

# *Longest Common Subsequence(LCS)*

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# *Substring and Subsequence*

A substring of a string S is another string S ' that occurs in S and all the letters are **contiguous** in S

E.g. Amanpreet

substring1 : Aman    substring2 : preet

A subsequence of a string S is another string S ' that occurs in S and all the letters **need not to be contiguous** in S

E.g Amanpreet

subsequence1 : Ant    subsequence 2 : mnet

# *Longest Common Subsequence*

The Longest Common Subsequence (LCS) problem is as follows. We are given two strings: string A of length x and string B of length y. We have to find the longest common subsequence: the longest sequence of characters that appear left-to-right in both strings.

Example, A= KASHMIR

B= CHANDIGARH

Example, A= KASHMIR

B= CHANDIGARH

LCS has 3 length and string is HIR

# ***LONGEST COMMON SUBSEQUENCE***

A subsequence is a sequence that appears in the same relative order, but not necessarily contiguous.

In LSC, we have to find Longest common Subsequence that is in same relative order.

String of length  $n$  has  $2^n$  different possible subsequences.

E.g.--

Subsequences of "ABCDEFGH".

"ABC", "ABG", "BDF", "AEG", "ACEFG", ...

## ***EXAMPLE***

LCS for input Sequences “ABCDGH” and “AEDFHR” is “ADH” of length 3.

LCS for input Sequences “AGGTAB” and “GXTXAYB” is “GTAB” of length 4.

# ***LONGEST COMMON SUBSEQUENCE***

Let the input sequences be  $X[0..m-1]$  and  $Y[0..n-1]$  of lengths  $m$  and  $n$  respectively.

let  $L(X[0..m-1], Y[0..n-1])$  be the length of LCS of the two sequences  $X$  and  $Y$ .

If last characters of both sequences match (or  $X[m-1] == Y[n-1]$ ) then  
 $\rightarrow L(X[0..m-1], Y[0..n-1]) = 1 + L(X[0..m-2], Y[0..n-2])$

If last characters of both sequences do not match (or  $X[m-1] != Y[n-1]$ ) then

$\rightarrow L(X[0..m-1], Y[0..n-1]) = \text{MAX} ( L(X[0..m-2], Y[0..n-1]), L(X[0..m-1], Y[0..n-2]) )$

# *LONGEST COMMON SUBSEQUENCE*

Consider the input strings “ABCDGH” and “AEDFHR”.

Last characters **do not match** for the strings. So length of LCS can be written as:

$$L(\text{“ABCDGH”, “AEDFHR”}) = \text{MAX} ( L(\text{“ABCDG”, “AEDFHR”}), L(\text{“ABCDGH”, “AEDFH”}) )$$

# EXAMPLE

Following is a partial recursion tree for input strings “AXYT” and “AYZX”

```

                    lcs("AXYT", "AYZX")
                   /      \
          lcs("AXY", "AYZX")  lcs("AXYT", "AYZ")
         /      \          /      \
lcs("AX", "AYZX") lcs("AXY", "AYZ") lcs("AXY", "AYZ") lcs("AXYT", "AY")
```



# ***EXAMPLE***

X=M,Z,J,A,W,X,U

Y=X,M,J,Y,A,U,Z

And Longest Common Subsequence is,

LCS(X,Y)=M,J,A,U

Now we will see table from which we can find LCS of Above sequences.

# *Brute Force Method*

Given two strings X of length m and Y of length n, find a longest subsequence common to both X and Y

STEP1 : Find all subsequences of 'X'.

STEP2: For each subsequence, find whether it is a subsequence of 'Y'.

STEP3: Find the longest common subsequence from available subsequences

# *Brute Force Method*

Given two strings X of length m and Y of length n, find a longest subsequence common to both X and Y

STEP1 : Find all subsequences of 'X'.  $2^m$

STEP2: For each subsequence, find whether it is a subsequence of 'Y'.  $n * 2^m$

STEP3: Find the longest common subsequence from available subsequences.

$$\text{T.C} = O(n2^m)$$


**To improve time complexity, we use dynamic programming**

# *Dynamic Programming*

## **Optimal substructure**

We have two strings

$$X = \{x_1, x_2, x_3, \dots, x_n\}$$

$$Y = \{y_1, y_2, y_3, \dots, y_m\}$$


- First compare  $x_n$  and  $y_m$ . If they matched, find the subsequence in the remaining string and then append the  $x_n$  with it.
- If  $x_n \neq y_m$ ,
  - Remove  $x_n$  from  $X$  and find LCS from  $x_1$  to  $x_{n-1}$  and  $y_1$  to  $y_m$
  - Remove  $y_m$  from  $Y$  and find LCS from  $x_1$  to  $x_n$  and  $y_1$  to  $y_{m-1}$

In each step, we reduce the size of the problem into the subproblems  
. It is **optimal substructure**.

# *Cont.*

## Recursive Equation

$$X = \{x_1, x_2, x_3, \dots, x_n\}$$

$$Y = \{y_1, y_2, y_3, \dots, y_m\}$$

$C[i, j]$  is length of LCS in  $X$  and  $Y$

$$c[i, j] = \begin{cases} 0 & ; i=0 \text{ or } j=0 \\ 1+c[i-1, j-1] & ; i, j>0 \text{ and } x_i=y_i \\ \max(c[i-1, j], c[i, j-1]) & ; i, j>0 \text{ and } x_i \neq y_i \end{cases}$$

# *CONDITIONS FOR ARROWS*

```
if  $x_i = y_j$ 
  then  $c[i, j] \leftarrow c[i - 1, j - 1] + 1$ 
       $b[i, j] \leftarrow \nwarrow$ 
else if  $c[i - 1, j] \geq c[i, j - 1]$ 
  then  $c[i, j] \leftarrow c[i - 1, j]$ 
       $b[i, j] \leftarrow \uparrow$ 
else  $c[i, j] \leftarrow c[i, j - 1]$ 
       $b[i, j] \leftarrow \leftarrow$ 
```

# *LCS Example*

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

		$Y_j$						
		0	1	2	3	4	5	6
$X_i$			B	D	C	A	B	A
0								
1	A							
2	B							
3	C							
4	B							
5	D							
6	A							
7	B							

# *LCS Example*

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

		$Y_j$ ↓	0	1	2	3	4	5	6
$X_i$ →				B	D	C	A	B	A
0			0	0	0	0	0	0	0
1	A		0						
2	B		0						
3	C		0						
4	B		0						
5	D		0						
6	A		0						
7	B		0						



# *LCS Example*

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

		$Y_j$ ↓						
		0	1	2	3	4	5	6
$X_i$ →			B	D	C	A	B	A
0		0	0	0	0	0	0	0
1	A	0						
2	B	0						
3	C	0						
4	B	0						
5	D	0						
6	A	0						
7	B	0						

$Z[i,j]$   
Here,  $i=1, j=1$   
 $\therefore Z[1, 1]$

# LCS Example

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

		$Y_j$						
		↓						
		0	1	2	3	4	5	6
$X_i$	→	$X_i$	B	D	C	A	B	A
0	$Y_j$	0	0	0	0	0	0	0
1	A	0						
2	B	0						
3	C	0						
4	B	0						
5	D	0						
6	A	0						
7	B	0						

Maximum of  
two box  
 $Z[i-1, j]$  and  
 $Z[i, j-1]$

X Y  
B A

Not Match

If the values are same we can take  
max from any one [ Either ↑ or ← ]  
But choose any one 1 direction for  
the whole table.

Arrow indicates from where you take the  
maximum

# LCS Example

		$Y_j$ ↓						
		0	1	2	3	4	5	6
$X_i$ →		$X_i$	B	D	C	A	B	A
0	$Y_j$	0	0	0	0	0	0	0
1	A	0	0 ↑					
2	B	0						
3	C	0						
4	B	0						
5	D	0						
6	A	0						
7	B	0						

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

X	Y	Max	Arrow
B	A	0	↑

Not Match

Let's take from upper one [ ↑ ]

Arrow indicates  
from where you take  
the maximum

# LCS Example

		$Y_j$ ↓						
$X_i$ →		0	1	2	3	4	5	6
	$X_i$		B	D	C	A	B	A
0	$Y_j$	0	0	0	0	0	0	0
1	A	0	0 ↑	0 ↑				
2	B	0						
3	C	0						
4	B	0						
5	D	0						
6	A	0						
7	B	0						

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

X	Y	Max	Arrow
D	A	0	↑

Not Match

Let's take from upper one [ ↑ ]

Arrow indicates  
from where you take  
the maximum

# LCS Example

		$Y_j$ ↓						
$X_i$ →		0	1	2	3	4	5	6
	$X_i$		B	D	C	A	B	A
0	$Y_j$	0	0	0	0	0	0	0
1	A	0	0 ↑	0 ↑	0 ↑			
2	B	0						
3	C	0						
4	B	0						
5	D	0						
6	A	0						
7	B	0						

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

X	Y	Max	Arrow
C	A	0	↑

Not Match

Let's take from upper one [ ↑ ]

Arrow indicates  
from where you take  
the maximum

# LCS Example

		$Y_j$						
		↓						
		0	1	2	3	4	5	6
$X_i$ →		$X_i$	B	D	C	A	B	A
0	$Y_j$	0	0	0	0	0	0	0
1	A	0	0 ↑	0 ↑	0 ↑			
2	B	0						
3	C	0						
4	B	0						
5	D	0						
6	A	0						
7	B	0						

Value =  
 $Z[i-1, j-1] + 1$

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

X	Y	Max	Arrow
A	A		↖

Match

If the character matches then,  
 $\text{value} = Z[i-1, j-1] + 1$   
 Arrow [ ↖ ]

Arrow indicates  
 from where you take  
 the maximum

# LCS Example

		$Y_j$ ↓						
		0	1	2	3	4	5	6
$X_i$ →		$X_i$	B	D	C	A	B	A
0	$Y_j$	0	0	0	0	0	0	0
1	A	0	0 ↑	0 ↑	0 ↑	1 ↖		
2	B	0						
3	C	0						
4	B	0						
5	D	0						
6	A	0						
7	B	0						

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

X	Y	Max	Arrow
A	A		↖

Match

If the character matches then,  
 $\text{value} = Z[i-1, j-1] + 1$   
 Arrow [ ↖ ]

Arrow indicates  
 from where you take  
 the maximum

# LCS Example

		$Y_j$ ↓	0	1	2	3	4	5	6
$X_i$ →		$X_i$	B	D	C	A	<b>B</b>	A	
0	$Y_j$	0	0	0	0	0	0	0	
1	<b>A</b>	0	0 ↑	0 ↑	0 ↑	1 ↖	1 ←		
2	B	0							
3	C	0							
4	B	0							
5	D	0							
6	A	0							
7	B	0							

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

X	Y	Max	Arrow
B	A	1	←

**Not Match**

Arrow towards the max value  
 Arrow [ ← ]  
 Value = max one

Arrow indicates  
 from where you take  
 the maximum



# LCS Example

		$Y_j$ ↓	0	1	2	3	4	5	6
$X_i$ →			$X_i$	B	D	C	A	B	A
0	$Y_j$	0	0	0	0	0	0	0	0
1	A	0	0 ↑	0 ↑	0 ↑	1 ↘	1 ←	1 ↘	
2	B	0							
3	C	0							
4	B	0							
5	D	0							
6	A	0							
7	B	0							

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

X	Y	Max	Arrow
A	A		↖

Match

If the character matches then,  
 $\text{value} = Z[i-1, j-1] + 1$   
 Arrow [ ↖ ]

Arrow indicates  
 from where you take  
 the maximum

# LCS Example

		$Y_j$						
		↓						
		0	1	2	3	4	5	6
$X_i$	→	$X_i$	B	D	C	A	B	A
0	$Y_j$	0	0	0	0	0	0	0
1	A	0	0 ↑	0 ↑	0 ↑	1 ↖	1 ←	1 ↖
2	B	0	1 ↖					
3	C	0						
4	B	0						
5	D	0						
6	A	0						
7	B	0						

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

X	Y	Max	Arrow
B	B		↖

Match

If the character matches then,  
 $\text{value} = Z[i-1, j-1] + 1$   
 Arrow [ ↖ ]

Arrow indicates  
 from where you take  
 the maximum

# LCS Example

		$Y_j$						
		↓						
		0	1	2	3	4	5	6
$X_i$	→	$X_i$	B	D	C	A	B	A
0	$Y_j$	0	0	0	0	0	0	0
1	A	0	0 ↑	0 ↑	0 ↑	1 ↖	1 ←	1 ↖
2	B	0	1 ↖	1 ←				
3	C	0						
4	B	0						
5	D	0						
6	A	0						
7	B	0						

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

X	Y	Max	Arrow
D	B	1	←

Not Match

Arrow towards the max value  
 Arrow [ ← ]  
 Value = max one

Arrow indicates  
 from where you take  
 the maximum

# LCS Example

		$Y_j$	0	1	2	3	4	5	6
$X_i$		$X_i$	B	D	C	A	B	A	
0	$Y_j$	0	0	0	0	0	0	0	
1	A	0	0 ↑	0 ↑	0 ↑	1 ↖	1 ←	1 ↖	
2	B	0	1 ↖	1 ←	1 ←				
3	C	0							
4	B	0							
5	D	0							
6	A	0							
7	B	0							

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

X	Y	Max	Arrow
C	B	1	←

Not Match

Arrow towards the max value  
 Arrow [ ← ]  
 Value = max one

Arrow indicates  
 from where you take  
 the maximum

# LCS Example

		$Y_j$	0	1	2	3	4	5	6
$X_i$		$X_i$	B	D	C	A	B	A	
0	$Y_j$	0	0	0	0	0	0	0	
1	A	0	0 ↑	0 ↑	0 ↑	1 ↙	1 ←	1 ↙	
2	B	0	1 ↙	1 ←	1 ←	1 ↑			
3	C	0							
4	B	0							
5	D	0							
6	A	0							
7	B	0							

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

X	Y	Max	Arrow
A	B	1	↑

Not Match

Taking from upper one for same value [ ↑ ]

Arrow indicates from where you take the maximum

# LCS Example

		$Y_j$ ↓	0	1	2	3	4	5	6
$X_i$ →			$X_i$	B	D	C	A	B	A
0		$Y_j$	0	0	0	0	0	0	0
1		A	0	0 ↑	0 ↑	0 ↑	1 ↖	1 ←	1 ↖
2		B	0	1 ↖	1 ←	1 ←	1 ↑	2 ↖	
3		C	0						
4		B	0						
5		D	0						
6		A	0						
7		B	0						

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

X	Y	Max	Arrow
B	B		↖

Match

If the character matches then,  
 $\text{value} = Z[i-1, j-1] + 1$   
 Arrow [ ↖ ]

Arrow indicates  
 from where you take  
 the maximum

# LCS Example

		$Y_j$ ↓	0	1	2	3	4	5	6
$X_i$ →			$X_i$	B	D	C	A	B	A
0	$Y_j$	0	0	0	0	0	0	0	0
1	A	0	0 ↑	0 ↑	0 ↑	1 ↖	← 1	1 ↖	
2	B	0	1 ↖	← 1	← 1	1 ↑	2 ↖	← 2	
3	C	0							
4	B	0							
5	D	0							
6	A	0							
7	B	0							

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

X	Y	Max	Arrow
A	B	2	←

Not Match

Arrow towards the max value  
 Arrow [ ← ]  
 Value = max one

Arrow indicates  
 from where you take  
 the maximum

# LCS Example

		$Y_j$ ↓						
		0	1	2	3	4	5	6
$X_i$ →		$X_i$	B	D	C	A	B	A
0	$Y_j$	0	0	0	0	0	0	0
1	A	0	0 ↑	0 ↑	0 ↑	1 ↖	1 ←	1 ↖
2	B	0	1 ↖	1 ←	1 ←	1 ↑	2 ↖	2 ←
3	C	0	1 ↑					
4	B	0						
5	D	0						
6	A	0						
7	B	0						

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

X	Y	Max	Arrow
B	C	1	↑

Not Match

Arrow towards the max value  
 Arrow [ ↑ ]  
 Value = max one

Arrow indicates  
 from where you take  
 the maximum



# LCS Example

		$Y_j$ ↓	0	1	2	3	4	5	6
$X_i$ →			$X_i$	B	D	C	A	B	A
0	$Y_j$	0	0	0	0	0	0	0	0
1	A	0	0 ↑	0 ↑	0 ↑	1 ↖	1 ←	1 ↖	
2	B	0	1 ↖	1 ←	1 ←	1 ↑	2 ↖	2 ←	
3	C	0	1 ↑	1 ↑					
4	B	0							
5	D	0							
6	A	0							
7	B	0							

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

X	Y	Max	Arrow
D	C	1	↑

Not Match

Taking from upper one for same value [ ↑ ]

Arrow indicates from where you take the maximum

# LCS Example

		$Y_j$						
		↓						
		0	1	2	3	4	5	6
$X_i$ →		$X_i$	B	D	C	A	B	A
0	$Y_j$	0	0	0	0	0	0	0
1	A	0	0 ↑	0 ↑	0 ↑	1 ↖	1 ←	1 ↖
2	B	0	1 ↖	1 ←	1 ←	1 ↑	2 ↖	2 ←
3	C	0	1 ↑	1 ↑	2 ↖			
4	B	0						
5	D	0						
6	A	0						
7	B	0						

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

X	Y	Max	Arrow
C	C		↖

Match

If the character matches then,  
 $\text{value} = Z[i-1, j-1] + 1$   
 Arrow [ ↖ ]

Arrow indicates  
 from where you take  
 the maximum

# LCS Example

		$Y_j$ ↓						
		0	1	2	3	4	5	6
$X_i$ →		$X_i$	B	D	C	A	B	A
0	$Y_j$	0	0	0	0	0	0	0
1	A	0	0 ↑	0 ↑	0 ↑	1 ↖	1 ←	1 ↖
2	B	0	1 ↖	1 ←	1 ←	1 ↑	2 ↖	2 ←
3	C	0	1 ↑	1 ↑	2 ↖	2 ←		
4	B	0						
5	D	0						
6	A	0						
7	B	0						

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

X	Y	Max	Arrow
A	C	2	←

**Not Match**

Arrow towards the max value  
 Arrow [ ← ]  
 Value = max one

Arrow indicates  
 from where you take  
 the maximum

# LCS Example

		$Y_j$						
		↓						
		0	1	2	3	4	5	6
$X_i$ →		$X_i$	B	D	C	A	B	A
0	$Y_j$	0	0	0	0	0	0	0
1	A	0	0 ↑	0 ↑	0 ↑	1 ↖	1 ←	1 ↖
2	B	0	1 ↖	1 ←	1 ←	1 ↑	2 ↖	2 ←
3	C	0	1 ↑	1 ↑	2 ↖	2 ←	2 ↑	
4	B	0						
5	D	0						
6	A	0						
7	B	0						

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

X	Y	Max	Arrow
B	C	2	↑

Not Match

Taking from upper one for same value [ ↑ ]

Arrow indicates from where you take the maximum

# LCS Example

		$Y_j$ ↓						
		0	1	2	3	4	5	6
$X_i$ →		$X_i$	B	D	C	A	B	A
0	$Y_j$	0	0	0	0	0	0	0
1	A	0	0↑	0↑	0↑	1↖	1←	1↖
2	B	0	1↖	1←	1←	1↑	2↖	2←
3	C	0	1↑	1↑	2↖	2←	2↑	2↑
4	B	0						
5	D	0						
6	A	0						
7	B	0						

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

X	Y	Max	Arrow
A	C	2	↑

Not Match

Taking from upper one for same value [↑]

Arrow indicates from where you take the maximum

# LCS Example

		$Y_j$						
		↓						
		0	1	2	3	4	5	6
$X_i$	→	$X_i$	B	D	C	A	B	A
0	$Y_j$	0	0	0	0	0	0	0
1	A	0	0 ↑	0 ↑	0 ↑	1 ↖	1 ←	1 ↖
2	B	0	1 ↖	1 ←	1 ←	1 ↑	2 ↖	2 ←
3	C	0	1 ↑	1 ↑	2 ↖	2 ←	2 ↑	2 ↑
4	B	0	1 ↖					
5	D	0						
6	A	0						
7	B	0						

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

X	Y	Max	Arrow
B	B		↖

Match

If the character matches then,  
 $value = Z[i-1, j-1] + 1$   
 Arrow [ ↖ ]

Arrow indicates  
 from where you take  
 the maximum

# LCS Example

		$Y_j$						
		↓						
		0	1	2	3	4	5	6
$X_i$	→	$X_i$	B	D	C	A	B	A
0	$Y_j$	0	0	0	0	0	0	0
1	A	0	0 ↑	0 ↑	0 ↑	1 ↖	1 ←	1 ↖
2	B	0	1 ↖	1 ←	1 ←	1 ↑	2 ↖	2 ←
3	C	0	1 ↑	1 ↑	2 ↖	2 ←	2 ↑	2 ↑
4	B	0	1 ↖	1 ↑				
5	D	0						
6	A	0						
7	B	0						

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

X	Y	Max	Arrow
D	B		↑

Not Match

Taking from upper one for same value [ ↑ ]

Arrow indicates from where you take the maximum

# LCS Example

		$Y_j$ ↓	0	1	2	3	4	5	6
$X_i$ →			$X_i$	B	D	C	A	B	A
0	$Y_j$	0	0	0	0	0	0	0	0
1	A	0	0 ↑	0 ↑	0 ↑	1 ↖	1 ←	1 ↖	
2	B	0	1 ↖	1 ←	1 ←	1 ↑	2 ↖	2 ←	
3	C	0	1 ↑	1 ↑	2 ↖	2 ←	2 ↑	2 ↑	
4	B	0	1 ↖	1 ↑	2 ↑				
5	D	0							
6	A	0							
7	B	0							

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

X	Y	Max	Arrow
C	B	2	↑

Not Match

Arrow towards the max value  
 Arrow [ ↑ ]  
 Value = max one

Arrow indicates  
 from where you take  
 the maximum



# LCS Example

		$Y_j$ ↓	0	1	2	3	4	5	6
$X_i$ →		$X_i$	B	D	C	A	B	A	
0	$Y_j$	0	0	0	0	0	0	0	
1	A	0	0 ↑	0 ↑	0 ↑	1 ↖	1 ←	1 ↖	
2	B	0	1 ↖	1 ←	1 ←	1 ↑	2 ↖	2 ←	
3	C	0	1 ↑	1 ↑	2 ↖	2 ←	2 ↑	2 ↑	
4	B	0	1 ↖	1 ↑	2 ↑	2 ↑			
5	D	0							
6	A	0							
7	B	0							

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

X	Y	Max	Arrow
A	B		↑

Not Match

Taking from upper one for same value [ ↑ ]

Arrow indicates from where you take the maximum

# LCS Example

		$Y_j$						
		↓						
		0	1	2	3	4	5	6
$X_i$	→	$X_i$	B	D	C	A	B	A
0		$Y_j$	0	0	0	0	0	0
1		A	0	0 ↑	0 ↑	0 ↑	1 ↖	1 ↖
2		B	0	1 ↖	1 ←	1 ←	1 ↑	2 ↖
3		C	0	1 ↑	1 ↑	2 ↖	2 ↑	2 ↑
4		B	0	1 ↖	1 ↑	2 ↑	2 ↑	3 ↖
5		D	0					
6		A	0					
7		B	0					

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

X	Y	Max	Arrow
B	B		↖

Match

If the character matches then,  
 $\text{value} = Z[i-1, j-1] + 1$   
 Arrow [ ↖ ]

Arrow indicates  
 from where you take  
 the maximum

# LCS Example

		$Y_j$ ↓						
		0	1	2	3	4	5	6
$X_i$ →		$X_i$	B	D	C	A	B	A
0	$Y_j$	0	0	0	0	0	0	0
1	A	0	0↑	0↑	0↑	1↖	1←	1↖
2	B	0	1↖	1←	1←	1↑	2↖	2←
3	C	0	1↑	1↑	2↖	2←	2↑	2↑
4	B	0	1↖	1↑	2↑	2↑	3↖	3←
5	D	0						
6	A	0						
7	B	0						

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

X	Y	Max	Arrow
A	B	3	←

Not Match

Arrow towards the max value  
 Arrow [ ← ]  
 Value = max one

Arrow indicates  
 from where you take  
 the maximum

# LCS Example

		$Y_j$ ↓						
		0	1	2	3	4	5	6
$X_i$ →		$X_i$	B	D	C	A	B	A
0	$Y_j$	0	0	0	0	0	0	0
1	A	0	0 ↑	0 ↑	0 ↑	1 ↖	1 ←	1 ↖
2	B	0	1 ↖	1 ←	1 ←	1 ↑	2 ↖	2 ←
3	C	0	1 ↑	1 ↑	2 ↖	2 ←	2 ↑	2 ↑
4	B	0	1 ↖	1 ↑	2 ↑	2 ↑	3 ↖	3 ←
5	D	0	1 ↑					
6	A	0						
7	B	0						

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

X	Y	Max	Arrow
B	D	1	↑

Not Match

Arrow towards the max value  
 Arrow [ ↑ ]  
 Value = max one

Arrow indicates  
 from where you take  
 the maximum

# LCS Example

		$Y_j$						
		↓						
		0	1	2	3	4	5	6
$X_i$	→	$X_i$	B	D	C	A	B	A
0	$Y_j$	0	0	0	0	0	0	0
1	A	0	0 ↑	0 ↑	0 ↑	1 ↖	1 ←	1 ↖
2	B	0	1 ↖	1 ←	1 ←	1 ↑	2 ↖	2 ←
3	C	0	1 ↑	1 ↑	2 ↖	2 ←	2 ↑	2 ↑
4	B	0	1 ↖	1 ↑	2 ↑	2 ↑	3 ↖	3 ←
5	D	0	1 ↑	2 ↖				
6	A	0						
7	B	0						

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

X	Y	Max	Arrow
D	D		↖

Match

If the character matches then,  
 $\text{value} = Z[i-1, j-1] + 1$   
 Arrow [ ↖ ]

Arrow indicates  
 from where you take  
 the maximum

# LCS Example

		$Y_j$						
		↓						
		0	1	2	3	4	5	6
$X_i$ →		$X_i$	B	D	C	A	B	A
0	$Y_j$	0	0	0	0	0	0	0
1	A	0	0 ↑	0 ↑	0 ↑	1 ↖	1 ←	1 ↖
2	B	0	1 ↖	1 ←	1 ←	1 ↑	2 ↖	2 ←
3	C	0	1 ↑	1 ↑	2 ↖	2 ←	2 ↑	2 ↑
4	B	0	1 ↖	1 ↑	2 ↑	2 ↑	3 ↖	3 ←
5	D	0	1 ↑	2 ↖	2 ↑			
6	A	0						
7	B	0						

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

X	Y	Max	Arrow
C	D	2	↑

Not Match

Taking from upper one for same value [ ↑ ]

Arrow indicates from where you take the maximum

# LCS Example

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

		$Y_j$ ↓	0	1	2	3	4	5	6
$X_i$ →		$X_i$	B	D	C	A	B	A	
0	$Y_j$	0	0	0	0	0	0	0	
1	A	0	0↑	0↑	0↑	1↖	1←	1↖	
2	B	0	1↖	1←	1←	1↑	2↖	2←	
3	C	0	1↑	1↑	2↖	2←	2↑	2↑	
4	B	0	1↖	1↑	2↑	2↑	3↖	3←	
5	D	0	1↑	2↖	2↑	2↑			
6	A	0							
7	B	0							

X Y Max Arrow

A D 2 ↑

Not Match

Taking from upper one for same value [ ↑ ]

Arrow indicates from where you take the maximum

# LCS Example

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

		$Y_j$						
		↓						
		0	1	2	3	4	5	6
$X_i$	→	$X_i$	B	D	C	A	B	A
0		$Y_j$	0	0	0	0	0	0
1	A	0	0 ↑	0 ↑	0 ↑	1 ↖	1 ←	1 ↖
2	B	0	1 ↖	1 ←	1 ←	1 ↑	2 ↖	2 ←
3	C	0	1 ↑	1 ↑	2 ↖	2 ←	2 ↑	2 ↑
4	B	0	1 ↖	1 ↑	2 ↑	2 ↑	3 ↖	3 ←
5	D	0	1 ↑	2 ↖	2 ↑	2 ↑	3 ↑	
6	A	0						
7	B	0						

X	Y	Max	Arrow
A	D		↑

**Not Match**

Arrow towards the max value  
 Arrow [ ↑ ]  
 Value = max one

Arrow indicates  
 from where you take  
 the maximum



# LCS Example

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

		$Y_j$ ↓	0	1	2	3	4	5	6
$X_i$ →		$X_i$	B	D	C	A	B	A	
0	$Y_j$	0	0	0	0	0	0	0	
1	A	0	0 ↑	0 ↑	0 ↑	1 ↖	1 ←	1 ↖	
2	B	0	1 ↖	1 ←	1 ←	1 ↑	2 ↖	2 ←	
3	C	0	1 ↑	1 ↑	2 ↖	2 ←	2 ↑	2 ↑	
4	B	0	1 ↖	1 ↑	2 ↑	2 ↑	3 ↖	3 ←	
5	D	0	1 ↑	2 ↖	2 ↑	2 ↑	3 ↑	3 ↑	
6	A	0							
7	B	0							

X Y Max Arrow

A D 3 ↑

Not Match

Taking from upper one for same value [ ↑ ]

Arrow indicates from where you take the maximum

# LCS Example

		$Y_j$ ↓	0	1	2	3	4	5	6
$X_i$ →			$X_i$	B	D	C	A	B	A
0	$Y_j$	0	0	0	0	0	0	0	0
1	A	0	0 ↑	0 ↑	0 ↑	1 ↖	1 ←	1 ↖	
2	B	0	1 ↖	1 ←	1 ←	1 ↑	2 ↖	2 ←	
3	C	0	1 ↑	1 ↑	2 ↖	2 ←	2 ↑	2 ↑	
4	B	0	1 ↖	1 ↑	2 ↑	2 ↑	3 ↖	3 ←	
5	D	0	1 ↑	2 ↖	2 ↑	2 ↑	3 ↑	3 ↑	
6	A	0	1 ↑						
7	B	0							

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

X	Y	Max	Arrow
B	A	1	↑

Not Match

Arrow towards the max value  
 Arrow [ ↑ ]  
 Value = max one

Arrow indicates  
 from where you take  
 the maximum

# LCS Example

		$Y_j$ ↓	0	1	2	3	4	5	6
$X_i$ →			$X_i$	B	D	C	A	B	A
0	$Y_j$	0	0	0	0	0	0	0	0
1	A	0	0 ↑	0 ↑	0 ↑	1 ↖	1 ←	1 ↖	
2	B	0	1 ↖	1 ←	1 ←	1 ↑	2 ↖	2 ←	
3	C	0	1 ↑	1 ↑	2 ↖	2 ←	2 ↑	2 ↑	
4	B	0	1 ↖	1 ↑	2 ↑	2 ↑	3 ↖	3 ←	
5	D	0	1 ↑	2 ↖	2 ↑	2 ↑	3 ↑	3 ↑	
6	A	0	1 ↑	2 ↑					
7	B	0							

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

X	Y	Max	Arrow
D	A	2	↑

Not Match

Arrow towards the max value  
 Arrow [ ↑ ]  
 Value = max one

Arrow indicates  
 from where you take  
 the maximum

# LCS Example

		$Y_j$ ↓	0	1	2	3	4	5	6
$X_i$ →			$X_i$	B	D	C	A	B	A
0	$Y_j$	0	0	0	0	0	0	0	0
1	A	0	0 ↑	0 ↑	0 ↑	1 ↖	1 ←	1 ↖	
2	B	0	1 ↖	1 ←	1 ←	1 ↑	2 ↖	2 ←	
3	C	0	1 ↑	1 ↑	2 ↖	2 ←	2 ↑	2 ↑	
4	B	0	1 ↖	1 ↑	2 ↑	2 ↑	3 ↖	3 ←	
5	D	0	1 ↑	2 ↖	2 ↑	2 ↑	3 ↑	3 ↑	
6	A	0	1 ↑	2 ↑	2 ↑				
7	B	0							

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

X	Y	Max	Arrow
C	A	2	↑

Not Match

Taking from upper one for same value [ ↑ ]

Arrow indicates from where you take the maximum

# LCS Example

		$Y_j$ ↓	0	1	2	3	4	5	6
$X_i$ →		$X_i$	B	D	C	A	B	A	
0	$Y_j$	0	0	0	0	0	0	0	
1	A	0	0 ↑	0 ↑	0 ↑	1 ↖	1 ←	1 ↖	
2	B	0	1 ↖	1 ←	1 ←	1 ↑	2 ↖	2 ←	
3	C	0	1 ↑	1 ↑	2 ↖	2 ←	2 ↑	2 ↑	
4	B	0	1 ↖	1 ↑	2 ↑	2 ↑	3 ↖	3 ←	
5	D	0	1 ↑	2 ↖	2 ↑	2 ↑	3 ↑	3 ↑	
6	A	0	1 ↑	2 ↑	2 ↑	3 ↖			
7	B	0							

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

X	Y	Max	Arrow
A	A		↖

Match

If the character matches then,  
 $\text{value} = Z[i-1, j-1] + 1$   
 Arrow [ ↖ ]

Arrow indicates  
 from where you take  
 the maximum

# LCS Example

		$Y_j$						
		↓						
		0	1	2	3	4	5	6
$X_i$ →		$X_i$	B	D	C	A	B	A
0	$Y_j$	0	0	0	0	0	0	0
1	A	0	0 ↑	0 ↑	0 ↑	1 ↖	1 ←	1 ↖
2	B	0	1 ↖	1 ←	1 ←	1 ↑	2 ↖	2 ←
3	C	0	1 ↑	1 ↑	2 ↖	2 ←	2 ↑	2 ↑
4	B	0	1 ↖	1 ↑	2 ↑	2 ↑	3 ↖	3 ←
5	D	0	1 ↑	2 ↖	2 ↑	2 ↑	3 ↑	3 ↑
6	A	0	1 ↑	2 ↑	2 ↑	3 ↖	3 ↑	
7	B	0						

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

X	Y	Max	Arrow
B	A		↑

Not Match

Taking from upper one for same value [ ↑ ]

Arrow indicates from where you take the maximum

# LCS Example

		$Y_j$ ↓	0	1	2	3	4	5	6
$X_i$ →		$X_i$	B	D	C	A	B	A	
0	$Y_j$	0	0	0	0	0	0	0	
1	A	0	0 ↑	0 ↑	0 ↑	1 ↖	1 ←	1 ↖	
2	B	0	1 ↖	1 ←	1 ←	1 ↑	2 ↖	2 ←	
3	C	0	1 ↑	1 ↑	2 ↖	2 ←	2 ↑	2 ↑	
4	B	0	1 ↖	1 ↑	2 ↑	2 ↑	3 ↖	3 ←	
5	D	0	1 ↑	2 ↖	2 ↑	2 ↑	3 ↑	3 ↑	
6	A	0	1 ↑	2 ↑	2 ↑	3 ↖	3 ↑	4 ↖	
7	B	0							

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

X	Y	Max	Arrow
A	A		↖

Match

If the character matches then,  
 $\text{value} = Z[i-1, j-1] + 1$   
 Arrow [ ↖ ]

Arrow indicates  
 from where you take  
 the maximum

# LCS Example

		$Y_j$						
		↓						
		0	1	2	3	4	5	6
$X_i$	→	$X_i$	B	D	C	A	B	A
0	$Y_j$	0	0	0	0	0	0	0
1	A	0	0 ↑	0 ↑	0 ↑	1 ↖	1 ←	1 ↖
2	B	0	1 ↖	1 ←	1 ←	1 ↑	2 ↖	2 ←
3	C	0	1 ↑	1 ↑	2 ↖	2 ←	2 ↑	2 ↑
4	B	0	1 ↖	1 ↑	2 ↑	2 ↑	3 ↖	3 ←
5	D	0	1 ↑	2 ↖	2 ↑	2 ↑	3 ↑	3 ↑
6	A	0	1 ↑	2 ↑	2 ↑	3 ↖	3 ↑	4 ↖
7	B	0	1 ↖					

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

X	Y	Max	Arrow
B	B		↖

Match

If the character matches then,  
 $\text{value} = Z[i-1, j-1] + 1$   
 Arrow [ ↖ ]

Arrow indicates  
 from where you take  
 the maximum



# LCS Example

		$Y_j$ ↓						
		0	1	2	3	4	5	6
$X_i$ →		$X_i$	B	D	C	A	B	A
0	$Y_j$	0	0	0	0	0	0	0
1	A	0	0 ↑	0 ↑	0 ↑	1 ↖	1 ←	1 ↖
2	B	0	1 ↖	1 ←	1 ←	1 ↑	2 ↖	2 ←
3	C	0	1 ↑	1 ↑	2 ↖	2 ←	2 ↑	2 ↑
4	B	0	1 ↖	1 ↑	2 ↑	2 ↑	3 ↖	3 ←
5	D	0	1 ↑	2 ↖	2 ↑	2 ↑	3 ↑	3 ↑
6	A	0	1 ↑	2 ↑	2 ↑	3 ↖	3 ↑	4 ↖
7	B	0	1 ↖	2 ↑				

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

X	Y	Max	Arrow
D	B	2	↑

Not Match

Arrow towards the max value  
 Arrow [ ↑ ]  
 Value = max one

Arrow indicates  
 from where you take  
 the maximum

# LCS Example

		$Y_j$ ↓	0	1	2	3	4	5	6
$X_i$ →			$X_i$	B	D	C	A	B	A
0	$Y_j$	0	0	0	0	0	0	0	0
1	A	0	0 ↑	0 ↑	0 ↑	1 ↖	1 ←	1 ↖	
2	B	0	1 ↖	1 ←	1 ←	1 ↑	2 ↖	2 ←	
3	C	0	1 ↑	1 ↑	2 ↖	2 ←	2 ↑	2 ↑	
4	B	0	1 ↖	1 ↑	2 ↑	2 ↑	3 ↖	3 ←	
5	D	0	1 ↑	2 ↖	2 ↑	2 ↑	3 ↑	3 ↑	
6	A	0	1 ↑	2 ↑	2 ↑	3 ↖	3 ↑	4 ↖	
7	B	0	1 ↖	2 ↑	2 ↑				

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

X	Y	Max	Arrow
C	B	2	↑

Not Match

Arrow towards the max value  
 Arrow [ ↑ ]  
 Value = max one

Arrow indicates  
 from where you take  
 the maximum

# LCS Example

		$Y_j$						
		↓						
		0	1	2	3	4	5	6
$X_i$ →		$X_i$	B	D	C	A	B	A
0	$Y_j$	0	0	0	0	0	0	0
1	A	0	0 ↑	0 ↑	0 ↑	1 ↖	1 ←	1 ↖
2	B	0	1 ↖	1 ←	1 ←	1 ↑	2 ↖	2 ←
3	C	0	1 ↑	1 ↑	2 ↖	2 ←	2 ↑	2 ↑
4	B	0	1 ↖	1 ↑	2 ↑	2 ↑	3 ↖	3 ←
5	D	0	1 ↑	2 ↖	2 ↑	2 ↑	3 ↑	3 ↑
6	A	0	1 ↑	2 ↑	2 ↑	3 ↖	3 ↑	4 ↖
7	B	0	1 ↖	2 ↑	2 ↑	3 ↑		

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

X	Y	Max	Arrow
A	B	3	↑

Not Match

Arrow towards the max value  
 Arrow [ ↑ ]  
 Value = max one

Arrow indicates  
 from where you take  
 the maximum

# LCS Example

		$Y_j$ ↓	0	1	2	3	4	5	6
$X_i$ →		$X_i$	B	D	C	A	B	A	
0	$Y_j$	0	0	0	0	0	0	0	0
1	A	0	0 ↑	0 ↑	0 ↑	1 ↖	1 ←	1 ↖	
2	B	0	1 ↖	1 ←	1 ←	1 ↑	2 ↖	2 ←	
3	C	0	1 ↑	1 ↑	2 ↖	2 ←	2 ↑	2 ↑	
4	B	0	1 ↖	1 ↑	2 ↑	2 ↑	3 ↖	3 ←	
5	D	0	1 ↑	2 ↖	2 ↑	2 ↑	3 ↑	3 ↑	
6	A	0	1 ↑	2 ↑	2 ↑	3 ↖	3 ↑	4 ↖	
7	B	0	1 ↖	2 ↑	2 ↑	3 ↑	4 ↖		

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

X	Y	Max	Arrow
B	B		↖

Match

If the character matches then,  
 $\text{value} = Z[i-1, j-1] + 1$   
 Arrow [ ↖ ]

Arrow indicates  
 from where you take  
 the maximum

# LCS Example

		$Y_j$ ↓	0	1	2	3	4	5	6
$X_i$ →			$X_i$	B	D	C	A	B	A
0	$Y_j$	0	0	0	0	0	0	0	0
1	A	0	0 ↑	0 ↑	0 ↑	1 ↖	1 ←	1 ↖	
2	B	0	1 ↖	1 ←	1 ←	1 ↑	2 ↖	2 ←	
3	C	0	1 ↑	1 ↑	2 ↖	2 ←	2 ↑	2 ↑	
4	B	0	1 ↖	1 ↑	2 ↑	2 ↑	3 ↖	3 ←	
5	D	0	1 ↑	2 ↖	2 ↑	2 ↑	3 ↑	3 ↑	
6	A	0	1 ↑	2 ↑	2 ↑	3 ↖	3 ↑	4 ↖	
7	B	0	1 ↖	2 ↑	2 ↑	3 ↑	4 ↖	4 ↑	

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

X	Y	Max	Arrow
B	B		↑

Not Match

Taking from upper one for same value [ ↑ ]

Arrow indicates from where you take the maximum



# *Traceback Approach*

# LCS Example

		$Y_j$ ↓						
		0	1	2	3	4	5	6
$X_i$ →		$X_i$	B	D	C	A	B	A
0	$Y_j$	0	0	0	0	0	0	0
1	A	0	0↑	0↑	0↑	1↖	1←	1↖
2	B	0	1↖	1←	1←	1↑	2↖	2←
3	C	0	1↑	1↑	2↖	2←	2↑	2↑
4	B	0	1↖	1↑	2↑	2↑	3↖	3←
5	D	0	1↑	2↖	2↑	2↑	3↑	3↑
6	A	0	1↑	2↑	2↑	3↖	3↑	4↖
7	B	0	1↖	2↑	2↑	3↑	4↖	4↑

$X=\{A, B, C, B, D, A, B\}$   
 $Y=\{B, D, C, A, B, A\}$

1. Firstly have to point out the highest value.

2. Then for left and up arrow we will just follow the direction.

3. For diagonal arrow we will point out the character for this cell.

# LCS Example

		$Y_j$ ↓	0	1	2	3	4	5	6
$X_i$ →			$X_i$	B	D	C	A	B	A
0	$Y_j$	0	0	0	0	0	0	0	0
1	A	0	0↑	0↑	0↑	1↖	1←	1↖	
2	B	0	1↖	1←	1←	1↑	2↖	2←	
3	C	0	1↑	1↑	2↖	2←	2↑	2↑	
4	B	0	1↖	1↑	2↑	2↑	3↖	3←	
5	D	0	1↑	2↖	2↑	2↑	3↑	3↑	
6	A	0	1↑	2↑	2↑	3↖	3↑	4↖	
7	B	0	1↖	2↑	2↑	3↑	4↖	4↑	

$X=\{A, B, C, B, D, A, B\}$   
 $Y=\{B, D, C, A, B, A\}$

1. Firstly have to point out the highest value.

2. Then for left and up arrow we will just follow the direction.

3. For diagonal arrow we will point out the character for this cell.



# LCS Example

		$Y_j$						
		↓						
		0	1	2	3	4	5	6
$X_i$	→	$X_i$	B	D	C	A	B	A
0	$Y_j$	0	0	0	0	0	0	0
1	A	0	0↑	0↑	0↑	1↖	1←	1↖
2	B	0	1↖	1←	1←	1↑	2↖	2←
3	C	0	1↑	1↑	2↖	2←	2↑	2↑
4	B	0	1↖	1↑	2↑	2↑	3↖	3←
5	D	0	1↑	2↖	2↑	2↑	3↑	3↑
6	A	0	1↑	2↑	2↑	3↖	3↑	4↖
7	B	0	1↖	2↑	2↑	3↑	4↖	4↑

LCS Z = < A >

$X=\{A, B, C, B, D, A, B\}$   
 $Y=\{B, D, C, A, B, A\}$

1. Firstly have to point out the highest value.

2. Then for left and up arrow we will just follow the direction.

3. For diagonal arrow we will point out the character for this cell.

# LCS Example

$X=\{A, B, C, B, D, A, B\}$   
 $Y=\{B, D, C, A, B, A\}$

		$Y_j$						
		$\downarrow$						
		0	1	2	3	4	5	6
$X_i \rightarrow$		$X_i$	B	D	C	A	B	A
0	$Y_j$	0	0	0	0	0	0	0
1	A	0	0 $\uparrow$	0 $\uparrow$	0 $\uparrow$	1 $\swarrow$	1 $\leftarrow$	1 $\swarrow$
2	B	0	1 $\swarrow$	1 $\leftarrow$	1 $\leftarrow$	1 $\uparrow$	2 $\swarrow$	2 $\leftarrow$
3	C	0	1 $\uparrow$	1 $\uparrow$	2 $\swarrow$	2 $\leftarrow$	2 $\uparrow$	2 $\uparrow$
4	B	0	1 $\swarrow$	1 $\uparrow$	2 $\uparrow$	2 $\uparrow$	3 $\swarrow$	3 $\leftarrow$
5	D	0	1 $\uparrow$	2 $\swarrow$	2 $\uparrow$	2 $\uparrow$	3 $\uparrow$	3 $\uparrow$
6	A	0	1 $\uparrow$	2 $\uparrow$	2 $\uparrow$	3 $\swarrow$	3 $\uparrow$	4 $\swarrow$
7	B	0	1 $\swarrow$	2 $\uparrow$	2 $\uparrow$	3 $\uparrow$	4 $\swarrow$	4 $\uparrow$

1. Firstly have to point out the highest value.

2. Then for left and up arrow we will just follow the direction.

3. For diagonal arrow we will point out the character for this cell.

LCS Z = < A >

Character insert in Z from right to left

# LCS Example

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

		$Y_j$						
		0	1	2	3	4	5	6
$X_i$		$X_i$	B	D	C	A	B	A
0	$Y_j$	0	0	0	0	0	0	0
1	A	0	0↑	0↑	0↑	1↖	1←	1↖
2	B	0	1↖	1←	1←	1↑	2↖	2←
3	C	0	1↑	1↑	2↖	2←	2↑	2↑
4	B	0	1↖	1↑	2↑	2↑	3↖	3←
5	D	0	1↑	2↖	2↑	2↑	3↑	3↑
6	A	0	1↑	2↑	2↑	3↖	3↑	4↖
7	B	0	1↖	2↑	2↑	3↑	4↖	4↑

1. Firstly have to point out the highest value.

2. Then for left and up arrow we will just follow the direction.

3. For diagonal arrow we will point out the character for this cell.

LCS  $Z = \langle B A \rangle$

Character insert in  $Z$  from right to left

# LCS Example

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

		$Y_j$ ↓	0	1	2	3	4	5	6
$X_i$ →			$X_i$	B	D	C	A	B	A
0		$Y_j$	0	0	0	0	0	0	0
1	A	0	0↑	0↑	0↑	1↖	1←	1↖	
2	B	0	1↖	1←	1←	1↑	2↖	2←	
3	C	0	1↑	1↑	2↖	2←	2↑	2↑	
4	B	0	1↖	1↑	2↑	2↑	3↖	3←	
5	D	0	1↑	2↖	2↑	2↑	3↑	3↑	
6	A	0	1↑	2↑	2↑	3↖	3↑	4↖	
7	B	0	1↖	2↑	2↑	3↑	4↖	4↑	

1. Firstly have to point out the highest value.

2. Then for left and up arrow we will just follow the direction.

3. For diagonal arrow we will point out the character for this cell.

LCS Z = < B A >

Character insert in Z from right to left

# LCS Example

$X=\{A, B, C, B, D, A, B\}$   
 $Y=\{B, D, C, A, B, A\}$

		$Y_j$ ↓	0	1	2	3	4	5	6
$X_i$ →		$X_i$	B	D	C	A	B	A	
0	$Y_j$	0	0	0	0	0	0	0	
1	A	0	0↑	0↑	0↑	1↖	1←	1↖	
2	B	0	1↖	1←	1←	1↑	2↖	2←	
3	C	0	1↑	1↑	2↖	2←	2↑	2↑	
4	B	0	1↖	1↑	2↑	2↑	3↖	3←	
5	D	0	1↑	2↖	2↑	2↑	3↑	3↑	
6	A	0	1↑	2↑	2↑	3↖	3↑	4↖	
7	B	0	1↖	2↑	2↑	3↑	4↖	4↑	

1. Firstly have to point out the highest value.

2. Then for left and up arrow we will just follow the direction.

3. For diagonal arrow we will point out the character for this cell.

LCS  $Z = \langle C B A \rangle$

Character insert in  $Z$  from right to left

# LCS Example

$X = \{A, B, C, B, D, A, B\}$   
 $Y = \{B, D, C, A, B, A\}$

		$Y_j$ ↓	0	1	2	3	4	5	6
$X_i$ →		$X_i$	B	D	C	A	B	A	
0	$Y_j$	0	0	0	0	0	0	0	
1	A	0	0↑	0↑	0↑	1↖	1←	1↖	
2	B	0	1↖	1←	1←	1↑	2↖	2←	
3	C	0	1↑	1↑	2↖	2←	2↑	2↑	
4	B	0	1↖	1↑	2↑	2↑	3↖	3←	
5	D	0	1↑	2↖	2↑	2↑	3↑	3↑	
6	A	0	1↑	2↑	2↑	3↖	3↑	4↖	
7	B	0	1↖	2↑	2↑	3↑	4↖	4↑	

1. Firstly have to point out the highest value.

2. Then for left and up arrow we will just follow the direction.

3. For diagonal arrow we will point out the character for this cell.

LCS  $Z = \langle C B A \rangle$

Character insert in Z from right to left

# LCS Example

$X=\{A, B, C, B, D, A, B\}$   
 $Y=\{B, D, C, A, B, A\}$

		$Y_j$	0	1	2	3	4	5	6
$X_i$		$X_i$	B	D	C	A	B	A	
0	$Y_j$	0	0	0	0	0	0	0	
1	A	0	0↑	0↑	0↑	1↖	1←	1↖	
2	B	0	1↖	1←	1←	1↑	2↖	2←	
3	C	0	1↑	1↑	2↖	2←	2↑	2↑	
4	B	0	1↖	1↑	2↑	2↑	3↖	3←	
5	D	0	1↑	2↖	2↑	2↑	3↑	3↑	
6	A	0	1↑	2↑	2↑	3↖	3↑	4↖	
7	B	0	1↖	2↑	2↑	3↑	4↖	4↑	

1. Firstly have to point out the highest value.

2. Then for left and up arrow we will just follow the direction.

3. For diagonal arrow we will point out the character for this cell.

LCS  $Z = \langle B C B A \rangle$

Character insert in Z from right to left

# LCS Example

	$Y_j$		0	1	2	3	4	5	6
$X_i$		$X_i$	B	D	C	A	B	A	
0	$Y_j$	0	0	0	0	0	0	0	
1	A	0	0↑	0↑	0↑	1↖	1←	1↖	
2	B	0	1↖	1←	1←	1↑	2↖	2←	
3	C	0	1↑	1↑	2↖	2←	2↑	2↑	
4	B	0	1↖	1↑	2↑	2↑	3↖	3←	
5	D	0	1↑	2↖	2↑	2↑	3↑	3↑	
6	A	0	1↑	2↑	2↑	3↖	3↑	4↖	
7	B	0	1↖	2↑	2↑	3↑	4↖	4↑	

LCS Z = < B C B A > : 4

X={A, B, C, B, D, A, B}  
Y={B, D, C, A, B, A}

1. Firstly have to point out the highest value.

2. Then for left and up arrow we will just follow the direction.

3. For diagonal arrow we will point out the character for this cell.



# LCS Example

		0	1	2	3	4	5	6	7
		∅	M	Z	J	A	W	X	U
0	∅	0	0	0	0	0	0	0	0
1	X	0	0	0	0	0	0	1	1
2	M	0	1	1	1	1	1	1	1
3	J	0	1	1	2	2	2	2	2
4	Y	0	1	1	2	2	2	2	2
5	A	0	1	1	2	3	3	3	3
6	U	0	1	1	2	3	3	3	4
7	Z	0	1	2	2	3	3	3	4

# ***LCS ALGORITHM***

```
LCS-LENGTH( $X, Y$ )
1   $m \leftarrow \text{length}[X]$ 
2   $n \leftarrow \text{length}[Y]$ 
3  for  $i \leftarrow 1$  to  $m$ 
4      do  $c[i, 0] \leftarrow 0$ 
5  for  $j \leftarrow 0$  to  $n$ 
6      do  $c[0, j] \leftarrow 0$ 
7  for  $i \leftarrow 1$  to  $m$ 
8      do for  $j \leftarrow 1$  to  $n$ 
9          do if  $x_i = y_j$ 
10             then  $c[i, j] \leftarrow c[i - 1, j - 1] + 1$ 
11                  $b[i, j] \leftarrow \nwarrow$ 
12             else if  $c[i - 1, j] \geq c[i, j - 1]$ 
13                 then  $c[i, j] \leftarrow c[i - 1, j]$ 
14                      $b[i, j] \leftarrow \uparrow$ 
15                 else  $c[i, j] \leftarrow c[i, j - 1]$ 
16                      $b[i, j] \leftarrow \leftarrow$ 
17  return  $c$  and  $b$ 
```

# *CONSTRUCTING AN LCS*

PRINT-LCS( $b, X, i, j$ )

```
1  if  $i = 0$  or  $j = 0$ 
2      then return
3  if  $b[i, j] = \text{"↖"}$ 
4      then PRINT-LCS( $b, X, i - 1, j - 1$ )
5          print  $x_i$ 
6  elseif  $b[i, j] = \text{"↑"}$ 
7      then PRINT-LCS( $b, X, i - 1, j$ )
8  else PRINT-LCS( $b, X, i, j - 1$ )
```

# *COMPLEXITY*

Complexity of Longest Common Subsequence is  $O(mn)$ .

Where  $m$  and  $n$  are lengths of the two Strings.

