Matrix Chain Multiplication

Given,

A_1	30 x 35	$= P_0 \times P_1$
A_2	35 x 15	$= P_1 \times P_2$
A_3	15 x 5	$= P_2 \times P_3$
A_4	5 x 10	$= P_3 \times P_4$
A_5	10 x 20	$= P_4 \times P_5$
A_6	20 x 25	$= P_5 \times P_6$

$$m[i,j] = \begin{cases} 0 & if\ i = j \\ \min\ \{ m[i,k] + m[k+1,j] + \ p_{i-1}p_kp_j\} & if\ i < j \end{cases}$$

$\boxed{\mathbf{i \leq k < j}}$

Length l =1, m[i,j] = 0 = m[i,i] = 0

Length l =2, m[i,j] = m[i,i+1]

$m[1,2] = m[1,1] + m[2,2] + \ P_0P_1P_2 = 0 + 0 + (30 * 35 * 15) = 15750$

$m[2,3] = m[2,2] + m[3,3] + \ P_1P_2P_3 = 0 + 0 + (35 * 15 * 5) = 2625$

$m[3,4] = m[3,3] + m[4,4] + \ P_2P_3P_4 = 0 + 0 + (15 * 5 * 10) = 750$

$m[4,5] = m[4,4] + m[5,5] + \ P_3P_4P_5 = 0 + 0 + (5 * 10 * 20) = 1000$

$m[5,6] = m[5,5] + m[6,6] + \ P_4P_5P_6 = 0 + 0 + (10 * 20 * 25) = 5000$

Length l =3, m[i,j] = m[i,i+2]

$$m[1,3] = min \begin{cases} m[1,1] + m[2,3] + \ P_0P_1P_3 = 0 + 2625 + (30 * 35 * 5) \quad = 7875 \\ m[1,2] + m[3,3] + \ P_0P_2P_3 = 15750 + 0 + (30 * 15 * 5) = 18000 \end{cases}$$

$= 7875$

$$m[2,4] = min \begin{cases} m[2,2] + m[3,4] + \ P_1P_2P_4 = 0 + 750 + (35 * 15 * 10) = 6000 \\ m[2,3] + m[4,4] + \ P_1P_3P_4 = 2625 + 0 + (35 * 5 * 10) = 4375 \end{cases}$$

$= 4375$

$$m[3,5] = min \begin{cases} m[3,3] + m[4,5] + \ P_2P_3P_5 = 0 + 1000 + (15 * 5 * 20) = 2500 \\ m[3,4] + m[5,5] + \ P_2P_4P_5 = 750 + 0 + (15 * 10 * 20) = 3750 \end{cases}$$

$= 2500$

$$m[4,6] = min \begin{cases} m[4,4] + m[5,6] + \ P_3P_4P_6 = 0 + 5000 + (5 * 10 * 25) = 6250 \\ m[4,5] + m[6,6] + \ P_3P_5P_6 = 1000 + 0 + (5 * 20 * 25) = 3500 \end{cases}$$

$= 3500$

Length l =4, m[i,j] = m[i,i+3]

$$m[1,4] = min \begin{cases} m[1,1] + m[2,4] + \ P_0P_1P_4 = 0 + 4375 + (30 * 35 * 10) & = 14875 \\ m[1,2] + m[3,4] + \ P_0P_2P_4 = 15750 + 750 + (30 * 15 * 10) & = 21000 \\ m[1,3] + m[4,4] + \ P_0P_3P_4 = 7875 + 0 + (30 * 5 * 10) & = 9375 \end{cases}$$

$= 9375$

$$m[2,5] = min \begin{cases} m[2,2] + m[3,5] + \ P_1P_2P_5 = 0 + 2500 + (35 * 15 * 20) & = 13000 \\ m[2,3] + m[4,5] + \ P_1P_3P_5 = 2625 + 1000 + (35 * 5 * 20) & = 7125 \\ m[2,4] + m[5,5] + \ P_1P_4P_5 = 4375 + 0 + (35 * 10 * 20) & = 11375 \end{cases}$$

$= 7125$

$$m[3,6] = min \begin{cases} m[3,3] + m[4,6] + \ P_2P_3P_6 = 0 + 3500 + (15 * 5 * 25) & = 5375 \\ m[3,4] + m[5,6] + \ P_2P_4P_6 = 750 + 5000 + (15 * 10 * 25) & = 9500 \\ m[3,5] + m[6,6] + \ P_2P_5P_6 = 2500 + 0 + (15 * 20 * 25) & = 10000 \end{cases}$$

$= 5375$

Length l =5, m[i,j] = m[i,i+4]

$$m[1,5] = min \begin{cases} m[1,1] + m[2,5] + \ P_0P_1P_5 = 0 + 7125 + (30 * 35 * 20) & = 28125 \\ m[1,2] + m[3,5] + \ P_0P_2P_5 = 15750 + 2500 + (30 * 15 * 20) & = 27250 \\ m[1,3] + m[4,5] + \ P_0P_3P_5 = 7875 + 1000 + (30 * 5 * 20) & = 11875 \\ m[1,4] + m[5,5] + \ P_0P_4P_5 = 9375 + 1000 + (30 * 10 * 20) & = 15375 \end{cases}$$

$= 11875$

$$m[2,6] = min \begin{cases} m[2,2] + m[3,6] + \ P_1P_2P_6 = 0 + 5375 + (35 * 15 * 25) & = 18500 \\ m[2,3] + m[4,6] + \ P_1P_3P_6 = 2625 + 3500 + (35 * 5 * 25) & = 10500 \\ m[2,4] + m[5,6] + \ P_1P_4P_6 = 4375 + 5000 + (35 * 10 * 25) & = 18125 \\ m[1,4] + m[5,5] + \ P_1P_2P_6 = 9375 + 0 + (35 * 15 * 25) & = 22500 \end{cases}$$

$= 10500$

Length l =6, m[i,j] = m[i,i+5]

$$m[1,6] = min \begin{cases} m[1,1] + m[2,6] + P_0P_1P_6 = 0 + 10500 + (30 * 35 * 25) & = 36750 \\ m[1,2] + m[3,6] + P_0P_2P_6 = 15750 + 5375 + (30 * 15 * 25) & = 32375 \\ m[1,3] + m[4,6] + P_0P_3P_6 = 7875 + 3500 + (30 * 5 * 25) & = 15125 \\ m[1,4] + m[5,6] + P_0P_4P_6 = 9375 + 5000 + (30 * 10 * 25) & = 21875 \\ m[1,5] + m[6,6] + P_0P_5P_6 = 11875 + 0 + (30 * 20 * 25) & = 26875 \end{cases}$$

=15125

m :

	1	2	3	4	5	6
1	0	15750	7875	9375	11875	15125
2		0	2625	4375	7125	10500
3			0	750	2500	5375
4				0	1000	3500
5					0	5000
6						0

s :

	1	2	3	4	5	6
1		1	1	3	3	3
2			2	3	3	3
3				3	3	3
4					4	5
5						5
6						

Parenthesizing the Matrices:

i. $(A_1\ A_2\ A_3\ A_4\ A_5\ A_6)$
ii. $((A_1\ A_2\ A_3)(\ A_4\ A_5\ A_6))$
iii. $(((A_1)(A_2\ A_3))\ (A_4\ A_5\ A_6))$
iv. $(((A_1)(A_2\ A_3))\ ((A_4\ A_5)(\ A_6)))$

Optimal Solution:

i. $(((A_1)(A_2\ A_3))\ ((A_4\ A_5)\ (\ A_6)))$: 15125