : What condition that b_1 , b_2 and b_3 should satisfy in order to solve the following system ?**

$$\left.\begin{aligned} x_1 + x_2 + x_3 &= b_1 \\ x_1 + 2x_3 &= b_2 \\ 2x_1 + x_2 + 3x_3 &= b_3 \end{aligned}\right\}$$

-----------------------------Solution-----------------------------------

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & b_1 \\ 1 & 0 & 2 & b_2 \\ 2 & 1 & 3 & b_3 \end{array}\right] \xrightarrow[-2R_1 + R_3 \to R_3]{-R_1 + R_2 \to R_2} \begin{bmatrix} 1 & 1 & 1 & b_1 \\ 0 & -1 & 1 & b_2 - b_1 \\ 0 & -1 & 1 & b_3 - 2b_1 \end{bmatrix}$$

$$\xrightarrow{-R_2 \to R_2} \begin{bmatrix} 1 & 1 & 1 & b_1 \\ 0 & 1 & -1 & b_1 - b_2 \\ 0 & -1 & 1 & b_3 - 2b_1 \end{bmatrix}$$

$$\xrightarrow{R_2 + R_3 \to R_3} \begin{bmatrix} 1 & 1 & 1 & b_1 \\ 0 & 1 & -1 & b_1 - b_2 \\ 0 & 0 & 0 & b_3 - b_2 - b_1 \end{bmatrix}$$

Condition : $b_3 - b_2 - b_1 = 0$