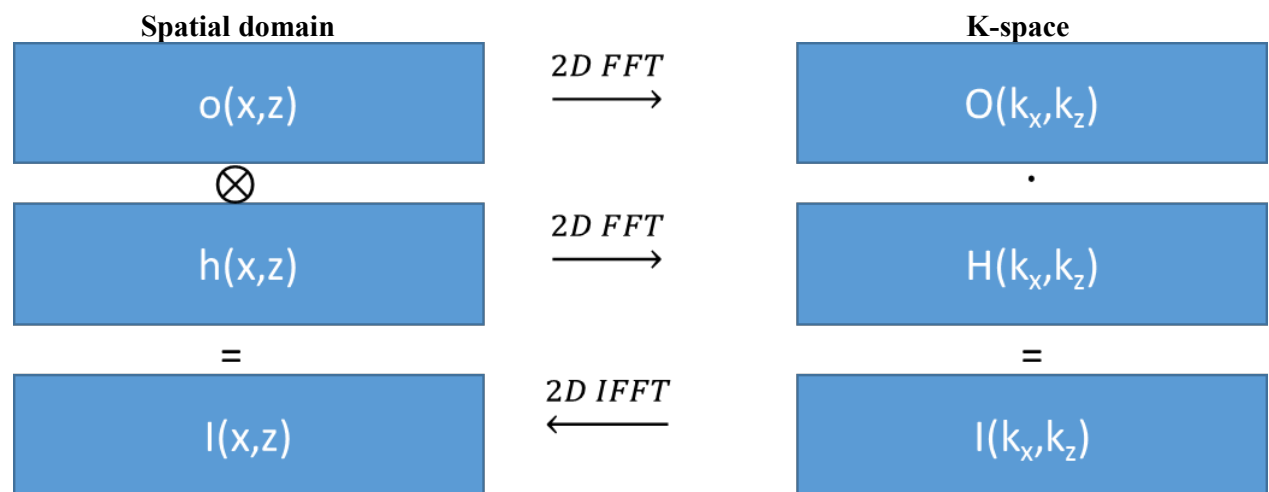# Exercise 8: K-space ultrasound simulation

Aim

Develop an understanding of the ultrasound imaging process in the spatial frequency domain, also referred to as *k-space*.

Background

In this exercise we will use a Matlab tool that simulates ultrasound imaging in the spatial frequency domain (in 2D). Using the simulation tool you will be able to change focus parameters, pulse frequency and pulse length for the imaging system, and the main part of the exercise is to comment on how these parameters affect the ultrasound image.

Spatial domain		K-space
$o(x,z)$	$\xrightarrow{2D\ FFT}$	$O(k_x,k_z)$
$\otimes$		$\cdot$
$h(x,z)$	$\xrightarrow{2D\ FFT}$	$H(k_x,k_z)$
$=$		$=$
$I(x,z)$	$\xleftarrow{2D\ IFFT}$	$I(k_x,k_z)$

Figure 1: Ultrasound imaging can be modelled as a convolution in space between the object function and the point spread function (left). Convolution in the spatial domain corresponds to multiplication in the spatial frequency domain (right). To go from one domain to the other we use the Fourier transform.

The transmitted pulse has a finite length that limits the resolution along the beam (the radial direction). The resolution in the azimuth direction is given by the transducer aperture size relative to the wavelength. This makes the ultrasound image a smeared version of the original object.

The point spread function in the frequency domain $H(k_x,k_z)$ can be approximated as the frequency response of the transmitted pulse (radial) multiplied with the frequency response of the aperture (azimuth).

According to the Fraunhofer approximation, the lateral variation of the continuous wave (CW) ultrasound field at the focal depth can be approximated as the Fourier transform of the aperture. The Fourier transform of this ultrasound field is thus a scaled version of the aperture. Extending this principle from CW to pulsed wave (PW) acquisition, containing more frequency components, the Frequency domain (K-space) representation of the ultrasound focal field is a lowpass-process in the lateral and elevation dimension and a bandpass process radially. This is illustrated in Fig. 2.
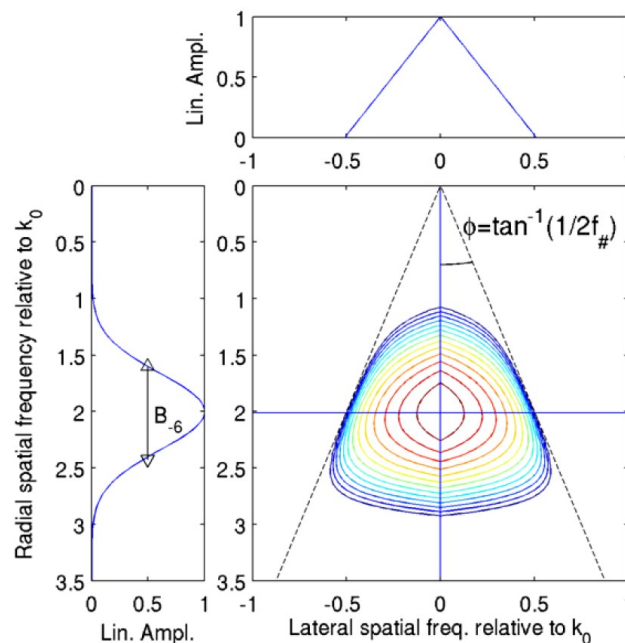
Figure 2: The bottom right pane shows a contour plot of a K-space point spread function (only showing positive radial frequencies); 3 dB separates each contour, with a total dynamic range of 40 dB. The top pane shows the aperture response at $k_z=2k_0$ in a linear scale, indicated by the horizontal line of the image. Likewise, the left pane shows the pulse response as indicated by the vertical line. The transmit- and receive apertures are equal and rectangular, which results in a triangular aperture response. The maximum lateral frequency content for each radial frequency is governed by the opening angle ϕ of the region of support.

Radially, the region of support is centered on $2k_0=2f_0/c$, and bounded by the pulse bandwidth. In the k_x and k_y directions, the region of support is bounded by the aperture function, where the lateral width increases linearly with increasing radial frequency, meaning that it is not a separable process.

Focusing parameters.
F-number: The F-number (f#) of a focused transducer gives the ratio between the focal distance and the aperture. A large F-number gives a long, wide focus, while a small F-number gives a short, narrow focus. The F-number can be different for transmission and reception.

Pulse parameters:
Pulse length: A short pulse provides the best resolution along the beam, which means it has a wide bandwidth in the frequency domain. Normally probes are limited to a relative bandwidth of about 60-70%, which means that the bandwidth of the pulse is 60-70% of the center frequency. This gives the lower limit for the transmitted pulse length.

Pulse frequency: A high frequency provides a better resolution both radially and laterally. Radially because the pulse length decreases when the frequency increases while holding the number of wavelengths constant. The lateral resolution increases because the aperture size is larger relative to the wavelength for higher frequencies.
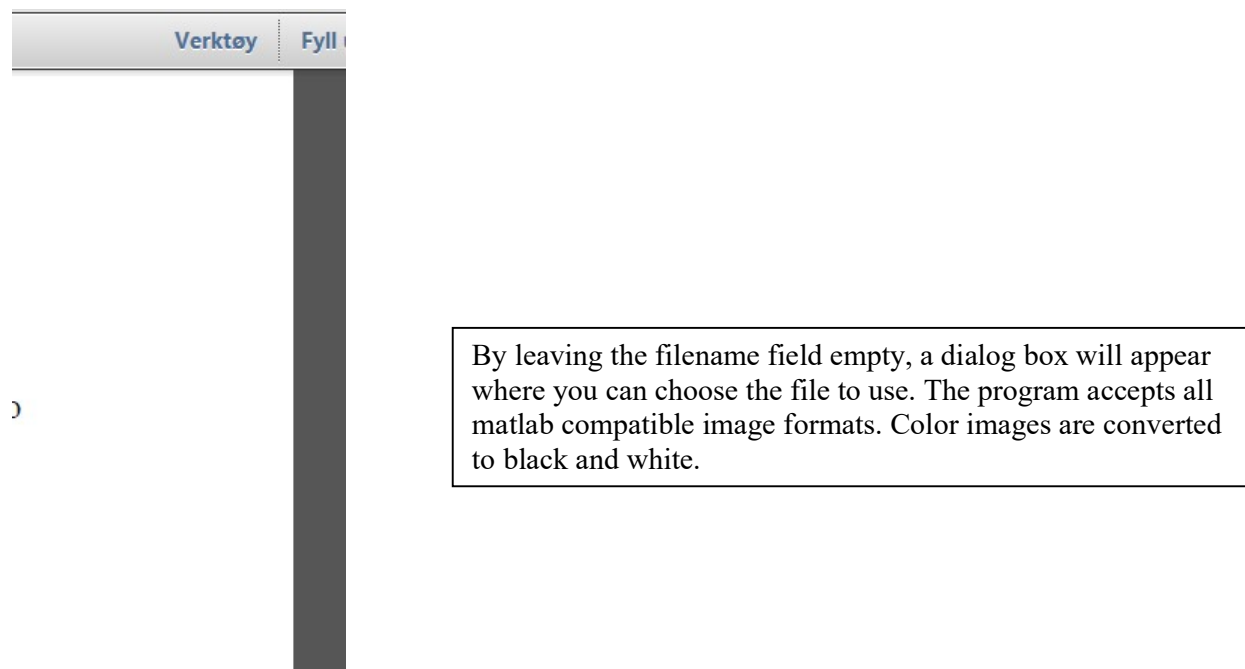
Exercises:
Download the zip-file with the matlab-code and example pictures from the course homepage. Unzip the files and start matlab. Go to the directory with the extracted files in matlab, and start the simulation program by typing
» echo2dsim
in the command window. The simulation program will store the previous parameters used. To start the program with the default parameters type:
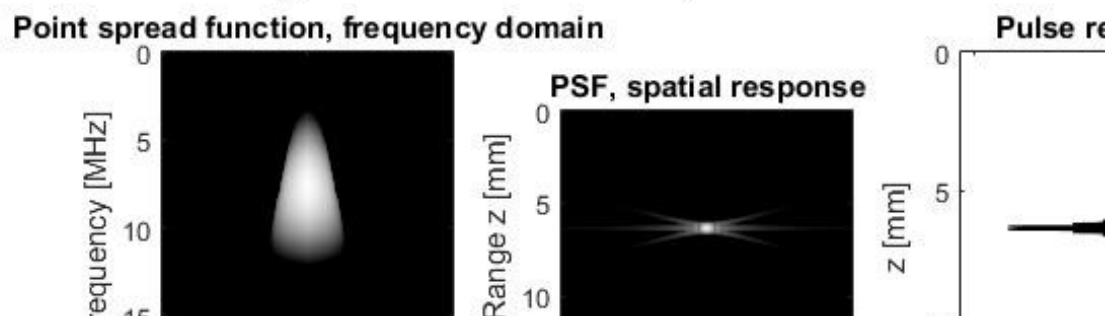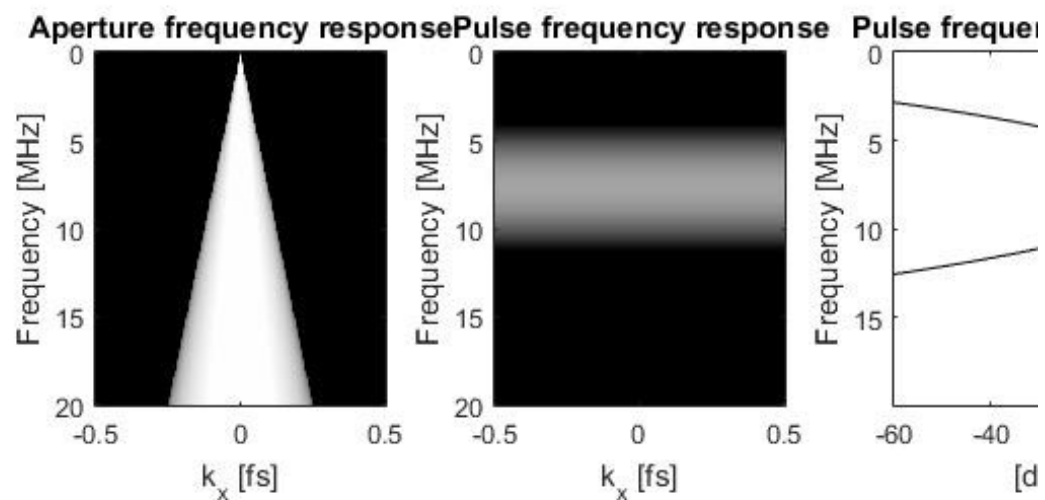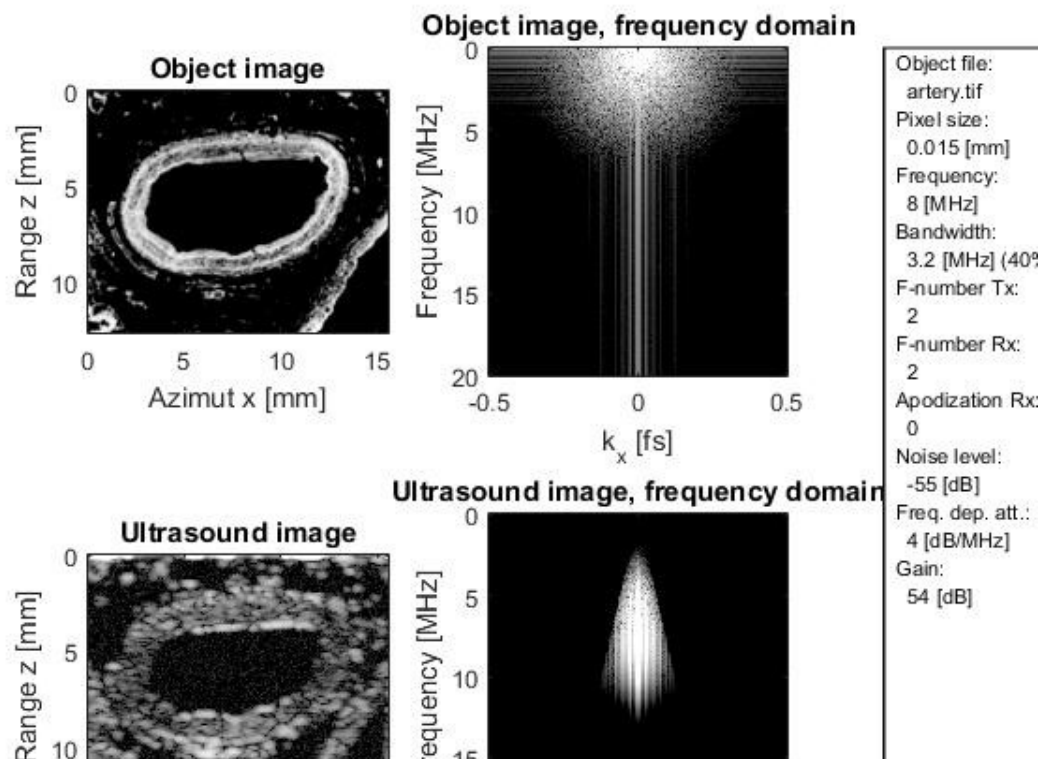**» echo2dsim(1)**

A dialog box will appear (Figure 3). You will be asked to specify the parameters for the simulation. All the fields have default settings, so the simulation will start when you press OK. Later in the exercise we will change the parameters.

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By leaving the filename field empty, a dialog box will appear where you can choose the file to use. The program accepts all matlab compatible image formats. Color images are converted to black and white.

Figure 3: Dialog box for specifying the simulation parameters.

In figure window 1 (Figure 4) the object image (upper) and the simulated ultrasound image (lower) will appear. To the left in spatial coordinates and in the middle in the frequency domain. If the resulting image is too bright or too dark use the buttons marked "+" and "-" to adjust the gain. The simulation parameters used are listed to the right in the figure window.

In figure window 2 (Figure 5) the response of the aperture (upper left corner) and the pulse in the frequency domain (upper middle right) will be shown. The point-spread function in the frequency domain (lower left) is the product of the aperture response and the pulse response. The point spread function in spatial coordinates is found by taking the inverse Fourier transform of the point spread function in the frequency domain. The lower right window shows the transmitted pulse. Use the buttons marked "Zoom+" and "Zoom-" to enlarge the pulse, for example to measure the pulse-length.

Object image

Object image, frequency domain

Object file:
  artery.tif
Pixel size:
  0.015 [mm]
Frequency:
  8 [MHz]
Bandwidth:
  3.2 [MHz] (40%
F-number Tx:
  2
F-number Rx:
  2
Apodization Rx:
  0
Noise level:
  -55 [dB]
Freq. dep. att.:
  4 [dB/MHz]
Gain:
  54 [dB]

Ultrasound image

Ultrasound image, frequency domain

Aperture frequency response

Pulse frequency response

Pulse frequen

Point spread function, frequency domain

PSF, spatial response

Pulse re

Part 1. Vary the pulse bandwidth. Comment on what happens with the resolution in the image when you:
i) Reduce the bandwidth to 25%
ii) Increase the bandwidth to 60%
Measure the length of the pulse in the two cases. What happens to the point spread function in the frequency domain?

Part 2. Set the bandwidth back to 40%, and vary the pulse frequency. Comment on what happens to the PSF in the frequency domain when you:
i) Reduce the frequency to 5 MHz.
ii) Increase the frequency to 10 MHz.
Use the buttons marked "+" and "-" to adjust the gain in the image.
Frequency dependent attenuation is included in the simulations. What effect will this have on the signal to noise ratio when you increase the frequency?

Part 3. Keep the frequency at 10 MHz and the bandwidth at 40%. What happens to the frequency response of the aperture when you:
i) Reduce the F-number for transmission and reception to 1, i.e. increase the aperture.
ii) Increase the F-number for transmission and reception to 3, i.e. reduce the aperture.
How does this affect the resolution in the image?

Part 4. Make an image with one horizontal, one vertical and 3-5 rotated lines at different angles. Use the included picture objects.bmp, and edit it in paint. Leave the background black, and the lines in different nuances of gray and white. Use wide lines. Run the simulation on the image and comment on how the simulated images look in space and in the frequency domain.

Part 5. Make another image with point noise, i.e. place white or gray dots on a black background. Use for example the spray can tool in Windows Paint. How does it look in the Fourier domain after imaging?

Part 6. Make another object with a circle, and comment on Fourier transform of it as well as the simulated image of the circle.

Part 7. For a one-way focused imaging system, the resolving power was first given by Lord Rayleigh as the distance from the peak to the first zero crossing in the diffraction pattern. This is called the Rayleigh criterion. The diffraction pattern of a one-way focused rectangular aperture is proportional to p(x)=sinc(x / f#λ), where sinc(x)=sin(π x)/(π x).

What is the lateral resolution of this system as given by the Rayleigh criterion?
How does this compare to the lateral resolution given by the two-sided -6dB beam width?
(Numerical solution is OK, for instance by plotting in matlab.)
What happens to the lateral resolution when the system is focused on both transmit and receive, using the same aperture?