Date:

(Assignment No 2)

NAME SAIF-UD-DIN
ID 32065 (BSCS)

Question No 1:

Evaluate the following integrals?

1) $\int \frac{1}{1-\sin x+\cos x}$

Sol:- Let $I = \int \frac{1}{1-\sin x+\cos x} dx$

As we know

$$I = \int \frac{1}{1-\sin x+\cos x} dx$$

We can further write $\sin x$ or $\cos x$ as

$$I = \int \frac{1+\tan^2 x/2}{1+\cancel{\tan^2 x/2} - 2\tan x/2 + 1 - \cancel{\tan^2 x/2}}$$

$$I = \int \frac{\sec^2 x/2}{2-2\tan x/2} dx$$

let

$$t = \tan\frac{x}{2}$$

$$\frac{dt}{dx} = \frac{1}{2}\sec^2\frac{x}{2}$$

$$dt = \frac{1}{2}\sec^2\frac{x}{2} \cdot dx$$

So the equation becomes

Scanned with CamScanner

$$I = \int \frac{\sec^2 x/2}{2(1-\tan x/2} \, dx$$

$$I = \int \frac{\sec^2 x/2}{2(1-\tan x/2)} \, dx$$

As ; $dt = \frac{1}{2} \sec^2 \frac{x}{2} \, dx$

So we get

$$I = \int \frac{dt}{1-t}$$

$\because$ $1 - t = u$

$$-1 = \frac{du}{dt}$$

$$-dt = du$$

$$dt = -du$$

Hence

$$I = -\int \frac{du}{u}$$

$$I = -\log u + c$$

$$I = \log |1-t| + c$$

$$I = -\log |1-t| + c \quad \text{Ans:-}$$

2- Evaluate :- $\int_{\pi/2}^{2\pi/3} \frac{\cos\theta}{\sin\theta\cos\theta + \sin\theta} \, d\theta$

Sol:- $\int_{\pi/2}^{2\pi/3} \frac{\cos\theta}{\sin\theta\cos\theta + \sin\theta}$

Dividing by '$\cos\theta$' on 'N' and 'D'

$$= \int_{\pi/2}^{2\pi/3} \frac{d\theta}{\sin\theta + \tan\theta}$$

Scanned with CamScanner

Date: .....

Substitute

$$z = \tan\left(\frac{x}{2}\right)$$

$$dz = \frac{1}{2}\sec^2\left(\frac{x}{2}\right)dx$$

$$dz = \frac{2\,dz}{1+dz^2}$$

limits change

$2\pi/3 \longrightarrow \tan(\pi/3)$

$\pi/2 \longrightarrow \tan(\pi/4)$

$$\int_1^{\sqrt{3}}$$

$$\sin\theta = \frac{2\tan(\theta/2)}{1+\tan^2(\theta/2)}$$

$$\sin\theta = \frac{2z}{1+z^2}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \Rightarrow \frac{2z}{1-z^2}$$

$$= \int_1^{\sqrt{3}} \frac{1}{\frac{2}{1+z^2} + \frac{2z}{1-z^2}} \times \left(\frac{2dz}{1+z^2}\right)$$

$$= \int_1^{\sqrt{3}} \frac{1}{\frac{2z(1-z^2)+2z(1+z^2)}{(1+z^2)(1-z^2)}} \times \left(\frac{2dz}{1+z^2}\right)$$

$$= \int_1^{\sqrt{3}} \frac{(1+z^2)(1-z^2)}{2z(1-z^2)+2z(1+z^2)} \times \left(\frac{2dz}{1+z^2}\right)$$

$$= \int_1^{\sqrt{3}} \frac{(1+z^2)}{2(1-z^2)+2(1+z^2)}\,dz$$

$$= \int_1^{\sqrt{3}} \frac{(1+z)^2}{2(1-z^2+1+z^2)}\,dz$$

$$= \int_1^{\sqrt{3}} \frac{1-z^2}{2z}\,dz$$

Scanned with CamScanner

$$= \int_1^{\sqrt{3}}$$

$$= \int_1^{\sqrt{3}} \frac{1}{2z} - \frac{z^{\cancel{2}}}{2\cancel{z}} \, dx$$

$$= \int_1^{\sqrt{3}} \frac{1}{2z} - \frac{z}{2} \, dz$$

$$= \left[\frac{1}{2} \ln z - \frac{z^2}{4} \right]_1^{\sqrt{3}}$$

Applying Limit

$$\left[\frac{1}{2} \ln(\sqrt{3}) - \frac{(\sqrt{3}^2)}{4} \right] - \left[\frac{1}{2} \ln(1) - \frac{(1^2)}{4} \right]$$

Multiplying and dividing by '2' we get

$$= \frac{2}{2 \times 2} \ln \sqrt{3} - \frac{3}{4} + \frac{1}{4}$$

$$= \frac{\ln (\sqrt{3})^2}{4} - \frac{1}{2}$$

$$= \frac{\ln 3}{4} - \frac{1}{2}$$

$$= \frac{1}{4} \ln 3 - \frac{1}{2} \quad \underline{\underline{\text{Ans}}}$$

3) Evaluate

$$\int_0^{\pi/2} \frac{dx}{2 + \cos x}$$

Sol:- $$\int_0^{\pi/2} \frac{dx}{2 + \cos x}$$

Scanned with CamScanner

M T W T F S

Date:

$$= \int_0^{\pi/2} \frac{dx}{(2) + \left(2\cos^2\frac{x}{2} - 1\right)}$$

$$= \int_0^{\pi/2} \frac{dx}{1 + 2\cos^2\left(\frac{x}{2}\right)}$$

$$\boxed{\because \cos^2\frac{x}{2} = \frac{1}{\sec^2 x/2}}$$

$$\cancel{\int_0^{\pi/2} \frac{dx}{1 + 2\cos^2\left(\frac{x}{2}\right)}}$$

$$\int_0^{\pi/2} \frac{dx}{1 + 2/\sec^2 x/2}$$

$$= \int_0^{\pi/2} \frac{\sec^2 x/2}{\sec^2 x/2 + 2}\, dx$$

$$= \int_0^{\pi/2} \frac{\sec^2 x/2}{3 + \tan^2 x/2}\, dx$$

$$\Rightarrow \boxed{\tan\frac{x}{2} = t}$$

By diff we get

$$\frac{1}{2}\sec^2\frac{x}{2}\, dx = dt$$

$$\sec^2\frac{x}{2}\, dx = 2dt$$

So limit equations becomes

$$\cancel{\int_0^1 \frac{2dt}{(\sqrt{3})^2 + (t)^2}}$$

$$\int_0^1 \frac{2dt}{3 + t^2}$$

Scanned with CamScanner

$$= \int_0^1 \frac{2\,dt}{(\sqrt{3})^2 + (t)^2}$$

$$= \left[\frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{t}{\sqrt{3}}\right)\right]_0^1$$

$$= \left[\frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right] - \left[\frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{0}{\sqrt{3}}\right)\right]$$

$$= \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) - 0$$

$$= \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$ Ans:-

Question No 4

Find the area enclosed by Circle, $x^2+y^2 = 25$ by using Integrals.

Sol:-

$$x^2+y^2 = 25 \quad \text{--- (i)}$$

As we know that

$\because \quad x = r\cos\theta$

$\because \quad y = r\sin\theta$

By putting the value of x and y in eq (i)

we get

$$\Rightarrow (r\cos\theta)^2 + (r\sin\theta)^2 = 25$$

$\because \quad x^2+y^2 = r^2$

$$r^2\cos^2\theta + r^2\sin^2\theta = 25$$

$$r^2(\cos^2\theta + \sin^2\theta) = 25$$

$\because \quad \cos^2\theta + \sin^2\theta = 1$

$$r^2(1) = 25$$

$$r^2 = 25$$

$$\sqrt{r^2} = \sqrt{25}$$

Scanned with CamScanner

$x = 5$

By putting the value of $(x=5)$ in the equation of Area, we get

$$A = \int_{\alpha}^{\beta} \frac{1}{2} x^2 \, d\theta$$

By putting values

$$A = \int_0^{2\pi} \frac{1}{2} (5)^2 \, d\theta$$

$$A = \frac{1}{2} \times 25 \int_0^{\pi/2} 1 \, d\theta$$

$$A = \frac{25}{2} \int_0^{\pi/2} 1 \, d\theta$$

$$A = \frac{25}{2} \left[\theta\right]_0^{\pi/2}$$

$$A = \frac{25}{\cancel{2}} \left[\cancel{2}\pi - 0\right]$$

$$\boxed{A = 2\pi \text{ Square}}$$ Ans.

Question No 5 :

Evaluate

$\int_0^1 \sqrt{x} \sin x \, dx$ to 6D, taking 5 sub intervals.

Sol.: $\int_0^1 \sqrt{x} \sin x \, dx$

As $n = 6$

$$\Delta x = \frac{b-a}{n}$$

$$\Delta x = \frac{1-0}{6} \Rightarrow \Delta x = \frac{1}{6}$$

Scanned with CamScanner

x	y
0	0
1/6	0.067727
2/6	0.001187
3/6	0.003887
4/6	0.004750
5/6	0.005937

$$\int_0^1 \sqrt{x}\sin x\,dx \approx \frac{6}{13}\left[0+4(0.067727)+2(0.001187)+4(0.004750)+2(0.004750)+4(0.005937+6\right]$$

$$\int_0^1 \sqrt{x}\sin x\,dx \approx \frac{1}{18}(6.393152)$$

$$\int_0^1 \sqrt{x}\sin x\,dx \approx 0.351286$$

Ans:

Question No 6 :-

Estimate $\int_0^1 \frac{1}{1+x^2}\,dx$ to 6D using the Simpson rule by taking 10 - sub integrals and hence approximate the value of π.

Sol:- $\int_0^1 \frac{1}{1+x^2}\,dx$

$x = 6$

Scanned with CamScanner

$$\Delta x = \frac{b-a}{n}$$

$$\Delta x = \frac{1-0}{6} \Rightarrow \Delta x = \frac{1}{6}$$

x	y	
0	0	$\to y_0$
1/6	1/36	$\to y_1$
2/6	4/36	$\to y_2$
3/6	9/36	$\to y_3$
4/6	16/36	$\to y_4$
5/6	25/36	$\to y_5$
6/6 = 1	1	$\to$

$$\int_0^1 \frac{1}{1+x^2}\,dx \approx \frac{1/6}{3}\left[0 + 4\left(\frac{1}{36}\right) + 2\left(\frac{4}{36}\right) + 4\left(\frac{4}{36}\right) + 4\left(\frac{9}{36}\right) + 2\left(\frac{16}{36}\right) + 4\left(\frac{25}{36}\right) + 1\right]$$

$$\int_0^1 \frac{1}{1+x^2}\,dx \approx \frac{1}{18}(6.55554)$$

$$\boxed{\int_0^1 \frac{1}{1+x^2}\,dx \approx 0.3641966666}$$ Ans

Question No 7:

The accompanying table gives the speeds, in miles per seconds, at various times for a test rocket that was first upward from the surface of the earth. Use these values to approximate the number of miles traveled during the first 180 s. Round your answer to the nearest tenth of a mile.

Scanned with CamScanner

time t (s)	0	30	60	90	120	150	180
Speed v	0.00	0.03	0.08	0.16	0.27	0.42	0.6

Sol:- $n = 180$

As we know that

$d = 30 - 60 = 60 - 90 = 30$

where 30 is a common difference.

By the Arithmatic series,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

By putting the values we get

$$S_{180} = \frac{\cancel{180}\,90}{\cancel{2}}\left[2(0) + (180-1)30\right]$$

$$S_{180} = 90\left[0 + (179)30\right]$$

$$S_{180} = 90\left[179 \times 30\right]$$

$$S_{180} = 90(5370)$$

$$S_{18} = 483300 \text{ Ans:-}$$

Question No 8:

Determine the slope of the tangent line to the Polar curve $r = 1 + \cos\theta$ at $\theta = \pi/4$.

Sol:- $r = 1 + \cos\theta$

As;

$\theta = \pi/4$

By putting values we get

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$$

$$\frac{dy}{dx} = \frac{(-\cos\theta)(\sin\theta) + r\cos\theta}{(-\cos\theta)(\cos\theta) + r\sin\theta}$$

Scanned with CamScanner

M T W T F S

Date:

$$\frac{dy}{dx} = \frac{(-\cos\theta)(\sin\theta) + (1+\cos\theta)(\cos\theta)}{(1-\cos\theta)(\cos\theta) + (1+\cos\theta)(\sin\theta)}$$

Putting the value of $\left[\theta = \frac{\pi}{4}\right]$

$$\frac{dy}{dx} = \frac{(1-\cos\pi/4)(\sin\pi/4) + (1+\cos\pi/4)(\cos\pi/4)}{(-\cos\pi/4)(\cos\pi/4) + (1+\cos\pi/4)(\sin\pi/4)}$$

$$\frac{dy}{dx} = \frac{(-0.9)(0.013) + (1.9)(0.9)}{(-0.9)(0.9) + (1.9)(0.013)}$$

$$\frac{dy}{dx} \quad \frac{-0.0117 + 1.71}{-0.81 + 0.0247}$$

$$\frac{dy}{dx} = \frac{\cancel{-}[(0.0117) - (1.71)]}{\cancel{-}[(0.81) - (0.0247)]}$$

$$\frac{dy}{dx} \quad \frac{0.0117 - 1.71}{0.81 - 0.0247}$$

$$\frac{dy}{dx} = \frac{-1.6983}{0.7853}$$

$$\boxed{\frac{dy}{dx} \quad -2.162 \text{ Ans}}$$

Question No 9:

Find the length of the curve $x = 8\sin^3(\theta/3)$ $0 \le \theta \le \pi/4$

Sol:- $x = 8\sin^3\left(\frac{\theta}{3}\right)$

Squaring bs we get

$$x^2 = 64\sin^6\left(\frac{\theta}{3}\right)$$

Scanned with CamScanner

$$\frac{dx}{d\theta} = 24 \sin^2\left(\frac{\theta}{3}\right) \times \cos\left(\frac{\theta}{3}\right) \times \left(\frac{1}{3}\right)$$

$$\frac{dx}{d\theta} = 8 \sin^2\left(\frac{\theta}{3}\right) \times \cos\left(\frac{\theta}{3}\right)$$

$$\left(\frac{dx}{d\theta}\right)^2 = 64 \sin^4\left(\frac{\theta}{3}\right) \cos^2\left(\frac{\theta}{3}\right)$$

As we know that:

$$L = \int_{\alpha}^{\beta} \sqrt{\left(x^2 + \frac{dx}{d\theta}\right)^2} \, d\theta \quad \text{— (i)}$$

$$= \int_0^{\pi/4} \sqrt{64 \sin^6\left(\frac{\theta}{3}\right) + \left[64 \sin^4\left(\frac{\theta}{3}\right) \cos^2\left(\frac{\theta}{3}\right)\right]} \, d\theta$$

$$= \int_0^{\pi/4} \sqrt{64 \sin^6\left(\frac{\theta}{3}\right) + \left[64 \sin^4\left(\frac{\theta}{3}\right) \cos^2\left(\frac{\theta}{3}\right)\right]} \, d\theta$$

$$= \int_0^{\pi/4} \sqrt{64 \sin^4\left(\frac{\theta}{3}\right)(1)} \, d\theta \qquad \boxed{\because \sin^2 x + \cos^2 x = 1}$$

$$= \int_0^{\pi/4} \sqrt{64} \cdot \sqrt{\sin^4\left(\frac{\theta}{3}\right)} \, d\theta$$

$$= 8 \int_0^{\pi/4} \sin^2\left(\frac{\theta}{3}\right) d\theta \qquad \boxed{\because \sin^2\theta = \frac{1 - \cos 2\theta}{2}}$$

$$= \cancel{8}^{4} \int_0^{\pi/4} \frac{1 - \cos 2(\theta/3)}{\cancel{2}} \, d\theta$$

$$= 4 \int_0^{\pi/4} \left(1 - \cos \frac{2\theta}{3}\right) d\theta$$

$$L = 4 \left[\theta - \frac{3}{2} \sin \frac{2\theta}{3}\right]_0^{\pi/4}$$

Scanned with CamScanner

AWP

M T W T F S

Date: .....

$$L = 4\left[\left(\frac{\pi}{4}\right) - \frac{3}{2}\sin\left(\pi/6\right)\right]$$

$$L = 4\left[\left(\frac{\pi}{4}\right) - \frac{3}{2}\cdot\frac{1}{2}\right]$$

$$\boxed{L = \pi - 3}$$ Ans:-

Scanned with CamScanner

Date:

Question No 10:

Find the area of the region inside one leaf of the three-leaved rose?

Sol:

$r = \sin\theta$

If $\sin 3\theta = 0$, then $\sin\theta = 0$

at $\theta = 0, \pi, 2\pi$

For

$\sin 3\theta$ it will be 0 at θ.

So,

$\pi/3$, $2\pi/3$

So the curve in the first quadrant vairs from

$\theta = 0$ to $\theta = \pi/3$

$$\text{Area} = \frac{1}{2}\int_{\alpha}^{\beta} r^2 \, d\theta$$

For one loop corresponding integral is

$$A = \frac{1}{2}\int_0^{\pi/3} (\sin 3\theta)^2 \, d\theta$$

$$A = \frac{1}{2} \times \frac{1}{2}\int_0^{\pi/3} (1 - \cos 6\theta) \, d\theta$$

$$A = \frac{1}{4}\int_0^{\pi/3} (1 - \cos 6\theta) \, d\theta$$

By Integrating

$$A = \frac{1}{4}\left[\theta - \frac{1}{6}\sin\theta\right]_0^{\pi/3}$$

$$A = \frac{1}{4}\left[\left(\frac{\pi}{3} - \frac{1}{6}\sin 2\pi\right) - \left(0 - \frac{1}{6}\sin 0\right)\right]$$

$$A = \frac{1}{4}\left[\frac{\pi}{3} - 0 - 0\right]$$

$$A = \frac{1}{4}\left(\frac{\pi}{3}\right) \Rightarrow \boxed{A = \frac{\pi}{12}}$$ Ans.

Abdul Wahab Paper

Scanned with CamScanner

Question No 11:

we know that
Area between the curve

$$A = \frac{1}{2}\int_{\alpha}^{B} (r_1^2 - r_2^2)\, d\theta$$

as $A = \frac{1}{2}\int_{\alpha}^{B} r_1^2\, d\theta - \frac{1}{2}\int_{\alpha}^{B} (r_2^2)\, d\theta \Rightarrow r = 2\cos\theta$

for finding limits of Integration, we have.

$r = \sqrt{\cos 2\theta}$

$\because 0 = \sqrt{\cos 2\theta}$

$\Rightarrow 0 = \cos 2\theta$

$\Rightarrow 2\theta = \cos^{-1}(0) = \pi/2$

$\therefore \theta = \frac{\pi}{4}$

$r = 2\cos\theta$

$\because \theta = 2\cos\theta$

$\Rightarrow 0 = \cos\theta$

$\Rightarrow \theta = \cos^{-1}(0)$

$\theta = \frac{\pi}{2}$

$$① \Rightarrow A = \frac{1}{2}\int_{0}^{\pi/2} (2\cos\theta)^2\, d\theta - \frac{1}{2}\int_{0}^{\pi/4} \left(\sqrt{\cos 2\theta}\right)^2 d\theta$$

Scanned with CamScanner

$$A = \frac{1}{2} \times 4^2 \int_0^{\pi/2} (2\cos\theta)^2 \, d\theta - \frac{1}{2} \int_0^{\pi/4} \left(\sqrt{\cos 2\theta}\right)^2 d\theta$$

$$A = 2 \int_0^{\pi/2} \left(\frac{1+\cos 2\theta}{2}\right) d\theta - \frac{1}{2} \left[\frac{\sin 2\theta}{2}\right]_0^{\pi/4}$$

$$A = \frac{2}{2} \int_0^{\pi/2} (1+\cos 2\theta) \, d\theta - \frac{1}{4} \left[\sin 2\left(\frac{\pi}{4}\right) - \sin(0)\right]$$

$$A = \int_0^{\pi/2} 1 \, d\theta + \int_0^{\pi/2} \cos 2\theta \, d\theta - \frac{1}{4}(1-0)$$

$$A = [\theta]_0^{\pi/2} + [\sin 2\theta/2]_0^{\pi/2} - \frac{1}{4}$$

$$A = \frac{\pi}{2} - 0 + \frac{1}{2}\left[\sin 2\left(\frac{\pi}{2}\right) - \sin 2(0)\right] - \frac{1}{4}$$

$$A = \frac{\pi}{2} - 0 + \frac{1}{2}\left[\sin 2\left(\frac{\pi}{2}\right) - \sin 2(0)\right] - \frac{1}{4}$$

$$A = \frac{\pi}{2} - 0 + \frac{1}{2}\left[\sin 2\left(\frac{\pi}{2}\right) - \sin 2(0)\right] - \frac{1}{4}$$

$$A = \frac{\pi}{2} + \frac{1}{2}(0-0) - \frac{1}{4}$$

$$\boxed{A = \frac{\pi}{2} - \frac{1}{4}}$$ Ans:-

Scanned with CamScanner

Date:

Question No 12:

Which of the following sequence $\{a_n\}$ shown below coverge, & which diverge? Find the limit of each convergent sequence?

12 - $a_n = \frac{(\ln n)^5}{\sqrt{n}}$

Sol:

As; $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{d/dn[\ln(n)^5]}{d/dn(\sqrt{n})}$

$\therefore \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{5\ln(n)}{1/2(n)^{-1/2}}$

$a_n = \frac{5\ln n}{1/2(n)^{-1/2}}$

$a_n = \frac{10\ln n}{(n)^{-1/2}}$

$a_n = \frac{10(1/n)}{(n)^{-1/2}}$

$a_n = \frac{10\sqrt{n}}{n}$

Applying limit $n\to\infty$ $\quad \because \frac{d}{dn}(n) = 1$

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{10 \times 1/2 \, (n)^{-1/2}}{1}$

$= \lim_{n\to\infty} \frac{5}{\sqrt{n}}$

$= \frac{5}{\infty}$

$= 0$ (Converging) Ans.

Scanned with CamScanner

M T W T F S

Question No : 13

Sol:- $a_n = \left(\frac{3n+1}{3n-1}\right)^n$

$\ln a_n = \ln\left(\frac{3n+1}{3n-1}\right)^n$

$\ln a_n = n \ln\left(\frac{3n+1}{3n-1}\right)$

Applying Limits

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} n \ln\left(\frac{3n+1}{3n-1}\right)$

$= \lim_{n\to\infty} n \ln\left(\frac{3n+1}{3n-1}\right)$

$= \lim_{n\to\infty} \frac{\ln\left(\frac{3n+1}{3n-1}\right)}{1/n}$

By applying L- Hapitals Rule:

$= \lim_{n\to\infty} \frac{d}{dn} \ln\left(\frac{3n+1}{3n-1}\right)$

$\frac{d}{dn} = \left(\frac{1}{n}\right)$

$= \lim_{n\to\infty} \frac{-2/(9x^2-1)}{+1/n^2}$

$= \lim_{n\to\infty} \frac{2n^2}{9n^2-1}$

$= \frac{2}{9} \lim_{n\to\infty} \frac{n^2}{n^2-1}$

$= \frac{2}{9}$

$= \boxed{0.22222 \text{ Ans}}$

Abdul Wahab

Scanned with CamScanner

Question No 14:-

Find the sum of the series.

$$\sum_{n=1} \frac{4an}{(2n-1)^2 (2n+1)^2}$$

Sol:

$$\frac{40}{(2n-1)^2(2n+1)^2} = \frac{A}{(2n-1)} + \frac{B}{(2n+1)^2} + \frac{C}{(2n+1)} + \frac{D}{(2n+1)^2} \rightarrow (1)$$

xing b.s by L.C.M

$$40 = A(2n-1)(2n+1)^2 + B(2n+1)^2 + C(2n+1)(2n-1)^2 + D(2n-1)^2 \rightarrow (ii)$$

put value of ('n') from eq

$$2n-1 = 0$$

$$2n = 1$$

$$\boxed{n = \frac{1}{2}}$$

put the eq (ii),

$$40 = A\left[2\left(\frac{1}{2}\right)-1\right]\left[2\left(\frac{1}{2}\right)+1\right]^2 + B\left[2\left(\frac{1}{2}\right)+1\right]^2$$

$$+ C\left[2\left(\frac{1}{2}\right)+1\right]\left[2\left(\frac{1}{2}\right)-1\right]^2 + D\left[2\left(\frac{1}{2}\right)-1\right]^2$$

$$40 = A[(0)(4)] + B(4) + C(3)(0) + D(0)$$

$$40 = B(4)$$

$$\boxed{B = 10}$$

Hence, from the equation

$$2n+1 = 0$$

$$n = -\frac{1}{2}$$

Scanned with CamScanner

put in eq (ii)

$$40 = A\left[2\left(\frac{-1}{2}\right)-1\right]\left[2\left(\frac{-1}{2}\right)+1\right]^2 + B\left[2\left(\frac{-1}{2}\right)+1\right]^2 +$$

$$C\left[2\left(\frac{-1}{2}\right)+1\right]\left[2\left(\frac{-1}{2}\right)-1\right]^2 + D\left[2\left(\frac{-1}{2}\right)-1\right]^2$$

$$40 = A[(-2)(0)] + B(0) + C(0)(4) + D(4)$$

$$40 = D(4)$$

$$\boxed{D = 10}$$

Comparing the co-efficient of A and C.

$$A = (2n-1)(2n+1)^2$$
$$= (2n-1)[(2n)^2 + 2(2n)(1) + (1)]$$
$$= (2n-1)(4n^2+4n+1)$$

$$C = (2n+1)(2n-1)^2$$
$$= (2n+1)(4n^2-4n+1)$$

$$A(2n-1) + C(2n+1) = 40$$
$$A\,2n - \cancel{A} + C\,2n + \cancel{C} = 40$$
$$\cancel{2}(A+C) = \cancel{40}$$
$$\boxed{A+C = 20}$$

$$A - C = -\frac{1}{2} + \frac{1}{2}$$

$$A - C = -\frac{1}{2} + \frac{1}{2}$$

$$\boxed{\therefore A = -10}$$

$$C = 20 + 10$$

$$\boxed{C = 30}$$

putting in eq (1)

$$\frac{40}{(2n-1)^2(2n+1)^2} = \frac{-10}{(2n-1)} + \frac{10}{(2n-1)^2} + \frac{30}{(2n+1)} + \frac{10}{(2n+1)^2}$$

Scanned with CamScanner

M T W T F S

Date: .....

$$= \frac{-10}{2n-1} + \frac{10}{(2n-1)^2} + \frac{20}{(2n+1)} + \frac{10}{(2n+1)^2}$$

$$\sum_{k=1}^{\infty} \frac{40}{(2n-1)^2(2n+1)^2} = \left[\frac{-10}{1} + \frac{10}{1} + \frac{10}{9} + \frac{10}{9}\right] + \left[\frac{-10}{3} + \frac{10}{9} + \frac{6}{5} + \frac{2}{5}\right] +$$

$$\left[\frac{-2}{5} + \frac{2}{7} + \frac{20}{7} + \frac{40}{49}\right] + \left[\frac{-10}{7} + \frac{10}{49} + \frac{10}{3} + \frac{10}{81}\right] +$$

$$\left[\frac{-10}{(2n-1)} + \frac{10}{(2n-1)^2} + \frac{10}{(2n+1)} + \frac{10}{(2n+1)^2} + \cdots\right]$$

$$= \left(\frac{100}{9}\right) + \left(\frac{188}{45}\right) + \left(\frac{708}{245}\right) + \left(\frac{8860}{3969}\right) + \cdots +$$

$$S_n = 10 - \frac{10}{(2n+1)^2}$$

$$= 10 - \frac{10}{(4n^2+4n+1)}$$

$$= \cancel{10} - \frac{\cancel{10}}{\cancel{4}(n^2+n)+1}$$

$$= 5 - \frac{5}{(n^2+n)+1}$$

$$= 5 - \frac{5}{\infty^2+\infty+1}$$

$$= 5 - 0 \rightarrow$$

$$\boxed{S_n = 5}$$

Scanned with CamScanner

Date:

Question No 15 :

a) $$\sum_{n=1}^{\infty} \frac{(n+3)}{3!\,n!\,3^n}$$

Sol:-

Let

$$a_n = \frac{(n+3)}{3!\,n!\,3^n}$$

use ratio test to find the $\lim_{n\to\infty}$

$$\Rightarrow \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{(n+4)}{3!\,(n+1)!\,3^{n+1}} \cdot \frac{(n+3)!}{3!\,(n)!\,3^n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(n+4) \cdot 3!\,n!\,3^n}{(n+3)!\;3!\,(n+1)!\,3^{n+1}} \right|$$

$$= \frac{1}{3} \cdot \lim_{n\to\infty} \left| \frac{n+4}{n+1} \right|$$

$$= \frac{1}{3} \cdot 1$$

$$= \frac{1}{3}$$ ~~Ans~~.

As limit is $\frac{1}{3} < 1$, so by the ratio test series

Scanned with CamScanner

Converges absolutely.

b) $$\sum_{n=1}^{\infty} \frac{(n!)^n}{(n^n)^2}$$

Sol:- $$\sum_{n=1}^{\infty} \frac{(n!)^n}{(n^n)^2}$$

For the limit $\lim_{n\to\infty} \sqrt[n]{a_n}$

$$\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\frac{(n!)^n}{(n^n)^2}}$$

$$= \lim_{n\to\infty} \frac{n!}{n^2}$$

$$= \infty \quad \text{Ans.}$$

Hence, From the equation due to root test, series diverges.