PID Controller Design For Servo Positioning Module

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Executive Summary:

In this project a PID controller was implemented for both a simulated and real-time DC servo motor, with the goal of improving the closed-loop response of the system to a square wave input. The effects of the nonlinearity of a system on the control system parameters were further investigated, and were used to calibrate the system for an optimal response.

For the first part, a simulation of a DC servo motor was used, which was set up according to a specific data set. The uncompensated response of the simulated servo in the nominal range was then observed, which meant the controller output was neither saturated nor in the deadzone. The controller parameters for this setting were the following: (Td= 0, Kp=1, Ti=infinity). The following response specifications were the result of these parameters: **Rise time=114.83 ms, maximum overshoot=23.4%, settling time= 307.1 ms, steady state error (step)=0.009, steady-state error (ramp)=3.3**. Next, an optimal PID controller for a compensated response of the DC servo module in the nominal range was designed. Of the several methods this design could have been achieved, the trial and error approach was selected. A second controller design which incorporated anti-windup gain was experimented with, but ultimately the optimal response was achieved with the system being under solely PID control. The controller parameters for this setting were the following: (Td= 0, Kp=0.52, Ti=5). The following response specifications were the result of these parameters: **Rise time=400.26 ms, maximum overshoot=0%, settling time= 282.15 ms, steady-state error (step)=0.14, steady-state error (ramp)=0.98**. Next, the effects of the compensated control parameters on a system operating outside the nominal range were observed. The experiment consisted of the following system responses: uncompensated, compensated and compensated with wind-up gain; while the system was in saturation (amplitude of square wave signal=200 degrees) and deadzone states (amplitude of square wave signal=20 degrees).

For the second part a real-time DC servo motor setup was used, with parameters matching ther dataset for the simulated servo motor, to reinforce the results from the first part. Experimenting with an uncompensated system response with the system operating in the nominal range resulted in the following specifications: **Rise time=400 ms, maximum overshoot=0%, settling time= 340 ms, steady state error (step)=3.38%, steady state error (ramp)=0.44**. The compensated system response in the nominal range resulted in the following specification: **Rise time=149 ms, maximum overshoot=0%, settling time= 180 ms, steady state error (step)=0.186, steady-state error (ramp)=0.296**. Finally, for the compensated system response in the non-nominal range, the effects of saturation and deadzone were observed. It was found to be difficult to observe the effects of saturation (when the input signal amplitude was set to 200 degrees) since there was no way in the lab to measure the control signal; however, an extremely delayed system response was observed which continued to delay as the experiment went from the benchmark system response to PID system response to PID+A system response. The effects of deadzone were distinct on the benchmark system response (with the input signal amplitude at 20 degrees); with the addition of PID control a drastic improvement was observed. It is important to note that optimal response specifications were not intended to be reached outside the nominal range.

Table. [1] Servo Motor Parameters:

Armature Inductance - La (H)	2.00e-04
Armature Resistance - Ra (ohm)	3.5
Motor Inertia - Jm (oz.in.sec^2)	3.47e-7
Motor Viscous Friction - Bm (N.m.sec)	1.3319e-07

Part A: Simulated Servo

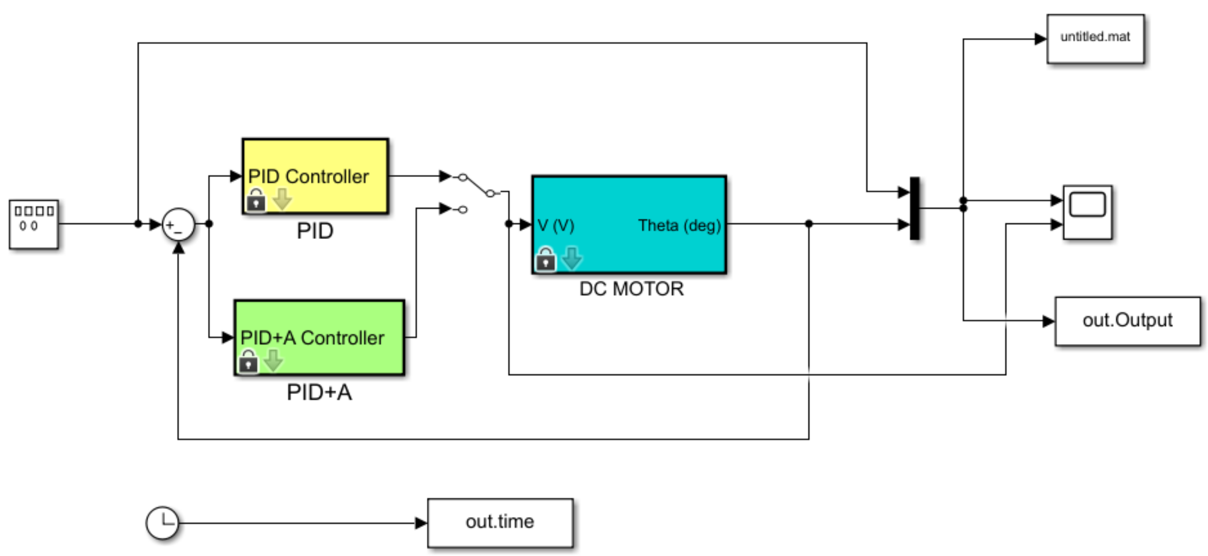

Fig. [1] Simulink Schematic of Simulated Servo Incorporating Variable PID Control Parameters

Part 1:

Uncompensated Simulated Servo Response In the Nominal Range:

To make the system uncompensated, the integral time constant was set to infinity, derivative time constant to 0 and proportional gain to 1.

After setting up a square wave input with an amplitude of 50° and frequency of 1 Hz, we got the following output: (The top graph consists of the reference input (purple) and system output (blue); the bottom graph consists of the control signal (brown) all plotted over 4 cycles and simulation run time of 4 seconds).

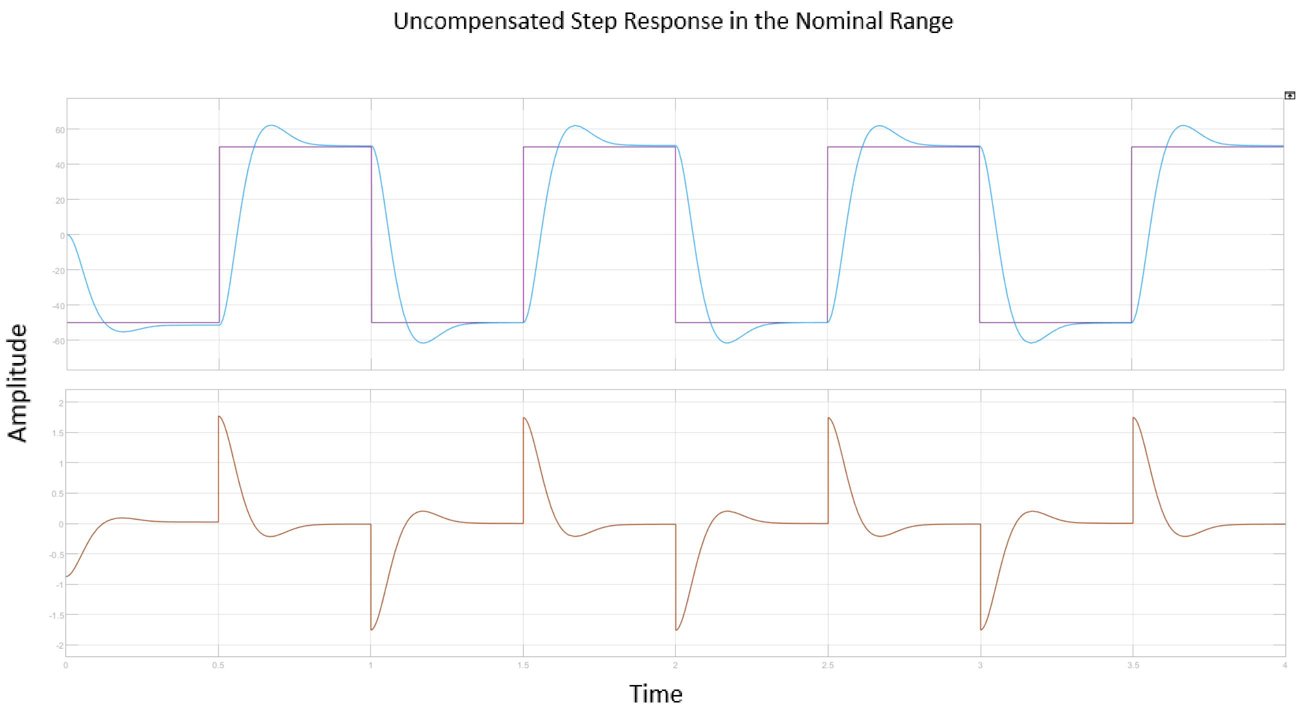

Fig. [2] Uncompensated Step Response

After setting up a sawtooth wave input with an amplitude of 50 and frequency of 1 rad/sec, we got the following output for **Kp=0.9**: (The top graph consists of the reference input (purple) and system output (red); the bottom graph consists of the control signal (brown) all plotted over 4 cycles and simulation run time of 26 seconds).

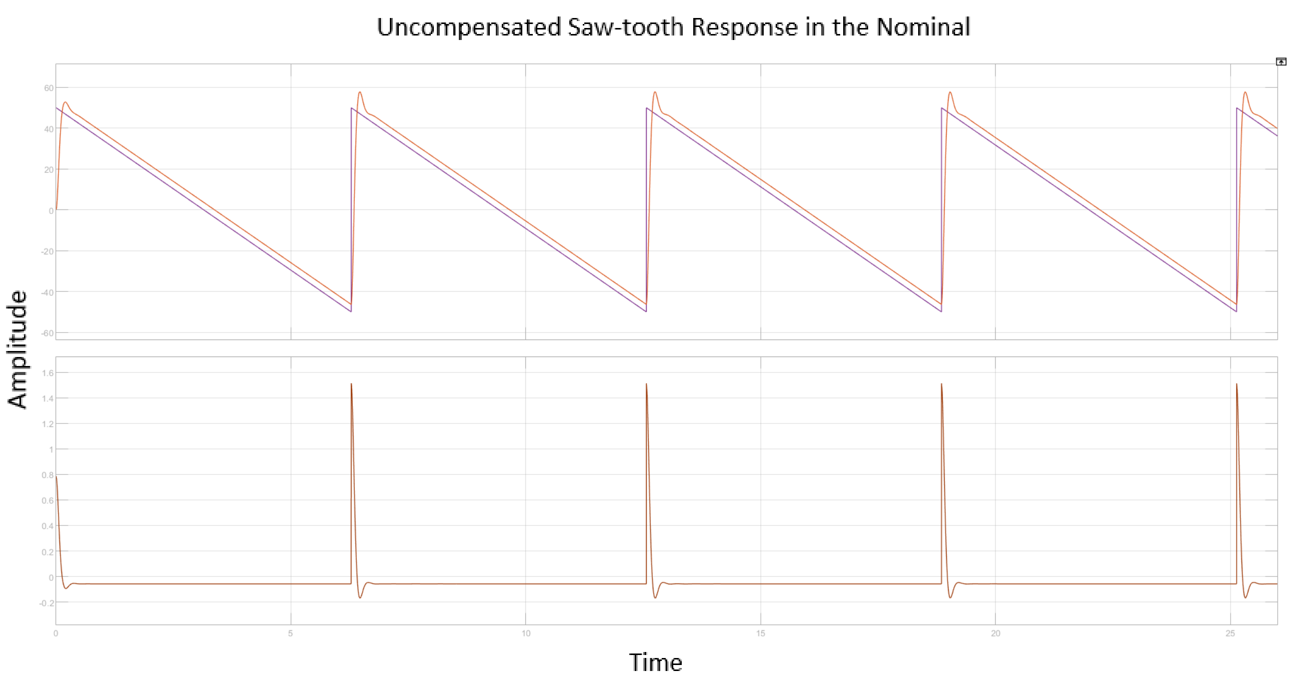

Fig. [3] Uncompensated sawtooth Response

Table. [2] The following parameters were measured from the graph:

Uncompensated System	Rise-time $t_{r(0\text{-}100\%)}$	Maximum Overshoot %O.S.	Settling-time $t_{s(+/-\ 2\%)}$	Steady- state error $e_{ss(step)}$	Steady- state error $e_{ss(ramp)}$
Proportional Controller Kp=1	114.829 ms	23.4	307.1 ms	0.009	3.3

Calculations for Maximum Overshoot:

$\frac{Ymax - Yss}{Yss} * 100\% = \frac{62.24-50.45}{50.45} * 100\% = 23.4\%$

Calculations for $e_{ss(step)}$:

$|\frac{Rss- Yss}{Rss}| * 100\% = |\frac{50-50.45}{50}| * 100\% = 0.009\%$

Calculations for $e_{ss(ramp)}$:

|Rss - Yss| = |48.14 - 44.84| = 3.3

For the rest of the specifications, the values were directly measured from the graph; no calculations were necessary.

Part 2:

Implementing Proportional Control on Simulated Servo:

After experimenting with the proportional gain, increasing and decreasing it slightly while making sure the system operates in the nominal range, we arrived at a Kp value which gives performance specifications closest to the requirements:

Kp = 0.9

It was found that as Kp was increased, the percent overshoot got worse, however, the rise time improved significantly. The settling time also improved up to a threshold after which it began to worsen. It is important to note that even a slight increase in Kp beyond 1 ended up saturating the control signal which meant that the system was not operating in the nominal range. As Kp decreased, the percent overshoot decreased and the settling time got worse. Kp was able to be lowered past 0.1 before the effects of dead-zone began to show.

The requirements were not met with solely proportional control, since in order to achieve a certain spec, another spec or two had to be compromised.

After setting up a square wave input with an amplitude of 50 and frequency of 1 Hz, we got the following output for **Kp=0.9**: (The top graph consists of the reference input (purple) and system output (red); the bottom graph consists of the control signal (brown) all plotted over 4 cycles and simulation run time of 4 seconds).

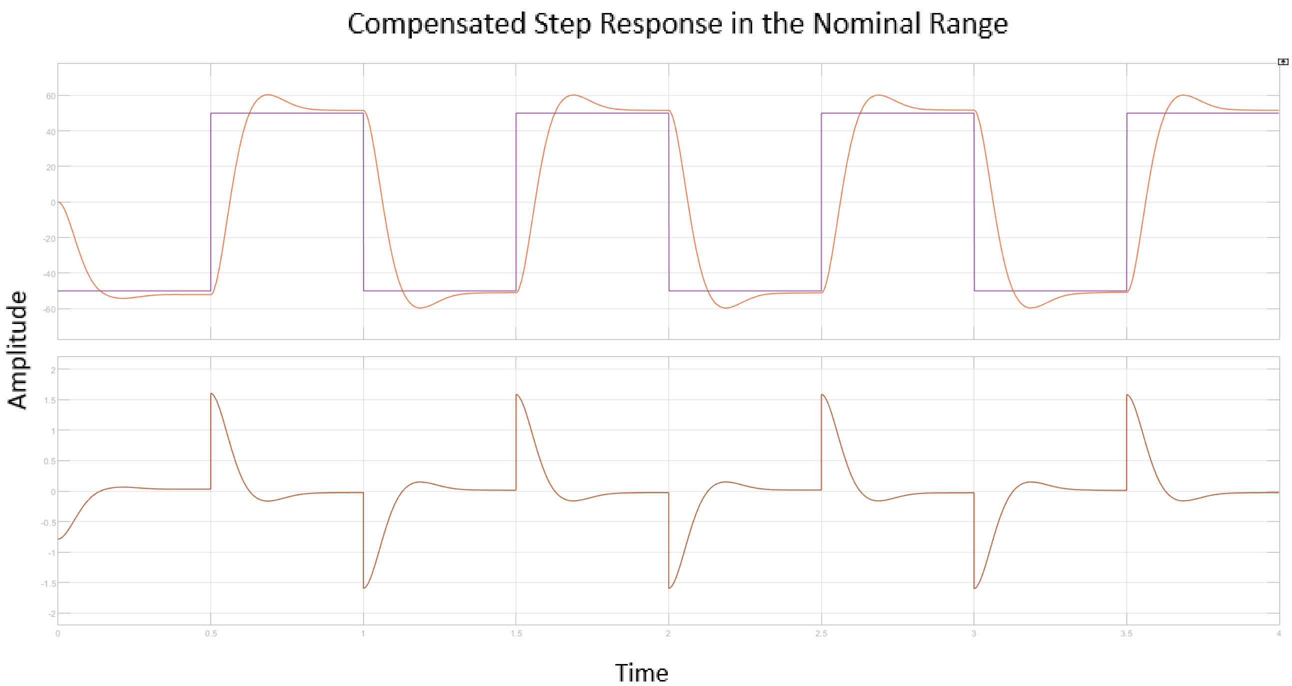

Fig. [4] Compensated Step Response

After setting up a sawtooth wave input with an amplitude of 50 and frequency of 1 rad/sec, we got the following output for **Kp=0.9**: (The top graph consists of the reference input (purple) and system output (red); the bottom graph consists of the control signal (brown) all plotted over 4 cycles and simulation run time of 26 seconds).

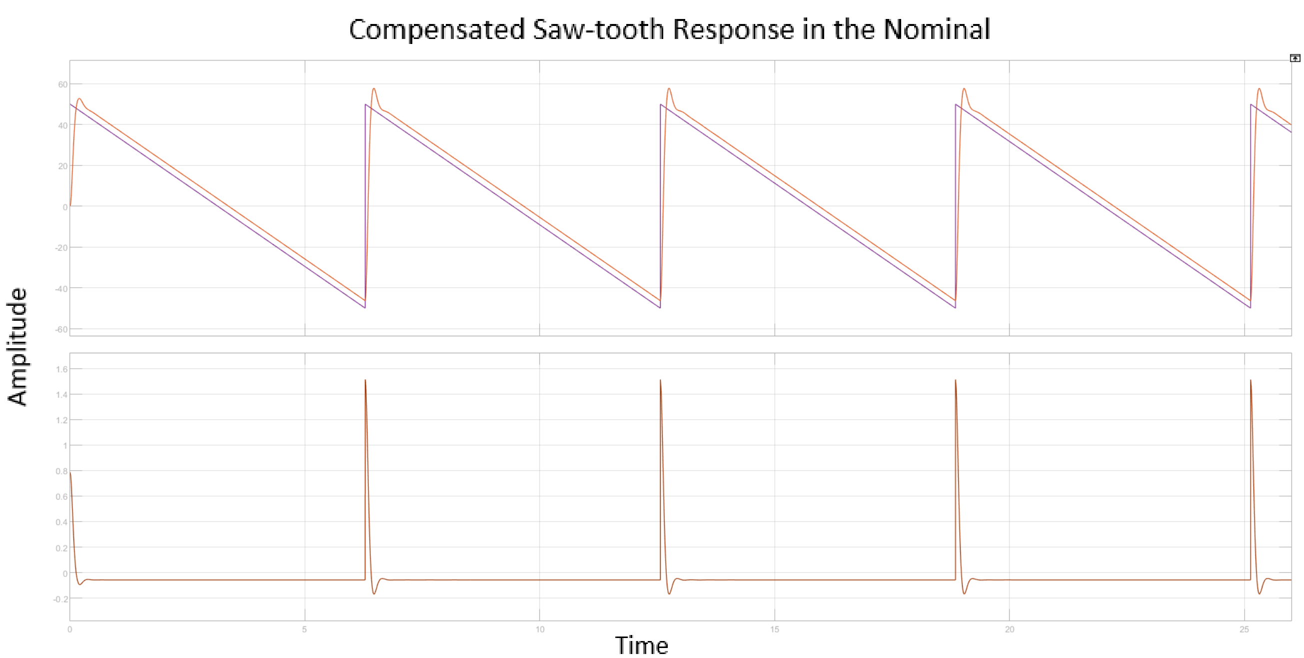

Fig. [5] Compensated Sawtooth Response

Table. [3] The following parameters were measured from the graph:

Compensated System (P-Control)	Rise-time $t_{r(0-100\%)}$	Maximum Overshoot %O.S.	Settling-time $t_{s(+/- 2\%)}$	Steady- state error $e_{ss(step)}$	Steady- state error $e_{ss(ramp)}$
Proportional Controller Kp=0.9	131.23 ms	8.5%	316.3 ms	3.6%	3.66

Calculations for Maximum Overshoot:

$$\frac{Ymax - Yss}{Yss} * 100\% = \frac{111.68-103.6}{103.6} * 100\% = 8.5\%$$

Calculations for $e_{ss(step)}$:

$$|\frac{Rss- Yss}{Rss}| * 100\% = |\frac{100-103.6}{100}| * 100\% = 3.6\%$$

Calculations for $e_{ss(ramp)}$:

|Rss - Yss| = |43.46 - 39.8| = 3.66

For the rest of the specifications, the values were directly measured from the graph; no calculations were necessary.

Implementing PID Control on Simulated Servo:

To find the best PID control parameters for the simulated servo, we decided to go the trial and error route to experimentally find the best values for ultimate system performance in the nominal range.

For optimal performance, the controller parameters we chose were the following:

Kp = 0.52, Ti = 5, Td = 0

Following is the output at the above set parameters after setting up a square wave input with an amplitude of 50 and frequency of 1 Hz: (The top graph consists of the reference input (purple) and system output (red); the bottom graph consists of the control signal (brown) all plotted over 4 cycles and simulation run time of 4 seconds).

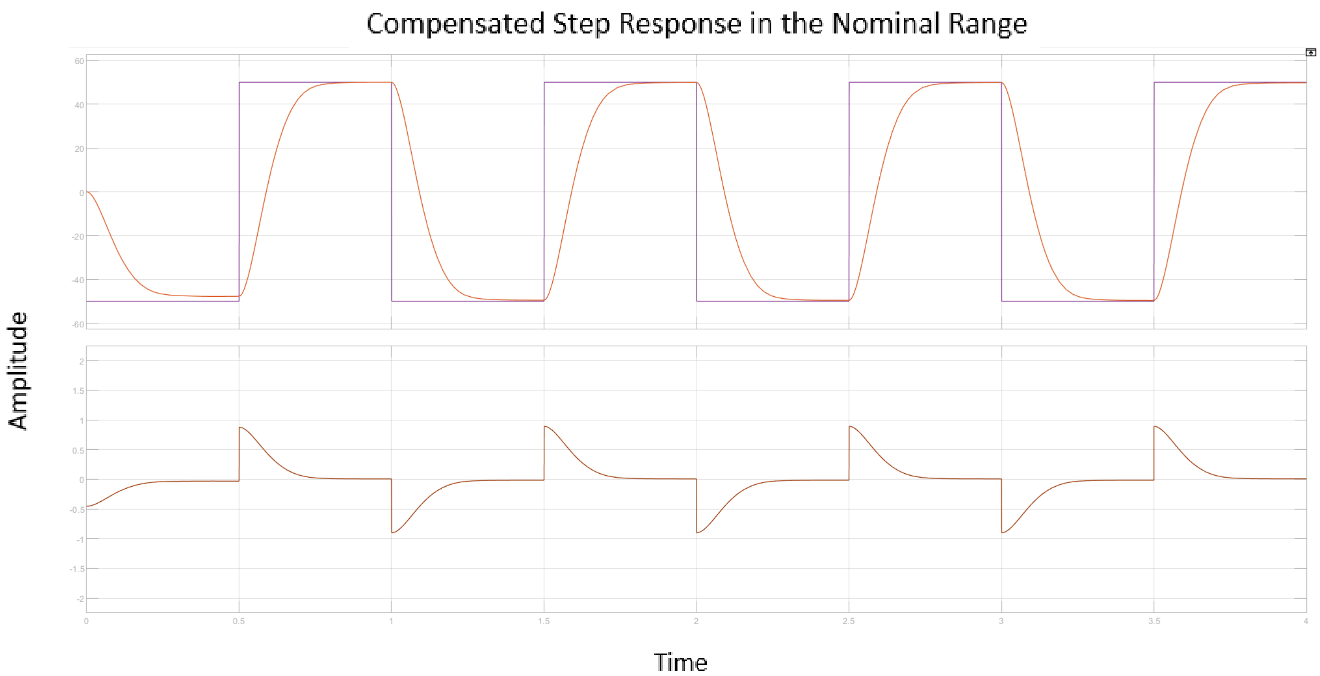

Fig. [6] Compensated Step Response

Following is the output at the above set parameters after setting up a sawtooth wave input with an amplitude of 50 and frequency of 1 rad/sec: (The top graph consists of the reference input (purple) and system output (red); the bottom graph consists of the control signal (brown) all plotted over 4 cycles and simulation run time of 26 seconds).

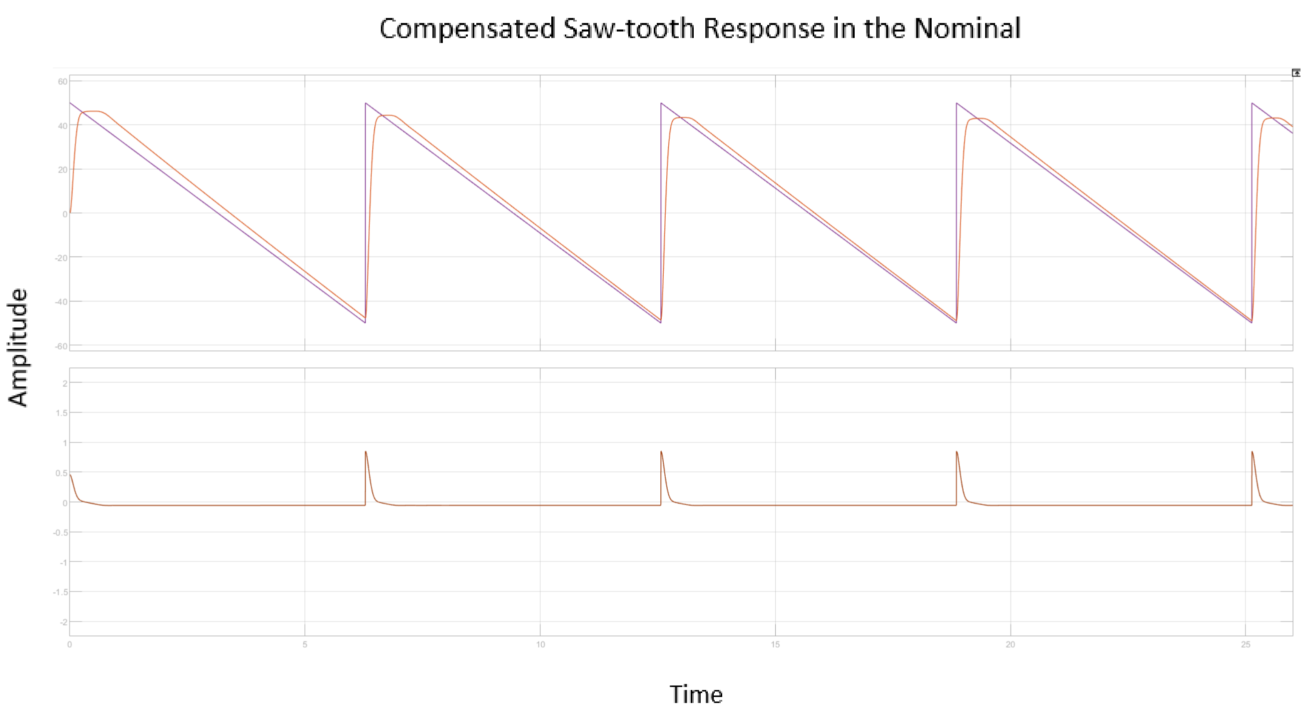

Fig. [7] Compensated Sawtooth Response

Using the additional Anti-Windup Gain in our PID controller we got the following results:
Kp = 0.45, Ti = 1.5, Td = 0, Kw = 20000

Following is the output at the above set parameters after setting up a square wave input with an amplitude of 50 and frequency of 1 Hz: (The top graph consists of the reference input (purple) and system output (red); the bottom graph consists of the control signal (brown) all plotted over 4 cycles and simulation run time of 4 seconds).

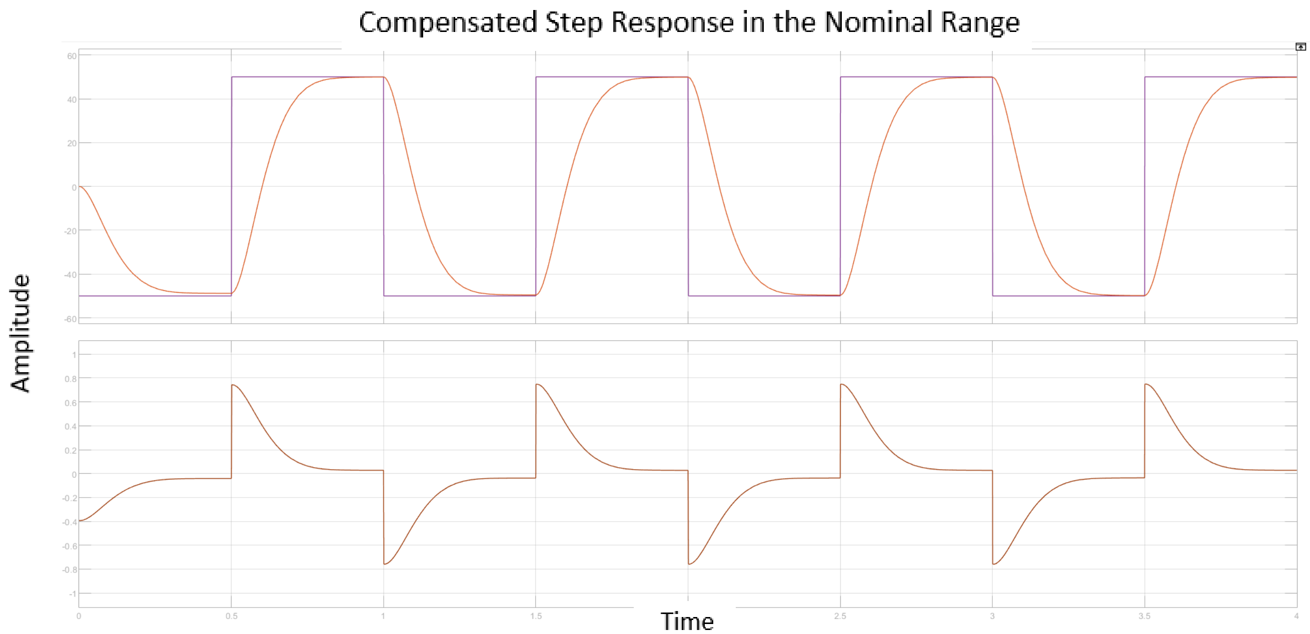

Fig. [8] Compensated Step Response

Following is the output at the above set parameters after setting up a sawtooth wave input with an amplitude of 50 and frequency of 1 rad/sec: (The top graph consists of the reference input (purple) and system output (red); the bottom graph consists of the control signal (brown) all plotted over 4 cycles and simulation run time of 26 seconds).

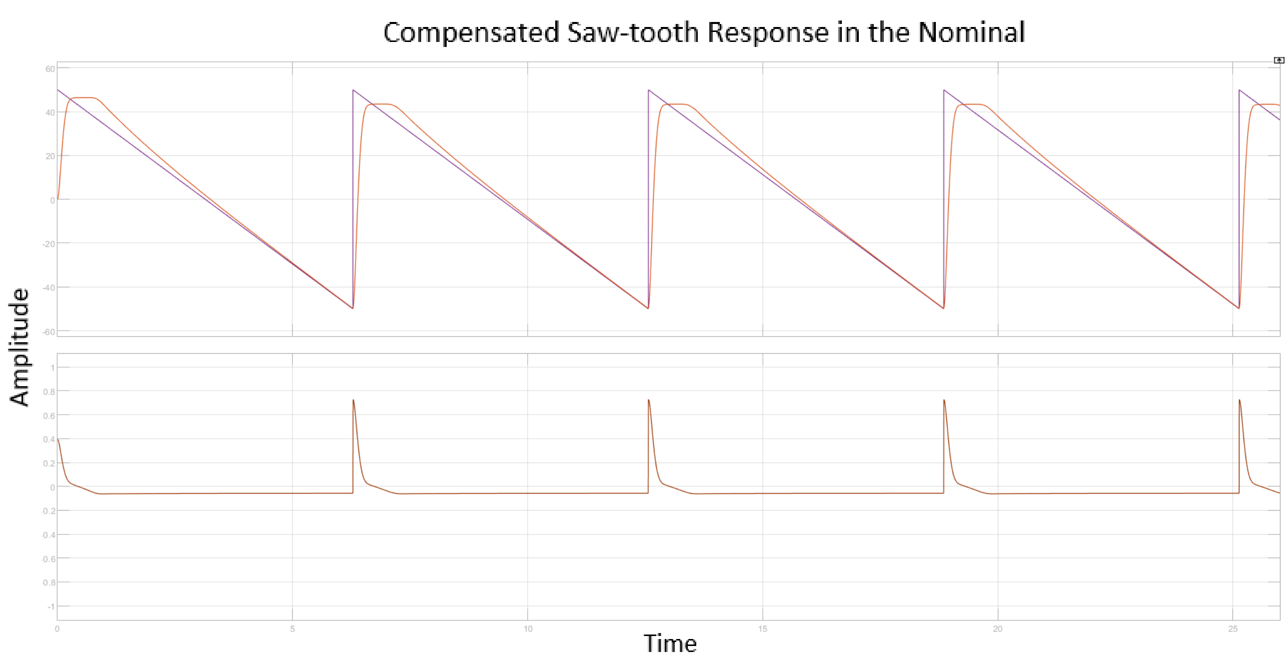

Fig. [9] Compensated Sawtooth Response

We decided to go with the PID controlled system as the "best" model.
The anti-windup gain setting allowed us to increase the proportional gain past the threshold, which would have put us outside the nominal range if we were solely using PID control. Similarly, it allowed us to decrease the integral time constant past a certain threshold without taking us out of the nominal range. We were also able to incorporate a bit of derivative control without completely disrupting the system- however it did not help our response. The Only benefit we saw with this setting in our best attainable response with it, was better steady-state error specifications; however, all other specs, apart from PO suffered.

Table. [4] The following parameters were measured from the graph for system under PID control:

Compensated System	Kp	Ti	Td	Rise-time $t_{r(0\text{-}100\%)}$	Maximum Overshoot %O.S.	Settling -time $t_{s(+/-\ 2\%)}$	Steady-state error $e_{ss(step)}$	Steady-state error $e_{ss(ramp)}$
PID Controller	0.52	5	0	400.26 ms	0	282.15 ms	0.14%	0.98

Calculations for $e_{ss(step)}$:
$|\frac{Rss - Yss}{Rss}| * 100\% = |\frac{100-99.86}{100}| * 100\% = 0.14\%$
Calculations for $e_{ss(ramp)}$:
|Rss - Yss| = |47.81 - 46.43| = 0.98
For the rest of the specifications, the values were directly measured from the graph; no calculations were necessary.

Part 3:

Operating the servo outside the nominal range using the optimal controller specs from previous experimentation:

<u>Saturation:</u>

PID control with specs (Kp = 0.52, Ti = 5 , Td = 0)

Amplitude 200, Frequency 1Hz (as step 1)

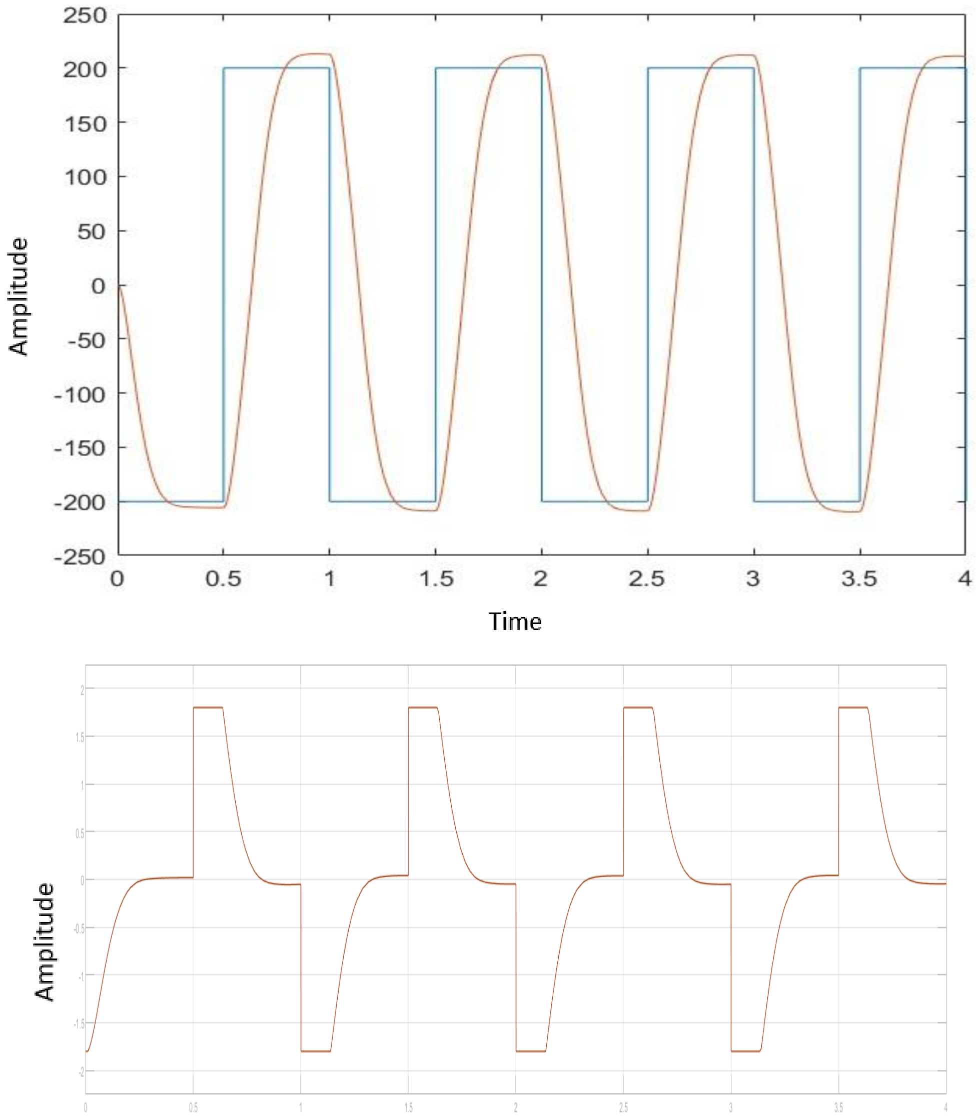

Fig. [11] Compensated PID Response Outside Nominal Range

Deadzone:

Nominal range values:
PID control with specs as step 2 (Kp = 0.52, Ti = 5 , Td = 0) Amplitude 20, Frequency 1Hz

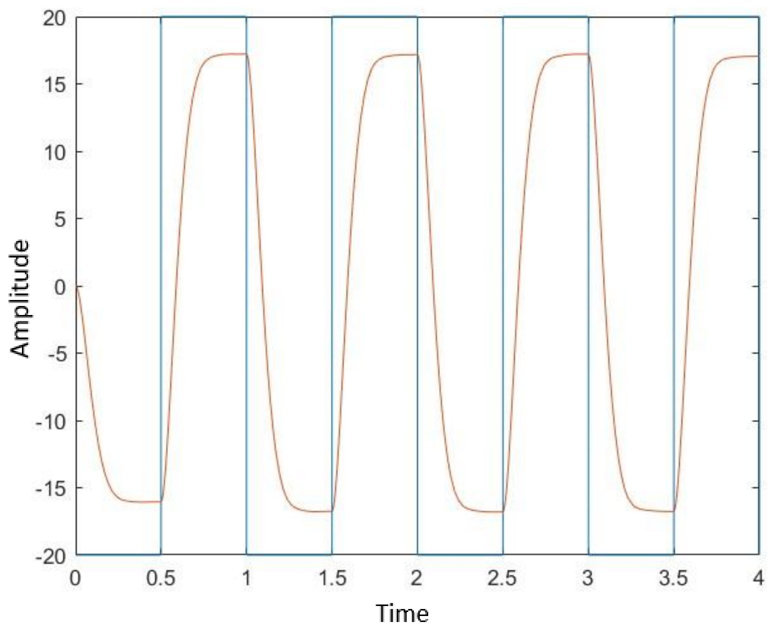

Fig. [15] Compensated PID Response Outside Nominal Range (Deadzone)

Reflection:

From our results it was observed that with the added PID control, the percent overshoot, steady-state error for both step and ramp input, and the settling time improved compared to the system under proportional control. On the other hand, the rise time was significantly worse. We used the trial and error approach to arrive at the PID controller settings. First, we attempted to get the best possible response using solely proportional control. The addition of the integral control was able to correct the percent overshoot and steady-state error. We ended up not using the derivative control since even a small value of the derivative time constant was highly disruptive to the system behavior.

The final setting of our PID controller that gave us the best response was:
Kp = 0.52, Ti = 5, Td = 0

To conclude our experiment on the simulated servo module, comparing the uncompensated specifications, proportional control compensated specifications, and the PID control compensated specifications, we were able to see mostly progressive improvement in the specs going down the line of uncompensated to PID control. The reason we might have seen degradation in some specs going from P to PID control, may have been due to our method of finding the controller parameters. It may have been better to use a more analytical method to arrive at more accurate parameters which would have been designed around the optimal parameters.

Looking at the servo operation outside the nominal range, once the input signal maximum and minimum value exceeded a certain threshold angle that was beyond the motors physical limits, the controller signal hit a plateau since it could not achieve that range. This phenomenon is called saturation. On the other hand during the deadzone state, when the input signal peak was beneath a certain threshold, the resulting control signal was negligible in its efforts to move the servo.

Part B: Real-Time Servo

Part 1: Uncompensated Servo Module Response in Nominal Range

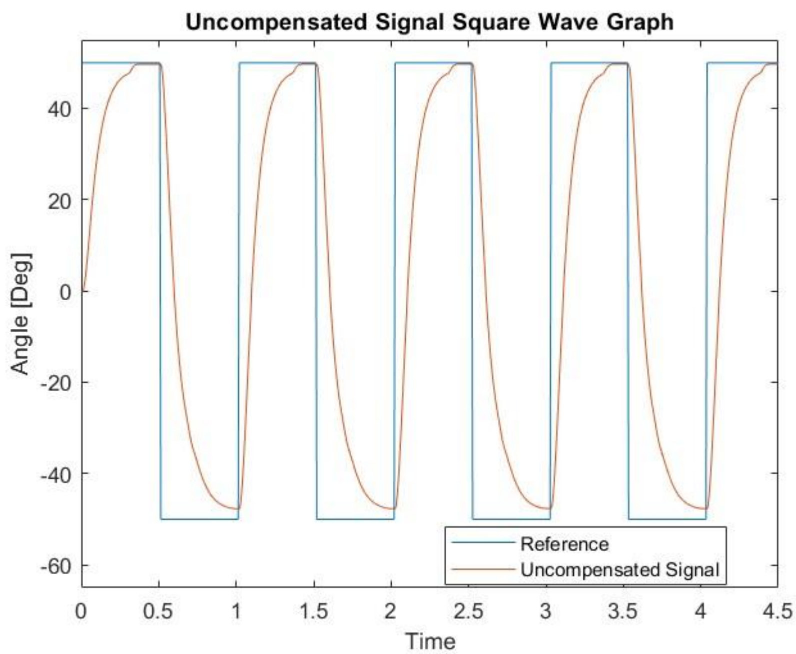

Fig. [20] Proportional Control Square Wave Graph

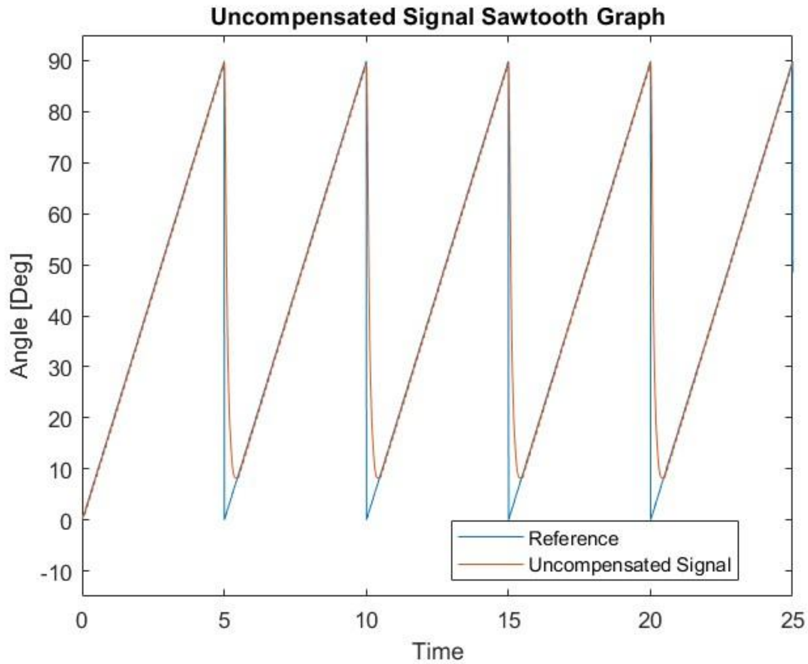

Fig. [21] Proportional Control Sawtooth Graph

Table. [7] Time Response Characteristics for Uncompensated System Under Proportional Control

Uncompensated System	Rise time $t_{r(10\text{-}90\%)}$	Maximum Overshoot %O.S	Settling time $t_{s(\mp 2\%)}$	Steady-state error $e_{ss(step)}$	Steady-state error $e_{ss(ramp)}$
Proportional Controller $K_p = 1$	0.400 s	0.000 %	0.340 s	3.380 %	0.44

Calculations for Time Response Characteristics for Uncompensated System Under Proportional Control:

Rise Time Calculations:
Point: (Time, Amplitude)
P1:(2.033s, -46.919); P2:(2.533, 48.912)
Rise time$_{(0\text{-}100\%)}$ = 2.533 - 2.033 = 0.5 s
Rise time$_{(10\text{-}90\%)}$ = 0.5 * 0.8 = 0.4 s

Max. Overshoot Calculations:
% O.S = Peak Value - Final Value/Total Variation of output signal
= 48.9122 - 48.9122 / 48.92 + 46.912
% O.S = 0 %
Peak Value = 48.9122; Final Value = 48.9122; Total Variation: 48.912 - (-46.919)

Settling - Time Calculations:
Steady State Value = 48.912
±2% Variation = 48.912 * 0.02 = 0.97824
$t_{s(+2\%)}$ = 48.89024
$t_{s(-2\%)}$ = 47.93376 = 2.373 s
$t_{s(/\pm 2\%)}$ = 2.373s - 2.033s = 0.34s

Steady State Error Calculations:
Input Variation = 100; Output Variation = 48.912 - (-47.7062) =96.6182
e_{ss} = 100 - 96.616/100 = 0.0338 * 100% = 3.38 %
Steady State Error (Ramp) Calculations:
Input = 13.08; Output = 12.64;
e_{ss} = 13.08 - 12.64 = 0.44

Part 2: Compensated Servo Module Response in Nominal Range [Proportional Controller]

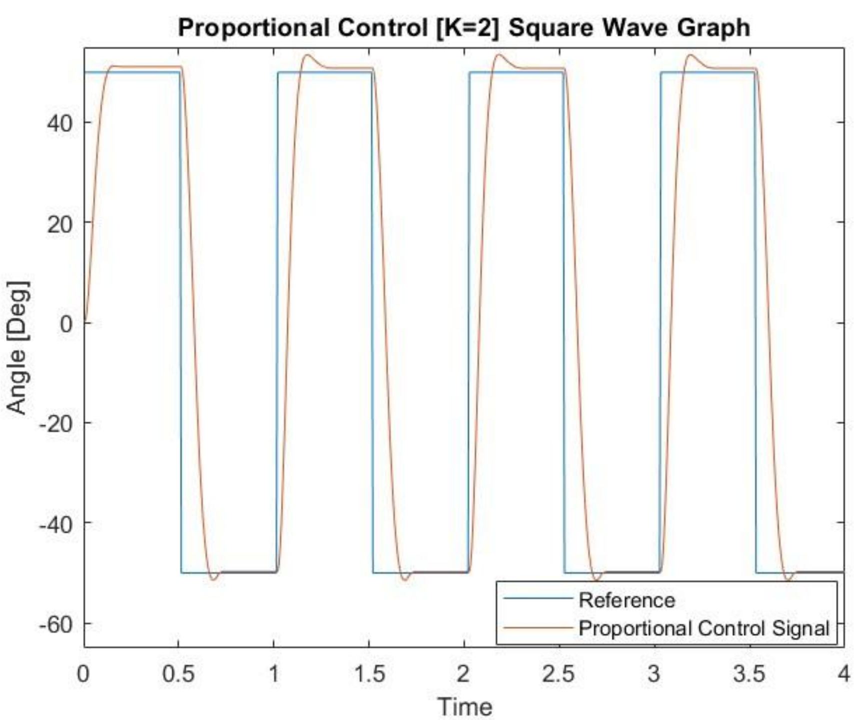

Fig. [22] Proportional Control (K=2) Square Wave Graph

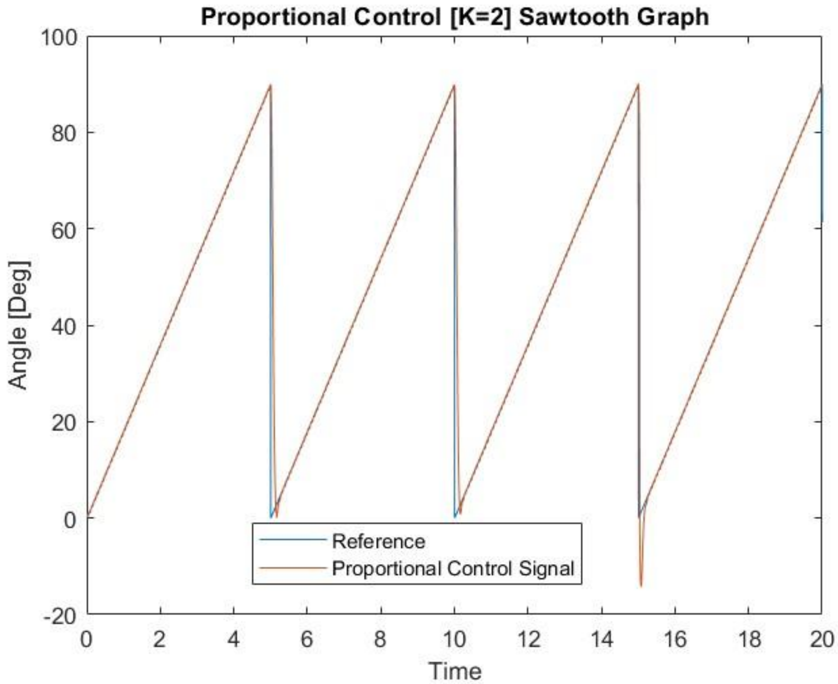

Fig. [23] Proportional Control (K=2) Sawtooth Graph

Table. [8] Time Response Characteristics for Compensated System Under Proportional Control

Compensated System	Proportional Gain	Rise time $t_{r(10\text{-}90\%)}$	Maximum Overshoot %O.S	Settling time $t_{s(\mp 2\%)}$	Steady-state error $e_{ss(step)}$	Steady-state error $e_{ss(ramp)}$
Proportional Controller	K_p = 2	0.128 s	2.664 %	0.206 s	0.4814 %	0.203

Part 2: Compensated Servo Module Response in Nominal Range [PID Controller]

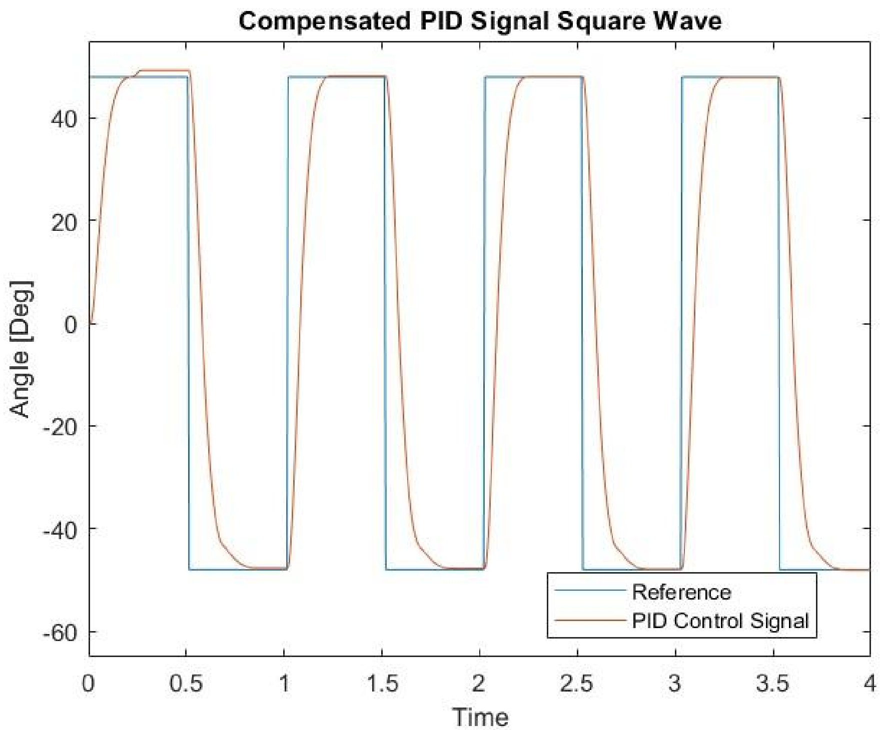

Fig. [24] PID Controller Square Wave Graph

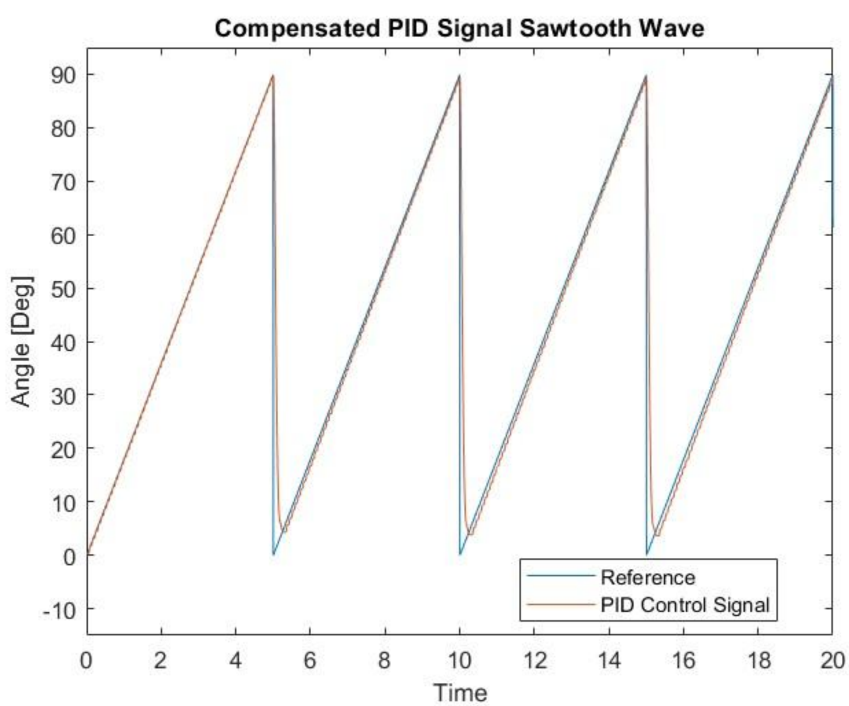

Fig. [25] PID Controller Sawtooth Wave Graph

Table. [9] Time Response Characteristics for Compensated System Under PID Control

Compensated System	Proportional Gain	Derivative Time Constant τ_d	Integral Time Constant τ_i	Rise time $t_{r(10\text{-}90\%)}$	Maximum Overshoot %O.S	Settling time $t_{s(''\mp 2\%)}$	Steady-state error $e_{ss(step)}$	Steady-state error $e_{ss(ramp)}$
Proportional Controller	K_p =1.5	0	5	0.149 s	0.000 %	0.180 s	0.186 %	0.296

Part 3: Compensated Servo Module Response Outside Nominal Range

Saturation Segment

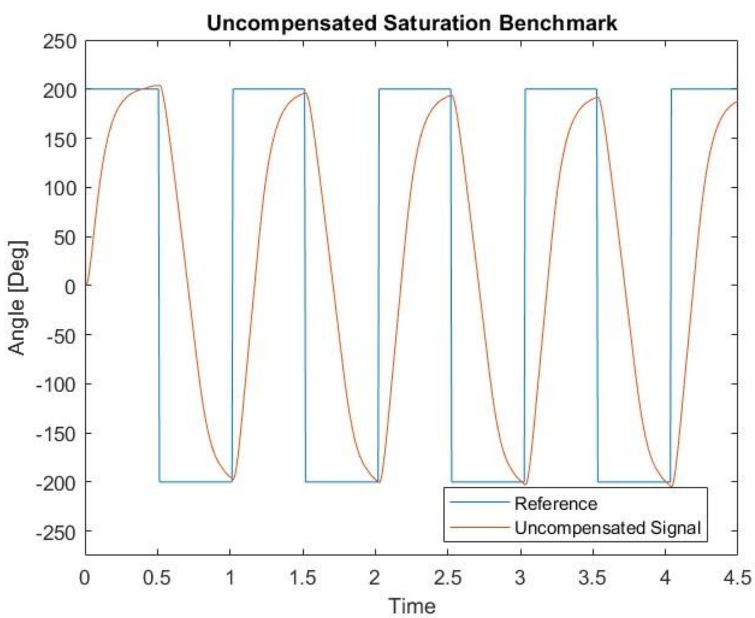

Fig. [26] Saturation Benchmark [200° Amplitude]

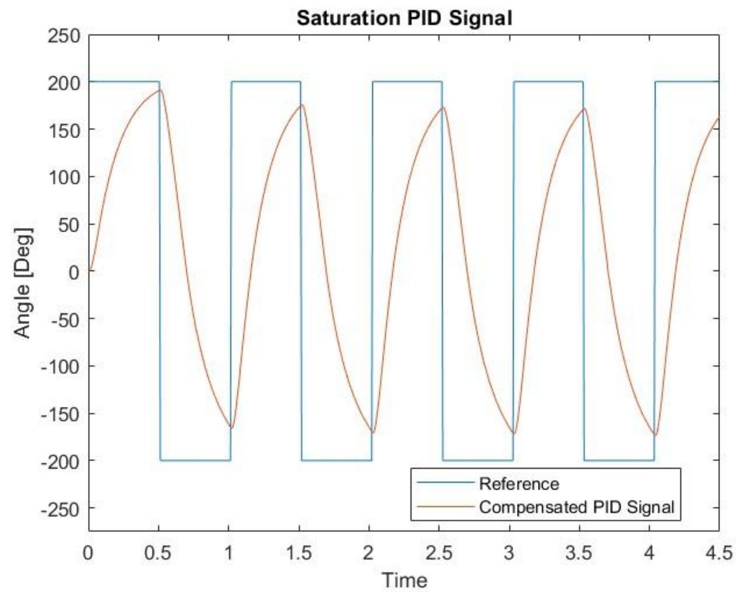

Fig. [27] Saturation PID Effects [200° Amplitude]

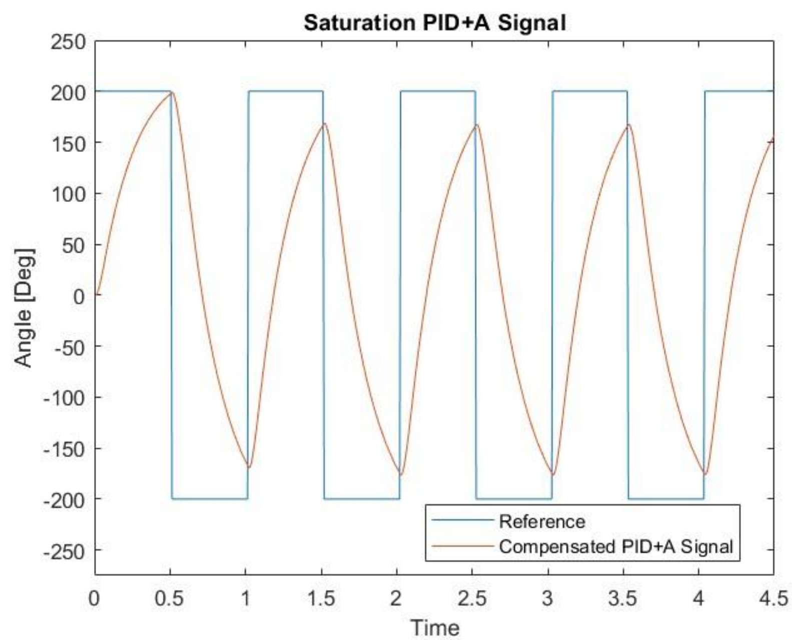

Fig. [28] Saturation PID+A Effects [200° Amplitude]

Note: In Fig.26 $K_P = 1$; $\tau_i = 10000$; $\tau_d = 0$
In Fig.27 $K_P = 1.5$; $\tau_i = 5$; $\tau_d = 0$
In Fig.28 $K_P = 1.5$; $\tau_i = 5$; $\tau_d = 0$; $K_W = 7$

Dead-Zone Segment

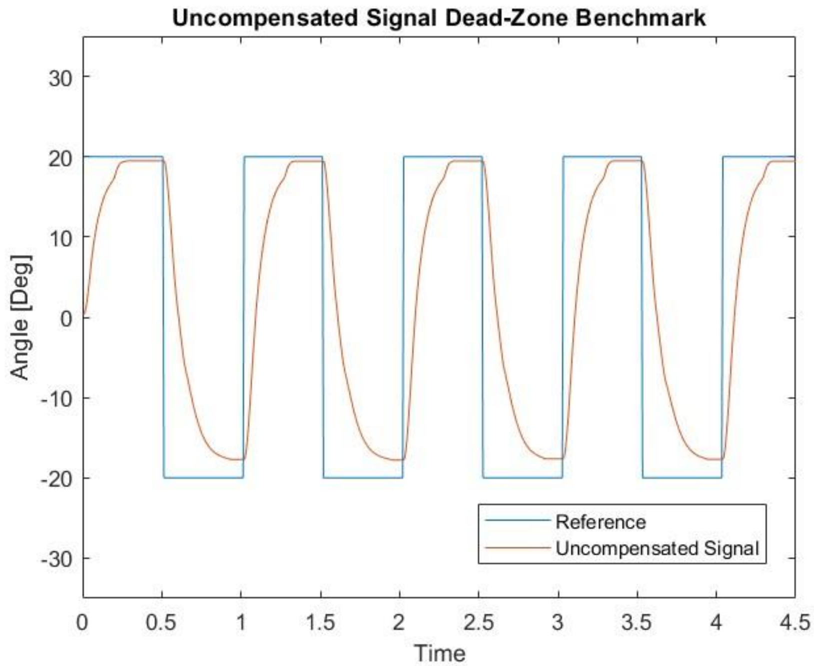

Fig. [29] Dead Zone Benchmark [20° Amplitude]

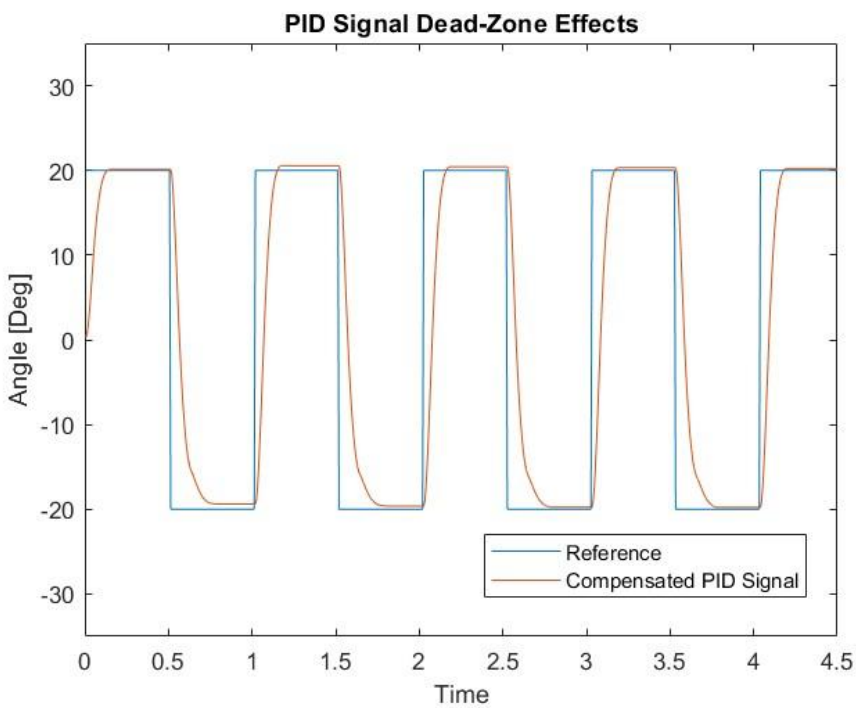

Fig. [30] Dead Zone PID Effects [20° Amplitude]

Note: In Fig.29 K_P = 1; $\boldsymbol{\tau}_i$ = 10000; $\boldsymbol{\tau}_d$ = 0
In Fig.30 K_P = 1.5; $\boldsymbol{\tau}_i$ = 5; $\boldsymbol{\tau}_d$ = 0

Reflecting on our simulation results for our optimal PID controller design operating in the nominal range, we were able to improve upon the percent overshoot, steady-state error for both step and ramp input and the settling time specifications; with controller parameters Kp=0.52, Ti=5 and Td=0. Compared to our servo module results, it took different controller parameters (Kp=1.5, Ti=7 and Td=0) to arrive at the optimal specifications tailored to the specific servo module we used in the lab. Most of our specifications from the real-time servo system response closely matched the improved specifications of the simulation; however, we noticed the settling time went up significantly for the real-time response. When looking at the operation outside the nominal range for both the real-time response and the simulated response, it was noted that both behaviors were really similar. It is important to note that for real-time, we were unable to visualize the control signal so our analysis of the saturation effects were solely based on our system response observations. The fact that our results were not identical yet similar for the most part, reinforces the fact that the parameters of the simulated motor may have slightly varied compared to the real time servo motor; thus, identical results were not expected.