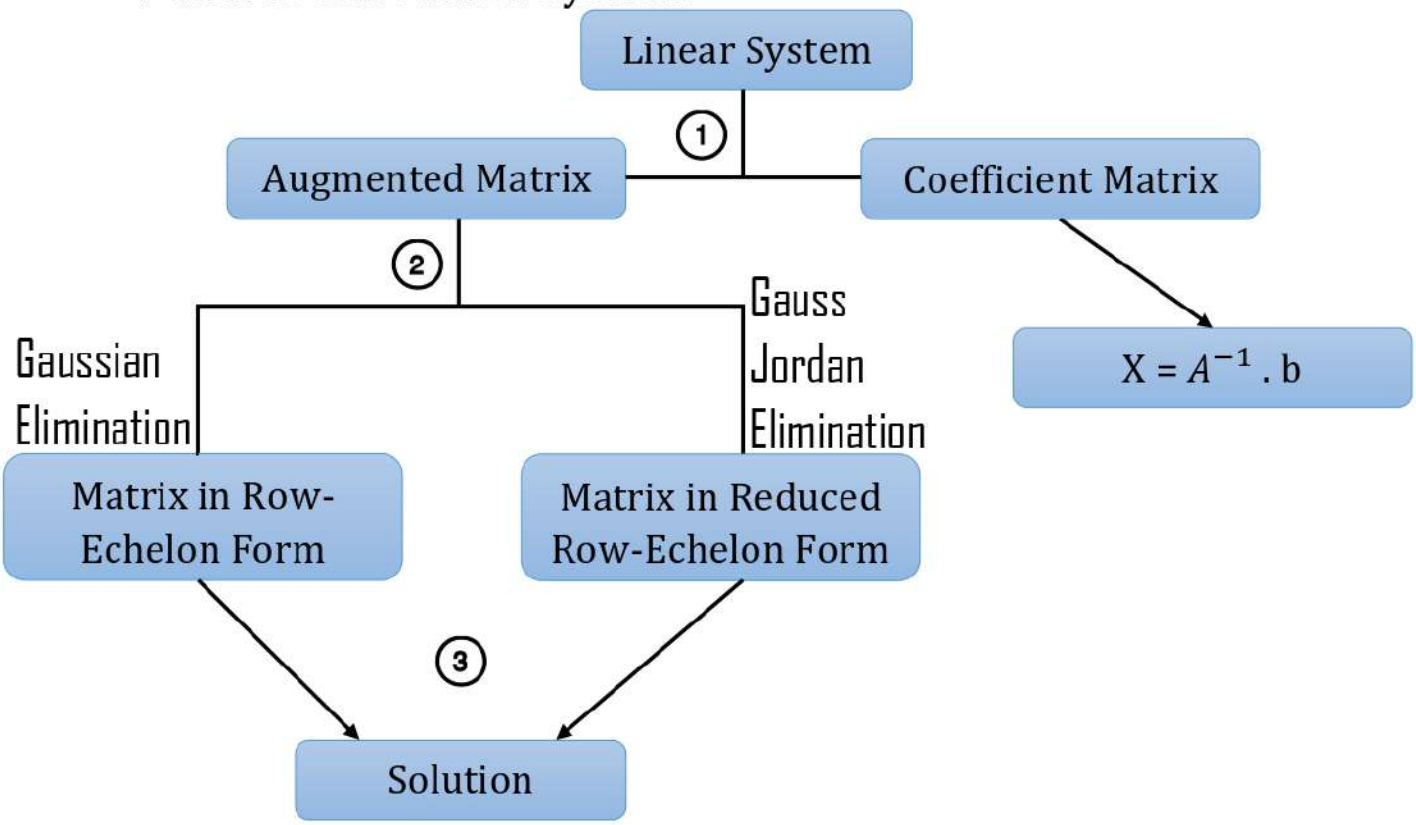
Chapter 1 :

Solve linear system by matrix

Section ④

How to solve linear system?



Solving linear system by matrix inversion

✓ If A is an invertible **n*n** matrix, then the system of equations **AX = b** has exactly one solution, namely **X = A^{-1} . b**

$$a_{11}X_1 + a_{12}X_2 = b_1$$

$$a_{21}X_1 + a_{22}X_2 = b_2$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

Example: solve the following linear system by matrix inversion

$$\left.\begin{matrix} X_1 + X_2 = 2 \\ 5X_1 + 6X_2 = 9 \end{matrix}\right\}$$

----------**Solution**----------

$$A = \begin{bmatrix} 1 & 1 \\ 5 & 6 \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} 6 & -1 \\ -5 & 1 \end{bmatrix} \qquad b = \begin{bmatrix} 2 \\ 9 \end{bmatrix}$$

$$AX = b$$

ضرب الطرفين فى A^{-1}

$$I\,X = A^{-1} \cdot b$$

$$X = A^{-1} \cdot b$$

$$X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 6 & -1 \\ -5 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 9 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

$X_1 = 3$ $\qquad$ $X_2 = -1$

Notes

- ✓ A is a coefficient matrix
- ✓ A is a square matrix
- ✓ A is invertible

Equivalent statements

✓ If A is an n*n matrix, then the following statements are equivalent

① A is invertible
② AX = 0 has only trivial solution
③ The Reduced Row-Echelon Form for A in I_n
④ AX = b is consistent
⑤ AX = b has exactly one solution

Homogeneous System AX = 0

له حل وحيد وهو الحل الصفرى

$$\left.\begin{aligned} 3x + y &= 0 \\ 2x + y &= 0 \end{aligned}\right\}$$

$X = 0$ $y = 0$

1) If A and B be real symmetric matrices of size n x n, then

A) $AA^T = 1$ B) $A = A^{-1}$ C) $AB = BA$ D) $(AB)^T = BA$

2) If, A, B, C are square matrices of the same order, then $(ABC)^{-1}$ is equal to

A) $C^{-1}A^{-1}B^{-1}$ B) $C^{-1}B^{-1}A^{-1}$

C) $A^{-1}B^{-1}C^{-1}$ D) $A^{-1}C^{-1}B^{-1}$

3) The system of equations

2x + 4y = 10,

x + 10y = 25 has

A) no solution B) only one solution

C) only two solutions D) infinite solutions

4) The system of simultaneous equations

x + 2y + z = 6

2x + y + 2z = 6

x + y + z = 5 has

A) unique solution B) infinite number of solutions

C) no solution D) exactly two solutions

5) $A=\begin{bmatrix} 5 & 0 & 2 \\ 0 & 3 & 0 \\ 2 & 0 & 1 \end{bmatrix}$**, The inverse of A is**

A) $\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1/3 & 0 \\ -2 & 0 & 5 \end{bmatrix}$ B) $\begin{bmatrix} 5 & 0 & 2 \\ 0 & -1/3 & 0 \\ 1/2 & 0 & 1 \end{bmatrix}$

C) $\begin{bmatrix} 1/5 & 0 & 1/2 \\ 0 & 1/3 & 0 \\ 1/2 & 0 & 1 \end{bmatrix}$ D) $\begin{bmatrix} 1/5 & 0 & -1/2 \\ 0 & 1/3 & 0 \\ -1/2 & 0 & 1 \end{bmatrix}$

6) If A= $\begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & 5 \\ -3 & -5 & 0 \end{bmatrix}$ **,then**

A) $A^T = -A$ B) $A^T = A$ C) $A^T = 2A$ D) *none* of this

7) The system of equations

4x + 6y = 8

7x + 8y = 9

3x + 2y = 1 has

A) no solution
B) only one solution
C) only two solutions
D) infinite solutions

8) If A = $\begin{bmatrix} 2 & -0.1 \\ 0 & 3 \end{bmatrix}$ **,And** $A^{-1} = \begin{bmatrix} 1/2 & a \\ 0 & b \end{bmatrix}$ **,then (a+b) equals**

A) 7/20 B) 3/20 C) 19/20 D) 11/20

9) Find the values of x, y, z and w from the below condition

$$5\begin{bmatrix} x & z \\ y & w \end{bmatrix} = \begin{bmatrix} 2 & 10 \\ 3 & 2x+y \end{bmatrix} + \begin{bmatrix} z & 5 \\ 7 & w \end{bmatrix}$$

A) x=1, y=3, z=4, w=0
B) x=2, y=3, z=8, w=1
C) x=1, y=2, z=3, w=1
D) x =1, y=2, z=4, w=1

10) Multiplication of a matrix with a scalar constant is called?

A) Complex multiplication
B) Linear multiplication
C) Scalar multiplication
D) Constant multiplication

11) Singular matrix are?

A) non-invertible
B) invertible
C) Both non-invertible and invertible
D) None Of the above

12) If A = $\begin{bmatrix} 1 & 3 & 1 \\ 2 & 7 & 3 \end{bmatrix}$ is an augmented matrix for linear system, then the system has.....

A) One solution
B) No solution
C) Infinitely many solutions

13) If A and B are both invertible nxn matrices, then AB is invertible.

A)True B)False

14) Let A and B be nxn matrices. Assume that AB =I_n. Then, BA = I_n.

A)True B)False

15) In diagonal matrix, all elements other than elements along primary diagonal are

A) equal to zero B) equal to two
C) equal to three D) equal to one

16) The dimension of row vector can be written as

A) n+1 B) 1-n C) 1+n D) 1*n

17) The dimension of column vector can be written as

A) m*1 B) 1*n C) m+1 D) n+1

18) In the transpose of matrix A, the columns of the matrix A becomes

A) multiple column B) rows
C) multiples D) divisors

19) The matrix A will not be transformed into an identity matrix if the matrix is

A) singular
B) non-singular
C) identified
D) unidentified

20) The product of matrix A and matrix A^{-1} results in the matrix classified as

A) identity matrix
B) matrix A
C) inverse matrix
D) both A and C

21) The product of identity matrix and any matrix A is equal to

A) product matrix
B) unidentified matrix
C) matrix A
D) identity matrix

22) The two matrices A = $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 4 & 1 \end{bmatrix}$ **and B =** $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 2 & 5 \\ 4 & 5 & 2 \end{bmatrix}$ **are**

A) Transposes of one another
B) Asymptotic
C) Parenthetical
D) Symmetric

23) Where the dimension of matrices is defined as rows×columns, you can multiply a 3×3 matrices on the left by a 2×3 matrix on the right

A)True
B)False

24) Where the dimension of matrices is defined as rows×columns, you can multiply a 3×2 matrices on the left by a 2×3 matrix on the right

A)True
B)False

25) An m×n matrix with m < n can be symmetric

A)True B)False

26) The system of linear equations

4x + 2y = 7

2x + y = 6 has

A) unique solution B) infinite number of solutions

C) no solution D) exactly two solutions

27) The two equations that have no values to satisfy both equations then this is called

A) consistent system B) inconsistent system

C) solution system D) constant system

28) The matrices can be added only if the matrices have

A) same dimension B) different dimension

C) multiple dimension D) divisor dimension

29) In the Gaussian Elimination method, the original equations are transformed by using

A) column operations B) row operations

C) mathematical operations D) subset dimension

30) If a matrix is in reduced row echelon form, then it is also in row echelon form

A)True B)False

31) Matrix having same number of columns and rows is classified as

A) Triangle matrix. B) Rectangle matrix.

C) Circle matrix. D) None of the above

32) Transpose of a rectangular matrix is a

A) Rectangular matrix. B) Diagonal matrix.
C) Square matrix. D) Scalar matrix.

33) In a matrix multiplication for A and B, $(AB)^T$ =

A) A B^T B) $B^T A^T$
C) 1/AB D) None of the above

34) If AB exists, then $(AB)^{-1}$ is

A) AB B) B^{-1} A
C) $B^{-1}\ A^{-1}$ D) None of the above

35) The system of equations 2x + 3y = 5, 6x + 9y = a has infinitely many solution if a is

A) 10 B) 2 C) 15 D) None of the above

36) What is a, if B = $\begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is a singular matrix?

A) 5 B) 8 C) 6 D) None of the above

37) For an n x n matrix $(A^T)^T$ =

A) A^T B) A^{-1} C) A D) None of the above

38) if A is a matrix, then AA^T is

A) not symmetric B) symmetric C) may be symmetric

39) suppose that A_{3*2}, B_{2*5}, C_{5*2} are three matrices. Which of the following expressions is true?

A) (A+B)C B)ABC C) (B+C)A D) BA

40) If A is an invertible matrix, then $(3A)^{-1}$ =

A) $\frac{1}{3}A^{-1}$ B) $\frac{1}{9}A^{-1}$ C) $3A^{-1}$ D) $\frac{1}{6}A^{-1}$

41) If A is a square matrix, then $(3A)^{T}$ =

A) $\frac{1}{3}A^{T}$ B) $\frac{1}{9}A^{T}$ C) $3A^{T}$ D) $\frac{1}{6}A^{T}$

42) $\left.\begin{matrix}(\mu^2 - 3)X + Y = 3 \\ X + Y = 5\end{matrix}\right\}$ **has one solution when μ = ...**

A) 2 B) 0 C) -2

43) $\begin{bmatrix}a & 3 & 6\\ 4 & 2 & 2\end{bmatrix}$ **is an augmented matrix, If a = 6, then the system has no solution?**

A) *True* B) *False*

44) If A = $\begin{bmatrix}2 & 4\\ 5 & 6\end{bmatrix}$ **, then** $8A^{-1} = \begin{bmatrix}6 & -4\\ -5 & 2\end{bmatrix}$ **?**

A) *True* B) *False*

45) If A = $\begin{bmatrix}a & b\\ c & d\end{bmatrix} = I_2$ **, then A is not invertible ?**

A) *True* B) *False*

46) If A = $\begin{bmatrix}a & b\\ c & d\end{bmatrix} = I_2$ **, then AX = B is .....**

A) *Consistent* B) *Inconsistent*

47) If A^T is invertible, then AX = b has one solution?

A) *True* B) *False*

48) If A is symmetric, then A^2 may be symmetric?

A) $True$ B) $False$

49) $\left.\begin{matrix}(\mu^2 - 3)X + Y = 3 \\ X + Y = 5\end{matrix}\right\}$ **has only one trivial solution when μ = 2 ?**

A) $True$ B) $False$

50) If $(AB)^{-1}$ = C, then the number of columns in A = number of rows in B ?

A) $True$ B) $False$

51) If the Reduced Row-Echelon-Form for A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ **is** I_2**, then AX = B is consistent?**

A) $True$ B) $False$

52) $\begin{bmatrix} a & 3 & 6 \\ 4 & 2 & 2 \end{bmatrix}$ **is an augmented matrix has no solution when a = …….**

A) 2 B) 4 C) 6

53) A = $\begin{bmatrix} 1 & 2 \\ 0 & d \end{bmatrix}$ **is in the Row-Echelon form, then d = …….**

A) 1 B) 0 C) 0 or 1