

## Part 1

- a) The Nyquist velocity is given as:

$$\text{Nyquist Velocity: } v_{Nyq} = \frac{\lambda \text{ PRF}}{4} = \frac{c \text{ PRF}}{4f_0}$$

Where the center frequency = 5MHz, C=1540m/s, and PRF=5KHz. The velocity is then:  
V\_Nyq=0.385m/s

- b) For velocity 0.2: We see that the power spectrum is more narrow and close to zero. It is easy to see if the object is moving close or away from the source. This is due because we have many samples on the received pulse from the scatter, because it is moving slow. We can also see that the shift between each sample is very small in the IQ slow time signal.  
For velocity 0.5: Now we see that we have less samples on the received pulse from the scatter. Which means that the shift between each sample is larger in the IQ slow time, and we have a large velocity distribution in the power spectrum.  
For velocity 1: At this velocity we get a phase shift of 180 degrees for each sample, and as seen from the power spectrum, it is difficult to tell if the scatter is moving toward or away from the source. The velocity distribution is leaking in the negative part of the power spectrum.  
For velocity 1.5: At this point we have exceed the nyquist limit, and get aliasing. The velocity distribution of the scatter is to high than what we can sample. Is moving way too fast than what we can sample.

- c) To increase the “velocity resolution”(ability to distinguish velocities), we can this proportionality:

$$\text{Proportionality: } \frac{v_d}{v_{Nyq}} = \frac{f_d}{\text{PRF}/2}$$

Which yields:

$$v_d = \frac{f_d v_{Nyq}}{\text{PRF}/2} = \frac{c f_d}{2f_0}$$

The velocity resolution in the spectrum is given as:

$$\Delta v = \frac{2v_{Nyq}}{N_{\text{window}}}$$

To increase the velocity resolution, we can increase the observation window. By doing this one decreases the temporal resolution. Because the window length determines the spectral width, so by decreasing the window length, one increases the radial resolution but reduces the velocity resolution. So by increasing the window length, one increases the velocity

resolution but decreases the radial resolution, because we get a broader spectral width. One can also compensate this with a weighted window function.

## Part 2

- a) The doppler shift is given as:

$$f_d = 2f_0 \frac{v \cos \theta}{c_0}$$

- b) Assume that we use Continuous Wave (CW) Doppler to measure the blood velocity in a vessel. Assume that: the transmit frequency is 2.5 MHz, the speed of sound  $c=1540$  m/s, and the blood velocity is 1 m/s normal to the ultrasound beam.

- 1) Assuming that  $c \gg v$  then the observed doppler shift is given by:

$$f_d = 2f_0 \frac{v}{c_0}$$

The doppler shift is then: 3246,75Hz

- 2) If we steer the angle 45 degrees, then we use the formula from 2a) and get that the doppler shift is: 1705.6Hz

- c) We shall measure the blood velocity in a constricted vessel using Pulsed Wave (PW) Doppler. The vessel lies  $r=7.7$ cm under the skin. We have positioned the beam  $\phi = 45^\circ$  relative to the bloodflow direction. Because of the constriction (the stenosis) in the blood vessel, the maximum velocity in the systolic part of the heart cycle is  $v_{\max} = 1.5$ m/s.

- 1) The maximum unambiguous velocity if the PW doppler system is:

$$v_{Nyq} = \frac{c \text{ PRF}}{4f_0}$$

To avoid interfering echoes from previous pulses sent from transducer, the time interval between each transmitted pulse must be larger than the roundtrip time for the previous pulse at a specific depth  $z$ . This means that the PRF must be limited to:

$$\text{PRF} < \frac{c}{2z}$$

This means that PRF can't exceed:  $\text{PRF} < 1540 / (2 \cdot 0.077) = 10\,000$ Hz

- 2) To avoid aliasing, the doppler shift can't exceed  $\text{PRF}/2$ . This means that the maximum frequency is given as:

$$f_d = (2 \cdot f_0 \cdot v \cdot \cos(\theta)) / c;$$

$$\text{which gives us that } f_0 = (c \cdot f_d) / (2 \cdot v \cdot \cos(45)) = (1540 \cdot 5\text{kHz}) / (2 \cdot 1.5 \cdot \cos(45)) = 3.629\text{MHz}$$

Another way of doing it:

$$V_{\max} = v_{\text{true}} \cdot \cos(45) = 1.5 \cdot \cos(45) = 1.06\text{m/s}$$

$$f_0 = (c \cdot \text{PRF}) / (4 \cdot V_{\max}) = 3.629\text{MHz}$$

- 3) To calculate the velocity and doppler for the 30 degree angle. We first need to calculate the doppler shift:

$$f_d = (2 \cdot f_0 \cdot v \cdot \cos(30)) / c = 6122.349\text{Hz}$$

and now the velocity in at the same angle, but with the new calculated doppler shift.

Note that the doppler shift exceeds the nyquist limit, meaning we are getting aliasing.

$$v = (c \cdot f_d) / (2 \cdot f_0 \cdot \cos(30)) = 1.5\text{m/s}$$

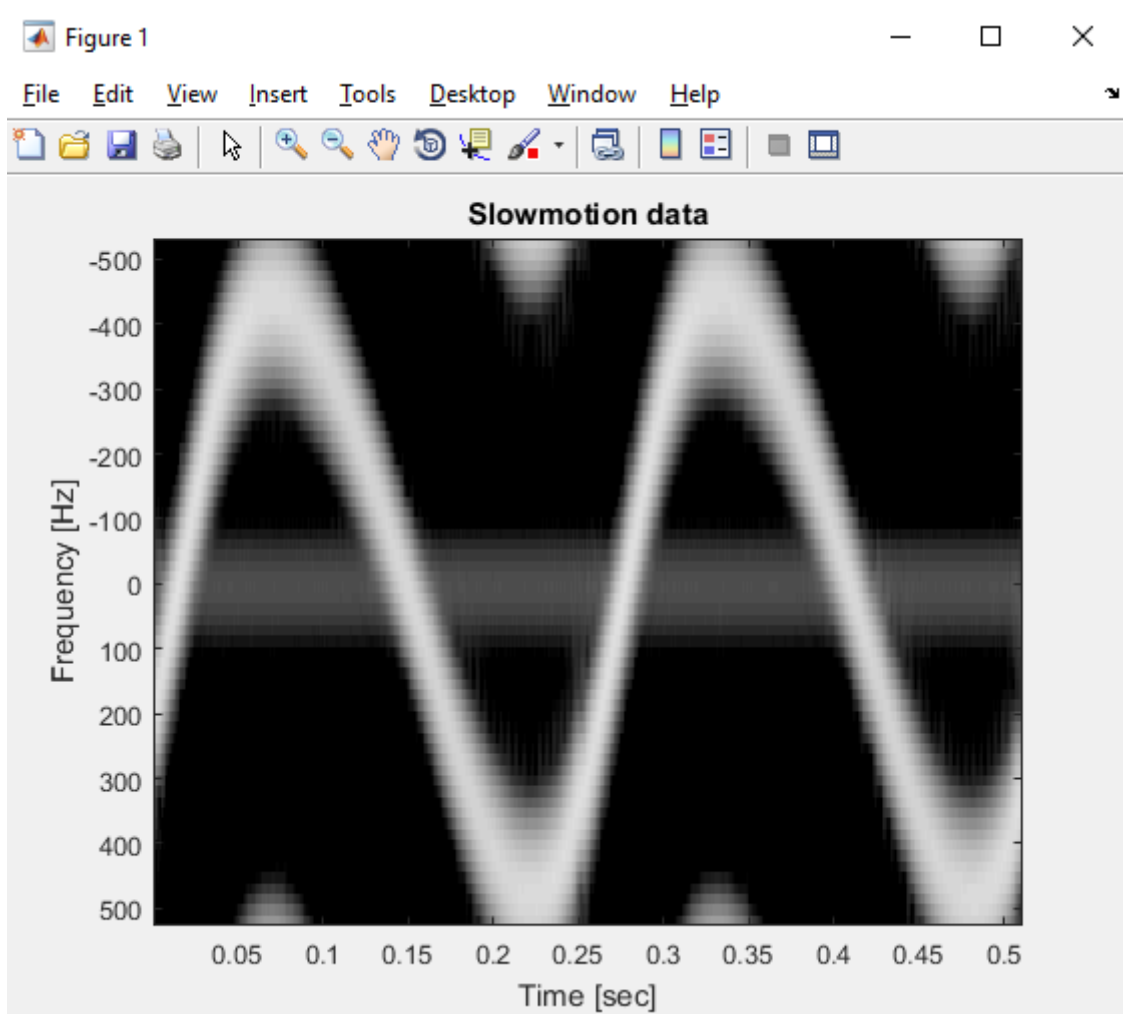
If we had used higher frequency, then alias would occur, and we would have trouble determining the direction of the velocity distribution.

- d) Argue whether these claims are true or untrue.

- 1) True: CW are continuously transmitting, giving no resolution. But since we have overlapping region from the receiving and transmitting aperture, we get a sensitivity.
- 2) True: Because the maximum velocity of the scatter is limited by the nyquist limit of the scatter. The maximum velocity is the nyquist velocity. And we also know that the  $PRF < c/2z$ . This gives us the inverse proportionality.
- 3) False: we have that  $v_{nyq} = (C \cdot PRF) / 4f_0$ , and the velocity resolution is given as:  $v_{res} = (2 \cdot v_{nyq}) / N_{window}$ . If we plunge in the formulas, we get the transmitted frequency in the denominator. This will then reduce the velocity resolution.
- 4) False: Color flow is a nice tool.
- 5) False: A short pulse is necessary to get a good spatial resolution. To increase the velocity resolution, one has to increase the length of the observation window.

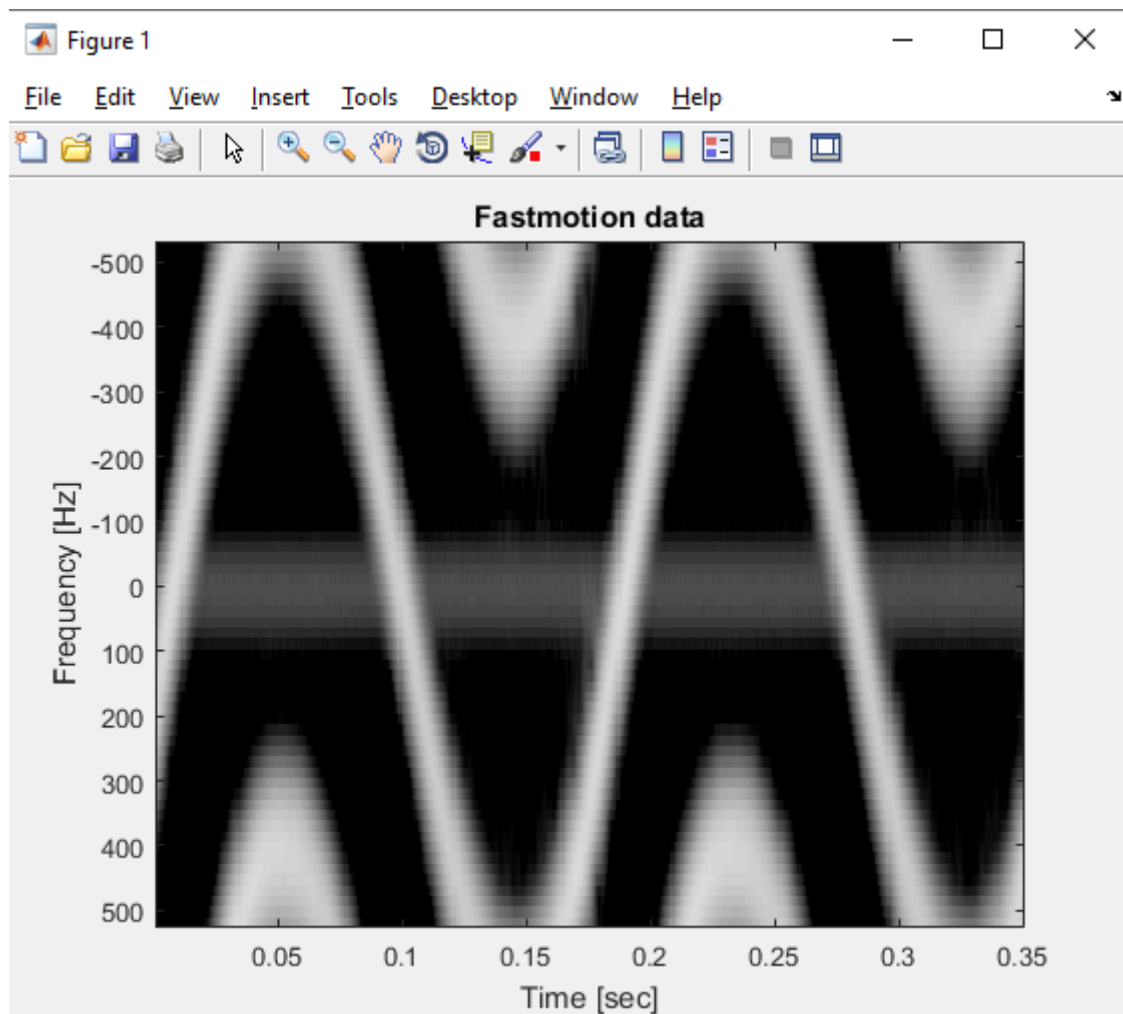
## Part 3

The middle beam for slow-time signal:

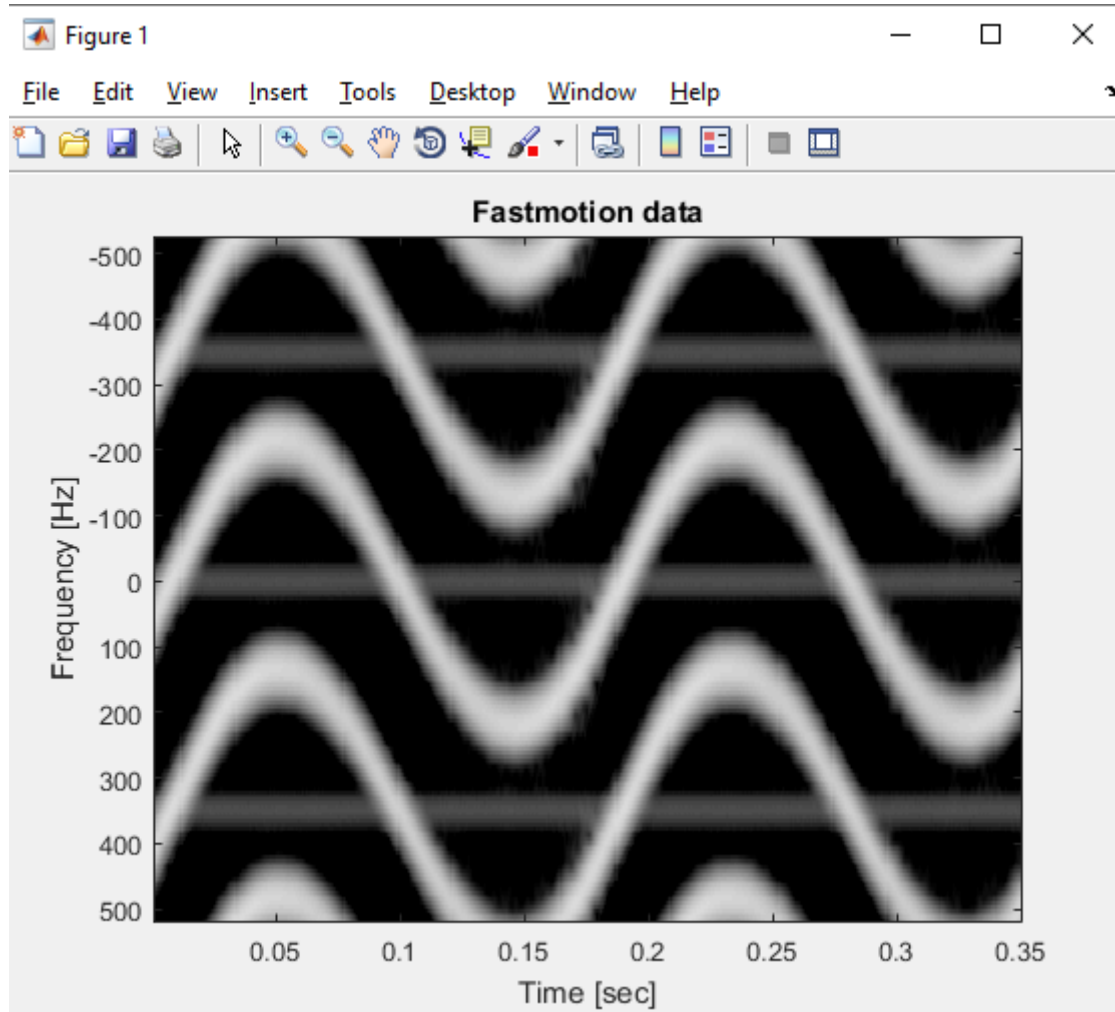


## Part 4

The middle beam for fast-time signal:



The middle beam from fast-time signal by stacking the doppler spectrum three times ( $P=[P,P,P]$ ), and extending the doppler axis:



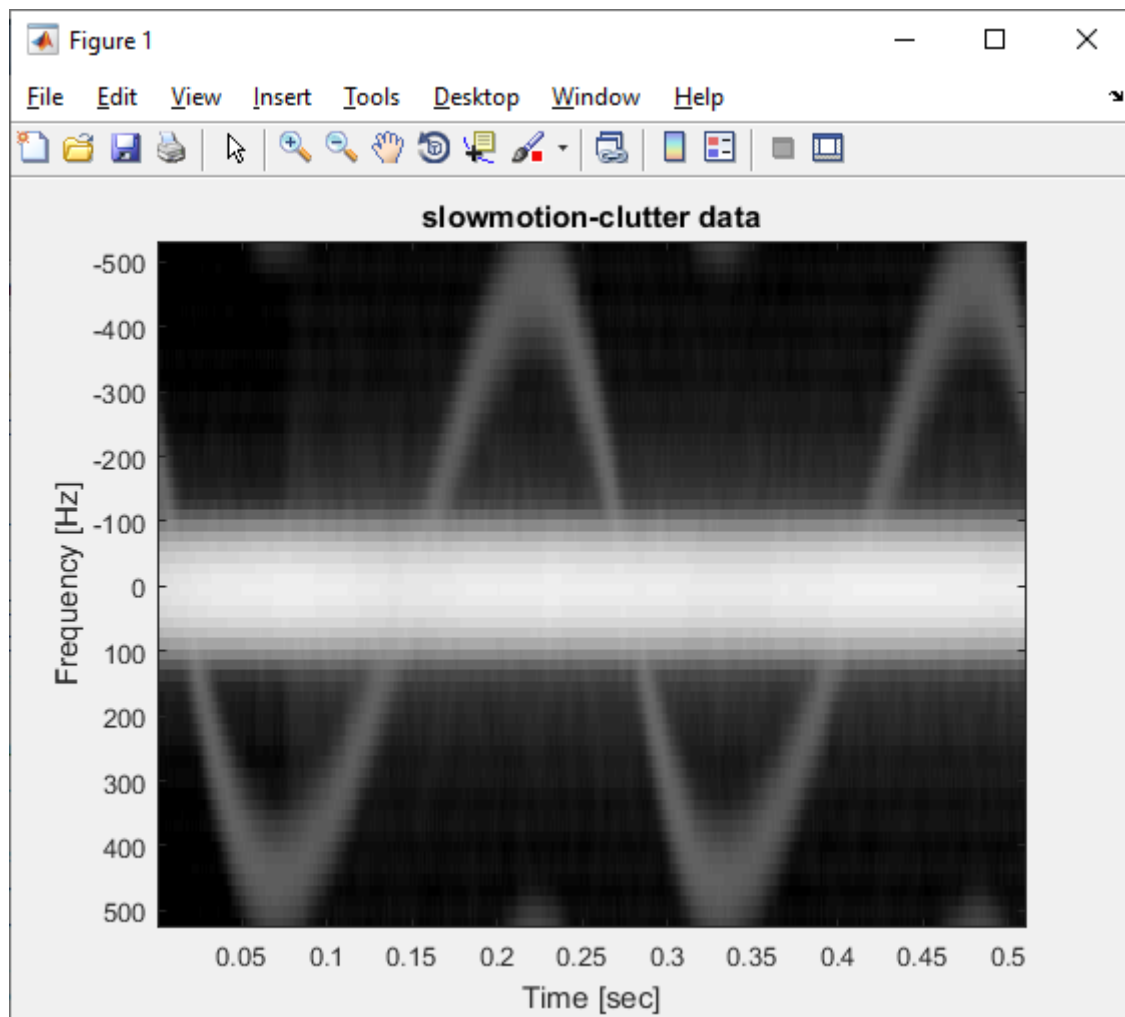
As we can see from the figure above, we get a maximum doppler shift frequency at roughly 300Hz. To calculate the Nyquist limit in this case:

$$v_{Nyq} = \frac{c PRF}{4f_0}$$

$$V_{nyq} = (1540 \cdot 3850) / (4 \cdot 2500000) = \pm 0.5929 \text{ m/s}$$

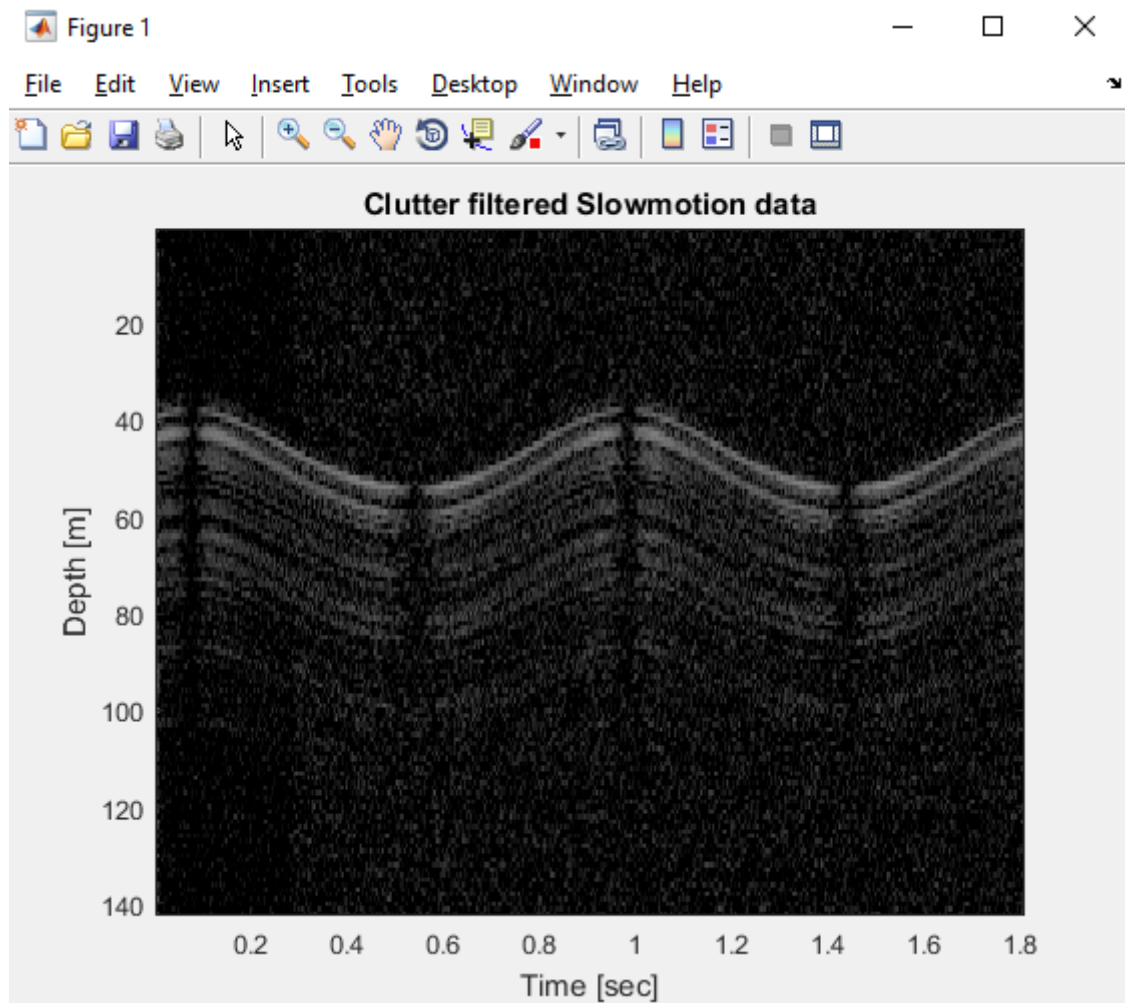
## Part 5

The signal from the slowmotion clutter file looks something like this:

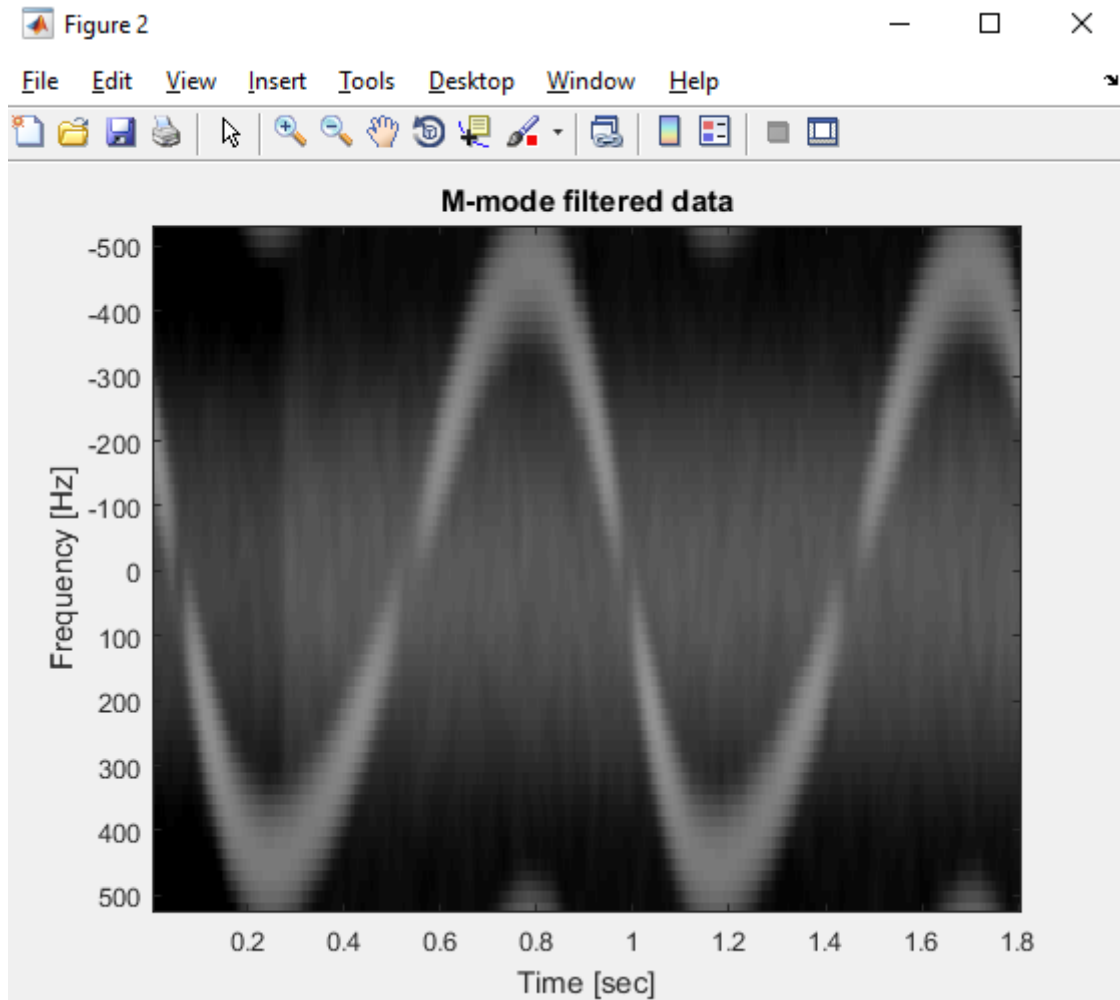


Here we can see a strong signal in the middle of the image. My guess is that the middle white line is the clutter, which may be a reflection from the tissue or reflection reverberation.

Here we can see the highpass filtered slowmotion\_clutter signal. In my case I have used the hamming when making the highpassfiler, and N=5 gave the best result:



The M-mode filtered signal looks like this:



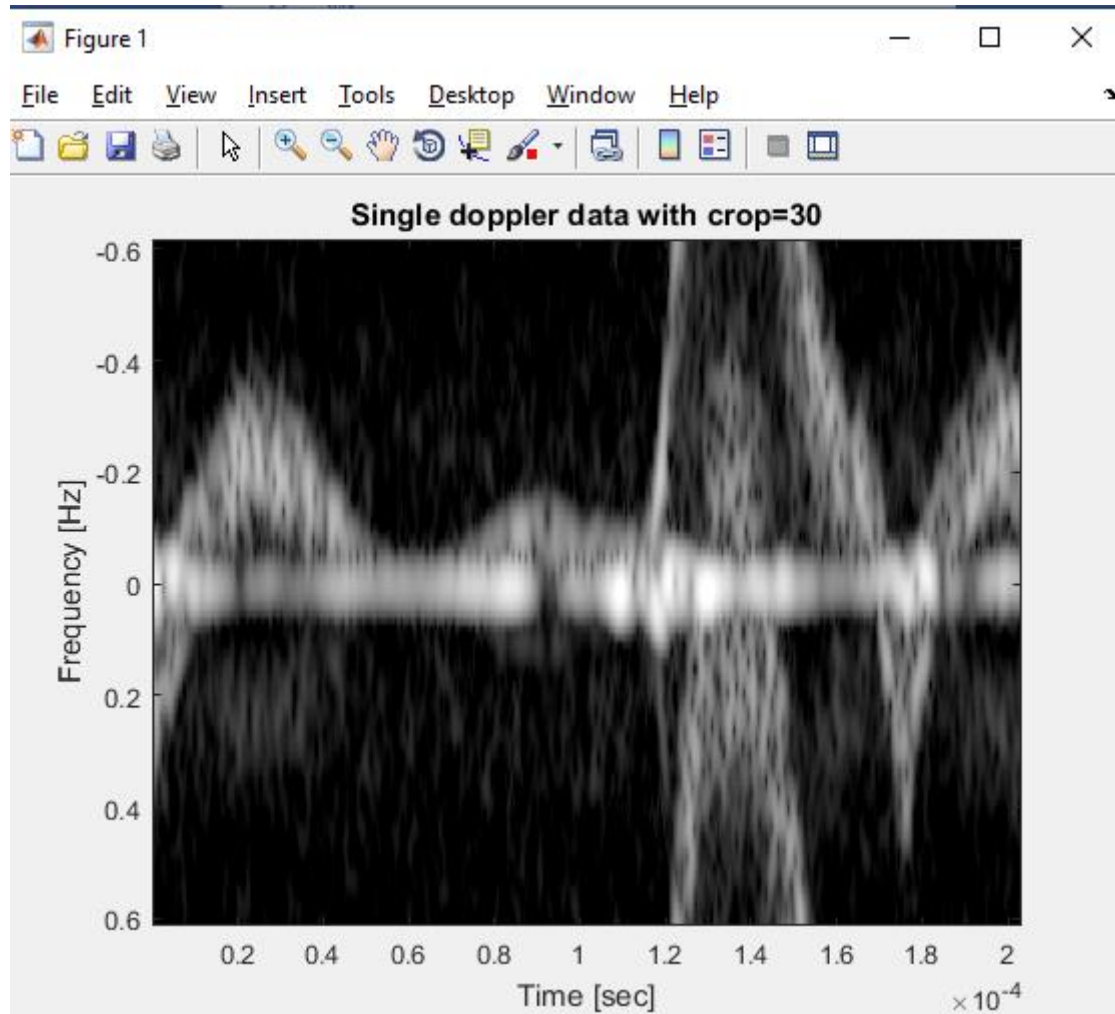
Here we can see that the clutter is removed from the signal.

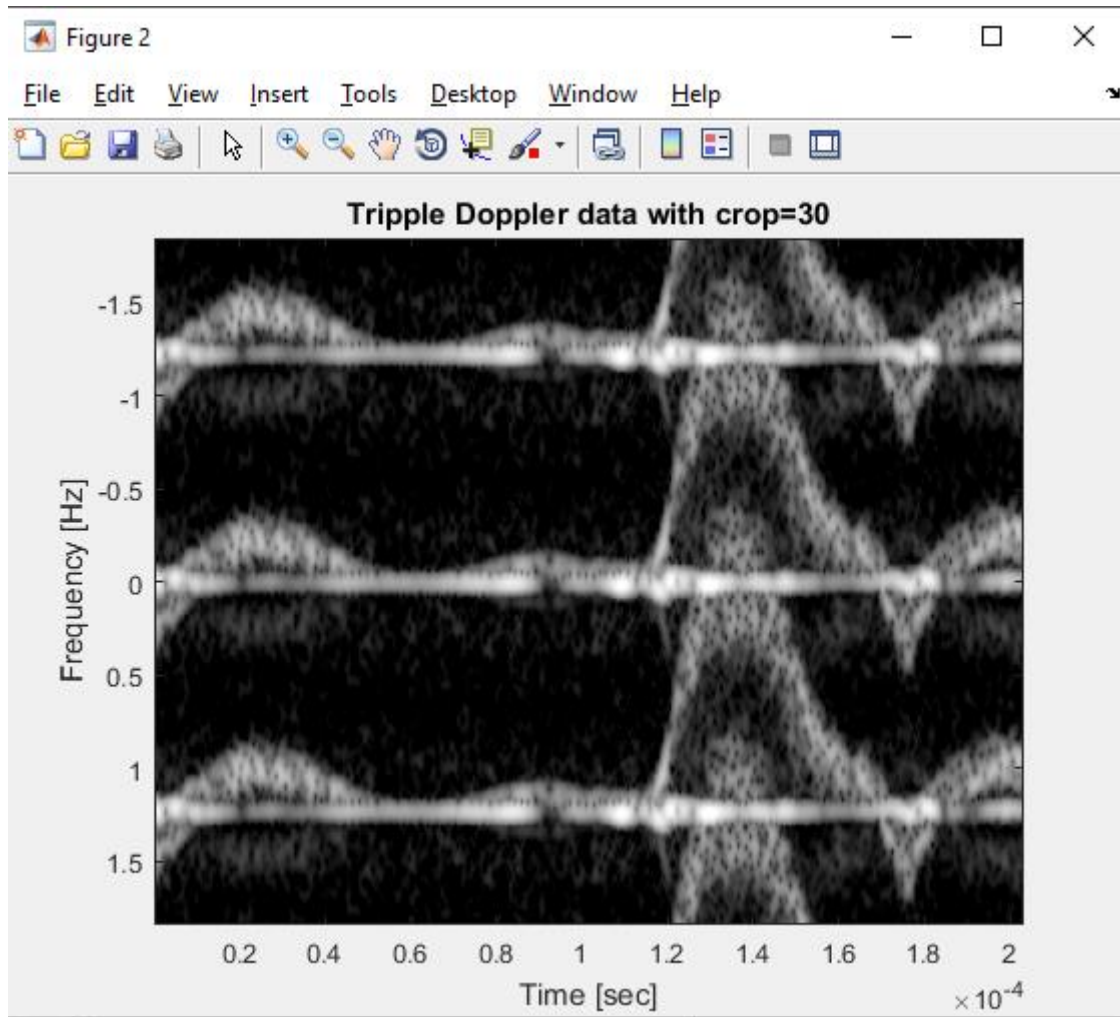
Understanding: What we do, is that we first high-pass-filter our received slowtime demodulated signal. We do this because we want to remove the clutters in the received signal from the body. We can do this because the frequency of the tissue/reverberation is close to the center frequency of the transmitted signal. While the received signal from blood has a higher frequency due to the velocity of the blood, which corresponds to a frequency shift in the frequency domain, hence the high-pass-filter.

After the high-pass-filter we use another hamming/hanning window to suppress the sidelobes of the received signal. We do this to get a better SNR, even though it decreases the radial resolution.

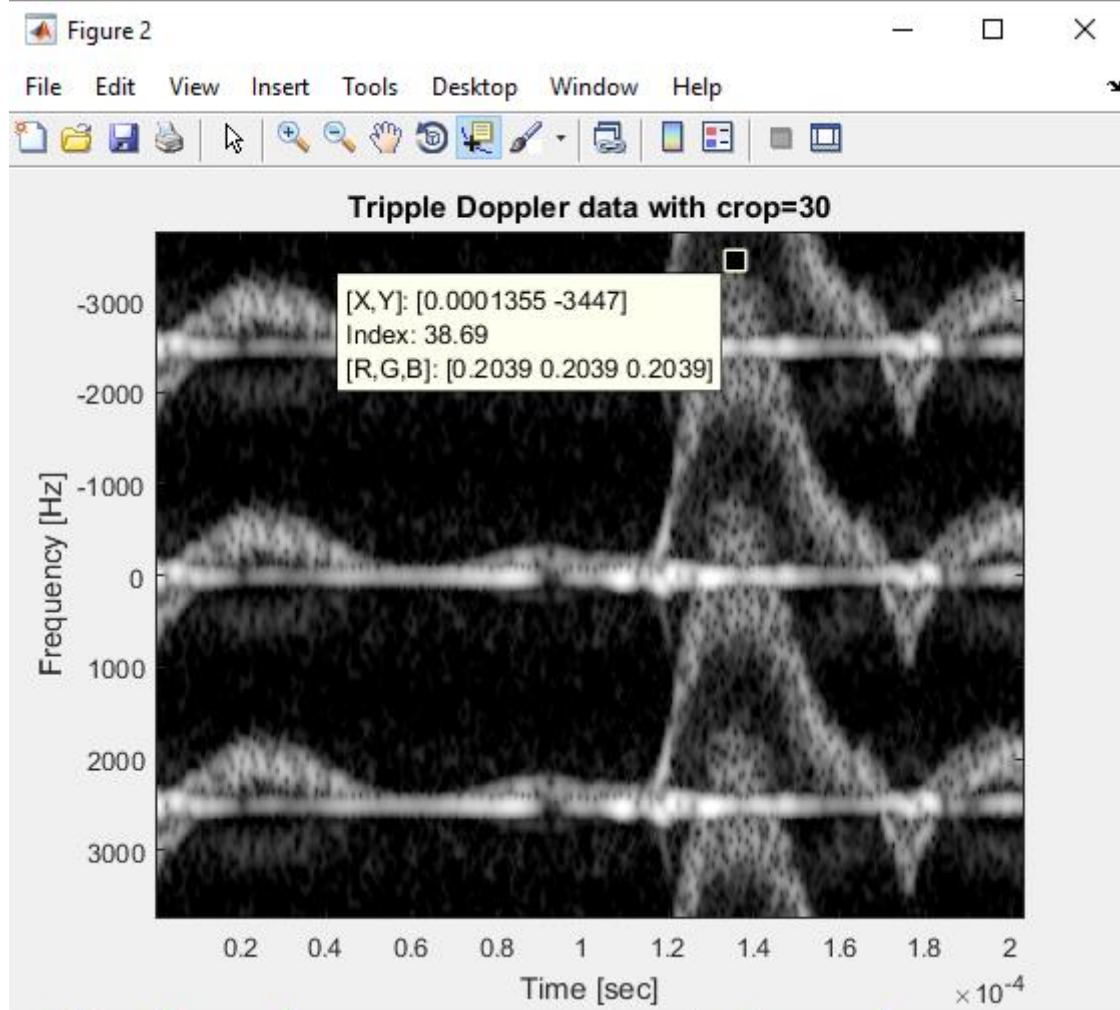


## Part 6





The maximum doppler shift obtained is given as:



Here we see that  $f_d = 3447$  Hz

So the maximum velocity is given as:

$$v_d = \frac{f_d v_{Nyq}}{PRF/2} = \frac{c f_d}{2 f_0}$$

The maximum velocity is  $v = (1540 \cdot 3447) / (2 \cdot 2500000) = 1.0617$  m/s

The Nyquist limit is given as:

$$v_{Nyq} = \frac{c PRF}{4 f_0}$$

Meaning that the  $v_{nyq} = (1540 \cdot 2.500000063155314 \cdot 10^3) / (4 \cdot 2500000) = 0.3850$  m/s

This means that our image contains alias!

The PRF should be within:

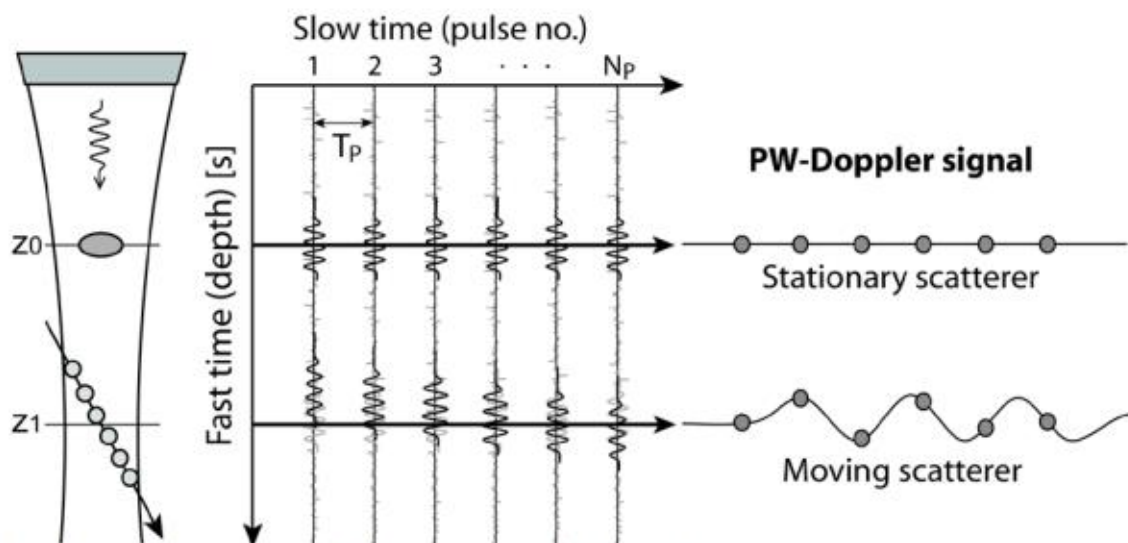
$$PRF < \frac{c}{2z}$$

To avoid aliasing, the doppler shift can't exceed  $PRF/2$ . Meaning that in our case, the PRF can't exceed:  $2.500000063155314 \cdot 10^3 / 2 \Rightarrow 2500\text{Hz} / 2 = 1250\text{Hz}$ .

In our case the doppler shift was measured to be  $3447\text{Hz}$ , meaning that we will get aliasing, which we do, because  $3447\text{ Hz} \geq 1250\text{Hz}$

## Key points

The pulse length used in PW-Doppler is way too short to allow measurement of this frequency shift based on the signal from one pulse emission. From equation (3), the required observation time  $\Delta t$  (which in this case equals the pulse length) should be several ms to achieve sufficient velocity resolution in the Doppler sonogram. However, but such a pulse length would not be compatible with a reasonable spatial resolution. The solution is to transmit several pulses in a sequence with constant time interval, and extract one sample from each echo signal as explained above. This sequence of sample resembles a signal in the audio range, similar to the Doppler signal achieved in CW-Doppler



**Figure 9:** The PW-Doppler signal acquisition model. The reflected echoes from moving targets are shifted in depth while being sampled at the same spatial position for multiple pulses. A stationary target will have zero output, while a moving target will yield a sampling of the Doppler signal.

In Figure 9, this principle is shown for a point scatterer moving through the ultrasound beam with constant velocity. The time delay of the received echo will then change with a constant time increment from pulse to pulse, which is converted into a phase shift in the sampled Doppler signal. By performing complex demodulation before sampling, a complex Doppler signal is obtained, where positive and negative Doppler shift can be separated in the Doppler spectrum.