



**STAMFORD  
UNIVERSITY  
BANGLADESH**

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**ID** : CSE 071 08128  
**Subject** : Algorithms (CSE - 231)  
**Batch** : 71 - A  
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Ans. the Q. No: 1

Given,  $[2, 2] m + [1, 1] m = [2, 1] m$

$$00P00 = (15 \times 25 \times 25) + 0 + 0 =$$

$$M_1 (25 \times 56) = P_0 \times P_1$$

$$M_2 (56 \times 71) = P_1 \times P_2$$

$$M_3 (71 \times 19) = P_2 \times P_3$$

$$M_4 (19 \times 71) = P_3 \times P_4$$

$$P0200 = (20 \times 01 \times 15) + 0 + 0 =$$

$$m[i, j] = \begin{cases} 0, & \text{if } i = j \\ \min \{ m[i, k] + m[k+1, j] + P_{i-1} P_k P_j \}, & \text{if } i < j \end{cases}$$

$$i \leq k < j$$

$$\text{Length } l = 1, m[i, j] = 0 = m[i, i] = 0$$

Length  $l = 2,$

$$m[1,2] = m[1,1] + m[2,2] + P_0 P_1 P_2$$

$$= 0 + 0 + (25 \times 56 \times 71) = 99400$$

$$m[2,3] = m[2,2] + m[3,3] + P_1 P_2 P_3$$

$$= 0 + 0 + (56 \times 71 \times 19) = 75594$$

$$m[3,4] = m[3,3] + m[4,4] + P_2 P_3 P_4$$

$$= 0 + 0 + (71 \times 19 \times 96) = 129504$$

Length  $l=3$ ,

$$\begin{aligned}
 m[1,3] &= \min \left\{ \begin{aligned} &m[1,1] + m[2,3] + P_0 P_1 P_3 \\ &= 0 + 75594 + (25 \times 56 \times 19) = 102144 \\ &m[1,2] + m[3,3] + P_0 P_2 P_3 \\ &= 99400 + 0 + (25 \times 71 \times 19) = 133125 \end{aligned} \right. \\
 &= 102144
 \end{aligned}$$

$$\begin{aligned}
 m[2,4] &= \min \left\{ \begin{aligned} &m[2,2] + m[3,4] + P_1 P_2 P_4 \\ &= 0 + 129504 + (56 \times 71 \times 96) = 511200 \\ &m[2,3] + m[4,4] + P_1 P_3 P_4 \\ &= 75594 + 0 + (56 \times 19 \times 96) = 177688 \end{aligned} \right. \\
 &= 177688
 \end{aligned}$$

$$\begin{aligned}
 m[3,5] &= \min \left\{ \begin{aligned} &m[3,3] + m[4,5] + P_2 P_3 P_5 \\ &= \\ &m[3,4] + m[5,5] + P_2 P_4 P_5 \\ &= \end{aligned} \right.
 \end{aligned}$$

P.T.O

Length  $l=4$ 

$$\begin{aligned}
 m[1,4] &= \min \left\{ \begin{aligned} &m[1,1] + m[2,4] + P_0 P_1 P_4 \\ &= 0 + 177688 + (25 \times 56 \times 96) = 312088 \\ &m[1,2] + m[3,4] + P_0 P_2 P_4 \\ &= 99400 + 129504 + (25 \times 71 \times 96) = 399304 \\ &m[1,3] + m[4,4] + P_0 P_3 P_4 \\ &= 102144 + 0 + (25 \times 19 \times 96) = 147744 \end{aligned} \right. \\
 &= 147744
 \end{aligned}$$

$$\begin{aligned}
 m[2,5] &= \min \left\{ \begin{aligned} &m[2,2] + m[3,5] + P_1 P_2 P_5 \\ &= \\ &m[2,3] + m[4,5] + P_1 P_3 P_5 \\ &= \\ &m[2,4] + m[5,5] + P_1 P_4 P_5 \\ &= \end{aligned} \right.
 \end{aligned}$$

m:

	1	2	3	4
1	0	99400	102144	147744
2	0	0	75544	177688
3	0		0	129504
4	0	(PM)	(PM)	0

(PM) (PM) (PM, M) (III)

S:

	1	2	3	4
1	1	1	1	3
2			2	3
3				3
4				

(1, M) (1, M) (1, M) (1)

Parentthesizing the Matrices:

$$(i) M_1 M_2 M_3 M_4$$

$$(ii) (M_1 M_2 M_3) (M_4)$$

$$(iii) ((M_1 M_2) (M_3)) (M_4)$$

Optimal solution:

$$(i) ((M_1 M_2) (M_3)) (M_4)$$

Am. the Q. No: 2

$$g(4, \emptyset) = c_{41} = 9$$

$$g(3, \emptyset) = c_{31} = 18$$

$$g(2, \emptyset) = c_{21} = 19$$

$$g(3, \{4\}) = c_{34} + g(4, \emptyset)$$

$$= 3 + 9 = 12$$

$$g(4, \{3\}) = c_{43} + g(3, \emptyset)$$

$$= 8 + 18 = 26$$

$$g(2, \{4\}) = c_{24} + g(4, \emptyset)$$

$$= 5 + 9 = 14$$



$$g(4, \{2\}) = c_{42} + g(2, \emptyset)$$

$$= 8 + 19$$

$$= 27$$

$$g(2, \{3\}) = c_{23} + g(3, \emptyset)$$

$$= 22 + 18$$

$$g(3, \{2\}) = c_{32} + g(2, \emptyset)$$

$$= 7 + 19$$

$$= 26$$

$$g(2, \{3, 4\}) = \min \{ c_{23} + g(3, \{4\}), c_{24} + g(4, \{3\}) \}$$

$$= \min \{ (22 + 12), (5 + 26) \} = 31$$

$$g(3, \{2, 4\}) = \min \{ c_{32} + g(2, \{4\}), \\ c_{34} + g(4, \{2\}) \}$$

$$= \min \{ (7+14), (3+27) \}$$

$$= 21$$

$$g(4, \{2, 3\}) = \min \{ \cancel{c_{42}} + g(2, \{3\}), \\ c_{43} + g(3, \{2\}) \}$$

$$= \min \{ (8+40), (8+26) \}$$

$$= 34$$

$$g(1, \{2, 3, 4\}) = \min \{ c_{12} + g(2, \{3, 4\}), \\ c_{13} + g(3, \{2, 4\}), c_{14} + g(4, \{2, 3\}) \}$$

$$= 28$$

(Path,  $A \rightarrow C \rightarrow B \rightarrow D \rightarrow A$ )

$\{ \{S, P\} \} \rightarrow \{P, S\}$

$\{ \{S+8\}, \{P+8\} \} \rightarrow \{P+8, S+8\}$

$1 \rightarrow 2$

$\{ \{S, P\} \} \rightarrow \{P, S\} \rightarrow \{ \{S, P\} \} \rightarrow \{P, S\}$

$\{ \{S, P\} \} \rightarrow \{P, S\}$

$\{ \{S+8\}, \{P+8\} \} \rightarrow \{P+8, S+8\}$

$P, S =$

$\{ \{P, S\} \} \rightarrow \{P, S\} \rightarrow \{ \{P, S\} \} \rightarrow \{P, S\}$

$\{ \{P, S\} \} \rightarrow \{P, S\} \rightarrow \{ \{P, S\} \} \rightarrow \{P, S\}$

$8 \rightarrow$

Ans. the Q. NO: 3

The steps of Dynamic

programming algorithm is —

- (i) Characterize optimal substructure
- (ii) Recursively define the value of an optimal solution
- (iii) Compute the value bottom up
- (iv) (if) needed construct an optimal solution.



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P.T.O



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$$\begin{aligned}
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 &= 102144 + 0 + (25 \times 19 \times 96) = 147744
 \end{aligned} \right. \\
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$$\begin{aligned}
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 &= \\
 &m[2,3] + m[4,5] + p_1 p_3 p_5 \\
 &= \\
 &m[2,4] + m[5,5] + p_1 p_4 p_5 \\
 &=
 \end{aligned} \right.
 \end{aligned}$$

Sub: \_\_\_\_\_

Day 

Sat	Sun	Mon	Tue	Wed	Thu	Fri
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Time: \_\_\_\_\_ Date: \_\_\_\_/\_\_\_\_/\_\_\_\_

m:

Parentizing the Motions

	1	2	3	4
1	0	99400	102144	147744
2	0	0	75544	177688
3	0		0	129504
4	0	(PM)	(PM)	0
	(PM)	(PM)	(PM, M)	(III)

S:

Optimal solution

	1	2	3	4
1	1	1	1	3
2			2	3
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$$= 28$$

Path,  $A \rightarrow C \rightarrow B \rightarrow D \rightarrow A$

$$(F_2 + \sigma)(P_1 + \tau) \text{ prime} =$$

7 (182) 67. 400

$$(25+3), (0+3) \text{ } \{ \text{min} =$$

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