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Sheet # 1

Question No. 1:-

(a) How many arrangements of the letters of the following words, taken all together, can be made?

(1) FASTING

Solution:-

Let $n=7$ and $r=7$

$${}^nP_r = \frac{n!}{(n-r)!} = \frac{7!}{(7-7)!} = \frac{7!}{(0!)} = \frac{7!}{1}$$

$${}^nP_r = 7! = 5040 \text{ Ans}$$

(2) MATHEMATICS

Solution:-

No. of expected letters $M = 2!$

No. of expected letters $A = 2!$

No. of expected letters $T = 2!$

Let $n = 11$

$$\text{Required no. of words} = \frac{11!}{2!\times 2!\times 2!} = \frac{11\times 10\times 9\times 8!}{2\times 1\times 2\times 1\times 2\times 1} = \frac{399168}{8}$$

$$= 4989600 \text{ Ans}$$

(b) Find the numbers greater than 230000 that can be formed from the digits 1, 2, 3, 4, 5, 6 without repeated any digit?

Solution:-

Let $n=6$ and $r=6$

$${}^nP_r = \frac{n!}{(n-r)!} \Rightarrow {}^6P_6 = \frac{6!}{(6-6)!} = \frac{6!}{0!} = \frac{6!}{1}$$

$${}^6P_6 = 720$$

Question No. 2:-

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(a) In how many ways can 5 boys and 4 girls be seated on a bench, that the girls and the boys occupy alternate seats?

Solution:-

There are 5 boys and 5 slots are available for them it means that they can be seated in 5! ways

Similarly, There are 4 girls and 4 slots are available for them it means that they can be seated in 4! ways

To find out their ways of alternate seats, we have to multiply the seated arrangement of boys and girls.

$B \times G = 5! \times 4! = (5\times4\times3\times2\times1)(4\times3\times2\times1)$

$B \times G = (120)(24) = 2880$ ways Ans

(b) There are 8 men and 10 women members of a club. How many committees of 7 can be formed, having:

(1) 4 women

Solution:-

Men = 8

Women = 10

Committees = 7

P(4 women) = 7 - 4 = 3 men

P(4 women) = ${}^{10}C_4 \times {}^{8}C_3 = 11760$ Ans

(2) At most 4 women

Solution:-

Men = 8

Women = 10

Committees = 7

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$P(\text{At most } 4 \text{ women}) = P(X \leq 4)$

$P(X \leq 4) = (3 \text{ men and } 4 \text{ women}) + (4 \text{ men and } 3 \text{ women}) + (5 \text{ men and } 2 \text{ women}) + (6 \text{ men and } 1 \text{ women}) + (7 \text{ men and } 0 \text{ women})$

$= (8C_3 \times 10C_4) + (8C_4 \times 10C_3) + (8C_5 \times 10C_2) + (8C_6 \times 10C_1) + (8C_7 \times 10C_0)$

$P(X \leq 4) = 11760 + 8400 + 2520 + 280 + 8$

$P(X \leq 4) = 22968$ Ans

(3) At least 4 women

Solution:-

Men = 8

Women = 10

Committees = 7

$P(\text{At least } 4 \text{ women}) = P(X \geq 4)$

$P(X \geq 4) = (3 \text{ men and } 4 \text{ women}) + (2 \text{ men and } 5 \text{ women}) + (1 \text{ men and } 6 \text{ women}) + (0 \text{ men and } 7 \text{ women})$

$= (8C_3 \times 10C_4) + (8C_2 \times 10C_5) + (8C_1 \times 10C_6) + (8C_0 \times 10C_7)$

$P(X \geq 4) = 11760 + 7056 + 1680 + 120$

$P(X \geq 4) = 20616$ Ans.

Question No. 3:-

The table below represents the college degrees awarded in a recent academic year by gender.

Date:

	Bachelor's B	Masters M	Doctorate D
Men	573,079	211,381	24,341
Women	775,424	301,264	21,683
	1348503	512645	46024

Total
808,801
1098371
1907172

Choose a degree at random. Find the probability that it is

(a) A bachelor's degree

Solution:-

$$P(B) = \frac{n(B)}{n(S)} = \frac{1348503}{1907172} = 0.707$$

(b) A doctorate or a degree awarded to a woman

Solution:-

$$P(D) = \frac{n(D)}{n(S)} = \frac{46024}{1907172} = 0.0241$$

$$P(W) = \frac{n(W)}{n(S)} = \frac{1098371}{1907172} = 0.576$$

$$P(D \text{ and } W) = \frac{21683}{1907172} = 0.0114$$

The probability is

$$P(D \text{ or } W) = P(D) + P(W) - P(D \cap W)$$

$$= 0.0241 + 0.576 - 0.0114$$

$$P(D \text{ or } W) = 0.5887$$

(c) A doctorate awarded to a woman

Solution:-

$$P(DW) = \frac{n(DW)}{n(S)} = \frac{21683}{1907172} = 0.0114$$

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(d) Not a master's degree

Solution:-

(1) Finding a master's degree and the probability is subtracted from 1.

$$P(M) = \frac{n(M)}{n(S)} = \frac{512625}{1907172} = 0.269$$

$$P(\overline{M}) = 1 - P(M) = 1 - 0.269 = 0.732$$

(2) Finding a Bachelor's and Doctrates degree. At th end The probability is added.

$$P(B) = \frac{n(B)}{n(S)}$$

$$= \frac{1348503}{1907172} = 0.707$$

$$P(D) = \frac{n(D)}{n(S)}$$

$$P(D) = \frac{46024}{1907172} = 0.0241$$

$$P(B \text{ or } D) = P(B) + P(D)$$
$$= 0.707 + 0.0241$$
$$= 0.731 \text{ Ans}$$

Question No. 4:-

(a) Two dice are thrown simultaneously. If the event A is that the sum of the number of dots shown in an odd number and the event B is that the number of dots shown on atleast one die is 3. Find P(A or B)

Solution:-

When two dice are thrown the possible outcomes are:-

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$n(S) = 36$

Since, A be the event that the sum of dots is an odd number then the favourite outcomes are:-

(1,2), (1,4), (1,6), (2,1), (2,3), (2,5), (3,2), (3,4), (3,6), (4,1), (4,3), (4,5), (5,2), (5,4), (5,6), (6,1), (6,3), (6,5).

$n(A) = 18$

Since, B be the event that the atleast one die has 3 dot on it then the outcomes are:-

(1,3), (2,3), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,3), (5,3), (6,3) $\Rightarrow n(B) = 11$

So, A and B have common outcomes (2,3), (3,2), (3,4), (3,6), (4,3), (6,3) $\Rightarrow n(A \cap B) = 6$

$P(A \text{ or } B) = P(A \cup B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{18}{36} + \frac{11}{36} - \frac{6}{36}$

$P(A \cup B) = \frac{23}{36}$ Ans.

(b) Two cards from a deck of 52 playing cards are drawn in such a way that the card is replaced after the first draw. Find the probabilities in the following cases:

(1) The first card is King and the second is Queen

Solution:-

The sample space is $n(S) = 52$

$P(A) = \frac{n(A)}{n(S)} = \frac{4}{52}$

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$$P(B) = \frac{n(B)}{n(S)} = \frac{4}{52}$$

$$P(A \cap B) = P(A) \cdot P(B)$$

$$= \frac{4}{52} \times \frac{4}{52} = \frac{16}{2704} = 0.6\%$$

(2) Both the card are faced cards

Solution:-

$$P(\text{Both are faced cards}) = \frac{16}{52} \times \frac{15}{51} = \frac{240}{2652}$$

$$= 9.05\%$$

Question No. 5:-

The gift basket store had the following premade gift baskets containing the following combination in stock.

	Cookies	Mugs	Candy	Total
Coffee	20	13	10	43
Tea	12	10	12	34
	32	23	22	77

Choose 1 basket at random. Find the probability that it contains

(a) Coffee or candy

Solution:-

$$P(\text{Coffee}) = \frac{n(\text{Coffee})}{n(S)} = \frac{43}{77} = 0.56$$

$$P(\text{Candy}) = \frac{n(\text{Candy})}{n(S)} = \frac{22}{77} = 0.28$$

$$P(\text{Coffee or Candy}) = \frac{n(C\&C)}{n(S)} = \frac{10}{77} = 0.13$$

$$P(\text{Coffee or Candy}) = P(\text{Coffee}) + P(\text{Candy}) - P(C\&C)$$

$$= \frac{43}{77} + \frac{22}{77} - \frac{10}{77}$$

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$= 0.56 + 0.28 - 0.13 = 0.71$

(b) Tea given that it contain mugs

Solution:-

$P(\text{Tea}/\text{Mugs}) = \dfrac{10}{77} = 0.13$

(c) Tea and Cookies

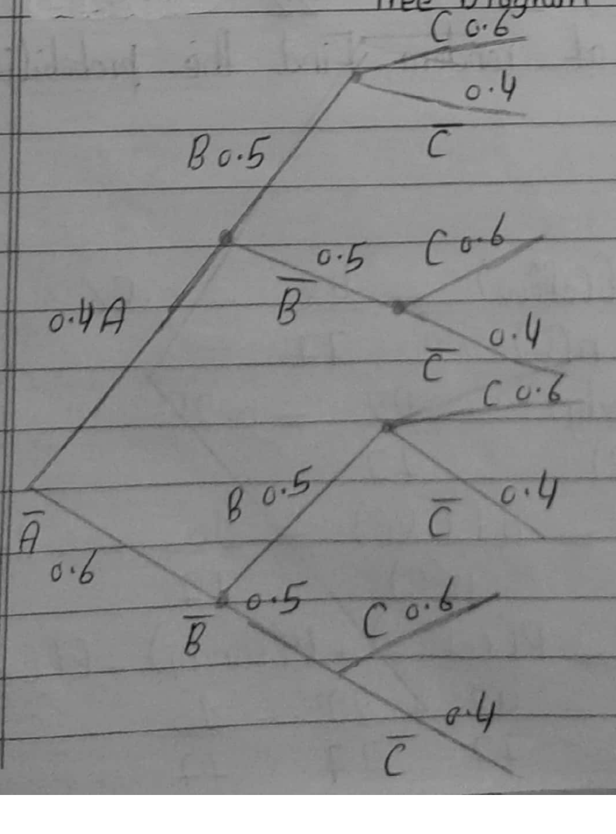
Solution:-

$P(\text{Tea and Cookies}) = \dfrac{12}{77} = 0.1558$

Question No. 6:-

Three missiles are fired at a target. If the probabilities of hitting the target are 0.4, 0.5 and 0.6 respectively and if the missiles are fired independently, what is the probability?

Tree Diagram



$ABC = 0.12$

$AB\bar{C} = 0.08$

$A\bar{B}C = 0.12$

$A\bar{B}\bar{C} = 0.08$

$\bar{A}BC = 0.18$

$\bar{A}B\bar{C} = 0.12$

$\bar{A}\bar{B}C = 0.18$

$\bar{A}\bar{B}\bar{C} = 0.12$

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Hitting Probability	Missing Probability
$A = 0.4$	$\bar{A} = 1 - 0.4 = 0.6$
$B = 0.5$	$\bar{B} = 1 - 0.5 = 0.5$
$C = 0.6$	$\bar{C} = 1 - 0.6 = 0.4$

(1) All the missiles hit the target

Solution:-

$$P(A \text{ and } B \text{ and } C) = P(A) \cdot P(B) \cdot P(C)$$
$$= (0.4)(0.5)(0.6)$$
$$= 0.12$$

(2) At least one of the three hits the target

Solution:-

Hit the target $= 0.12$

Not hit the target $= 1 - 0.12$

Atleast one hit the target $= 0.88$

(3) Exactly one hits the target

Solution:-

$$= 0.08 + 0.12 + 0.18$$
$$= 0.38$$

(4) Exactly 2 hit the target

Solution:-

$$= 0.08 + 0.12 + 0.18$$
$$= 0.38 \text{ Ans.}$$

Question No. 7:-

(a) Given $P(A) = 0.60$, $P(B) = 0.40$, $P(A \cap B) = 0.24$. Find $P(A \setminus B)$, $P(A \cup B)$, $P(A \setminus \bar{B})$, $P(B \setminus A)$, $P(\bar{B})$. What is the relation between A and B?

Solution:-

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$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$P(A \cup B) = 0.60 + 0.40 - 0.24$
$P(A \cup B) = 0.76$

$P(\bar{B}) = 1 - P(B)$
$P(\bar{B}) = 1 - 0.40 = 0.6$

$$P(A \backslash B') = \frac{P(A \cap B)'}{P(B)'}$$
$$= \frac{P(A) - P(A \cap B)}{1 - P(B)}$$
$$P(A \backslash B') = \frac{0.36}{0.6} = 0.6$$

$$P(A \backslash B) = \frac{P(A \cap B)}{P(B)}$$
$$P(A \backslash B) = \frac{0.24}{0.40} = 0.6$$

$$P(B \backslash A) = \frac{P(A \backslash B)(PB)}{P(A)}$$
$$P(B \backslash A) = \frac{(0.6)(0.40)}{0.60} = 0.4$$

(b) Given $P(A) = 0.5$ and $P(A \cup B) = 0.6$. Find $P(B)$ if

(1) A & B are mutually exclusive

Solution:–

$P(A \text{ or } B) = P(A \cup B)$
$P(A \cup B) = P(A) + P(B)$
$P(B) = P(A) - P(A \cup B)$
$-P(B) = 0.5 - 0.6 \rightarrow +P(B) = +0.1$

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$P(B) = 0.1$ Ans

(2) A & B are independent

Solution:-

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

$$P(A \text{ and } B) = (0.5) \cdot (0.1) = 0.05$$

(3) $P(A|B) = 0.4$

Solution:-

$$P(A) = \frac{P(A|B)}{P(B)}$$

$$P(B) = \frac{P(A|B)}{P(A)} = \frac{0.4}{0.5} = 0.8 \text{ Ans}$$

Question No. 8:-

Hospital records indicated that knee replacement patients stayed in the hospital for the number of days shown in the distribution.

Number of days stayed	Frequency
3	15
4	32
5	56
6	19
7	5
	F = 127

Find these probabilities

(a) A patient stayed exactly 5 days

Solution:-

$$P(E) = \frac{f}{n} = \frac{56}{127} = 0.4409$$

(b) A patient stayed less than 6 days

Solution:-

$$P(E) = \frac{f}{n} = \frac{15}{127} + \frac{32}{127} + \frac{56}{127} = \frac{103}{127}$$

$$= 0.8110$$

(c) A patient stayed at most 4 days

Solution:-

$$P(E) = \frac{f}{n} = \frac{15}{127} + \frac{32}{127} = \frac{47}{127} = 0.3700$$

(d) A patient stayed at least 5 days

Solution:-

$$P(E) = \frac{f}{n} = \frac{56}{127} + \frac{19}{127} + \frac{5}{127} = \frac{80}{127}$$

$$P(E) = 0.6299 \text{ Ans.}$$

Question No. 9:-

Sony produces its TV sets in 3 manufacture plants A, B & C

- Plant A produces 50% of the TV sets & the probability that a TV set manufactured here is defective is 0.02.
- Plant B produces 30% of the T.V sets & the probability of defective is 0.05.
- Plant C produces 20% of the TV sets & the probability that a TV set manufactured here is defective is 0.01.

(a) Make the probability Tree of this situation

(b) If a sony TV is randomly selected. What is the probability that it is defective?

(c) If a TV is selected at random & found to be defective then what is the probability

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that it was manufactured in plant B?

Solution:-

$$P(A) = 50\% = \frac{50}{100} = 0.5$$

$$P(D|A) = 0.02$$

$$P(B) = 30\% = \frac{30}{100} = 0.3$$

$$P(D|B) = 0.05$$

$$P(C) = 20\% = \frac{20}{100} = 0.2$$

$$P(D|C) = 0.01$$

Using total probability theorem

$$P(D) = P(D|A) \cdot P(A) + P(D|B) \cdot P(B) + P(D|C) \cdot P(C)$$

$$P(D) = (0.02) \cdot (0.5) + (0.05)(0.3) + (0.01) \cdot (0.2)$$

$$P(D) = 0.01 + 0.015 + 0.002$$

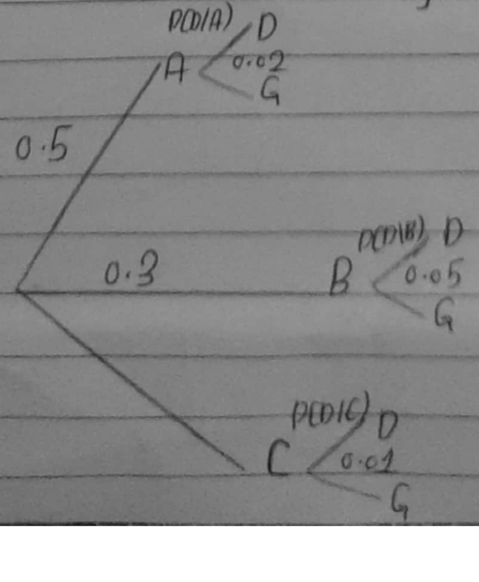
$$P(D) = 0.027$$

Now, By using Baye's Theorem

$$P(B|D) = \frac{P(B) \cdot P(D|B)}{P(D)} = \frac{(0.3)(0.05)}{0.027}$$

$$P(B|D) = \frac{0.015}{0.027} = 0.5$$

Tree Diagram



Join Events

$P(A) \cdot P(D|A)$

$P(B) \cdot P(\ |\ )$

$P(C) \cdot P(D|C)$

Here G = Good

D = Defective