Question 1 (Part A) :-

(a) $S \cdot S = \{ aaf \quad aaa \quad afa \quad aff \quad faa \quad faf \quad ffa \quad fff \}$.

(b) $Z_F = \{ aaf, aff, faf, fff \}$

$X_A = \{ aaf, aaa, afa, aff \}$.

(c) $Z_F \cap X_A = \{ aaf, afa \}$ no they are not mutualey exclusive as $Z_F \cap X_A \neq \phi$

(d) $Z_F \cup X_A = \{ aaf, aaa, afa, aff, faf, fff \}$ no they're not collectively exhausted.

(PART B)

	H_0	H_1	H_2	
L	0.1	0.1	0.2	0.4
B	0.4	0.1	0.1	0.6
	0.5	0.2	0.3	

$P[H_0] = 0.5$, $P[B] = 0.6$

$P[H_2 L] = 0.2$

$$P[L|H_0] = \frac{P[LH_0]}{P[H_0]} = \frac{0.1}{0.5} = 1/5$$

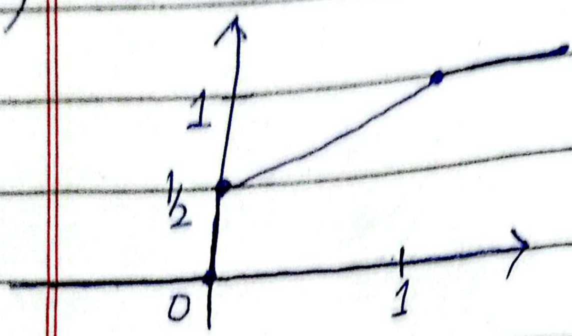
$$P[B|H_0] = \frac{P[BH_0]}{P[H_0]} = \frac{0.4}{0.5} = 4/5$$

Question 2 (Part A)

$$F_X(x) = \begin{cases} 0 & x < -1 \\ \frac{x+1}{2} & -1 \leq x < 1 \\ 1 & x \geq 1 \end{cases}$$

a) Sketch

x	$\frac{x+1}{2}$
-1	0
0	$1/2$
1	1



(b) $P[X = 1/2]$

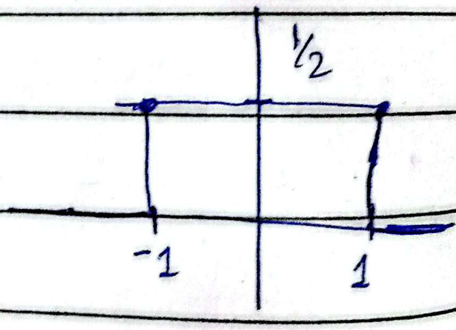
$$F_X(1/2) = \frac{(1/2)+1}{1} = 3/2$$

(c) $P[X \leq 0]$

$$F_X(0) = \frac{0+1}{2} = 1/2$$

(d) pdf

$$f_X(x) = \frac{dF_X(x)}{dx}$$



$$f_X(x) = \begin{cases} 1/2 & -1 \leq x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

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$X \sim \text{Gaussian}(3, 0.5)$

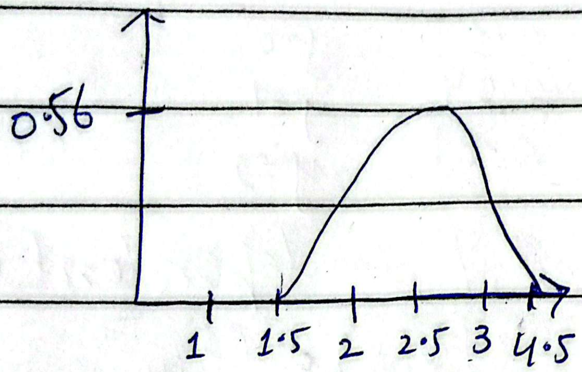
$\mu = 3, \quad \sigma = 0.5$

a) $f_X(x) = \dfrac{1}{\sqrt{2\pi}\sigma} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2}$

$f_X(x) = \dfrac{1}{\sqrt{\pi}} e^{-1/2\left(\frac{x-3}{0.5}\right)^2}$

b) $\mu - 3\sigma$ to $\mu + 3\sigma$

1.5 to 4.5



c) $P[2 \le X \le 4]$

$= \Phi\left(\frac{4-3}{0.5}\right) - \Phi\left(\frac{2-3}{0.5}\right)$

$= \Phi(2) - \Phi(-2)$

$= \Phi(2) - (1 - \Phi(2))$

$= 2\Phi(2) - 1$

$= 2(0.97725) - 1$

$= 0.9545$

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Question # 3:

$P_{XY}(x,y)$	$y=0$	$y=1$	$y=2$	$P_X(x)$
$x=0$	0.01	0	0	0.01
$x=1$	0.09	0.09	0	0.18
$x=2$	0	0	0.81	0.81
$P_Y(y)$	0.1	0.09	0.81	1

a) $P_X(x) = \begin{cases} 0.01 & x=0 \\ 0.18 & x=1 \\ 0.81 & x=2 \end{cases}$

b) $P_Y(y) = \begin{cases} 0.1 & y=0 \\ 0.09 & y=1 \\ 0.81 & y=2 \end{cases}$

c) Are X and Y independent.

$P_{XY}(x,y) = P_X(x) * P_Y(y)$

let's check:

$P_{XY}(1,1) = P_X(1) \cdot P_Y(1)$

$0.09 = 0.18 \times 0.09$

$0.09 \neq 0.0162$

No, they're not.

d) $Var[X] = E[X^2] - (E[X])^2$

$E[X] = 1.8$, $E[X^2] = 3.42$

$Var[X] = 3.42 - (1.8)^2 \Rightarrow$ 0.18

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e) $Var[Y] = E[Y^2] - (E[Y])^2$

$E[Y] = 1.71$, $E[Y^2] = 3.33$

$Var[Y] = 3.33 - (1.71)^2 = 0.4059$

f) corelation.

$E[XY] = 3.33$

g) No as $E[XY] \neq 0$ so they're not orthogonal.

h) Covariance.

$$COV[X,Y] = E[XY] - E[X]E[Y]$$
$$= 3.33 - (1.71)(1.8)$$
$$COV[X,Y] = 0.252$$

i) Corelation coffienct

$$P_{XY} = \frac{COV[X,Y]}{\sqrt{VAR[X]VAR[Y]}} = \frac{0.252}{\sqrt{(0.18)(0.4059)}}$$

$$P_{XY} = 0.9322$$

j) ~~No~~ Yes, as $COV[X,Y] \neq 0$ so they're corelated.

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Question # 5 :-

$$P_X(x) = \begin{cases} 0.1 & x=0 \\ 0.9 & x=1 \\ 0 & \text{otherwise} \end{cases}$$

(a) $E[X]$ and $P_X(1)$

$E[X] = 0.1\times 0 + 0.9\times 1 \Rightarrow 0.9$

$P_X(1) = 0.9$

(b) $n = 90$, $P_X(1) = 0.9$.

$P[|M_{90}(X) - P_X(1)| \geq 0.05] \leq a$

We know that :-

$$P[|M_n(X) - P_X(x)| \geq c] \leq \frac{VAR[X]}{nc^2}$$

$VAR[X] = E[X^2] - (E[X])^2$

$VAR[X] = 0.9 - (0.9)^2 \Rightarrow 0.09$.

$$P[|M_{90}(X) - P_X(1)| \geq 0.05] \leq \frac{(0.09)}{(90)(0.05)^2}$$

$$a = \frac{0.09}{(90)(0.05)^2} = 0.4$$

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(c) $P[|M_n(X) - P_X(1)| \geq 0.03] \leq 0.1$

$P[|M_n(X) - P_X(u)| \geq c] \leq \frac{VAR[X]}{nc^2}$

$$\frac{0.09}{n(0.03)^2} \neq 0.1$$

$$n = \frac{0.09}{(0.1)(0.03)^2} \Rightarrow 1000$$

so;

$$n \leq 1000$$