

## A PATH FROM LOGIC TO METAPHYSICS\*

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There is a certain order among our concepts, not only our mathematical concepts, but all of our concepts, which I will call the order of conceptual priority, one concept being prior to another if the definition of the second concept refers back to the first concept. Thus, if you start deep in mathematics with some complex notions, like real number or analytic function or locally compact group or something like that, and define them properly, you are eventually reduced to the most fundamental notions that you have in mathematics, which, I think it is generally agreed, are the notions of set and element of a set, the logical notions of proposition and truth, and the notion of function. The purpose of this talk is to see where we are led if we continue in this process of asking ourselves backwards, to see where we end up, that is, what are the most fundamental notions of all, the notions which are such that it no longer seems reasonable to ask for anything further behind them.

I would like to start, not deep in mathematics, of course, but with the logical notions of proposition and truth. Intuitionistically, truth of a proposition is analyzed as existence of proof: a proposition is true if there exists a proof of it. Now, I will not dwell upon the notion of proof of a proposition, because a proposition is defined precisely by explaining what a proof of it looks like: so, once we know the proposition, we certainly know what a proof of the proposition is. But look at the other component that I use to define the notion of truth, namely, the notion of existence. It is quite clear that the notion of existence that enters here is not the notion of existence that is expressed by means of the existential quantifier: rather, the notion of existence that enters here is the traditional philosophical notion of existence of a concept, or existence of an essence, if you prefer, where by saying that a concept has existence I mean that there exists an object which falls under the concept. So to say that a proposition is true is the same as to say that the concept proof of the proposition has existence in the traditional philosophical sense. Now, as already

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Aristotle pointed out, it is of vital importance to distinguish between actual and potential existence. And so it is for us, dealing with the notions of proposition and truth, because, prefixing the attributes actual and potential to existence as it enters into the definition of truth of a proposition, we arrive at the notions of actual truth and potential truth of a proposition, respectively.

What does it mean for a proposition to be actually true? Well, that a proposition *A* is actually true means that *A* has been proved, that is, that a proof of *A* has been constructed, which we can also express by saying that *A* is known to be true, whereas to say that *A* is potentially true is to say that *A* can be proved, that is, that a proof of *A* can be constructed, which is the same as to say, in usual terminology, simply that *A* is true. Thus the notion of potential truth is the notion of truth which appears in the writings of Dummett, Prawitz and Putnam from the late seventies. Now, it has often been pointed out that it is very counter-intuitive to say that a proposition becomes true when it is proved, and it has often been held against the intuitionists that they construe the notion of truth in that way. The notion of truth then concerned is of course the notion of actual truth: before a proposition has been proved, it is not actually true, and it becomes actually true, which is the same as to say that its truth becomes actualized, when the proof is constructed. But, of course, even an intuitionist cannot fail to understand this objection, and ought to answer it by saying that there is, not only the notion of actual truth, but also the notion of potential truth, and that, even before the proposition was proved, it could be proved, which is to say that, although not yet actually true, it was potentially true. Thus the notion of potential truth is not tensed in the way the notion of actual truth is. On this analysis of the notion of truth, potential truth, that is, it is clear that there are no propositions which are true but which cannot be proved, because potential truth is simply analyzed as potential existence of proof.

We may characterize the difference between the notions of actual truth and potential truth by saying that actual truth is knowledge dependent and tensed, whereas potential truth is knowledge independent and tenseless, but observe that, even if I say that it is knowledge independent, it does not mean that it is altogether independent of the notion of knowledge, because it is still conceptually dependent upon the notion of knowledge. And this is because actuality precedes potentiality in the order of conceptual priority. Again, this is something which was stated with every conceivable clarity by Aristotle already, and which was taken over by Thomas, the scholastic formulation being, *Actus est prior potentia*. The reason is that to say that something is potentially is to say that it can be actually: in our case, to say that *A* is potentially true is to say that *A* can be actually true, that is, that *A* can be known to be true. So, clearly, the notion of actuality precedes potentiality in the conceptual order, and hence, in this sense, not even the notion of potential truth is knowledge independent: it is conceptually dependent upon the notion of knowledge. Putnam has expressed this point by saying that truth is independent of justification here and now, but not independent of *all* justification. In this formulation, he means by truth justified or warranted assertability, which is the same as what I have called potential truth.

The second point I want to make is that, in the definition of potential truth, we cannot change the words *A* can be proved into *A* has been, is being or will be proved, that is, will be proved at some time in the course of history, because the conceptual relation between saying that something has been, is being or will be done and saying that it can be done is that we have an entailment in the direction, If something has been, is being or will be done, then it can be done, but not in the converse direction. In the case of proving a proposition, this means that, if a proposition has been, is being or will be proved, then certainly it can be proved, that is, it is potentially true, but there is absolutely no reason to believe that we can go in the opposite direction. The principle just spelled out is again a principle which had a succinct scholastic formulation: it is the principle, *Ab esse ad posse valet consequentia* (illatio).

In the case of proving a proposition, the point that I have just made is due to Prawitz, who brought it up in a paper ten years ago, saying precisely that it is impossible to replace can be by has been, is being or will be in the definition of the notion of potential truth. What I would like to add is only that, again, this is a very old question, and that it was stated with complete clarity by Aristotle, and repeated in scholastic times by Thomas, that there is this gap between saying that something has been, is being or will be done and that it can be done. And you arrive at the same standpoint, that there is such a gap, if you think in terms of possible worlds, because to say that a proposition has been, is being or will be proved is to say that it is proved at some stage in the development of the actual world, whereas to say that it can be proved is to say that it is proved at some stage in the development of some possible world, not necessarily the actual one. Now, I am not of the opinion that one should try to reduce the notion of possibility or potentiality to that of actuality in some possible world, but it is at least reassuring that possible worlds intuition yields the same conclusion on this point. The opposite view, and there is such an opposite view, namely, that every possibility will be actualized in the course of time, is the principle which has been called the principle of plenitude by Lovejoy. But, as I have said, I can see no basis whatever for that principle. If it were to be justified, it would have to be on purely conceptual grounds, and there just are no such grounds.

Next, I would like to leave the notions of proposition and truth and pass on to a notion which is more fundamental than those two, namely, the notion of judgement. In logic, we say such things as that *A* is a proposition and that a proposition *A* is true, and we may begin to wonder what kind of things these are. The terminology that I have found it convenient to revive here is to call them judgements. Of course, *A* is true is the only form of judgement that Frege considered, but it turns out that also *A* is a proposition has to be considered a form of judgement, and those who are familiar with type theory will know that there are other forms of judgement as well that we have been led to consider, but I will not dwell upon them. Now, when analyzing the notion of judgement, it turns out to be necessary to distinguish clearly between judgement, on the one hand, in the sense of the act of judging and, on the other hand, in the sense of that which is judged. If you ask what a judgement in either of these two senses is, it seems that the only answer you can give amounts to no more than a change of word really, a change from one word to a more

fundamental word, or more venerable word, namely, that to judge is the same as to know, more precisely, to get to know, which is to say that the act of judging is the very act of knowing, and that that which is judged is that which is known, that is, the object of knowledge. And knowing is of course to be taken here as a primitive concept: you can clarify it in various ways, but you cannot reduce it to any other kind of act.

With respect to the notion of judgement, it is important to distinguish between what I will use Kantian terminology to refer to, since I do not have a better one, namely, to distinguish between a logically possible judgement, an actual judgement and a really possible judgement. This terminology, logically possible as opposed to really possible, was used by Kant, as I said, but it seems to go further back to Duns Scotus. A judgement is logically possible, or a judgement simpliciter, if you prefer, as soon as it has been laid down what it means, that is, what you mean by making it. This is what is required for it to make sense to talk about making the judgement in question. For example, whatever proposition *A* that we take, *A* is true is a logically possible judgement, because we have defined what it means, namely, that there exists a proof of *A*. An actual judgement, on the other hand, is a judgement that has been made, Ger. *gefälltes Urteil*, and a really possible, or potential, judgement is a judgement that can be made. Again, you see that there is a characteristic order here between the notions of logical possibility, actuality and real possibility in that logical possibility precedes actuality in the conceptual order, whereas actuality precedes real possibility, because, certainly, if we say that a judgement is actual if it has been made, then it must first of all make sense to talk about making that judgement, that is, it has to be a logically possible judgement, whereas I have already, referring back to Aristotle and Thomas, dwelled upon the fact that real possibility, which is the same as potentiality, is posterior to actuality in the conceptual order. If we go back to the distinction between actual truth and potential truth that I started by making, it is clear that to say that *A* is actually true is the same as to say that the judgement *A* is true is actual, and to say that *A* is potentially true is to say that the judgement *A* is true is really possible, or potential, whereas, as I said before, for any proposition *A* whatever, *A* is true is a logically possible judgement.

The next point I need to say a few words about is the fallibility of knowledge. Now, knowledge is fallible in the sense that even a completely unreserved or unqualified knowledge claim that we make, a theorem that we prove, typically, in mathematics, may lead us into difficulties which show to us later that we will have to detract that knowledge claim, or withdraw the theorem that we previously put forward, by publishing an admission of the error or a correction or something like that. And the possibility of mistake or error seems never to be excluded, which is to say that, if we were to include in the definition of knowledge that it should be infallible, then there would be no knowledge that is humanly accessible. Thus the conceptual connection here seems to be, not that knowledge is infallible, because knowledge is fallible in the sense that I have just explained, but that true knowledge, or real knowledge, is infallible. And that true or real knowledge is infallible is a mere conceptual truth, because infallibility means impossibility of going wrong, and to say that a piece of knowledge is true or real is exactly saying that it

cannot go wrong under any circumstances. So truth or reality in this sense is indistinguishable from the notion of infallibility itself. Here I have used the words true or real with their opposites false or apparent, but I might also just as well use the words right or correct here on the one hand and wrong or incorrect on the other. So, of course, right knowledge cannot go wrong: that is a wholly conceptual truth.

The notion of truth of a proposition that I started by explaining is certainly distinct from the notion of truth which is applied to knowledge in the compound true knowledge, and that is why I prefer to use the word rightness or correctness for this latter notion of truth, to distinguish it terminologically. Now, there has been a long discussion, at least since the appearance of Joachim's book *The Nature of Truth* in the beginning of this century, in which one has contrasted a correspondence theory of truth with a coherence theory of truth. This is a contrast which is correlated to the distinction between the notion of truth of a proposition that I started with and the notion of truth as applied to knowledge, because, when you are concerned with the notion of truth of a proposition, some form of a correspondence theory is certainly right: even intuitionistically, the correspondence definition of truth is correct for truth of a proposition, if by correspondence with reality you simply mean the existence of a proof object. On the other hand, when you come to the notion of truth in the sense of rightness or correctness of a judgement, then it seems quite clear, and has been known since Kant, at least, that the correspondence theory breaks down and that the only notion which is left to us is a coherence notion of truth, because we discover our errors precisely by discovering, in the worst cases, outright inconsistencies or, in milder cases, at least incoherencies of some kind. If you think of my work on type theory, for instance, there have been errors in the system from time to time, and those errors have always been discovered by seeing the inconsistencies or other incoherencies that they have given rise to.

This much about the notion of judgement, which we have identified as the notion of knowledge, but I would like to continue even from this point, although we now leave the domain of logic, because certainly logic, as we at least traditionally understand it, studies reasoning or demonstration, which is nothing but the act or process through which we acquire knowledge, and tries to discover the rules governing that activity, which are the rules of inference. But, of course, the notions of act and object that have turned out to be crucial in the analysis of the notion of judgement or knowledge, ambiguous as it is between the act of judging or knowing and that which is judged or known, that is, the object of knowledge, those two notions are not at all limited to acts of knowing, because the concept of act can be understood in the widest possible sense, comprising in the traditional division both thoughts, words and deeds, that is to say, mental acts, verbal acts and concrete acts, like baking a cake or scrubbing a floor or something like that, concrete acts or real acts, whatever you prefer. And acts even of this more general kind, that is, acts like, if you think of mental acts, imagining something, or verbal acts, like commanding something, or real acts, like producing something, they have just like acts of knowing the characteristic that they have an object, an object towards which they are directed, which is the result of the action.

With respect to the distinction between act and object, the history is a bit curious, because in the classical languages it so to say was not even necessary to draw attention to this distinction, because it was built into the grammatical structure of the languages themselves, like in Greek, for instance, you have *πράξις*, the act of doing, and *πράγμα*, that which is done, *ποίησις*, the act of making, and *ποίημα*, that which is made, *αἴσθησις*, the act of perceiving, and *αἴσθημα*, that which is perceived, *νόησις*, the act of thinking, and *νόημα*, that which is thought, and so on, and in Latin, similarly, *actio*, the act of doing, and *actum*, that which is done, *factio*, the act of making, and *factum*, that which is made, *perceptio*, the act of perceiving, and *perceptum*, that which is perceived, *conceptio*, the act of conceiving, and *conceptum*, that which is conceived, *cognitio*, the act of knowing, and *cognitum*, that which is known, and you can make the list as long as you please, whereas in the modern languages we have arrived at a situation where the noun of action is used both for the act of doing something and for that which is done, like construction, for instance, is ambiguous between the act of constructing and that which is constructed. And it seems that, in this way, simply by the change of the structure of the languages, the distinction between act and object has lost the central position that it had when one still philosophized in the classical languages, and it is only within the phenomenological tradition, starting with Bolzano and Brentano, that the distinction between act and object has regained the central position that it indeed deserves.

It turns out that several of the notions that I have already discussed can be lifted to the level of acts and objects in general. This is so, in particular, for the notions of logical possibility, actuality and real possibility: they certainly make sense for any objects whatsoever, not only for objects of knowledge. Indeed, an object is logically possible if it merely has been laid down what is meant by doing it, that is, by performing an act with that object, and an object is actual if it has been done, that is, if an act with that object has been performed, and an object is really possible if it can be done, that is, if an act with that object can be performed. Moreover, the remarks that I made earlier, in connection with acts of knowing, on the order of conceptual priority between logical possibility, actuality and real possibility apply of course also in this more general case. And it is similar with the notion of rightness or correctness: it certainly applies, not only to our acts of knowing, but to all of our acts. Indeed, the combination of words right action is so commonplace, and hence fundamental, that it hardly needs mentioning.

The notion of rightness or correctness gives rise to an interesting question concerning the order of conceptual priority between the rightness of an act and the rightness of an object. Which notion is the conceptually prior one? Is it that an act is right if the object of that act is right, or is it that an object is right if it has been rightly done? And I think it suffices here to consider the case of a mathematical theorem, which is an object of knowledge, to see that the order of conceptual priority is that rightness applies primarily to the action and only derivatively to the object, because if we ask whether a theorem is correct, we certainly go to the proof, and the proof is the act or process through which we get to know the theorem, so that we have to check this act or process for its correctness and thereby arrive at the correctness of the theorem, and not the other way round.

The duality between act and object is almost the same as the duality between life and world, where I take it that life and activity are synonymous, because if you consider just a single act and its object, then of course we use the terms act and object, but if you consider the whole stream or flux of actions that I perform, then it is that stream or flux which is my life or my activity, and if we think, not only of the actions that I perform, but the actions that all of us, living beings, perform, then this stream is our life, and, correlatively, we speak of my world as the totality of everything, that is, all the objects, that I have done, and of our world as the world that consists of everything that we have done together. If you understand the notion of world in this way, as the totality of all objects that have been done, that is, objects of all actions that have been performed, then it is clear that the world is by its very nature our life world, Ger. *Lebenswelt*, in Husserl's terminology. On this conception, it simply does not make sense to speak of world in any other sense than that of life world, of world in which we live.

Now you see where we have arrived by this process of relentlessly asking ourselves backwards, what we have arrived at as the most fundamental notions, namely, the notions of act and object, or if we think, not only of a single act and its object, but the whole flux of actions, life and world. On our way, we have passed the notions of actuality and potentiality, of past, present and future, and so on, but certainly these are the two most fundamental notions that we are thrown back upon, the notions of act and object, and we also need the notion of rightness of an action to account for the error phenomenon. What has struck me is that there is a considerable similarity between this result and the fundamental structure of traditional rationalist metaphysics, namely, God, soul and world, as it appears in Wolff's *Vernünftige Gedanken von Gott, der Welt und der Seele des Menschen, auch allen Dingen überhaupt* and was taken over by Kant in the *Kritik der reinen Vernunft* and the *Prolegomena*. The similarity is this, of course, that world corresponds to world, soul corresponds to activity or life, since the ego, self or soul is the performer of its acts, and God corresponds to the notion of rightness. Normally, following Augustine, God is identified with truth in the sense of reality as opposed to falsehood in the sense of mere appearance, but I have already remarked, in my discussion of the fallibility of knowledge, that to say of an object of knowledge that it is true or real is the same as to say that it is right, that is, that it is rightly known, or rightly apprehended.

What about the notion of creation? If we understand creation in the usual way as causing to come into existence or bringing into being, and understand the notion of being or existence as actuality, which is how the exalted notion of being traditionally has been understood, then we arrive at what seems to be a very heretical conclusion here, because what is it that gives actuality to the objects of the world? Well, I have already said that what makes an object actual is that there is an act that is being performed with that object, so on this conception it is our activity that gives the objects of the actual world their actuality, which means that it is our activity that becomes the process of creation. This may seem disturbing, of course, since it seems that this kind of thinking would put us in the position of God, if we think of God primarily as the creator, but there are in our tradition really two different conceptions of God, one is of God as truth, which is the

Augustinian conception that I have already touched upon, and the other is of God as being, understood in the sense of actuality, which is the Thomistic conception. Now, Thomas certainly did not have in mind putting forward a notion of God which was different from or in conflict with the Augustinian notion of God, and he achieved this by interpreting the notions of being and truth in such a way that they could be identified: for Thomas, being and truth were convertible terms in scholastic terminology, that is, terms having the same meaning. But, on the analysis of these notions that I have given here, it is quite clear that this identification can no longer be upheld, because being is identified with actuality and truth with rightness, and the notion of actuality is certainly to be distinguished from the notion of rightness: that something is actual, that is, that something has been done, does not necessarily mean that it is right: that is precisely why we have the notion of error, to give room for the fact that we make wrong actions, and when we make a wrong action, there is something which is actual although it is not right. So the notion of actuality and the notion of rightness certainly have to be distinguished, and it seems to me that, at least if we want to interpret the traditional triple of God, soul and world, it is quite clear that God must correspond to the notion of rightness and not to the notion of actuality, which is essentially the notion of activity or life: more precisely, activity or life is what lends actuality to whatever is actual.

There now remain only two final remarks on the problems of the evil and the freedom of the will. If we ask for the source of the evil, first of all, it is clear that, following this way of thinking, what is the source of the evil? Well, it is of course we who are the source of the evil, because the evil in the world is precisely the result of our errors or mistakes, that is, of our wrong actions, and hence it is from us that the evil is coming. You see here very clearly the fundamental difference from the view of the world as God's creation, which is the world view that gave rise to the problem of the evil. Indeed, if we view the world as being created by God, and take the world to include, not only trees and mountains and so on, but also us and all other living beings, then of course there becomes the problem, Where does the evil in the world come from? If God has created the world, it seems that he must be responsible for the evil in the world, since he has made the world the way it is, whereas the different interpretation of the process of creation, as our activity, shifts the responsibility for the evil from God to ourselves. And it is precisely the same way with the related problem of the freedom of the will. Now, the problem of the freedom of the will has both an old form and a modern form, I mean, modern in the sense of the modern time. The old form of the problem of the free will again has to do with the fact that one thought of the world, including us and all other living beings, as being created by God. And not only the world at this particular moment, but the world from the beginning of time till the end of time, that is, the world in all its time development was thought of as being created by God: first he had it in his mind, and then he created it in all its time development by an act of his will. On that view of the world, it seems of course that God must be responsible for everything that is happening in this world that he has created, and so where is there any room for the freedom of the will that we are all familiar with? If he has arranged everything, he must have arranged also the actions that are going

to be performed in this world of his.

That was the old problem of the freedom of the will, but it has also a closely related modern version, namely, the problem that you have with the free will in the mechanistic world view. If you view the world as a whole as a physical system, whether evolving according to deterministic laws or to some probabilistic laws or to some laws which are probabilistic in the sense of quantum theory, you have the same problem, that if we belong to the world that is governed by these physical laws, deterministic or probabilistic, where is there any room for the freedom of choosing to do this or that that we all experience? It is to be governed by those laws on that conception. Again, you see that the problem comes from the characteristic way of looking at the world, including us and all other living beings, as a whole, so to say from the outside, a whole that is regarded, in the latter case, as governed by physical laws and, in the former case, as brought forth by God's act of creation. It is precisely this view of the world that becomes changed when you structure the concepts in the way that I have suggested. We are not part of a world which depends for its existence on the activity of an extramundane agent: rather, we are ourselves the agents who by our activity lend actuality to the objects of the world. The purpose of my discussion of the problems of the evil and the freedom of the will was to show that perhaps the conclusion that we have reached, shocking as it may seem at first sight, that it is our activity which is the process of creation, solves old problems rather than creates new ones.