Box Intersection

```
boxIntersection[θ_, size_, shape_, cornerRadius_] := Module[
   {list, circleOrigin, edge, circleEquation, combinedEquation, x, y, R, cutRadius},
   If[shape == True,
    (*Circle*)
    (*Easy: Just get the point on the circle at the given angle.*)
        list = {size/2.0 + size/2.0 Cos[θ], -size/2.0 + size/2.0 Sin[θ]}
    ,
        (*Square*)

    (*Calculate if the point ends up on the top edge, corner, or right edge.*)
        circleOrigin = {size - cornerRadius, 0.0 - cornerRadius};
        edge = {size/2.0 + size/2.0 * Cos[θ], -size/2.0 + size/2.0 * Sin[θ]};
        If[edge[[1]] ≤ circleOrigin[[1]],
            (*On the top edge*)
            list = {edge[[1]], 0};
         ,
            If[edge[[2]] ≤ circleOrigin[[2]],
                (*On the right edge*)
                list = {size, edge[[2]]};
              ,
                (*On the corner*)
                circleEquation =
        (x - circleOrigin[[1]])^2 + (y - circleOrigin[[2]])^2 == cornerRadius^2;
                combinedEquation = circleEquation /.
         {x → size/2.0 + R Cos[θ], y → -size/2.0 + R Sin[θ]};
                cutRadius = R /. Solve[combinedEquation, R][[2]];
                list = {size/2.0 + (cutRadius Cos[θ]), -size/2.0 + (cutRadius Sin[θ])};
             ];
        ];
    ];
    list
   ];
```

In[39]:=
```
Clear[circleOrigin, edge, size, cornerRadius, θ];
circleOrigin = {size - cornerRadius, 0 - cornerRadius};
edge = {size/2 + size/2 * Cos[θ], -size/2 + size/2 * Sin[θ]};
circleEquation = (x - circleOrigin[[1]])^2 + (y - circleOrigin[[2]])^2 == cornerRadius^2;
combinedEquation = circleEquation /. {x → size/2 + R Cos[θ], y → -size/2 + R Sin[θ]};
cutRadius = R /. Solve[combinedEquation, R][[2]];
assumptions = {cornerRadius ≥ 0, size > 0, cornerRadius ≥ size/2, 0 < θ < π/2};
output = {FullSimplify[size/2 + (cutRadius Cos[θ]), Assumptions → assumptions],
  FullSimplify[-size/2 + (cutRadius Sin[θ]), Assumptions → assumptions]}
output = output /. {size (3 + Cos[2 θ] + Sin[2 θ]) → a};
output = output /. {-2 cornerRadius (Cos[θ] + Sin[θ]) → b};
output =
  output /. {(4 cornerRadius - size) size + (-2 cornerRadius + size)^2 Sin[2 θ] → c};
output = output /. {size Cos[θ] (-Cos[θ] + Sin[θ]) → d}
```

Out[46]=
$$\left\{\frac{1}{4}\left(\text{size}\left(3+\text{Cos}[2\theta]+\text{Sin}[2\theta]\right)+2\,\text{Cos}[\theta]\left(-2\,\text{cornerRadius}\left(\text{Cos}[\theta]+\text{Sin}[\theta]\right)+\sqrt{\left(4\,\text{cornerRadius}-\text{size}\right)\text{size}+\left(-2\,\text{cornerRadius}+\text{size}\right)^2\text{Sin}[2\theta]}\right)\right),\right.$$
$$\left.\frac{1}{2}\left(\text{size}\,\text{Cos}[\theta]\left(-\text{Cos}[\theta]+\text{Sin}[\theta]\right)-2\,\text{cornerRadius}\,\text{Sin}[\theta]\left(\text{Cos}[\theta]+\text{Sin}[\theta]\right)+\text{Sin}[\theta]\sqrt{\left(4\,\text{cornerRadius}-\text{size}\right)\text{size}+\left(-2\,\text{cornerRadius}+\text{size}\right)^2\text{Sin}[2\theta]}\right)\right\}$$

Out[50]=
$$\left\{\frac{1}{4}\left(a+2\left(b+\sqrt{c}\right)\text{Cos}[\theta]\right),\ \frac{1}{2}\left(d+b\,\text{Sin}[\theta]+\sqrt{c}\,\text{Sin}[\theta]\right)\right\}$$

Middle Point

```
middlePoint[size_, radius_] := Module[{},
  {size/2.0, -size/2.0 - size * radius}
 ];
```

Short Point

```
shortPoint[size_, radius_] := Module[{},
  {size/2.0 - size * radius, -size/2.0}
 ];
```

Long Arm End Point

```
longArmEndPoint[θ_, size_, r_, boxIntersectionEndPoint_, middleEndPoint_] :=
 Module[{lineEquation, circleEquation, circleX, circleY, lineX, lineY, x, y},
  (*The line between the middle and long end points.*)
  lineEquation = y - boxIntersectionEndPoint[[2]] ==
    (middleEndPoint[[2]] - boxIntersectionEndPoint[[2]])/
     (middleEndPoint[[1]] - boxIntersectionEndPoint[[1]])
      (x - boxIntersectionEndPoint[[1]]);
  (*The circle where the mid-point needs to lie.*)
  circleEquation = (x - size/2.0)^2 + (y + size/2.0)^2 == r^2;
  (*Calculate the intersection of the circle and line.*)
  circleX = NSolve[circleEquation, x][[2]];
  circleY = NSolve[circleEquation, y][[2]];
  lineY = NSolve[lineEquation /. circleX, y];
  lineX = NSolve[lineEquation /. circleY, x];
  {x /. lineX[[1]], y /. lineY[[1]]}
 ]
```

In[199]:=
```
Clear[x, y, x1, y1, x2, y2, lineEquation,
 circleEquation, size, r, circleX, circleY, lineY, lineX]
lineEquation = y - y1 == (y2 - y1)/(x2 - x1) (x - x1);
(*The circle where the mid-point needs to lie.*)
circleEquation = (x - size/2.0)^2 + (y - size/2.0)^2 == r^2;
(*Calculate the intersection of the circle and line.*)
circleX = NSolve[circleEquation, x][[2]];
circleY = NSolve[circleEquation, y][[2]];
lineY = NSolve[lineEquation /. circleX, y];
lineX = NSolve[lineEquation /. circleY, x];
assumptions = {size > 0, 0 < θ < π/2, x1 > x2 > 0, y1 > y2 > 0};
output = {FullSimplify[x /. lineX[[1]], Assumptions → assumptions],
  FullSimplify[y /. lineY[[1]], Assumptions → assumptions]}
output = output /. {1.` size x1^2 - 2.` size x1 x2 + 1.` size x2^2 +
      1.` size x1 y1 - 1.` size x2 y1 + 2.` x2 y1^2 - 1.` size x1 y2 +
      1.` size x2 y2 - 2.` x1 y1 y2 - 2.` x2 y1 y2 + 2.` x1 y2^2 → a};
output = output /. {-16.` (1.` x1^2 - 2.` x1 x2 + 1.` x2^2 + 1.` y1^2 - 2.` y1 y2 + 1.` y2^2) → b};
output =
  output /. {r^2 (-1.` x1^2 + 2.` x1 x2 - 1.` x2^2) + size^2 (0.5` x1^2 - 1.` x1 x2 + 0.5` x2^2) → c};
output = output /. {1.` x2^2 y1^2 - 2.` x1 x2 y1 y2 + 1.` x1^2 y2^2 +
      size (1.` x1 x2 y1 - 1.` x2^2 y1 - 1.` x1^2 y2 + 1.` x1 x2 y2) → d};
output = output /. {(x1 (4.` y1 - 4.` y2) y2 + x2 y1 (-4.` y1 + 4.` y2) +
       size (-2.` x1^2 + x2 (-2.` x2 + 2.` y1 - 2.` y2) + x1 (4.` x2 - 2.` y1 + 2.` y2))) → e};
output = output /. {1.` x1^2 - 2.` x1 x2 + 1.` x2^2 + 1.` y1^2 - 2.` y1 y2 + 1.` y2^2 → f};
output = output /.
   {0.5` size x1 y1 - 0.5` size x2 y1 - 1.` x1 x2 y1 + 1.` x2^2 y1 + 0.5` size y1^2 - 0.5` size x1
       y2 + 1.` x1^2 y2 + 0.5` size x2 y2 - 1.` x1 x2 y2 - 1.` size y1 y2 + 0.5` size y2^2 → g};
output = output /. {(-4.` x2^2 y1 - 4.` x1^2 y2 + x1 x2 (4.` y1 + 4.` y2) + size
        (-2.` x1 y1 + 2.` x2 y1 - 2.` y1^2 + 2.` x1 y2 - 2.` x2 y2 + 4.` y1 y2 - 2.` y2^2)) → h};
output = output /. {1.` x2^2 y1^2 - 2.` x1 x2 y1 y2 + 1.` x1^2 y2^2 +
      r^2 (-1.` y1^2 + 2.` y1 y2 - 1.` y2^2) → i};
output = output /. {size^2 (0.5` y1^2 - 1.` y1 y2 + 0.5` y2^2) +
     size (x1 (1.` y1 - 1.` y2) y2 + x2 y1 (-1.` y1 + 1.` y2)) → j}
```

Out[206]=
$$\left\{\left(0.5\left(1.\,\text{size}\,x1^2-2.\,\text{size}\,x1\,x2+1.\,\text{size}\,x2^2+1.\,\text{size}\,x1\,y1-1.\,\text{size}\,x2\,y1+2.\,x2\,y1^2-1.\,\text{size}\,x1\,y2+1.\,\text{size}\,x2\,y2-2.\,x1\,y1\,y2-2.\,x2\,y1\,y2+2.\,x1\,y2^2-0.5\sqrt{\left(-16.\left(1.\,x1^2-2.\,x1\,x2+1.\,x2^2+1.\,y1^2-2.\,y1\,y2+1.\,y2^2\right)\left(r^2\left(-1.\,x1^2+2.\,x1\,x2-1.\,x2^2\right)+\text{size}^2\left(0.5\,x1^2-1.\,x1\,x2+0.5\,x2^2\right)+1.\,x2^2\,y1^2-2.\,x1\,x2\,y1\,y2+1.\,x1^2\,y2^2+\text{size}\left(1.\,x1\,x2\,y1-1.\,x2^2\,y1-1.\,x1^2\,y2+1.\,x1\,x2\,y2\right)\right)+\left(x1\,(4.\,y1-4.\,y2)\,y2+x2\,y1\,(-4.\,y1+4.\,y2)+\text{size}\left(-2.\,x1^2+x2\,(-2.\,x2+2.\,y1-2.\,y2)+x1\,(4.\,x2-2.\,y1+2.\,y2)\right)\right)^2\right)\right)\right)\Big/\left(1.\,x1^2-2.\,x1\,x2+1.\,x2^2+1.\,y1^2-2.\,y1\,y2+1.\,y2^2\right),$$

$$\left(1.\left(0.5\,\text{size}\,x1\,y1-0.5\,\text{size}\,x2\,y1-1.\,x1\,x2\,y1+1.\,x2^2\,y1+0.5\,\text{size}\,y1^2-0.5\,\text{size}\,x1\,y2+1.\,x1^2\,y2+0.5\,\text{size}\,x2\,y2-1.\,x1\,x2\,y2-1.\,\text{size}\,y1\,y2+0.5\,\text{size}\,y2^2-0.25\sqrt{\left(\left(-4.\,x2^2\,y1-4.\,x1^2\,y2+x1\,x2\,(4.\,y1+4.\,y2)+\text{size}\left(-2.\,x1\,y1+2.\,x2\,y1-2.\,y1^2+2.\,x1\,y2-2.\,x2\,y2+4.\,y1\,y2-2.\,y2^2\right)\right)^2-16.\left(1.\,x1^2-2.\,x1\,x2+1.\,x2^2+1.\,y1^2-2.\,y1\,y2+1.\,y2^2\right)\left(1.\,x2^2\,y1^2-2.\,x1\,x2\,y1\,y2+1.\,x1^2\,y2^2+r^2\left(-1.\,y1^2+2.\,y1\,y2-1.\,y2^2\right)+\text{size}^2\left(0.5\,y1^2-1.\,y1\,y2+0.5\,y2^2\right)+\text{size}\,(x1\,(1.\,y1-1.\,y2)\,y2+x2\,y1\,(-1.\,y1+1.\,y2))\right)\right)\right)\Big/\left(1.\,x1^2-2.\,x1\,x2+1.\,x2^2+1.\,y1^2-2.\,y1\,y2+1.\,y2^2\right)\right\}$$

Out[216]=
$$\left\{\frac{0.5\left(a-0.5\sqrt{b\,(c+d)+e^2}\right)}{f},\ \frac{1.\left(g-0.25\sqrt{h^2+b\left(i+j\right)}\right)}{f}\right\}$$

Testing

```
Manipulate[
 angle = θ π / 180;
 Graphics[
  {
   Thick, Black,
   If[circle,
    Circle[{size/2.0, -size/2.0}, size/2.0],
    Rectangle[{0, 0}, {size, -size}, RoundingRadius → cornerRadius * size]
   ],
   PointSize[Medium],
   Point[{size/2.0, -size/2.0}],
   Red, Point[boxIntersection[angle, size, circle, size * cornerRadius]],
   Green, Point[middlePoint[size, .135]],
   Blue, Point[shortPoint[size, .185]],
   Orange, Point[longArmEndPoint[angle, size, size .33, boxIntersection[
      angle, size, circle, size * cornerRadius], middlePoint[size, .135]]]
  },
  PlotRange → All,
  Axes → True
 ]

 , {{θ, 36}, 0, 90, 0.1}, {size, 1, 100, 1},
 {circle, {True, False}}, {cornerRadius, 0, 0.5, 0.01}]
```