GRAPH SEARCH ALGORITHMS

➢DFS(Depth First Search)

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Depth First Search

- DFS, go as far as possible along a single path until it reaches a dead end (that is a vertex with no edge out or no neighbor unexplored) then backtrack
- As the name implies the DFS search deeper in the graph whenever possible. DFS explores edges out of the most recently discovered vertex **v** that still has unexplored edges leaving it. Once all of v's edges have been explored, the search backtracks to explore edges leaving the vertex from which **v** was discovered.

Depth-First Search

- Graph G = (V, E) directed or undirected
- Adjacency list representation
- Goal: Systematically explore every vertex and every edge
- Idea: search deeper whenever possible
  - Using a Stack(LIFO)

Information Needed to Maintain in DFS Algorithm

- Input graph G = (V,E) is represented using adjacency list.

- Each vertex is colored **white, gray or black**.

    (1) **Vertex with White Color**: undiscovered vertex.

    (2) **Vertex with Gray Color**: Consider to be discovered.

    (3) **Vertex with Black Color:** Adjacency-list of Vertex has been discovered .

- Additional Data Structures

    (1) color[u] : Store the color of each vertex $u \varepsilon V$

    (2) Π[u]   : Store the predecessor of u.

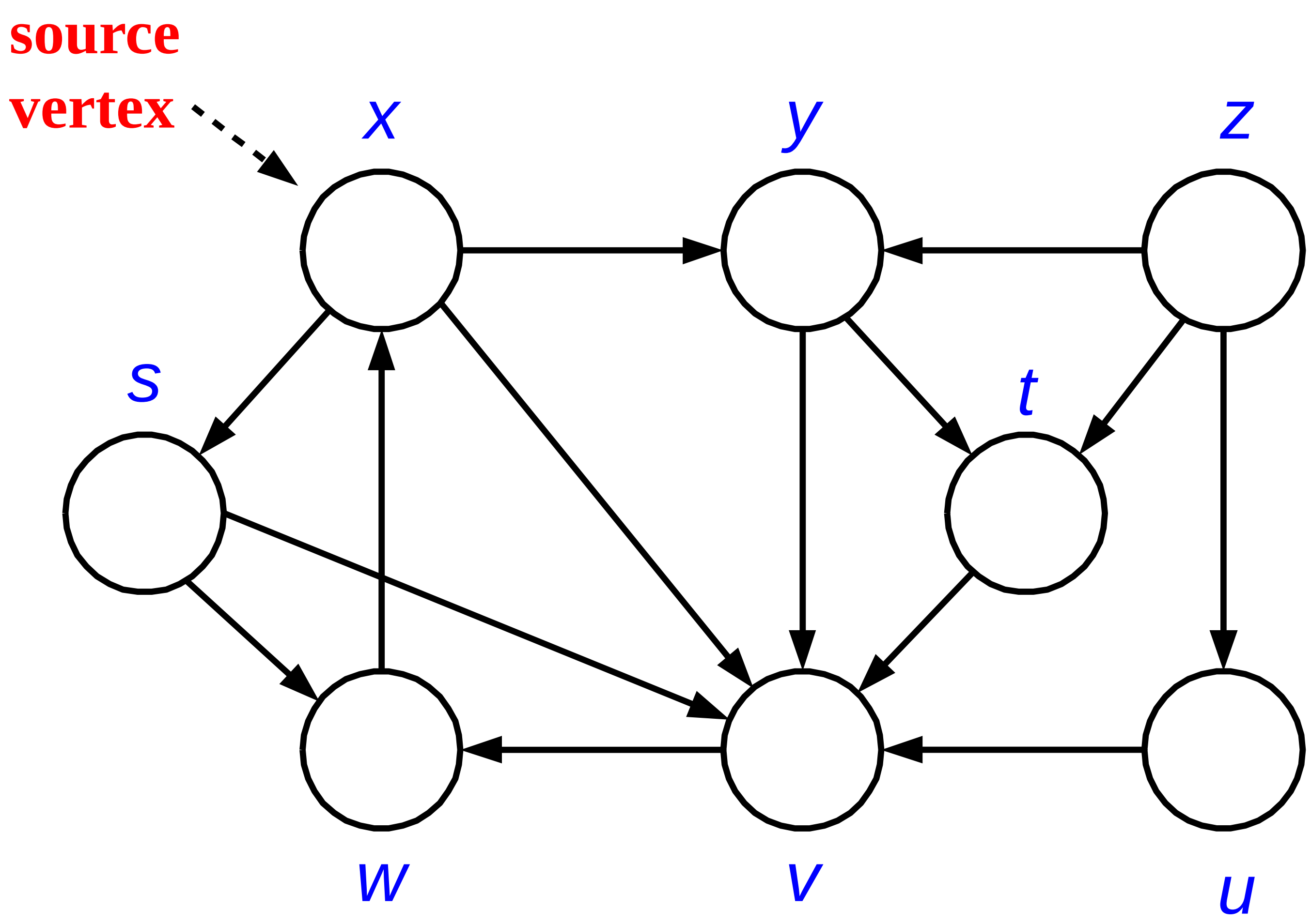
- Besides creating depth first forest DFS also timestamps each vertex. Each vertex goes through two time stamps:

    (3) d[u]   : Store when u is first discovered and grayed (discovery time).

    (4) f[u]   : Store when the search finishes examining u's adjacency-list and blackens u (finishing time).

    - $f[u] > d[u]$

Depth-First Search: Example



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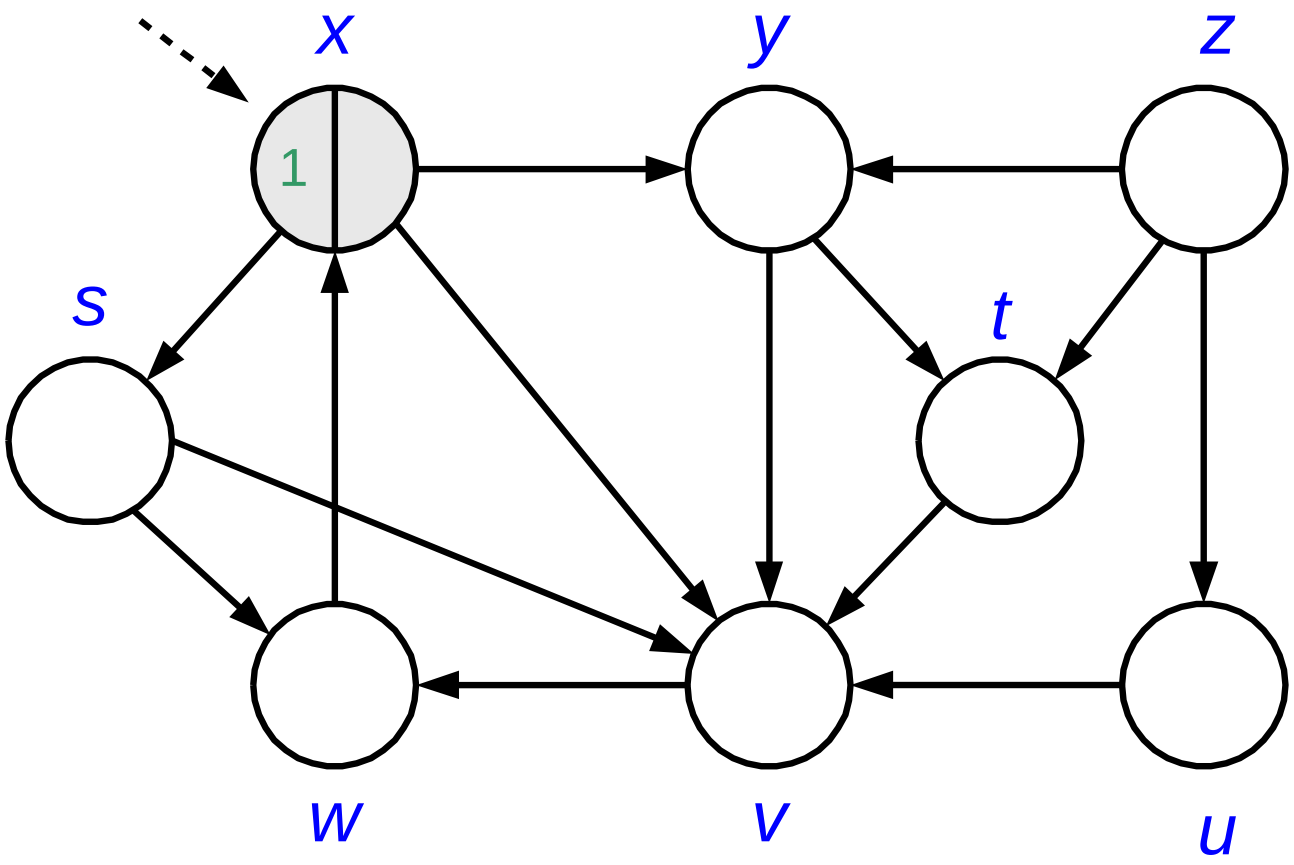

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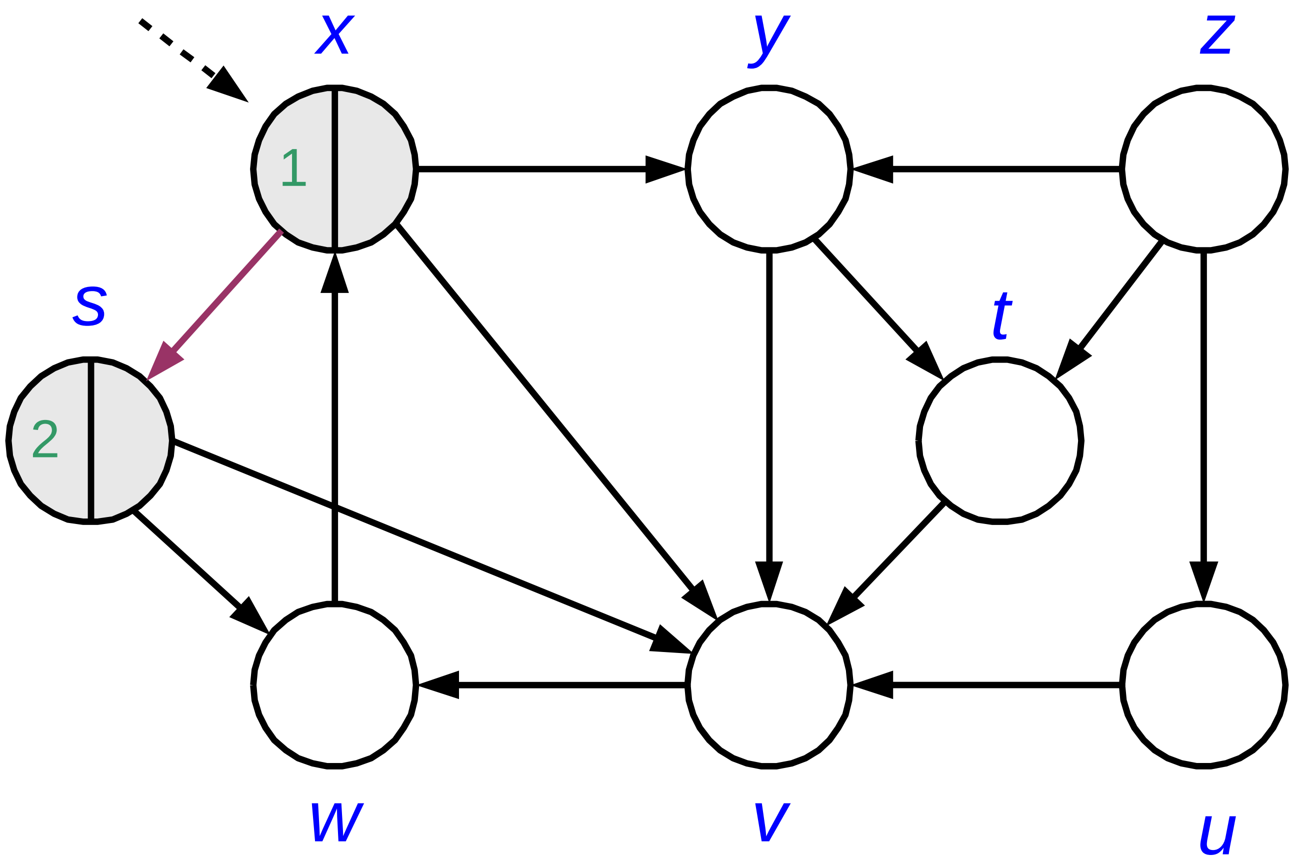
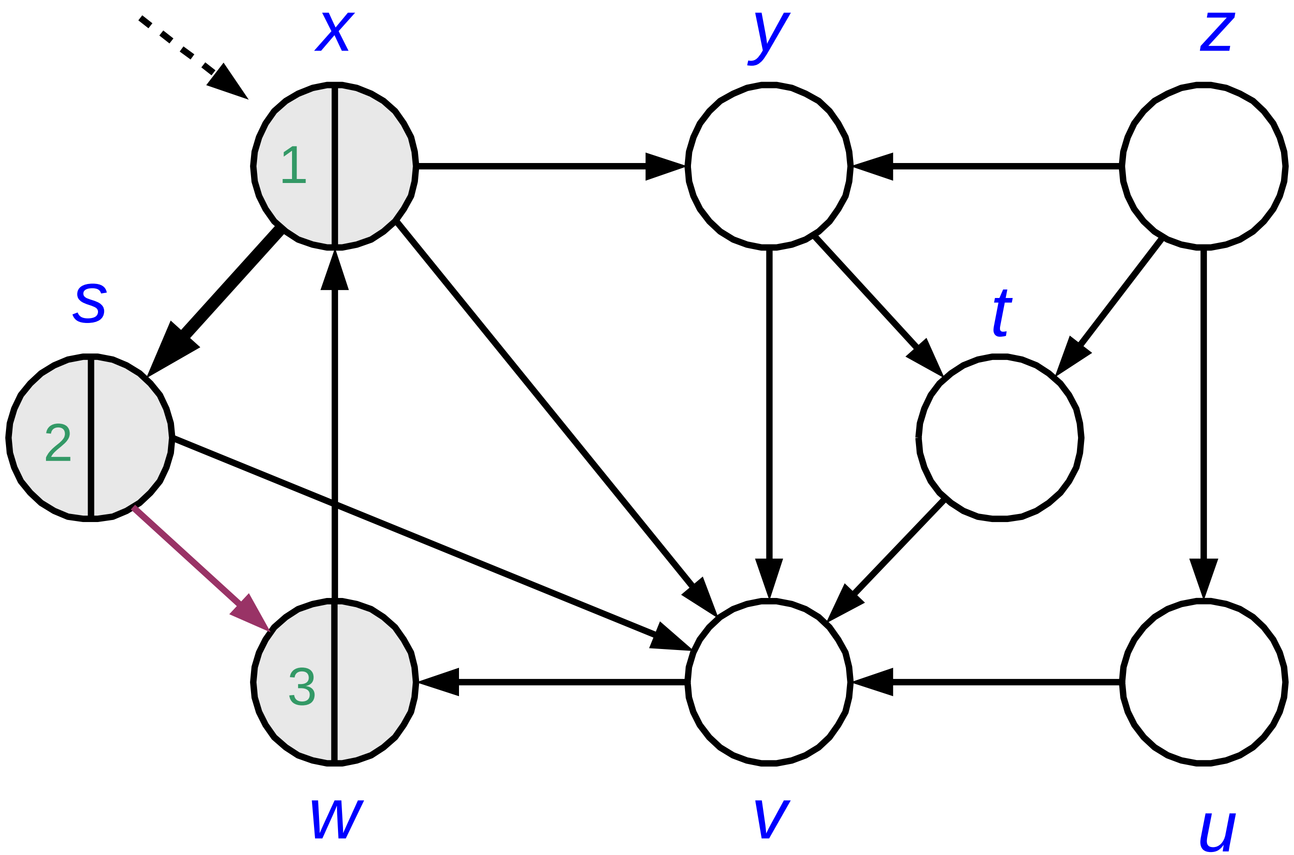


Depth-First Search: *Example*



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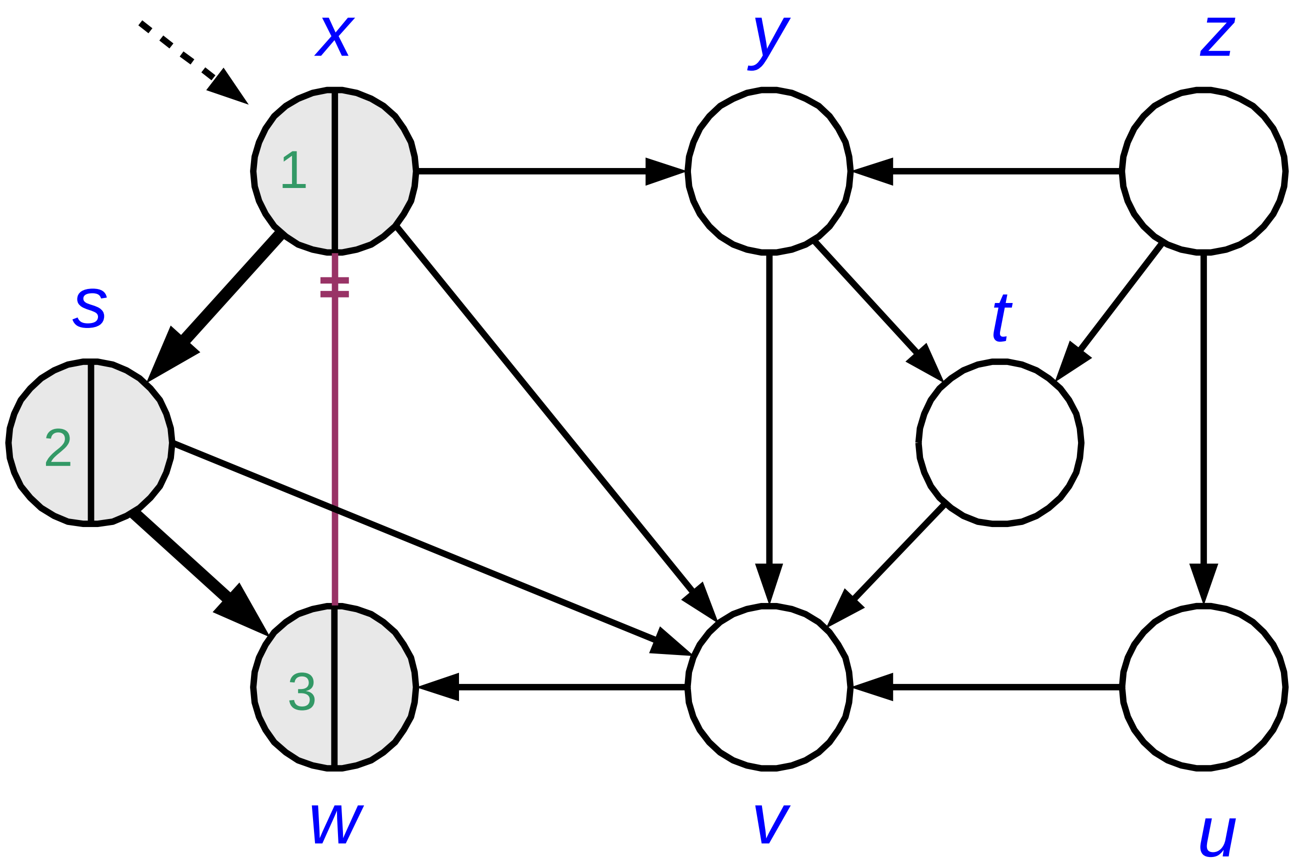

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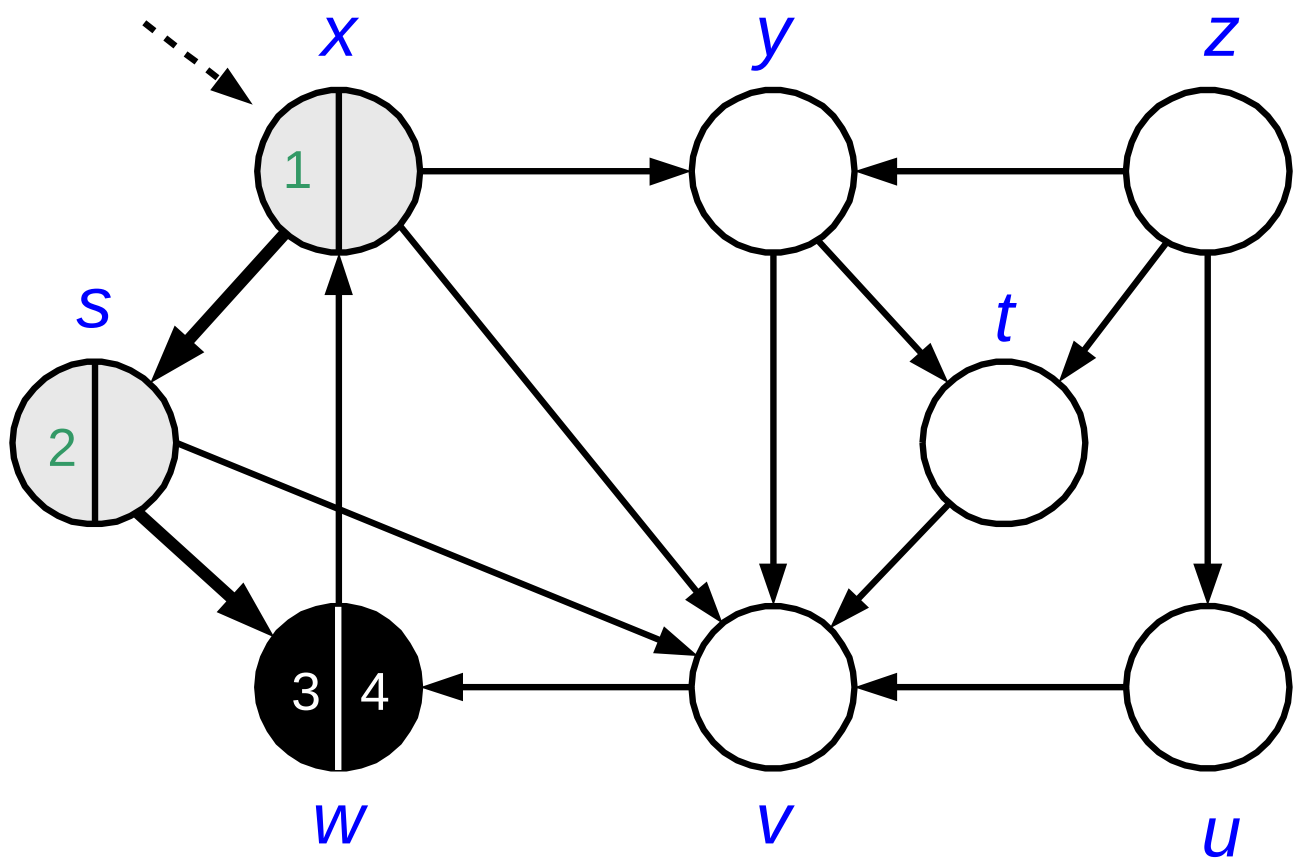

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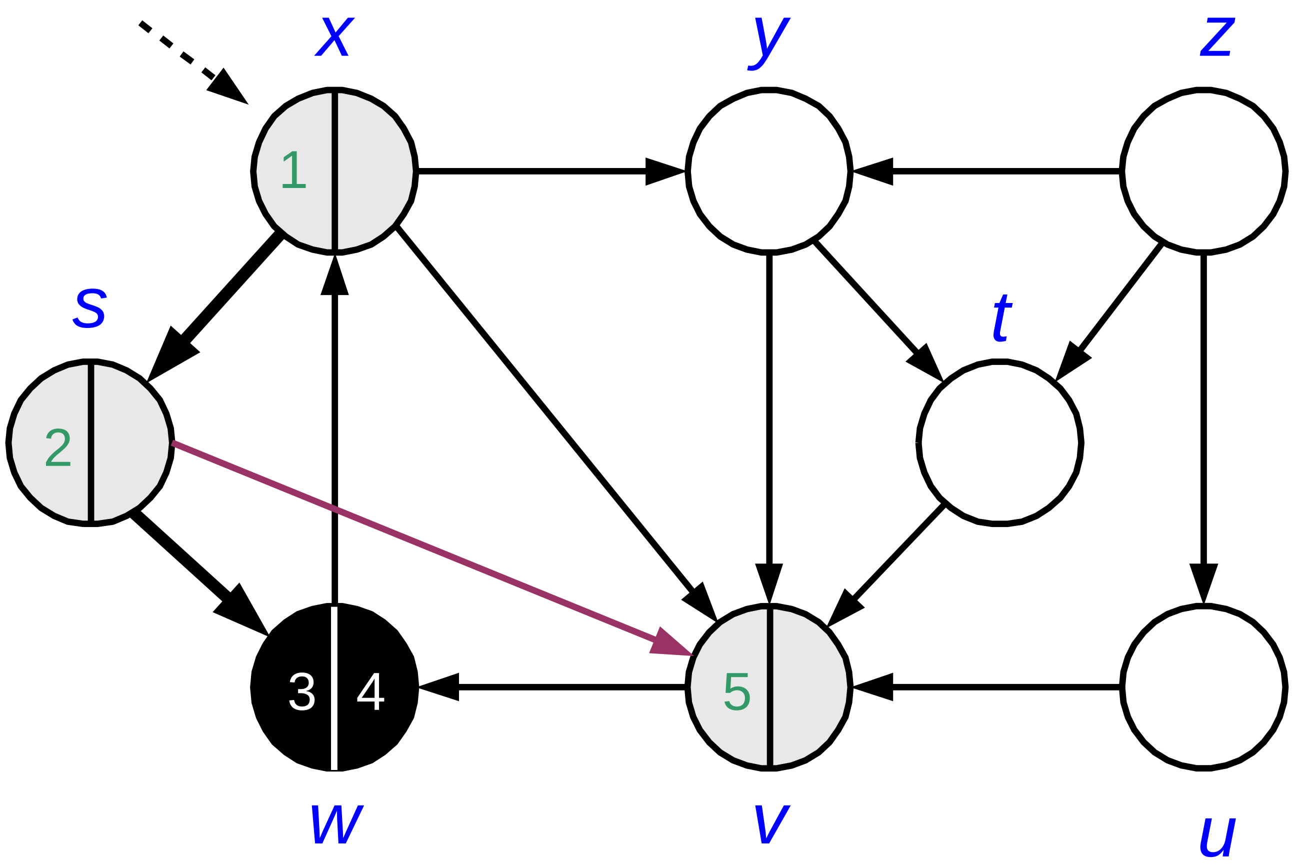

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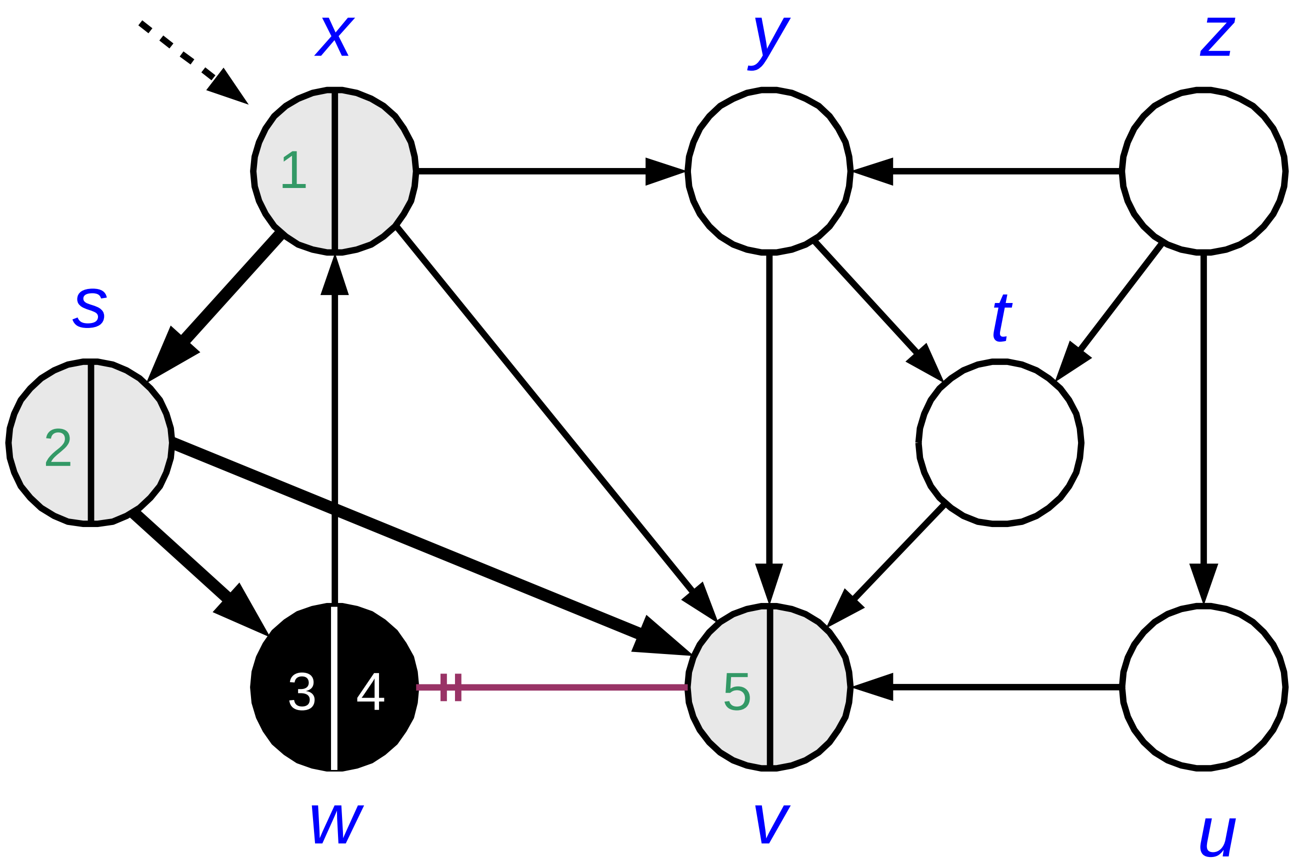

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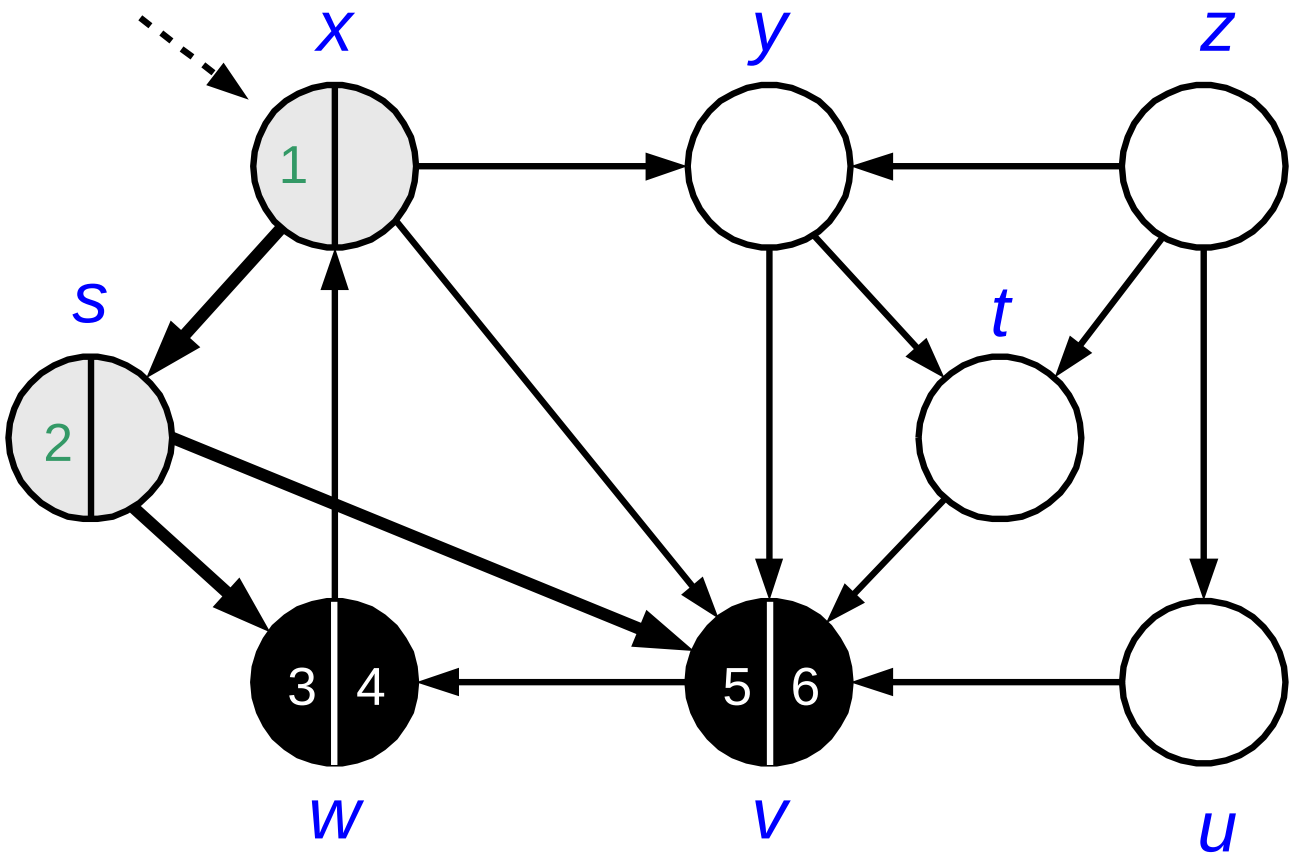

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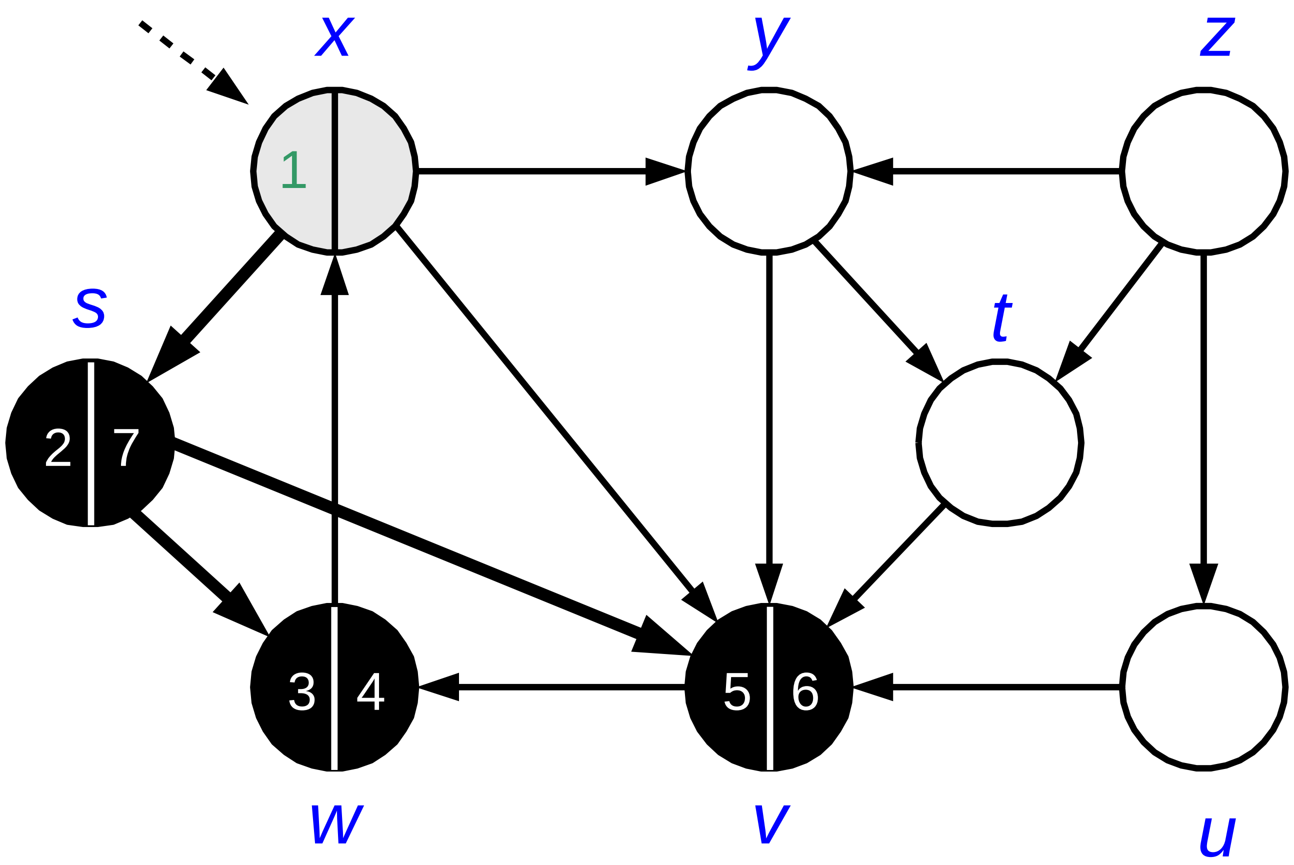

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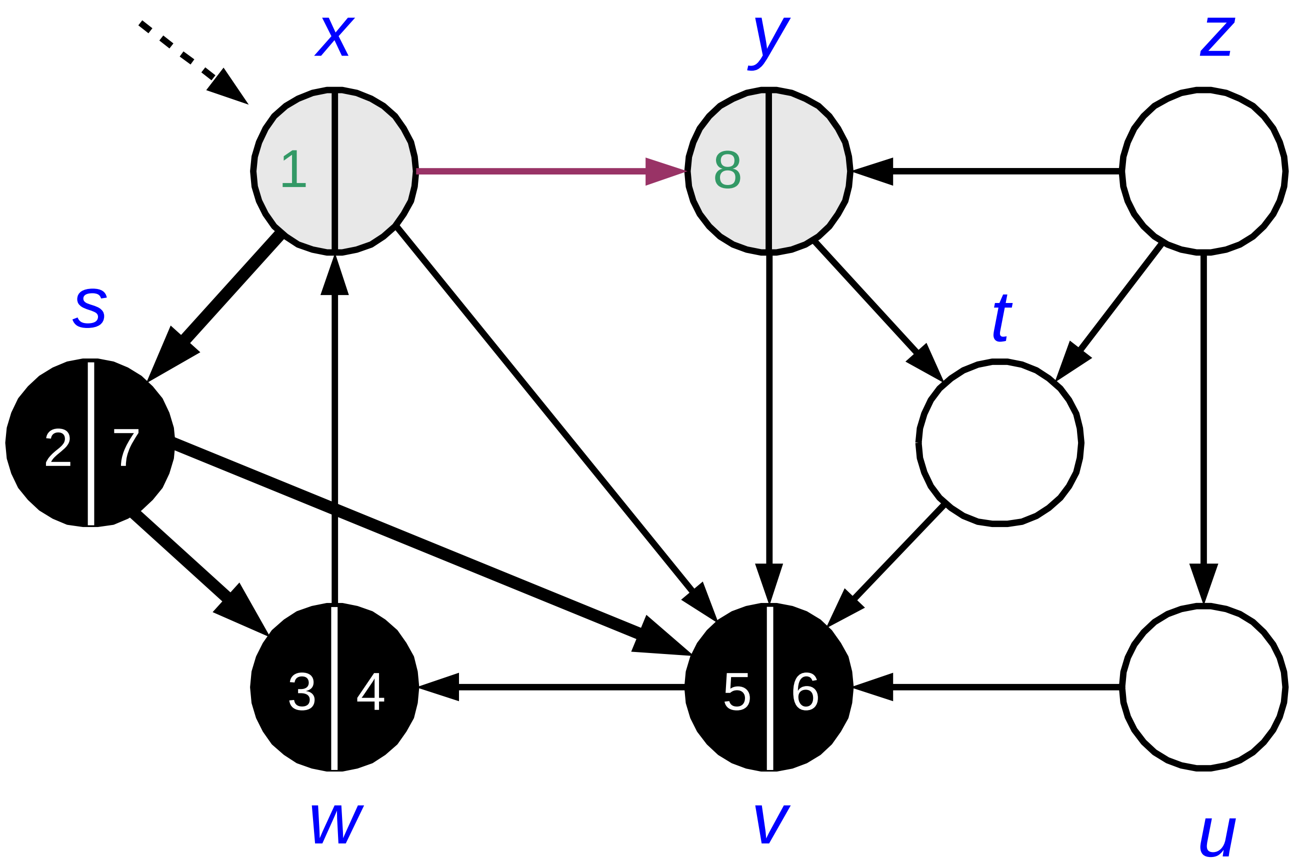

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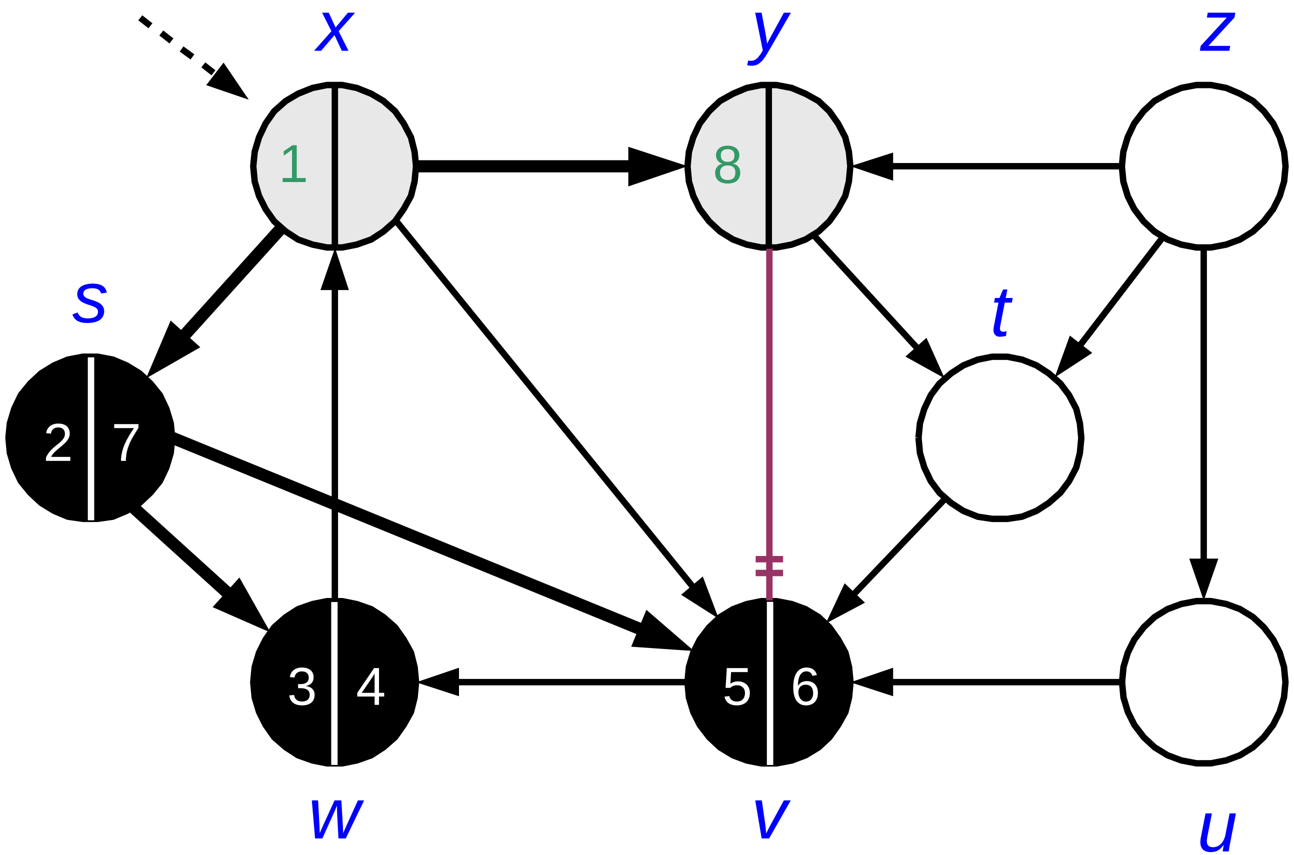

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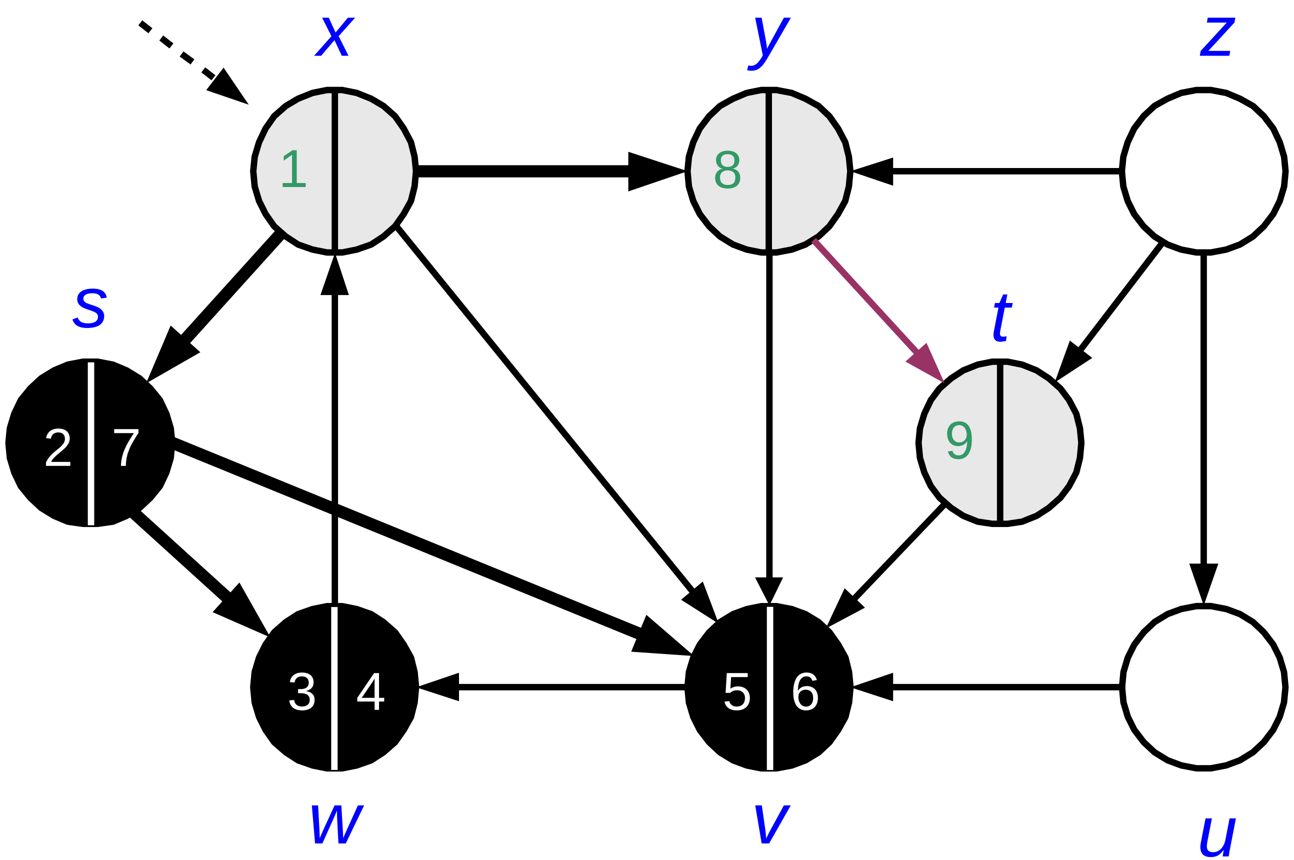

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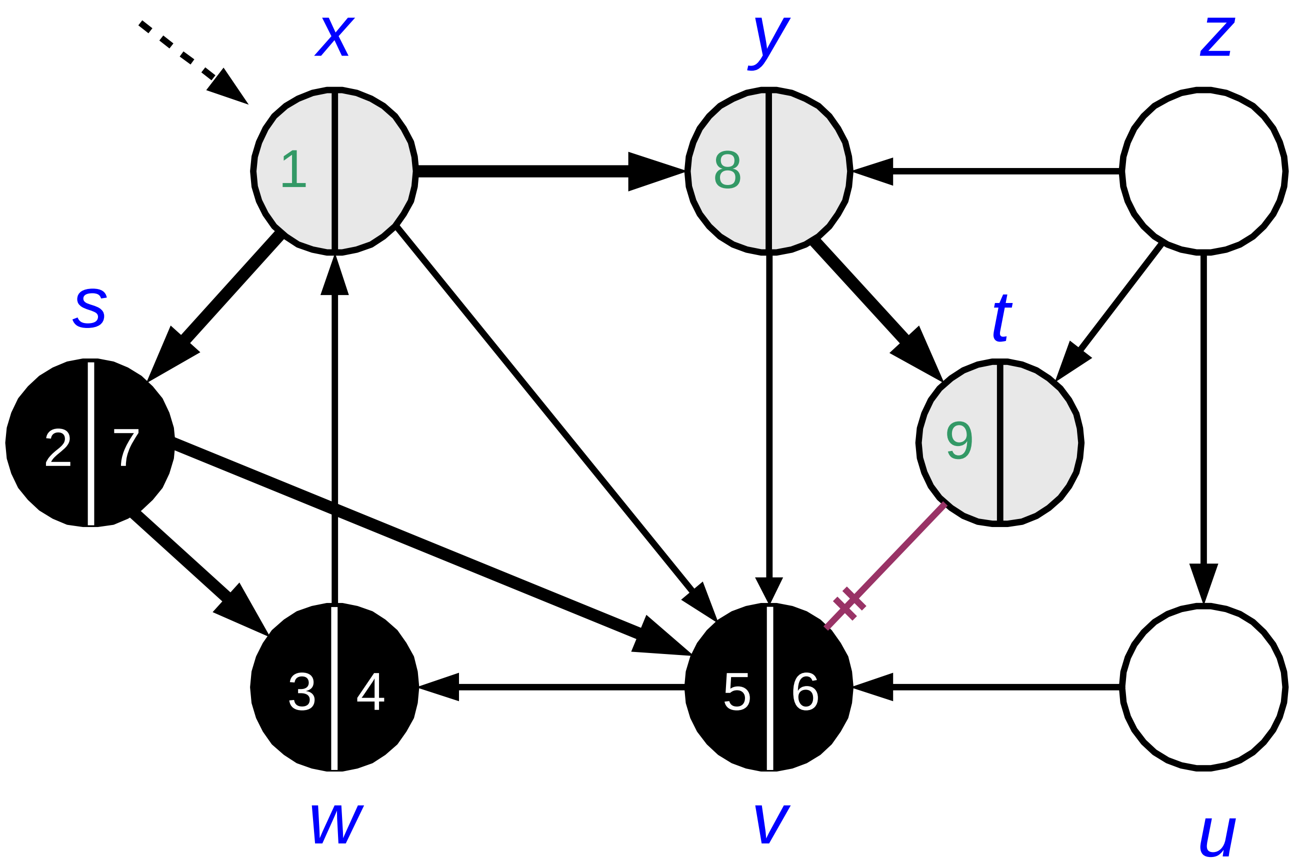

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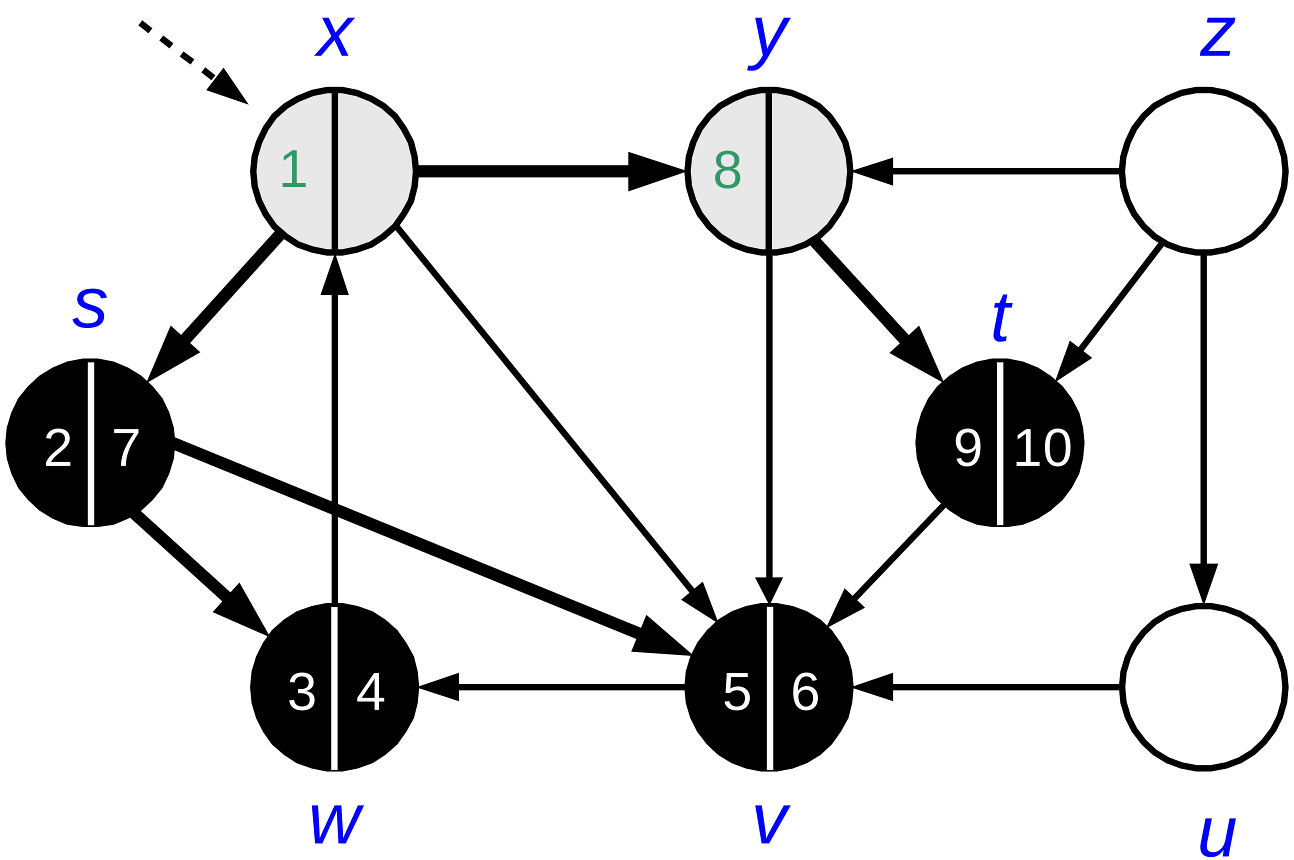

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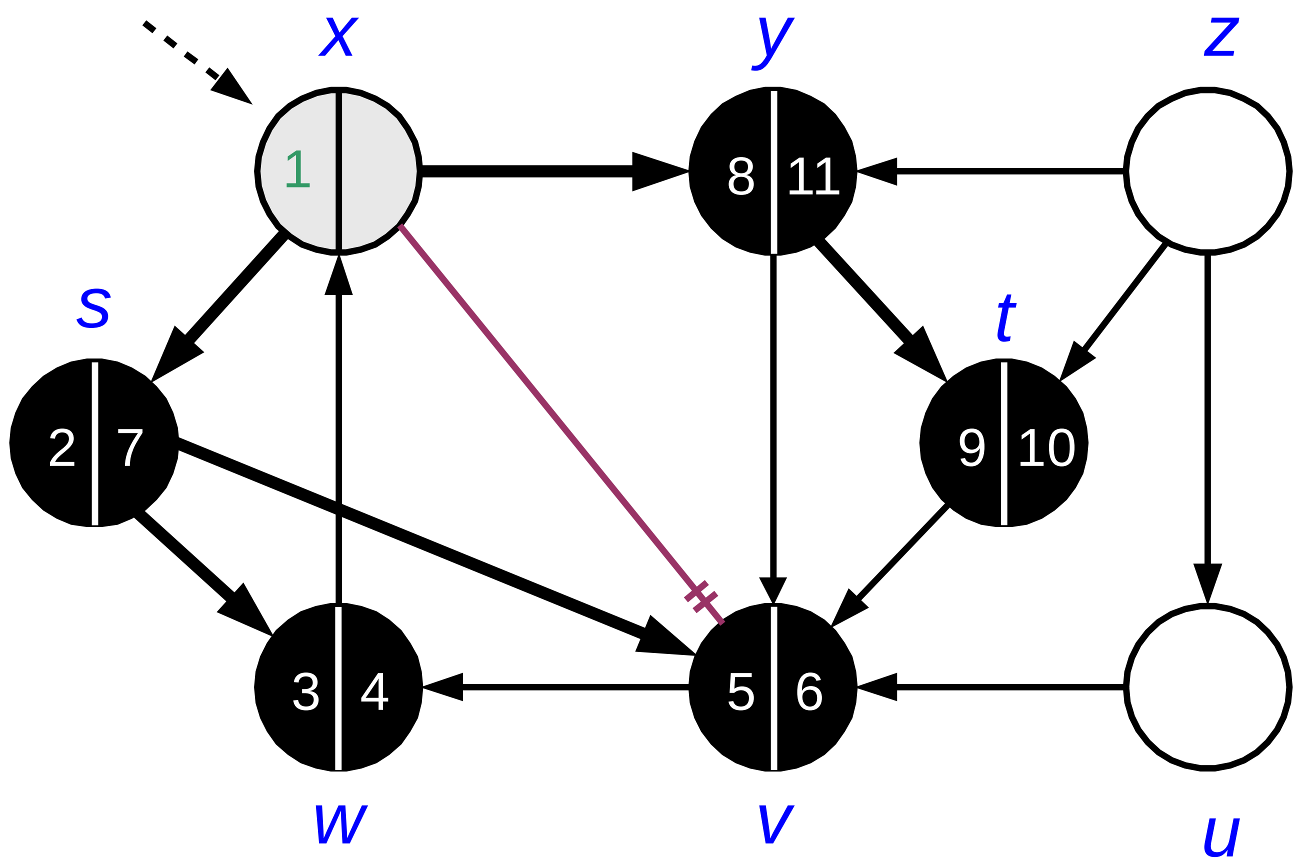

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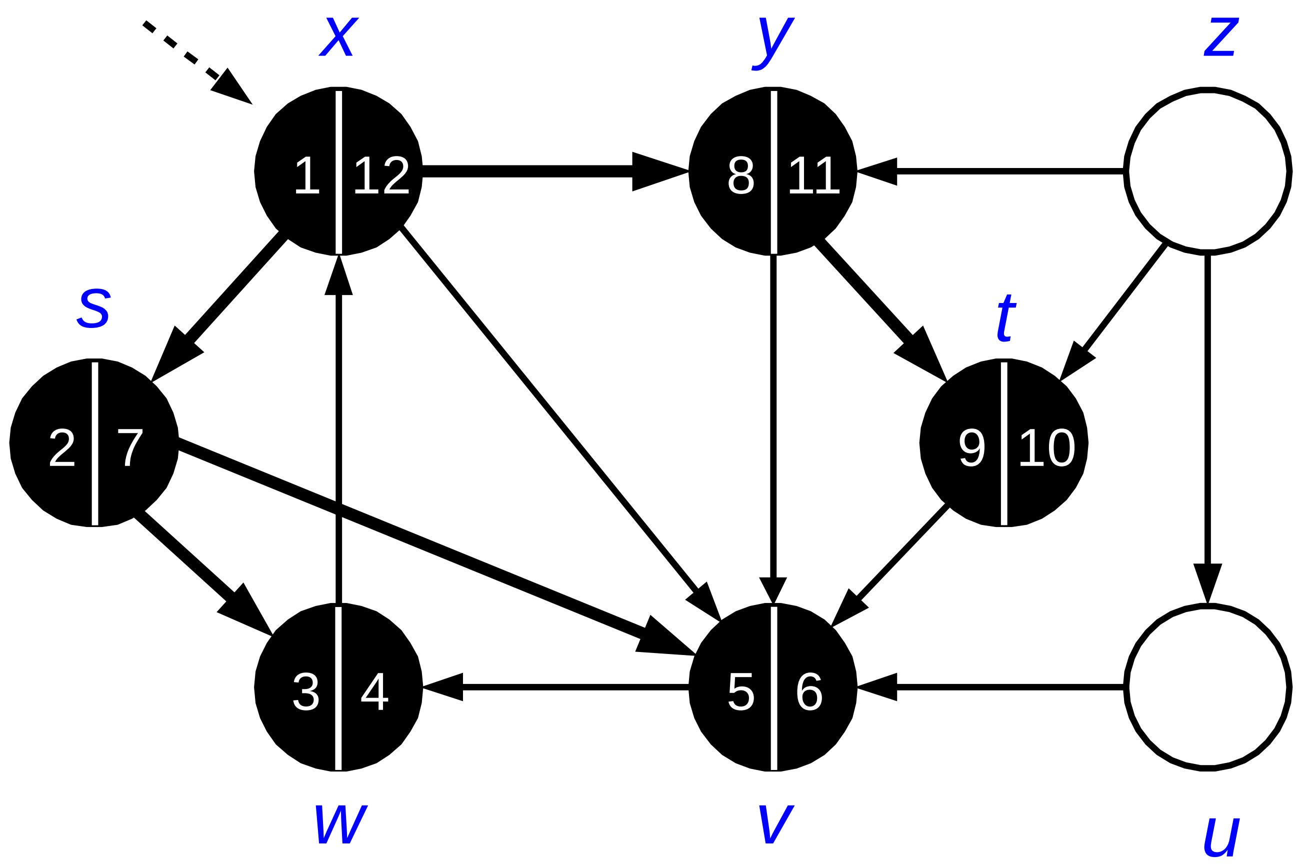

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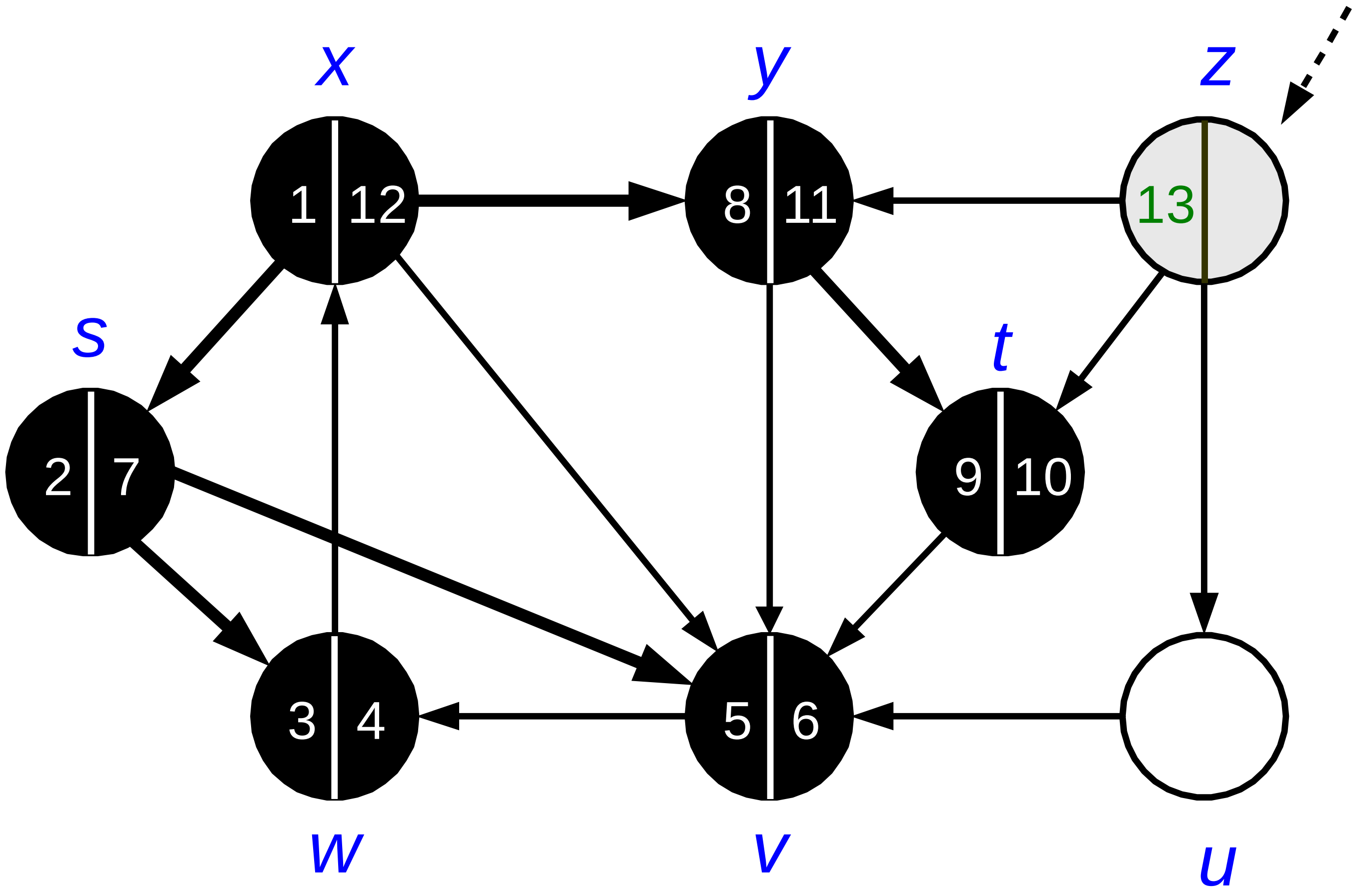

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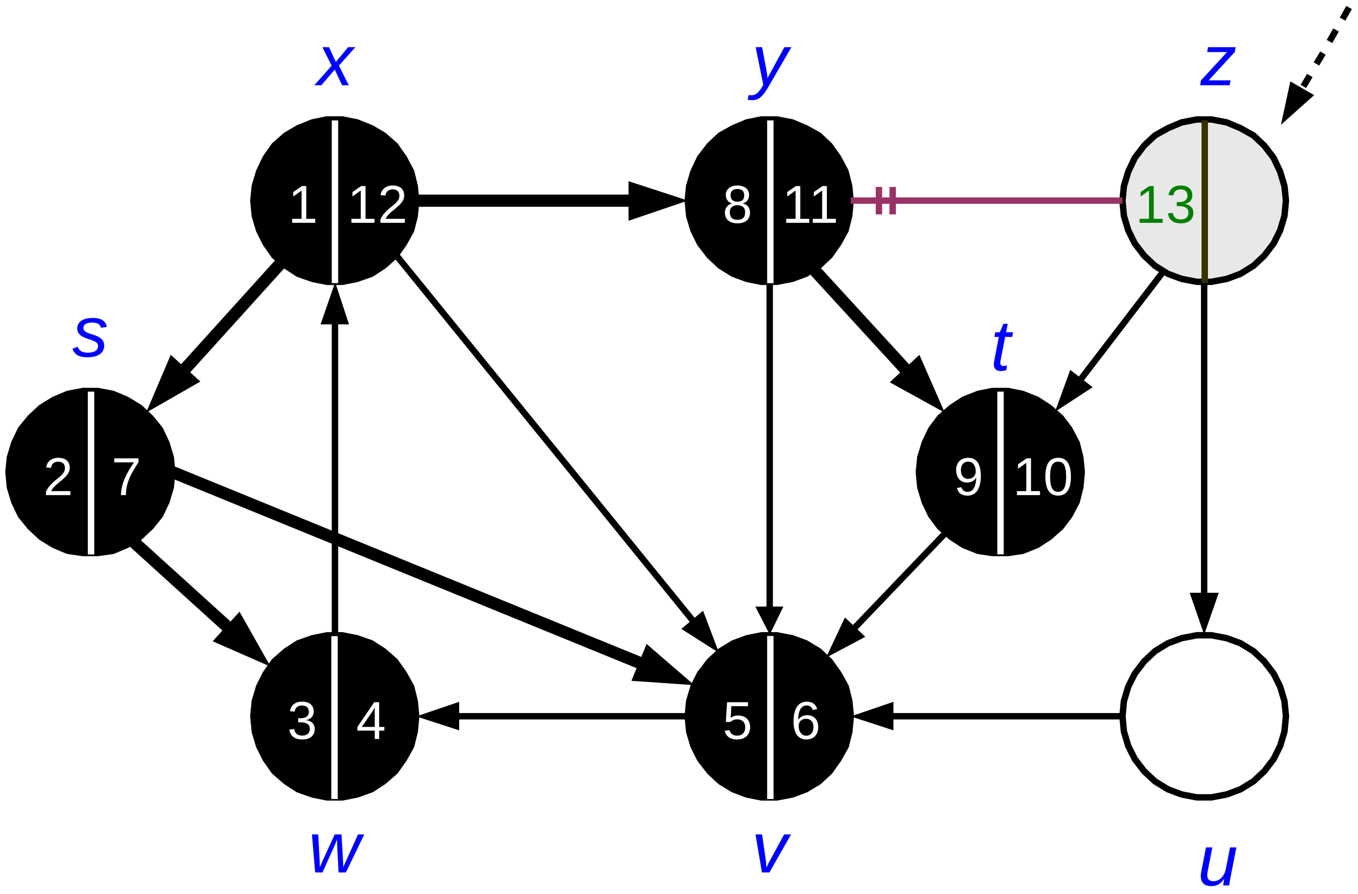

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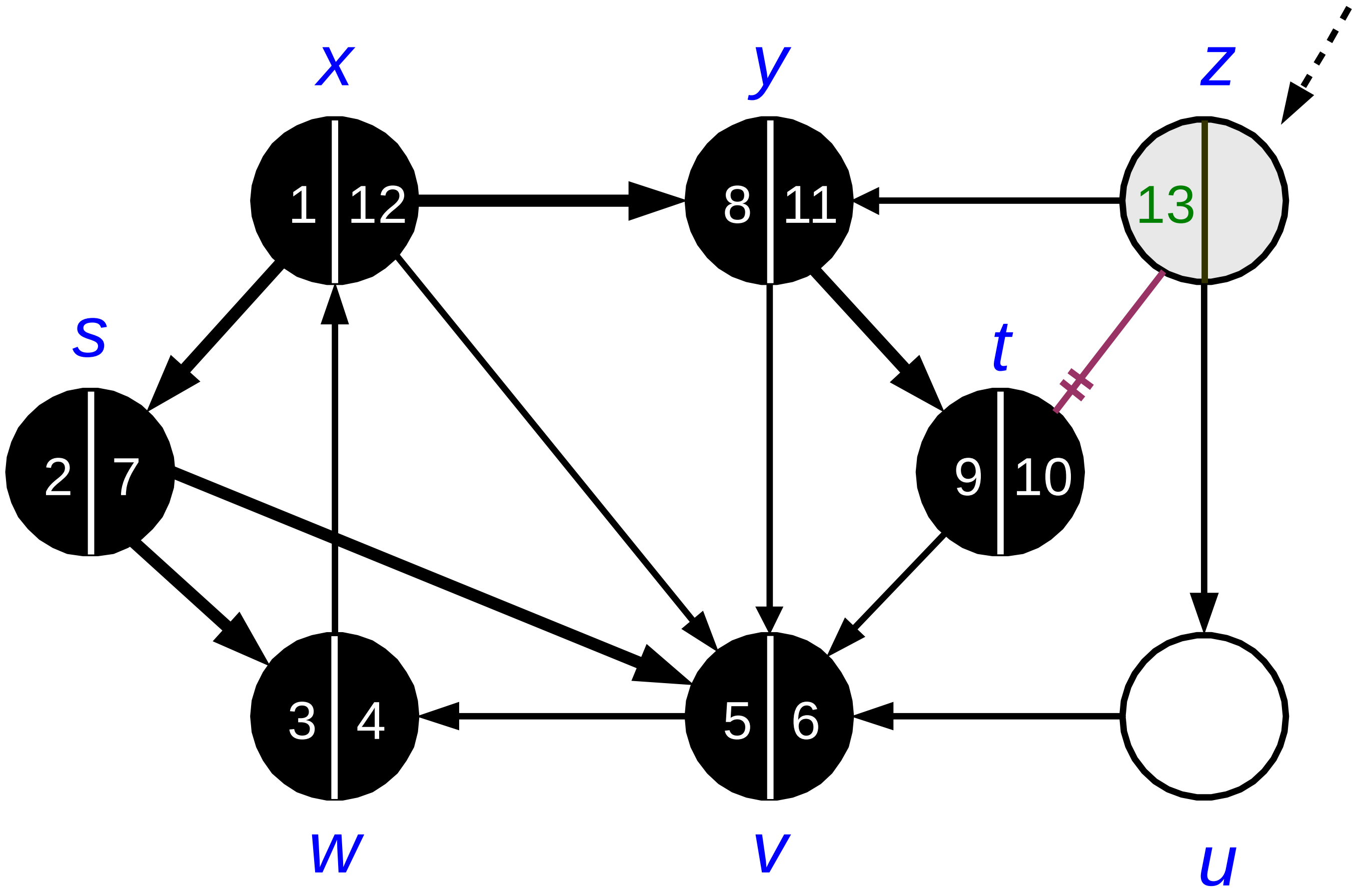

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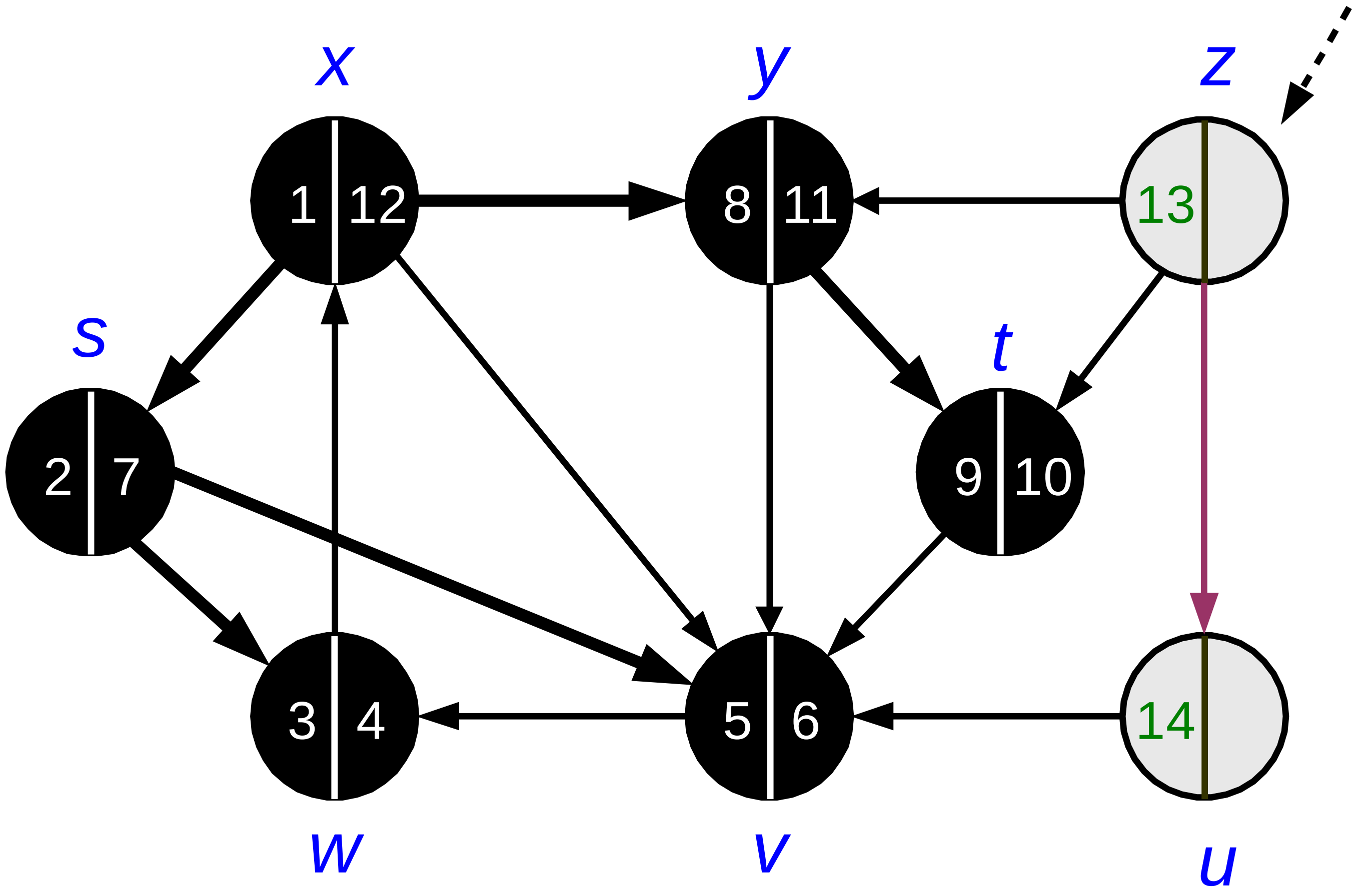

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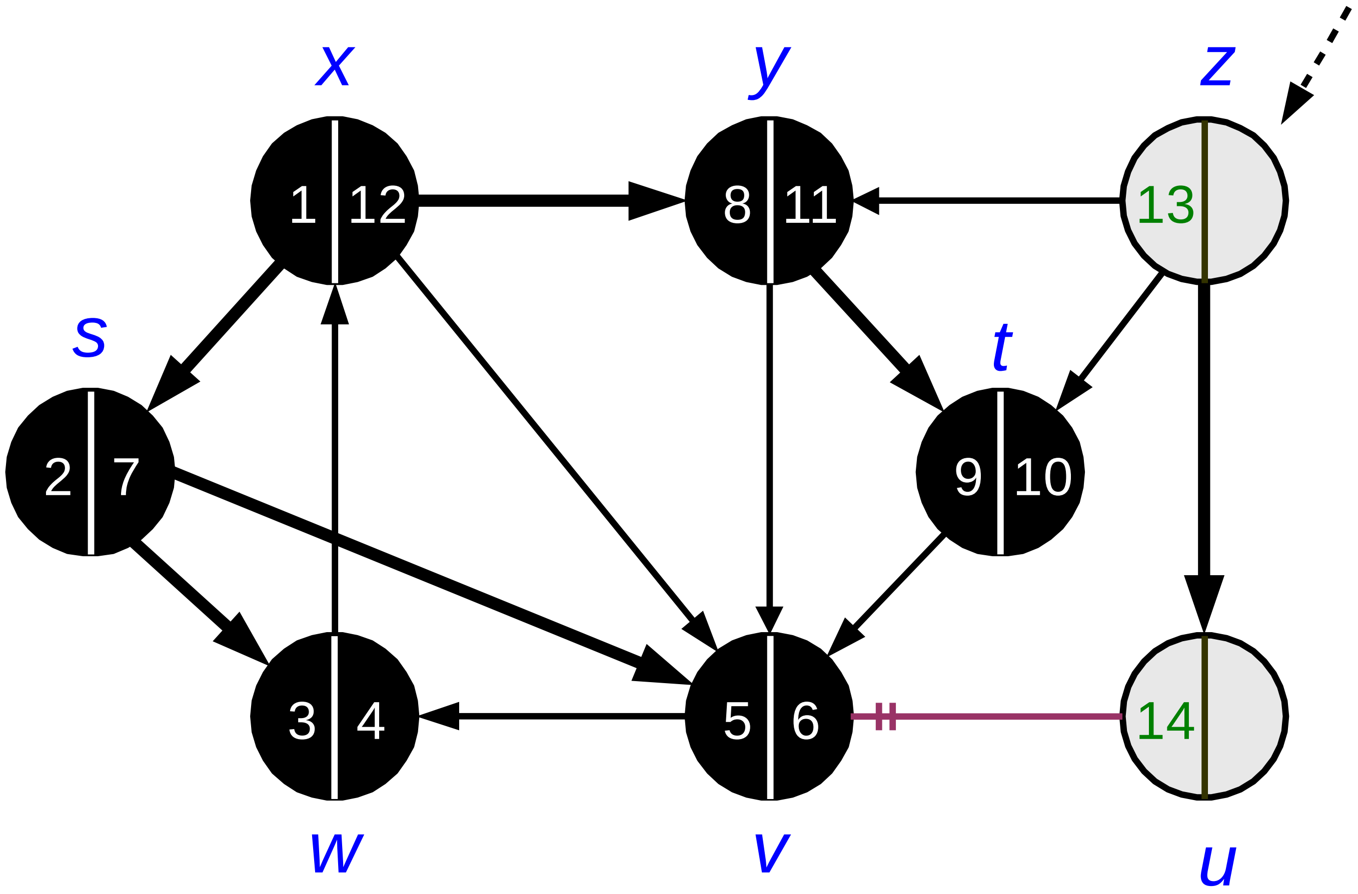

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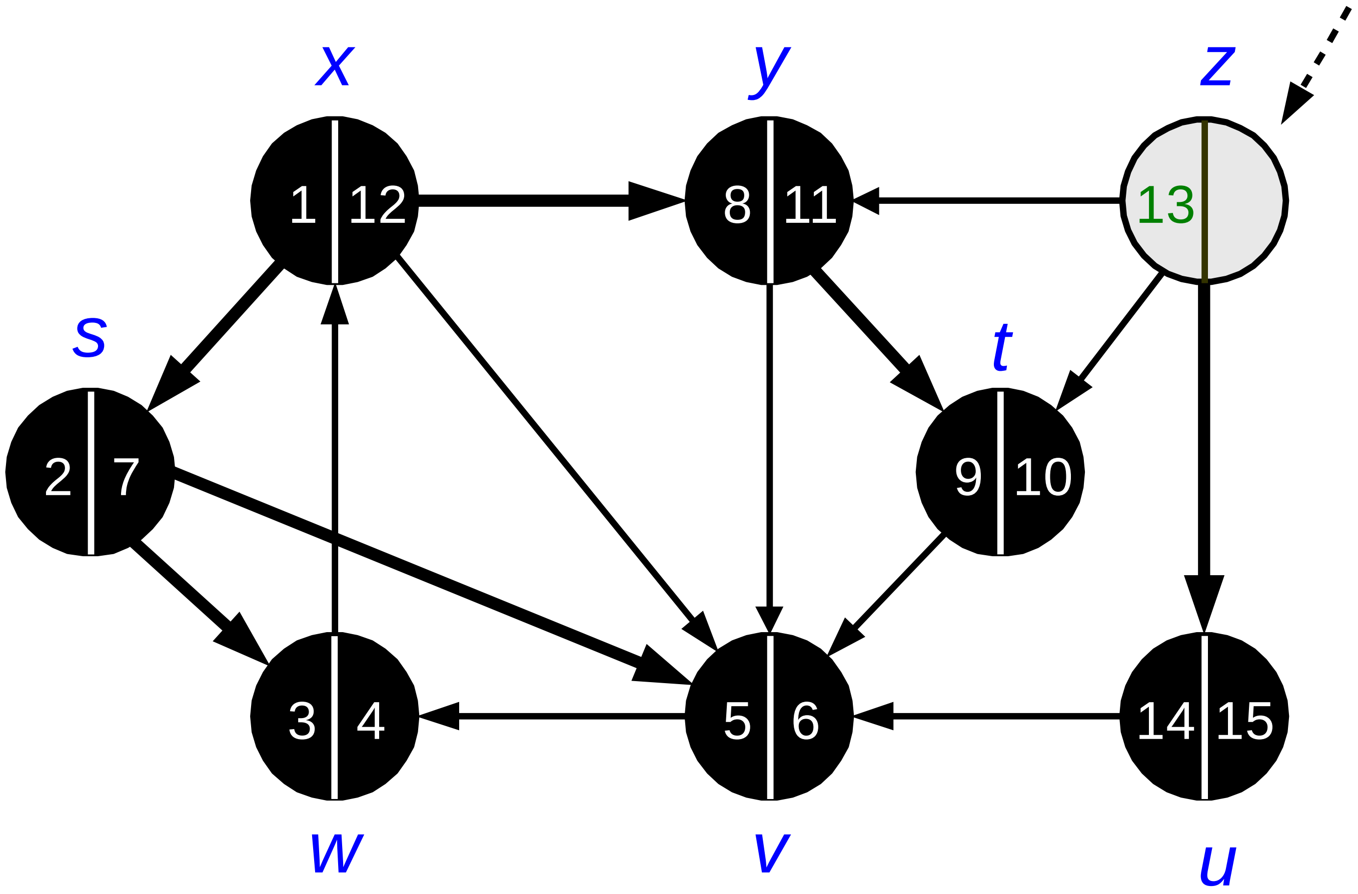

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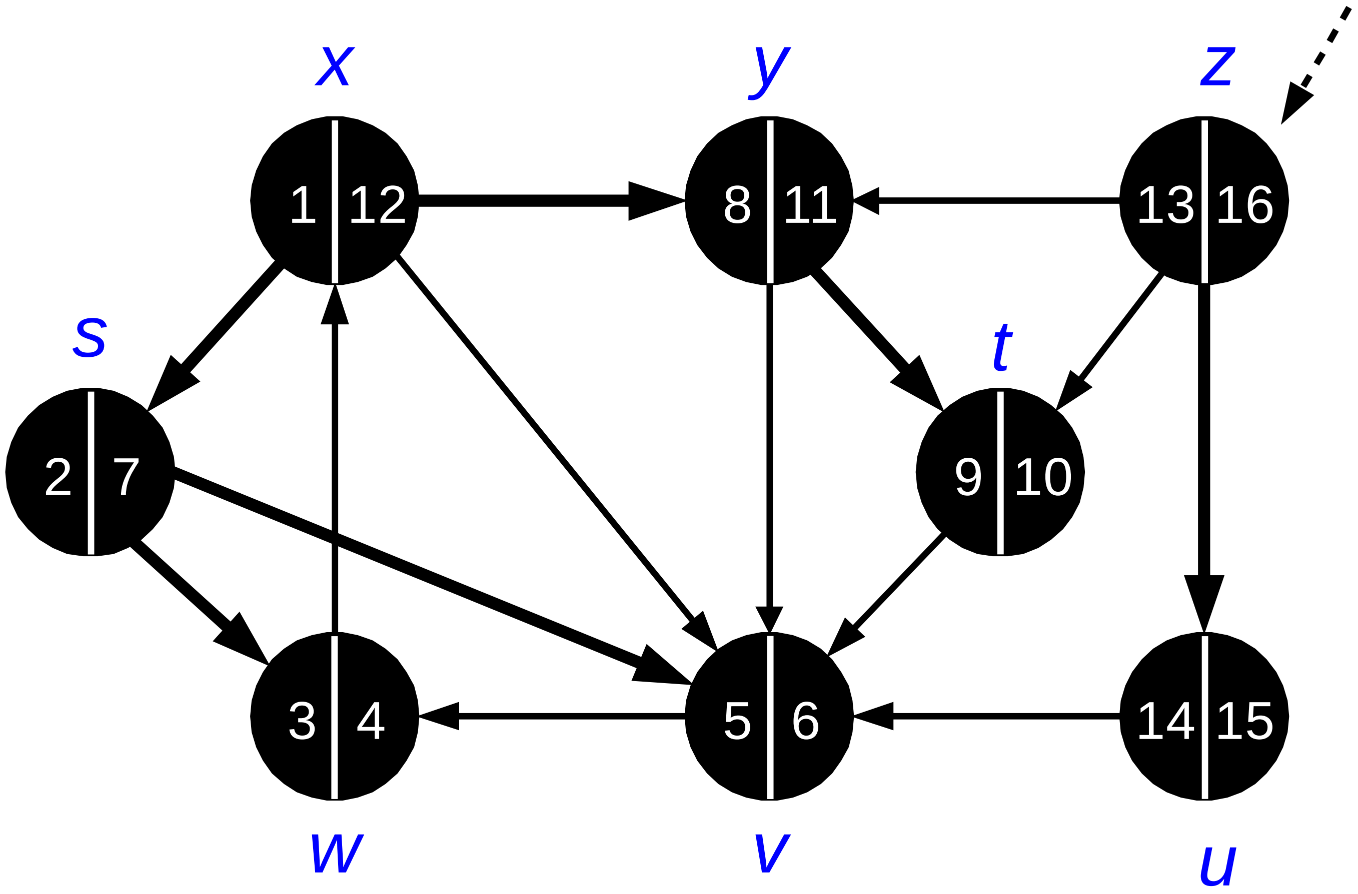
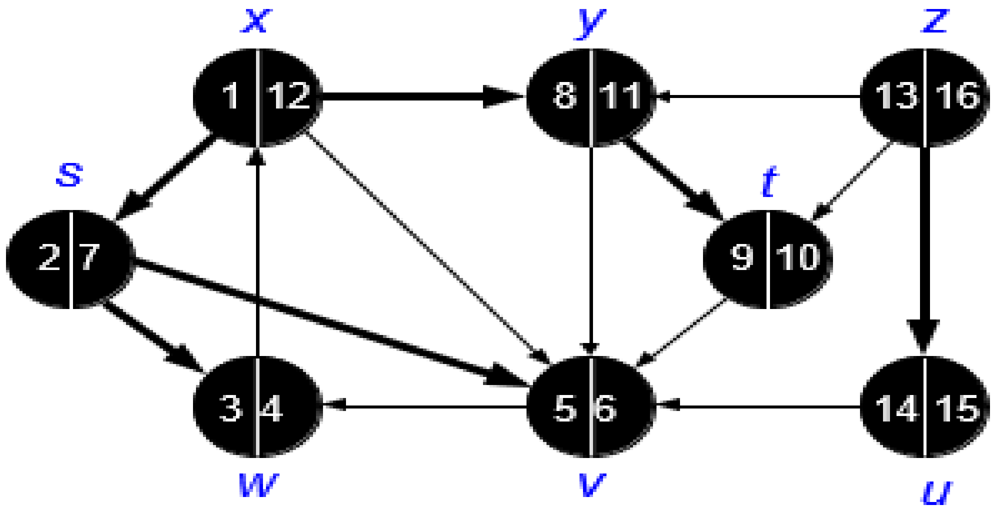
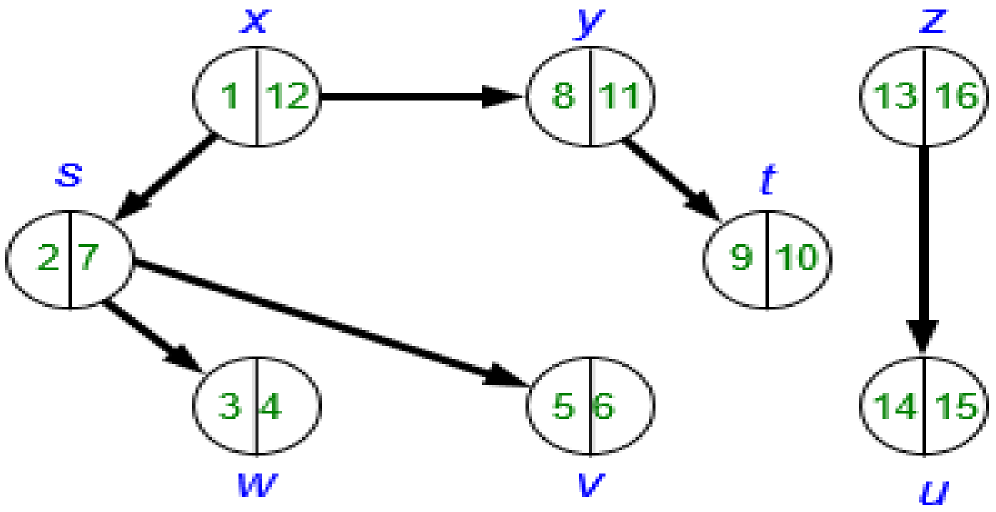


Depth-First Search: Example

DFS(G) terminated	Depth-first forest (DFF)
	

DFS: Algorithm

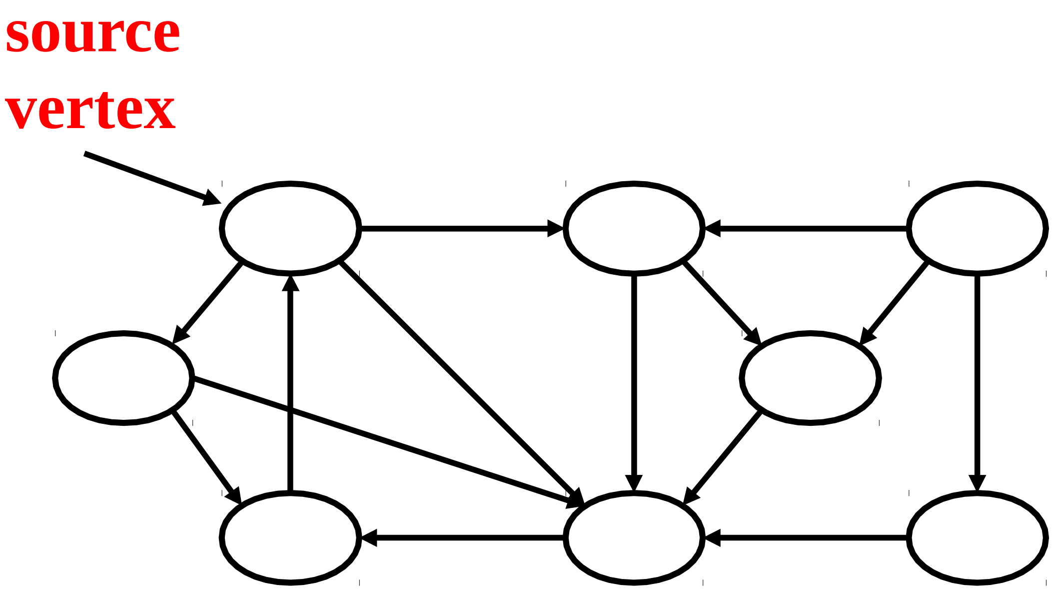
- DFS(G)

1. for each vertex u € V[G]
2. color[u]=white
3. π[u]=NIL
4. time = 0
5. for each vertex u € V[G]
6. if (color[u]= =white)
7. DFS-VISIT(G,u)

DFS: Algorithm (Cont.)

- DFS-VISIT(u)

1. time = time + 1
2. d[u] = time
3. color[u] = gray
4. for each v € Adj(u) in **G** do
5. if (color[v] == white)
6. π [v] = u;
7. DFS-VISIT(G,v);
8. color[u] = black
9. time = time + 1;
10. f[u] = time;



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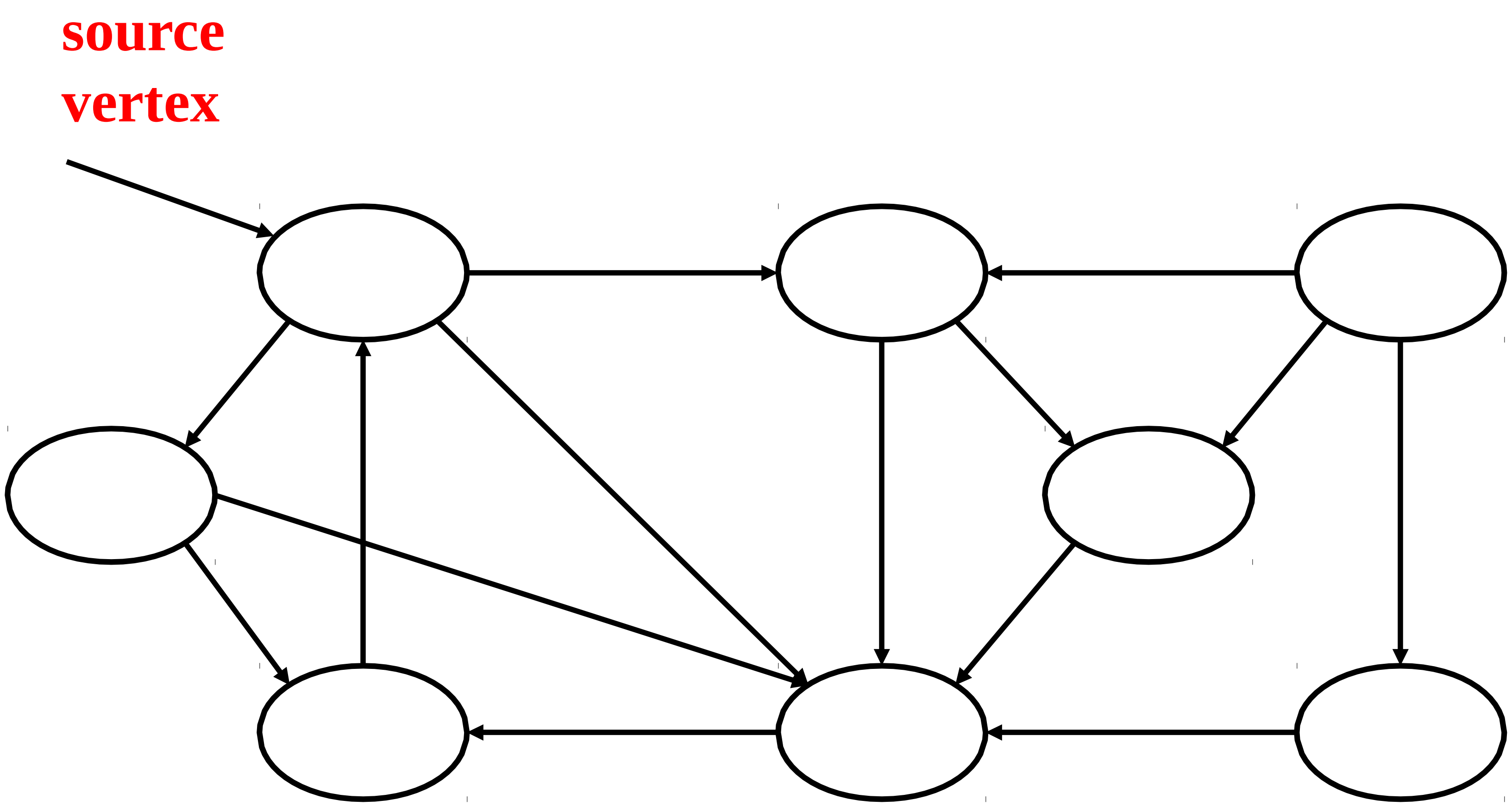

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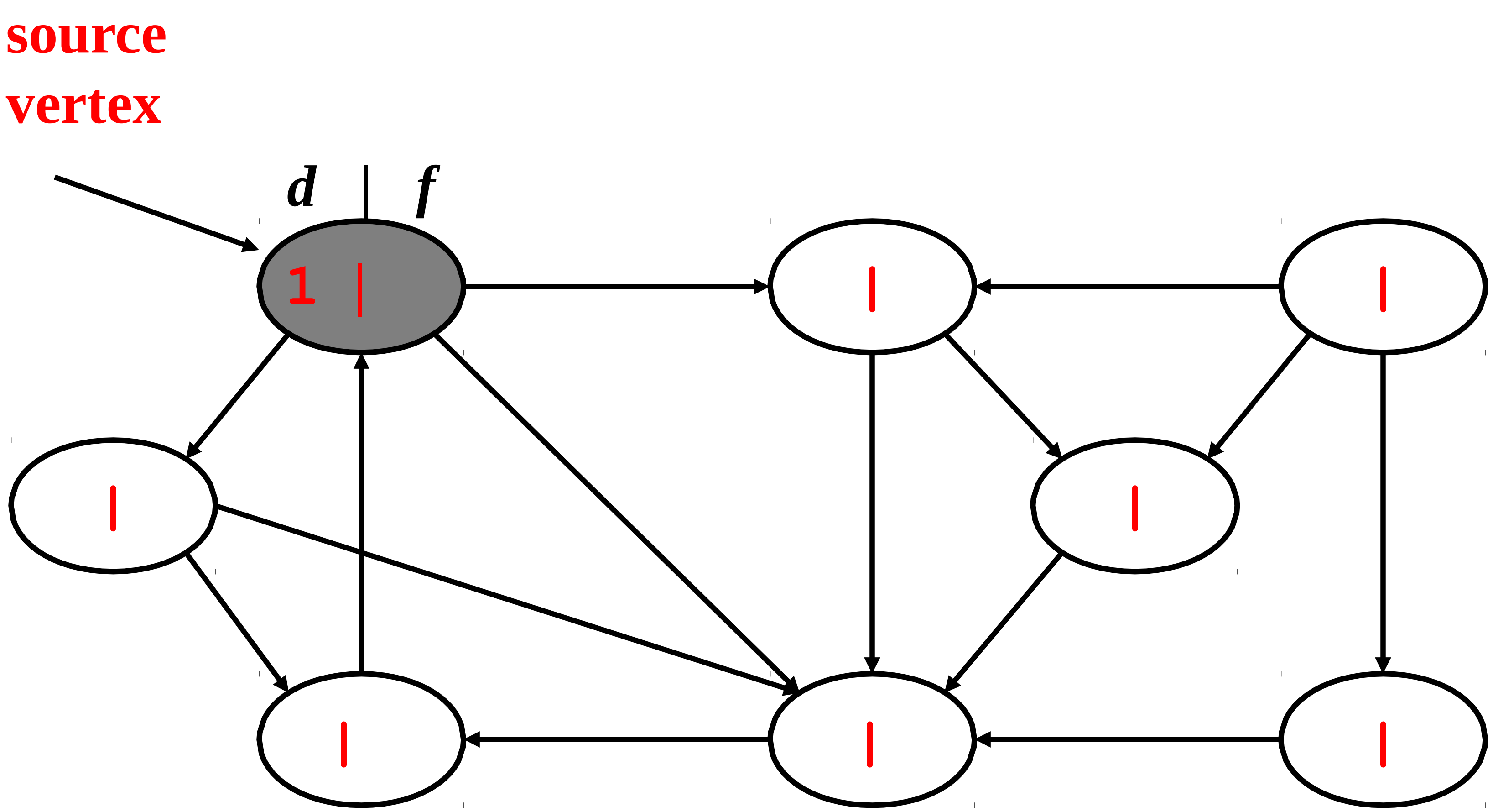

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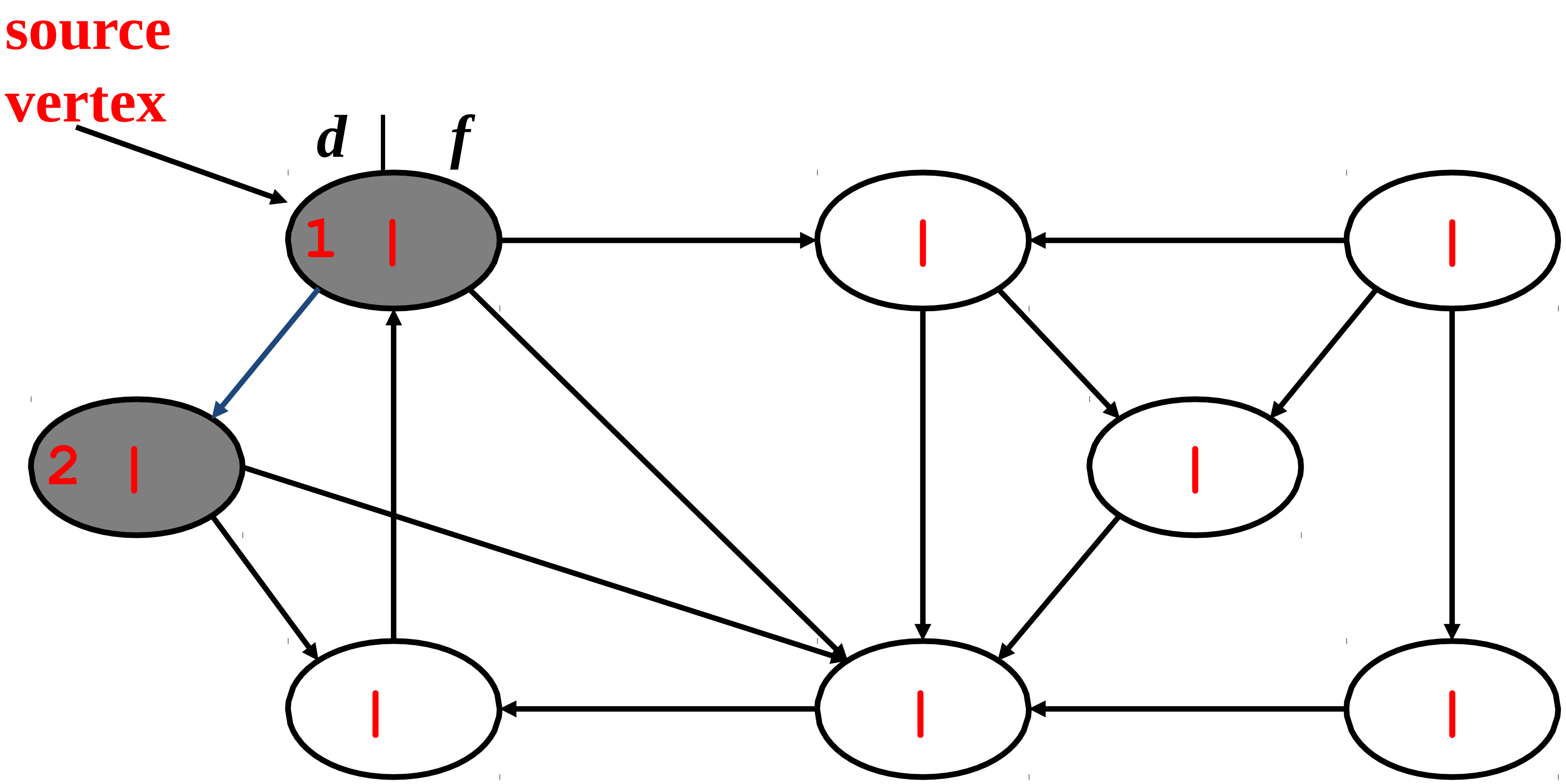

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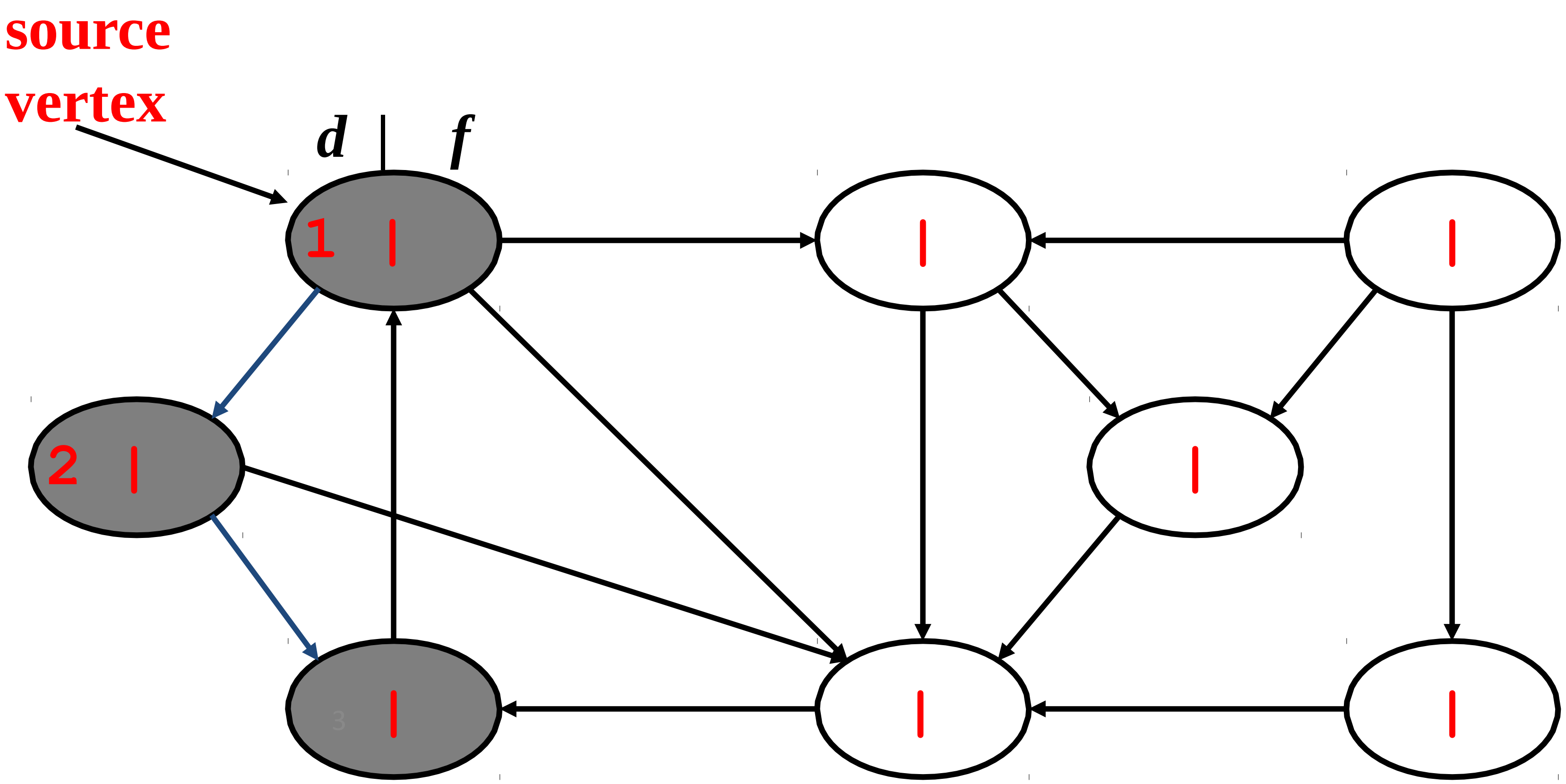

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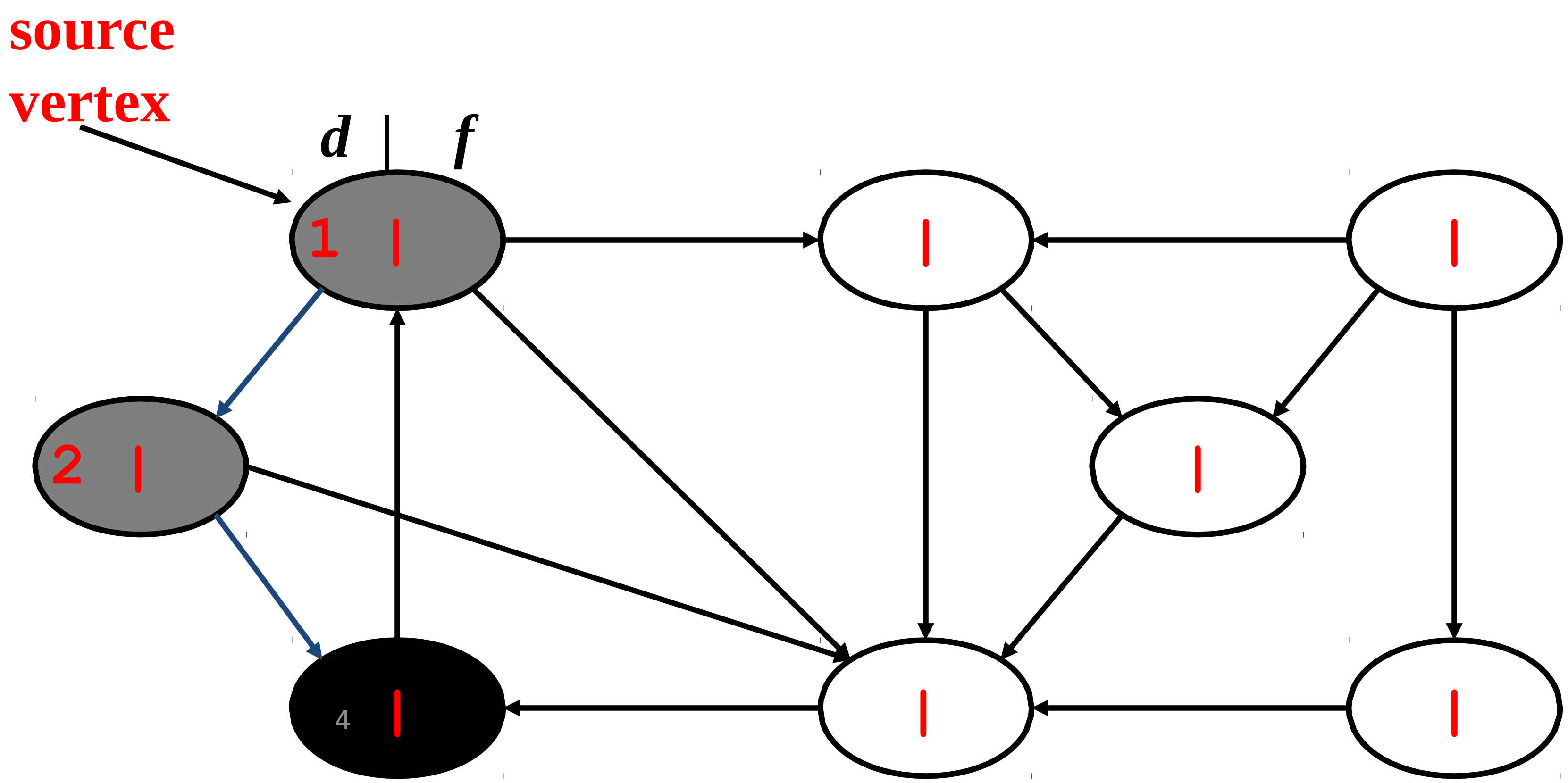

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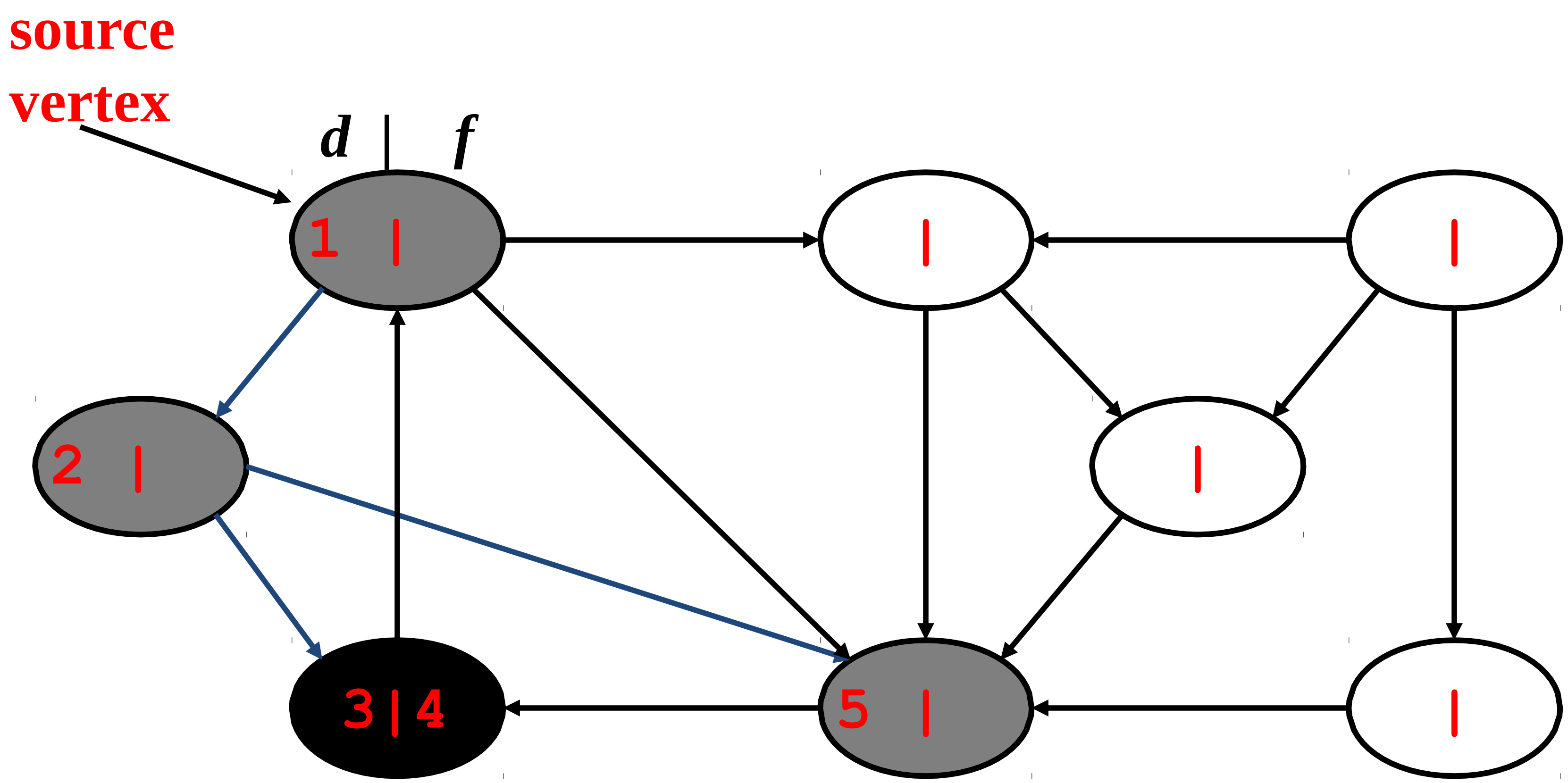

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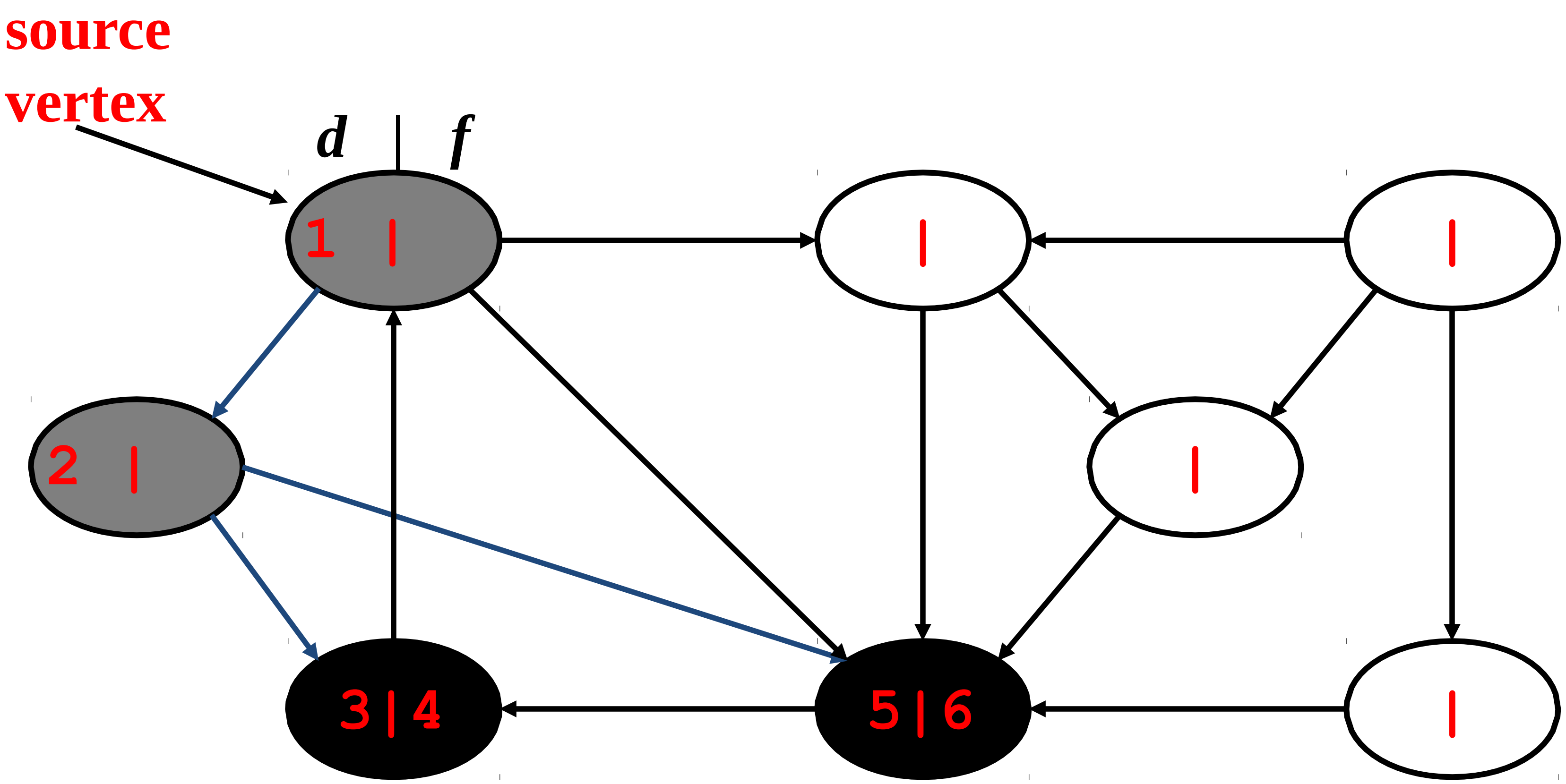

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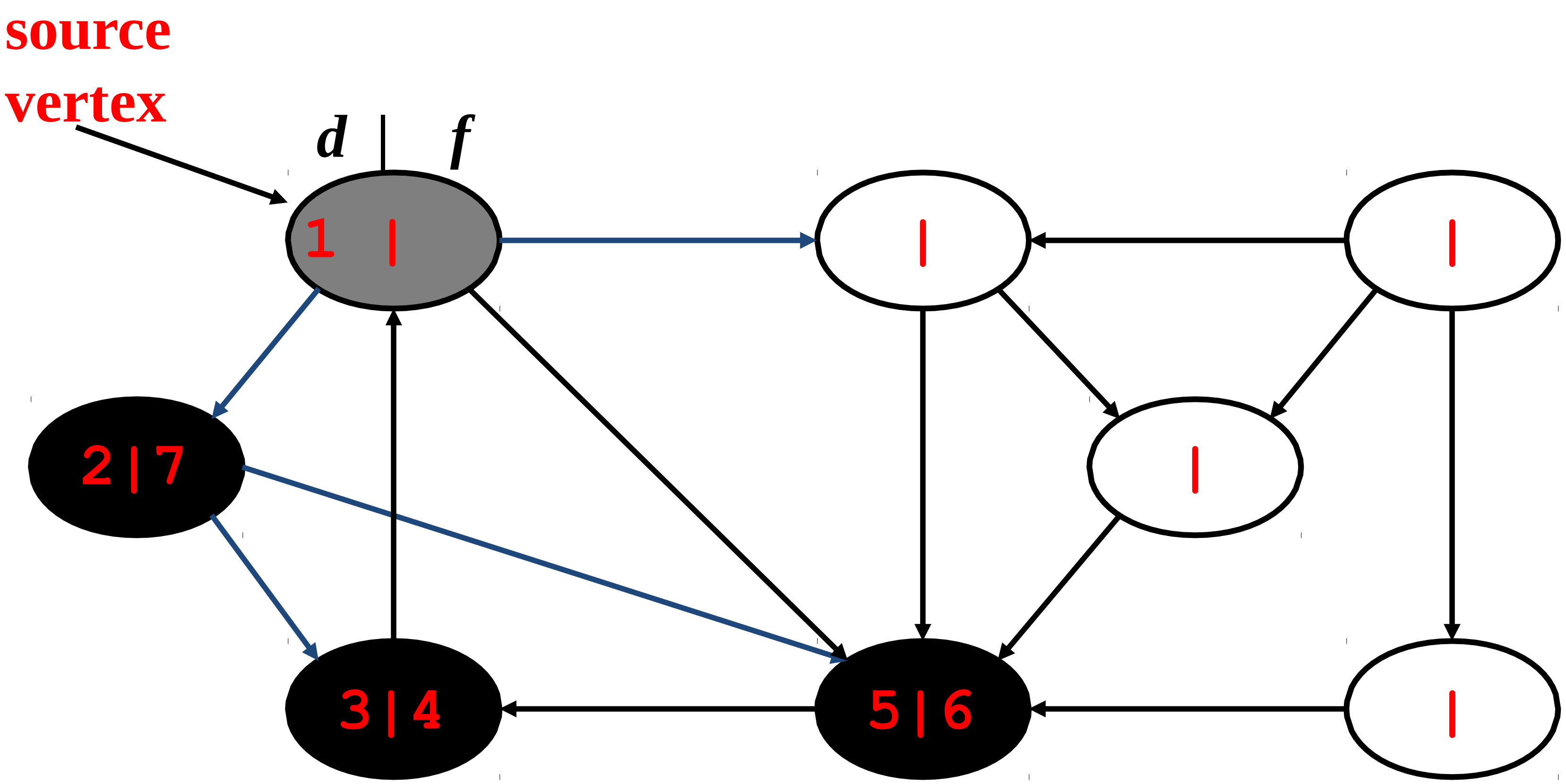

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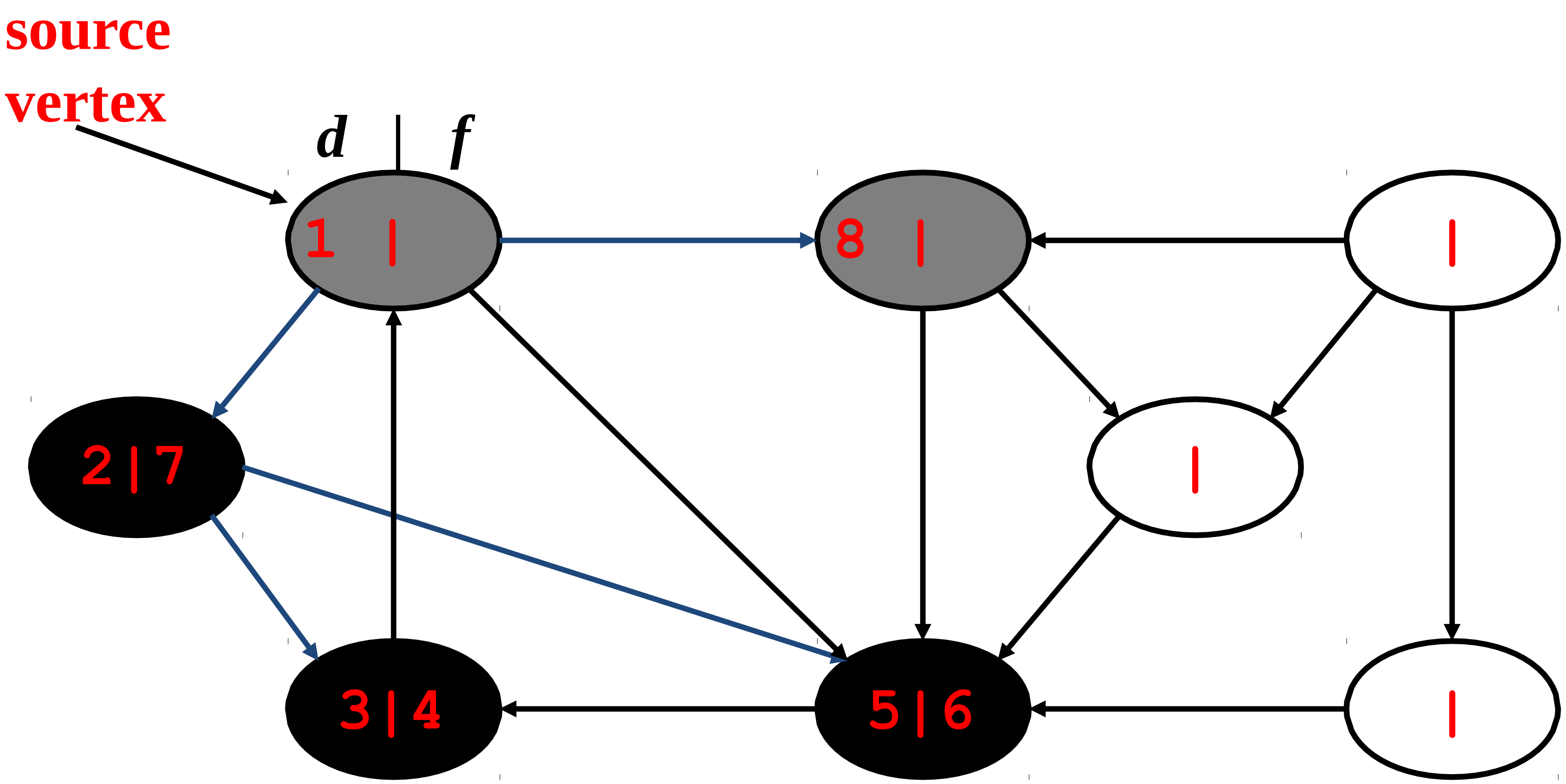

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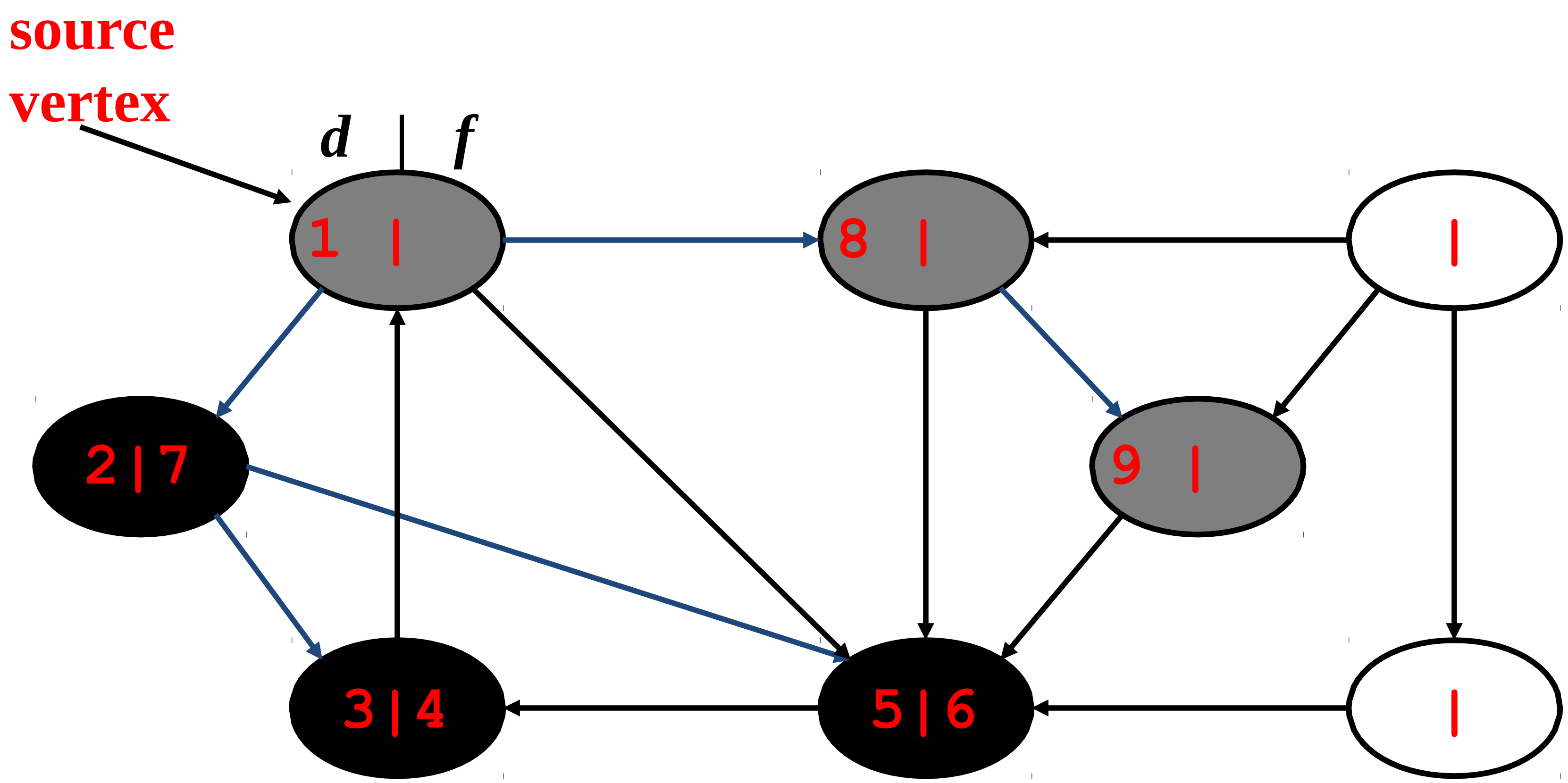

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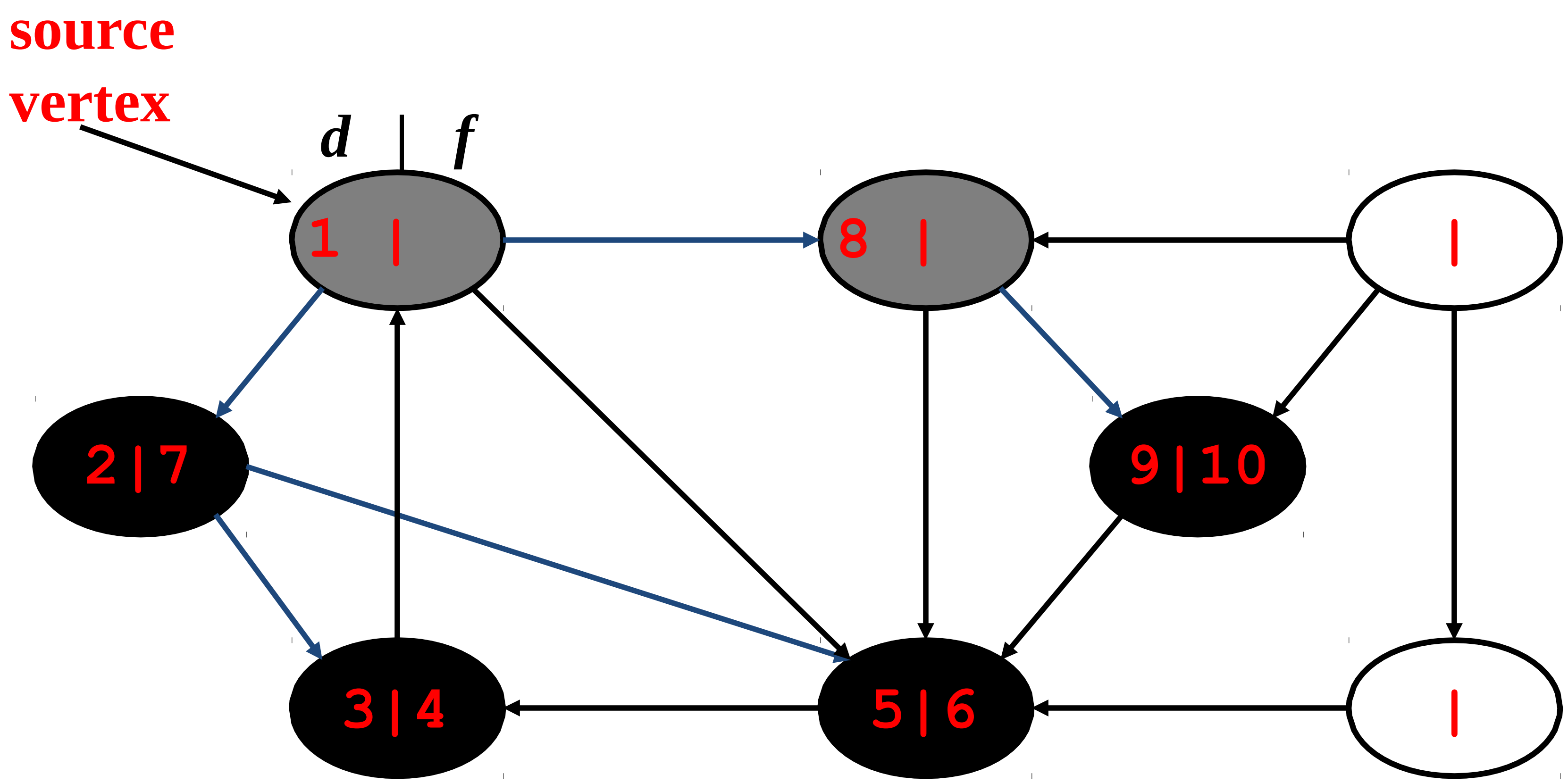

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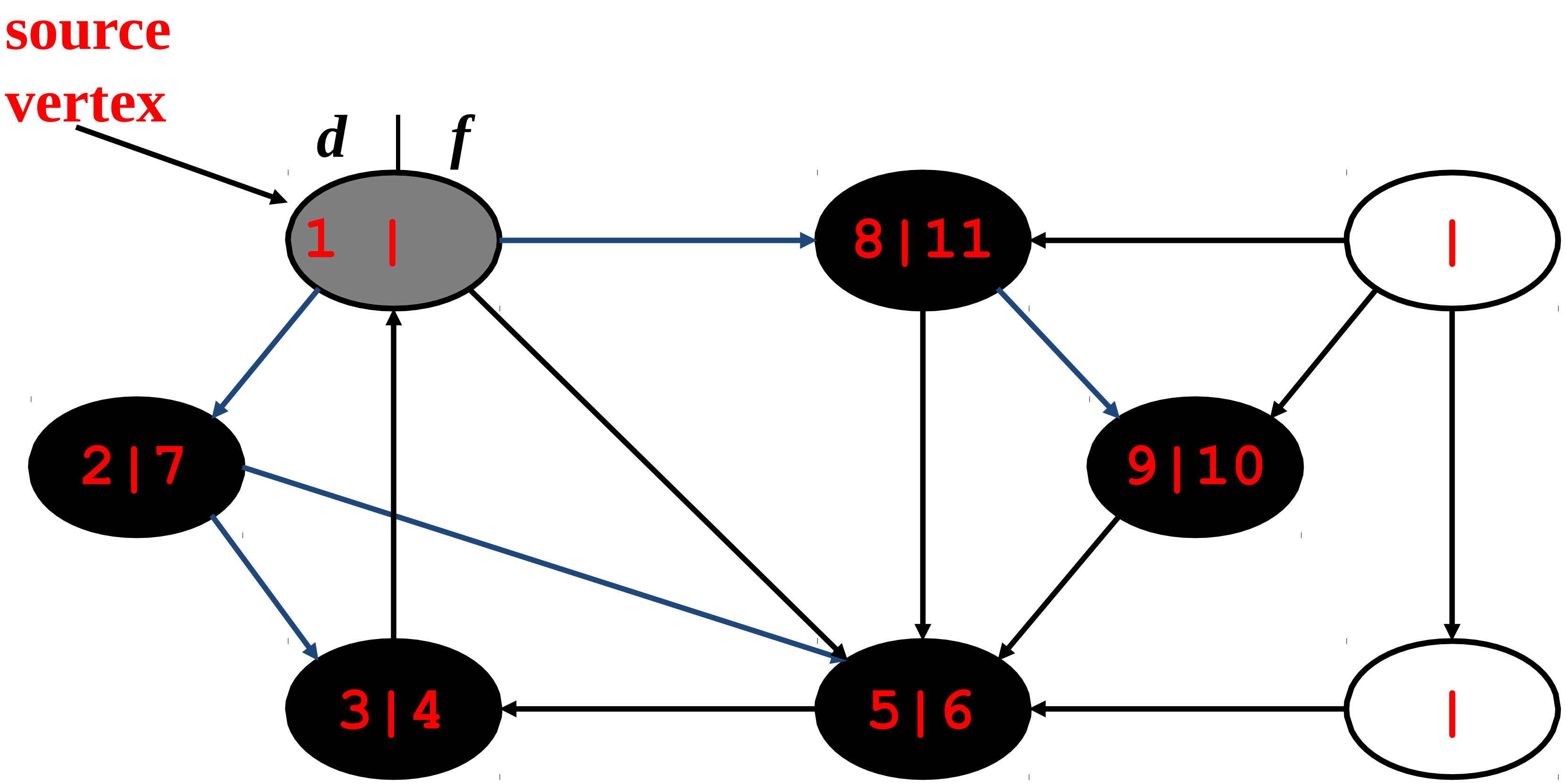

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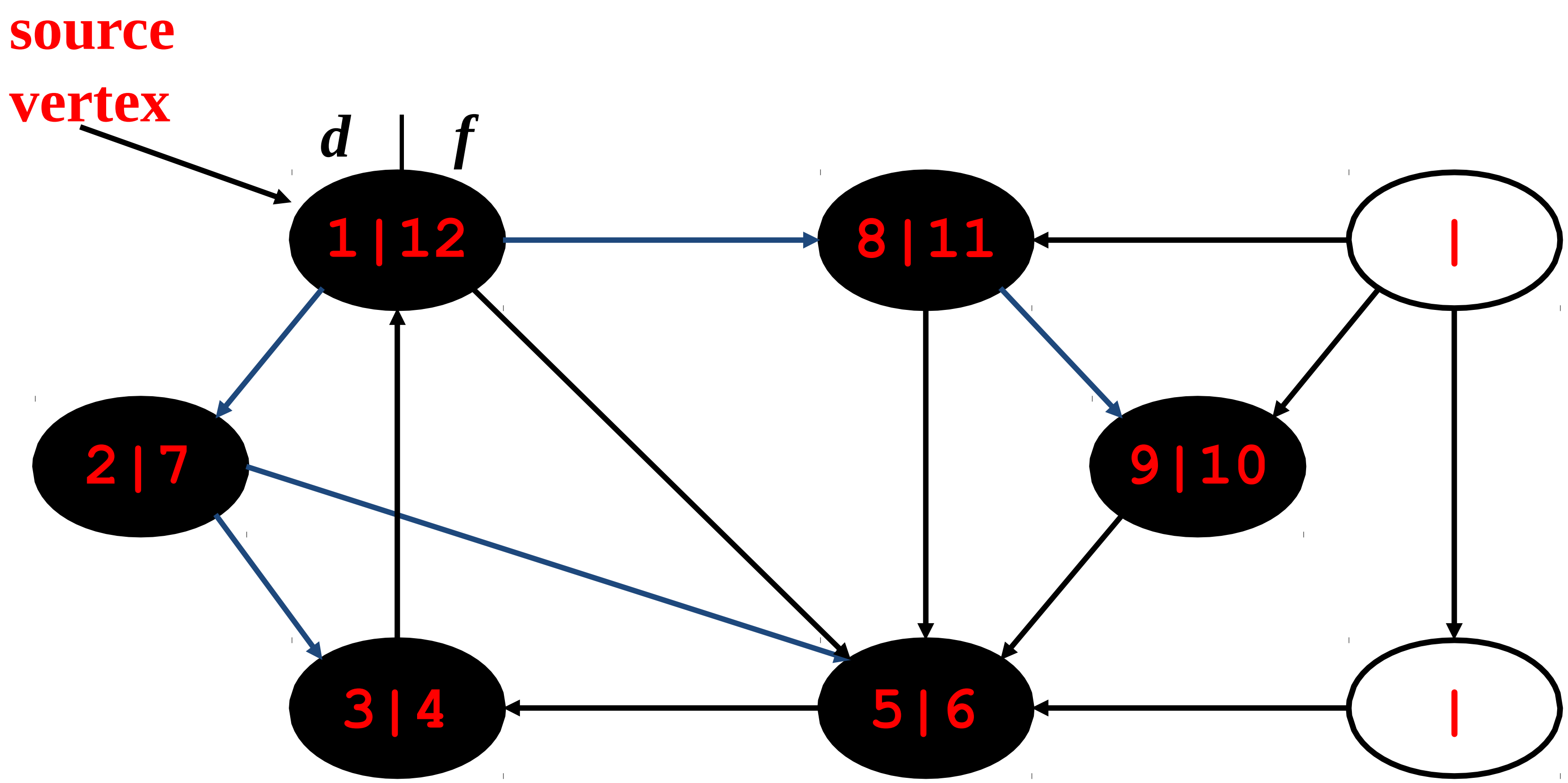

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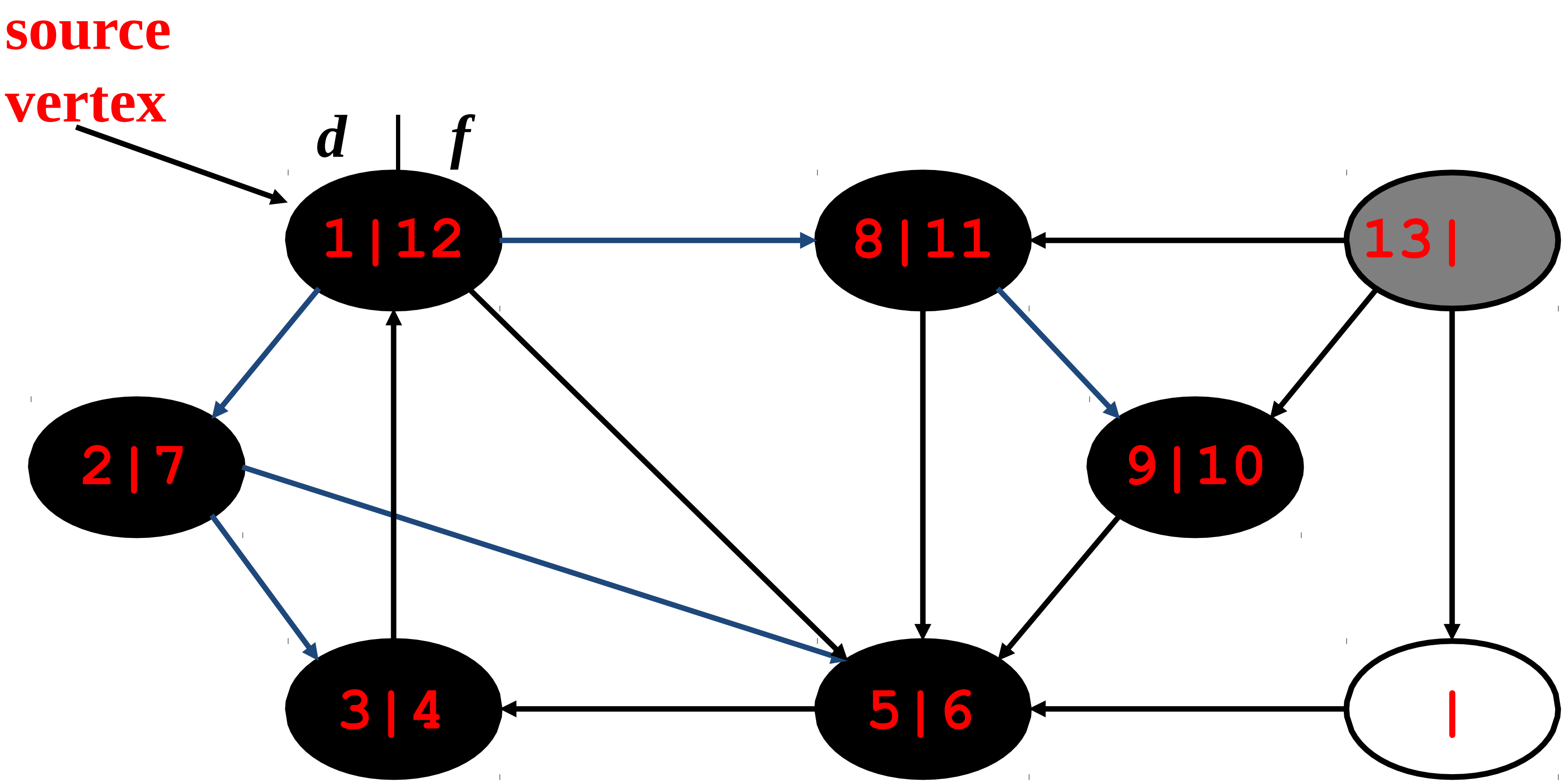

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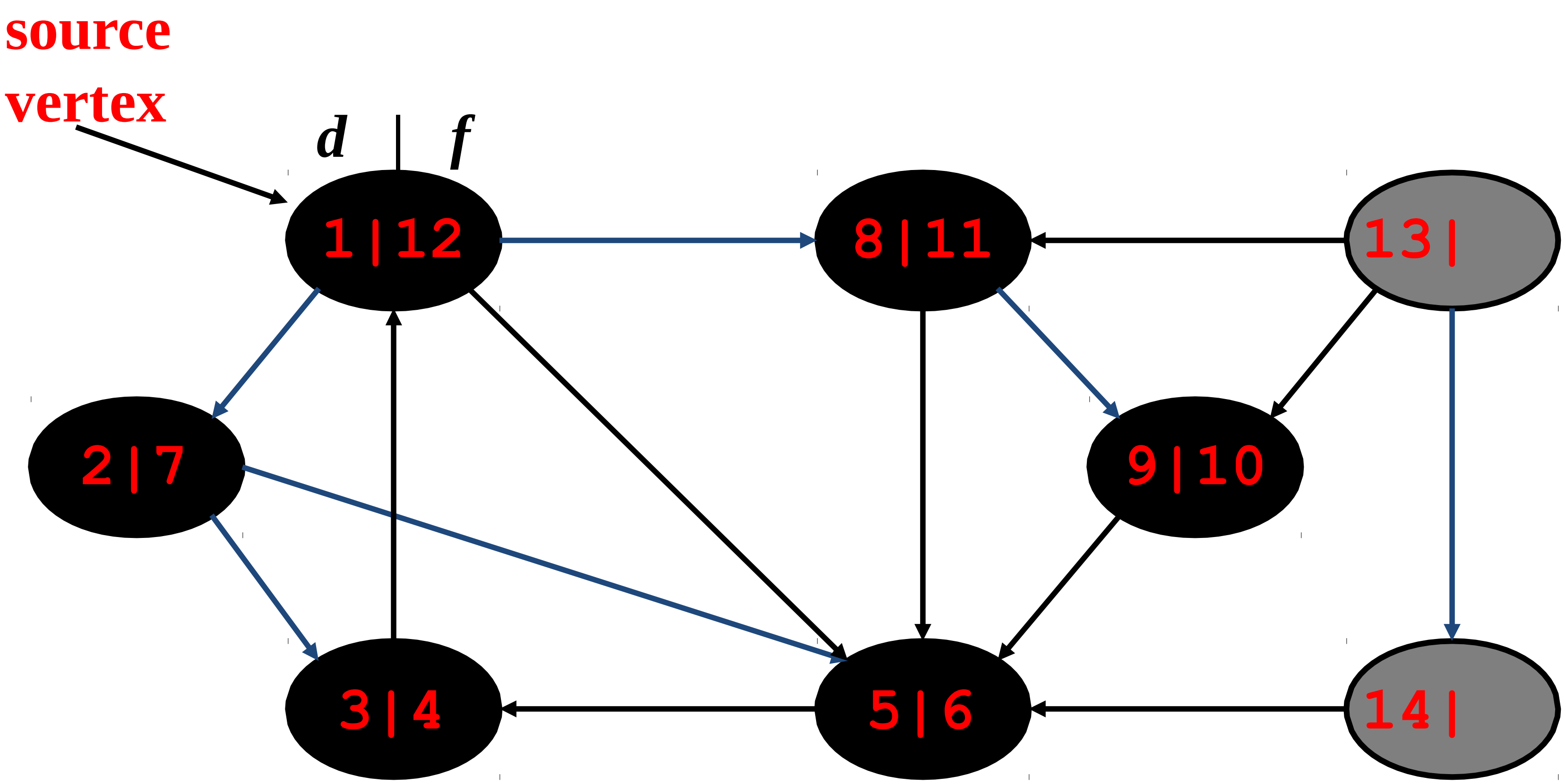

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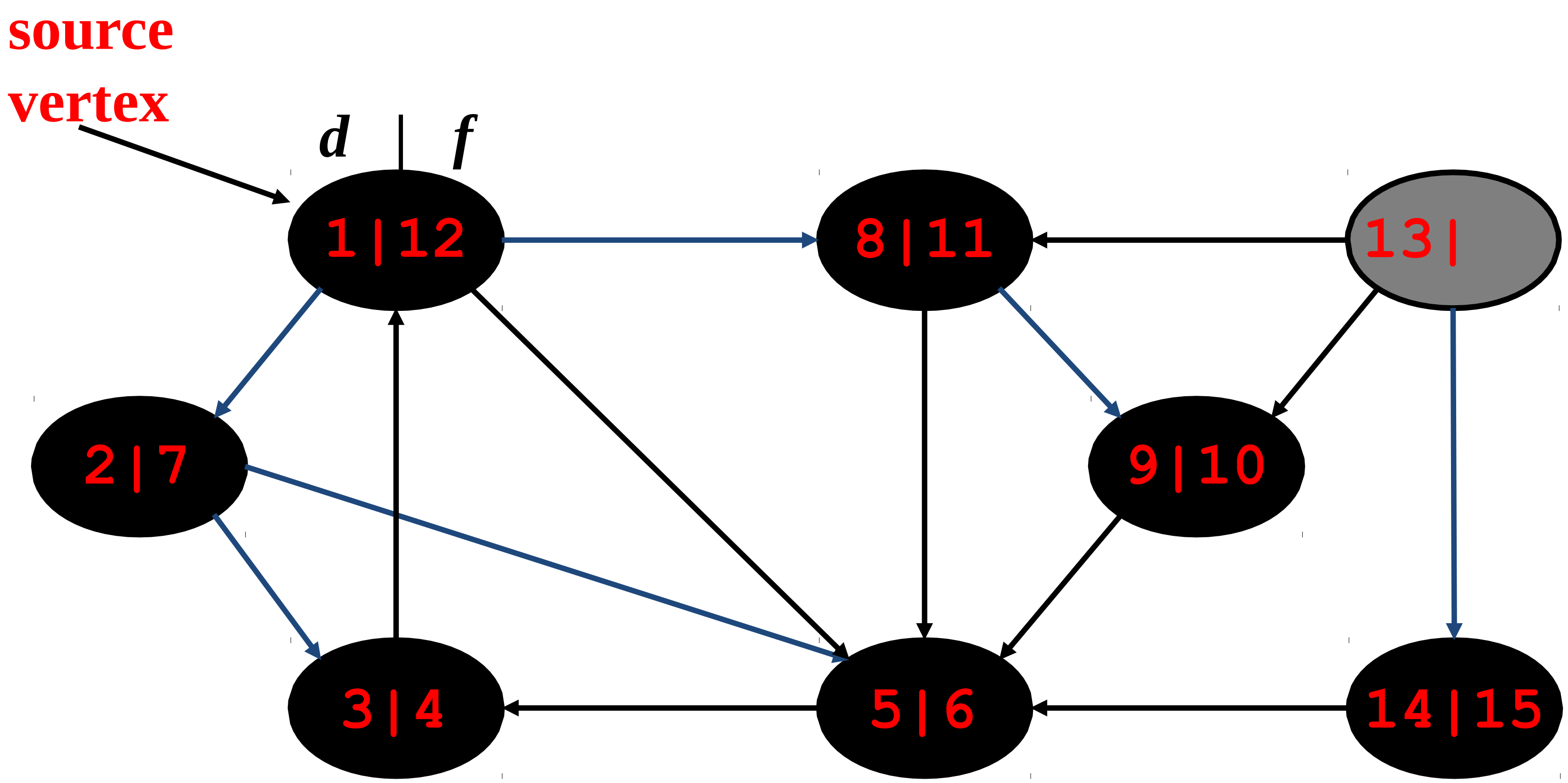

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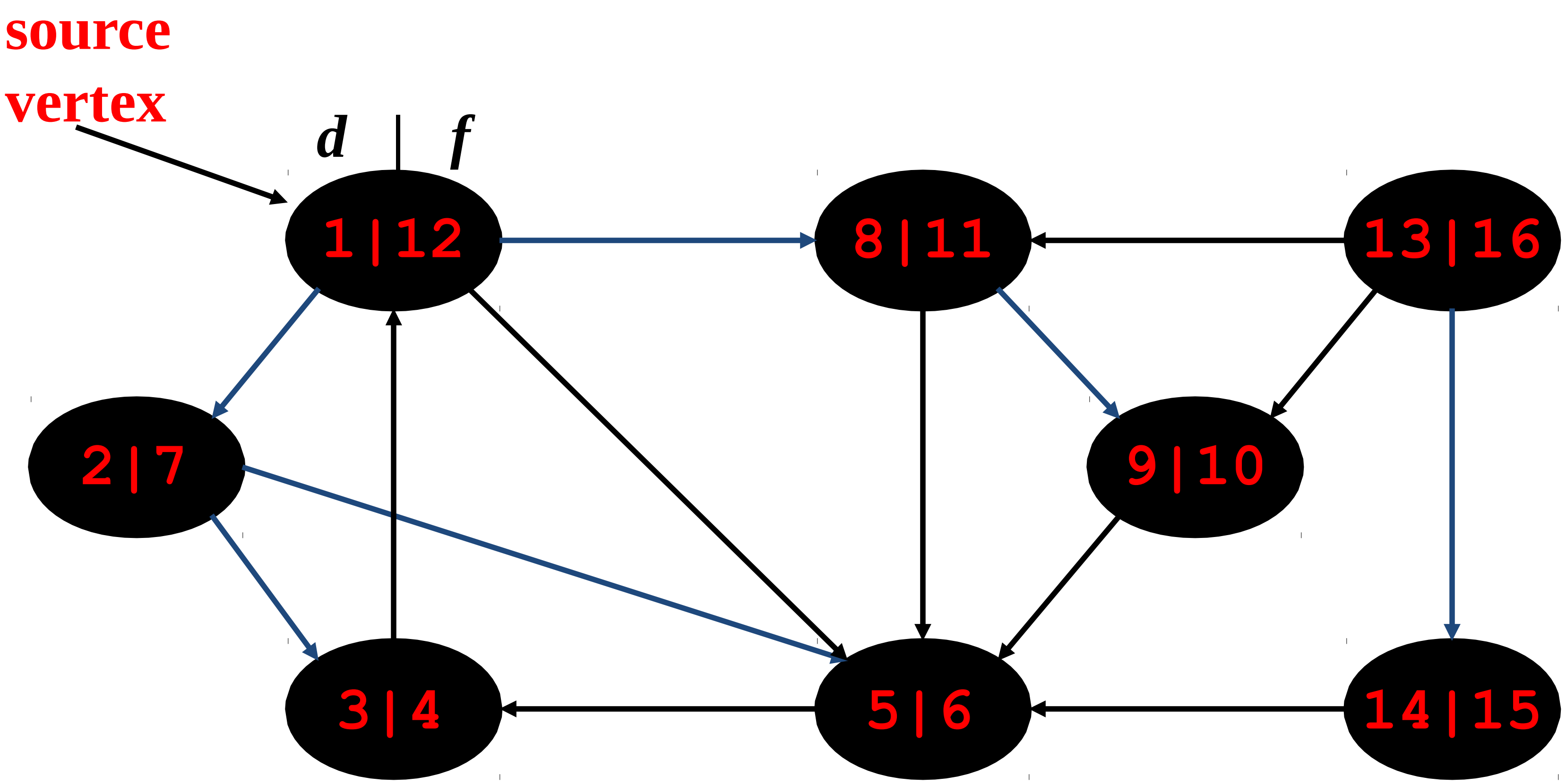
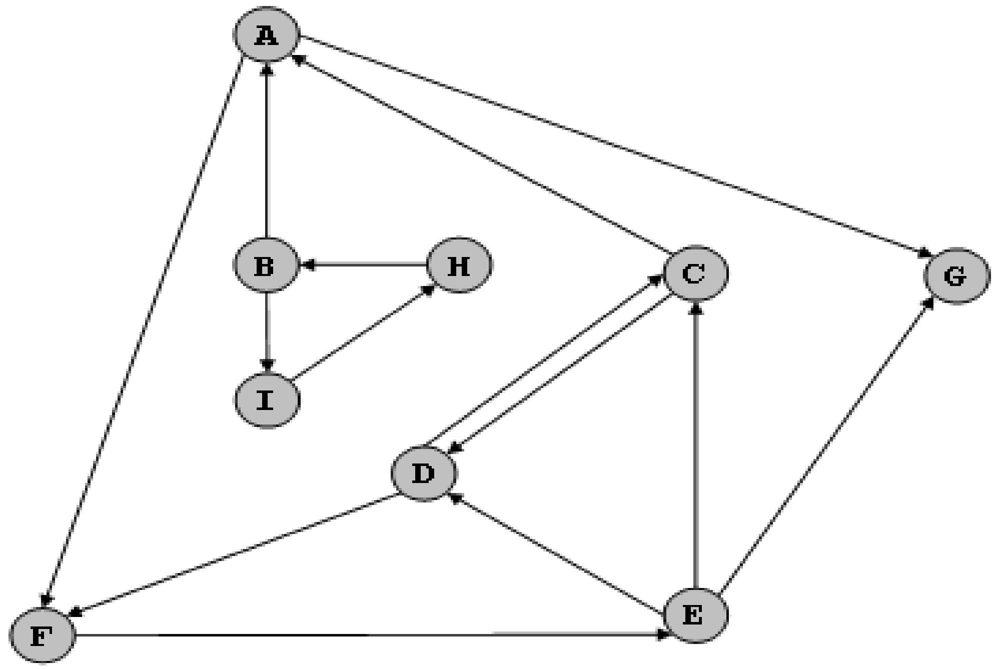


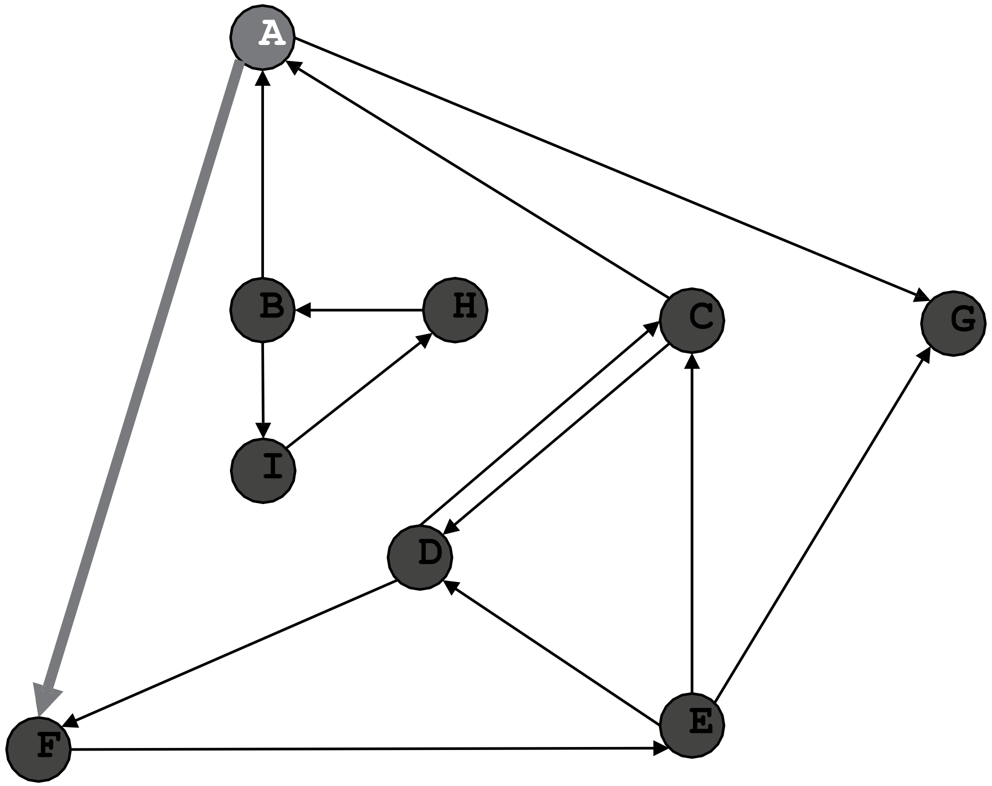
Directed Depth First Search



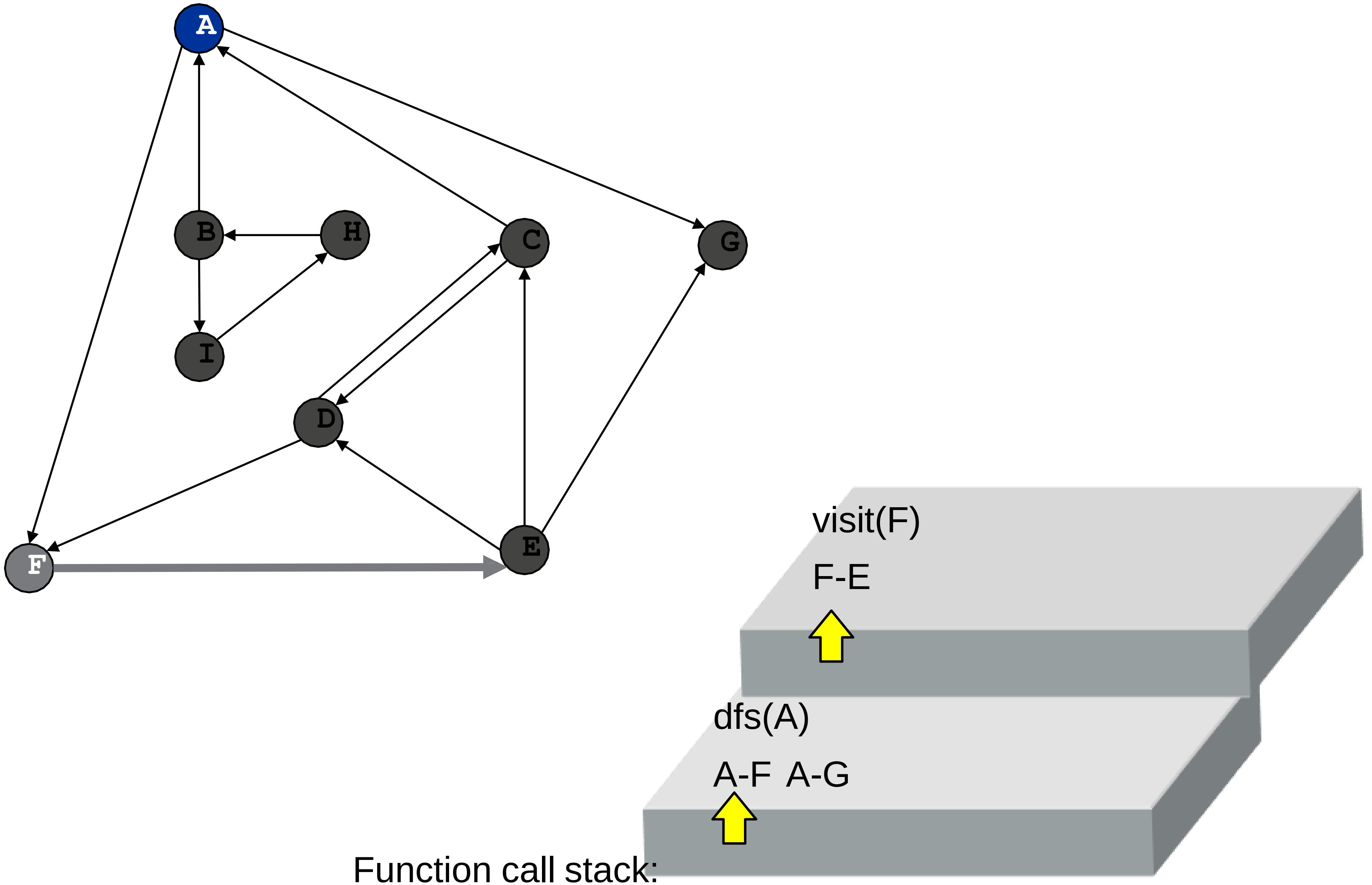
Adjacency Lists

A : F G
B : A H
C : A D
D : C F
E : C D G
F : E
G :
H : B
I : H

DIRECTED DEPTH FIRST SEARCH



Function call stack:



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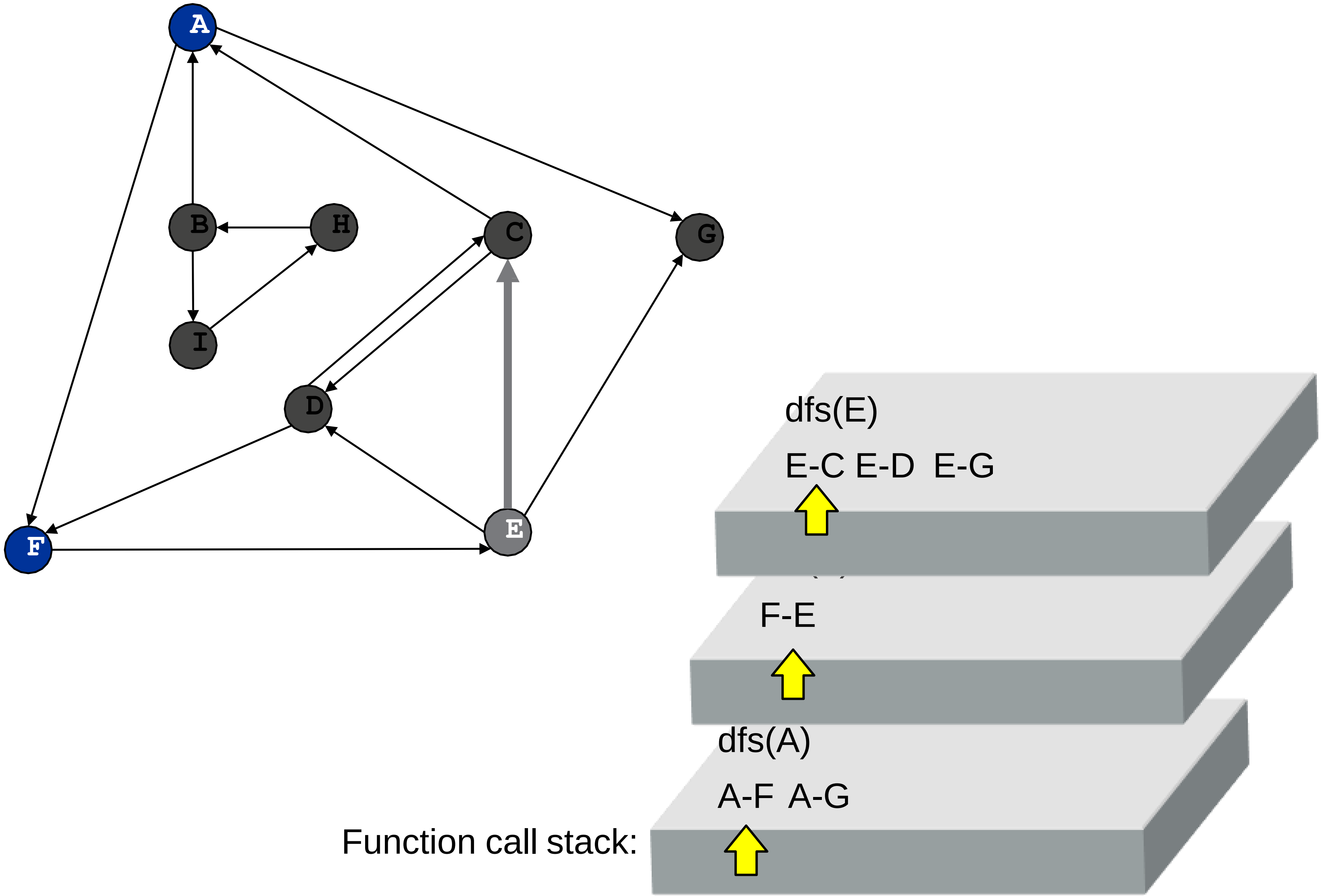
A
B
H
C
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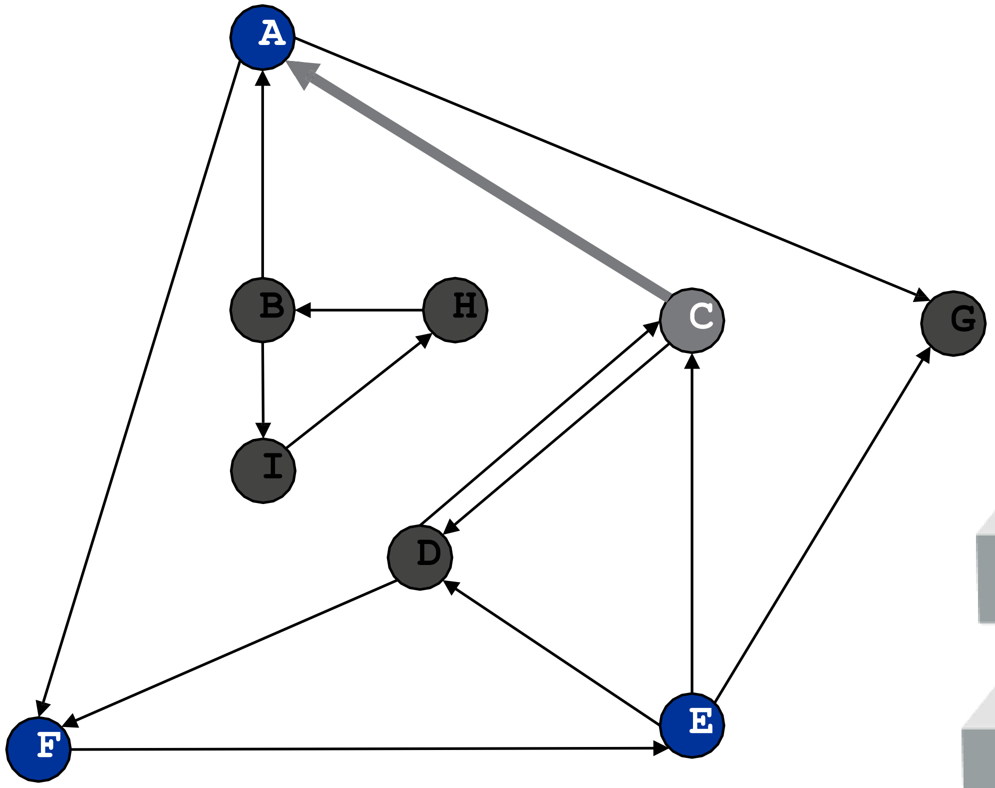
I
D
E
F
dfs(E)
E-C E-D E-G
F-E
dfs(A)
A-F A-G
Function call stack:



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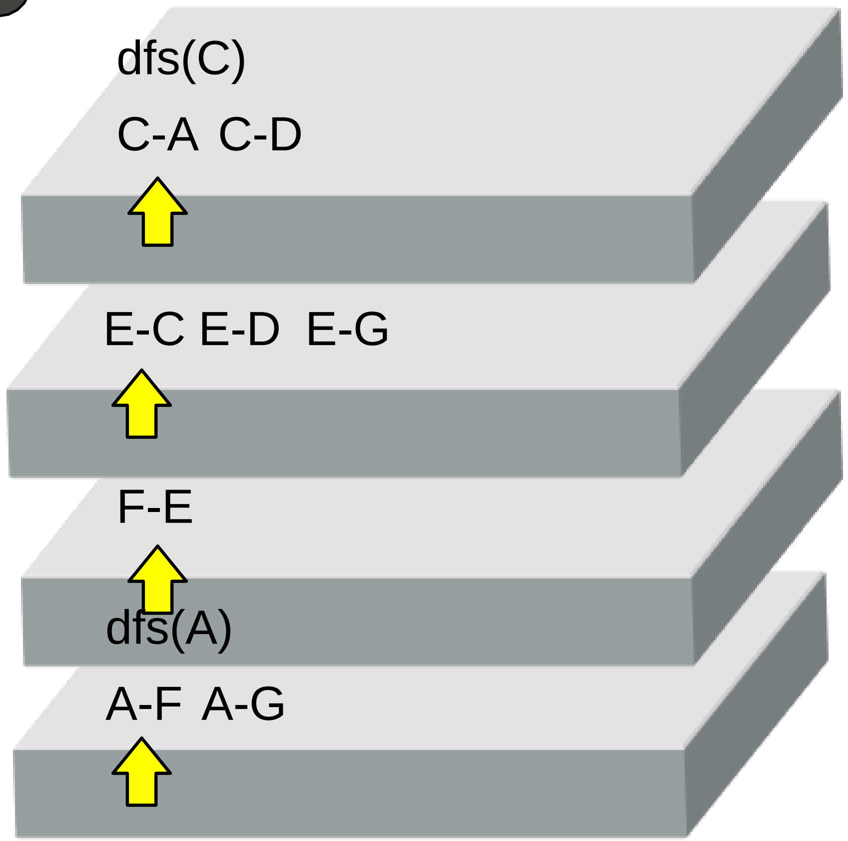

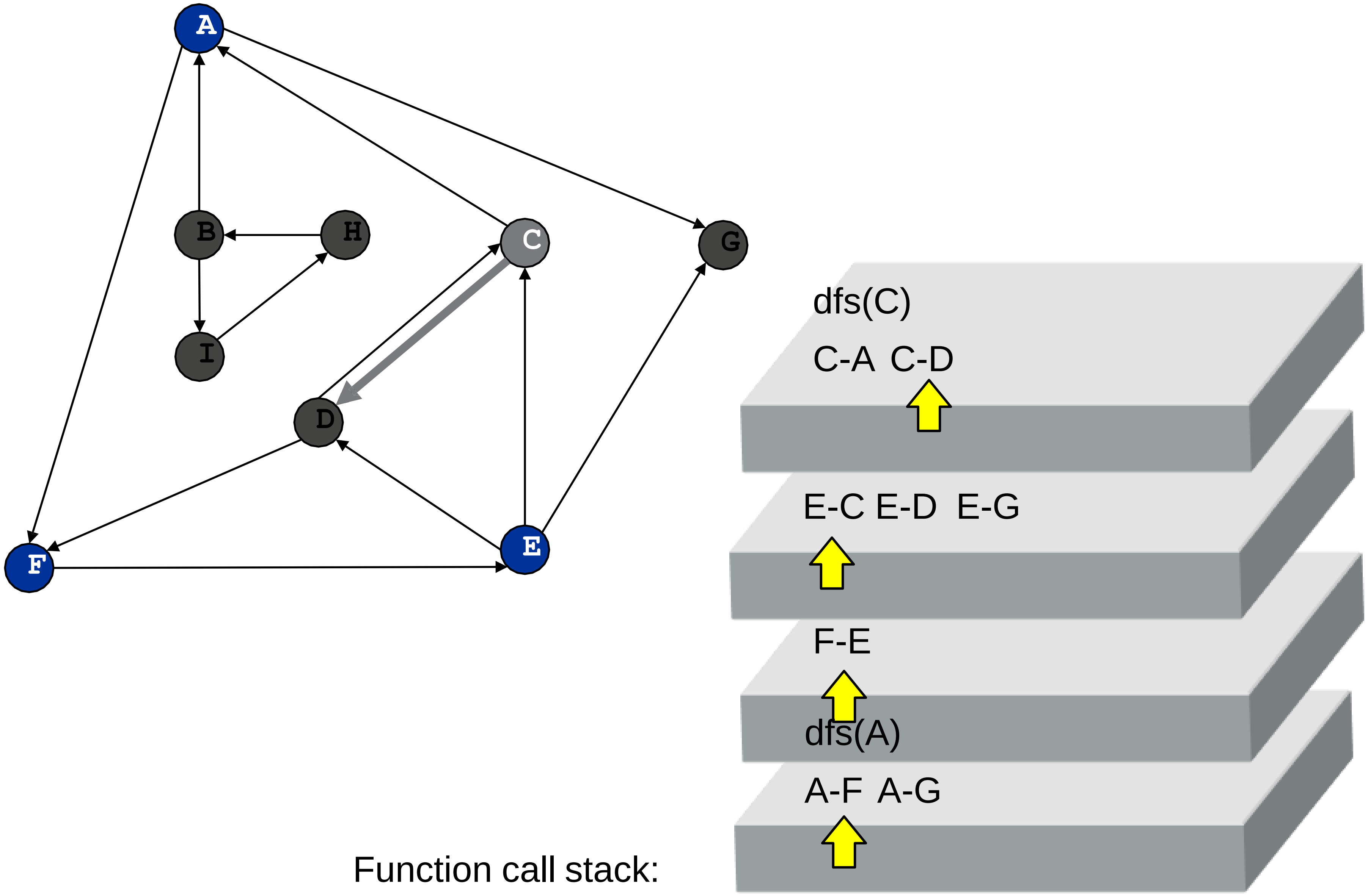

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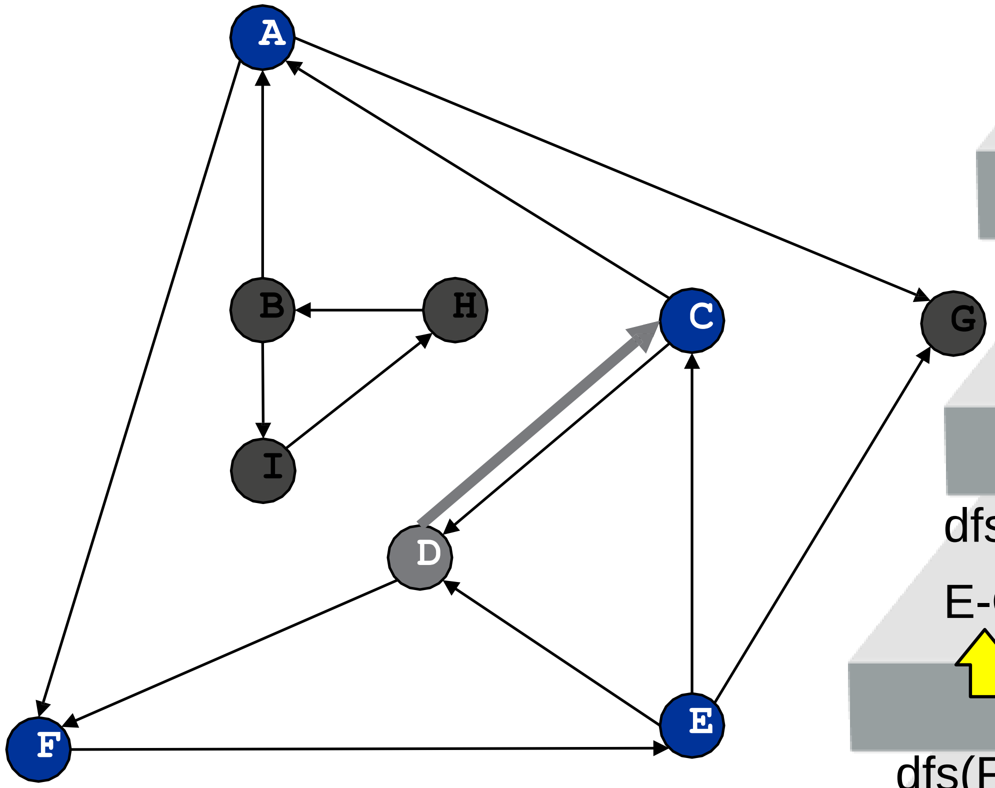

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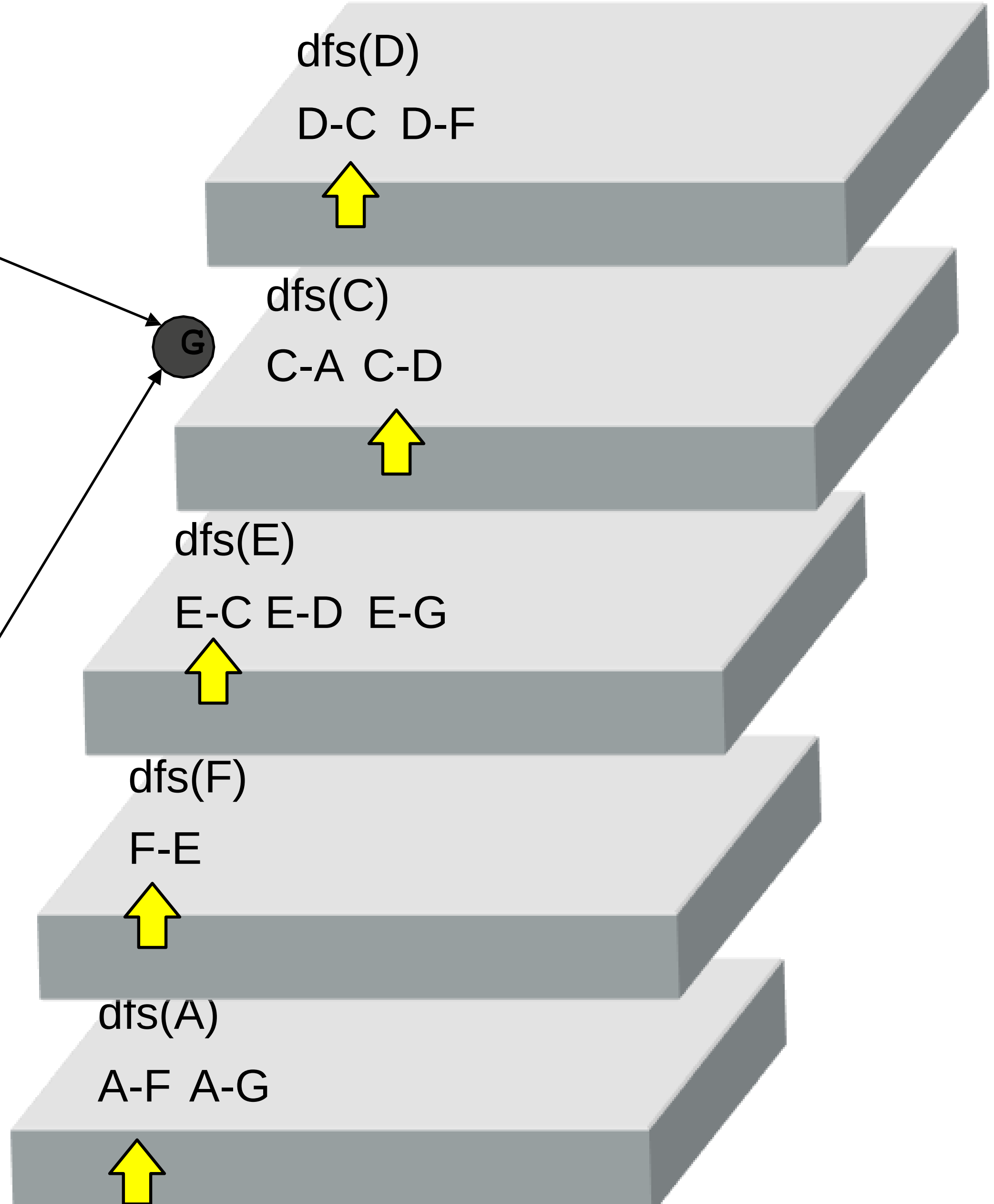

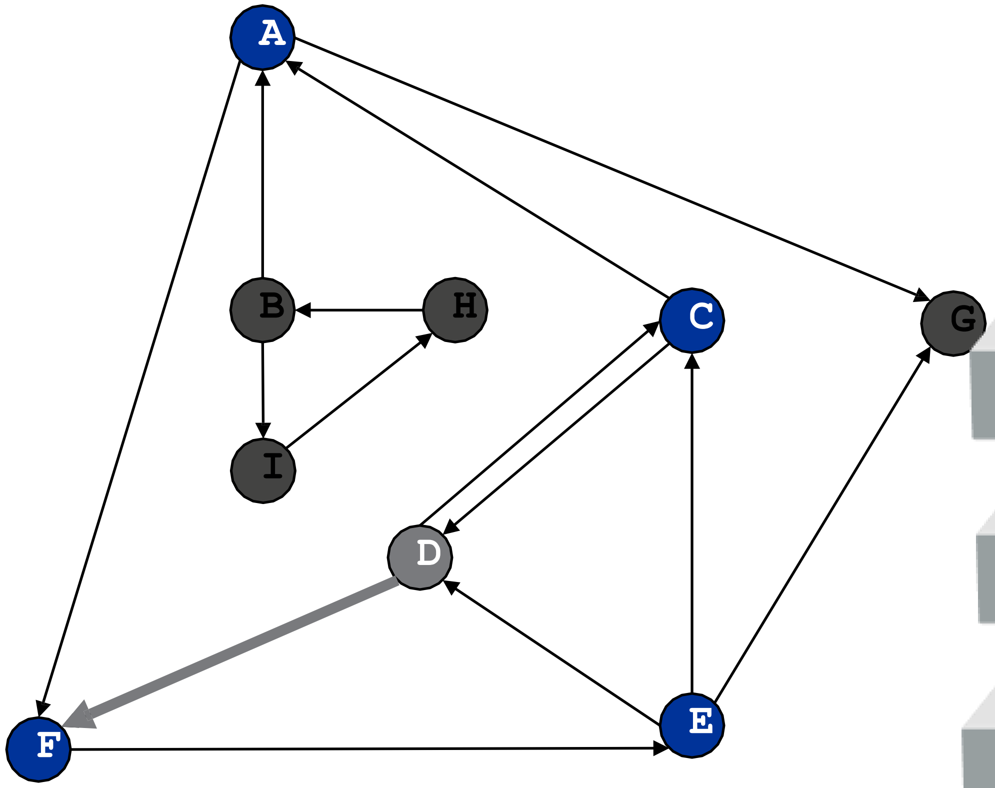

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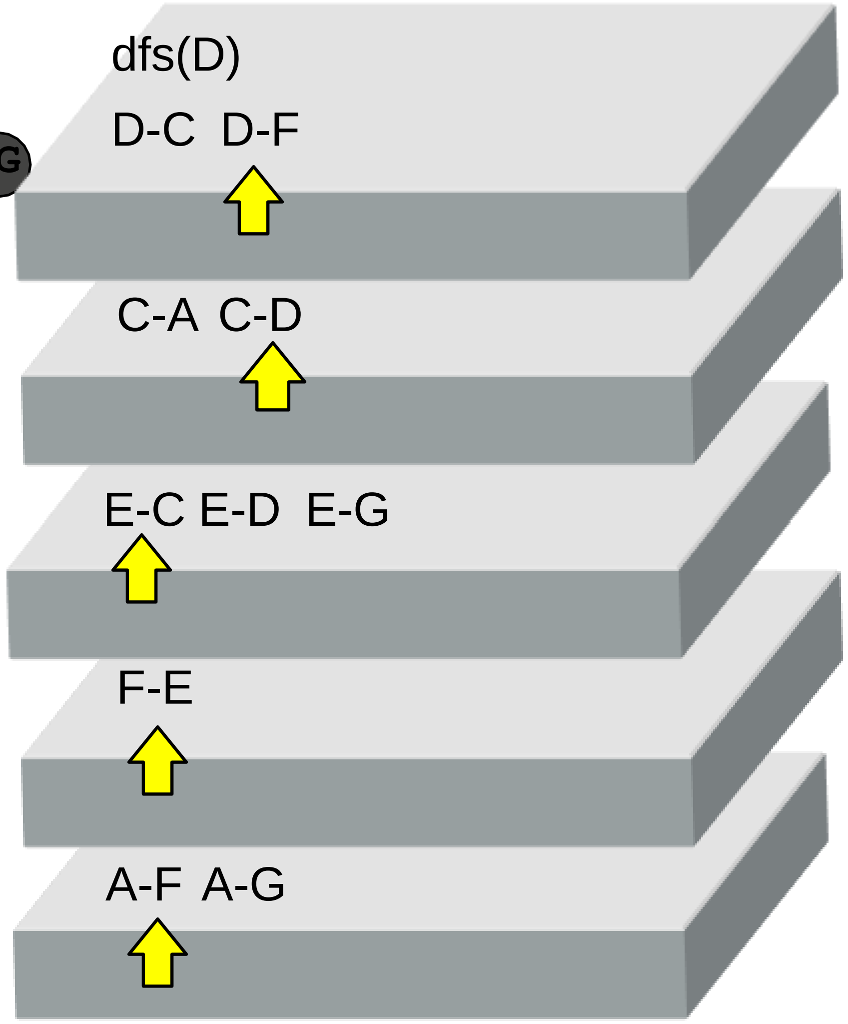

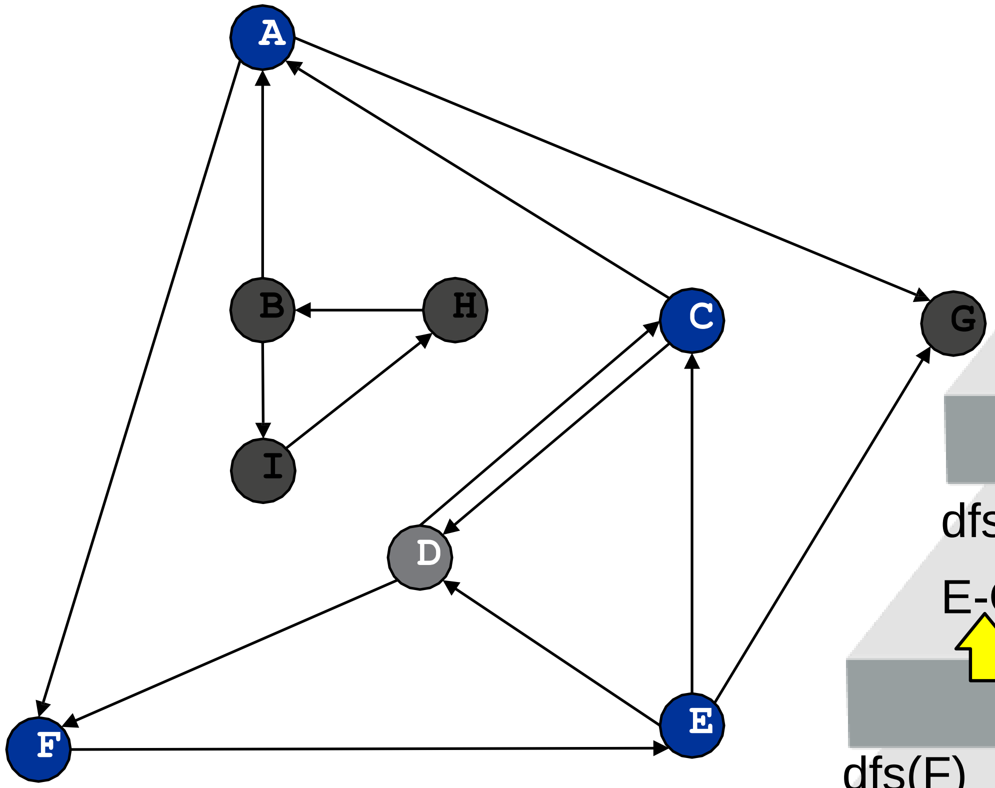

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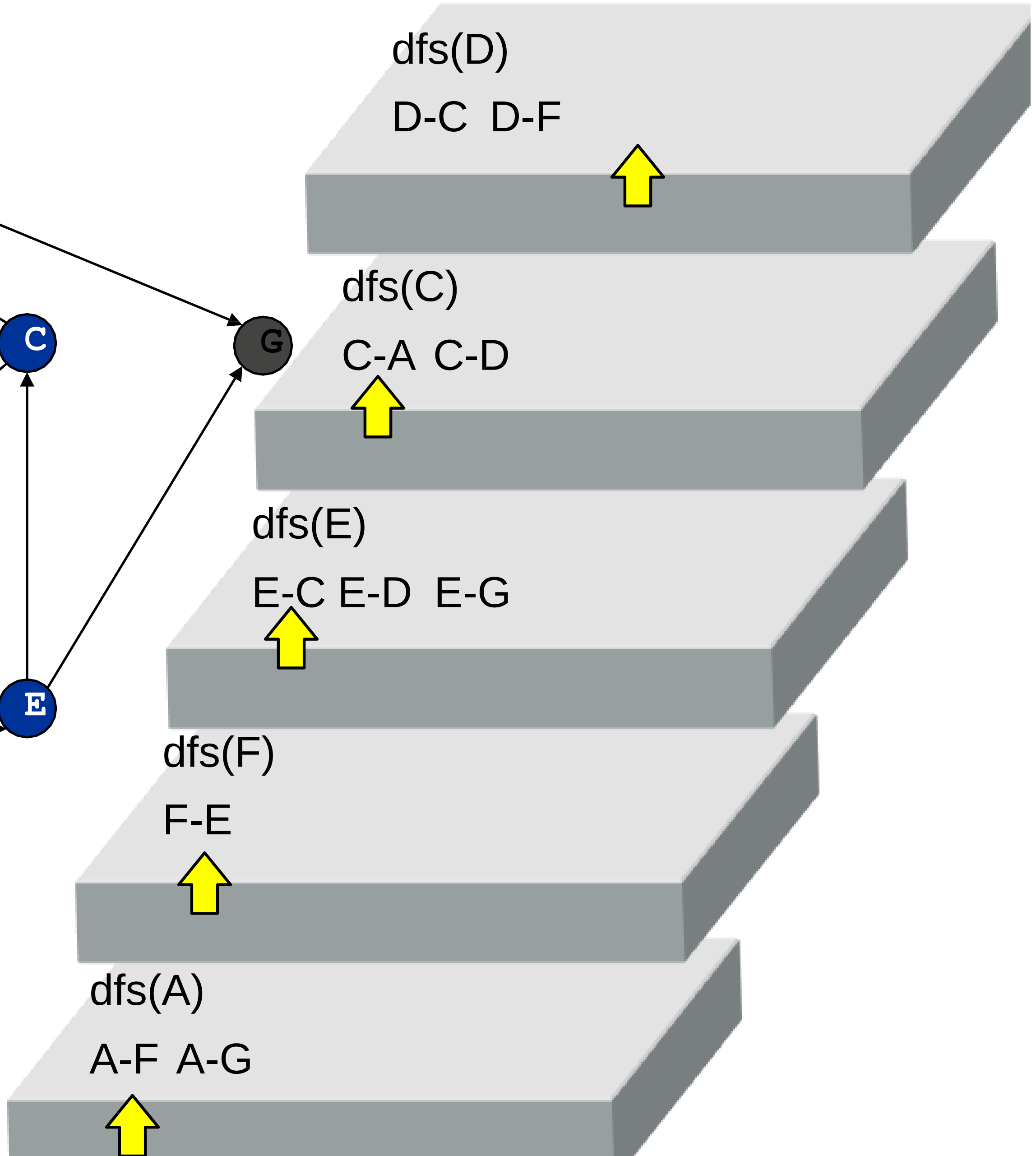

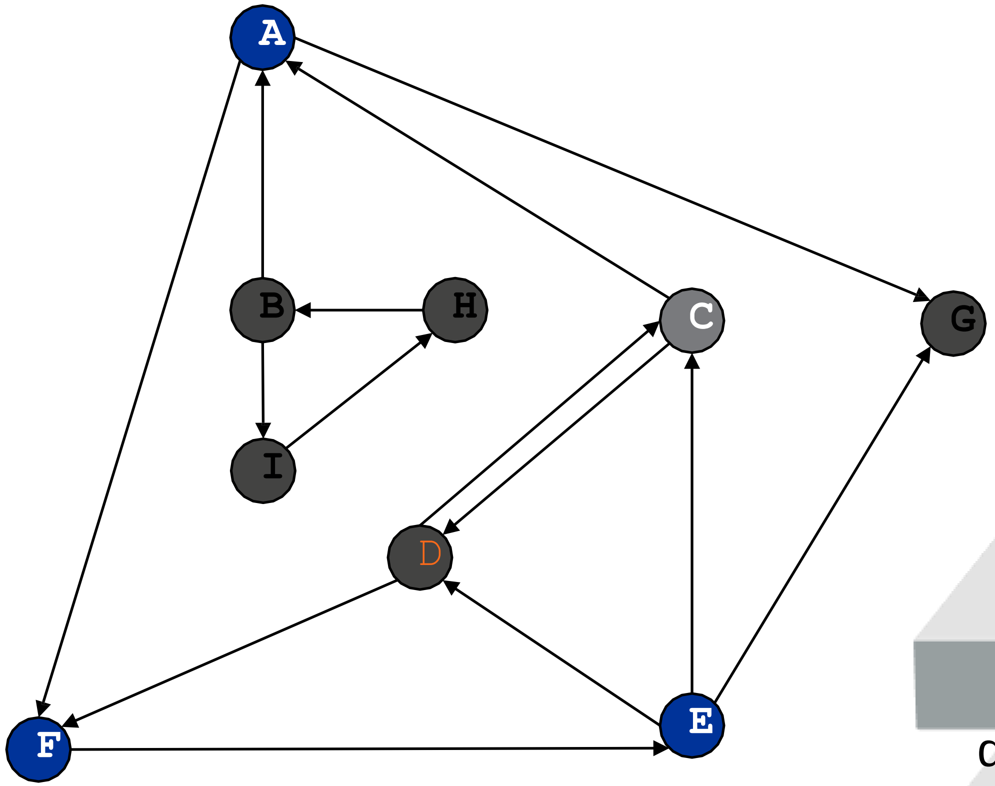

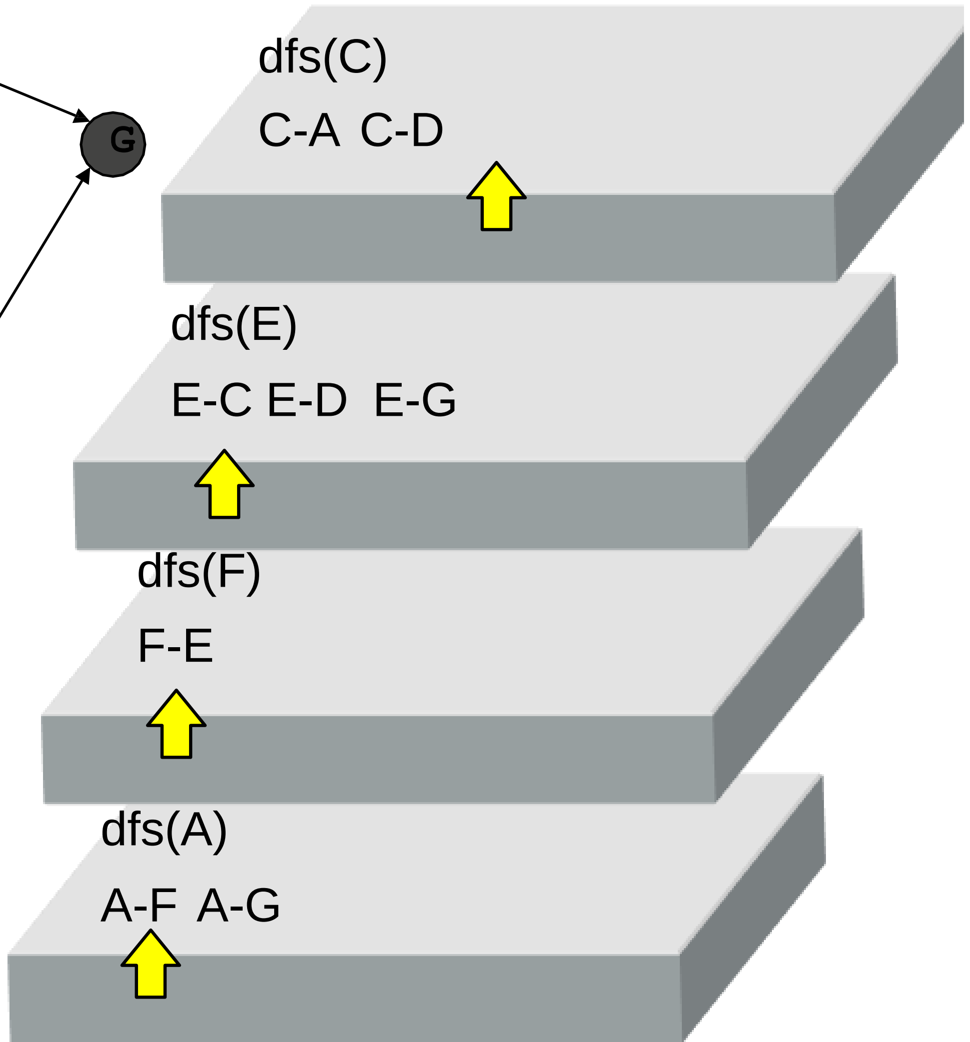

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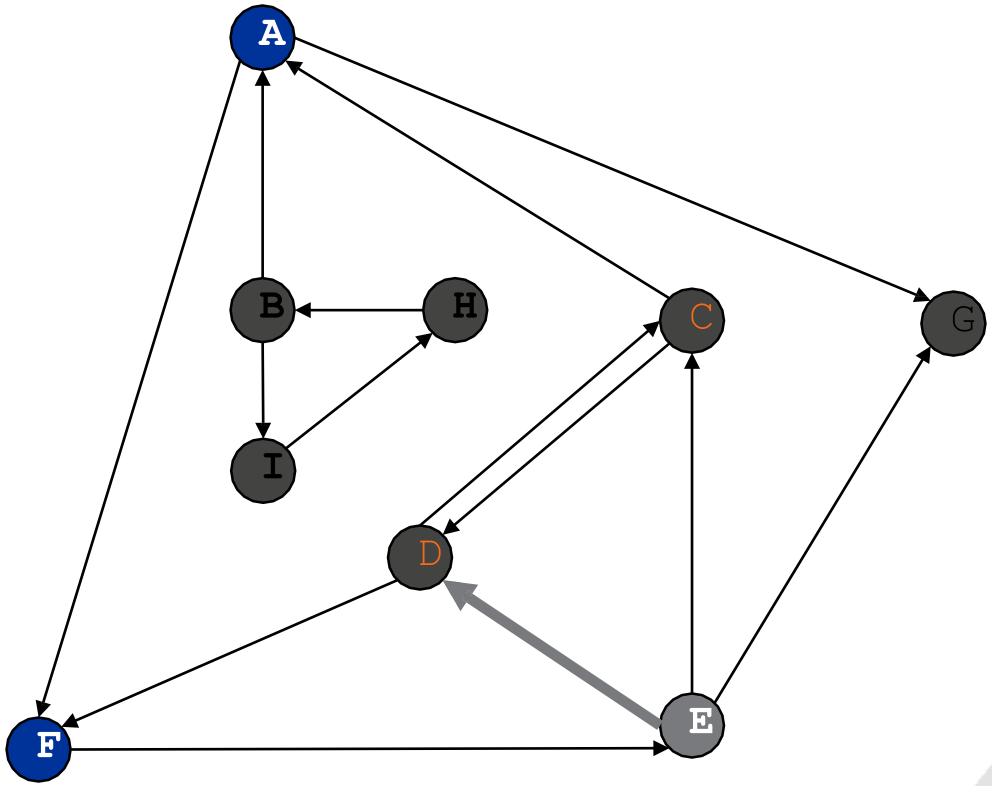

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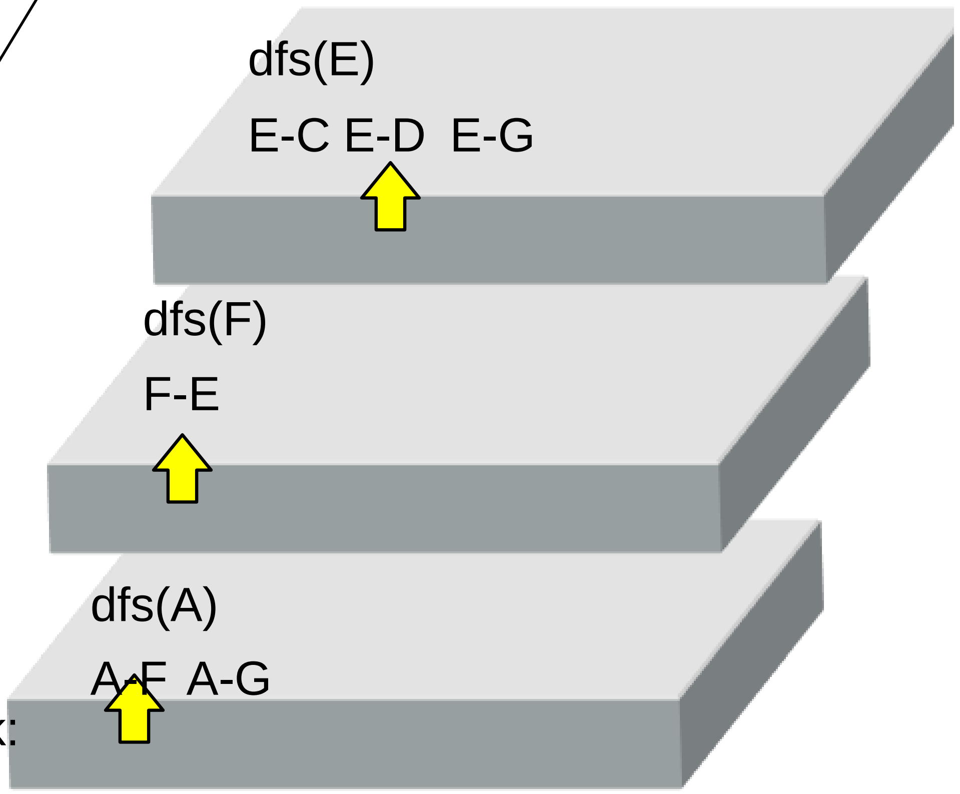
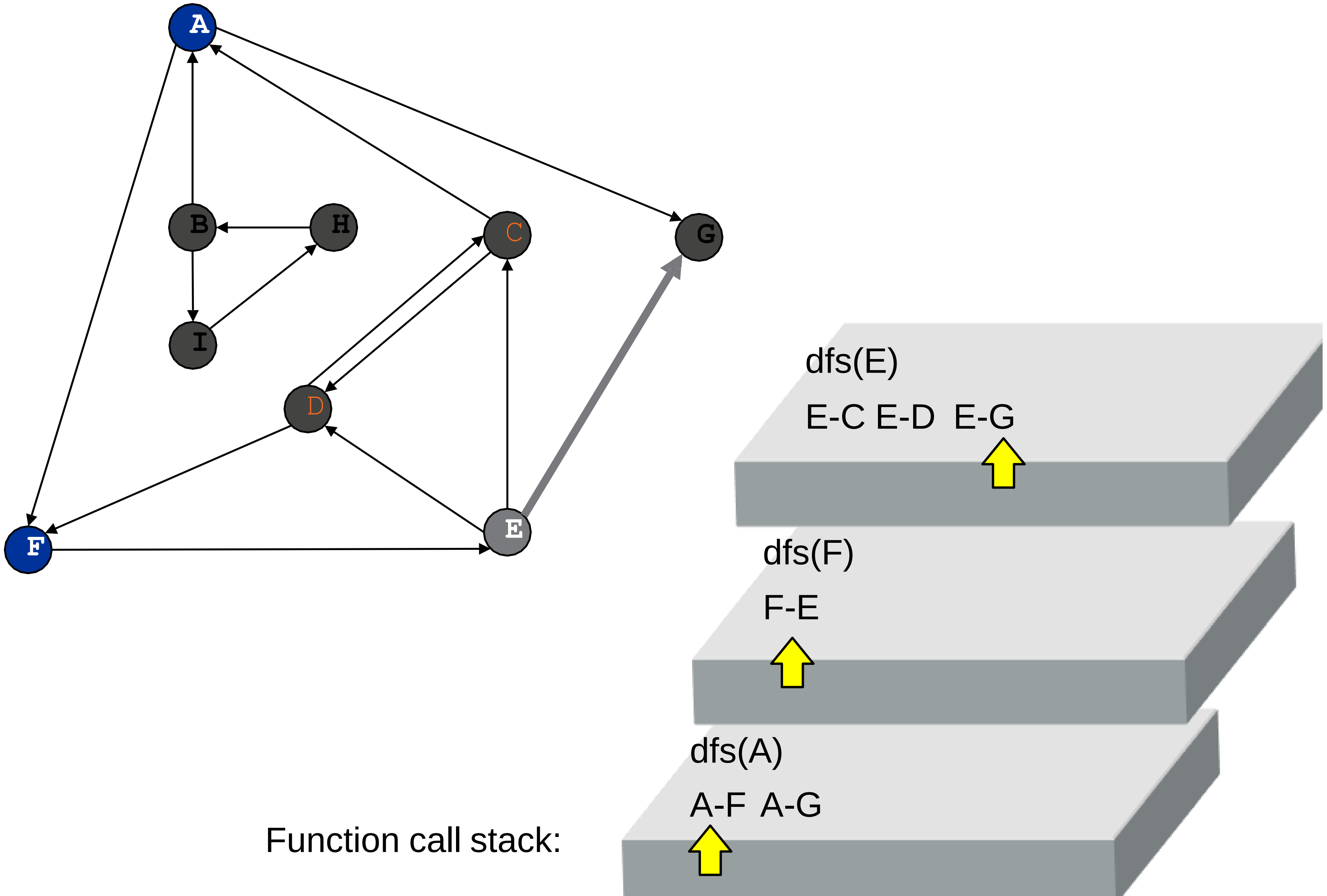
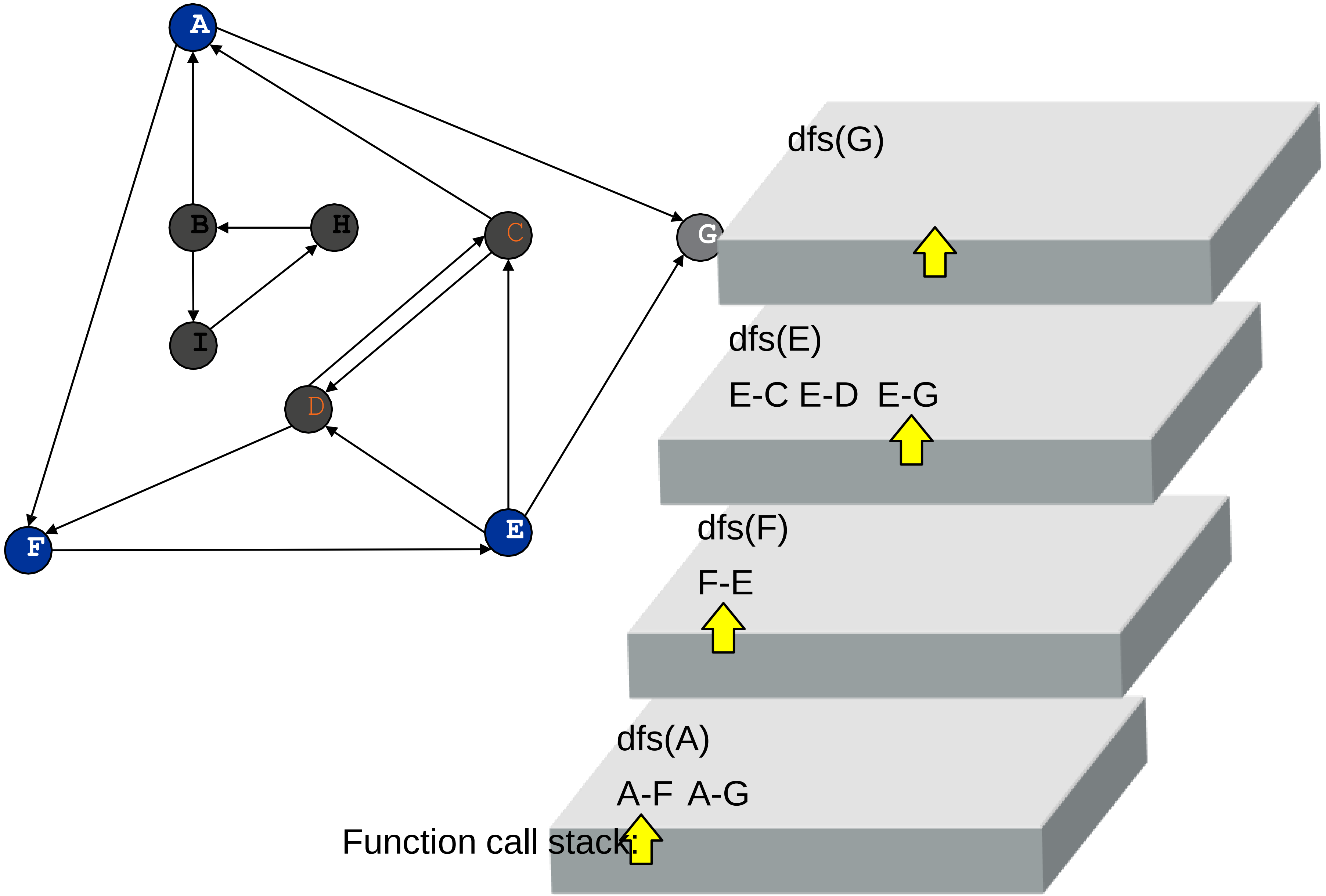



A
B
H
C
G
I
D
E
F
dfs(E)
E-C E-D E-G
dfs(F)
F-E
dfs(A)
A-F A-G
Function call stack:


A
B
H
C
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I
D
E
F
dfs(G)
dfs(E)
E-C E-D E-G
dfs(F)
F-E
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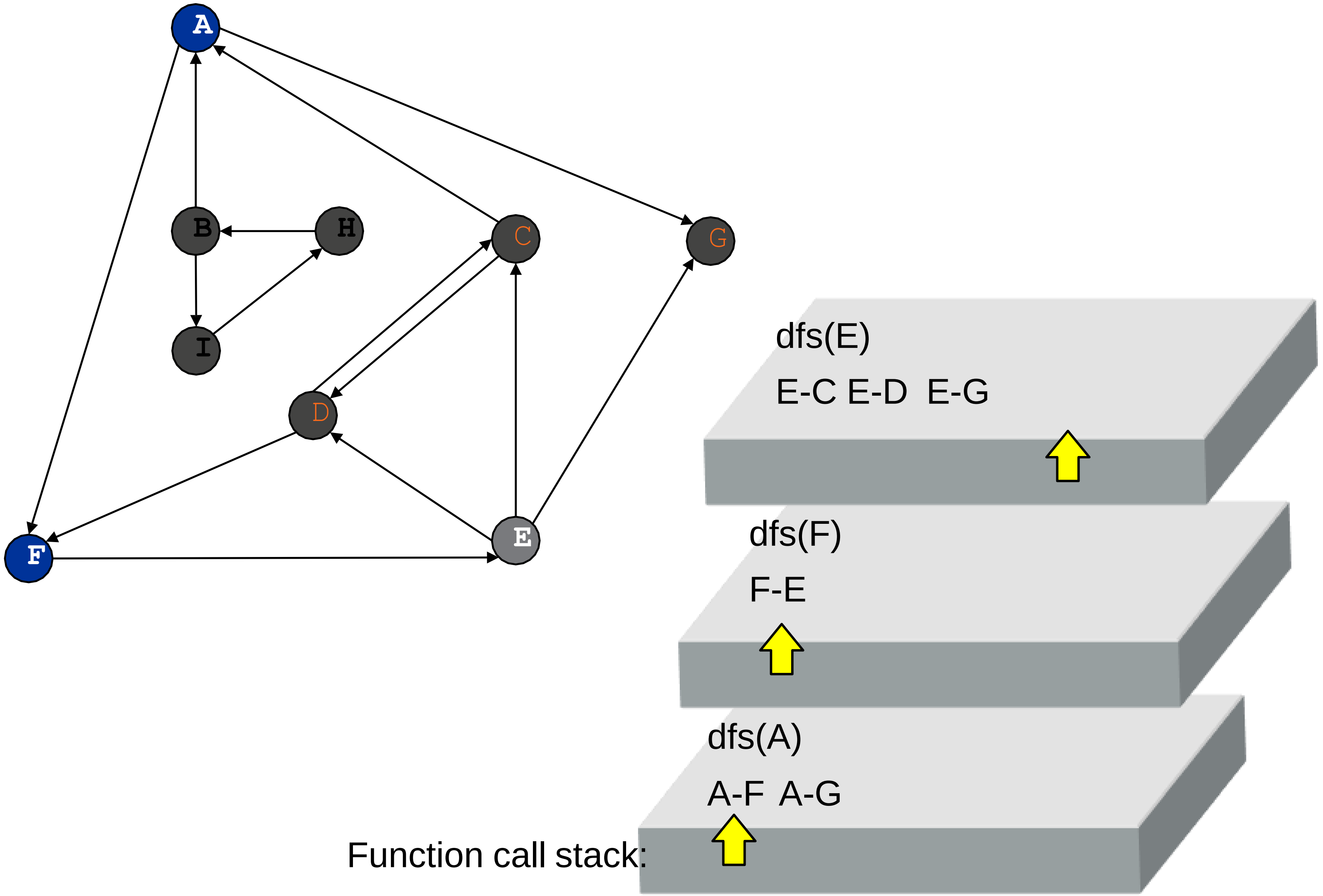
Function call stack:


A
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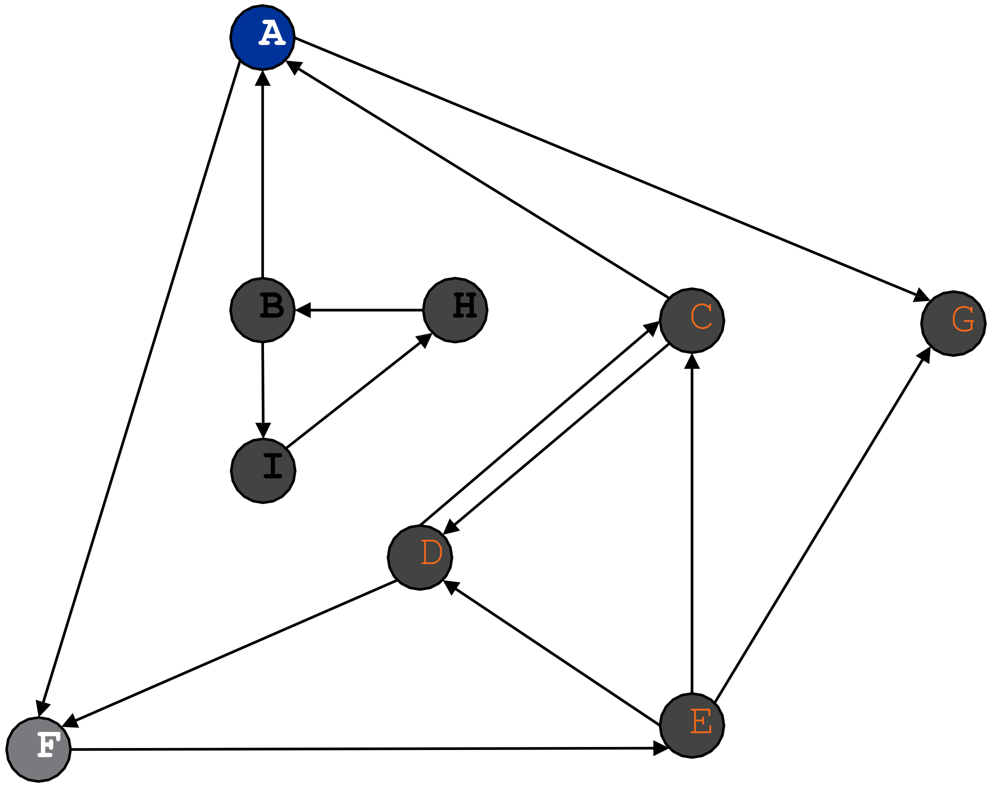
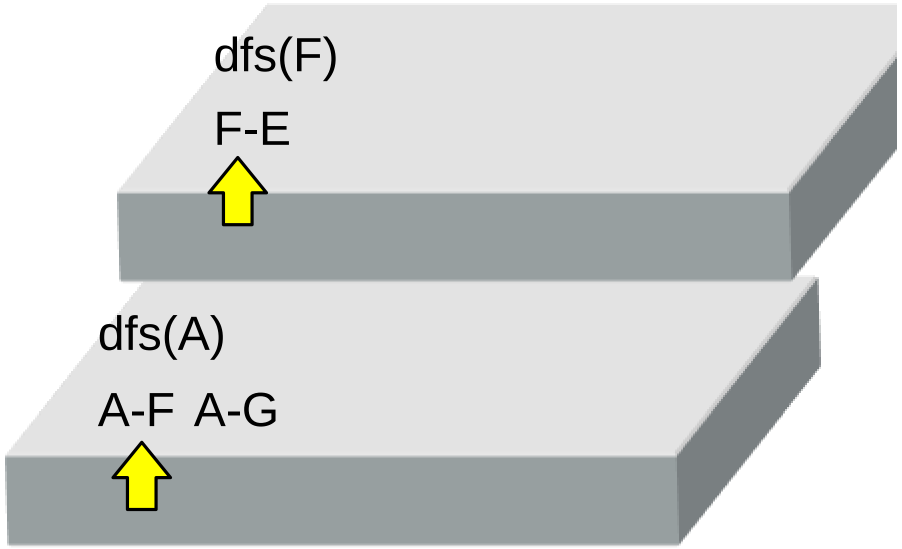
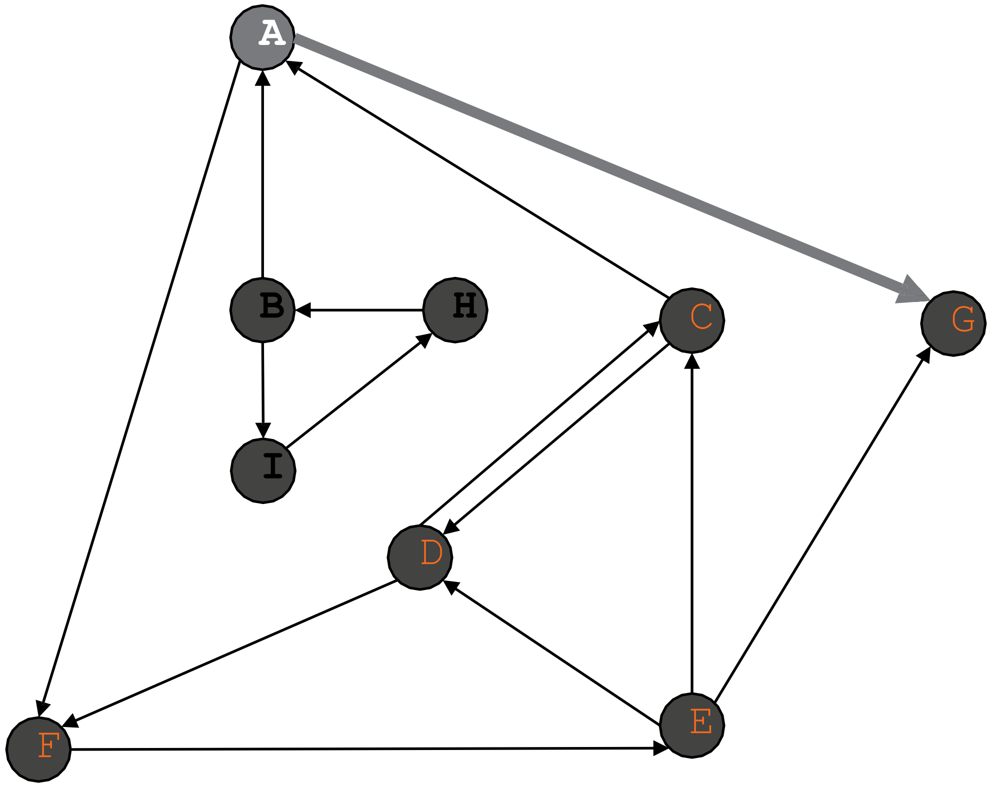
E
dfs(E)
E-C E-D E-G
dfs(F)
F-E
dfs(A)
A-F A-G
Function call stack:

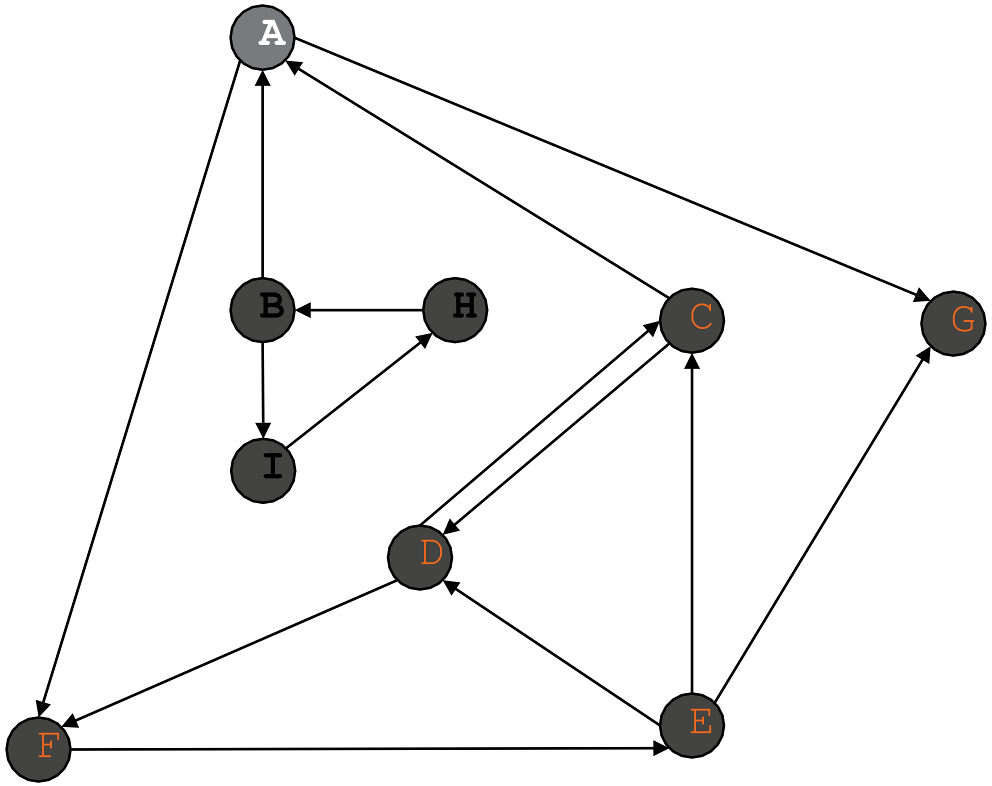


Function call stack:

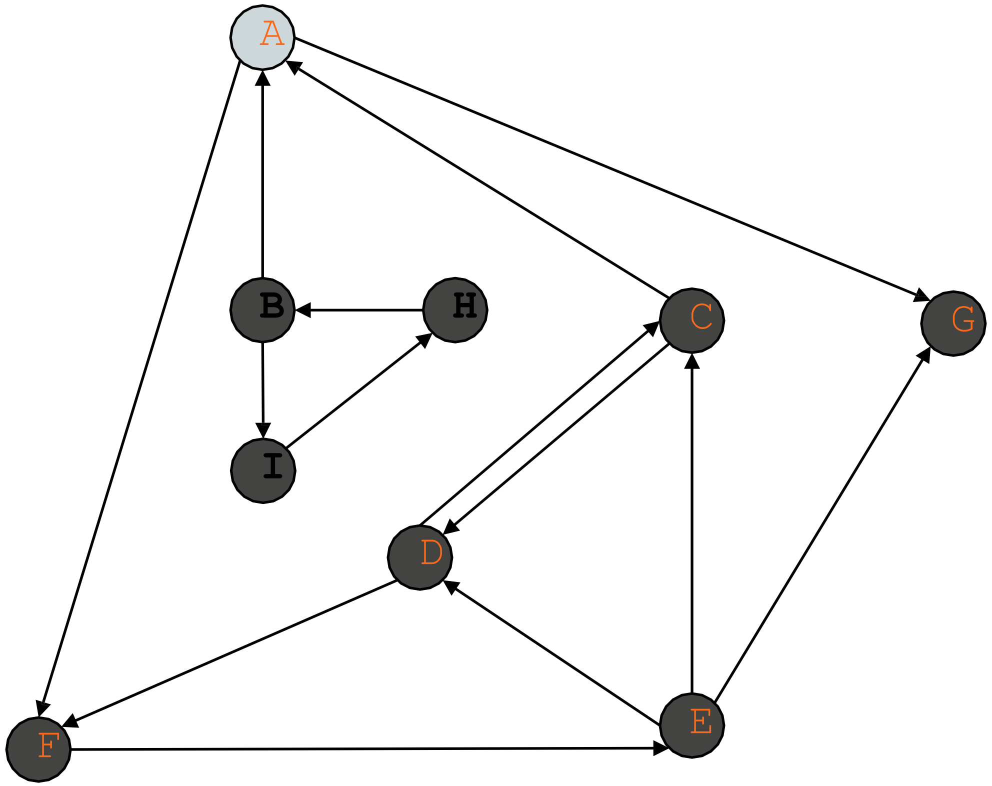




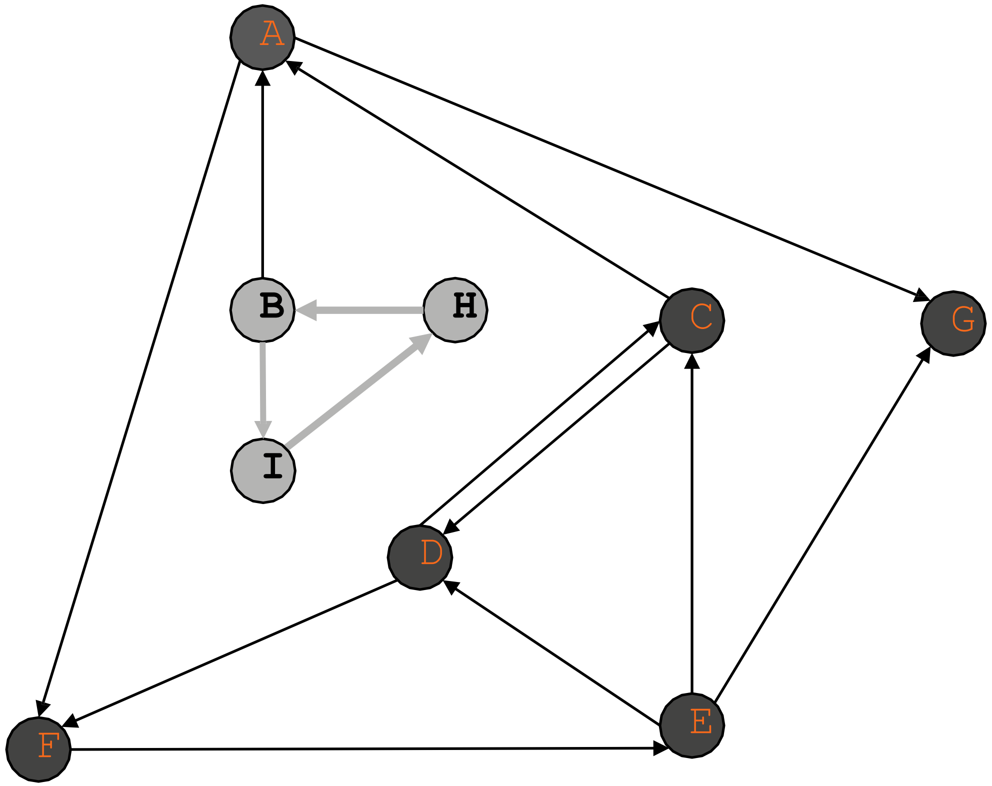
Function call stack:



Function call stack:



Nodes reachable from A: A, C, D, E, F, G



Nodes reachable from A: A, C, D, E, F, G

Running Time of DFS Algorithm

Lines 1-2 and lines 4-5 of DFS take time $\Theta(V)$, exclusive of the time to execute the calls to DFS-VISIT.

The procedure DFS_VISIT is called exactly once for each vertex $v \in V$. During an execution DFS_VISIT(v), the loop on lines 4-7 is executed $|Adj[v]|$ times. Since $\Sigma|Adj[v]| = \Theta(E)$, the total cost of executing lines 2-6 of DFS-VISIT is $\Theta(E)$.

So the total running time of DFS is $\Theta(V+E)$.

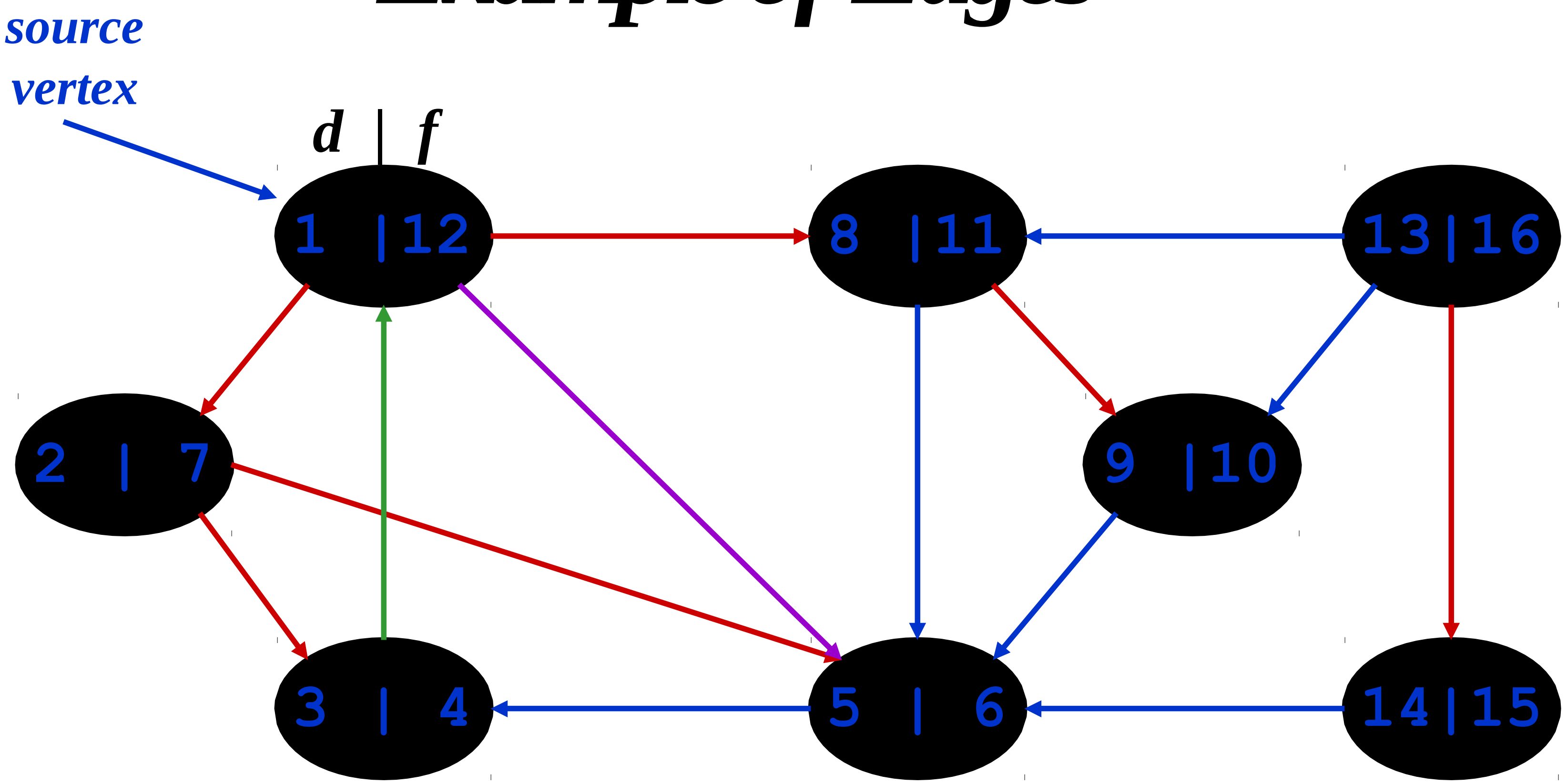
DFS: Application

- Topological Sort
- Strongly Connected Component

Classification of Edges

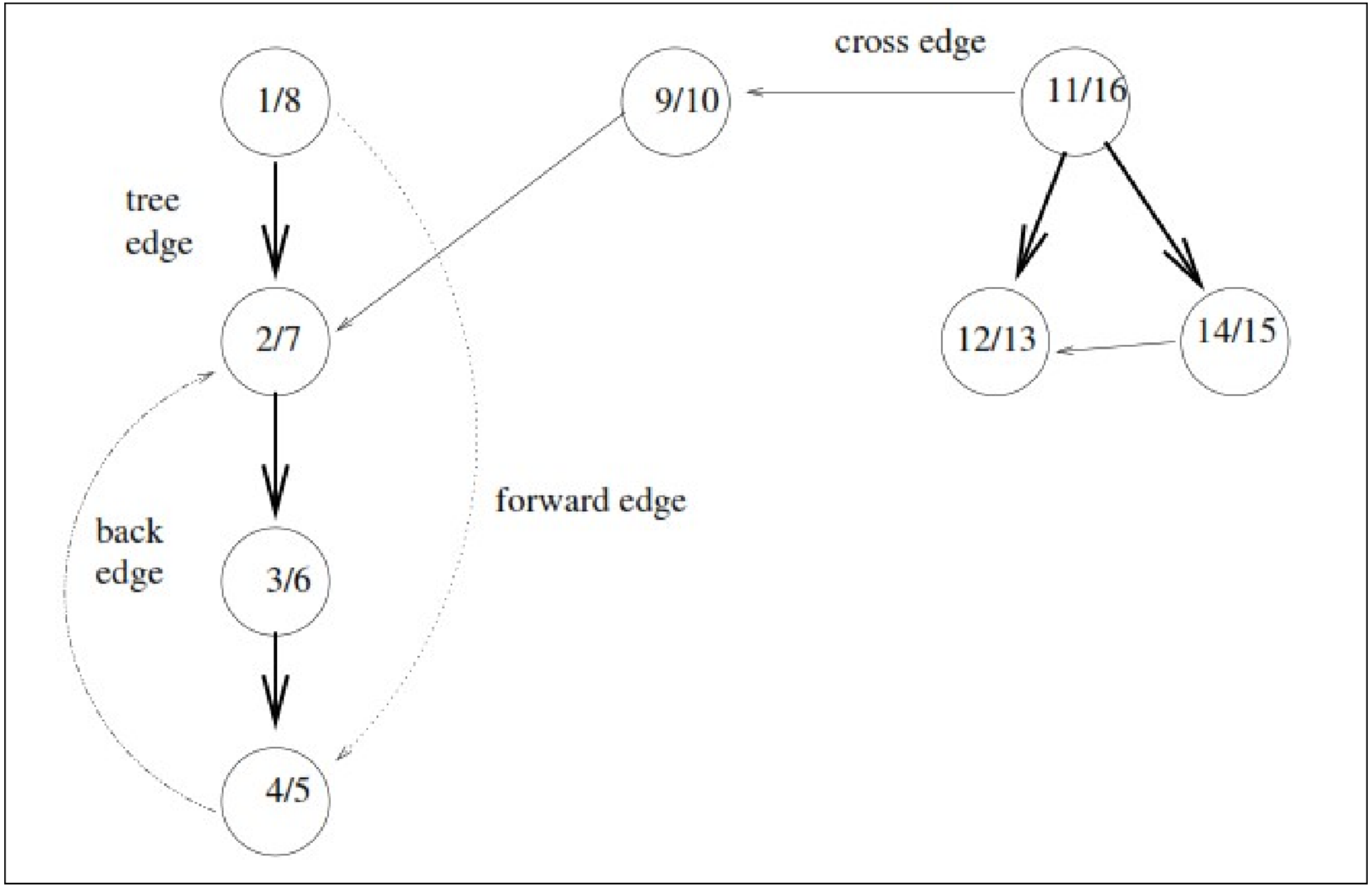
- Another interesting property of DFS is that the DFS can be used to classify the edges of the input graph G=(V,E). The DFS creates depth first forest which can have four types of edges:
- **Tree edge:** Edge (u,v) is a tree edge if **v** was first discovered by exploring edge (u,v). **White** color indicates tree edge.
- **Back edge:** Edge (u,v) is a back edge if it connects a vertex **u** to a ancestor **v** in a depth first tree. **Gray** color indicates back edge.
- **Forward edge:** Edge (u,v) is a forward edge if it is non-tree edge and it connects a vertex **u** to a descendant **v** in a depth first tree. **Black** color indicates forward edge.
- Cross edge: rest all other edges are called the cross edge. **Black** color indicates forward edge.

Example of Edges



Tree edges   Back edges   Forward edges   Cross edges

Example of Edges



Thank you