Dijkstra's Algorithm

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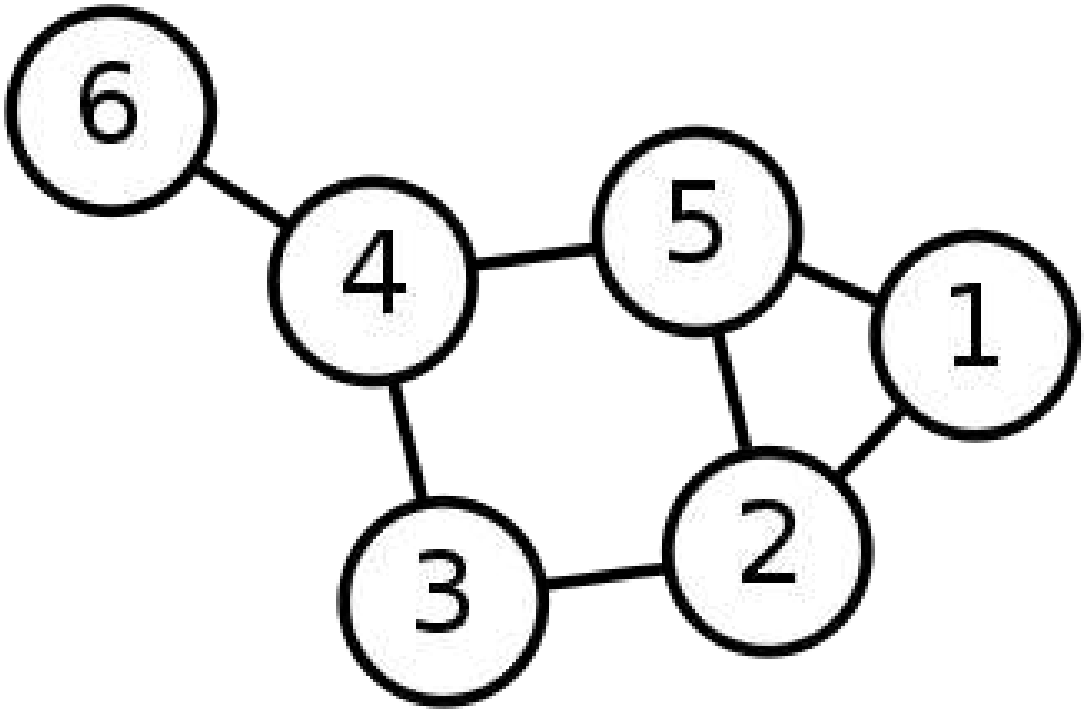
Single-Source Shortest Path Problem

Single-Source Shortest Path Problem - The problem of finding shortest paths from a source vertex *v* to all other vertices in the graph.



Dijkstra's algorithm

Dijkstra's algorithm - is a solution to the single-source shortest path problem in graph theory.

Works on both directed and undirected graphs. However, all edges must have nonnegative weights.

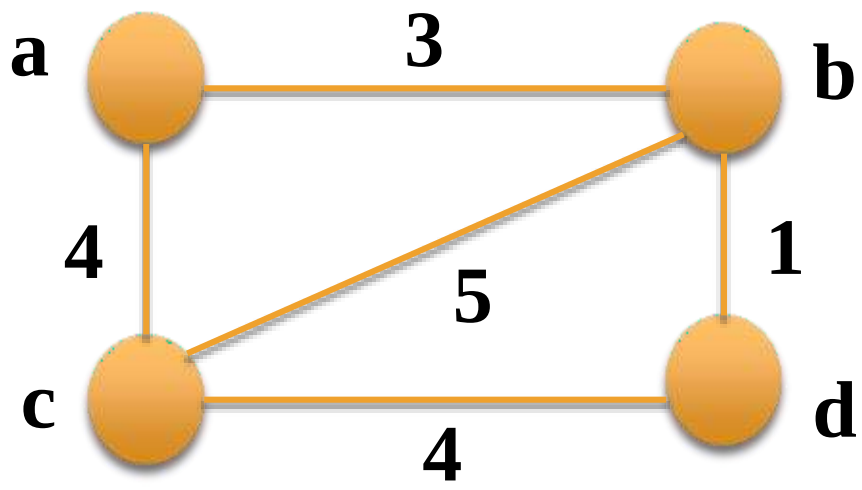
Approach: Greedy

Input: Weighted graph G={E,V} and source vertex $v \in V$, such that all edge weights are nonnegative

Output: Lengths of shortest paths (or the shortest paths themselves) from a given source vertex $v \in V$ to all other vertices

Rules of Dijkstra Algorithm

- It can be applied on both directed and undirected graph.
- It is applied on weighted graph.
- For the algorithm to be applied the graph must be connected.



Rules of Dijkstra Algorithm

Dijkstra's algorithm does not work for graphs with negative weight edges.
For graphs with negative weight edges, Bellman_ford algorithm can be used.

- In Dijkstra's algorithm always assign source vertex to zero distance.
- Assign every other node a tentative distance.

Dijkstra's algorithm - Pseudocode

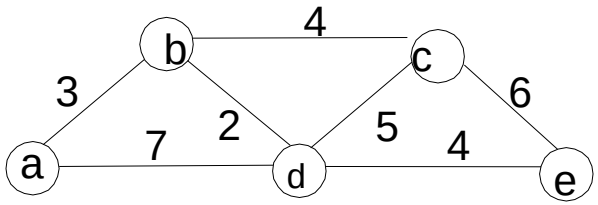
```
DIJKSTRA(G, w, s)
1  INITIALIZE-SINGLE-SOURCE(G, s)
2  S = ∅
3  Q = G.V
4  while Q ≠ ∅
5      u = EXTRACT-MIN(Q)
6      S = S ∪ {u}
7      for each vertex v ∈ G.Adj[u]
8          RELAX(u, v, w)
```

```
INITIALIZE-SINGLE-SOURCE(G, s)
1  for each vertex v ∈ G.V
2      v.d = ∞
3      v.π = NIL
4  s.d = 0
```

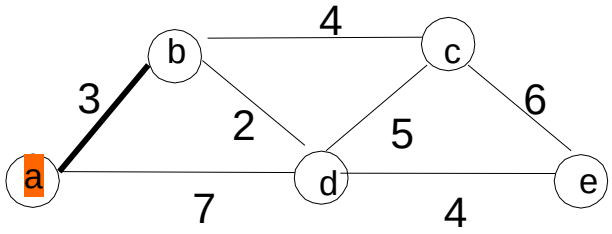
```
RELAX(u, v, w)
1  if v.d > u.d + w(u, v)
2      v.d = u.d + w(u, v)
3      v.π = u
```

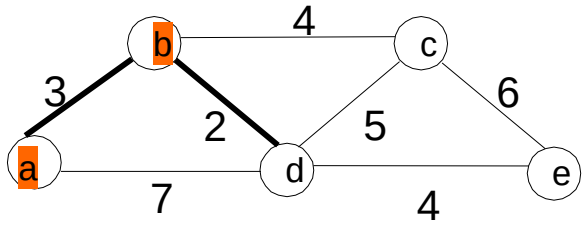
Relaxation. If the new path from s to v is shorter than $d[v]$, then update $d[v]$ to the length of this new path.

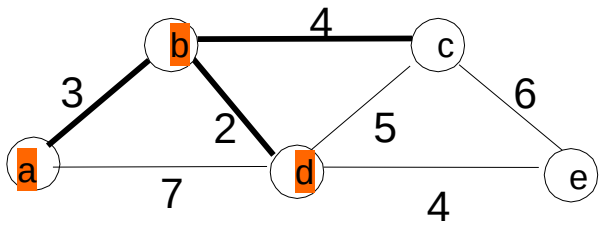
Example

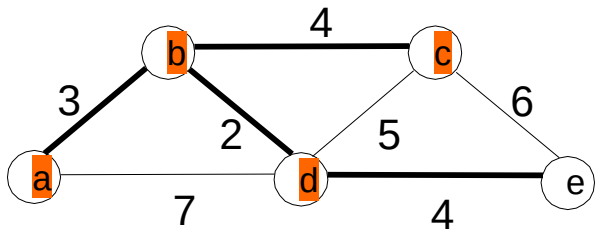


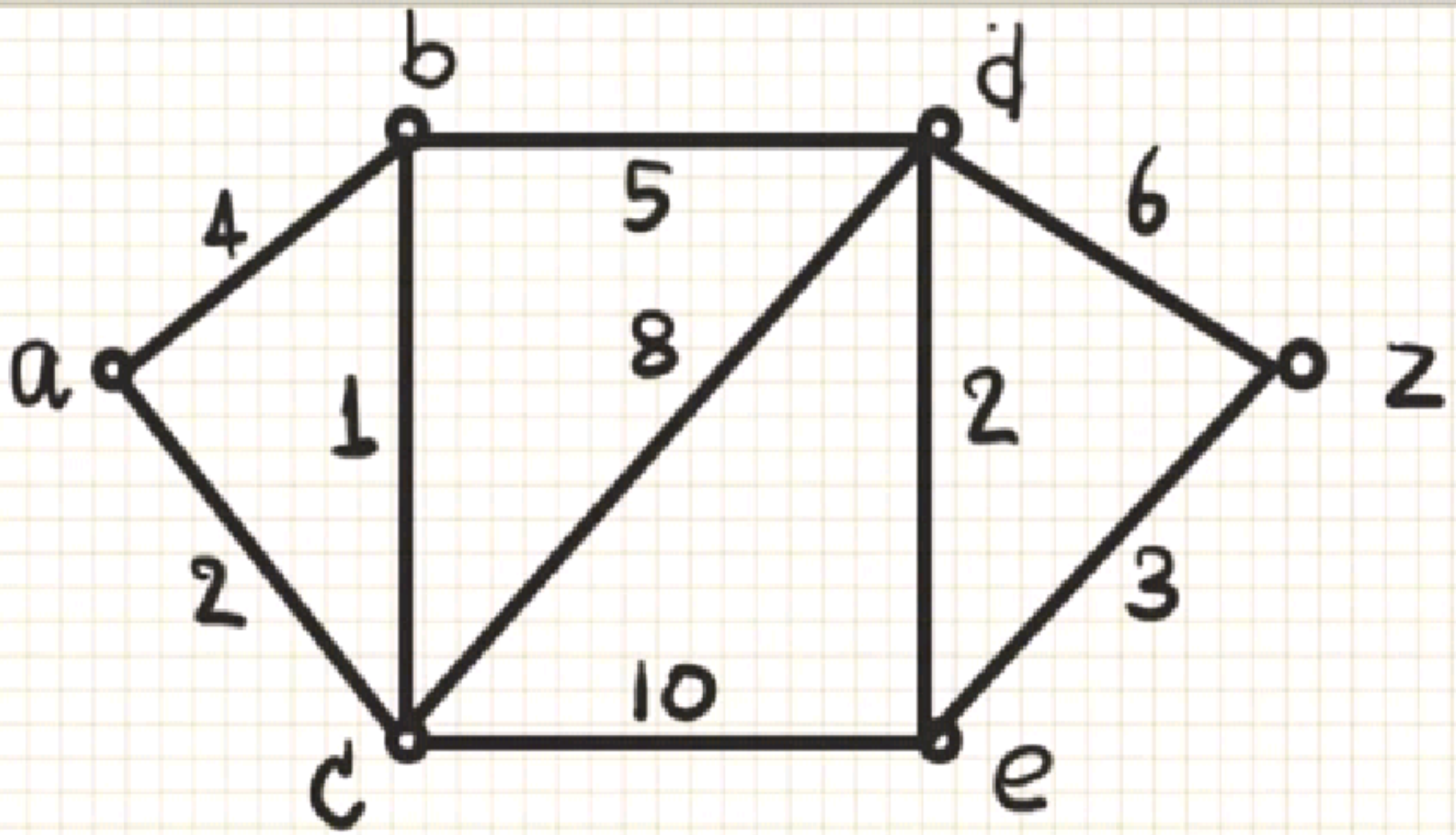
Tree vertices	Remaining vertices
a(-,0)	<u>b(a,3)</u> c(-,∞) d(a,7) e(-,∞)
b(a,3)	c(b,3+4) <u>d(b,3+2)</u> e(-,∞)
d(b,5)	<u>c(b,7)</u> e(d,5+4)
c(b,7)	<u>e(d,9)</u>
e(d,9)	





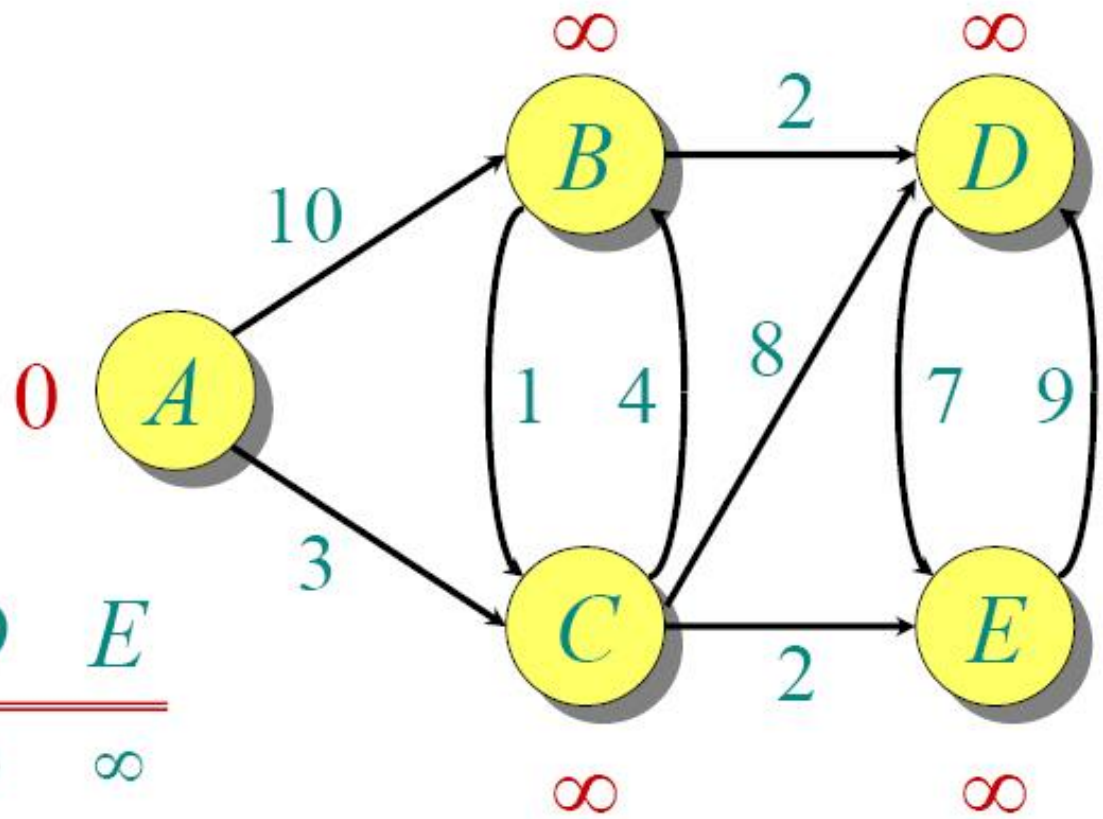





b
d
4
5
6
a
8
z
1
2
2
3
10
c
e

Dijkstra Animated Example

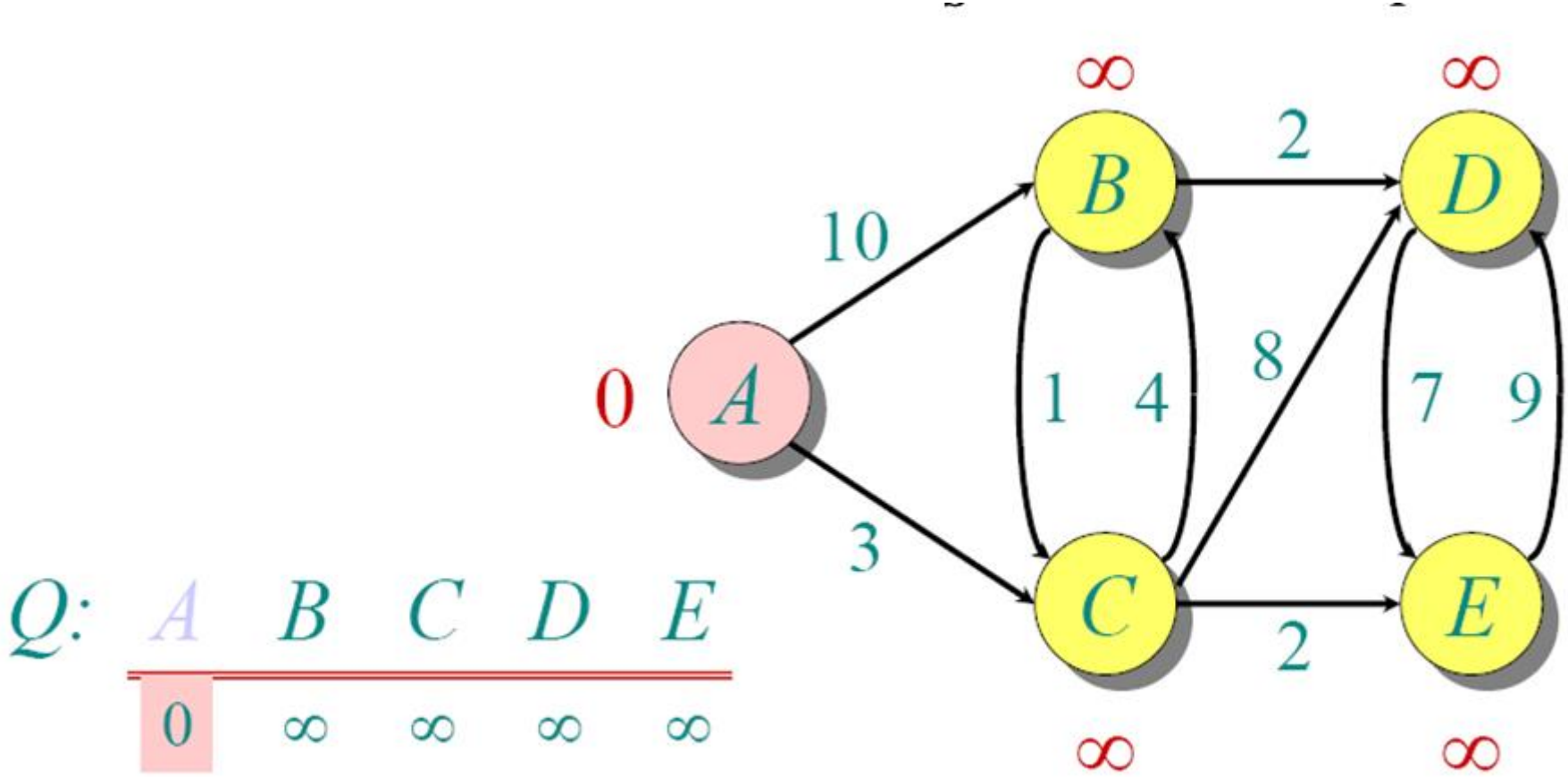
Initialize:



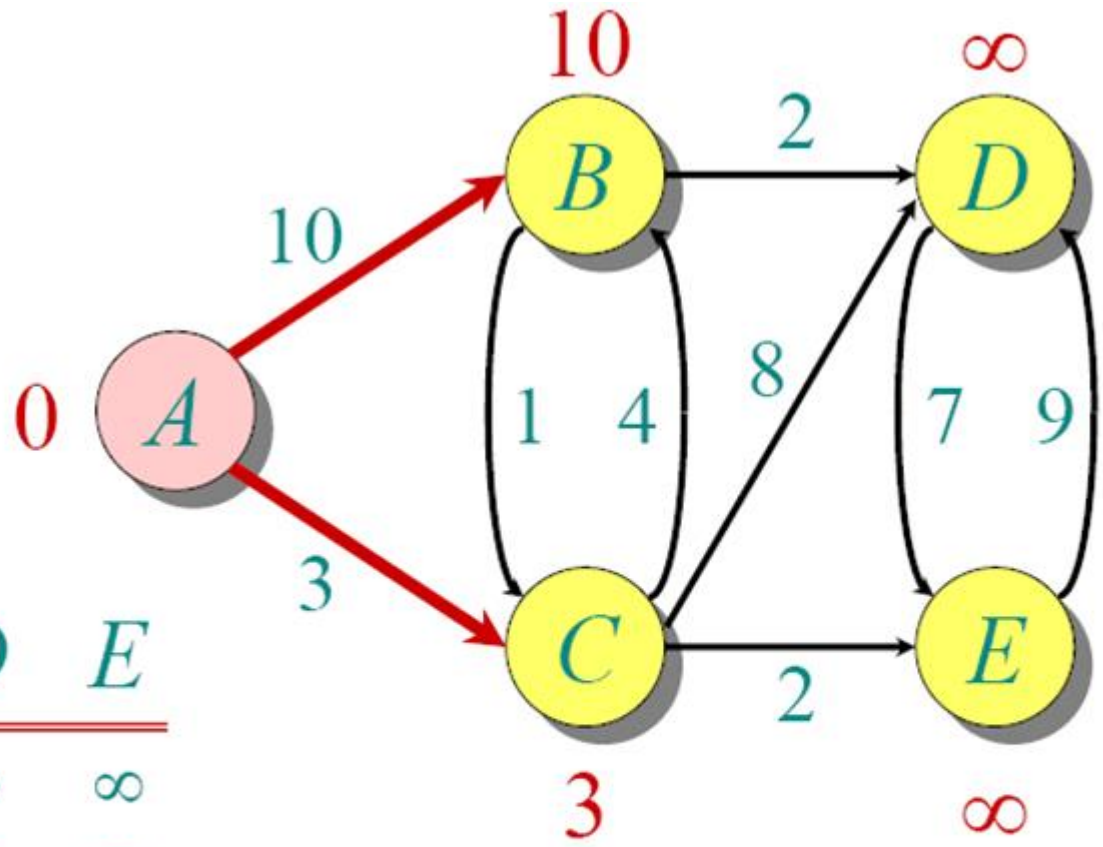
Q:	*A*	*B*	*C*	*D*	*E*
	0	∞	∞	∞	∞

S: {}

Dijkstra Animated Example



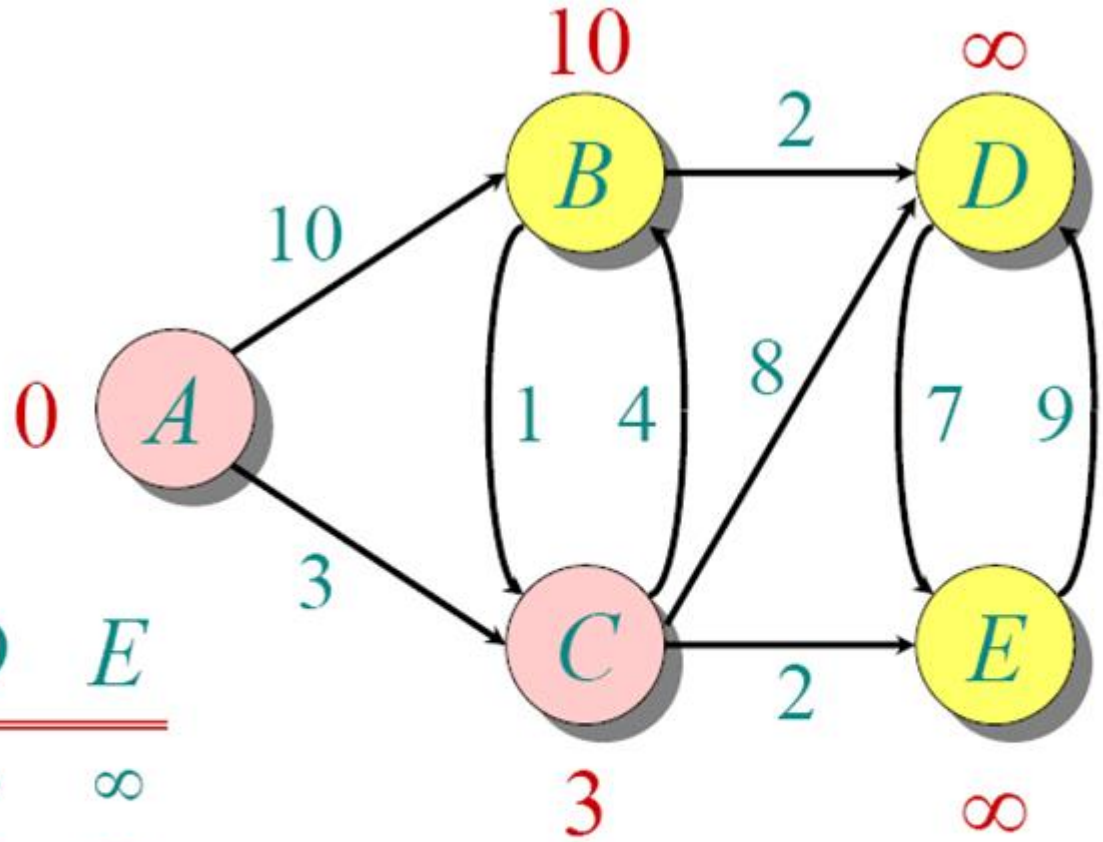
Dijkstra Animated Example



Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞

S: { A }

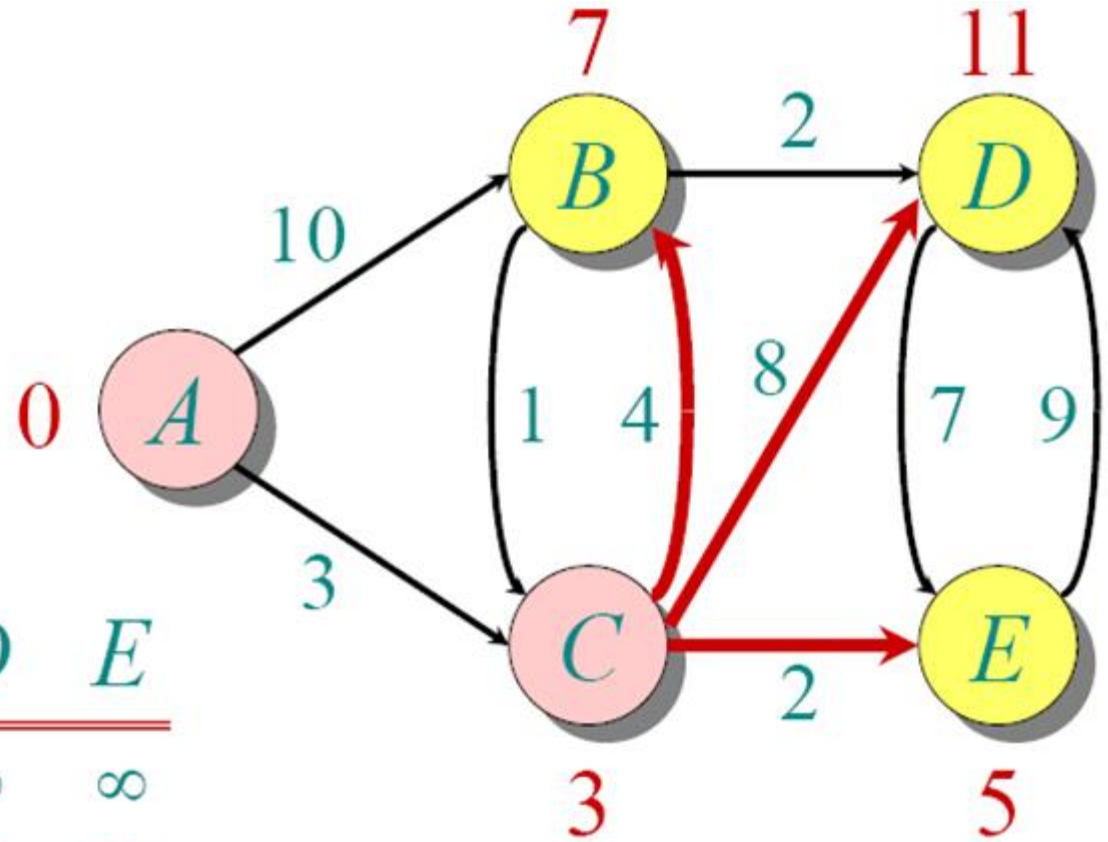
Dijkstra Animated Example



Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞

S: { *A*, *C* }

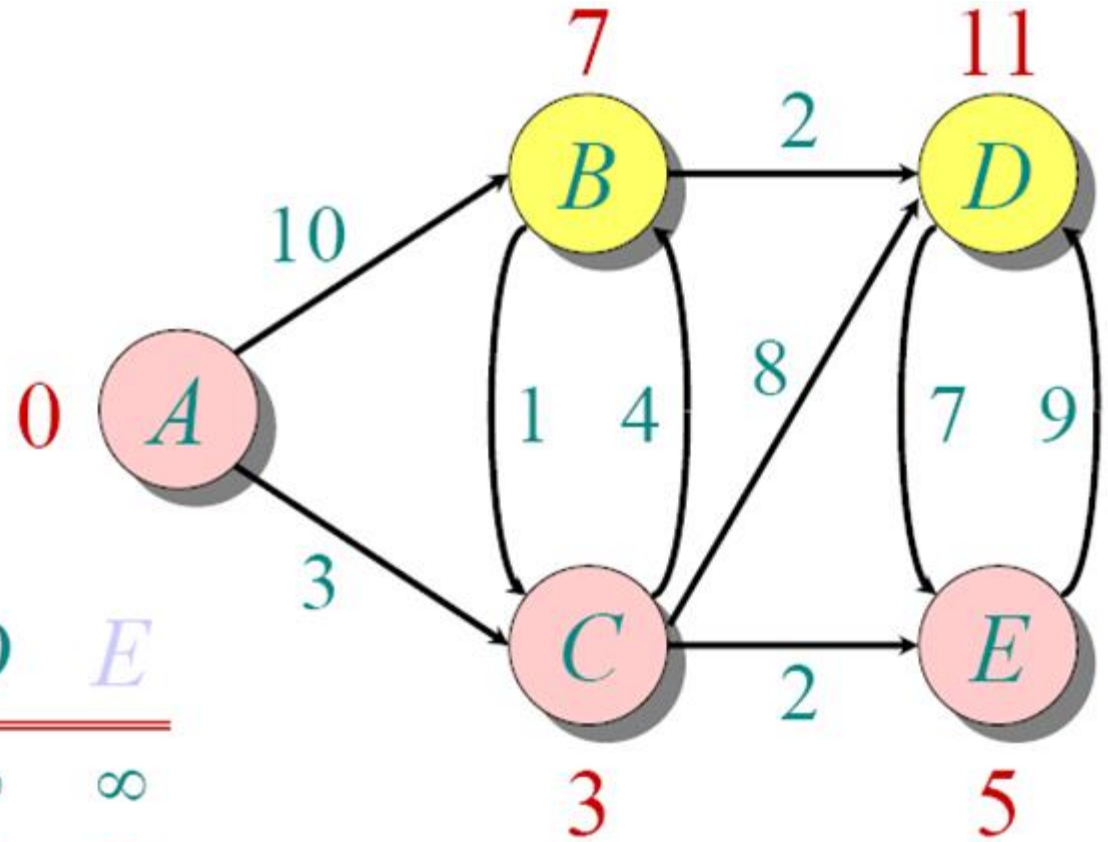
Dijkstra Animated Example



Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5

S: { A, C }

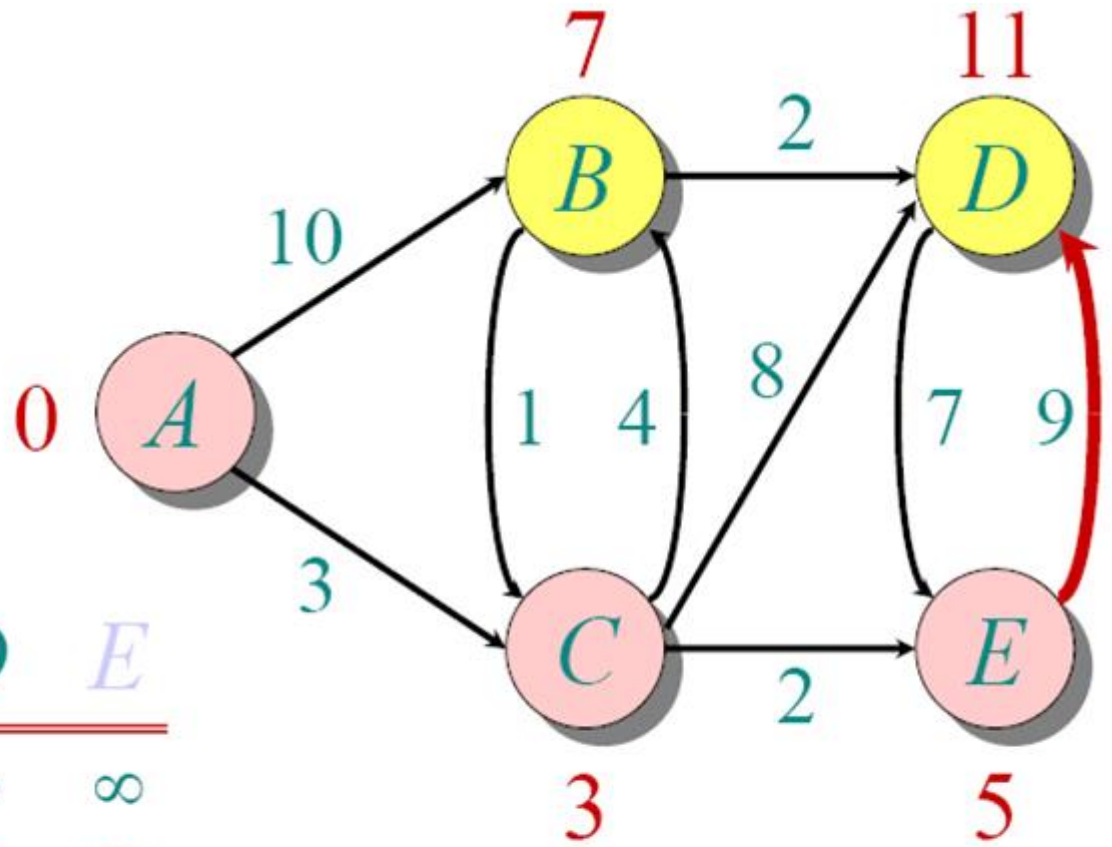
Dijkstra Animated Example



Q:	*A*	*B*	*C*	*D*	*E*
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5

S: { *A*, *C*, *E* }

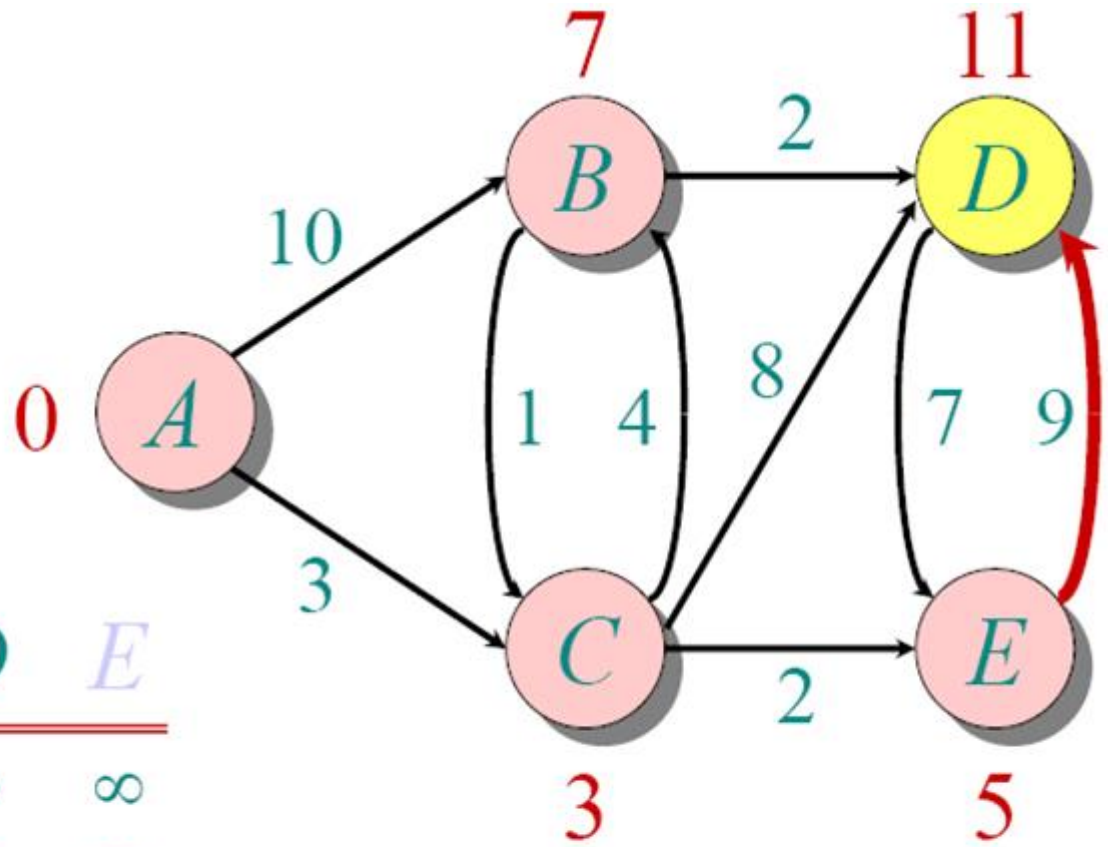
Dijkstra Animated Example



Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5
		7		11	

S: { *A*, *C*, *E* }

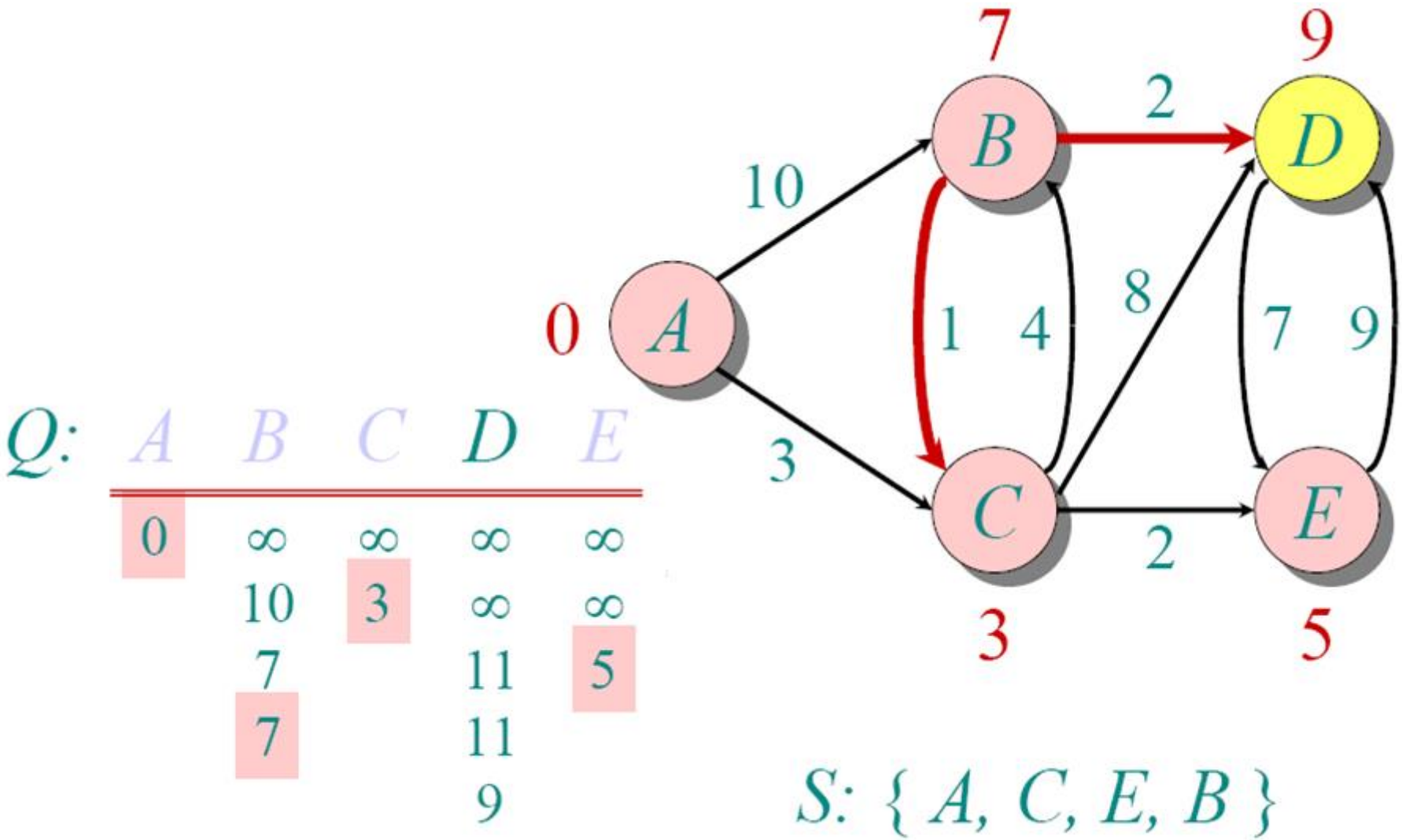
Dijkstra Animated Example



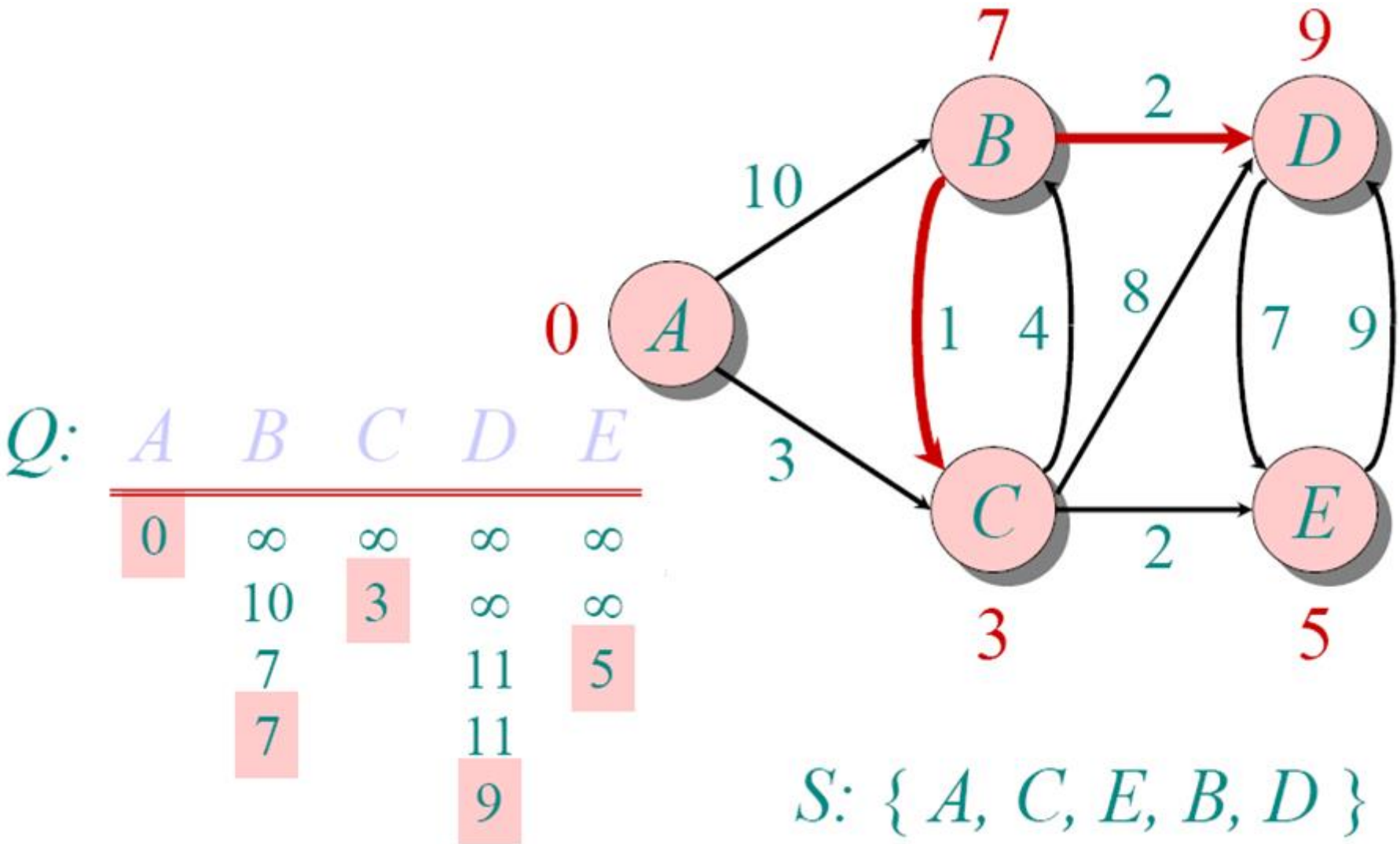
Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5
		7		11	

$S: \{ A, C, E, B \}$

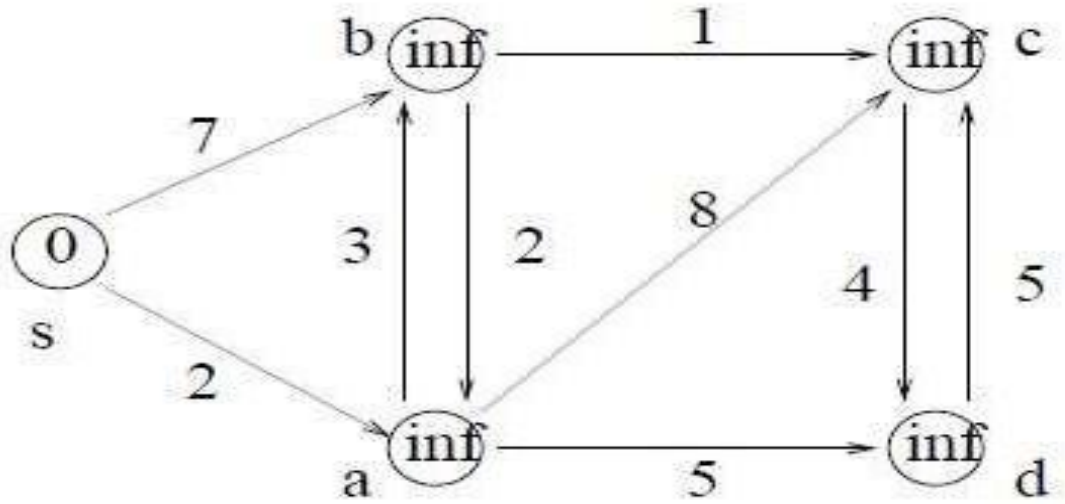
Dijkstra Animated Example



Dijkstra Animated Example



Dijkstra Animated Example-1



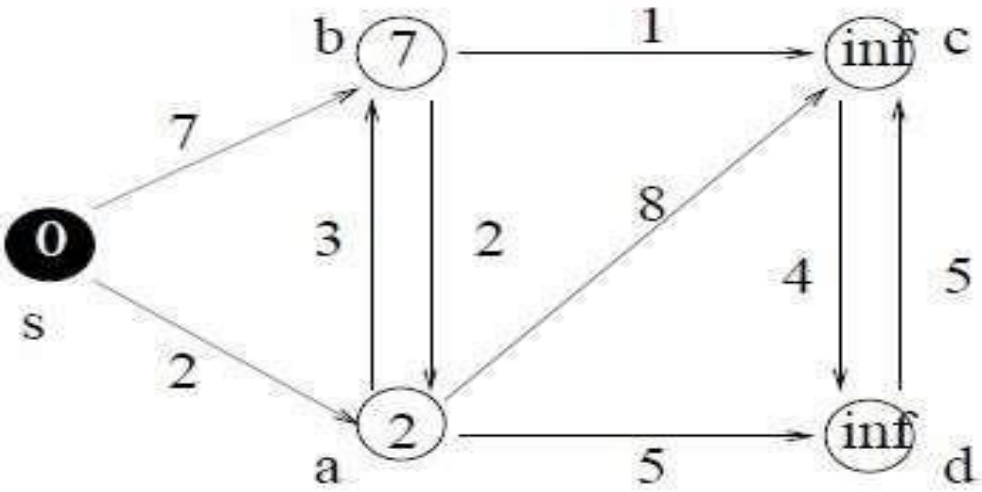
Step 0: Initialization.

v	s	a	b	c	d
$d[v]$	0	∞	∞	∞	∞
$pred[v]$	nil	nil	nil	nil	nil
$color[v]$	W	W	W	W	W

Priority Queue:

v	s	a	b	c	d
$d[v]$	0	∞	∞	∞	∞

Dijkstra Animated Example-1



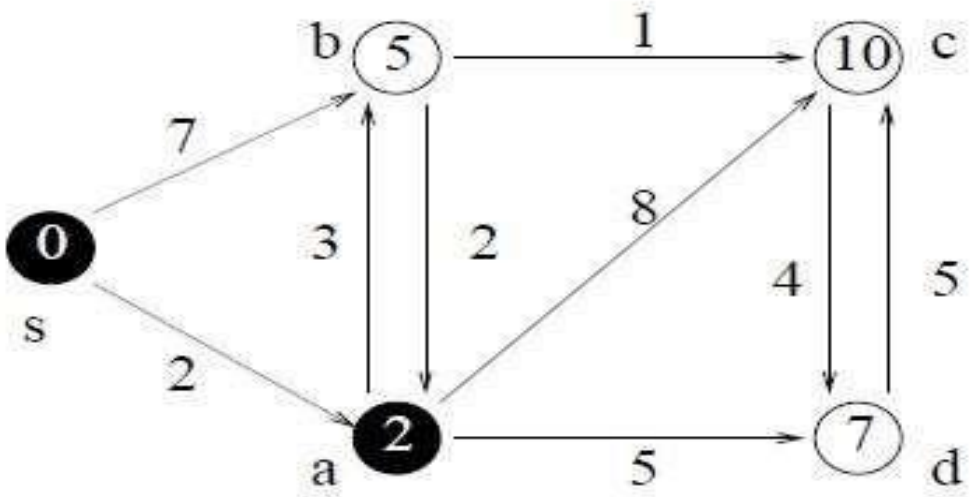
Step 1: As $Adj[s] = \{a, b\}$, work on a and b and update information.

v	s	a	b	c	d
$d[v]$	0	2	7	∞	∞
$pred[v]$	nil	s	s	nil	nil
$color[v]$	B	W	W	W	W

Priority Queue:

v	a	b	c	d
$d[v]$	2	7	∞	∞

Dijkstra Animated Example-1



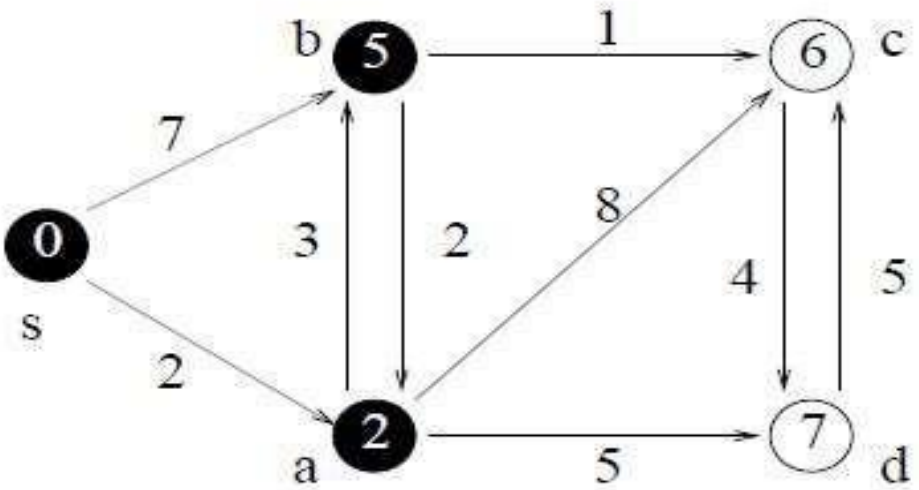
Step 2: After Step 1, a has the minimum key in the priority queue. As $Adj[a] = \{b, c, d\}$, work on b, c, d and update information.

v	s	a	b	c	d
$d[v]$	0	2	5	10	7
$pred[v]$	nil	s	a	a	a
$color[v]$	B	B	W	W	W

Priority Queue:

v	b	c	d
$d[v]$	5	10	7

Dijkstra Animated Example-1



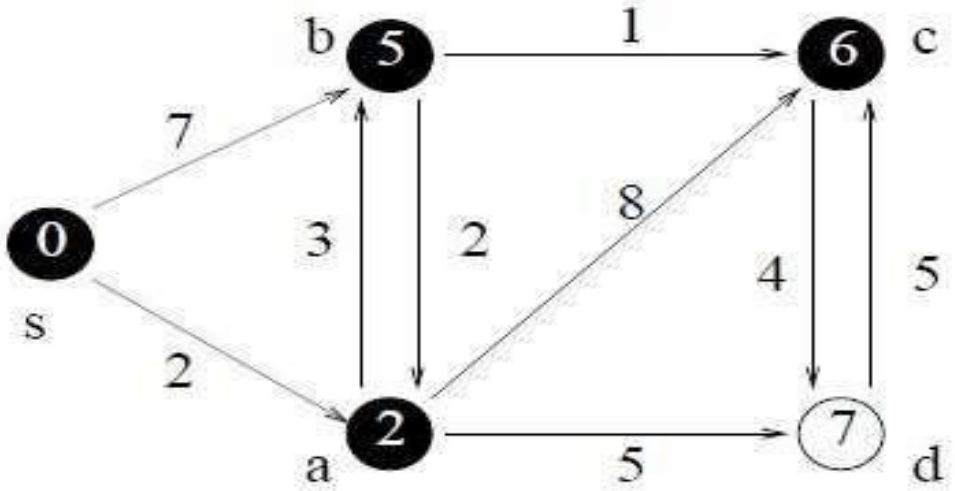
Step 3: After Step 2, b has the minimum key in the priority queue. As $Adj[b] = \{a, c\}$, work on a, c and update information.

v	s	a	b	c	d
$d[v]$	0	2	5	6	7
$pred[v]$	nil	s	a	b	a
$color[v]$	B	B	B	W	W

Priority Queue:

v	c	d
$d[v]$	6	7

Dijkstra Animated Example-1



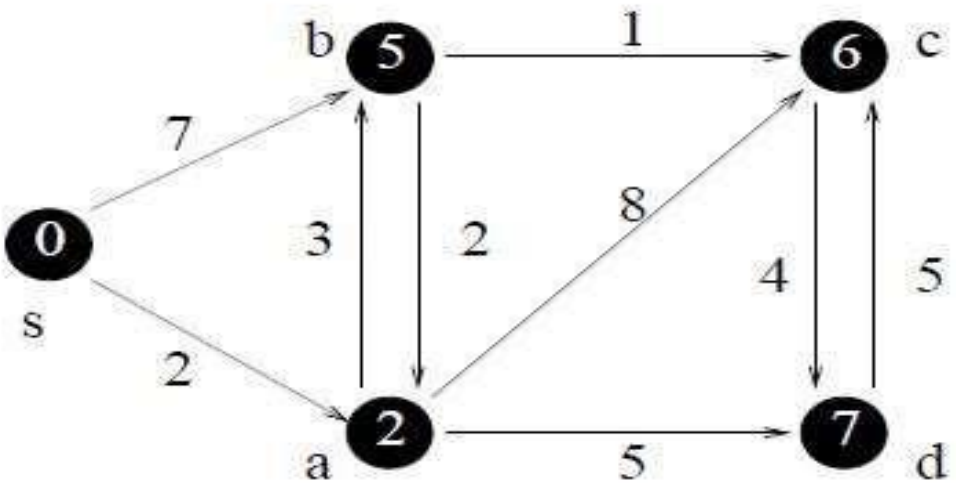
Step 4: After Step 3, c has the minimum key in the priority queue. As $Adj[c] = \{d\}$, work on d and update information.

v	s	a	b	c	d
$d[v]$	0	2	5	6	7
$pred[v]$	nil	s	a	b	a
$color[v]$	B	B	B	B	W

Priority Queue:

v	d
$d[v]$	7

Dijkstra Animated Example-1



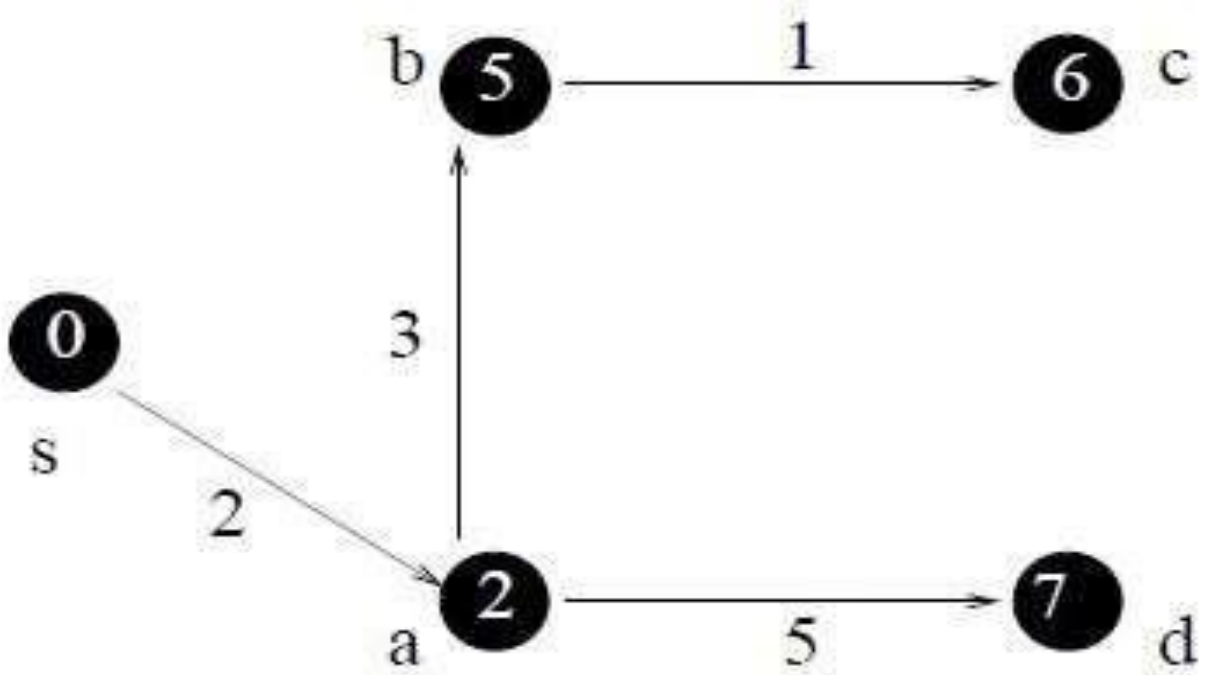
Step 5: After Step 4, d has the minimum key in the priority queue. As $Adj[d] = \{c\}$, work on c and update information.

v	s	a	b	c	d
$d[v]$	0	2	5	6	7
$pred[v]$	nil	s	a	b	a
$color[v]$	B	B	B	B	B

Priority Queue: $Q = \emptyset$.

We are done.

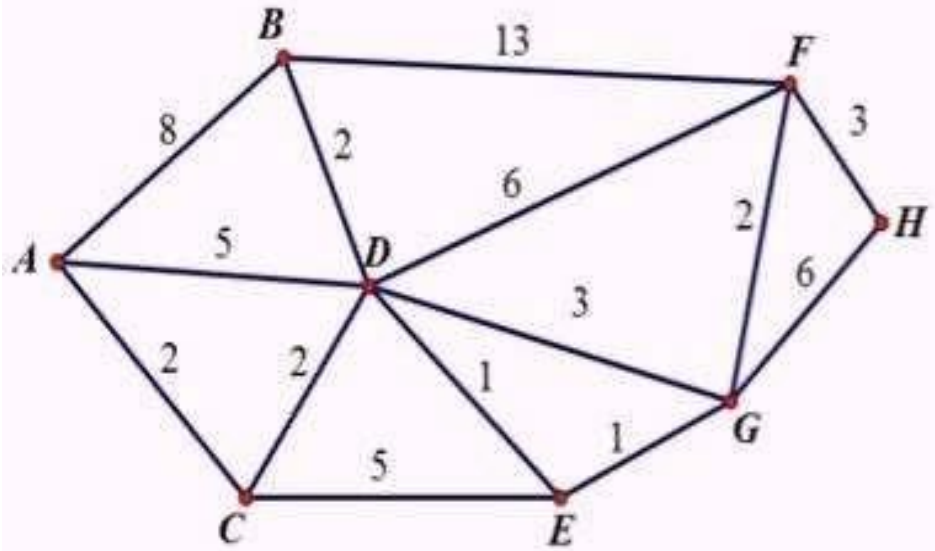
Dijkstra Animated Example-1



Example:

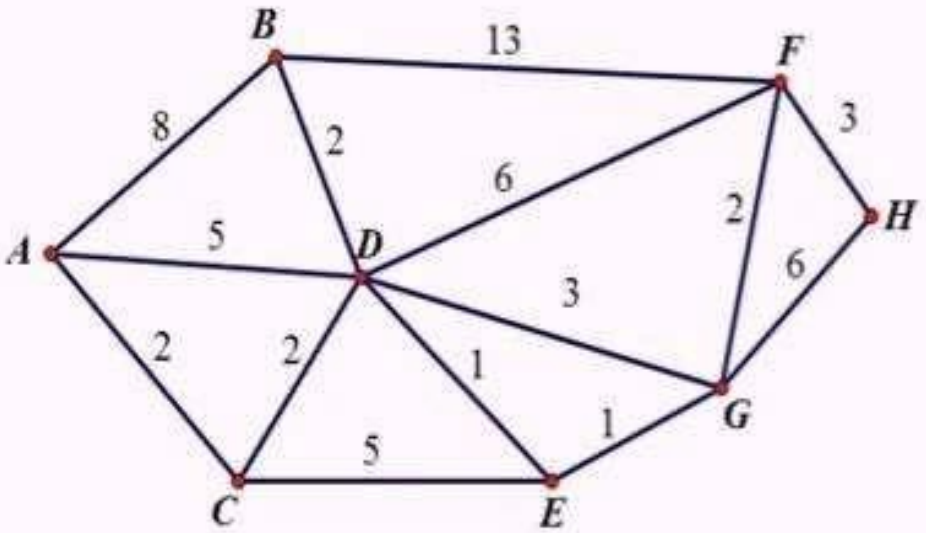
v	s	a	b	c	d
$d[v]$	0	2	5	6	7
$pred[v]$	nil	s	a	b	a

Dijkstra Animated Example-2



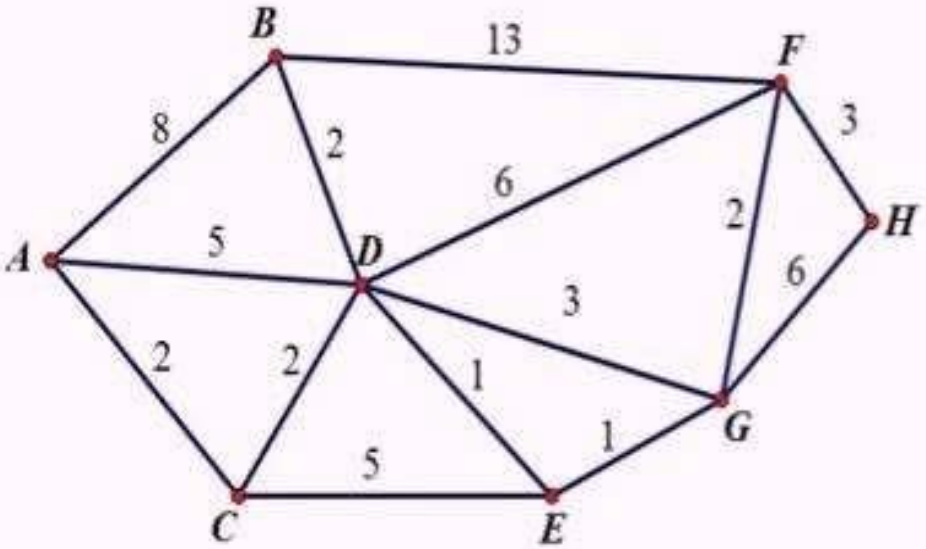
V	A	B	C	D	E	F	G	H
	0_A	∞	∞	∞	∞	∞	∞	∞

Dijkstra Animated Example-2



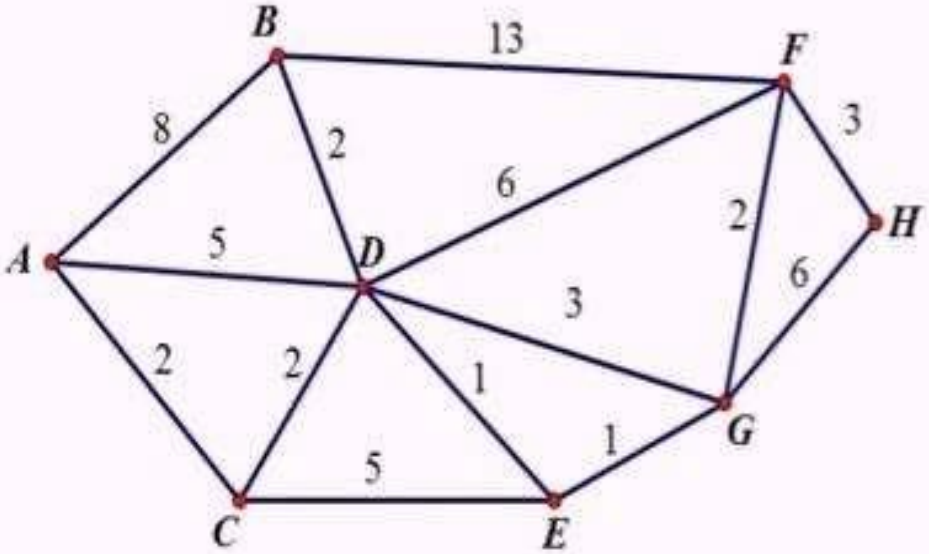
V	A	B	C	D	E	F	G	H
A	0_A	8_A	2_A	5_A	∞	∞	∞	∞

Dijkstra Animated Example-2



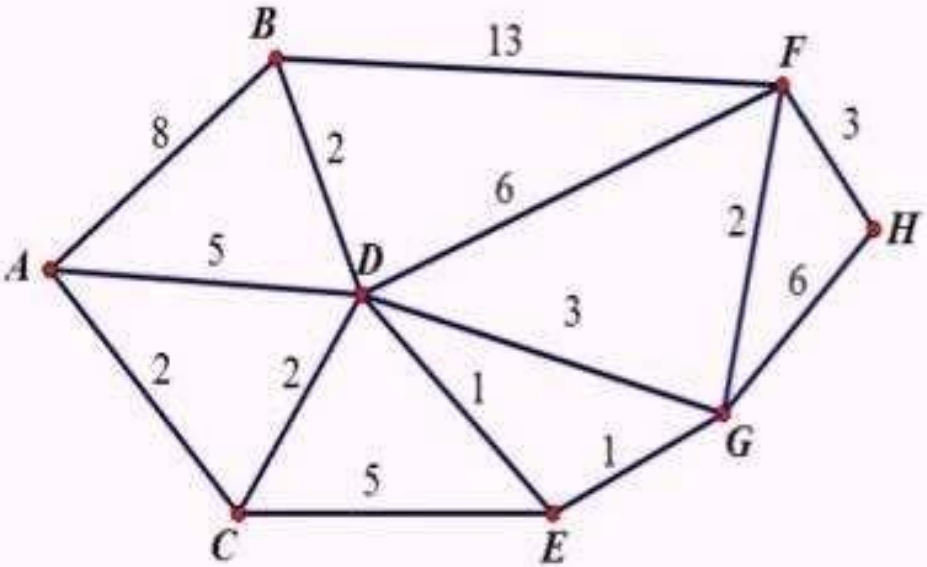
V	A	B	C	D	E	F	G	H
A	$\boxed{0_A}$	8_A	2_A	5_A	∞	∞	∞	∞
C		8_A	$\boxed{2_A}$	4_C	7_C	∞	∞	∞

Dijkstra Animated Example-2



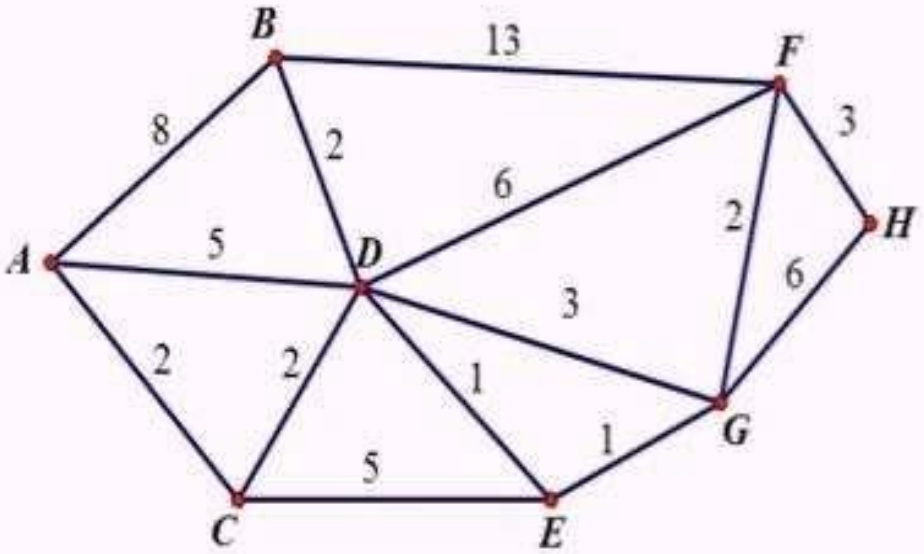
V	A	B	C	D	E	F	G	H
A	$\boxed{0_A}$	8_A	2_A	5_A	∞	∞	∞	∞
C		8_A	$\boxed{2_A}$	4_C	7_C	∞	∞	∞
D		6_D		$\boxed{4_C}$	5_D	10_D	7_D	∞

Dijkstra Animated Example-2



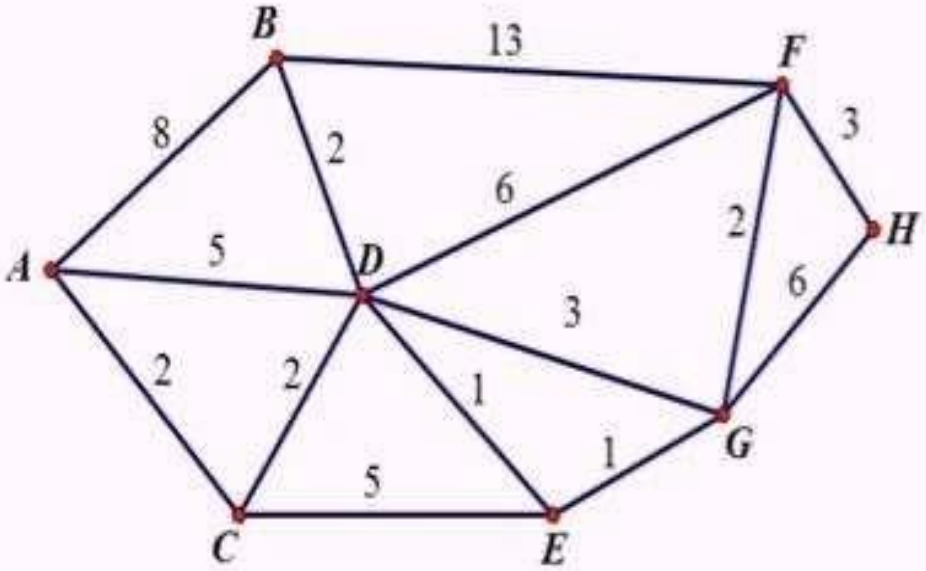
V	A	B	C	D	E	F	G	H
A	$\boxed{0_A}$	8_A	2_A	5_A	∞	∞	∞	∞
C		8_A	$\boxed{2_A}$	4_C	7_C	∞	∞	∞
D		6_D		$\boxed{4_C}$	5_D	10_D	7_D	∞
E		6_D			$\boxed{5_D}$	10_D	6_E	∞

Dijkstra Animated Example-2



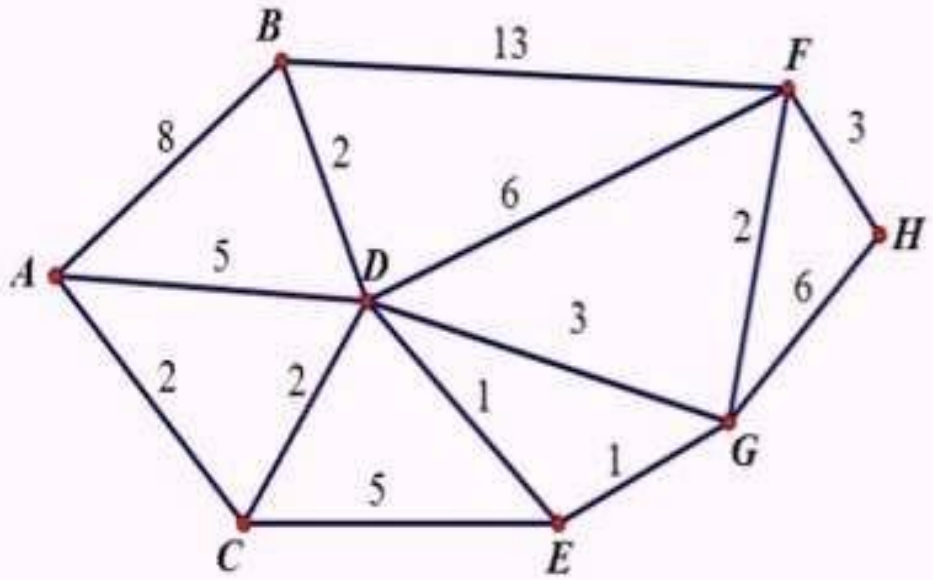
V	A	B	C	D	E	F	G	H
A	$\boxed{0_A}$	8_A	2_A	5_A	∞	∞	∞	∞
C		8_A	$\boxed{2_A}$	4_C	7_C	∞	∞	∞
D		6_D		$\boxed{4_C}$	5_D	10_D	7_D	∞
E		6_D			$\boxed{5_D}$	10_D	6_E	∞
B		$\boxed{6_D}$				10_D	6_E	∞

Dijkstra Animated Example-2



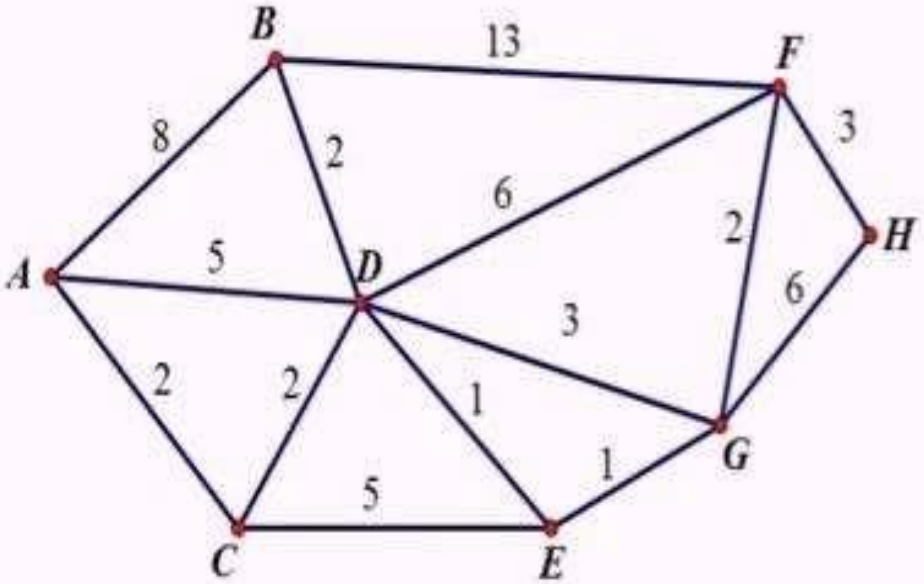
V	A	B	C	D	E	F	G	H
A	0_A	8_A	2_A	5_A	∞	∞	∞	∞
C		8_A	2_A	4_C	7_C	∞	∞	∞
D		6_D		4_C	5_D	10_D	7_D	∞
E		6_D			5_D	10_D	6_E	∞
B		6_D				10_D	6_E	∞
G						8_G	6_E	12_G

Dijkstra Animated Example-2



V	A	B	C	D	E	F	G	H
A	0_A	8_A	2_A	5_A	∞	∞	∞	∞
C		8_A	2_A	4_C	7_C	∞	∞	∞
D		6_D		4_C	5_D	10_D	7_D	∞
E		6_D			5_D	10_D	6_E	∞
B		6_D				10_D	6_E	∞
G						8_G	6_E	12_G
F						8_G		11_F

Dijkstra Animated Example-2



V	A	B	C	D	E	F	G	H
A	0_A	8_A	2_A	5_A	∞	∞	∞	∞
C		8_A	2_A	4_C	7_C	∞	∞	∞
D		6_D		4_C	5_D	10_D	7_D	∞
E		6_D			5_D	10_D	6_E	∞
B		6_D				10_D	6_E	∞
G						8_G	6_E	12_G
F						8_G		11_F
								11_F

Shortest Path from A to H,
A to C, C to D, D to E, E to G, G to F, F to H

Implementations and Running Times

The simplest implementation is to store vertices in an array or linked list. This will produce a running time of

O(|V|^2 + |E|)

For sparse graphs, or graphs with very few edges and many nodes, it can be implemented more efficiently storing the graph in an adjacency list using a binary heap or priority queue. This will produce a running time of

$O((|E|+|V|) \log |V|)$

Dijkstra's Algorithm - Why It Works

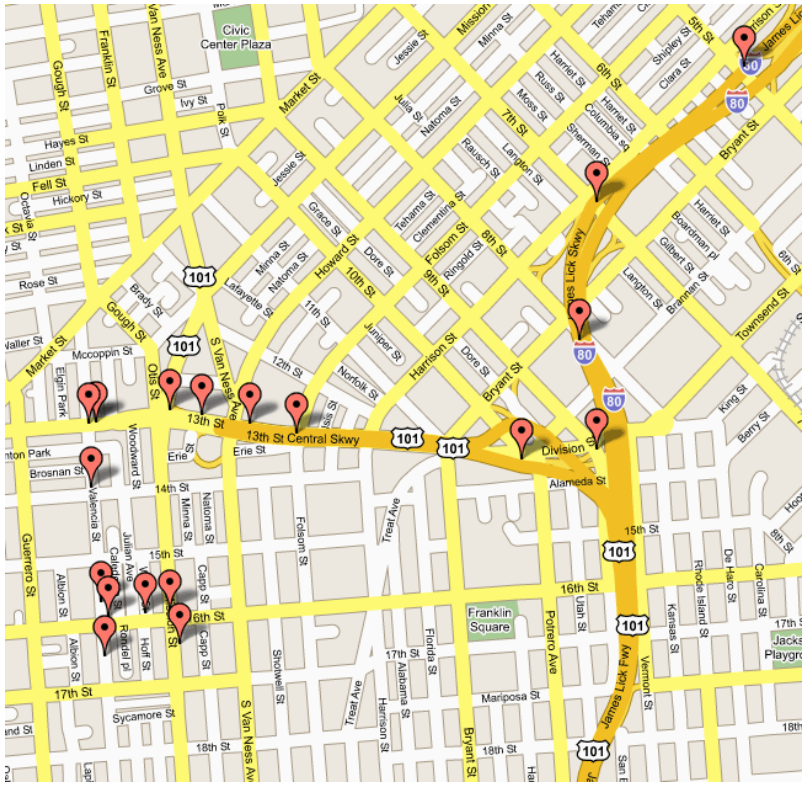
As with all greedy algorithms, we need to make sure that it is a correct algorithm (e.g., it *always* returns the right solution if it is given correct input).

A formal proof would take longer than this presentation, but we can understand how the argument works intuitively.

If you can’t sleep unless you see a proof, see the second reference or ask us where you can find it.

Applications of Dijkstra's Algorithm

- Traffic Information Systems are most prominent use
- Mapping (Map Quest, Google Maps)
- Routing Systems



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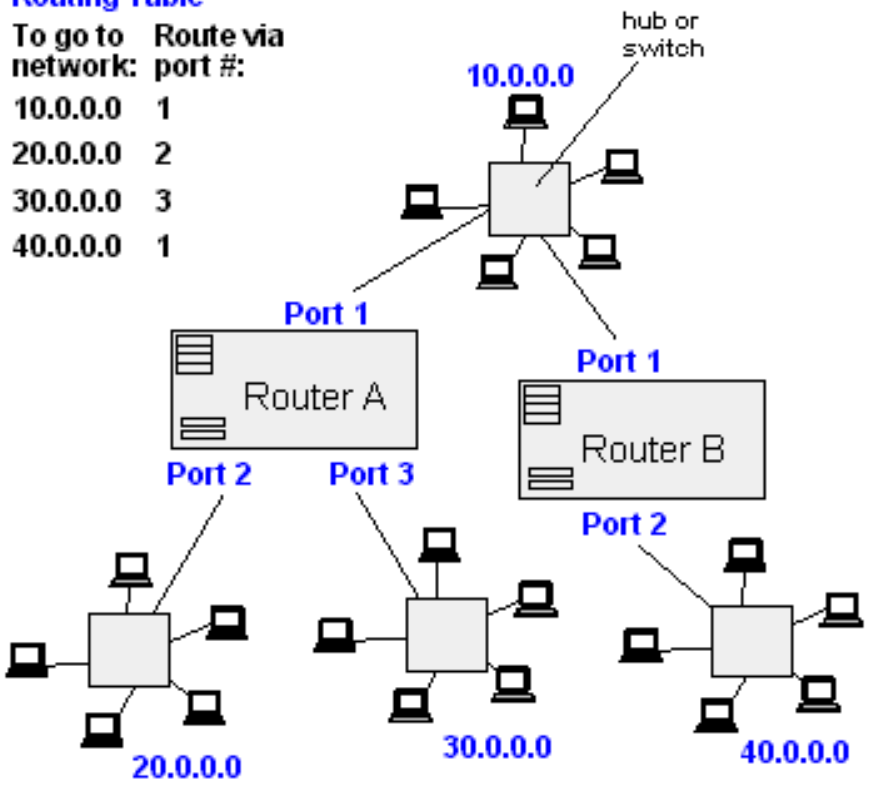
Router A
Routing Table

To go to network:	Route via port #:
10.0.0.0	1
20.0.0.0	2
30.0.0.0	3
40.0.0.0	1




Thank
you