Discrete Assignment #2

24K-0013

Q1(a) R is reflexive as all (x,x) are present

(b) R is not symmetric as for example $(a,c) \in R$ but $(c,a) \notin R$

(c) R is not atisymmetric as for example both (b,c) and $(a,b) \notin R$

(e) R is not irreflexieve as diagonals are present

(f) R is not asymmetric as diagonals and mutal pairs exist

Q2(a) $\{(0,0),(1,1),(2,2),(3,3)\}$

(b) $\{(1,3),(2,2),(3,1),(4,0)\}$

(c) $\{(1,0),(2,0),(2,1),(3,1),(3,2),(4,0),(4,2),(4,3)\}$

(d) $\{(1,0),(1,1),(1,2),(1,3),(2,0),(2,3),(3,3),(4,0)\}$

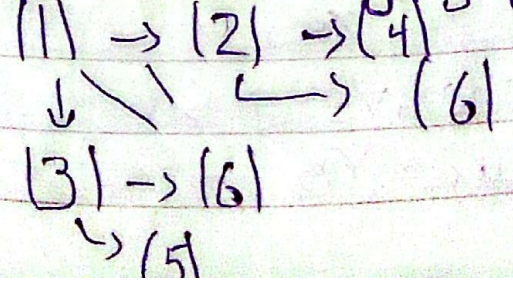
(e) $\{(0,1),(1,0),(1,1),(1,2),(1,3),(2,1),(2,3),(3,1),(3,2),(4,1),(4,3)\}$

(f) $\{(1,2),(2,1),(2,2)\}$

Q3: $(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,2),(2,4),(2,6),(3,3),(3,6),(4,4),(5,5),(6,6)$

Partial order Relation

Divisibilinty graph (1..6)



Discrete Assignment #2

24K - 0013

Q1(a) R is reflexive as all (x,x) are present

(b) R is not symmetric as for example $(a,c) \in R$ but $(c,a) \notin R$

(c) R is not atisymmetric as for example both (b,c) and $(a,b) \notin R$

(e) R is not irreflexieve as diagonals are present

(f) R is not asymmetric as diagonals and mutal pairs exist

Q2(a) $\{(0,0),(1,1),(2,2),(3,3)\}$

(b) $\{(1,3),(2,2),(3,1),(4,0)\}$

(c) $\{(1,0),(2,0),(2,1),(3,1),(3,2),(4,0),(4,2),(4,3)\}$

(d) $\{(1,0),(1,1),(1,2),(1,3),(2,0),(2,3),(3,3),(4,0)\}$

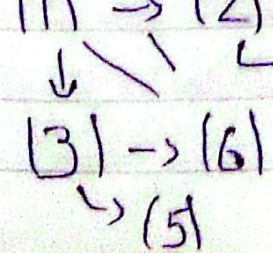
(e) $\{(0,1),(1,0),(1,1),(1,2),(1,3),(2,1),(2,3),(3,1),(3,2),(4,1),(4,3)\}$

(f) $\{(1,2),(2,1),(2,2)\}$

Q3: $(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,2),(2,4),(2,6),(3,3),(3,6),(4,4),(5,5),(6,6)$

Partial order Relation

Divisibilinty graph (1..6)



Adjacency matrix

$$\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array}$$

Q4(a) Reflexive : No
Symmetric : No
Asymmetric : No

(b) Reflexive Yes
Symmetric : Yes
Antisymmetric : No
Transitive : Yes

(c) Reflexive : No
Symmetric : Yes
Antisymmetric : No
Trasitive : No

(d) Reflexive . No
Symmetric : No
Antisymmetric : Yes
Transitive : yes

(e) ~~Refex~~ Reflexive : yes
Symmetric : Yes

Transitive : Yes

(f) Reflexive: No

Symmetric : No

Antisymmetric: No

Transitive : No

Q5 (a) Reflexive : No

Irreflexive : Yes

Symmetric : No

Antisymmetric: Yes

Transitive : Yes

Asymmetric : Yes

(b) Reflexive : Yes

Irreflexive : ~~Yes~~ No

Symmetric : Yes

Transitive : ~~No~~ Yes

Antisymmetric: No

Asymmetric : No

Irreflexive ~~Transitive~~ :

Transitive : Yes

(c) Reflexive : Yes

Symmetric : Yes

Transitive : Yes

Antisymmetric : No

Asymmetric : No

Irreflexive : No

(d) Reflexive: Yes
Symmetric: Yes
Transitive: No
Antisymmetric: No
Asymmetric: No
Irreflexive: No

Q6 (a) equality relation $\{(m,n) \mid m \in \omega\}$

(b) $R = \{(1,2),(2,1),(3,4)\}$

(c) equality relation $\{(n,n)\}$

Q7 (a) $A \times A$ (all 4 pairs)

(b) R6

(c) R3

(d) R3

(e) $\emptyset$

(f) R1

(g) R4

(h) R4

(i) R1

(j) $A \times A$ (all 9 Pairs)

Q8 a (i) Matrix = [[1, 1, 1], [0, 0, 0], [0, 0, 0]]

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

a (ii) [[0, 1, 0], [1, 1, 0], [0, 0, 1]]

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

a (iii) [[1, 1, 1], [0, 1, 1], [0, 0, 1]]

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

a (iv) [[0, 0, 1], [0, 0, 0], [1, 0, 0]]

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Q8 (b) (i) (1, 1), (1, 3), (2, 2), (3, 1), (3, 3)

b (ii) : (1, 2), (2, 2), (3, 2)

~~(1, 1), (1, 2), (1, 3), (2, 1), (2, 3), (1, 2), (2, 2), (3, 2)~~

b (iii) (1, 1), (1, 2), (1, 3), (2, 1), (2, 3), (3, 1), (3, 2), (3, 3)

Q.9 (a) Yes

(b) Let $m > 1$ and define $R = \{(a,b) : a \equiv b \pmod{m}\}$ on $\mathbb{Z}$

Reflexive: $a - a = 0$ is divisible by m

So $a \equiv a \pmod{m}$.

Symmetric: If $a \equiv b \pmod{m}$ $m \mid (a-b)$ so $m \mid (b-a)$ hence $b \equiv a \pmod{m}$

Transitive: If $a \equiv b \pmod{m}$ and $b \equiv c \pmod{m}$, then $m \mid (a-b)$ and $m \mid (b-c)$

So $m \mid (a-c)$ and $a \equiv c \pmod{m}$

Thus congruence modulo m is an equivalence relation. Its equivalence (congruence) classes are residue classes $[0]_m, [1]_m, \ldots, [m-1]_m$.

Q.10 (a) (i) $\pm 1, 3, 5, 7, 9$

(a)(ii) $8.5, 7, 5.5, 4, 2.5$

(a)(iii) $-1, 1/4, -1/9, 1/16, -1/25$

a (iv) $7, 10.66, 13.75, 16.92, 18.71$

b (i) Arithmetic Sequence

$d = -7$, $T_{11} = -85$

(b) (ii). Arithmetic Sequence

$d = 3b$, $T_{15} = a + 0$

1b)(iii): Geometric ~~Seqence~~ Sequence

$r = 3/4$, $T_{17} = 4\left(\frac{3}{4}\right)^{16}$

1b)(iv): Geometric Sequence

$r = 1/2$, $T_{a} = 1/4$

Q11(a x i): $T_3 = ar^2 = 10$, $T_5 = ar^4 = 5/2$

Divide: $r^2 = \frac{5/2}{10} = \frac{1/4}{4} = r = \pm\frac{1}{2}$

$a = \frac{10}{r^2} = \frac{10}{1/4} = 40$

$a = 40$, $r = 1/2$ or $= -1/2$

40, 20, 10, 5, 2, 5 .... or

40, -20, 10, -5, 2.5, ...

a(ii): $T_5 = ar^4 = 8$, $T_8 = ar^7 = -64/27$

Divide: $r^3 = \frac{-64/27}{} = -\frac{8}{27} \Rightarrow r = \frac{-2}{3}$

$r^4 = \left(\frac{2}{3}\right)^4 = \frac{16}{81}$ a $\Rightarrow \frac{8}{1681} = \frac{81}{2}$

$a = \frac{81}{2}$, $r = \frac{-2}{3}$

Q11(b)(i) $T_5 = a + 3d = 7$, $T_8 = a + 15 = 31$

$12d = 24 \Rightarrow d = 2$, Then $a = 7 - 32$

]

1b)(ii) $T_5 = a + 4d = 86$, $T_{10} = a + 9d = 146$

Subtract: $5d = 60 \Rightarrow d = 12$ Then $a =$

$86 - 4 \times 12 = 38$

$a = 38$, $d = 12$ A M

Q12(a)(i) Smallest multiple 7, 256

$7 \times 37 = 784, 259$

Largest Multiple of 784

$7 \times 12 = 784$

Number terms:

$n = 12 - 37 + 1 = 76$

These form an AP, with $a = 2$

$a_n = 784, d = 7$

$a_n : S_n = \frac{n}{2}(a_1 + a_n)$

$\frac{76(259 + 784)}{2} = 38 \times 1043$

$= 39,634$

$n = 76$, Sum $= 39,634$ A7

Q12 (a)(ii) $a_1 = 1/n$

$a_n = n^2 - n + 1$

$S_n = n/2 \left(\frac{1}{n} + n^2 - n + 1 \right)$

$n/2 \left(\frac{n^2 - n + 1}{n} \right)$

$= n^2 - n + 1$ A7

Q13(a): $\sum_{i=1}^{100} \left(\frac{1}{i} \right)$

b(i) 1

b(ii) 55

Q14 (a) Drop = 12 ft Total = 130 ft m

(b) $P = 70000(1.07)^{4}$

$P \cong 105t$ or $P = 105{,}051$

Q15 (a) (i) $-1, 2, -4, 8, -16, 32$

(a) (ii) $2, -1, -3, -2, 1, 3$

(a) (iii) $1, 3, 27, \cancel{81}\ 187, 143.48$ Lacs, 617.67

(a) (iv) $-1, 0, 1, 3, \cancel{2}\ 13, 74$

(b) (i) $1, -2, 4, -8$

(b) (ii) $3, 3, 3, 3$

(b) (iii) $8, 11, 28\ \cancel{2187, 143.48}, 71$

(b) (iv) $2, 0, 8, 0$

Q16 (a) Edges: Undirected Multiple

Edge: Yes

Loops: No

Type: Undirected Multiple ((b,a-b,

(b) Edges Undirected Multiple b-d)

Edges: No

Loop: No

Type: simple Graph

(c) Edges: Undirected Multiple edges: Yes

(a-b) c-d)

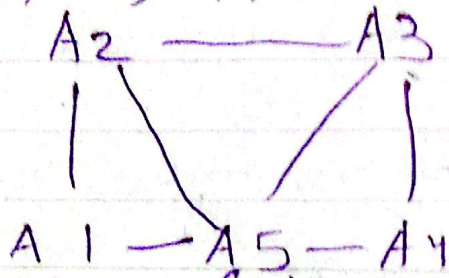
Loops Yes

(a, b, c, d)

Type: Directed Multigraph

Q17(a)

Edges (A1, A2), (A1, A4), (A1, A5), (A2, A2), (A2, A3), (A3, A4), (A3, A5) (A4, A5)



(b) Vertices {A1, A2, A3, A4, A5}

A1. Non Positive

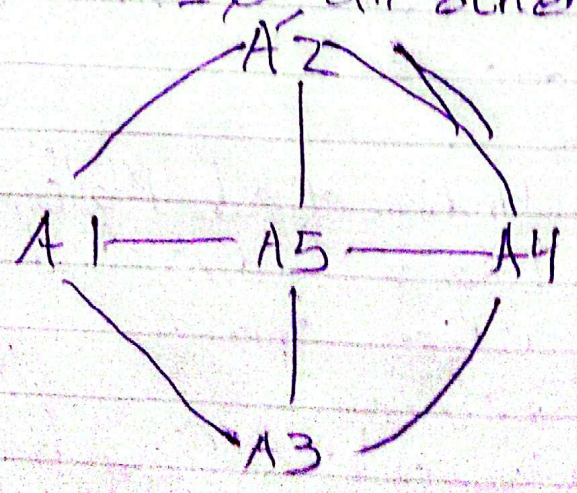
A2. All Intergers

A3: Even

A4: Odd

A5: Multiples of 3

A3 n A4 = ∅ intersec all other pairs.



Q18 (a)(i) Vertices : 5

Edges : 8 (loops + double edge)

Degress : a = 4, b = 3, c = 5, d = 4, e = 0

Neighborhood a = { a, b, d }

b = { a, c, d }

c = { b, c, d }

d = { a, b, c }

e = { }

Q18 (a)(ii) Vertices : 6, edge : 6

Degress : d = 0, e = 4, f = 0, g = 3, h = 3, i = 3

Neighborhood : d = { }

e = { c, g, h }

f = { }

g = { e, h, i }

h = { e, g, i }

i = { g, h }

Q18 (b)(i) Vertices : 5, Edges : 13 (+ loops)

a, b, c, d all in = 3, out = 3 e in = 1

out = 1

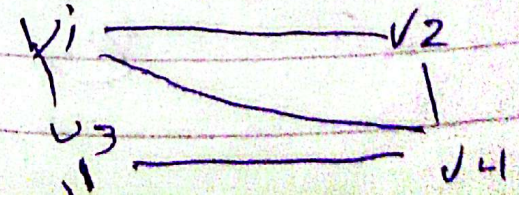
Q18 b (ii) Vertices : ~~15~~ 5

Edges : ~~13 (+ loops)~~ 8 (loops at it)

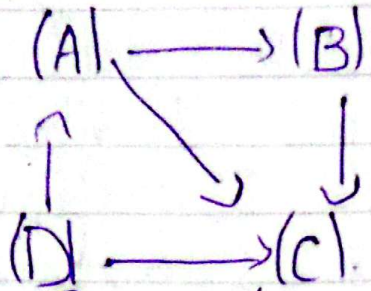
In loop degrees vary

a = 2/2, b = 2/3, c = 2/2, d = 2/1

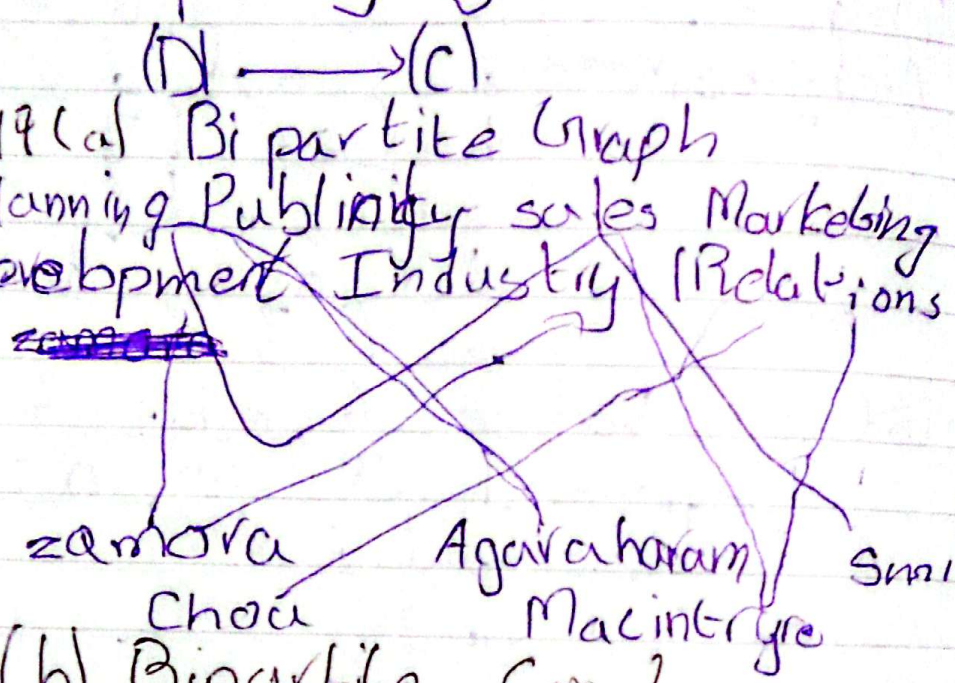
(ii)



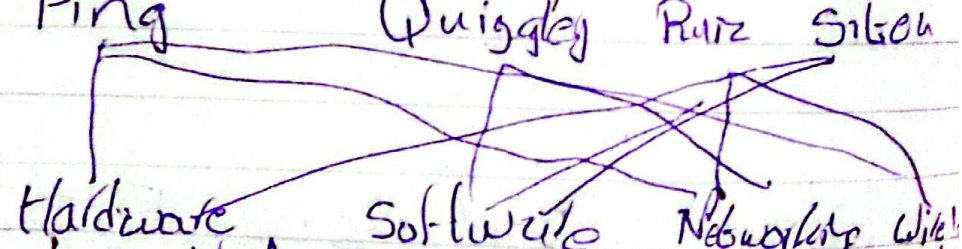
(ii)



Q19 (a) Bipartite Graph



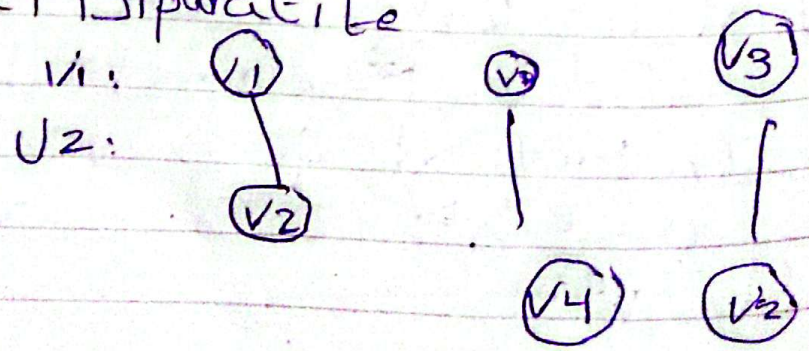
(b) Bipartite Graph



Q12 (a) Not Biparatitte (odd cycle)

(b) Not Biparatite (odd cycle)

(c) Biparatize



(d) Bipartite

Q2 (a) only the 3rd (the 8-cycle labelled A) and the 4th (the star labelled B) are bipartite

A Graph is bipartite ⟺ it contains no odd cycle

Graph A (octagon) - it is an 8-cycle (all cycles shown have even lenght) An even cycle is bipartite (partition vertices alternately around the cycle) ⟹ bipartite

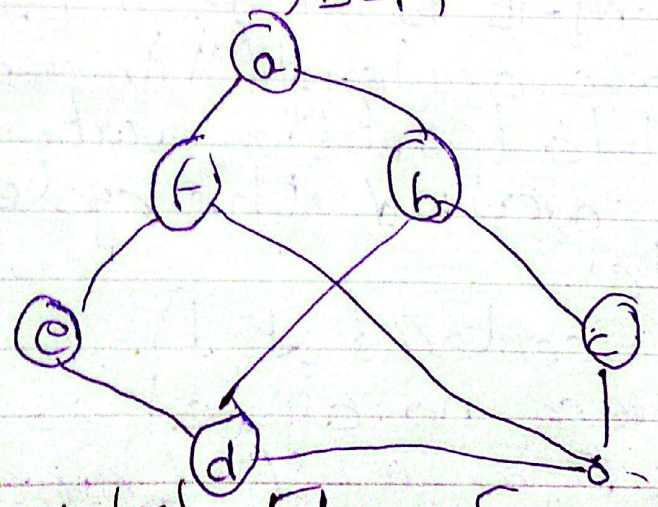
Graph B (Star centered at e) - a star is a tree no cycle) as shown in the figure (e.g the three vertices around the top corner form an odd cycle presence of this 3-cycle ⟹ not bipartite

Leftmost graph (4) - contains a 3-cycle (mising b) as shown in the figure e.g.

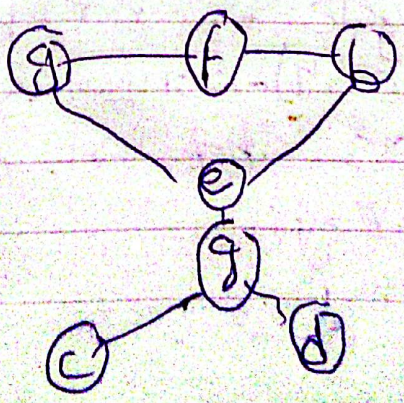
~~Q21(b)~~. The Three vertices around the top corners form an odd cycle. Presence of the 3-cycle => not bipartite

· Second graph (H) contain a triangle (three mutally connected vertices visible in the drawing, e.g. b, e, d) => has an odd cycle => not bipartite

Q21(b)(i) Edges, Hexagons & (f-b, b-c, b-f)



b(ii) Edges. {a-a, b, c, d, f, g}
u (a-f, f-e, e-g, f-c, g-d)



Q22 (a) 1, 1, 2, 3 → No graph (sum is 7 which is odd)

(b) 1, 1, 3, 3 → Exists (non simple)

$v_1 - v_3 = - v_4 - v_2$

(c) 1, 1, 3, 3 (simple) → impossible

Q23 (a) Handshaking Theorem

$15 \times 3 = 45$ (odd)

$45 \neq 2e$ No!

(b) Graph not possible

Handshaking Theorem

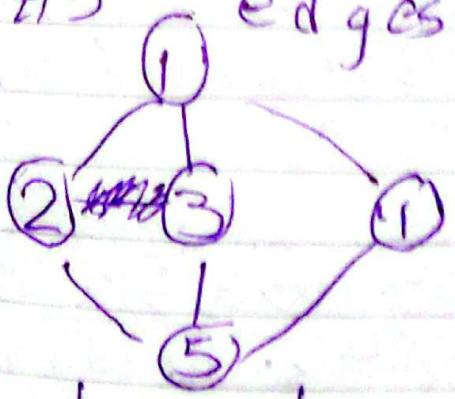
$4 \times 3 = 12$ (even)

$12 = 2e$ Yes!

total degree is even exists possible

(c) If 10 people each simple shake hand with each other

$\frac{1}{2}$ 45 edges



Each node connects for 10 total edges = c(10, 5) = 45

(d) For 5 people having 2 friend

Yes! $5 \times 2 = 10$ (even)

For 3 friends No
5 × 3 = 15 (odd)
5 vertices
Using Handshaking lemma
sum of degrees = 2×E V×4 = 2
⇒ 4V = 20 ⇒ V = 5 A]

Q 24(a) Graph x and z are connected graph is not connected
The connected components of and {B, C}

(b) None of the graph are graph free

(c) Graph x has a loop at vertex
Graph x and y are loop free

(d) Graph y is simple graph.
Graph x has loop graph z has multiple edges between A and C

Q25(a) Vertices 5, edges 7
degree sequence → 4, 3, 3, 2, 2
Isomorphic → satisfied condition
g: V(G) → V(H) ⇒
g(V1) = U5, g(V2) = U3, g(V3) = U1, g(V4) = U2, g(V5) = U4
Isomorphic.

Q25 (b) Vertices: 7
edges: 9
degree sequence =>
4, 3, 3, 2, 2, 2, 2 → Conditions Satisfied

Isomorphic

$g: v(a) \to v(4) = g(v_1) = U_5$, $g(V_2) = U_4$
$g: (V_3) = V_3$, $g(V_4) = U_6$, $g(V_4) = V_2$,
$g: (V_5) = U_7$, $g(V_6) = U_1$

(d): vertices: ~~7~~ 5, edge: 7
degree sequence different
in g $d(V_2) = 4$ whereas no
such vertex in G for $d = 4$

Not Isomorphic.

Q26: Isomorphic graph
Group 1: {F, K, T}
Invariations E = 4, degree
Sequence: (3, 2, 1, 1)
Group 2: {M, S, V, Z}
Invariants: E = 4 degree
Sequence: (2, 2, 2, 1, 1)

Non Isomorphic graph:-
A: E = 5, degree Sequence
(3, 3, 2, 2, 1)
R: E = 5 degree Sequence
(3, 2, 2, 2, 1)

X.E - 4 degree Seqence
(4, 1, 1, 1)

Q28 (a). Using Hamiltorian - Circuit through Q28 Graph (a)

Route: A -> B -> C -> D -> A

Cost: 30+30+25+40 -> 125

(b) Graph (b)

Route: A -> B -> C -> D A

Cost: 20+30+12+35 => 97

(c) Graph (c)

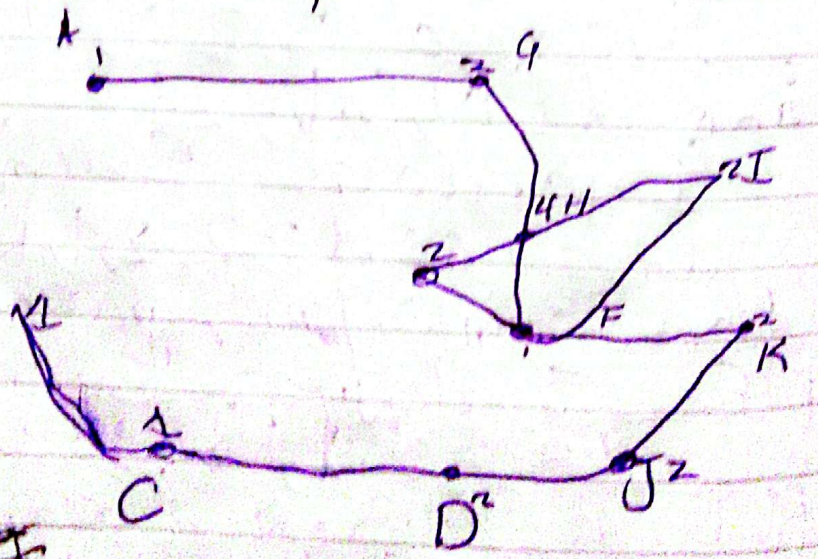
Route: A -> B -> D -> C -> A

Cost: 10+25+30+15 => 80

Q29 (a) Yes this is possible

A -> H -> G -> B -> C -> D -> E -> F

(b) we can represent the floor as



$d(A) = d(B) = 1 \Rightarrow$ odd

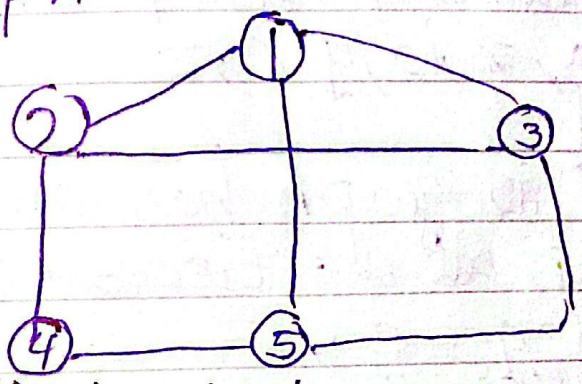
$d(C) = d(D) = d(E) = d(F) = d(G) =$
$d(H) = d(I) = d(T) = d(U) \Rightarrow$ even.

It is an Euler path.

$A \to G \to H \to F \to E \to I \to H \to E \to K \to J$
$\to D \to C \to B$

Domino Set

Modeling it as a complete graph



Q30(a)(i) Hamiltonian Circuit
$V_0, V_1, V_2, V_6, V_5, V_4, V_7, V_3, V_0$

(a)(ii) Hamiltonian Circuit not exist (closed loop cannot form).

Hamiltonian path
b, c, f, g, h, e, a, d

a(iii) Hamiltonian Circuit
d, c, b, a, g, f, e, d

Hamitonian path
d, c, b, a, g, f, p

Q30 (b) (i) Hamiltonian Circuit:
non existent (number of vertices)
Hamiltonian path
f, g, a, b, h, e, d, c

(b) (ii) Hamiltonian Circuit:
non existent (number of vertieces)
Hamiltonian path:
Yes, a, h, j, b, c, d, e, f, g

(b) (iii) Hamiltonian path
a, x, b, y, c, z

Q31 a) (i)
This graph has an Euler Circu
because all vertieces have an
even degree
$V_1 \to V_2 \to V_3 \to V_4 \to V_2 \to V_1$

a (ii) This graph has an euler
Circuit:
$V_0 \to V_1 \to V_2 \to V_3 \to V_4 \to V_5 \to V_6 \to V_7 \to V_8$
$\to V_5 \to V_1 \to V_0$

a (iii). This graph has an euler circ
$a \to b \to c \to d \to e \to f \to g \to h \to i \to c \to g$

Q31 (b) (i) This has no euler path Expl
ation. The graph has two vertic
of odd degree (d and g), so
any Eular path must

started and end at those vertices
Since U and W both have even degrees, an Eular path between them is not possible

(b)(ii): This graph has no Eular path from u to w. All verticesces in the graph only contains Eular circuit. An Eular path must start and end at the same vertices; there is no Eular path from u to w are distinct vertices, there is no Eular path from ~~u to w~~ u to w

Q35(a) 2
(b) 0
(c) 4
(d) u and v
~~Q35~~(e) d
(f) K and l
(g) m, s, t, x and y
(h) a, b, c, d, e, f, g, h, i, k, m, n, t

Q35(i) a, d, n and w

Q35(ii) a, o, j, l, s, x, y, u, d, e, p

Q36(a)(i) The order of edges added is: $(v_1, v_2), (v_2, v_4), (v_3, v_4)$
$(v_0, v_5), (v_5, v_6), (v_6, v_7)$

Q36(b)(i) The order of edges added is:
(b, d), (a, c), (a, h), (d, e), (e, f)

(b)(ii) (a, g), (a, b), (a, c), (e, f), (d, e), (c, e)
This is the order of edges added

Q38(a)(i) Preorder: a, b, e, f, g, c, h, j, k, d, k, l, m
Inorder: e, b, f, g, a, h, i, j, c, k, d, l, d, a
Postorder: e, f, g, b, h, i, j, c, k, l, m, d, a
(ii) Preorder. a, b, e, L, m, f, n, c, g, h, p, d, i, j, q, o, k
Inorder. L, e, m, b, n, f, a, g, c, o, h, p, i, q, j, o, k
Postorder: L, m, e, n, f, b, g, o, p, h, i, q, o, j, k, d, a

Q38(b)(i) Removed edges (b, c), (c, e), (g, h)
(ii) Removed edges. (e, f), (c, d), (a, d)
(b)(iii) Removed edges. (a, d) (b, c), (c, i), (e, f), (L) (g, h), (h, i)

Q39 (a) 1999

(b) 2000

(c) 991

(d) 67

Q40 (a)(i) Pre(x, + x * xy / xy

Post(x, + x / y * x / y

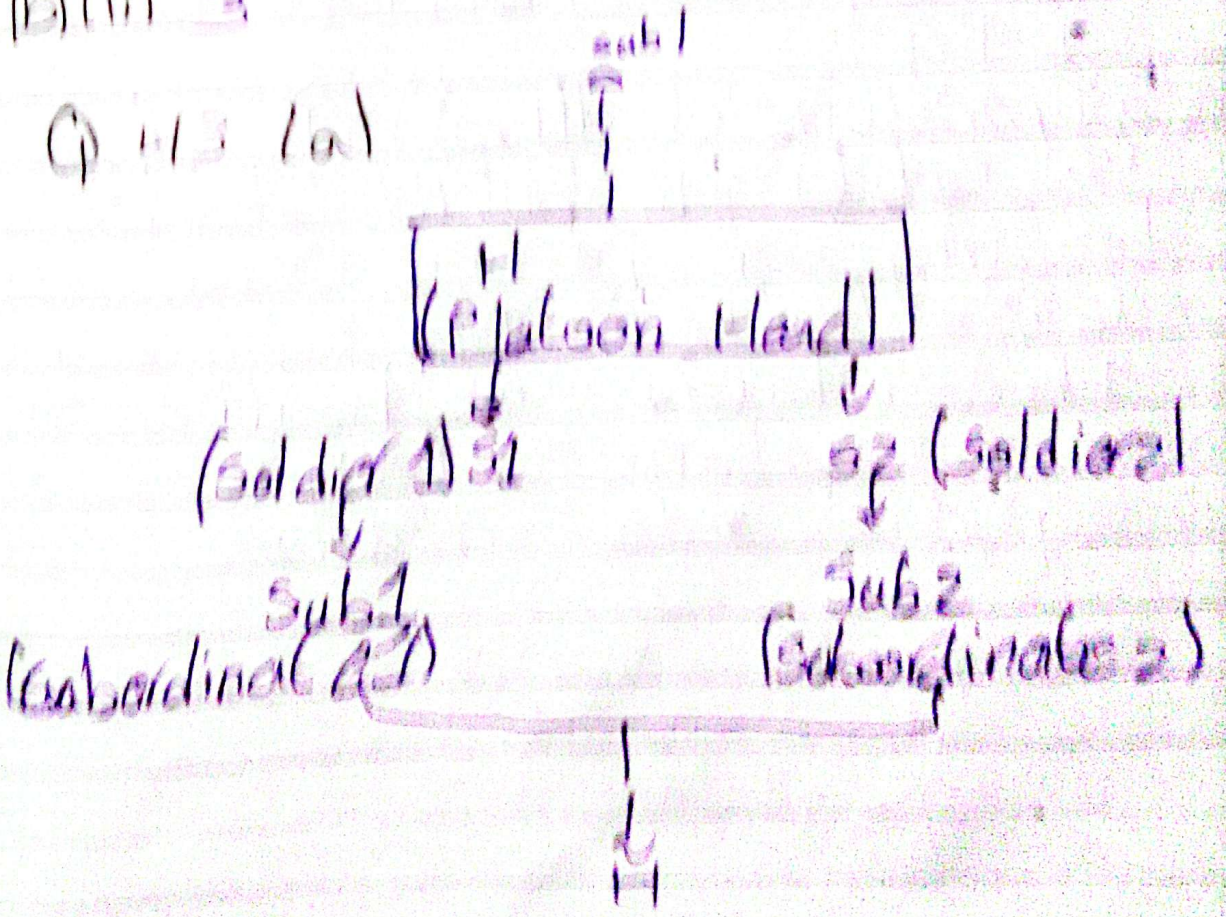
(ii) Pre(x, + x / + x * xy * y

Post(x, x xy * x + y * +

(b)(i) 1

(ii) 3

Q41 (a)



Q43(b) Relation R on Vertices (a, b) ∈ R if - there's a directed edge from a to b

R = {(H, S1), (H, S2), (S1, Sub1), (S2, Sub2), (Sub1, H), (Sub2, H)}

Testing for Equivalence:

1. Reflexive? No (H, H) ∉ R

2. Symmetric? No ~~(H, H)~~ (H, S1) ∈ R and (S1, Sub1) ∈ R, but (H, Sub1) ∉ R

~~Testing for practical order:~~

~~1. Ref~~

3. Transitive? No (H, S1) ∈ R and (S1, Sub1) ∈ R, but (H, Sub1) ∉ R

Testing for Partial Order

1. Reflexive? No

The relation is neither an equivalence relation nor a partial order relation

Q4/(a). Graph edges (undirected):

- Home - A : 3, Home - 8, 2, Home - C, 5
- A-B: 1, A-D: 3
- B-C: 6, B-D: 1, B-E: 1
- C-E: 2
- D-F: 4
- E-F 1, E - School: 4
  F - school: 2

Dijkstra step:

Step	Vertex	Distance	previous	Unvisited
0	Home	0	-	All
1	Home	0	-	A(3), B(2), C(5)
2	B	2	Home	
3	A	3	Home	C(5), A(3), D(3), E(3)
4	D	3	B	C(5), D(3), E(3)
5	E	3	B	C(5), E(3), F(7)
6	F	4	E	C(5), School(7), F(4)
7	school	6	F	C(5), School(6)
				C(5)

Shortest path: Home → B → E → F → School

Cost: 2+1+1+2 = 6

Q44(b): Degree of each vertex:

- Home: 3 (odd)
- A : 3 (odd)
- B : 5 (odd)
- C : 3 (odd)
- D : 3 (odd)
- E : 4 (even)
- F : 3 (odd)
- School: 2 (even)

Eular path: Does not exist (requires exactly 0 or 2 vertieces with odd degree, but we have 6 odd-degree vertieces)

Q45 (a) Graph & edges (from Figure 1)

- A : W, X, Y
- B : W, X, Y, Z
- C : W, X, Y, Z
- D : Z

Graph H edge (from Figure 2)

- 1 : 2, 5
- 2 : 1, 4, 6
- 3 : 4, 8
- 4 : 2, 3
- 5 : 1, 8

- 6 : 2, 7
- 7 : 6, 8
- 8 : 3, 5, 7

Degree Sequences
Graph G : A(3), B(4), C(4), D(1), W(3), X(3), Y(3), Z(3) →
Stored : [1, 3, 3, 3, 3, 3, 4, 4]
Graph H : 1(2), 2(3), 3(2), 4(2), 5(2), 6(2), 7(2), 8(3) →
Sorted : [2, 2, 2, 2, 2, 2, 3, 3]
Result : Graph are not isomorphic (different degree sequences)
Reason : Degree Sequences don't match. Graph G has Vertices with degrees [1, 3, 3, 3, 3, 3, 4, 4] while Graph H has [2, 2, 2, 2, 2, 2, 3, 3]

Q45(b)(i) : Set 1 : {A, B, C, D}
Set 2 : {W, X, Y, Z}
Complete bipartite tu requires : Every vertex in Set 1 connects to all in Set 2
Actual Connections
- A → W, X, Y (missing Z)
- D → only (missing W, X, Y)

Result: Graph 1 is not a Complete bipartite graph. It's bipartite but not Complete bipartite

(b) (ii) Degrees of vertices in H:

- 1: degree 2
- 2: degree 3 (odd)
- 3: degree 2
- 4: degree 2
- 5: degree 2
- 6: degree 2
- 7: degree 2
- 8: degree 3 (odd)

Euler Circuit: Does not exist (2 vertices with odd degree; Euler Circuit requires all even degree)

Eular path: Exists exactly 2 odd degree vertices: starts at 2, ends at 8 or vice versa)

Hamiltonian Circuit Exists, 1 -> 2 -> 4 -> 3 -> 8 -> 7 -> 6 (needs 5) -> 5 -> 1

Q46(a)(i) Domain $\{a, b, c, d\}$
Co-domain: $B = \{a, b, c, d\}$
Range: $\{a, b, c, d\}$
(a)(ii) This function is injective, surjective and ~~biject~~ bijective.
a(iii) $f^{-1} = \{(b, a), (a, b), (c, c), (d, d)\}$
b(i) Domain $A = \{a, b, c, d\}$
Range $\{b, c, d\}$
(b)(ii) This function is not injective, not surjective, and not bijective
(b)(iii) Inverse does not exist.
(c)(i) Domain: $A = \{a, b, c, d\}$
Co-domain: $B = \{a, b, c, d\}$
Range $\{b, c, d\}$
(c)(ii) This function is not injective, not surjective, and not bijective
(c)(iii) Inverse does not exist
~~(d)(ii)~~ (d)(i) Domain: $A = \{a, b, c, d\}$
Co-domain: $B = \{a, b, c, d\}$
Range $= \{a, b, c, d\}$
(d)(ii) This function is injective, surjective and bijective
(d)(iii) $f^{-1} = \{(c, d), (a, b), (b, c), (d, d)\}$

Q47(a)(i) : $F(S) = \{0, 1, 3\}$

(a)(ii) : $F(S) = \{0, 1, 3, 5, 8\}$

(a)(iii) $F(S) = \{0, 8, 16, 40\}$

(a)(iv) $F(S) = \{1, 12, 33, 65\}$

(b)(i) : 0

(b)(ii) : 1

(b)(iii) : -1

(b)(iv) 0

(b)(v) 3

(b)(vi) -1

(b)(vii) 1

(b)(viii) 4

Q47(~~b~~c): Let x be a real number. We can write $x = n + \delta$ where n is an interger and $0 \leq \delta < 1$

From this, we have.

$$[x] = n$$

$[x] = n$ if $\delta = 0$, and $[x] = n+1$ if $0 < \delta < 1$

Now Let's consider $-x = -n - \delta$

case 1. $\delta = 0$

$$[-x] = [-n] = -n$$

$$-[x] = -n$$

In this case, $\lceil -x \rceil = -\lfloor x \rfloor$

Case 2: $0 < \delta < 1$

$\lceil -x \rceil = \lceil -n - \delta \rceil$. Since $-1 < -\delta < 0$, we have $\lceil -n - \delta \rceil = -n-1$

$-\lfloor x \rfloor = -(n+1) = -n-1$

In this case, $\lceil -x \rceil = -\lfloor x \rfloor$

Both ~~sides~~ cases hold, so the identity is proven.

Let $x = n + \delta$, where is an integer and $0 \leq \delta \leq 1$.

$\lfloor x \rfloor = n$

From this we have:

$\lceil x \rceil = n$

Now Let's consider $-x = -n - \delta$

Cause 1: $\delta = 0$

$\lceil -x \rceil = \lceil -n \rceil = -n$

$-\lfloor x \rfloor = -n$

In this case $\lceil -x \rceil = \lfloor x \rfloor$.

Case 2: $0 < \delta < 1$

$\lceil -x \rceil = \lceil -n - \delta \rceil$ Since $-1 < -\delta < 0$ we have $\lceil -n - \delta \rceil = -n$

$-\lfloor x \rfloor = -n$

$\lceil -x \rceil = -\lfloor x \rfloor$

In this case, $\lceil -x \rceil = -\lfloor x \rfloor$
Both cases hold, so the identity is proven The statement that if x is a real number, then $\lceil -x \rceil = -\lfloor x \rfloor$ and $\lceil -x \rceil = -\lfloor x \rfloor$ true

Q48(a) The composition of f and g is $(f \circ g)(a) = 6a + 7$.
The composition of g and f is $(g \circ f)(a) = 6a + 11$.
(b) Both f and g are one-to-one functions but are not onto functions
(c) Neither f ~~and~~ nor g are onto functions, So they are not invertible

Q (a) 1 Modeling physical phenoma Computer Programming
(b) 1. Data buses
2. Social networks
(c) 1 ~~Fun~~ Financial Calculations
2 Genetics
(d) Calculating Compound int
2 Signal ~~processing~~ processing

Tel 1. Navigation and GPS
2 Socal network Analysis
(A) 1 File System
Decision-making process

X ——————————————— X