Predicting the Future: SNe Ia and TRGB Data

M.L. Smith, Retired
mlsmith55@gmail.com

UMEÅ UNIVERSITY

November 19, 2024

Contents

Introduction

Scientists often use graphs to represent observations and mathematics to explain correlations.

It is common to propose several mathematical descriptions then choose the best model using statistical and prediction results as guides to worthiness.

One should select the proper metrics for data analysis. The experiment of Galileo is a good example where free fall distance is correlated to time.

One would select meters and seconds for the Galileo experiment; miles and days would be inappropriate.

Introduction

Sometimes presentations are better received when a metric is transformed; compress widely spread data (the dynamic range problem) for ease of presentation.

Metric transformation is often a mistake when applied to analysis.

We explore some problems when inappropriate metrics are chosen. One example presents astronomical observations of distant supernovae.

Another example, presented in the **Appendix**, follows cancer cell replication over time.

We find that investigators select the proper metrics for one experiment but inappropriate metrics for the other.

Introduction

A common fault is transforming data by applying the log function, log(x), then using the “new” data. Some reasons why using log(data) often leads to incorrect results:

1. Data are measured using metrics (m, km, seconds, etc.), but $\log(x)$ has no metric.
2. Standard deviations as log(σ), are distorted, which often worsens with increasing distance between the signal and observer.
3. The best data pair, sometimes the origin (0,0) without significant error, cannot be used because $\log(0) = -\infty$. Exclusion of any data pair with insignificant error can never be justified.
4. Since the σ values are distorted, computerized goodness of fit estimates are improperly estimated; χ^2 and r^2 are often incorrect, complicating model discrimination.

Standard Deviations

The the standard deviation, σ, estimates the dependent variable uncertainty [3]. This often means the real value has a $\approx 67\%$ probability of residing between the upper, lower limits of σ, presuming a Gaussian distribution of observations.

Consider a function $f_{(x)}$ where the x value has an estimated error of $\pm x_{\text{err}}$. The error for the y values at any arbitrary x value is

$$y_{\text{err}} = \frac{f(x + x_{\text{err}}) - f(x - x_{\text{err}})}{2}$$

Therefore σ estimates the y value deviation from the mean, so $\sigma = y_{\text{err}}$. The dependent variable is usually the variable with greater σ so $\sigma_{(x)}$ is often ignored.

Chi-squared test, χ^2

Pearson's χ^2 is a common measure of compliance with the data and calculated as

$$\chi^2 = \sum \left(\frac{(\text{Observed value} - \text{Expected value})^2}{\text{Denom}} \right).$$

where the Denom is either the expected value or $\sigma^2_{(y)}$. A smaller χ^2 usually indicates a better model [4].

Some investigators use the *reduced* χ^2, which is simply the "normalized" χ^2 as

$$\chi^2_{red} = \chi^2/(N - P)$$

where N is the number of data pairs and P the parameter count. Models sporting many parameters will display slightly larger values of χ^2_{red}.

r-squared test, r^2

The r-squared, r^2, statistic is a commonly used measure of the correlation between the dependent and independent variables [5,6]. It is calculated as

$$r^2 = 1 - \frac{\sum(y_{\text{observed}} - y_{\text{expected}})^2}{\sum(y_{\text{observed}} - y_{\text{mean}})^2}.$$

Formally, r^2 only applies to linear models, for more general application the *adjusted* r^2 is used as

$$r^2_{adj} = 1 - \frac{(1 - r^2)(N - 1)}{(N - P - 1)}$$

where N is the number of data pairs and P the parameter count. There are many other statistical tests which are useful to estimate model worth, the F-statistic being very good.

Codes and data

The popular Python 3 programming language used here is available for free and one helpful website is
https://www.anaconda.com/products/distribution

We checked our routines with the Jupyter Notebook and Spyder IDE using both Apple® Macintosh and Windows® 11 on a PC laptop. The publicly available data are saved as .csv files for use with *scipy*, *pandas* and *numpy* Python libraries.

Where to find the data? Where to find the programs?

All programs and data we use are available in at GitHub -

https://github.com/MLSResearchGroup/About-Transforming-Data

Data

The data are from online sources and articles [1], [11].

For the first demonstration, we use observational data of supernovae type Ia (SNe Ia) and TRGB emissions reported by the Riess and Freedman teams [1,11]. We use the *mag* (or *distance mag*) and σ as given and "back-calculate" the supernovae distances, D_L, and associated σ values.

The second experiment (**Appendix**) are counts of cancer cells observed every 4 hours for 236 hours during unhindered replication [2]. Ten independent counts were made at each time and the mean and σ were calculated.

Supernovae Type Ia Cosmology Test
background

Astrophysicists commonly correlate galactic distances with redshifts (*z*). Strong correlation is considered evidence supporting claims about Universe expansion rate, spacetime geometry, matter density, dark matter and dark energy (Λ).

Intergalactic distances are often measured from light intensity, using a metric termed μ or *distance mag*. This historic method follows the logarithmic scaling of luminosity as perceived by the human eye. The actual luminosity distance is correctly based on D_L, as parsec (pc), kiloparsec (Kpc) or megaparsec (Mpc) which are linear measures of large distances.

The best data currently comes from measuring the luminosity of distant supernovae (SNe Ia) explosions and the associated galaxy redshift, z. These emissions are considered standard candles.

Supernovae Type Ia Cosmology Test

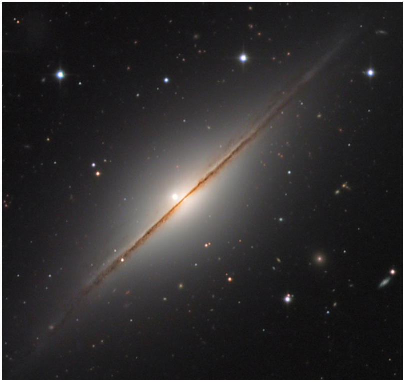


Figure: The supernova near the Little Sombrero galaxy. The bright light just to the left of the galactic center is a type Ia supernova and an example of light and possibly dust contaminations. July 17, 2021; Image from NASA.

The *distance mag* (μ) is a function of the luminosity distance, D_L, as

$$distance\ mag(\mu) = 5\log_{10} D_L + 25$$

and the luminosity distance, D_L in Mpc units, is back-calculated from *distance mag* as

$$D_L = 10^{(\mu-25)/5}.$$

Astrophysicists use the redshift, z, linearly related to galactic recession velocity, which is measured with little error as the independent variable.

The Einstein-DeSitter (E-DS) model considers our Universe being primarily spacetime without matter or dark energy [7]. To find H_0 with the *distance mag vs. z* correlation we use

magE-DS model

$$distance\ mag = 5\log_{10}\left[\frac{c(1+z)}{H_0}\sinh\left(\frac{z}{1+z}\right)\right] + 25.$$

With the D_L and z data we use the simpler and correct

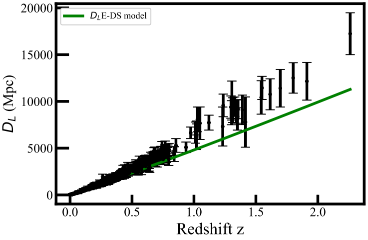
D_LE-DS model

$$D_L = \frac{c(1+z)}{H_0}\sinh\left(\frac{z}{1+z}\right)$$

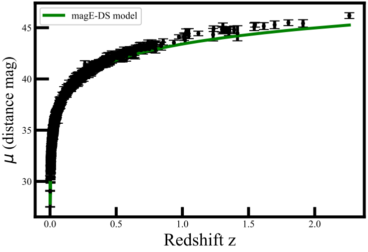
here c is lightspeed as km s^{-1} and H_0 as km s^{-1} Mpc^{-1}.

Supernovae Type Ia Cosmology Test

E-DS model using the 1701 SNe Ia with 18 TRGB data [1,11]



(a) luminosity distance vs. velocity



(b) distance mag vs. velocity

The E-DS model was suggested before the tremendous size of our Universe was known. The best data available to Einstein was published by Hubble in 1929 [8].

Supernovae Type Ia Cosmology Test
Normalized parameters

Cosmologists use *normalized* parameters to calculate properties of our Universe: for matter density Ω_m;
Ω_k for the portion which is spacetime;
Ω_Λ to describe Universe expansion [9].

normalized Ω parameters

Normalized parameters means the parameters sum to 1 as

$$\Omega_m + \Omega_\Lambda = 1$$

for the ΛCDM model, *standard model* of cosmology, but with

$$\Omega_m + \Omega_k = 1$$

for the *Arctanh* models; the Universe without dark energy.
Remember that $1 - \Omega_m = \Omega_\Lambda$ or $1 - \Omega_m = \Omega_k$.

In terms of the normalized parameters; the *standard model*

luminosity distance *vs.* expansion factor $\xi = 1/(1+z)$

$$D_L = \frac{c}{H_0 \xi} \sinh \left\{ \int_{\xi_1}^{1} \frac{d\xi}{\xi \sqrt{\frac{\Omega_m}{\xi} + \Omega_\Lambda \xi^2}} \right\}$$

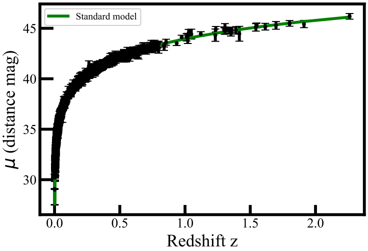
The *standard model* in terms of

distance mag vs. z [9,10]

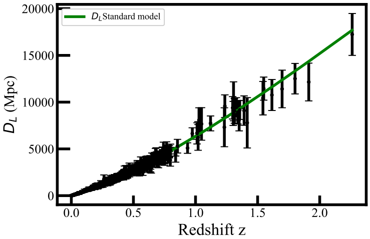
$$distance\ mag = \frac{c(1+z)}{H_0} \sinh \left\{ \int_0^z \frac{dz}{\sqrt{(1+z)^2(1+\Omega_m z) - z(2+z)(\Omega_\Lambda)}} \right\}$$

Supernovae Type Ia Cosmology Test

The *standard* models, *distance mag vs z* and D_L *vs z*



(c) distance mag vs. velocity



(d) luminosity distance vs. velocity

Two standard model plots. Note the height difference of σ values between plots.

This model describes a universe containing matter in quasi-Euclidean space geometry without the cosmological constant (dark energy) [7]. The description is

Arctanh model, luminosity distance vs. z

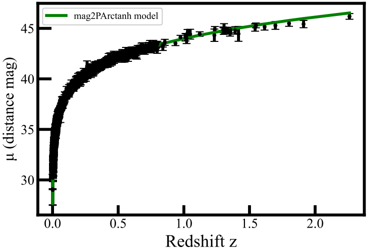
$$D_L = \frac{c}{H_0 \xi \sqrt{|\Omega_k|}} \sinh \left\{ \sqrt{|\Omega_k|} \int_{\xi_1}^{1} \frac{d\xi}{\xi \sqrt{\frac{\Omega_m}{\xi} + \Omega_k}} \right\}$$

and this can be integrated to become (with $\xi = 1/(1+z)$)

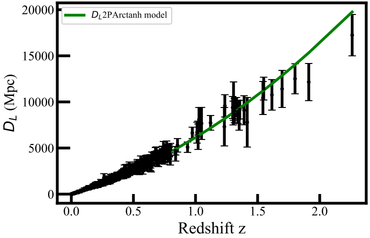
$$D_L = \frac{c(1+z)}{H_0 \sqrt{|\Omega_k|}} \sinh \left\{ 2 \left(\operatorname{arctanh}(\sqrt{|\Omega_k|}) - \operatorname{arctanh}\left(\frac{\sqrt{|\Omega_k|}}{\sqrt{\Omega_m (1+z) + \Omega_k}} \right) \right) \right\}$$

Supernovae Type Ia Cosmology Test

The Arctanh models, *distance mag vs z* and D_L *vs z*



(e) distance mag vs. velocity



(f) luminosity distance vs. velocity

These plots are appear similar to the *Standard* model plots though the meanings are quite different.

Results from the Einstein-DeSitter, *standard* and *Arctanh* models using the 1701 SNe Ia with 18 TRGB data. All calculations use relative weights of σ values.

model	H_o (km s^{-1} Mpc^{-1})	Ω_m with Ω_k or (Ω_Λ)	χ^2_{red}
D_LE-DS	66.2±0.2	≈ 0	1.38
*mag*E-DS	65.2±0.2	≈ 0	1.43
standard	70.7±0.2	(0.49±0.02)	0.72
D_L3P*standard*	71.2±0.2	0.001±0.15	0.69
D_L2PArctanh	70.8±0.2	0.05±0.03	0.69
*mag*2PArctanh	70.1±0.2	0.08±0.03	0.70

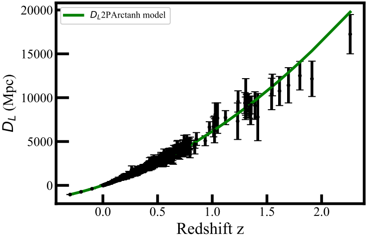
The D_L3P*standard* model includes the H_0, Ω_m and Ω_Λ as free parameters. Note the σ of Ω_m for the D_L3P*standard* model is very large due to the use of 3 free parameters.

Proper science describes the present and predicts the future. Most interesting, calculation of the origin at $z = 0$, our present situation, is impossible with the *mag*arctanh or *standard* models with the *distance mag vs. z* correlation

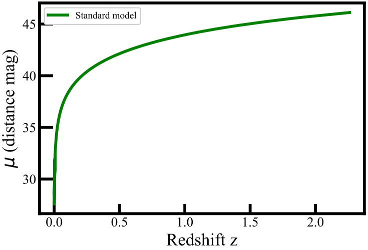
- the log of our position at the origin 0 is $-\infty$
- but our position and recession velocity are known with 100% certainty

Our present and future conditions are presented in the next two figures.

Also - a plot of *distance mag vs z* can neither locate the Milky Way at this moment nor predict any future position.



(g) luminosity distance vs. velocity



(h) distance mag vs. velocity

The calculated curve (left) exists at $z < 0$. Python returns error messages for a value of *distance mag* at $z = 0$ (right).

Supernovae Type Ia Cosmology Test
Results

- The E-DS model functions well for nearby TRGB and SNe but fails at larger distances*
- The values for Ω_m from the *standard* model is significantly higher than the *Arctanh* models
- The Ω_k is calculated to be very large (≈1) for the *Arctanh* models but presumed to be ≈0 for the *standard* model
- Because σ values are smaller after log transformation, resulting χ^2_{red} values of should be considered doctored
- Parameter values and statistics differ between metrics; statistics are distorted by the *distance mag* transformation [12,13]

*The statistics calculated using these data do not necessarily support a preference for the *standard* model over the *arctanh* model but only over the E-DS model

Conclusions

We demonstrate that data transformation shall be avoided

- Transforming data just to visualize a straight line may lead investigators towards incorrect results
- The σ values associated with TRGB and SNe Ia *distance mag* values [1, 10] suffer contraction
- Discarding the origin when employing the *distance mag* transformation cannot be justified
- Calculated parameters and σ should be similar using either metric values or transformed values but are not
- The example following cancer cell counts during replication showed that transforming good data, especially the σ values, can lead to faulty statistics and is a waste of time (**Appendix**)

Bibliography

1 D. Brout, et al., The Pantheon+ Analysis: Cosmological Constraints. *Astrophys. J.* **938**, 110 (2022). DOI 10.3847/1538-4357/ac8e04

2 K.E. Johnson, et al., Cancer cell population growth kinetics at low densities deviate from the exponential growth model and suggest an allee effect. *PLoS Biology* **17**, e3000399 (2019).

3 M. Hargrave, Standard Deviation Formula and Uses vs. Variance. *Investopedia* (2022, July 6). https://www.investopedia.com/terms/s/standarddeviation.asp

4 A. Hayes, Chi-Square (χ^2) Statistic. *Investopedia* (2022, October 23). https://www.investopedia.com/terms/c/chi-square-statistic.asp

5 J. Fernando, R-Squared Formula, Regression, and Interpretations. *Investopedia*(2021, September 12). https://www.investopedia.com/terms/r/r-squared.asp

6 S. Gupta, R-Squared: Formula Explanation - Analytics Vidhya. *Medium* (2021, December 28). https://medium.com/analytics-vidhya/r-squared-formula-explanation-6dc0096ce3ba

Bibliography

7 A.M. Oztas, et al., Spacetime Curvature is Important for Cosmology Constrained with Supernova Emissions. *Int. J. Theor. Phys.* **47**, 2474 (2008). DOI 10.1007/s10773-008-9680-7

8 E. Hubbel, A relation between distance and radial velocity among extra-galactic nebulae. *Proc. Nat. Acad. Sci. USA* **15**, 168. DOI 10.1073/pnas.15.3.168

9 S.M. Carroll, et al., The Cosmological Constant. *Ann. Rev. Astron. Astrophys.* **30**, 499 (1992). DOI 10.1146/annurev.aa.30.090192.002435

10 A.G. Riess, et al, Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant. *Astronom. J.* **116**, 1009 (1998). DOI 10.1086/300499

11 W.L. Freedman, et al., The Carnegie-Chicago Hubble Program. VIII. *Astrophys. J.* **882**, 34. DOI 10.3847/1538-4357/ab2f73

12 M.L. Smith & A.M. Oztas, Log-transformation problems: astrophysics examples. (2017) https://www.youtube.com/watch?v=Y1nEQmg2yJA&feature=youtu.be

13 M.L. Smith & A.M. Oztas, Lawrence Krause Do Your Homework. (2017) https://www.youtube.com/watch?v=IN0jqhqjW3Y&feature=youtu.be

Notes on cosmology terms

Some terms applied to spacetime geometry

The following terms are used interchangeably by many physicists and astronomers though the exact meanings are not identical

- elliptical (concave) geometry and closed universe
- flat geometry and Euclidean and quasi-Euclidean geometry
- hyperbolic (convex) geometry and open universe*
- *hyperbolic geometry is impossible in a universe with matter - only possible in a universe with anti-gravity

Cancer cell replication is a good example of exponential growth. We model the cell count increase over time as

Exponential model (Exp)

$$N(t) = N_0 e^{\beta t} \tag{1}$$

Here N_0 is the initial cell number, β the replication rate and t, the time (hours). To transform the dependent data we take the natural logarithm of equation (1) on both sides yielding

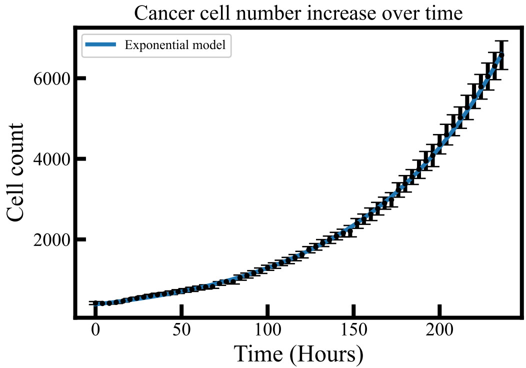
logarithmic model (Log)

$$\ln[N(t)] = \ln N_0 + \beta t \tag{2}$$

our logarithmic model is an example of coordinate transformation; the ordinate data are transformed into log_e*(data)*.

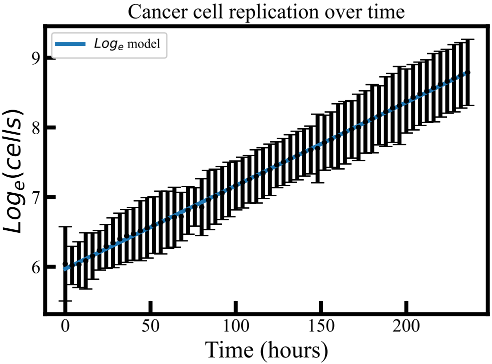
Cancer Cell Replication
Exponential Model Plot

Data taken from electronic publication



Note the steady increase of standard deviation with time.

Cancer Cell Replication
Log Model Plot



The semi-log plot example. Note the smaller increase of $Log_e(\sigma)$ with time.

Cancer Cell Replication

Table: Results from the Exp and Log models of cancer cell replication data using curve_fit regression routine. Both calculations use relative weights of σ values.

Model	N_0 (initial cell count)	β (hour^{-1}) growth rate	r^2_{adj}	χ^2
Exp	389.6± 2.3	0.012$\pm 4E^{-5}$	0.9995	10.1
Log_e	391.4± 1.0	0.012$\pm 5E^{-5}$	0.9992	0.22

- Both r^2_{adj} values are ≈1 but χ^2 values are dissimilar; these two regressions should be comparable but are not
- Distortion of the $Log_e(\sigma)$ values is reflected in the very small χ^2; warning that these results are suspect
- The value for N_0 are smaller than the cell count at $t = 0$, indicating some cells died during experiment initiation

Some questions may be asked

- Derive the well known equation for standard deviation

$$\sigma = \sqrt{\frac{\sum(x_i - \mu)}{N}}$$

  where μ is the mean, x_i the observed values and N the number of observations, from the equation on page 6.
- Why does the Log model presents a slightly larger estimate of initial cell count (420) than the Exp model?
- Does this justify using a Log model, or is it just happenstance?
- Evaluate the local Hubble law $\frac{c}{\xi} = H_0 D_L$ beginning with the E-DS or arctanh model?
- Which universe model do you think is closer to reality? Why?

Some more questions

- The expansion factor, $\xi = 1$, is our relative recession velocity and the position of our earth, $D_L = 0$, a certainty. Now consider our situation when the recession velocity $z \to 0$ and *mag*$= -\infty$.
  - Is it justified to discard the position of the earth, the only position and velocity known with full certainty, when using the *mag* models?
- The σ values associated with the *mag* data are relatively smaller than the σ values associated with D_L. Do we always want smaller errors, explain?
- When Einstein derived his cosmological model (E-DS), it was thought the universe was only the Milky Way and all stars are motionless.
  - Do you think this is reflected in the E-DS model?
  - Historical question - Why did Einstein and Stephen Hawking consider our Universe matter dominated?
- Difficult question - edit either the Arctanh or magArctanh model to evaluate the data with one more free parameter - the Ω_k. The code will now have 3 parameters. If successful note the standard deviations.