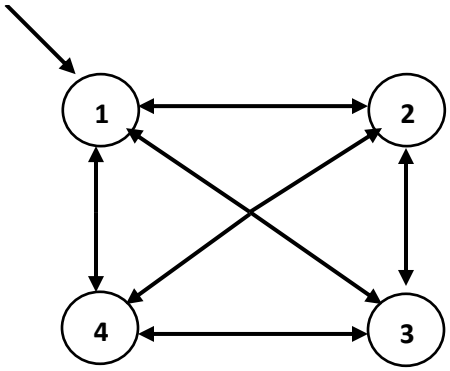
Travelling Salesperson Problem

Start



$$\begin{bmatrix} 0 & 10 & 15 & 20 \\ 5 & 0 & 9 & 10 \\ 6 & 13 & 0 & 12 \\ 8 & 8 & 9 & 0 \end{bmatrix}$$

$C_{ij} = Cost\ for\ tarvelling\ from\ i\ to\ j$

$g(i,s)$
$= Cost\ of\ travelling\ from\ \ i\ to\ the\ starting\ node$ [$through\ \ s\ set\ of\ nodes$]

[Means, Starting from i node then travelling through s set of nodes and get back to the starting node]

Solution:

$g(2,\emptyset) = C_{21} = 5$

$g(3,\emptyset) = C_{31} = 6$

$g(4,\emptyset) = C_{41} = 8$

$g(2,\{3\}) = C_{23} + \ g(3,\emptyset) = \ C_{23} + C_{31} = 9 + 6 = 15$

$g(2,\{4\}) = C_{24} + \ g(4,\emptyset) = \ C_{24} + C_{41} = 10 + 8 = 18$

$g(3,\{2\}) = C_{32} + \ g(2,\emptyset) = \ C_{32} + C_{21} = 13 + 5 = 18$

$g(3,\{4\}) = C_{34} + \ g(4,\emptyset) = \ C_{34} + C_{41} = 12 + 8 = 20$

$g(4,\{2\}) = C_{42} + \ g(2,\emptyset) = \ C_{42} + C_{21} = 8 + 5 = 13$

$g(4,\{3\}) = C_{43} + \ g(3,\emptyset) = \ C_{43} + C_{31} = 9 + 6 = 15$

$$g(2,\{3,4\}) = min\{\ C_{23} + \ g(3,\{4\}),\ \ C_{24} + \ g(4,\{3\})$$
$$= min\{(9 + 20), (10 + 15\ )\}$$
$$= min\{29\ , 25\}$$
$$= 25$$

$$g(3,\{2,4\}) = min\{\ C_{32} + \ g(2,\{4\}),\ \ C_{34} + \ g(4,\{2\})$$
$$= min\{(13 + 18), (12 + 13\ )\}$$
$$= min\{31\ , 25\}$$
$$= 25$$

$$g(4,\{2,3\}) = min\{\ C_{42} + \ g(2,\{3\}),\ \ C_{43} + \ g(3,\{2\})$$
$$= min\{(8 + 15), (9 + 18\ )\}$$
$$= min\{23\ , 27\}$$
$$= 23$$

$g(1,\{2,3,4\})$

$= min\{\ C_{12} + \ g(2,\{3,4\}),\ \ C_{13} + \ g(3,\{2,4\}),\ \ C_{14} + \ g(4,\{2,3\}\ )\ \}$

$= min\{(10 + 25), (15 + 25\ ), (20 + 23)\}$

$= \min\{35,40,43\}$

$= 35$

Path:

$1 \rightarrow 2 \rightarrow 4 \rightarrow 3 \rightarrow 1 : 35$