COMSATS University Islamabad, Lahore Campus

Department of Electrical and Computer Engineering

Terminal – Spring 2021

Degree Program: BS Electrical Engineering, BS Computer Engineering
Course: Probability Methods in Engineering
Date of Examination: 01.07.2021
Resource Persons: Dr. Ejaz Ansari, Dr. Muhammad Farooq-i-Azam, Syed Ahmad Faran
Student's name: _____

Course Code: EEE251
Class: FA19-BEE-A, B, C FA19-BCE-A, B
Semester: 4
Time duration: 3 hours This includes submission time.
Total Marks: 100
Roll No: _____

Instructions

- Solve the following questions on paper in legible handwriting.
- Do not use large number of pages unnecessarily, it will increase the size of soft file.
- Sign each page of solution at the bottom.
- Scan the pages using camscanner, clear scanner or Microsoft Lens.
- Save the file as Terminal-FA19-BEE-000.pdf replacing 000 with your roll number.
- Scan and save the pdf file using minimum size option.
- Keep the hard paper copy of solution safe with you. It may be required to be submitted in paper form in future when you attend university.
- Total time is 3 hours, including 15 minutes for saving, scanning, making PDF and finally uploading the file on CUonline. Late submissions will not be accepted and will be dealt with as per decision of the Departmental Examination Committee.
- R is your roll number in the following questions.

Q.1) (CLO1–PLO1) 20 marks

An electronic machine generates tokens for customers having serial numbers ranging randomly from 1 to $R + 60$. Suppose you are the first person to use this machine and you take two tokens from the machine.

(a) If the machine generates one serial number only once, compute the probability that you will get serial numbers 5 and 60. 7 marks

(b) If the machine can generate one serial number repeatedly, compute the probability that you will get serial numbers 5 and 60. 7 marks

(c) Identify with reason, whether the experiment in part (a) or part (b) is experiment with independent trials. 6 marks

Q.2) (CLO2–PLO2) 20 marks

Part A

A random variable X has the following PMF

$$P_X(x) = \begin{cases} \frac{c}{x}, & x = R,\, 2R,\, 4R, \\ 0, & \text{otherwise.} \end{cases}$$

Determine and compute the following

(a) The constant c, 4 marks

(b) Probability $P[3R \leq X \leq 5R]$. 6 marks

Part B

A random variable X has the following probability density function (PDF)

$$f_X(x) = \begin{cases} cx, & 0 \leq x \leq 2R, \\ 0, & \text{otherwise.} \end{cases}$$

Determine and compute the following

(a) The constant c, 4 marks

(b) Probability $P[0 \leq X \leq R]$. 6 marks

Q.3) (CLO2–PLO2) 20 marks

The probability density function of a random variable X is

$$f_X(x) = \begin{cases} \frac{1}{R} e^{-\frac{x}{R}}, & x \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

Analyze and compute the following

(a) Probability $P[X < 0]$. 2 marks

(b) Probability $P[X < R]$. 4 marks

(c) Moment generating function (MGF) $\phi_x(s)$. 10 marks

(d) First moment using the MGF. 4 marks

Q.4) (CLO2–PLO2) 20 marks

The number of messages sent by a student during a day is a random variable X, with the mean value $\mu_X = 10R$, and variance $\sigma_x^2 = 4R$.

(a) Use the Markov inequality to compute bounds on the probability that the number of messages sent in a day lies between $8R$ and $12R$. 10 marks

(b) Use the Chebyshev inequality to compute a lower bound on the probability that the number of messages sent in a day lies between $8R$ and $12R$. 10 marks

Q.5) (CLO3–PLO4) 20 marks

The random variables X and Y have joint probability density function (PDF)

$$f_{X,Y}(x,y) = \begin{cases} c(x+y), & 0 \le x \le y \le R, \\ 0, & \text{otherwise.} \end{cases}$$

Analyze and determine the following

(a) The constant c. 2 marks

(b) Marginal PDF $f_X(x)$. 2 marks

(c) Marginal PDF $f_Y(y)$. 2 marks

(d) Variance $Var[X] = \sigma_x^2$. 3 marks

(e) Variance $Var[Y] = \sigma_y^2$. 3 marks

(f) Covariance of X and Y, $COV[X,Y] = \sigma_{XY}$. 3 marks

(g) Correlation coefficient, ρ_{XY}. 2 marks

(h) The linear minimum mean square error estimator of X given Y i.e. $\hat{X}_L(Y)$. 3 marks