



COMSATS University Islamabad, Lahore Campus  
Department of Electrical and Computer Engineering  
Terminal Examination – Spring 2024

Course: Probability Methods in Engineering  
Degree Program: BS Electrical Engineering,  
BS Computer Engineering  
Date of Examination: 11.06.2024  
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Course Code: EEE251, CPE251  
Credit Hours: 3(3,0)  
Class: FA22-BEE-A, B  
FA22-BCE-A, B  
Time duration: 3 hours  
Total Marks: 100  
Roll No: [REDACTED]

Question 1

(C3-CLO1-PL01)

[20 marks]

Part A

An integrated circuit factory has three machines X, Y, and Z. Test one integrated circuit produced by each machine. Either a circuit is acceptable ( $a$ ) or it fails ( $f$ ). An observation is a sequence of three test results corresponding to the circuits from machines X, Y, and Z, respectively. For example,  $aaf$  is the observation that the circuits from X and Y pass the test and the circuit from Z fails the test.

- (a) Show the sample space of the experiment. [2 marks]
- (b) Show the elements of the following sets, [2 marks]  
 $Z_F = \{\text{circuit from Z fails}\},$   
 $X_A = \{\text{circuit from X is acceptable}\}.$
- (c) Discover and show whether  $Z_F$  and  $X_A$  are mutually exclusive. [3 marks]
- (d) Show whether  $Z_F$  and  $X_A$  are collectively exhaustive. [3 marks]

Part B

Mobile telephones perform handoffs as they move from cell to cell. During a call, a telephone either performs zero handoffs ( $H_0$ ), one handoff ( $H_1$ ), or more than one handoff ( $H_2$ ). In addition, each call is either long ( $L$ ), if it lasts more than three minutes, or brief ( $B$ ). The following table describes the probabilities of the possible types of calls.

	$H_0$	$H_1$	$H_2$
$L$	0.1	0.1	0.2
$B$	0.4	0.1	0.1

Solve to find the following probabilities,

- (a)  $P[H_0]$  i.e. a phone makes no handoffs. [2 marks]
- (b)  $P[B]$  i.e. a call is brief. [2 marks]
- (c) a call is long or there are at least two handoffs. [2 marks]

(d)  $P[L|H_0]$ 

[2 marks]

(e)  $P[B|H_0]$ 

[2 marks]

**Question 2**

(C4-CLO2-PLO2)

[20 marks]

**Part A**The cumulative distribution function (CDF) of a random variable  $X$  is given by

$$F_X(x) = \begin{cases} 0 & x < -1, \\ \frac{x+1}{2} & -1 \leq x < 1 \\ 1 & x \geq 1. \end{cases}$$

(a) Sketch to illustrate the CDF.

[2 marks]

(b) Compute  $P[X = \frac{1}{2}]$ .

[2 marks]

(c) Identify  $P[X \leq 0]$ .

[2 marks]

(d) Analyze and compute the probability density function (PDF) of  $X$ .

[2 marks]

(e) Sketch to illustrate the PDF.

[2 marks]

**Part B**A lead acid battery lasts, on average, 3.0 years with a standard deviation of 0.5 year. Let the battery life be a random variable  $X$  which follows Gaussian distribution.(a) Identify and write down the probability density function (PDF) of  $X$ .

[3 marks]

(b) Sketch to illustrate the PDF of  $X$ .

[3 marks]

(c) Analyze and compute the probability that the battery will last between 2 and 4 years.

[4 marks]

**Question 3**

(C4-CLO2-PLO2)

[20 marks]

The random variables  $X$  and  $Y$  have the following joint probability mass function (PMF),

$P_{X,Y}(x,y)$	$y = 0$	$y = 1$	$y = 2$
$x = 0$	0.01	0	0
$x = 1$	0.09	0.09	0
$x = 2$	0	0	0.81

Identify the followings using the knowledge of pairs of random variables.

(a) The marginal PMF  $P_X(x)$ 

[3 marks]

(b) The marginal PMF  $P_Y(y)$ 

[3 marks]

(c) Are  $X$  and  $Y$  independent?

[2 marks]

(d)  $\text{Var}[X]$ 

[2 marks]

(e)  $\text{Var}[Y]$ 

[2 marks]

(f) Correlation,  $r_{XY}$ 

[2 marks]

- (g) Are  $X$  and  $Y$  orthogonal? [1 mark]  
 (h) Covariance,  $COV[X, Y]$  [2 marks]  
 (i) Correlation coefficient,  $\rho_{XY}$  [2 marks]  
 (j) Are  $X$  and  $Y$  correlated? [1 mark]

**Question 4**

(C4-CLO2-PLO2)

[20 marks]

Random variables  $X$  and  $Y$  have the joint probability mass function as given below:

$P_{X,Y}(x,y)$	$y = -1$	$y = 0$	$y = 1$
$x = -2$	0.42	0.12	0.16
$x = -1$	0.28	0.08	0.04

- (a) *Identify and compute* the marginal PMFs  $P_X(x)$  and  $P_Y(y)$ . [4 marks]  
 (b) *Illustrate and compute* the moment generating functions (MGFs) of  $X$  and  $Y$ . [6 marks]  
 (c) *Analyze and identify* the MGF of  $Z = X + Y$ . [6 marks]  
 (d) *Compute* the PMF of  $Z$ . [4 marks]

**Question 5**

(C4-CLO3-PLO4)

[20 marks]

$X_1, X_2, \dots, X_n$  are  $n$  independent identically distributed samples of random variable  $X$  with the following PMF:

$$P_X(x) = \begin{cases} 0.1 & x=0, \\ 0.9 & x=1, \\ 0 & \text{otherwise.} \end{cases}$$

- (a) *Identify and compute*  $E[X]$  and  $P_X(1)$ . [4 marks]  
 (b) *Analyze and identify* the confidence level  $\alpha$  such that  $M_{90}(X)$ , the estimate based on 90 observations, is within 0.05 of  $P_X(1)$ . Stated in other words, find  $\alpha$ , such that [8 marks]

$$P[|M_{90}(X) - P_X(1)| \geq 0.05] \leq \alpha$$

- (c) *Analyze and illustrate* how many samples  $n$  are necessary to have  $M_n(X)$  within 0.03 of  $P_X(1)$  with confidence level 0.1. In other words, find  $n$ , such that, [8 marks]

$$P[|M_n(X) - P_X(1)| \geq 0.03] \leq 0.1$$



$z$	$\Phi(z)$	$z$	$\Phi(z)$	$z$	$\Phi(z)$	$z$	$\Phi(z)$	$z$	$\Phi(z)$	$z$	$\Phi(z)$
0.00	0.5000	0.50	0.6915	1.00	0.8413	1.50	0.9332	2.00	0.97725	2.50	0.99379
0.01	0.5040	0.51	0.6950	1.01	0.8438	1.51	0.9345	2.01	0.97778	2.51	0.99396
0.02	0.5080	0.52	0.6985	1.02	0.8461	1.52	0.9357	2.02	0.97831	2.52	0.99413
0.03	0.5120	0.53	0.7019	1.03	0.8485	1.53	0.9370	2.03	0.97882	2.53	0.99430
0.04	0.5160	0.54	0.7054	1.04	0.8508	1.54	0.9382	2.04	0.97932	2.54	0.99446
0.05	0.5199	0.55	0.7088	1.05	0.8531	1.55	0.9394	2.05	0.97982	2.55	0.99461
0.06	0.5239	0.56	0.7123	1.06	0.8554	1.56	0.9406	2.06	0.98030	2.56	0.99477
0.07	0.5279	0.57	0.7157	1.07	0.8577	1.57	0.9418	2.07	0.98077	2.57	0.99492
0.08	0.5319	0.58	0.7190	1.08	0.8599	1.58	0.9429	2.08	0.98124	2.58	0.99506
0.09	0.5359	0.59	0.7224	1.09	0.8621	1.59	0.9441	2.09	0.98169	2.59	0.99520
0.10	0.5398	0.60	0.7257	1.10	0.8643	1.60	0.9452	2.10	0.98214	2.60	0.99534
0.11	0.5438	0.61	0.7291	1.11	0.8665	1.61	0.9463	2.11	0.98257	2.61	0.99547
0.12	0.5478	0.62	0.7324	1.12	0.8686	1.62	0.9474	2.12	0.98300	2.62	0.99560
0.13	0.5518	0.63	0.7357	1.13	0.8708	1.63	0.9484	2.13	0.98341	2.63	0.99573
0.14	0.5558	0.64	0.7389	1.14	0.8729	1.64	0.9495	2.14	0.98382	2.64	0.99585
0.15	0.5598	0.65	0.7422	1.15	0.8749	1.65	0.9505	2.15	0.98422	2.65	0.99598
0.16	0.5638	0.66	0.7454	1.16	0.8770	1.66	0.9515	2.16	0.98461	2.66	0.99609
0.17	0.5678	0.67	0.7486	1.17	0.8790	1.67	0.9525	2.17	0.98500	2.67	0.99621
0.18	0.5718	0.68	0.7517	1.18	0.8810	1.68	0.9535	2.18	0.98537	2.68	0.99632
0.19	0.5758	0.69	0.7549	1.19	0.8830	1.69	0.9545	2.19	0.98574	2.69	0.99643
0.20	0.5798	0.70	0.7580	1.20	0.8849	1.70	0.9554	2.20	0.98610	2.70	0.99653
0.21	0.5838	0.71	0.7611	1.21	0.8869	1.71	0.9564	2.21	0.98645	2.71	0.99664
0.22	0.5878	0.72	0.7642	1.22	0.8888	1.72	0.9573	2.22	0.98679	2.72	0.99674
0.23	0.5918	0.73	0.7673	1.23	0.8907	1.73	0.9582	2.23	0.98713	2.73	0.99683
0.24	0.5958	0.74	0.7704	1.24	0.8925	1.74	0.9591	2.24	0.98745	2.74	0.99693
0.25	0.5998	0.75	0.7734	1.25	0.8944	1.75	0.9599	2.25	0.98778	2.75	0.99702
0.26	0.6038	0.76	0.7764	1.26	0.8962	1.76	0.9608	2.26	0.98809	2.76	0.99711
0.27	0.6078	0.77	0.7794	1.27	0.8980	1.77	0.9616	2.27	0.98840	2.77	0.99720
0.28	0.6118	0.78	0.7823	1.28	0.8997	1.78	0.9625	2.28	0.98870	2.78	0.99728
0.29	0.6158	0.79	0.7852	1.29	0.9015	1.79	0.9633	2.29	0.98899	2.79	0.99736
0.30	0.6198	0.80	0.7881	1.30	0.9032	1.80	0.9641	2.30	0.98928	2.80	0.99744
0.31	0.6238	0.81	0.7910	1.31	0.9049	1.81	0.9649	2.31	0.98956	2.81	0.99752
0.32	0.6278	0.82	0.7939	1.32	0.9066	1.82	0.9656	2.32	0.98983	2.82	0.99760
0.33	0.6318	0.83	0.7967	1.33	0.9082	1.83	0.9664	2.33	0.99010	2.83	0.99767
0.34	0.6358	0.84	0.7995	1.34	0.9099	1.84	0.9671	2.34	0.99036	2.84	0.99774
0.35	0.6398	0.85	0.8023	1.35	0.9115	1.85	0.9678	2.35	0.99061	2.85	0.99781
0.36	0.6438	0.86	0.8051	1.36	0.9131	1.86	0.9686	2.36	0.99086	2.86	0.99788
0.37	0.6478	0.87	0.8078	1.37	0.9147	1.87	0.9693	2.37	0.99111	2.87	0.99795
0.38	0.6518	0.88	0.8106	1.38	0.9162	1.88	0.9699	2.38	0.99134	2.88	0.99801
0.39	0.6558	0.89	0.8133	1.39	0.9177	1.89	0.9706	2.39	0.99158	2.89	0.99807
0.40	0.6598	0.90	0.8159	1.40	0.9192	1.90	0.9713	2.40	0.99180	2.90	0.99813
0.41	0.6638	0.91	0.8186	1.41	0.9207	1.91	0.9719	2.41	0.99202	2.91	0.99819
0.42	0.6678	0.92	0.8212	1.42	0.9222	1.92	0.9726	2.42	0.99224	2.92	0.99825
0.43	0.6718	0.93	0.8238	1.43	0.9236	1.93	0.9732	2.43	0.99245	2.93	0.99831
0.44	0.6758	0.94	0.8264	1.44	0.9251	1.94	0.9738	2.44	0.99266	2.94	0.99836
0.45	0.6798	0.95	0.8289	1.45	0.9265	1.95	0.9744	2.45	0.99286	2.95	0.99841
0.46	0.6838	0.96	0.8315	1.46	0.9279	1.96	0.9750	2.46	0.99305	2.96	0.99846
0.47	0.6878	0.97	0.8340	1.47	0.9292	1.97	0.9756	2.47	0.99324	2.97	0.99851
0.48	0.6918	0.98	0.8365	1.48	0.9306	1.98	0.9761	2.48	0.99343	2.98	0.99856
0.49	0.6958	0.99	0.8389	1.49	0.9319	1.99	0.9767	2.49	0.99361	2.99	0.99861