Mechanical, Industrial, and Aerospace Engineering
Concordia University

Course: AERO 482, ENGR 6461
Deadline: December 10th 23:59, 2025

Project: Deterministic Strapdown INS on an Ellipsoidal Earth with Error Ellipse Analysis

For AERO 482, the project must be completed in groups of 3 (three) people. For grouping, go to the Moodle page of the course. For ENGR 6461, the project is to be done individually.

Important: For AERO 482, each team member must submit a confidential peer assessment detailing each member's contribution (in %) with a brief justification. Submit the peer assessment via Moodle together with the project.

Note: The project report must be limited to **10 pages**, including figures, appendix, and references (IEEE format). Include:

1. **Introduction:** Begin with a brief review of relevant literature, and provide any assumptions you've made, including the reasoning behind these choices.
2. **Background:** Provide an overview of the essential mathematical concepts and equations required to understand the project (e.g., matrix notations, Earth reference frame, change of coordinates, etc.).
3. **Methodology:** Describe the step-by-step methodology you use with clear explanations of each step and the algorithms applied to address the problem.
4. **Results and Discussion:** Present the results you obtained, with graphs and figures to show the outputs. Discuss these findings in depth and why/how certain results have been achieved.
5. **Conclusions:** Provide the summary and future insights and directions based on the weakness of your method, results, and discussions and areas for improvement.

Objective

Design and implement a deterministic strapdown INS on an ellipsoidal Earth to propagate position and velocity from simulated inertial sensors. Use correct frame transformations, model gravity and specific force, examine bias sensitivity, and characterize horizontal position uncertainty via covariance and error ellipses.

Problem Statement

Consider a 100 s maneuver with constant speed and gentle descent, approximated in local navigation axes as:

$$x_{\text{true}}(t) = R\cos(\omega t), \quad y_{\text{true}}(t) = R\sin(\omega t), \quad z_{\text{true}}(t) = V_z t, \qquad t \in [0, 100] \text{ s},$$

with $V = 200$ m/s, $R = 5000$ m, $\omega = V/R$, and $V_z = -3$ m/s. Roll and pitch are constant; yaw increases as $\psi(t) = \omega t$.

For the earth model use an ellipsoidal Earth (WGS-84 parameters acceptable) for kinematic sanity checks (e.g., radii of curvature) but implement the INS in local navigation coordinates. Treat gravity g consistently (constant 9.80665 m/s^2 is acceptable if stated).

Sensors (body frame).

- **Gyros:** true yaw rate $\dot{\psi} = \omega$; add a constant bias $b_g = 5 \times 10^{-5}$ rad/s and white noise $\sigma_{\text{gyro}} = 1 \times 10^{-5}$ rad/s (per axis; roll/pitch axes with zero-mean noise only).
- **Accelerometers:** measure specific force $\mathbf{f}^b$; inject constant bias $b_a = 0.01$ m/s^2 (each axis) and white noise $\sigma_{\text{acc}} = 0.05$ m/s^2.

Tasks

T1: Frames & Transformations (Ch. 2). Implement $R_z(\psi)$ to rotate body→navigation. Verify orthogonality $R_z^\top R_z = I$. Use the orthogonal projection $P = I - \mathbf{n}\mathbf{n}^\top$ to separate vertical/horizontal components for sanity checks.

T2: Gravity & Specific Force (Ch. 4). Model gravity $\mathbf{g}^n$ in the navigation frame. Convert measured body-frame accelerations to specific force, rotate to navigation $\mathbf{f}^n = R_z(\psi)\mathbf{f}^b$, then recover inertial acceleration via

$$\mathbf{a}^n = \mathbf{f}^n + \mathbf{g}^n.$$

T3: Strapdown INS Mechanization (Deterministic). Integrate gyro rate to update yaw ψ; then integrate acceleration to velocity and position:

$$\psi_k = \psi_{k-1} + \dot{\psi}_{k-1}\Delta t, \quad \mathbf{v}_k = \mathbf{v}_{k-1} + \mathbf{a}_k^n \Delta t, \quad \mathbf{r}_k = \mathbf{r}_{k-1} + \mathbf{v}_k \Delta t.$$

Use $\Delta t = 0.1$ s.

T4: Bias Characterization. Insert two stationary windows (e.g., $t \in [0, 10]$ s and $[50, 60]$ s) where the aircraft is assumed not moving in the navigation frame. Estimate constant gyro and accelerometer biases by time-averaging those windows; re-run the INS with/without bias compensation and compare drift (plots and RMSE).

T5: Uncertainty & Error Ellipses (Chs. 2 & 5). Quantify the 2-D horizontal position uncertainty at $t = 100$ s using one of:

- **(A) Monte Carlo:** ≥ 200 runs with fresh sensor noise; compute sample covariance K_{xy} of (x, y).
- **(B) First-order propagation:** derive a Jacobian J from sensor errors to (x, y) and use the covariance "sandwich" $K_{xy} \approx J\,\Sigma_{\text{sens}}\,J^\top$.

Plot the **95% error ellipse** from K_{xy} (label axes and principal directions).

T6: Sanity Checks & Reporting. Provide: (i) 3-D truth vs INS (bias-off vs bias-compensated), (ii) N–E path with error ellipse at selected times, (iii) time histories of position error components, and (iv) a short discussion linking drift trends to the sign/magnitude of injected biases.

Hints

- Use consistent frames: accelerometer outputs are in *body*;
- For ellipsoidal checks, you may compute radii of curvature $R_M(\phi), R_N(\phi)$ at a nominal latitude ϕ to compare N/E arc-lengths (keep concise; the INS itself may remain local).
- For error ellipse: eigen-decompose $K_{xy} = W\Lambda W^\top$; the semi-axes are $\sqrt{\chi^2_{2,0.95}\,\lambda_i}$.
- Initial conditions: set $\mathbf{r}(0)$ and $\mathbf{v}(0)$ equal to truth at $t = 0$; set $\psi(0) = 0$.
- Keep roll/pitch constant (e.g., zero).

Deliverables

- MATLAB scripts/functions for simulation and plotting.
- A $\leq$**10-page** report (IEEE style) including the sections listed on page 1. Cite the course text equations you used.