Stamford University Bangladesh

Department of Computer Science and Engineering

Midterm Examination

Trimester: Spring 2021 | **Batch: CSE-S-71-A**
Course Code: MATH 319 | **Time: 2 Hours and 30 Minutes**
Course Title: Fourier Analysis & Laplace Transformation | **Full marks: 30**

(**There are eight short questions and two long questions. You have to answer all the questions)**

Part-I (Short Questions):

1. Write down the basic difference between Fourier series and Fourier transformation? **01**

2. Examine whether the following functions are even, odd or neither: **01**

(a) $\sin 5t$

(b) $e^{2t} - t$. Justify your answer for both cases.

3. Write down the main directions of Dirichlet's conditions for Fourier transformations. **01**

4. Determine the coefficient a_0 for the Fourier series of $f(x) = \begin{cases} 0, & for\ -\pi < x < 0 \\ 2, & for\ 0 < x < \pi \end{cases}$. **01**

5. What will be the Parseval's identity for half-range Fourier sine and cosine series? **01**

6. Obtain the frequency domain by using Fourier sine transform for the function $f(x) = e^{-2x}$ for $x \geq 0$. **01**

7. Sketch the graph of the aperiodic waveform in the time domain **02**

$f(t) = \begin{cases} 4, when & -3 \leq x \leq 0 \\ 0, & otherwise \end{cases}$. Also calculate it's Fourier transformations.

8. Find the Fourier cosine series for the function $f(x) = \begin{cases} 1, & for\ 0 < x < \frac{\pi}{2} \\ 0, & for\ \frac{\pi}{2} < x < \pi \end{cases}$. **02**

(Please go to 2nd page)

Part-II (Subjective Questions):

9. (a) Express $f(x) = x$ as a half range Fourier cosine series in the interval $0 < x < 2$. **03**

(b) Consider $f(t) = t^2$ is a signal whose period lies on $-\pi \leq t \leq \pi$. Find the Fourier expression for this function. Also, deduce that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \ldots\ldots\ldots\ldots = \frac{\pi^2}{12}$. **03**

(c) Show that the complex Fourier series for $f(x) = cos\ ax$ (where $-\pi < x < \pi$) is **04**

$$f(x) = \frac{a \sin a\pi}{\pi} \sum_{n=-\infty}^{\infty} \frac{(-1)^n}{a^2 - n^2} e^{inx}.$$

10.(a) If $f(t) = \begin{cases} 3, & for\ |t| < a \\ 0, & for\ \ |t| > a \end{cases}$ and $F(\omega) = \frac{6 \sin a\omega}{\omega}$, by using Parseval's identity of Fourier transformation, prove that $\int_0^\infty \frac{sin^2 at}{\omega^2} dt = \frac{a\pi}{6}$. **03**

(b) Find the Fourier cosine transformation of the function $f(x) = \begin{cases} 0 & when\ 0 < x < 1 \\ 2 - x & when\ \ 1 < x < 2. \\ 2 & when\ x > 2 \end{cases}$ **03**

(c) Find the Fourier sine integral for the function: $f(x) = e^{-3\alpha x}$, where α is a constant. **04**

Hence, show that $\int_0^\infty \frac{u\ sin\ ux}{9\alpha^2 + u^2} du = \frac{\pi}{2} e^{-3\alpha x}$.

NB: After 2:30 hours you will get extra 30 minutes to send your answer script as a single PDF file to **edmodo** class (**CSE 71A (MATH 319)_FALT (Spring 2021)**) whose class code: 3hnm4w

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