Longest Common Subsequence(LCS)

Mashiwat Tabassum Waishy

Lecturer



Substring and Subsequence

A substring of a string S is another string S' that occurs in S and all the letters are **contiguous** in S

E.g. Amanpreet substring1: Aman substring2: preet

A subsequence of a string S is another string S ' that occurs in S and all the letters **need not to be contiguous** in S

E.g Amanpreet

subsequence1: Ant subsequence 2: mnet

Longest Common Subsequence

The Longest Common Subsequence (LCS) problem is as follows. We are given two strings: string A of length x and string B of length y. We have to find the longest common subsequence: the longest sequence of characters that appear left-to-right in both strings.

Example, A= KASHMIR
B= CHANDIGARH

Example, A= KASHMIR

B= CHANDIGARH

LCS has 3 length and string is HIR

LONGEST COMMON SUBSEQUENCE

A subsequence is a sequence that appears in the same relative order, but not necessarily contiguous.

In LSC, we have to find Longest common Subsequence that is in same relative order.

String of length n has 2ⁿ different possible subsequences.

E.g.--

Subsequences of "ABCDEFG".

"ABC", "ABG", "BDF", "AEG", "ACEFG", ...

EXAMPLE

LCS for input Sequences "ABCDGH" and "AEDFHR" is "ADH" of length 3.

LCS for input Sequences "AGGTAB" and "GXTXAYB" is "GTAB" of length 4.

LONGEST COMMON SUBSEQUENCE

Let the input sequences be X[0..m-1] and Y[0..n-1] of lengths m and n respectively.

let L(X[0..m-1], Y[0..n-1]) be the length of LCS of the two sequences X and Y.

If last characters of both sequences match (or X[m-1] == Y[n-1]) then $\rightarrow L(X[0..m-1], Y[0..n-1]) = 1 + L(X[0..m-2], Y[0..n-2])$

If last characters of both sequences do not match (or X[m-1] != Y[n-1] then

 \rightarrow L(X[0..m-1], Y[0..n-1]) = MAX (L(X[0..m-2], Y[0..n-1]), L(X[0..m-1], Y[0..n-2])

LONGEST COMMON SUBSEQUENCE

Consider the input strings "ABCDGH" and "AEDFHR".

Last characters do not match for the strings. So length of LCS can be written as:

L("ABCDGH", "AEDFHR") = MAX (L("ABCDG", "AEDFHR"), L("ABCDGH", "AEDFH"))

EXAMPLE

Following is a partial recursion tree for input strings "AXYT" and "AYZX"

EXAMPLE

X=M,Z,J,A,W,X,U

Y=X,M,J,Y,A,U,Z

And Longest Common Subsequence is,

LCS(X,Y)=M,J,A,U

Now we will see table from which we can find LCS of Above sequences.

Brute Force Method

Given two strings X of length m and Y of length n, find a longest subsequence common to both X and Y

STEP1: Find all subsequences of 'X'.

STEP2: For each subsequence, find whether it is a subsequence of 'Y'.

STEP3: Find the longest common subsequence from available subsequences

Brute Force Method

Given two strings X of length m and Y of length n, find a longest subse quence common to both X and Y

STEP1: Find all subsequences of 'X'. 2^m

STEP2: For each subsequence, find whether it is a subsequence of 'Y'. n*2^m

STEP3: Find the longest common subsequence from available subsequences.

$$T.C = O(n2^{m})$$

To improve time complexity, we use dynamic programming

Dynamic Programing

Optimal substructure

We have two strings

$$X = \{ x1,x2,x3,...,xn \}$$

 $Y = \{y1,y2,y3,...,ym \}$

- •First compare xn and ym. If they matched, find the subsequence in the remaining string and then append the xn with it.
- •If $xn \neq ym$,
 - Remove xn from X and find LCS from x1 to xn-1 and y1 to ym
 - Remove ym from Y and find LCS from x1 to xn and y1 to ym-1

In each step, we reduce the size of the problem into the subproblems . It is optimal substructure.

Cont.

Recursive Equation

$$X = \{ x1, x2, x3, \dots, xn \}$$

$$Y = \{y1, y2, y3, \dots, ym\}$$

C[i,j] is length of LCS in X and Y

$$c[i,j] = \begin{cases} 0 & \text{; } i=0 \text{ or } j=0 \\ 1+c[i-1,j-1] & \text{; } i, j>0 \text{ and } xi=yi \\ \max(c[i-1,j],c[i,j-1]) & \text{; } i, j>0 \text{ and } xi\neq yi \end{cases}$$

CONDITIONS FOR ARROWS

```
if x_i = y_j

then c[i, j] \leftarrow c[i-1, j-1] + 1

b[i, j] \leftarrow \text{``} \text{``}

else if c[i-1, j] \ge c[i, j-1]

then c[i, j] \leftarrow c[i-1, j]

b[i, j] \leftarrow \text{``} \text{``}

else c[i, j] \leftarrow c[i, j-1]

b[i, j] \leftarrow \text{``} \leftarrow \text{``}
```

	Y_{j}							
'		0	1	2	3	4	5	6
$X_i \rightarrow$			В	D	С	Α	В	Α
0								
1	Α							
2	В							
3	С							
4	В							
5	D							
6	Α							
7	В							

X={A, B, C, B, D, A, B} Y={B, D, C, A, B, A}

	Y_j							
ı	\	0	1	2	3	4	5	6
$X_i \longrightarrow$			В	D	С	Α	В	Α
0		0	0	0	0	0	0	0
1	Α	0						
2	В	0						
3	С	0						
4	В	0						
5	D	0						
6	Α	0						
7	В	0						

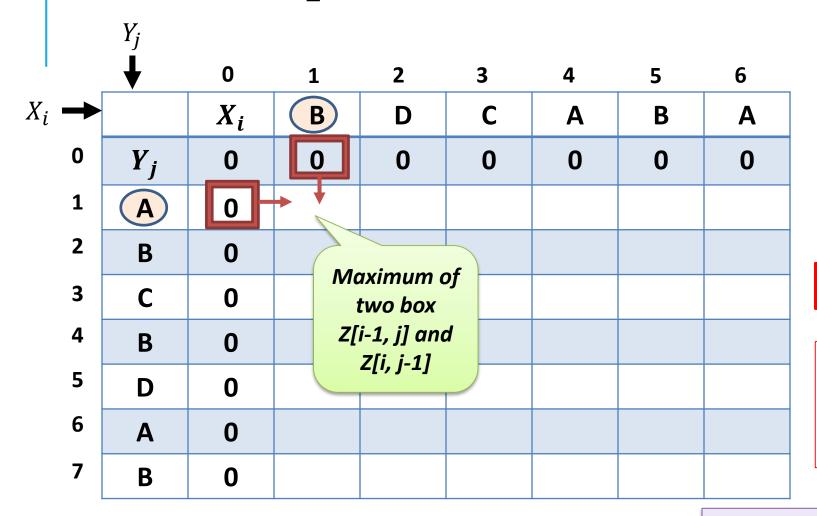
X={A, B, C, B, D, A, B} Y={B, D, C, A, B, A}

	Y_j							
ı	\	0	1	2	3	4	5	6
$X_i \longrightarrow$			В	D	С	Α	В	Α
0		0	0	0	0	0	0	0
1	Α	0						
2	В	0						
3	С	0						
4	В	0						
5	D	0						
6	Α	0						
7	В	0						

X={A, B, C, B, D, A, B} Y={B, D, C, A, B, A}

Z[i,j] Here, i=1, j=1 ∴ Z[1, 1]



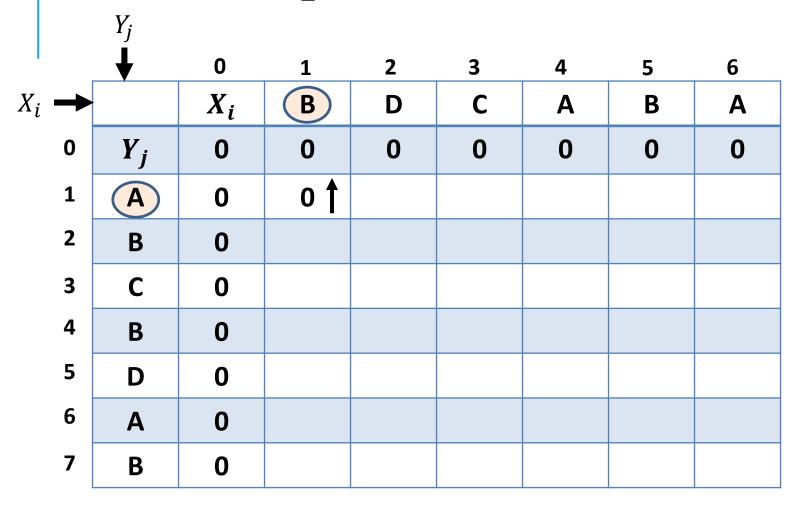


XY

ВА

Not Match

If the values are same we can take max from any one[Either ↑ or ←] But choose any one 1 direction for the whole table.

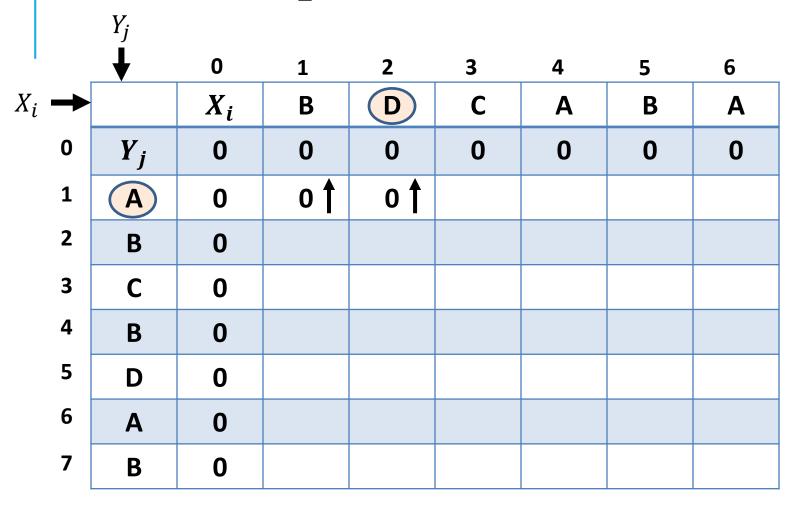


X={A, B, C, B, D, A, B} Y={B, D, C, A, B, A}

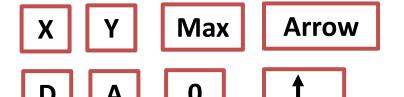


Not Match

Let's take from upper one [1]

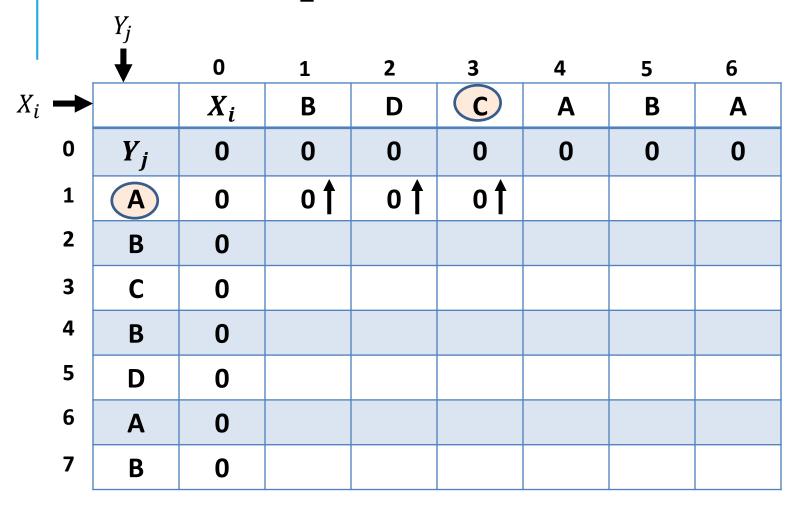


X={A, B, C, B, D, A, B} Y={B, D, C, A, B, A}

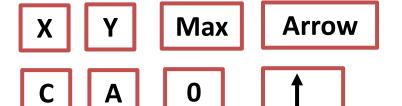


Not Match

Let's take from upper one [1]

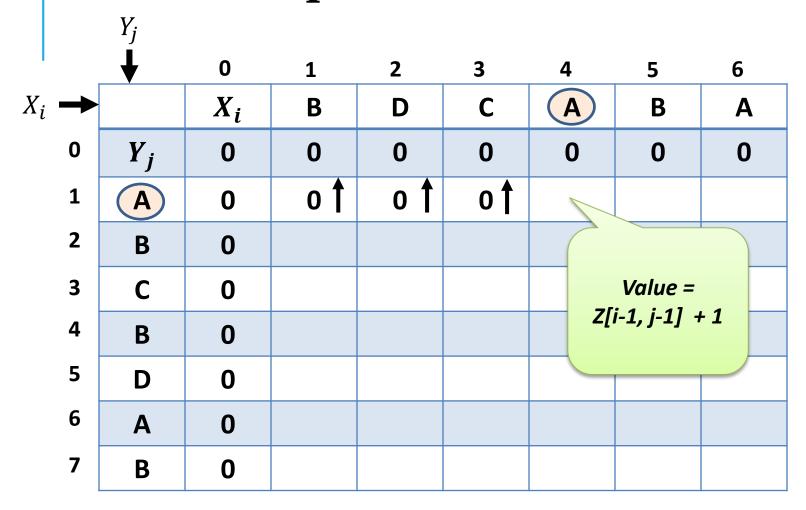


X={A, B, C, B, D, A, B} Y={B, D, C, A, B, A}



Not Match

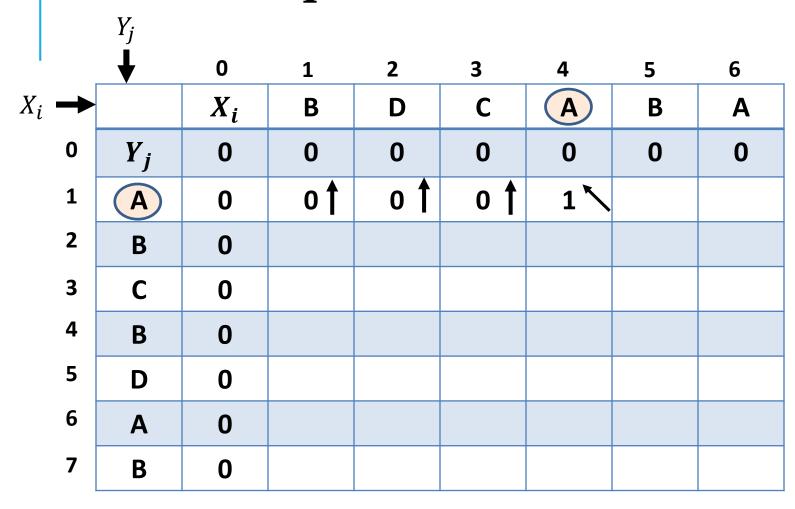
Let's take from upper one [1]



 $X=\{A, B, C, B, D, A, B\}$ $Y=\{B, D, C, A, B, A\}$



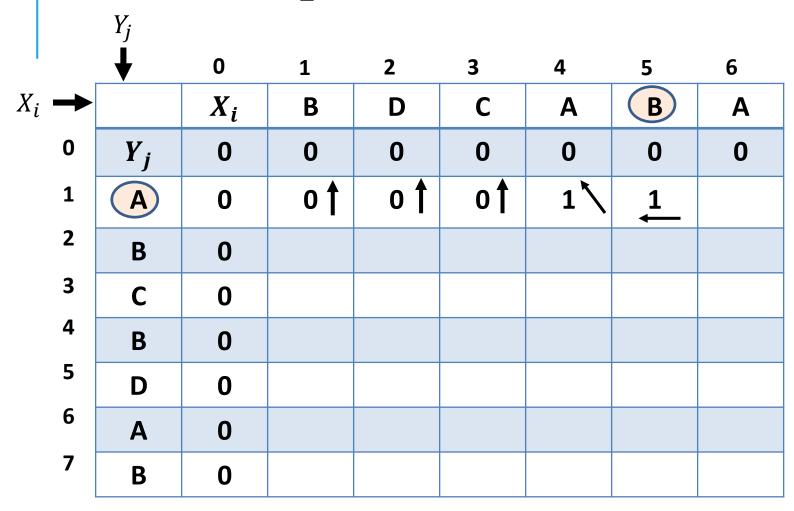
Match



X={A, B, C, B, D, A, B} Y={B, D, C, A, B, A}



Match

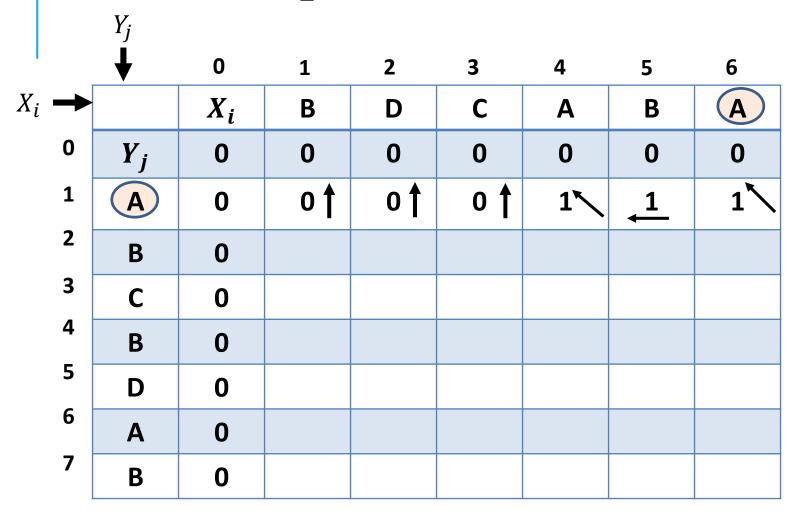


X={A, B, C, B, D, A, B} Y={B, D, C, A, B, A}



Not Match

Arrow towards the max value
Arrow [←]
Value = max one



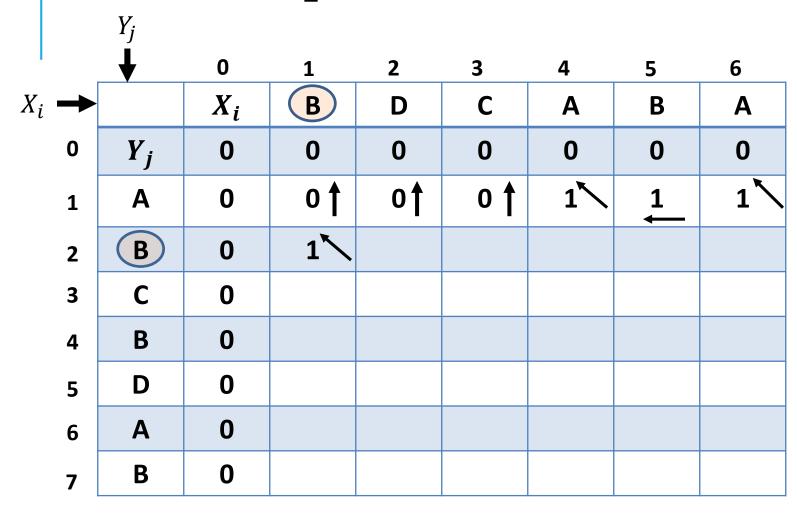
X={A, B, C, B, D, A, B} Y={B, D, C, A, B, A}





Match

If the character matches then, value = Z[i-1, j-1] +1
Arrow []

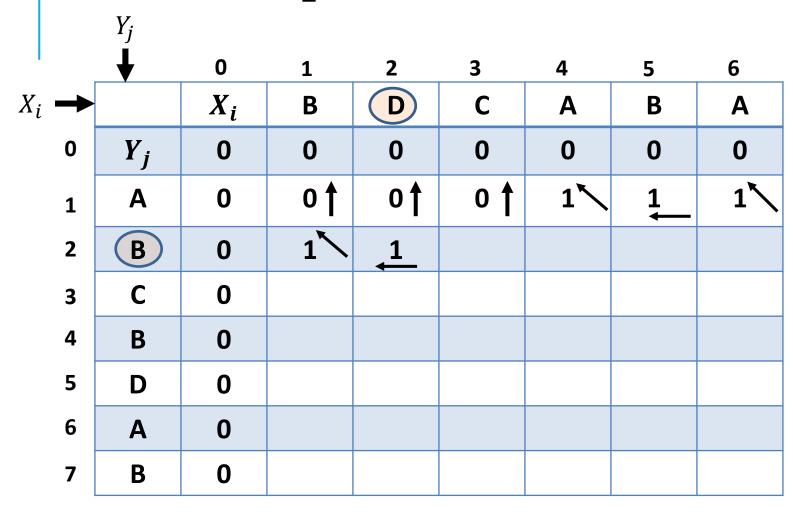


X={A, B, C, B, D, A, B} Y={B, D, C, A, B, A}



Match

If the character matches then, value = Z[i-1, j-1] +1
Arrow [\sum]

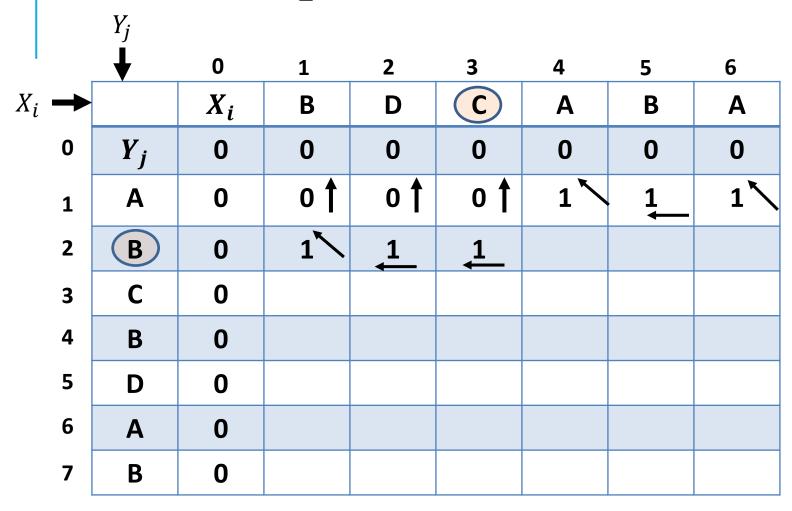


X={A, B, C, B, D, A, B} Y={B, D, C, A, B, A}



Not Match

Arrow towards the max value Arrow [←]
Value = max one

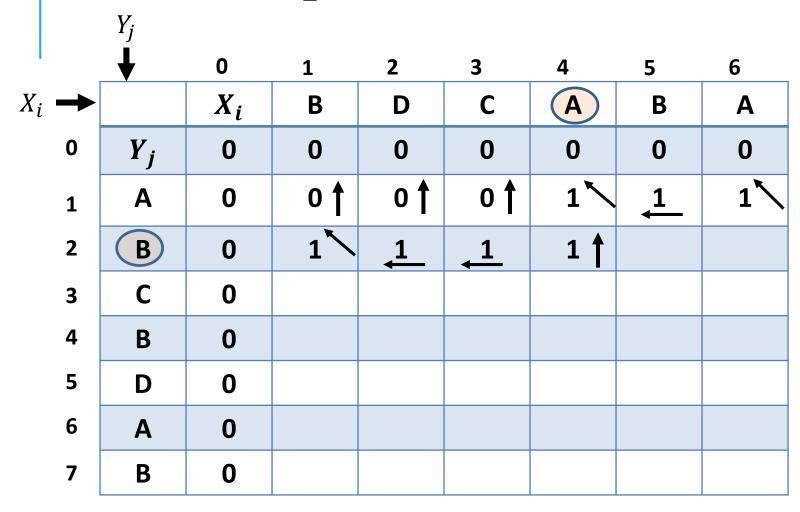


X={A, B, C, B, D, A, B} Y={B, D, C, A, B, A}



Not Match

Arrow towards the max value Arrow [←]
Value = max one

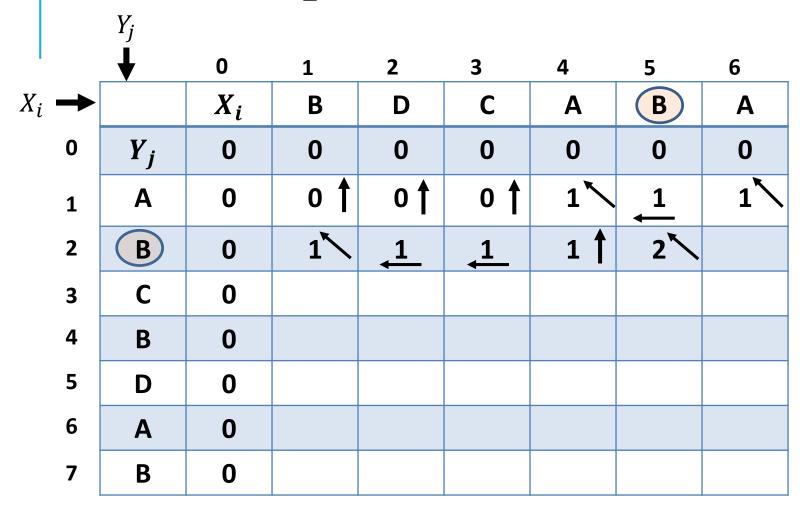


X={A, B, C, B, D, A, B} Y={B, D, C, A, B, A}



Not Match

Taking from upper one for same value [1]



X={A, B, C, B, D, A, B} Y={B, D, C, A, B, A}



Match

If the character matches then, value = Z[i-1, j-1] +1
Arrow []

	Y_j							
•	<u> </u>	0	1	2	3	4	5	6
$X_i \rightarrow$		X_{i}	В	D	С	Α	В	A
0	Y_{j}	0	0	0	0	0	0	0
1	Α	0	0 🕇	0 🕇	0 🕇	1	_1	1
2	В	0	1	1_	1	1 🕇	2	2
3	С	0						
4	В	0						
5	D	0						
6	Α	0						
7	В	0						

X={A, B, C, B, D, A, B} Y={B, D, C, A, B, A}



Not Match

Arrow towards the max value Arrow [←] Value = max one

	Y_{j}							
'	+	0	1	2	3	4	5	6
$X_i \longrightarrow$	•	X_{i}	B	D	С	Α	В	Α
0	Y_{j}	0	0	0	0	0	0	0
1	Α	0	0 🕇	0 🕇	0 🕇	1	1	1
2	В	0	1	_1_	1	1 🕇	2	2
3	C	0	1 🕇					
4	В	0						
5	D	0						
6	Α	0						
7	В	0						

X={A, B, C, B, D, A, B} Y={B, D, C, A, B, A}

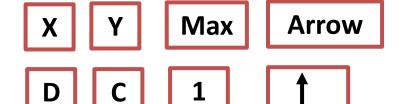


Not Match

Arrow towards the max value Arrow [†]
Value = max one

		Y_{j}							
1		\	0	1	2	3	4	5	6
X_i	->		X_{i}	В	D	С	Α	В	Α
	0	Y_{j}	0	0	0	0	0	0	0
	1	Α	0	0 🕇	0 🕇	o †	1	_1	1
	2	В	0	1	_1	<u>1</u>	1 🕇	2 \	2
	3	C	0	1 🕇	1				
	4	В	0						
	5	D	0						
	6	Α	0						
	7	В	0						

X={A, B, C, B, D, A, B} Y={B, D, C, A, B, A}



Not Match

Taking from upper one for same value [1]

		Y_j							
		\	0	1	2	3	4	5	6
X_i	→		X_i	В	D	C	Α	В	Α
	0	Y_{j}	0	0	0	0	0	0	0
	1	Α	0	0 🕇	0 🕇	0 🕇	1	1	1
	2	В	0	1	_1_	_1	1 🕇	2	2
	3	C	0	1 🕇	1 🕇	2			
	4	В	0						
	5	D	0						
	6	Α	0						
	7	В	0						

 $X={A, B, C, B, D, A, B}$ $Y={B, D, C, A, B, A}$



Match

If the character matches then, value = Z[i-1, j-1] +1
Arrow []

		Y_j							
		\	0	1	2	3	4	5	6
X_i	→		X_{i}	В	D	С	A	В	Α
	0	Y_{j}	0	0	0	0	0	0	0
	1	Α	0	0 🕇	0 🕇	0 🕇	1	1	1
	2	В	0	1	<u>1</u>	_1	1 🕇	2	<u>2</u>
	3	C	0	1 🕇	1 🕇	2 \	2 _		
	4	В	0						
	5	D	0						
	6	Α	0						
	7	В	0						

 $X={A, B, C, B, D, A, B}$ $Y={B, D, C, A, B, A}$



Not Match

Arrow towards the max value Arrow [←]
Value = max one

	Y_i		•					
	↓	0	1	2	3	4	5	6
$X_i \longrightarrow$		X_i	В	D	С	Α	B	Α
0	Y_j	0	0	0	0	0	0	0
1	Α	0	0 🕇	0 🕇	0 🕇	1	1	1
2	В	0	1	<u>1</u>	_1	1 🕇	2	2
3	C	0	1 🕇	1 🕇	2 \	2	2 🕇	
4	В	0						
5	D	0						
6	Α	0						
7	В	0						

X={A, B, C, B, D, A, B} Y={B, D, C, A, B, A}





Not Match

Taking from upper one for same value [1]

X={A, B, C, B, D, A, B} Y={B, D, C, A, B, A}

	$Y_{\underline{j}}$							
ı	+	0	1	2	3	4	5	6
$X_i \longrightarrow$		X_{i}	В	D	С	Α	В	A
0	Y_{j}	0	0	0	0	0	0	0
1	Α	0	0 🕇	0 🕇	0 🕇	1	1	1
2	В	0	1	_1_	_1	1 🕇	2	2
3	C	0	1	1 🕇	2	<u>2</u>	2	2 🕇
4	В	0						
5	D	0						
6	Α	0						
7	В	0						

X Y Max Arrow
A C 2 1

Not Match

Taking from upper one for same value [1]

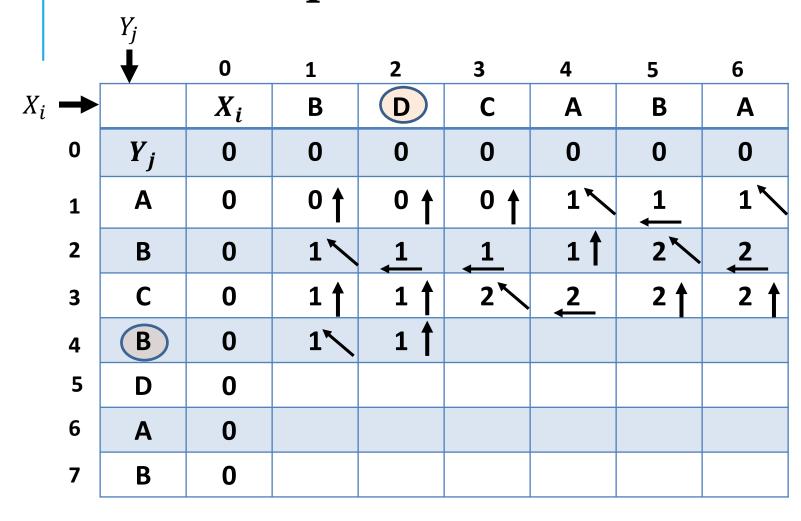
	Y_{j}							
ı		0	1	2	3	4	5	6
$X_i \rightarrow$	•	X_{i}	B	D	С	Α	В	Α
0	Y_j	0	0	0	0	0	0	0
1	Α	0	0 🕇	0 🕇	0 🕇	1	1	1
2	В	0	1	1	1	1 🕇	2 \	2
3	С	0	1 🕇	1 🕇	2	2	2 🕇	2 🕇
4	В	0	1					
5	D	0						
6	Α	0						
7	В	0						

X={A, B, C, B, D, A, B} Y={B, D, C, A, B, A}

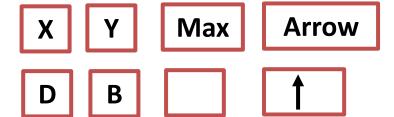


Match

If the character matches then, value = Z[i-1, j-1] +1
Arrow [\sqrt{}]



 $X=\{A, B, C, B, D, A, B\}$ $Y=\{B, D, C, A, B, A\}$

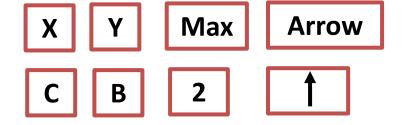


Not Match

Taking from upper one for same value [1]

	Y_j							
	\	0	1	2	3	4	5	6
$X_i \longrightarrow$		X_i	В	D	C	Α	В	Α
0	Y_{j}	0	0	0	0	0	0	0
1	Α	0	0 🕇	0 🕇	0 🕇	1	1	1
2	В	0	1	1	<u>1</u>	1 🕇	2 🔨	2
3	С	0	1 🕇	1 🕇	2 🔨	2_	2 🕇	2 🕇
4	В	0	1	1 1	2 🕇			
5	D	0						
6	Α	0						
7	В	0						

 $X=\{A, B, C, B, D, A, B\}$ $Y=\{B, D, C, A, B, A\}$

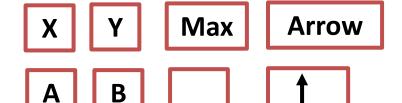


Not Match

Arrow towards the max value Arrow [1]
Value = max one

	Y_j							
ı	\	0	1	2	3	4	5	6
X_i	>	X_i	В	D	С	A	В	Α
0	Y_{j}	0	0	0	0	0	0	0
1	Α	0	0 🕇	0 🕇	0 🕇	1	1	1
2	В	0	1	1	1	1 🕇	2 \	2
3	С	0	1 🕇	1 🕇	2 \	2	2 🕇	2 🕇
4	B	0	1	1 1	2 🕇	2 🕇		
5	D	0						
6	Α	0						
7	В	0						

X={A, B, C, B, D, A, B} Y={B, D, C, A, B, A}



Not Match

Taking from upper one for same value [1]

	Y_{j}							
1		0	1	2	3	4	5	6
$X_i \longrightarrow$		X_i	В	D	С	Α	B	Α
0	Y_{j}	0	0	0	0	0	0	0
1	Α	0	0 🕇	0 🕇	0 🕇	1	1	1
2	В	0	1	1	1	1 🕇	2 \	2
3	С	0	1 🕇	1 🕇	2 🔨	2	2 🕇	2 🕇
4	B	0	1	1 1	2 🕇	2 🕇	3 🔨	
5	D	0						
6	Α	0						
7	В	0						

 $X={A, B, C, B, D, A, B}$ $Y={B, D, C, A, B, A}$



Match

If the character matches then, value = Z[i-1, j-1] +1
Arrow [\sqrt{}]

X={A, B, C, B, D, A, B} Y={B, D, C, A, B, A}

	Y_j							
•	*	0	1	2	3	4	5	6
$X_i \longrightarrow$		X_i	В	D	С	Α	В	A
0	Y_{j}	0	0	0	0	0	0	0
1	Α	0	0 🕇	0 🕇	0 🕇	1	1	1
2	В	0	1	_1	_1	1 🕇	2	2
3	С	0	1 🕇	1 🕇	2	2	2 🕇	2 🕇
4	B	0	1	1 1	2 🕇	2 🕇	3/	3
5	D	0						
6	Α	0						
7	В	0						

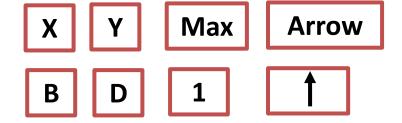
X Y Max Arrow
A B 3 ←

Not Match

Arrow towards the max value Arrow [←]
Value = max one

	Y_j							
'	+	0	1	2	3	4	5	6
$X_i \longrightarrow$		X_i	B	D	С	Α	В	Α
0	Y_{j}	0	0	0	0	0	0	0
1	Α	0	0 🕇	0 🕇	0 🕇	1	1	1
2	В	0	1	1	<u>1</u>	1 🕇	2 🔨	2
3	С	0	1 🕇	1 🕇	2 🔨	2	2 🕇	2 🕇
4	В	0	1	1 1	2 🕇	2 🕇	3 🔨	3_
5	D	0	1 🕇					
6	Α	0						
7	В	0						

 $X={A, B, C, B, D, A, B}$ $Y={B, D, C, A, B, A}$



Not Match

Arrow towards the max value Arrow [1]
Value = max one

	Y_j							
•	▼	0	1	2	3	4	5	6
$X_i \longrightarrow$		X_{i}	В	D	С	Α	В	Α
0	Y_{j}	0	0	0	0	0	0	0
1	Α	0	0 🕇	0 🕇	0 🕇	1	1	1
2	В	0	1	1_	1	1 🕇	2 🔨	2
3	С	0	1 🕇	1 🕇	2 🔨	2	2 🕇	2 🕇
4	В	0	1	1 🕇	2 🕇	2 🕇	3 🔨	<u>3</u>
5	D	0	1 🕇	2				
6	Α	0						
7	В	0						

X={A, B, C, B, D, A, B} Y={B, D, C, A, B, A}

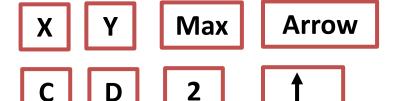


Match

If the character matches then, value = Z[i-1, j-1] +1
Arrow []

	Y_j							
ı	\	0	1	2	3	4	5	6
$X_i \longrightarrow$		X_{i}	В	D	C	Α	В	Α
0	Y_{j}	0	0	0	0	0	0	0
1	Α	0	0 🕇	0 🕇	0 🕇	1	1	1
2	В	0	1	_1_	1	1 🕇	2 \	2
3	С	0	1 🕇	1 🕇	2 \	2	2 🕇	2 🕇
4	В	0	1	1 1	2 🕇	2 🕇	3	3_
5	D	0	1 🕇	2	2 🕇			
6	Α	0						
7	В	0						

 $X=\{A, B, C, B, D, \overline{A}, B\}$ $Y=\{B, D, C, A, B, A\}$



Not Match

Taking from upper one for same value [↑]

	Y_j							
ı	+	0	1	2	3	4	5	6
$X_i \rightarrow$		X_i	В	D	С	A	В	Α
0	Y_{j}	0	0	0	0	0	0	0
1	Α	0	0 🕇	0 🕇	0 🕇	1	1	1
2	В	0	1	1	<u>1</u>	1 🕇	2	2
3	С	0	1 🕇	1 🕇	2	2	2 🕇	2 🕇
4	В	0	1	1 1	2 🕇	2 🕇	3/	3
5	D	0	1 🕇	2	2 🕇	2 🕇		
6	Α	0						
7	В	0						

 $X=\{A, B, C, B, D, \overline{A}, B\}$ $Y=\{B, D, C, A, B, A\}$

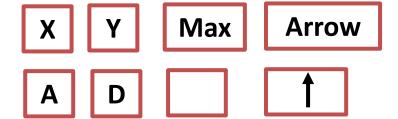


Not Match

Taking from upper one for same value [↑]

	Y_j							
1	\	0	1	2	3	4	5	6
$X_i \longrightarrow$		X_{i}	В	D	С	Α	B	Α
0	Y_{j}	0	0	0	0	0	0	0
1	Α	0	0 🕇	0 🕇	0 🕇	1	1	1
2	В	0	1	<u>1</u>	<u>1</u>	1 🕇	2	2
3	С	0	1 🕇	1 🕇	2 \	2	2 🕇	2 🕇
4	В	0	1	1 🕇	2 🕇	2 🕇	3	3_
5	D	0	1 🕇	2	2 🕇	2 🕇	з 🕇	
6	Α	0						
7	В	0						

 $X={A, B, C, B, D, A, B}$ $Y={B, D, C, A, B, A}$



Not Match

X={A, B, C, B, D, A, B} Y={B, D, C, A, B, A}

	Y_j				_		_	_
	_ ▼	0	1	2	3	4	5	6
$X_i \longrightarrow$		X_i	В	D	С	Α	В	A
0	Y_{j}	0	0	0	0	0	0	0
1	Α	0	0 🕇	0 🕇	0 🕇	1	1	1
2	В	0	1	<u>1</u>	_1	1 🕇	2 \	2
3	С	0	1 🕇	1 🕇	2	2	2 🕇	2 🕇
4	В	0	1	1 1	2 🕇	2 🕇	3	3
5	D	0	1 🕇	2	2 🕇	2 🕇	3 🕇	3 🕇
6	Α	0						
7	В	0						

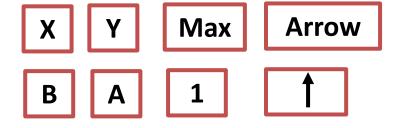
X Y Max Arrow
A D 3 †

Not Match

Taking from upper one for same value [1]

	Y_j	0	1	2	3	4	5	6
$X_i \longrightarrow$	V	X_i	B	D	C	A	В	A
0	Y_j	0	0	0	0	0	0	0
1	Α	0	0 🕇	0 †	0 🕇	1	1	1
2	В	0	1	1	1	1 🕇	2 🔨	2
3	С	0	1 🕇	1 🕇	2 🔨	2	2 🕇	2 🕇
4	В	0	1	1 🕇	2 🕇	2 🕇	3 🔨	3
5	D	0	1 🕇	2	2 🕇	2 🕇	3 🕇	3 🕇
6	A	0	1 🕇					
7	В	0						

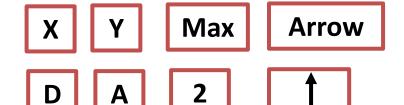
 $X={A, B, C, B, D, A, B}$ $Y={B, D, C, A, B, A}$



Not Match

	Y_{j}							
'	+	0	1	2	3	4	5	6
$X_i \longrightarrow$		X_{i}	В	D	С	Α	В	Α
0	Y_{j}	0	0	0	0	0	0	0
1	Α	0	0 🕇	0 🕇	0 🕇	1	1	1
2	В	0	1	1_	1	1 🕇	2 🔨	2
3	С	0	1 🕇	1 🕇	2 🔨	2_	2 🕇	2 🕇
4	В	0	1	1	2 🕇	2 🕇	3 🔨	3
5	D	0	1 🕇	2 \	2 🕇	2 🕇	3 🕇	3 🕇
6	A	0	1 🕇	2 🕇				
7	В	0						

 $X={A, B, C, B, D, A, B}$ $Y={B, D, C, A, B, A}$



Not Match

	Y_j							
1	+	0	1	2	3	4	5	6
$X_i \longrightarrow$		X_i	В	D	C	Α	В	Α
0	Y_j	0	0	0	0	0	0	0
1	Α	0	0 🕇	0 🕇	0 🕇	1	1	1
2	В	0	1	1_	1	1 🕇	2 🔨	2
3	С	0	1 🕇	1 🕇	2 🔨	2	2 🕇	2 🕇
4	В	0	1	1 1	2 🕇	2 🕇	3 🔨	<u>3</u>
5	D	0	1 🕇	2	2 🕇	2 🕇	3 🕇	3 🕇
6	A	0	1 🕇	2 🕇	2 🕇			
7	В	0						

X={A, B, C, B, D, A, B} Y={B, D, C, A, B, A}



Not Match

Taking from upper one for same value [1]

	$Y_{\underline{j}}$							
1	+	0	1	2	3	4	5	6
$X_i \longrightarrow$		X_i	В	D	С	A	В	Α
0	Y_{j}	0	0	0	0	0	0	0
1	Α	0	0 🕇	0 🕇	0 🕇	1	1	1
2	В	0	1	1	<u>1</u>	1 🕇	2 🔨	2_
3	С	0	1 🕇	1 🕇	2 🔨	2_	2 🕇	2 🕇
4	В	0	1	1	2 🕇	2 🕇	3 \	3_
5	D	0	1 🕇	2 🔨	2 🕇	2 🕇	3 🕇	3 🕇
6	A	0	1 🕇	2 🕇	2 🕇	3 🔨		
7	В	0						

 $X={A, B, C, B, D, A, B}$ $Y={B, D, C, A, B, A}$

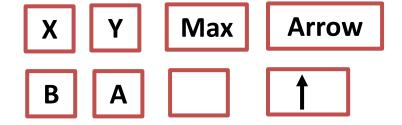


Match

If the character matches then, value = Z[i-1, j-1] +1
Arrow [\sum]

	Y_j							
•	▼	0	1	2	3	4	5	6
$X_i \rightarrow$		X_i	В	D	С	Α	B	Α
0	Y_{j}	0	0	0	0	0	0	0
1	Α	0	0 🕇	0 🕇	0 🕇	1	1	1
2	В	0	1	1_	<u>1</u>	1 🕇	2 🔨	2
3	С	0	1 🕇	1 🕇	2 🔨	2_	2 🕇	2 🕇
4	В	0	1	1 🕇	2 🕇	2 🕇	3 🔨	3
5	D	0	1 🕇	2 🔨	2 🕇	2 🕇	3 🕇	3 🕇
6	A	0	1 🕇	2 🕇	2 🕇	3 🔨	3 🕇	
7	В	0						

 $X=\{A, B, C, B, D, \overline{A}, B\}$ $Y=\{B, D, C, A, B, A\}$



Not Match

Taking from upper one for same value [†]

	Y_j							
•	+	0	1	2	3	4	5	6
$X_i \longrightarrow$		X_{i}	В	D	С	Α	В	A
0	Y_{j}	0	0	0	0	0	0	0
1	Α	0	0 🕇	0 🕇	0 🕇	1	1	1
2	В	0	1	1_	<u>1</u>	1 🕇	2 \	2_
3	С	0	1 🕇	1 🕇	2 🔨	2_	2 🕇	2 🕇
4	В	0	1	1 🕇	2 🕇	2 🕇	3 \	3
5	D	0	1 🕇	2 🔨	2 🕇	2 🕇	3 🕇	3 🕇
6	A	0	1 🕇	2 🕇	2 🕇	3 🔨	3 🕇	4
7	В	0						

X={A, B, C, B, D, A, B} Y={B, D, C, A, B, A}



Match

If the character matches then, value = Z[i-1, j-1] +1
Arrow [\sqrt{}]

	Y_j							
	▼	0	1	2	3	4	5	6
$X_i \longrightarrow$		X_i	B	D	С	Α	В	Α
0	Y_{j}	0	0	0	0	0	0	0
1	Α	0	0 🕇	0 🕇	0 🕇	1	1	1
2	В	0	1	1	_1_	1 🕇	2 🔨	2_
3	С	0	1 🕇	1 🕇	2 🔨	2_	2 🕇	2 🕇
4	В	0	1	1 🕇	2 🕇	2 🕇	3 🔨	3_
5	D	0	1 🕇	2 🔨	2 🕇	2 🕇	3 🕇	з 🕇
6	Α	0	1 🕇	2 🕇	2 🕇	3 🔨	3 🕇	4
7	B	0	1					

X={A, B, C, B, D, A, B} Y={B, D, C, A, B, A}



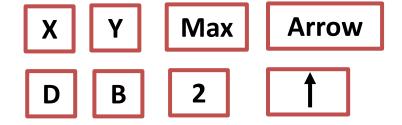
ВВ

Match

If the character matches then, value = Z[i-1, j-1] +1
Arrow []

	Y_j							
'	+	0	1	2	3	4	5	6
$X_i \rightarrow$		X_i	В	D	С	Α	В	Α
0	Y_j	0	0	0	0	0	0	0
1	Α	0	0 🕇	0 🕇	0 🕇	1	1	1
2	В	0	1	1	1	1 🕇	2 🔨	2
3	С	0	1 🕇	1 🕇	2 🔨	2_	2 🕇	2 🕇
4	В	0	1	1	2 🕇	2 🕇	3 🔨	3
5	D	0	1 🕇	2 🔨	2 🕇	2 🕇	3 🕇	3 🕇
6	Α	0	1 🕇	2 🕇	2 🕇	3 🔨	3 🕇	4
7	B	0	1	2 🕇				

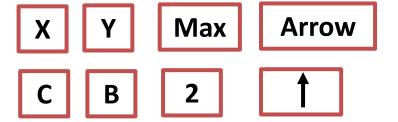
 $X={A, B, C, B, D, A, B}$ $Y={B, D, C, A, B, A}$



Not Match

	Y_j							
		0	1	2	3	4	5	6
$X_i \longrightarrow$		X_{i}	В	D	C	Α	В	Α
0	Y_j	0	0	0	0	0	0	0
1	Α	0	0 🕇	0 🕇	0 🕇	1	1	1 🔨
2	В	0	1	1_	1	1 🕇	2 🔨	2
3	С	0	1 🕇	1 🕇	2 🔨	2	2 🕇	2 🕇
4	В	0	1	1 🕇	2 🕇	2 🕇	3 🔨	3
5	D	0	1 🕇	2 🔨	2 🕇	2 🕇	3 🕇	3 🕇
6	Α	0	1 🕇	2 🕇	2 🕇	3 🔨	3 🕇	4
7	B	0	1	2 🕇	2 🕇			

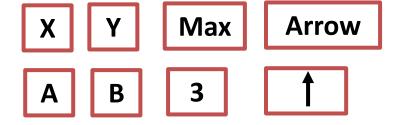
 $X={A, B, C, B, D, A, B}$ $Y={B, D, C, A, B, A}$



Not Match

	$Y_{\underline{j}}$							
ı	\	0	1	2	3	4	5	6
$X_i \longrightarrow$		X_{i}	В	D	С	A	В	Α
0	Y_{j}	0	0	0	0	0	0	0
1	Α	0	0 🕇	0 🕇	0 🕇	1	1_	1
2	В	0	1	1	_1_	1 🕇	2 🔨	2
3	С	0	1 🕇	1 🕇	2 \	2_	2 🕇	2 🕇
4	В	0	1	1 🕇	2 🕇	2 🕇	3 🔨	3_
5	D	0	1 🕇	2 🔨	2 🕇	2 🕇	3 🕇	3 🕇
6	Α	0	1 🕇	2 🕇	2 🕇	3 🔨	з 🕇	4
7	В	0	1	2 🕇	2 🕇	3 🕇		

 $X=\{A, B, C, B, D, \overline{A}, B\}$ $Y=\{B, D, C, A, B, A\}$



Not Match

Arrow towards the max value Arrow [1]
Value = max one

	Y_j							
•		0	1	2	3	4	5	6
$X_i \longrightarrow$		X_i	В	D	С	Α	В	Α
0	Y_{j}	0	0	0	0	0	0	0
1	Α	0	0 🕇	0 🕇	0 🕇	1	1	1
2	В	0	1	1_	1	1 🕇	2 🔨	2
3	С	0	1 🕇	1 🕇	2 🔨	2	2 🕇	2 🕇
4	В	0	1	1 🕇	2 🕇	2 🕇	3 🔨	3_
5	D	0	1 🕇	2 🔨	2 🕇	2 🕇	3 🕇	3 🕇
6	Α	0	1 🕇	2 🕇	2 🕇	3	3 🕇	4
7	B	0	1	2 🕇	2 🕇	3 🕇	4	

X={A, B, C, B, D, A, B} Y={B, D, C, A, B, A}





Match

If the character matches then, value = Z[i-1, j-1] +1
Arrow [\stacksquare]

	Y_j	0	1	2	3	4	_	6
	V	U	Ŧ		3	4	5	0
$X_i \longrightarrow$		X_i	В	D	С	Α	В	A
0	Y_{j}	0	0	0	0	0	0	0
1	Α	0	0 🕇	0 🕇	0 🕇	1	1	1
2	В	0	1	1	_1	1 🕇	2 \	2_
3	С	0	1 🕇	1 🕇	2 🔨	2	2 🕇	2 🕇
4	В	0	1	1 🕇	2 🕇	2 🕇	3 \	3
5	D	0	1 🕇	2 🔨	2 🕇	2 🕇	3 🕇	3 🕇
6	Α	0	1 🕇	2 🕇	2 🕇	3	3 🕇	4
7	В	0	1	2 🕇	2 🕇	3 🕇	4	4 🕇

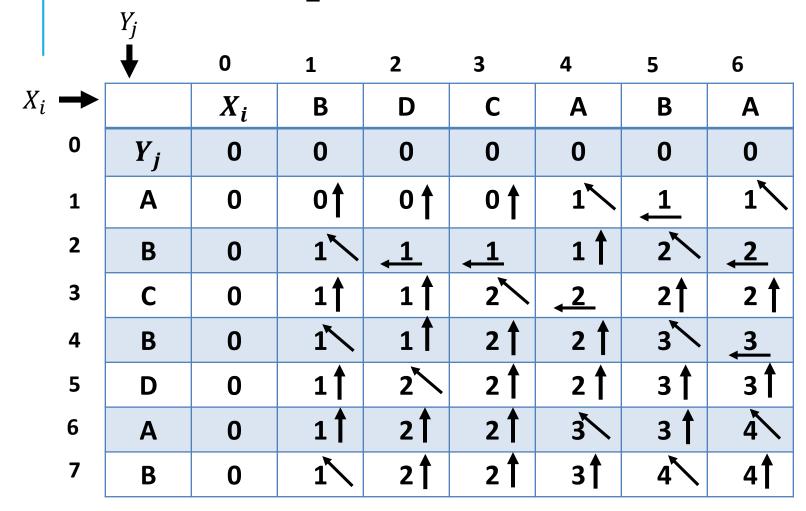
 $X={A, B, C, B, D, A, B}$ $Y={B, D, C, A, B, A}$



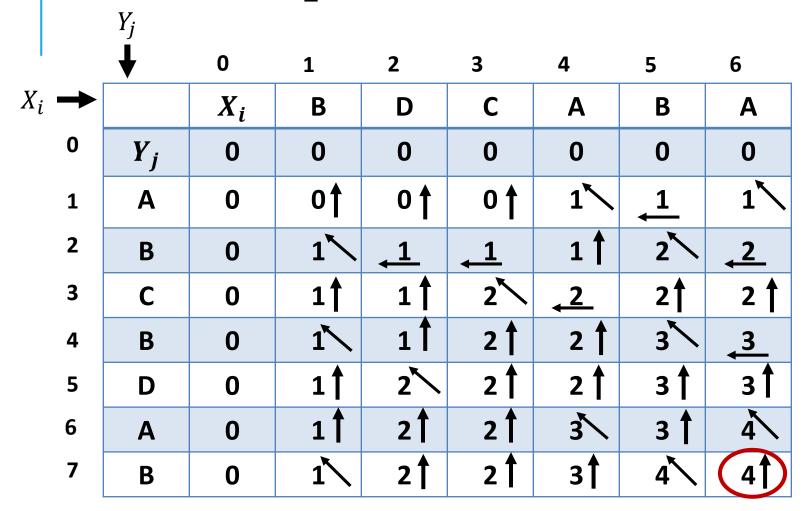
Not Match

Taking from upper one for same value [1]

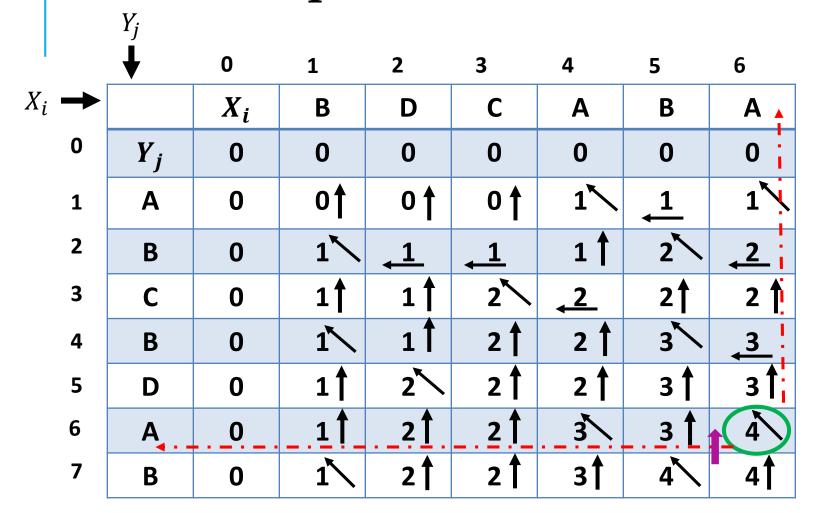
Traceback Approach



- 1. Firstly have to point out the highest value.
- 2. Then for left and up arrow we will just follow the direction.
- 3. For diagonal arrow we will point out the character for this cell.



- 1. Firstly have to point out the highest value.
- 2. Then for left and up arrow we will just follow the direction.
- 3. For diagonal arrow we will point out the character for this cell.



- 1. Firstly have to point out the highest value.
- 2. Then for left and up arrow we will just follow the direction.
- 3. For diagonal arrow we will point out the character for this cell.

X={A, B, C, B, D, A, B} Y={B, D, C, A, B, A}

	\bigvee_{j}	0	1	2	3	4	5	6
$X_i \longrightarrow$		X_i	В	D	С	Α	В	Α
0	Y_{j}	0	0	0	0	0	0	0
1	Α	0	0	0 🕇	0 🕇	1	1	1
2	В	0	1	_1	_1	1 🕇	2	2
3	С	0	1	1 🕇	2	2	2	2 🕇
4	В	0	1	1 1	2 🕇	2 🕇	3	_3_
5	D	0	1 🕇	2	2 🕇	2 🕇	3	3 1
6	Α	0	1 1	2 1	2 1	3	3 🕇	4
7	В	0	1\	2 🕇	2 🕇	3	4	4

- 1. Firstly have to point out the highest value.
- 2. Then for left and up arrow we will just follow the direction.
- 3. For diagonal arrow we will point out the character for this cell.

LCS $Z = \langle A \rangle$

	Y_j	0	1	2	3	4	5	6
$X_i \longrightarrow$	•	X_i	В	D	С	Α	В	Α
0	Y_j	0	0	0	0	0	0	0
1	Α	0	0	0 🕇	0 🕇	1	1	1
2	В	0	1	_1	_1	1 🕇	2	2
3	С	0	1	1 🕇	2	_2_	2	2 🕇
4	Β₄	0_	_1	<u>1 </u>	2	2	43	3
5	D	0	1 🕇	2	2 🕇	2 🕇	3	3 1
6	Α	0	1 1	2 1	2 🕇	3	3 🕇	4
7	В	0	1\	2 🕇	2 🕇	3	4	4

- 1. Firstly have to point out the highest value.
- 2. Then for left and up arrow we will just follow the direction.
- 3. For diagonal arrow we will point out the character for this cell.

$$LCS Z = < B A >$$

X={A, B, C, B, D, A, B} Y={B, D, C, A, B, A}

	Y_j							
'	\	0	1	2	3	4	5	6
$X_i \longrightarrow$		X_i	В	D	С	Α	В	Α
0	Y_{j}	0	0	0	0	0	0	0
1	Α	0	0	0 🕇	0 🕇	1	1	1
2	В	0	1	_1_	_1	1 🕇	2	2
3	С	0	1	1 🕇	2	2	2	2 🕇
4	В	0	1\	1 1	2 🕇	2 🕇	4 3	3
5	D	0	1 🕇	2	2 🕇	2 🕇	3	3 🕇
6	Α	0	1 1	2 1	2 🕇	3	3 🕇	4
7	В	0	1\	2 🕇	2 🕇	3	4	41

- 1. Firstly have to point out the highest value.
- 2. Then for left and up arrow we will just follow the direction.
- 3. For diagonal arrow we will point out the character for this cell.

LCS Z = < B A >

 $X=\{A, B, C, B, D, A, B\}$ $Y=\{B, D, C, A, B, A\}$

	Y_j	•					_	
		0	1	2	3	4	5	6
$X_i \longrightarrow$		X_i	В	D	C 🛕	Α	В	Α
0	Y_j	0	0	0	0	0	0	0
1	Α	0	0	0 🕇	0 🕇	1	1	1
2	В	0	1	_1	_1	1 🕇	2	_2_
3	C	0	11	<u>1</u>	2		2	2 🕇
4	В	0	1\	1 1	2 🕇	2 🕇	4 3	_3
5	D	0	1 🕇	2	2 🕇	2 🕇	3	3 1
6	Α	0	1	2 🕇	2 🕇	3	3 🕇	4
7	В	0	1	2 🕇	2 🕇	3	4	4

- 1. Firstly have to point out the highest value.
- 2. Then for left and up arrow we will just follow the direction.
- 3. For diagonal arrow we will point out the character for this cell.

LCS Z = < C B A >

X={A, B, C, B, D, A, B} Y={B, D, C, A, B, A}

	Y_j							
'	\	0	1	2	3	4	5	6
$X_i \longrightarrow$		X_i	В	D	С	Α	В	Α
0	Y_{j}	0	0	0	0	0	0	0
1	Α	0	0†	0 🕇	0 🕇	1	1	1
2	В	0	1	_1	1	1 🕇	2	2
3	С	0	1	1 🕇 🐪	2	<u>2</u>	2	2 🕇
4	В	0	1	1	2 🕇	2 🕇	4 3	3
5	D	0	1 🕇	2	2 🕇	2 🕇	3	3 🕇
6	Α	0	1 🕇	2 1	2 🕇	3	3 🕇	4
7	В	0	1\	2 🕇	2 🕇	3	4	4

- 1. Firstly have to point out the highest value.
- 2. Then for left and up arrow we will just follow the direction.
- 3. For diagonal arrow we will point out the character for this cell.

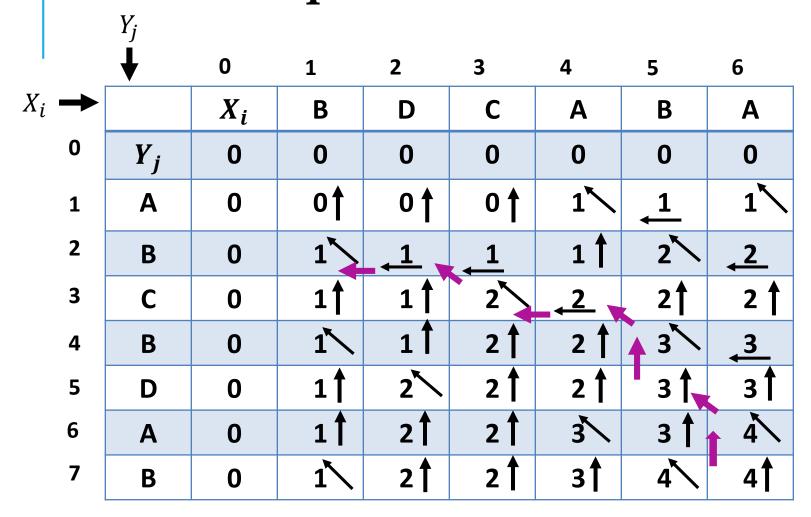
LCS Z = < C B A >

X={A, B, C, B, D, A, B} Y={B, D, C, A, B, A}

	Y_j	0	1	2	3	4	5	6
$X_i \longrightarrow$	•	X_i	В	D	С	Α	В	Α
0	Y_j	0	0	0	0	0	0	0
1	Α	0	01	0 🕇	0 🕇	1	1	1
2	В	0	1	<u>1</u>	1	1 🕇	2	<u>2</u>
3	С	0	1	1 🕇	2	2	2	2 🕇
4	В	0	1	1 1	2 🕇	2 🕇	4 3	3
5	D	0	1 🕇	2	2 🕇	2 🕇	3	3 1
6	Α	0	1 1	2 1	2 1	3	3 🕇	4
7	В	0	1	2 🕇	2 🕇	3	4	4

- 1. Firstly have to point out the highest value.
- 2. Then for left and up arrow we will just follow the direction.
- 3. For diagonal arrow we will point out the character for this cell.

LCS Z = < B C B A >



X={A, B, C, B, D, A, B} Y={B, D, C, A, B, A}

- 1. Firstly have to point out the highest value.
- 2. Then for left and up arrow we will just follow the direction.
- 3. For diagonal arrow we will point out the character for this cell.

 $LCS Z = \langle B C B A \rangle : 4$

		0	1	2	3	4	5	6	7
		Ø	М	Z	J	Α	w	x	U
0	Ø	0	0	0	0	0	0	0	0
1	X	0	0	0	0	0	0	1	1 ←
2	М	0	1	1	1←—	1 ←	1 ←	1 ←	1 ←
3	J	0	1 †	1	2	_2 ←	2 ←	2 ←	2 ←
4	Y	0	1	1	2	2	2	2	2
5	Α	0	1	1 🕇	2	3	3	3 ←	3 ←
6	U	0	1 🕇	1	2	3	3	3	4
7	Z	0	1	2	2	3	3	3	4

LCS ALGORITHM

```
LCS-LENGTH(X, Y)
 1 m \leftarrow length[X]
 2 n \leftarrow length[Y]
 3 for i \leftarrow 1 to m
            do c[i, 0] \leftarrow 0
 5 for j \leftarrow 0 to n
            do c[0, j] \leftarrow 0
      for i \leftarrow 1 to m
            do for j \leftarrow 1 to n
                      do if x_i = y_i
10
                             then c[i, j] \leftarrow c[i - 1, j - 1] + 1
11
                                   b[i,j] \leftarrow "\"
                             else if c[i - 1, j] \ge c[i, j - 1]
13
                                      then c[i, j] \leftarrow c[i-1, j]
14
                                             b[i, j] \leftarrow "\uparrow"
15
                                      else c[i, j] \leftarrow c[i, j-1]
                                             b[i, j] \leftarrow "\leftarrow"
16
      return c and b
```

CONSTRUCTING AN LCS

```
PRINT-LCS(b, X, i, j)
   if i = 0 or j = 0
      then return
   if b[i, j] = "\\"
      then PRINT-LCS(b, X, i-1, j-1)
           print x_i
   elseif b[i, j] = "\uparrow"
      then PRINT-LCS(b, X, i - 1, j)
   else PRINT-LCS(b, X, i, j - 1)
```

COMPLEXITY

Complexity of Longest Common Subsequence is O(mn).

Where m and n are lengths of the two Strings.

Thank You!