Navigation Using Kalman Filter Derivation (Autopilot)

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ABSTRACT

The robot localization problem is a key problem in making truly autonomous robots. One of the main challenges in creating fully autonomous robots is the localization problem for robots. It can be challenging for a robot to figure out what to do next if it is lost. Robots are able to locate themselves by using relative and precise measures that provide the robot with input regarding its driving maneuvers and the state of its surroundings. The robot needs to pinpoint its location as precisely as possible using this information. The fact that there is ambiguity in the robot's sensing and driving makes this challenging. It is necessary to combine the unclear information in the best possible way.[1] There are still numerous difficult difficulties to be solved in the design of dependable navigation and control systems for unmanned aerial vehicles (UAVs) that solely rely on visual cues and inertial data. These problems range from basic control theory to hardware and software development. In order to overcome these problems, this study develops and implements an autopilot for tiny and mini rotorcraft UAVs that is based on adaptive vision. The suggested autopilot has a nonlinear control system for target tracking and flight control, as well as a Visual Odometer (VO) for navigation in areas where GPS is not available. With the use of a single camera installed on the vehicle, the VO detects and tracks visual cues in the surrounding area to estimate the rotorcraft ego-motion.[2]

An adaptive mechanism has been added to the VO, combining inertial and optic flow measurements to ascertain the Implementing LQR controller to take decision based on estimated values and Implementing EKF to estimate the state space. Finding the nominal trajectory using minimization to avoid obstacle collision.

Keywords: LQR controller, Implementing EKF, Visual navigation, Adaptive control, Rotorcraft UAV, Visual odometry, Visual servoing.

INTRODUCTION

One of the intriguing areas of study that contributes to the performance of a genuinely autonomous mobile robot is mobile robot localization. In many circumstances, multiple autonomously operating robots can complete tasks more quickly. Even greater productivity increases, though, might be possible if robots can cooperate. The ability of the robots to connect and communicate with one another is a prerequisite for cooperation. The authors of this work examine the simultaneous localization and mapping (SLAM) process. Two filters are used for this purpose: the Extended Kalman Filter (EKF) with SLAM and the linear Kalman Filter (KF) with SLAM [1, 2]. In the field of robotics, and particularly in a mobile robot system, SLAM is essential. Cooperatively measuring the robot's position and the surrounding map model is the main goal of SLAM [3-4]. This information is critical for the safe interaction of robots within the operation area. Over the past ten years, several iterations of the SLAM algorithm have been proposed. A laser rangefinder [5] served as the primary sensor node in the majority of the early SLAM algorithms. Currently, active or passive visual sensor nodes are the most popular choice [6, 7]. Compared to a laser rangefinder, compact, lightweight, and reasonably priced cameras may now provide more precise data and almost limitless estimation. The environment map is doubled or tripled needed in SLAM [8, 9]. The first is the frequently used map. vital to support or supplement other duties; a map, for instance, can alert a track arrangement or provide a worker with an initiative to imagine. Second, the plot or map is used to constrain the error made when determining the robot's state. In the absence of a map, the dead reckoning would point energetically and quickly. However, by reentering the familiar areas, the robot can reorganize its localization error by using a map, for example, which contains a set of distinct landmarks. SLAM applications are therefore more helpful in these circumstances where a previous plan is lacking and needs to be created. The first subset of filter-applying methods is the focus of KF derivatives [10, 11]. The KFs believe that data is impacted by Gaussian noises, which in our case may not be entirely accurate. Although KFs have excellent convergence properties, they are rarely used for SLAM because they are designed to address the issues with linear systems in their most basic form. However, the EKF is a frequently used tool in nonlinear filtering systems like SLAM. For the nonlinear systems, EKF introduces a step of linearization, and linearization around the current estimate is carried out by a first-order Taylor expansion. EKFs are shown to be optimal as long as linearization is carried out around the precise value of the state vector. In practical terms, it is the value to estimate [12]. Combinations of autonomous mobile robots and SLAM have become increasingly significant in the controlling domain in recent years. In particular, autonomous robots are utilized extensively. for the upkeep and rescue efforts during the catastrophe management, including radioactive leaks. Since the area is inaccessible, it is essential to map the environment and localize the robot at the same time in order to pinpoint the precise source spot [13–14]. Because active mapping determines the degree of localization, SLAM has become a crucial problem. Nevertheless, the approximation of the sensor noises and the real-time stochastic system as Gaussian make the SLAM implementation using the EKF quite interesting. As a result, improper modification of the noise covariance may eventually cause filter divergence, which would make the system as a whole unstable[15].

Related Work

Islam Alaa: This paper A number of environmental factors, including signal interference and outages, can impact GPS navigation capabilities. On the other hand, it can offer decent navigational accuracy. in the long run. It goes without saying that INS and GPS complement each other, and combining them will result in a reliable navigation system over time. The quality of the system's error covariance parameter tuning has a significant impact on the yielded performance and integration accuracy. This paper described the use of an EKF to integrate the GPS and INS. Brief reviews of several tuning techniques were conducted using a qualitative approach[1].

Le Chang, Xiaoji Niu: This paper When GNSS position correction is required continuously, observational updates are performed using the GNSS position by the GNSS/INS integrated navigation system. The GNSS positioning accuracy primarily determines the navigation accuracy, particularly the position accuracy. Thus, analyzing the navigation error during GNSS signal interruption is how the accuracy of the GNSS/INS integrated navigation system is determined. The GNSS, IMU, and LiDAR-SLAM integrated navigation system accuracy was assessed in the same manner in this paper[2].

Farid Kendoul: In this paper, Our vision-based autopilot, which is intended for small UAVs and MAVs flying at low altitudes, is presented in this study. The created system primarily uses visual signals and is based on a downward-looking camera. to complete a variety of navigational tasks in uncharted territory. The proposed vision-based autopilot expands optic flow-based control capabilities to complex navigational tasks like precise hovering, arbitrary trajectory tracking, stationary and moving target tracking, etc., in contrast to some existing bio-inspired flight controllers that are typically used for reactive navigation. These features are mostly attributable to the integrated adaptive visual odometer, which enables height estimate and rotorcraft position and velocity recovery[3].

Lin Feng: In this paper, the gyro sensor drifts over time, the attitude calculation's cumulative error is minimized, which lowers the flying attitude estimation's accuracy and dependability. An accelerometer is susceptible to vibration, which can lead to inaccurate measurement results. The external magnetic field can easily disrupt the electronic compass, causing the output attitude information to appear more erroneous. It is challenging to eliminate a single sensor's inherent flaws; nevertheless, an attitude estimate fusion method based on several sensors may balance the benefits and drawbacks of multiple sensors, resulting in the resolution of high precision and high reliability attitude information[4].

In this paper Readers who are unfamiliar with the origins of SLAM may find it helpful to consult conventional and recent work on SLAM. The initial technique for the SLAM problem was presented in 1986–1991 years. In 1986, Smith and Chesseman In this paper that addressed SLAM issues. To address this issue, they provide the EKF. They provided a numerical foundation in that study to explain the relationship between landmarks and the geometric uncertainty[5].

In this paper Numerous additional scholars have focused on different aspects of SLAM. For instance, the authors of introduced a novel design that uses a single monocular SLAM system to track the mobile robot's unrestricted mobility. The benchmarks for creating a network feature detection process are displayed by the enhanced oriented FAST and rotated BRIEF (ORB) features. The function matching strength of an improved matching feature system is improved. Rather than using the regular EKF, the upgraded EKF measures the several dimensional states of the free-moving visual sensor. Additionally, the authors discuss the topic of SLAM applications for navigational issues in. An EKF or particle filter (PF) algorithm is commonly used for data processing in the solution of high-accuracy problems[6].

In this paper The authors proposed three methods for reducing the linearization error of KF algorithms, and SLAM is used to evaluate each method's viability and efficiency. Because of the Taylor method, the linearization error in the derivative-based techniques of the KF system is undetectable. expansion for the nonlinear motion process' linearization. The three methods that are being described lower the linearization error by using novel formulations in place of the Jacobian observation matrix. Similarly, a SLAM with limited sensing through the use of EKF is presented in. SLAM refers to the robot's difficulty in mapping an unknown atmosphere while modifying its specific location, which is based on data from sensors and a comparable map[7].

In this paper The authors introduced an AUV vision-based SLAM that uses submerged non-natural markers to sense downward and forward camera motion. Together with a number of navigation sensor nodes, including a depth sensor node, a Doppler velocity log (DVL), and an inertial measurement unit (IMU), the camera may also estimate the AUV location data. The landmark detection algorithm is set up to monitor the landmark and robot state within the traditional EKF SLAM framework. Moreover, investigate the partial observability of mobile robots based on EKF in an effort to create a solution that can prevent inaccurate measurements[8].

In this paper The computational expenses of mobile robots can be reduced by taking into account only specific 3 Wireless Communications and Mobile Computing environmental landmarks; however, this will result in an increase in device uncertainties. Though some of the landmarks have been removed for reference, the fuzzy logic process is presented to ensure that the computation has produced the intended output. Fuzzy logic is employed in the measuring invention of KF to pinpoint the precise location of the mobile robots and any detected landmarks during the observation procedure[9].

Methodology

Dynamics of the Model

The thrust force exerted by the motors are always perpendicular to the frame. Hence the net thrust will always be sum of T1 and T2. Thus, we get the relation:

$$u1 = T1 + T2$$

Since the forces act at different locations on the same frame, there will be a net torque exerted by the two forces. We hence define u2 as the net moment about the centre of the frame as:

$$u2 = L/2\,(T1 - T2)$$

The suggested SLAM algorithms based on KF and EKF are shown in this section. The ensuing subsections examine and assess the suggested algorithms.

Writing Newton's 2nd Law of motion in the horizontal and vertical directions, we get:

$$\ddot{x} = -\frac{u_1 sin(\phi)}{m}$$

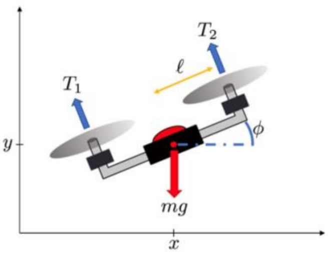
$$\ddot{y} = -g + \frac{u_1 cos(\phi)}{m}$$

Writing the torque equation about the centre of mass, we get:

$$\ddot{\phi} = \frac{u_2}{J}$$

There will be translational drag and rotational drag because of the fluid (air). To account that we need to add translational drag coefficient and rotational drag coefficient.

$$\begin{bmatrix} x \\ v_x \\ y \\ v_y \\ \phi \\ \omega \end{bmatrix} = \begin{bmatrix} v_x \\ \frac{-(T_1+T_2)\sin\phi - C_D^v v_x}{m} \\ v_y \\ \frac{(T_1+T_2)\cos\phi - C_D^v v_y}{m} - g \\ \omega \\ \frac{(T_2-T_1)\ell - C_D^\phi \omega}{I_{yy}} \end{bmatrix}$$



Proposed Simultaneous Localization and Mapping Algorithms

Kalman Filter-Assisted Simultaneous Localization and Mapping. The authors discussed the SLAM theory, which leads to effective localization and mapping in WSNs, in the section that follows. The author specifically provides the operating environment analysis before discussing the suggested algorithm and contrasting it with alternative SLAM techniques[1]. The KF offers a well-designed and statically optimal explanation for the linear systems in the presence of Gaussian white noise. This method creates the gain known as the Kalman gain by utilizing linear estimates linked to the states and error covariance matrices. An estimate of a posteriori is produced by adding this advantage to the assessment of a preceding condition[2]. The dynamic model of is defined by the equations below.

$$Xk+1 = f(Xk, Uk, Wk), \qquad (1)$$

$$Zk+1 = h(Xk, Vk), \qquad (2)$$

which manage state proliferation and state measurements; the discrete-time is represented by k, the state and measurement noise vectors are W and V, and the process input is U.

Nominal Trajectory

Parameters used

1. cost function
2. z_guess
3. Bounds
4. inequality constraints
5. equality constraints

Extended Kalman filter

$$X_{k+1/k} = F_k \times X_{k/k}, \qquad (3)$$

$$P_{k+1/k} = F_k \times P_{k/k} \times F_k^T + A_k \times Q_k \times A_k^T, \qquad (4)$$

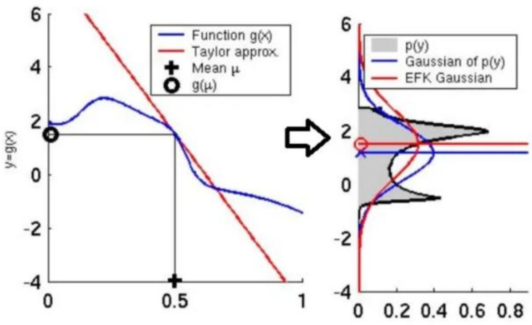
$$K_{k+1} = P_{k+1/k} \times H_k^T \left[H_{k+1} \times P_{k+1/k} \times H_{k+1}^T + R_k\right]^{-1}, \qquad (5)$$

$$X_{k+1/k+1} = X_{k+1/k} + K_{k+1}[Z_{k+1} - H_{k+1} \times X_{k+1/k}], \qquad (6)$$

$$P_{k+1/k+1} = [I - K_{k+1} \times H_{k+1}] \times P_{k+1/k}. \qquad (7)$$

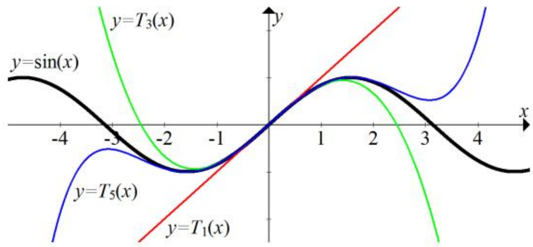
Equations (1) and (2) are utilized iteratively in conjunction with these KF-based approach equations. The previous state estimate is generalized in Equation (3), and Equation (4) stands for the corresponding state covariance error. Equation (5) can be used to estimate the gain of Kalman and is used to update the covariance and state approximation. mistake, which is respectively specified by Equations (6) and (7). In practical terms, the iterative KF technique and EKF are similar. additionally, it is occasionally applied to nonlinear systems. First-order partial derivatives like the Jacobian are used to linearize the nonlinear system Fk matrices and measurement Hk+1.

Linear / Non-Linear function vs gaussian distribution



For an immobile robot, a one-dimensional SLAM with KF is used, and the measurement is regarded as absolute. A mobile robot with one degree of freedom (DoF) is employed, which remains stationary in a straight path. The mobile robot is employed to identify fixed or inactive landmarks. By using SLAM with linear KF, the mobile robot's velocity (v) and position (p) at the landmarks are determined. Ten landmark locations are taken into consideration. The robot is stationary at a specific location on the real trajectory, which is v = 0 m/s.

Taylor Series

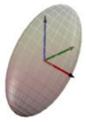


Let f(x) be a real or composite function that is a differentiable function of a real or composite neighborhood number. The following power series are then described by the Taylor series,

Furthermore, a one-dimensional SLAM with KF is
applied for a motionless robot, and the measurement is considered a relative measurement. Here, a 1-DoF mobile robot is used in a motionless and fixed position of a straight lane that detects the motionless/stationary landmarks. The robot velocity and the position/location landmarks are calculated by using the SLAM with a KF, see Figure 4. The fourth one is a one-dimensional SLAM with linear KF. In this case, a moving vehicle is considered with a relative measurement and a 1-DoF robot is traveling on a straight line that detects the motionless/stationary landmarks. The robot position/location, velocity, and landmark position are calculated through SLAM with linear KF.

The Kalman Filter Assumptions are:

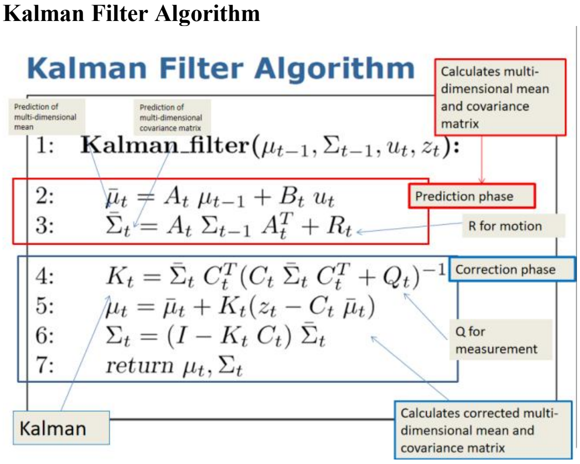
1. Gaussian distributions
2. Gaussian noise
3. Linear motion
4. Linear observation model

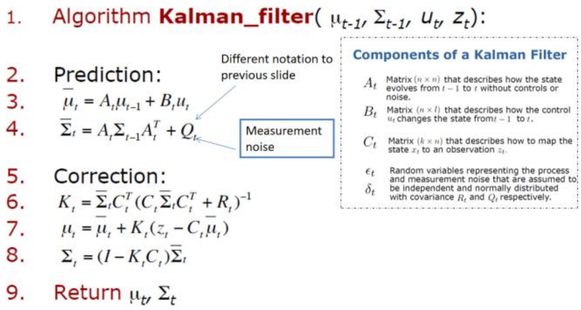


$$x_t = A_t x_{t-1} + B_t u_t + \epsilon_t$$

$$z_t = C_t x_t + \delta_t$$

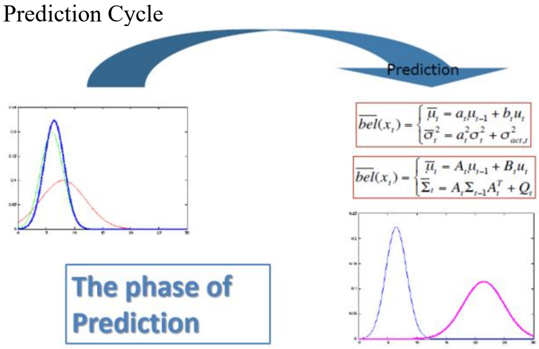
Kalman Filter Algorithm



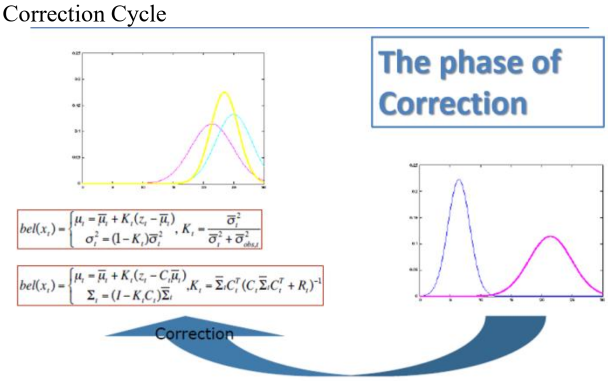


The Prediction-Correction-Cycle

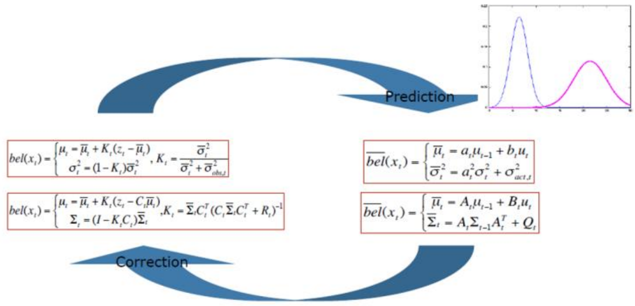
Prediction Cycle



Correction Cycle



The Prediction-Correction-Cycle



Nonlinear Dynamic Systems

Most realistic robotic problems involve nonlinear functions

$$\boxed{x_t = g(u_t, x_{t-1})}$$

$$\boxed{z_t = h(x_t)}$$

Simultaneous Localization and Mapping with Extended Kalman Filter:

Localization and Mapping at the Same Time Using an Extended Kalman Filter. EKF is widely recognized as a popular solution to the SLAM issue for localizing mobile robots. The EKF SLAM-based algorithm for a mobile robot that follows a predetermined trajectory was realized by the authors in this part. For EKF SLAM to adjust the robot's position, it uses existing landmarks in the navigation system. A mobile robot uses EKF SLAM on a designated field with a particular feature. The EKF state and observation model, which are shown below, are two fundamental mathematical models that the authors took into consideration.

$$X_{k+1} = f(X_k, U_k, w_k),$$

$$Z_{k+1} = h(X_{k+1}, v_{k+1}),$$

The estimated state vector at time k + 1 is indicated by Xk+1 in this case. Assuming all noise to be wk, the time is the discrete time for a known input Uk. The estimated measurement vector at time instant k + 1 is denoted by Zk+1 in Equation (9), where vk is the observation noise. The covariance matrix of observation and prediction is represented by the symbols Rk and Qk, respectively. An estimate of the optimal state is provided by EKF. The Zk+1 measurement states that the EKF-SLAM goal is to estimate the landmark state Xk recursively. EKF is essentially broken down into multiple phases, each of which is represented by the state vector Xk+1 at the beginning state.

$$X_{k+1} = f(X_k, U_{k+1}) + \nabla F_x \times \left(X_k - X_{\frac{k}{k}}\right).$$

In the prediction stage, the covariance matrix for prediction Pk+1/k can be represented as:

$$P_{k+1/k} = \nabla F_x \times P_{k/k} \times \nabla F_x^T + \nabla F_u \times Q_k \times \nabla F_u^T.$$

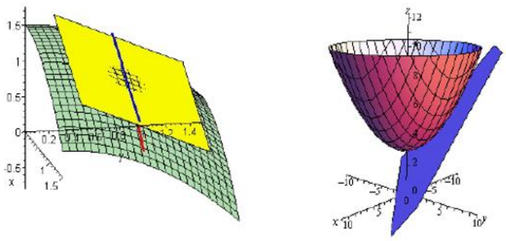
Regarding the state vector, which is Xk+1, the Jacobian matrices of the function f are shown by the symbols ΔFx and ΔFu in the equation above. B stands for the state transition matrix, and F is the state equation, which has the following representation:

$$B = \begin{bmatrix} dt \times \cos\left(x(3)\right) & 0 \\ dt \times \cos\left(x(3)\right) & 0 \\ 0 & dt \end{bmatrix}$$

$$F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Therefore, the Jacobian of the state equation will become

$$JF = \begin{bmatrix} 0 & 0 & -dt \times u(1) \times \sin\left(x(3)\right) \\ 0 & 0 & dt \times u(1) \times \cos\left(x(3)\right) \\ 0 & 0 & 0 \end{bmatrix},$$



Jacobian Matrix represents the gradient of a scalar valued function

LQR controller:

Linear System Model:
LQR is applicable to linear time-invariant systems, which can be described by state-space equations. The state-space representation of a system captures its dynamics in terms of state variables and control inputs.

Quadratic Cost Function:
The goal of the LQR controller is to minimize a quadratic cost function, which typically includes terms related to both the system's state and control input. The cost function is defined as a weighted sum of the squares of the state variables and control inputs.

State Feedback Control:
LQR employs state feedback control, meaning that the control input is a linear function of the system's state. The controller computes the control input based on the current state of the system.

Weighting Matrices:
The cost function includes weighting matrices, Q and R, that allow the designer to assign relative importance to the state variables and control inputs, respectively. The choice of these matrices affects the performance and behavior of the controller.

Optimal Control Solution:
The LQR controller seeks to find the optimal state feedback gains that minimize the cost function. The optimal gains are determined by solving the continuous-time algebraic Riccati equation, which is a matrix equation involving the system matrices, Q, and R.

Closed-Loop System:
Once the optimal gains are obtained, the closed-loop system is formed by applying the state feedback control. The closed-loop system is stable and designed to minimize the defined cost function.

Discrete-Time LQR:
The same principles can be applied to discrete-time systems, where the controller minimizes a discrete-time cost function. In this case, the solution involves solving a discrete-time algebraic Riccati equation.

Results

The results we present now (and also the example in section 6) were performed without using odometry. This should demonstrate the robustness of the approach. Of course, as Thrun already noted in, this is only possible as long as any 2D structure is present in the world. The built map presented in fig. 2 was acquired by driving the robot out of its lab, up the corridor, down the corridor and then back into the lab.

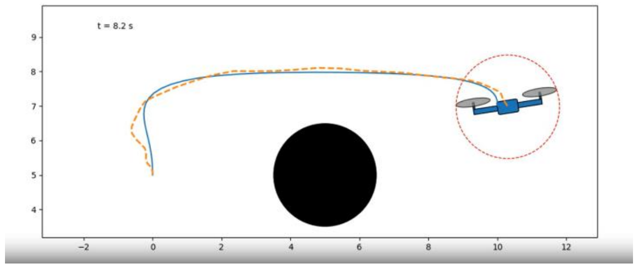


Fig: Drone path obtained using LQR with states based on measurement only.

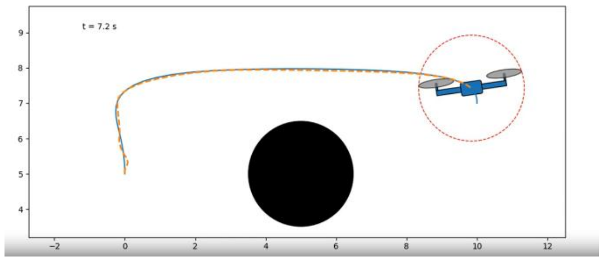
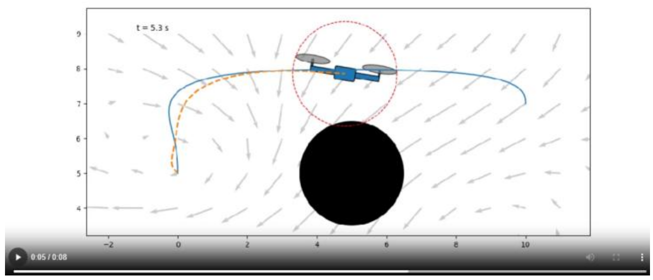


Fig: Drone path obtained using LQR with states based on EKF estimated states.

Wind field code, including definition of apply wind disturbance.



Conclusion

In summary, our text generator seamlessly blends the capabilities of LSTM-based deep learning with the elegance of Markov Chains[1]. The LSTM model enhances the generation process by discerning intricate patterns, while Markov Chains introduce diversity and coherence[2]. The visualization of transition probabilities provides a nuanced understanding of the learned sequential relationships[3]. This hybridized approach offers a dynamic text generation mechanism, underscoring the symbiosis of advanced neural network architectures and probabilistic modeling[4]. The outcome represents a promising stride toward crafting contextually rich and diverse textual content, showcasing the potency of merging deep learning and traditional probabilistic methods.

Future Scope

- Simulate external disturbances and test the robustness of the system.
- Optimize the nominal trajectory path computation.
- Work with state covariance matrix, errors in measurements and prediction to improve prediction of better Kalman gain.

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