Trained Born Iterative Method for Two-Dimensional Microwave Imaging

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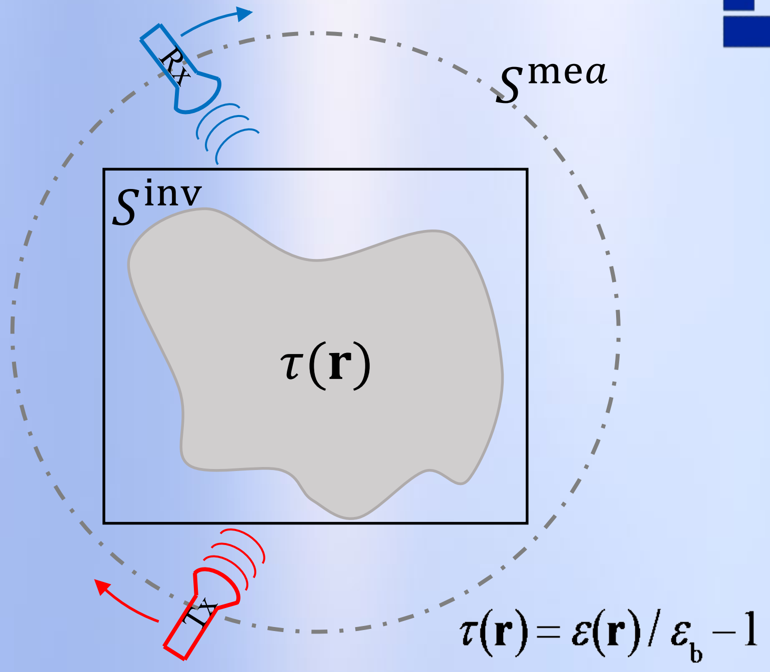
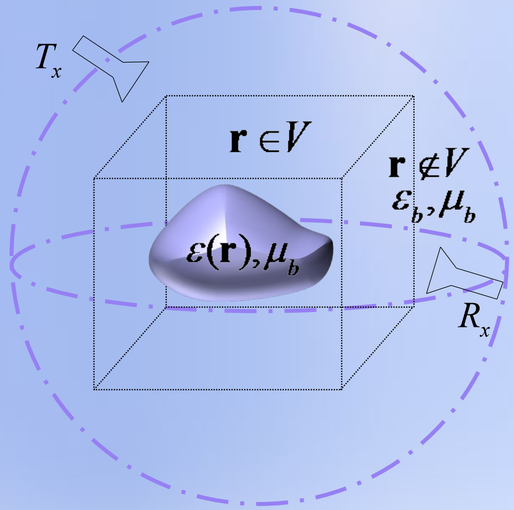
Problem configuration

- Investigation domain S is surrounded by receivers and transmitters
- Material properties of the domain are unknown
- It is assumed that each transmitter excites S individually
- Scattered field are measured at the receivers
- $\|\mathbf{e}^{sca} - f(\tau)\|_2^2$ is minimized to find $\tau(\mathbf{r})$

$$\underbrace{e^{\mathrm{sca}} = f(\tau)}_{\text{Mathematical forward model}}$$





Problem description

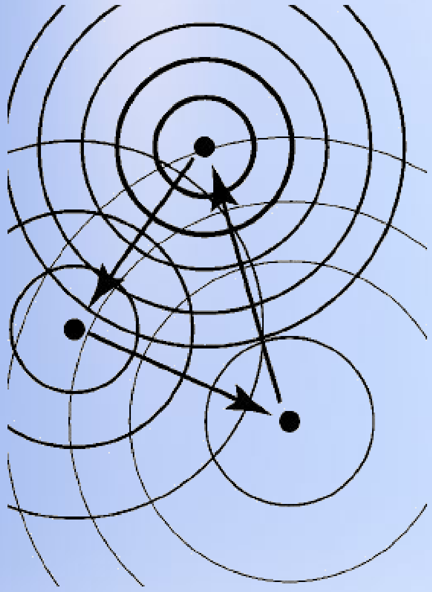
$$e^{\mathrm{sca}} = f(\tau)$$

Mathematical forward model

Challenges

- Nonlinearity:
  - The forward operator is a nonlinear function of the medium parameters
  - The strength of nonlinearity is related to multiscatering

- Ill-posedness:
  - The forward operator contains smoothening integrals
  - Noisy measurements
  - Measurements are valid over a finite set



Rehr, John J., and Robert C. Albers. "Theoretical approaches to x-ray absorption fine structure." *Reviews of modern physics* 72.3 (2000): 621.

Forward Model

$$e^{\mathrm{inc}}(\mathbf{r}) = e^{\mathrm{tot}}(\mathbf{r}) + k_0^2 \int_{S^{\mathrm{inv}}} \tau(\mathbf{r}') e^{\mathrm{tot}}(\mathbf{r}') g(\mathbf{r}, \mathbf{r}') ds', \quad \mathbf{r} \in S^{\mathrm{inv}}$$

$$e^{\mathrm{sca}}(\mathbf{r}) = -k_0^2 \int_{S^{\mathrm{inv}}} \tau(\mathbf{r}') e^{\mathrm{tot}}(\mathbf{r}') g(\mathbf{r}, \mathbf{r}') ds', \quad \mathbf{r} \in S^{\mathrm{mea}}$$

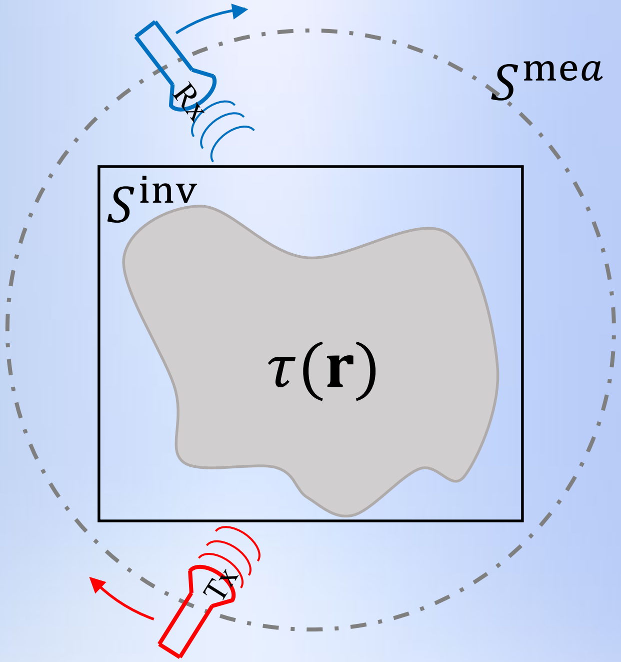
Apply Discretization

$$\mathbf{e}_{tr}^{\mathrm{inc}} = \left(\mathbf{I} + \mathbf{A}\mathcal{D}(\mathbf{t})\right) \mathbf{e}_{tr}^{\mathrm{tot}}$$

$$\mathbf{e}_{tr}^{\mathrm{sca}} = \mathbf{G}\mathcal{D}(\mathbf{e}_{tr}^{\mathrm{tot}})\mathbf{t}$$

$$\mathbf{G}_{n,m} = k_0 \int_{S_n} g(\mathbf{r}', \mathbf{r}_m) ds'$$

2D scaler green function



Existing methods, sparse Born iterative method (SBIM)

$$e^{\mathrm{inc}}(\mathbf{r}) = e^{\mathrm{tot}}(\mathbf{r}) + k_0^2 \int_{S^{\mathrm{inv}}} \tau(\mathbf{r}')e^{\mathrm{tot}}(\mathbf{r}')g(\mathbf{r},\mathbf{r}')ds', \quad \mathbf{r} \in S^{\mathrm{inv}}$$

$$e^{\mathrm{sca}}(\mathbf{r}) = -k_0^2 \int_{S^{\mathrm{inv}}} \tau(\mathbf{r}')e^{\mathrm{tot}}(\mathbf{r}')g(\mathbf{r},\mathbf{r}')ds', \quad \mathbf{r} \in S^{\mathrm{mea}}$$

Apply Discretization

$$\mathbf{e}_{tr}^{\mathrm{inc}} = \left(\mathbf{I} + \mathbf{A}\mathcal{D}(\mathbf{t})\right)\mathbf{e}_{tr}^{\mathrm{tot}}$$

$$\mathbf{e}_{tr}^{\mathrm{sca}} = \mathbf{G}\mathcal{D}(\mathbf{e}_{tr}^{\mathrm{tot}})\mathbf{t}$$

Algorithm 1 Sparse Born Iterative Method

Input: l_1

1: Initialization: $\{\mathbf{e}_{tr,1}^{\mathrm{tot}} = \mathbf{e}_{tr}^{\mathrm{inc}}, \ \forall tr\}$
2: **for** $i = 1$ to N^{bim} **do**
3: $\quad \mathbf{H}_i = cas(\mathbf{G}\mathcal{D}(\mathbf{e}_{1,i}^{\mathrm{tot}}), ..., \mathbf{G}\mathcal{D}(\mathbf{e}_{N^{tra},i}^{\mathrm{tot}}))$
4: $\quad \mathbf{t}_i = min_{\mathbf{t}} ||\mathbf{H}_i\mathbf{t} - \mathbf{e}^{\mathrm{mea}}||_2^2$ s.t. $|\mathbf{t}| < l_1$
5: $\quad$ **for** $tr = 1$ to N^{tra} **do**
6: $\quad\quad \mathbf{e}_{tr,i+1}^{\mathrm{tot}} = \left(\mathbf{I} + \mathbf{A}\mathcal{D}(\mathbf{t}_i)\right)^{-1}\mathbf{e}_{tr}^{\mathrm{inc}}$
7: $\quad$ **end for**
8: **end for**
9: **return** $\bar{\mathbf{t}}_{N^{\mathrm{bim}}}$

$$\mathbf{t}_{i,l} = \mathcal{T}^{\delta}\left(\mathbf{t}_{i,l-1} - \gamma_i \mathbf{H}_i^{\dagger}\left(\mathbf{H}_i\mathbf{t}_{i,l-1} - \mathbf{e}^{\mathrm{mea}}\right)\right)$$

$$\{\mathcal{T}^{\delta}(\mathbf{t})\}_n = \{\mathbf{t}\}_n \frac{max(|\{\mathbf{t}\}_n| - \delta, 0)}{max(|\{\mathbf{t}\}_n| - \delta, 0) + \delta}$$

Proposed approach, Trained Born Iterative Method (TBIM)

Proposed solution

- Trained regularization network. Trained to enhance images containing target(s) that inherit features from the training dataset

Algorithm 1 Sparse Born Iterative Method

Input: l_1

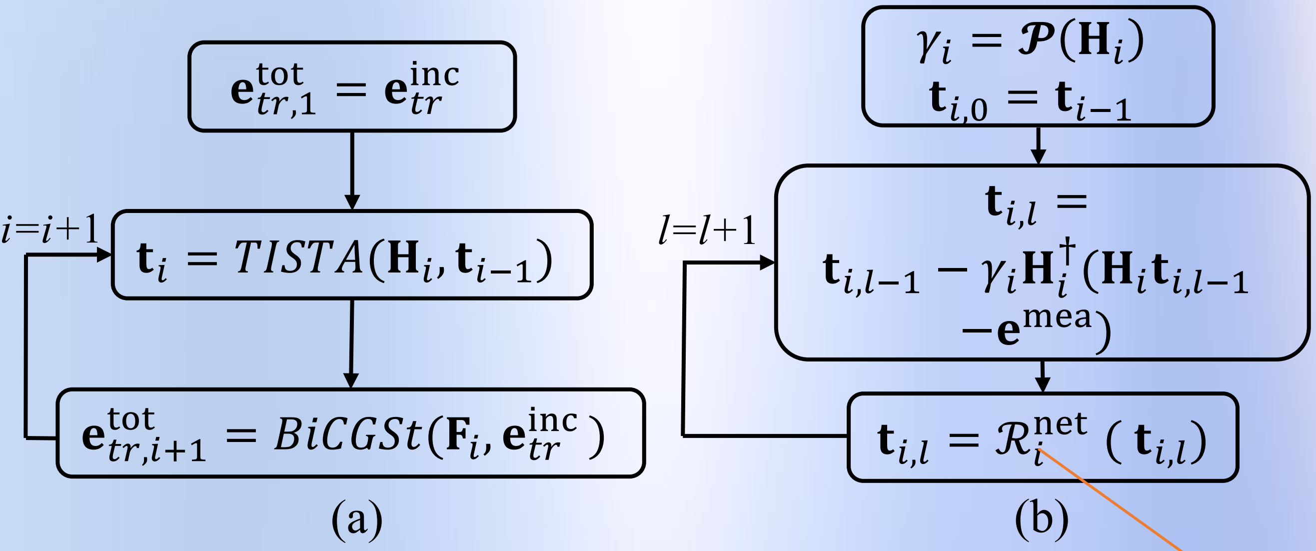
Initialization: $\{\mathbf{e}^{\text{tot}}_{tr,1} = \mathbf{e}^{\text{inc}}_{tr},\ \forall tr\}$
for $i = 1$ to N^{bim} **do**
  $\mathbf{H}_i = cas(\mathbf{G}\mathcal{D}(\mathbf{e}^{\text{tot}}_{1,i}), ..., \mathbf{G}\mathcal{D}(\mathbf{e}^{\text{tot}}_{N^{tra},i}))$
  $\mathbf{t}_i = min_{\mathbf{t}} \|\mathbf{H}_i\mathbf{t} - \mathbf{e}^{\text{mea}}\|_2^2$ s.t. $|\mathbf{t}| < l_1$
  for $tr = 1$ to N^{tra} **do**
    $\mathbf{e}^{\text{tot}}_{tr,i+1} = (\mathbf{I} + \mathbf{A}\mathcal{D}(\mathbf{t}_i))^{-1}\mathbf{e}^{\text{inc}}_{tr}$
  end for
end for
return $\bar{\mathbf{t}}_{N^{\text{bim}}}$

Algorithm 2 Trained Born Iterative Method

Input: l_1

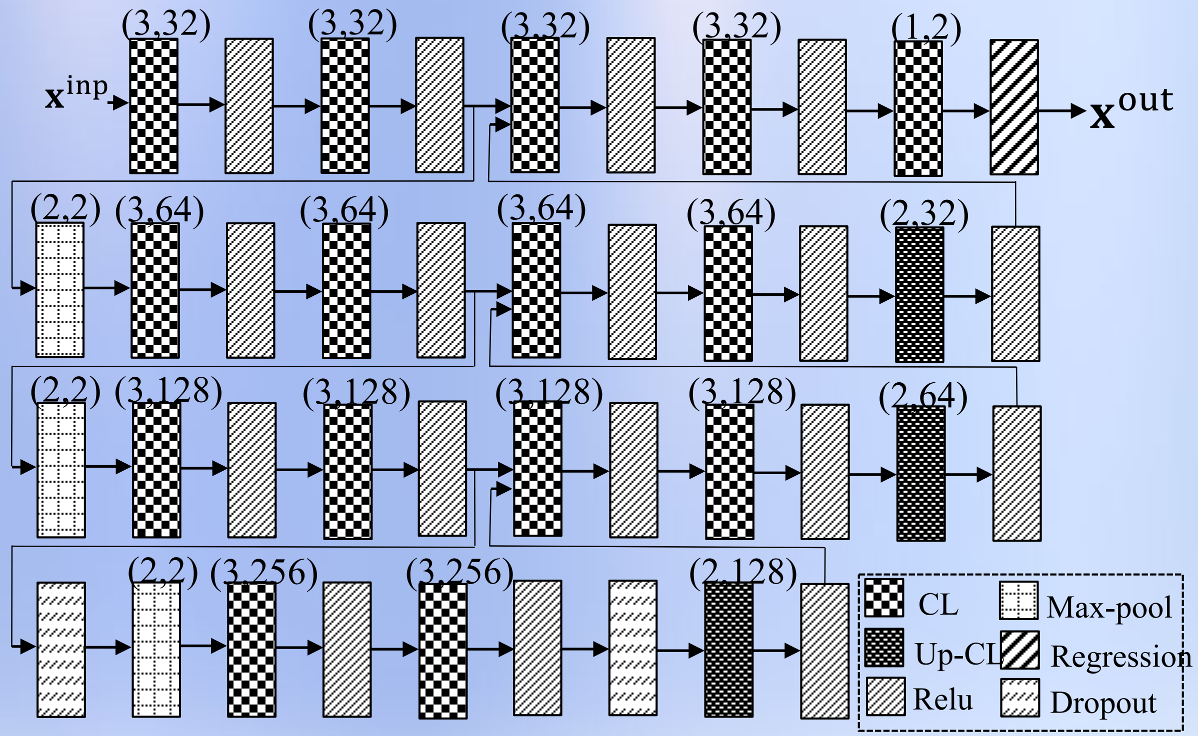
Initialization: $\{\mathbf{e}^{\text{tot}}_{tr,1} = \mathbf{e}^{\text{inc}}_{tr},\ \forall tr\}$
for $i = 1$ to N^{bim} **do**
  $\mathbf{H}_i = cas(\mathbf{G}\mathcal{D}(\mathbf{e}^{\text{tot}}_{1,i}), ..., \mathbf{G}\mathcal{D}(\mathbf{e}^{\text{tot}}_{N^{tra},i}))$
  $\gamma_i = \mathcal{P}(\mathbf{H}_i)$
  for $l = 1$ to N^{lwb} **do**
    $\mathbf{t}_{i,l} = \mathcal{R}^{\text{net}}_i\left(\mathbf{t}_{i,l-1} - \gamma_i\mathbf{H}_i^{\dagger}(\mathbf{H}_i\mathbf{t}_{i,l-1} - \mathbf{e}^{\text{mea}})\right)$
  end for
  for $tr = 1$ to N^{tra} **do**
    $\mathbf{e}^{\text{tot}}_{tr,i+1} = BiCGSt(\mathbf{F}_i, \mathbf{e}^{\text{inc}}_{tr})$
  end for
end for
return $\bar{\mathbf{t}}_{N^{\text{bim}}}$

Proposed approach, (TBIM)



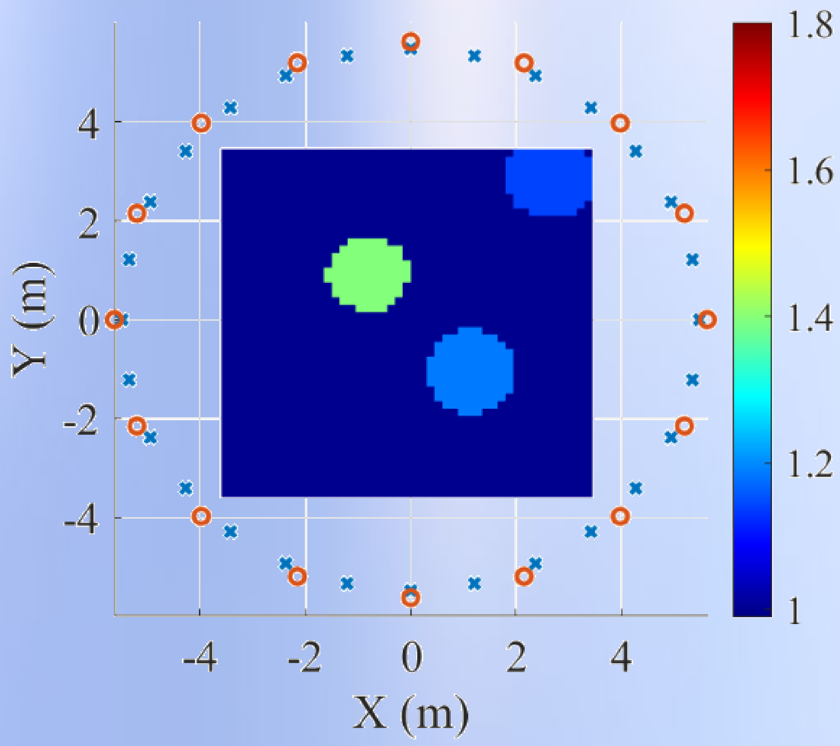
Block diagram for the trained Born iterative method

U-net regularization network

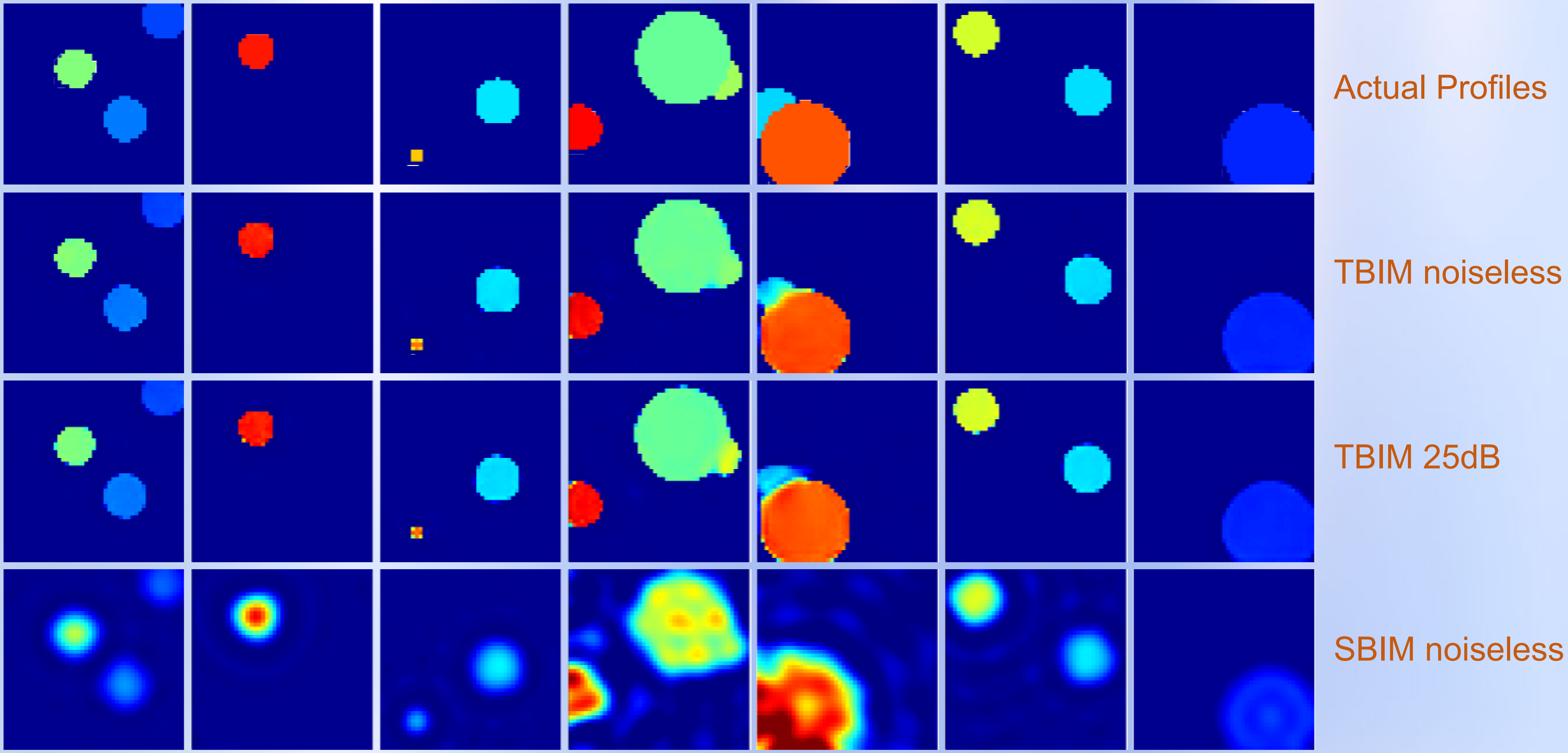


Numerical Results

- Two training datasets were created:
  i) Cylindrical targets
  ii) Multiple shape targets
- Each set contains 70000 example
- 2000 example used for validation
- 2000 example used for testing
- 64000 for training
- The figure show the receiver-transmitter configuration and the colormap



Cylindrical targets



Results from the first seven examples of the testing set reconstructed at 3 BIM iterations and using 7 TISTA iteration

TABLE I

RNEs of the examples shown in Fig. 5. In addition, 15 dB TBIM and 10 dB TBIM results are included.

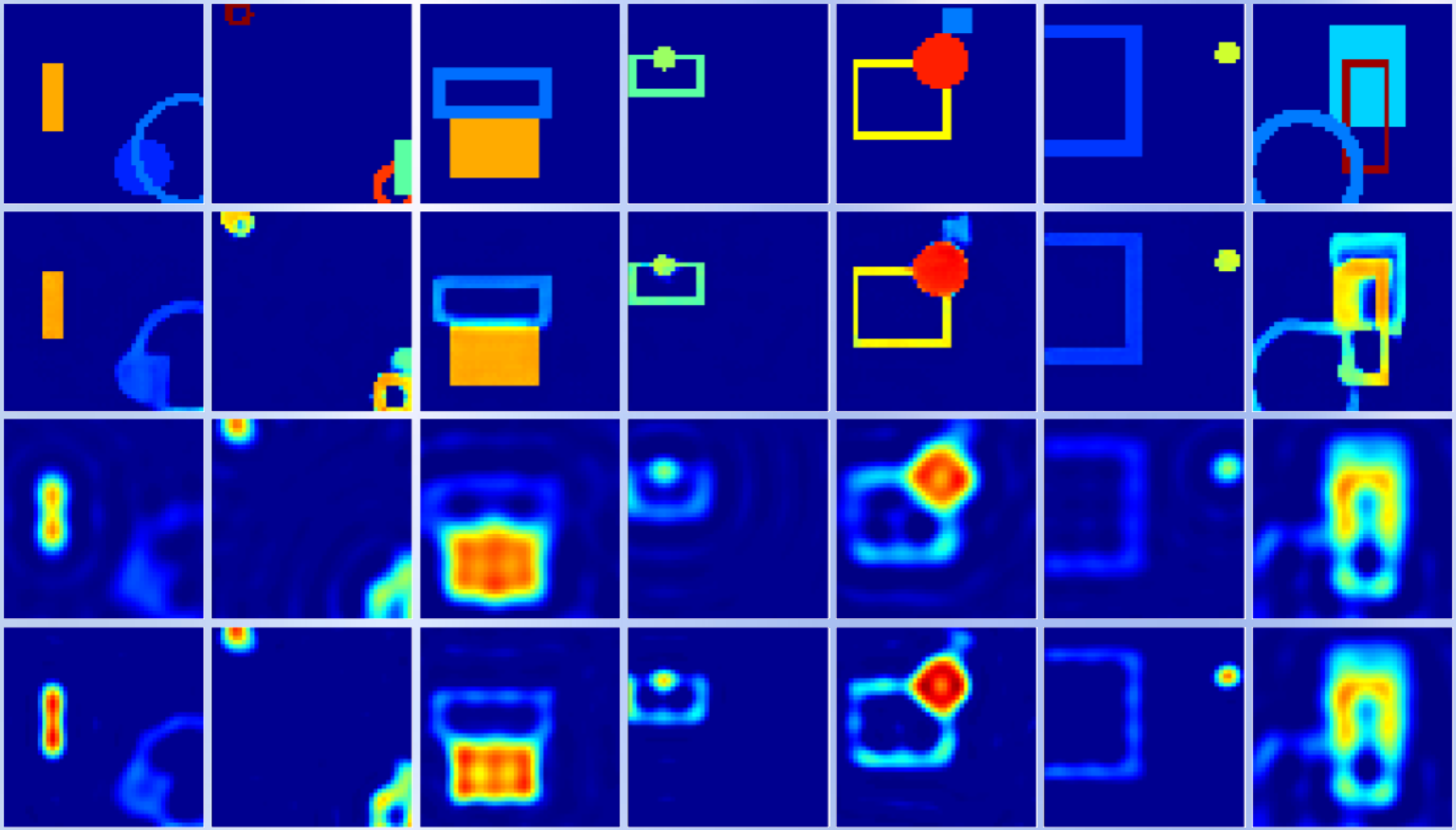
Ex. No.	Noiseless TBIM	25 dB TBIM	15 dB TBIM	10 dB TBIM	Noiseless SBIM
Ex. 1	6.55%	11.40%	15.64%	17.13%	42.14%
Ex. 2	2.34%	3.56%	13.07%	18.71%	48.83%
Ex. 3	7.96%	11.85%	12.68%	16.07%	50.73%
Ex. 4	5.73%	10.44%	13.69%	21.14%	55.33%
Ex. 5	10.29%	10.70%	15.08%	15.66%	86.88%
Ex. 6	4.05%	6.40%	12.43%	14.98%	41.14%
Ex. 7	6.65%	9.75%	14.12%	11.28%	21.03%

TABLE II

Testing set MRNE per TBIM iteration count at different SNR levels for cylindrical targets set.

SNR	$i = 1$	$i = 2$	$i = 3$
Noiseless	13.74%	9.46%	7.64%
25 dB	14.54%	11.56%	11.17%
15 dB	16.70%	14.55%	14.13%
10 dB	19.11%	17.31%	17.26%

Multiple shape targets



Results from the first seven examples of the testing set reconstructed using 7 TISTA iterations, all at 25dB noise level

TABLE III

RNEs OF THE EXAMPLES SHOWN IN FIG. 6.

Ex. No.	$N^{\mathrm{bim}} = 3$, TBIM	$N^{\mathrm{bim}} = 3$, TBIM	$N^{\mathrm{bim}} = 25$, SBIM
Ex. 1	13.82%	48.24%	33.97%
Ex. 2	51.79%	60.93%	55.60%
Ex. 3	9.06%	61.28%	25.91%
Ex. 4	17.58%	64.49%	49.67%
Ex. 5	8.44%	65.10%	42.60%
Ex. 6	3.98%	49.23%	32.05%
Ex. 7	51.85%	63.79%	47.99%

TABLE IV

TESTING SET MRNE PER TBIM ITERATION COUNT AT 25DB SNR LEVEL FOR MULTIPLE-SHAPE TARGETS SET.

SNR	$i = 1$	$i = 2$	$i = 3$
25 dB	21.34%	14.18%	12.74%

Thank you