2nd order partial derivative

$$z = x^3 + 2xy$$

first order partial derivative

$$\frac{dz}{dx} = 3x^2 + 2(1)y$$

$$= 3x^2 + 2y$$

$$\frac{dz}{dy} = 0 + 2x(1)$$

$$= 2x$$

2nd order p.d:

$$\frac{\partial^2 z}{\partial x^2} = 3 \cdot 2x + 0$$

$$= 6x$$

2nd order:

$$\frac{\partial^2 z}{\partial y^2} = 0$$

Mixed/cross p.d.

$$\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$= \frac{\partial}{\partial y}\underset{x \text{ const}}{\left(3x^2 + 2y\right)}$$

$$= 0 + 2 \cdot 1$$

$$= 2$$

mixed:

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)$$

$$= \frac{\partial}{\partial y}\underset{x \text{ const}}{(2x)}$$

$$= 0$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)$$

$f(x,y) = x + y + xy$

1. $\frac{\partial^2 f}{\partial x^2}$ f_{xx}

2. $\frac{\partial^2 f}{\partial y^2}$ f_{yy}

3. $\frac{\partial^2 f}{\partial y \partial x}$ f_{xy}

4. $\frac{\partial^2 f}{\partial x \partial y}$ f_{yx}

1. y constant (x যেটা আছে সেটা variable)

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}(x+y+xy)\right)$$
$$= \frac{\partial}{\partial x}(1+0+y)$$
$$= \frac{\partial}{\partial x}(1+y)$$
$$= 0+0$$
$$= 0$$

2. x constant (y যেটা আছে সেটা constant)

$$\frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}(x+y+xy)\right)$$
$$= \frac{\partial}{\partial y}(0+1+x\cdot 1)$$
$$= \frac{\partial}{\partial y}(1+x)$$
$$= 0+0$$
$$= 0$$

3. $\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)$
$$= \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}(x+y+xy)\right)$$ x যেটা আছে সেটা constant, y variable
$$= \frac{\partial}{\partial y}(1+0+1\cdot y)$$
$$= \frac{\partial}{\partial y}(1+y)$$ y যেটা আছে সেটা variable, x constant.
$$= 0+1$$
$$= 1.$$

4. $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)$
$$= \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}(x+y+xy)\right)$$ x constant
$$= \frac{\partial}{\partial x}(0+1+x\cdot 1)$$
$$= \frac{\partial}{\partial x}(1+x)$$ y constant
$$= 0+1\cdot$$
$$= 1.$$