CSEN 1002

Task 8: LL(1) Parsing

Outline

Example Grammar 1 - Parsing Table

Input

S;A;B# a;b;c;d# S/AaS,d;A/BbBaSc,e;B/e# S/ab,d;A/b,e;B/e# S/$c;A/a;B/ab

$$
\begin{aligned}
S &\rightarrow AaS \mid d \\
A &\rightarrow BbBaSc \mid \varepsilon \\
B &\rightarrow \varepsilon
\end{aligned}
$$

Example Grammar 1 - Parsing Table

Input

S;A;B# a;b;c;d# S/AaS,d;A/BbBaSc,e;B/e# S/ab,d;A/b,e;B/e# S/$c;A/a;B/ab

$$
\begin{aligned}
S &\rightarrow AaS \mid d \\
A &\rightarrow BbBaSc \mid \varepsilon \\
B &\rightarrow \varepsilon
\end{aligned}
$$

LHS	RHS	First	Follow
S	AaS	a,b	$,c
	d	d	
A	BbBaSc	b	a
	e	e	
B	e	e	a,b

Example Grammar 1 - Parsing Table

LHS	RHS	First	Follow
S	AaS	a,b	$,c
	d	d	
A	BbBaSc	b	a
	e	e	
B	e	e	a,b

	a	b	c	d	$
S					
A					
B					

Example Grammar 1 - Parsing Table

LHS	RHS	First	Follow
S	AaS	a,b	$,c
	d	d	
A	BbBaSc	b	a
	e	e	
B	e	e	a,b

	a	b	c	d	$
S	AaS	AaS			
A					
B					

Example Grammar 1 - Parsing Table

LHS	RHS	First	Follow
S	AaS	a,b	$,c
	d	d	
A	BbBaSc	b	a
	e	e	
B	e	e	a,b

	a	b	c	d	$
S	AaS	AaS		d	
A					
B					

Example Grammar 1 - Parsing Table

LHS	RHS	First	Follow
S	AaS	a,b	$,c
	d	d	
A	BbBaSc	b	a
	e	e	
B	e	e	a,b

	a	b	c	d	$
S	AaS	AaS		d	
A		$BbBaSc$			
B					

Example Grammar 1 - Parsing Table

LHS	RHS	First	Follow
S	AaS	a,b	$,c
	d	d	
A	BbBaSc	b	a
	e	e	
B	e	e	a,b

	a	b	c	d	$
S	AaS	AaS		d	
A	e	$BbBaSc$			
B					

Example Grammar 1 - Parsing Table

LHS	RHS	First	Follow
S	AaS	a,b	$,c
	d	d	
A	BbBaSc	b	a
	e	e	
B	e	e	a,b

	a	b	c	d	$
S	AaS	AaS		d	
A	e	$BbBaSc$			
B	e	e			

Example 1

	a	b	c	d	\$
S	AaS	AaS		d	
A	e	$BbBaSc$			
B	e	e			

Derivation:
S

Initialized

a	b	a	d	c	a	d	\$

↑

S
\$

Example 1

	a	b	c	d	$\$$
S	AaS	AaS		d	
A	e	$BbBaSc$			
B	e	e			

Derivation:

S; AaS

> $M(S, a) = AaS$. The stack content is updated and a new step in the derivation is added.

a	b	a	d	c	a	d	$

↑

Stack:

A
a
S
$\$$

Example 1

	a	b	c	d	$\$$
S	AaS	AaS		d	
A	e	$BbBaSc$			
B	e	e			

Derivation:

S; AaS; aS

$M(A, a) = e$. The stack content is updated and a new step in the derivation is added.

a	b	a	d	c	a	d	$

↑ (at a)

Stack:

a
S
$\$$

Example 1

	a	b	c	d	\$
S	AaS	AaS		d	
A	e	$BbBaSc$			
B	e	e			

Derivation:
S; AaS; aS

Matching the top-of-the-stack terminal with input symbol

a	b	a	d	c	a	d	\$

↑

S
\$

Example 1

	a	b	c	d	$\$$
S	AaS	AaS		d	
A	e	$BbBaSc$			
B	e	e			

Derivation:

S; AaS; aS; aAaS

> $M(S, b) = AaS$. The stack content is updated and a new step in the derivation is added.

a	b	a	d	c	a	d	$
	↑						

A
a
S
$\$$

Example 1

	a	b	c	d	\$
S	AaS	AaS		d	
A	e	$BbBaSc$			
B	e	e			

Derivation:
S; AaS; aS; aAaS; aBbBaScaS

$M(A, b) = BbBaSc$. The stack content is updated and a new step in the derivation is added.

a	b	a	d	c	a	d	\$
	↑						

B
b
B
a
S
c
a
S
\$

Example 1

	a	b	c	d	\$
S	AaS	AaS		d	
A	e	$BbBaSc$			
B	e	e			

Derivation:
S; AaS; aS; aAaS; aBbBaScaS; abBaScaS

$M(B, b) = e$. The stack content is updated and a new step in the derivation is added.

a	b	a	d	c	a	d	\$
	↑						

b
B
a
S
c
a
S
\$

Example 1

	a	*b*	*c*	*d*	$
S	*AaS*	*AaS*		*d*	
A	*e*	*BbBaSc*			
B	*e*	*e*			

Derivation:
S; AaS; aS; aAaS; aBbBaScaS; abBaScaS

> Matching the top-of-the-stack terminal with input symbol

a	b	a	d	c	a	d	$
		↑					

Stack:

B
a
S
c
a
S
$

Example 1

	a	b	c	d	$\$$
S	AaS	AaS		d	
A	e	$BbBaSc$			
B	e	e			

Derivation:

S; AaS; aS; aAaS; aBbBaScaS; abBaScaS; abaScaS

$M(B, a) = e$. The stack content is updated and a new step in the derivation is added.

a	b	a	d	c	a	d	$

↑ (pointer under the third cell, a)

Stack (top to bottom):

a
S
c
a
S
$\$$

Example 1

	a	*b*	*c*	*d*	$
S	*AaS*	*AaS*		*d*	
A	*e*	*BbBaSc*			
B	*e*	*e*			

Derivation:
S; AaS; aS; aAaS; aBbBaScaS; abBaScaS; abaScaS

> Matching the top-of-the-stack terminal with input symbol

a	b	a	d	c	a	d	$
			↑				

Stack:

S
c
a
S
$

Example 1

	a	b	c	d	$\$$
S	AaS	AaS		d	
A	e	$BbBaSc$			
B	e	e			

Derivation:

S; AaS; aS; aAaS; aBbBaScaS; abBaScaS; abaScaS; abadcaS

$M(S, d) = d$. The stack content is updated and a new step in the derivation is added.

a	b	a	d	c	a	d	$

↑ (under d)

Stack:

d
c
a
S
$\$$

Example 1

	a	b	c	d	\$
S	AaS	AaS		d	
A	e	$BbBaSc$			
B	e	e			

Derivation:

S; AaS; aS; aAaS; aBbBaScaS; abBaScaS; abaScaS; abadcaS

Matching the top-of-the-stack terminal with input symbol

a	b	a	d	c	a	d	\$

↑

c
a
S
\$

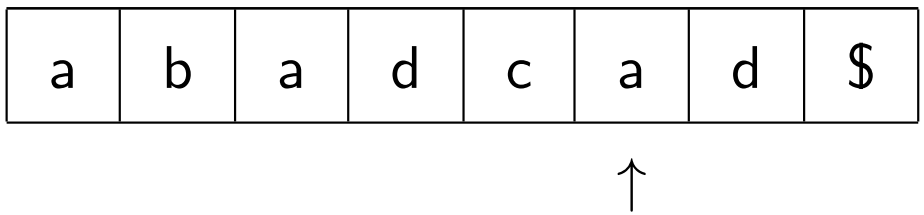
Example 1

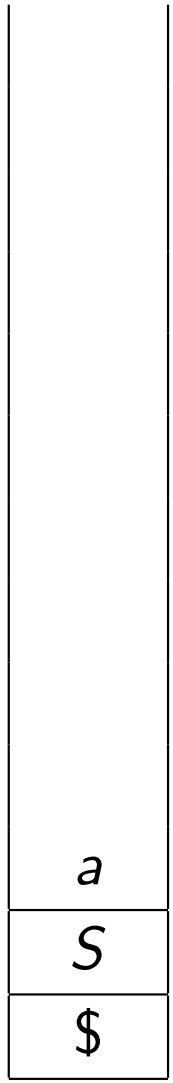
	a	b	c	d	\$
S	AaS	AaS		d	
A	e	$BbBaSc$			
B	e	e			

Derivation:

S; AaS; aS; aAaS; aBbBaScaS; abBaScaS; abaScaS; abadcaS

Matching the top-of-the-stack terminal with input symbol





Example 1

	a	b	c	d	\$
S	AaS	AaS		d	
A	e	$BbBaSc$			
B	e	e			

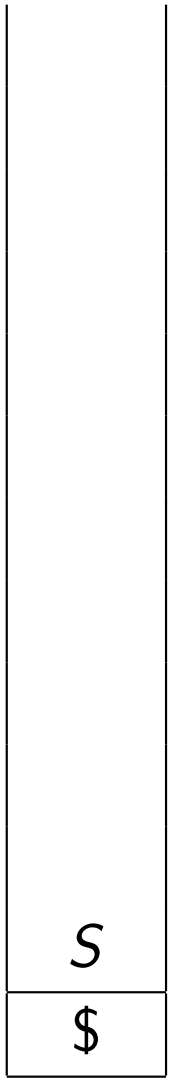
Derivation:

S; AaS; aS; aAaS; aBbBaScaS; abBaScaS; abaScaS; abadcaS

Matching the top-of-the-stack terminal with input symbol

a	b	a	d	c	a	d	\$

↑

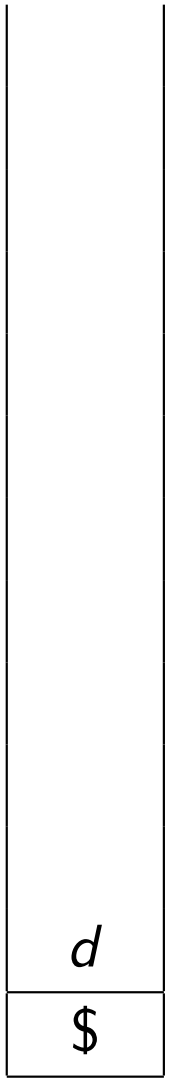


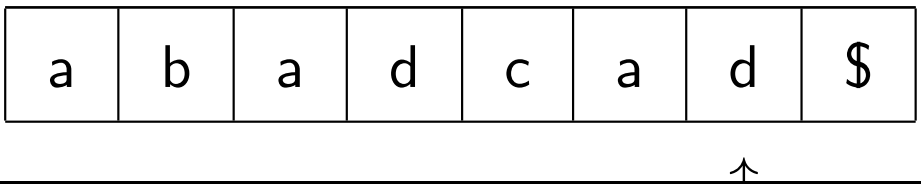
Example 1

	a	b	c	d	\$
S	AaS	AaS		d	
A	e	$BbBaSc$			
B	e	e			

Derivation:
S; AaS; aS; aAaS; aBbBaScaS; abBaScaS; abaScaS; abadcaS; abadcad

$M(S, d) = d$. The stack content is updated and a new step in the derivation is added.



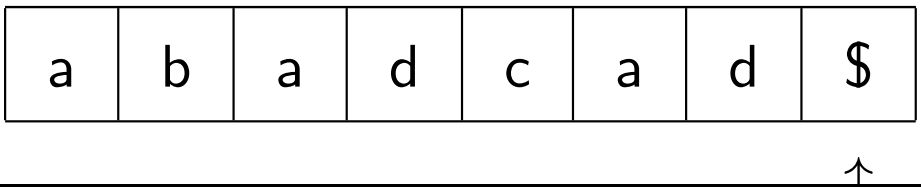


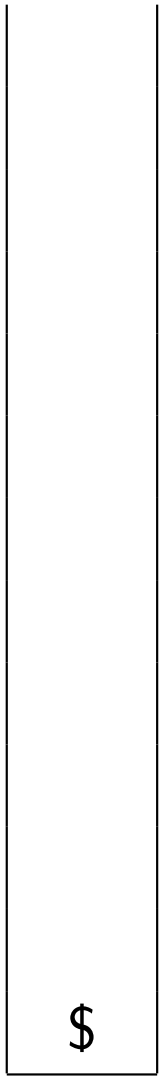
Example 1

	a	b	c	d	\$
S	AaS	AaS		d	
A	e	$BbBaSc$			
B	e	e			

Derivation:
S; AaS; aS; aAaS; aBbBaScaS; abBaScaS; abaScaS; abadcaS; abadcad

Matching the top-of-the-stack terminal with input symbol





Example 1

	a	*b*	*c*	*d*	$
S	*AaS*	*AaS*		*d*	
A	*e*	*BbBaSc*			
B	*e*	*e*			

Derivation:
S; AaS; aS; aAaS; aBbBaScaS; abBaScaS; abaScaS; abadcaS; abadcad

Parsed

a	b	a	d	c	a	d	$

Example 2

	a	b	c	d	\$
S	AaS	AaS		d	
A	e	$BbBaSc$			
B	e	e			

Derivation:
S

Initialized

b	b	b	d	\$

↑

S
\$

Example 2

	a	b	c	d	\$
S	AaS	AaS		d	
A	e	$BbBaSc$			
B	e	e			

Derivation:
S; AaS

$M(S, b) = AaS$. The stack content is updated and a new step in the derivation is added.

b	b	b	d	\$

↑

A
a
S
\$

Example 2

	a	b	c	d	\$
S	AaS	AaS		d	
A	e	$BbBaSc$			
B	e	e			

Derivation:
S; AaS; BbBaScaS

$M(A, b) = BbBaSc$. The stack content is updated and a new step in the derivation is added.

b	b	b	d	\$
↑				

B
b
B
a
S
c
a
S
\$

Example 2

	a	b	c	d	\$
S	AaS	AaS		d	
A	e	$BbBaSc$			
B	e	e			

Derivation:
S; AaS; BbBaScaS; bBaScaS

$M(B, b) = e$. The stack content is updated and a new step in the derivation is added.

b	b	b	d	\$

↑

b
B
a
S
c
a
S
\$

Example 2

	a	b	c	d	\$
S	AaS	AaS		d	
A	e	$BbBaSc$			
B	e	e			

Derivation:

S; AaS; BbBaScaS; bBaScaS

Matching the top-of-the-stack terminal with input symbol

b	b	b	d	\$

↑

B
a
S
c
a
S
\$

Example 2

	a	b	c	d	$
S	AaS	AaS		d	
A	e	$BbBaSc$			
B	e	e			

Derivation:

S; AaS; BbBaScaS; bBaScaS; baScaS

$M(B, b) = e$. The stack content is updated and a new step in the derivation is added.

b	b	b	d	$
	↑			

Stack:

a
S
c
a
S
$

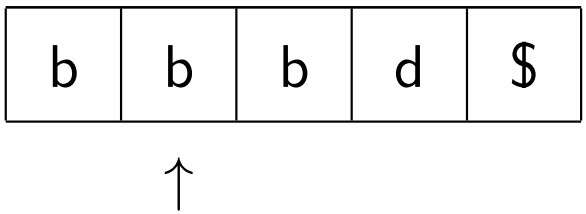
Example 2

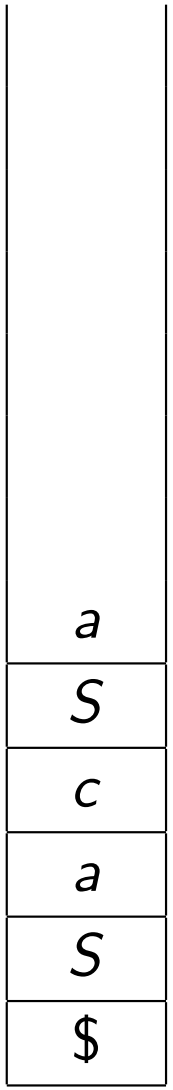
	a	b	c	d	\$
S	AaS	AaS		d	
A	e	$BbBaSc$			
B	e	e			

Derivation:

S; AaS; BbBaScaS; bBaScaS; baScaS; ERROR

ERROR as the top of the stack symbol doesn't match the next symbol on the tape





Example 3

	a	b	c	d	\$
S	AaS	AaS		d	
A	e	$BbBaSc$			
B	e	e			

Derivation:
S

Initialized

a	b	a	c	\$

↑

S
\$

Example 3

	a	b	c	d	\$
S	AaS	AaS		d	
A	e	$BbBaSc$			
B	e	e			

Derivation:
S; AaS

$M(S, a) = AaS$. The stack content is updated and a new step in the derivation is added.

a	b	a	c	\$

↑

A
a
S
\$

Example 3

	a	b	c	d	$\$$
S	AaS	AaS		d	
A	e	$BbBaSc$			
B	e	e			

Derivation:

S; AaS; aS

> $M(A, a) = e$. The stack content is updated and a new step in the derivation is added.

a	b	a	c	$

↑

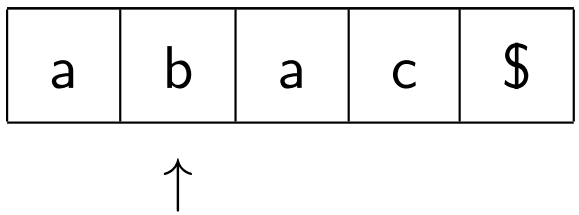
a
S
$\$$

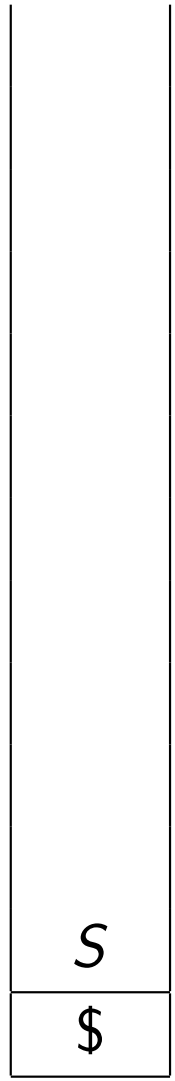
Example 3

	a	b	c	d	\$
S	AaS	AaS		d	
A	e	$BbBaSc$			
B	e	e			

Derivation:
S; AaS; aS

Matching the top-of-the-stack terminal with input symbol



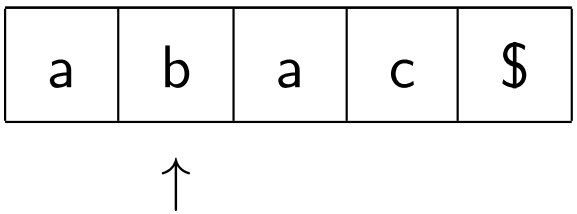


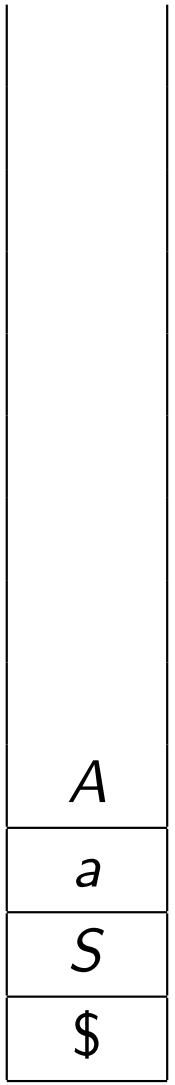
Example 3

	a	b	c	d	\$
S	AaS	AaS		d	
A	e	$BbBaSc$			
B	e	e			

Derivation:
S; AaS; aS; aAaS

$M(S, b) = AaS$. The stack content is updated and a new step in the derivation is added.





Example 3

	a	b	c	d	\$
S	AaS	AaS		d	
A	e	$BbBaSc$			
B	e	e			

Derivation:
S; AaS; aS; aAaS; aBbBaScaS

$M(A, b) = BbBaSc$. The stack content is updated and a new step in the derivation is added.

a	b	a	c	\$
	↑			

B
b
B
a
S
c
a
S
\$

Example 3

	a	b	c	d	\$
S	AaS	AaS		d	
A	e	$BbBaSc$			
B	e	e			

Derivation:
S; AaS; aS; aAaS; aBbBaScaS; abBaScaS

$M(B, b) = e$. The stack content is updated and a new step in the derivation is added.

a	b	a	c	\$
	↑			

b
B
a
S
c
a
S
\$

Example 3

	a	b	c	d	\$
S	AaS	AaS		d	
A	e	$BbBaSc$			
B	e	e			

Derivation:

S; AaS; aS; aAaS; aBbBaScaS; abBaScaS

Matching the top-of-the-stack terminal with input symbol

a	b	a	c	\$

↑

B
a
S
c
a
S
\$

Example 3

	a	b	c	d	\$
S	AaS	AaS		d	
A	e	$BbBaSc$			
B	e	e			

Derivation:

S; AaS; aS; aAaS; aBbBaScaS; abBaScaS; abaScaS

$M(B, a) = e$. The stack content is updated and a new step in the derivation is added.

a	b	a	c	\$

↑

a
S
c
a
S
\$

Example 3

	a	b	c	d	$
S	AaS	AaS		d	
A	e	$BbBaSc$			
B	e	e			

Derivation:

S; AaS; aS; aAaS; aBbBaScaS; abBaScaS; abaScaS

> Matching the top-of-the-stack terminal with input symbol

a	b	a	c	$

↑ (under c)

Stack:

S
c
a
S
$

Example 3

	a	b	c	d	\$
S	AaS	AaS		d	
A	e	$BbBaSc$			
B	e	e			

Derivation:

S; AaS; aS; aAaS; aBbBaScaS; abBaScaS; abaScaS; ERROR

ERROR as $M(S, c)$ is $\varnothing$.

a	b	a	c	\$

↑ (under c)

Stack:

S
c
a
S
\$

Example Grammar 2 - Parsing Table

Input

S;T# a;c;i# S/iST,e;T/cS,a# S/i,e;T/c,a# S/$ac;T/$ac

$$
\begin{aligned}
S &\rightarrow iST \mid \varepsilon \\
T &\rightarrow cS \mid a
\end{aligned}
$$

Example Grammar 2 - Parsing Table

Input

S;T# a;c;i# S/iST,e;T/cS,a# S/i,e;T/c,a# S/$ac;T/$ac

$$
\begin{aligned}
S &\rightarrow iST \mid \varepsilon \\
T &\rightarrow cS \mid a
\end{aligned}
$$

LHS	RHS	First	Follow
S	iST	i	$,c,a
	e	e	
T	cS	c	$,c,a
	a	a	

Example Grammar 2 - Parsing Table

LHS	RHS	First	Follow
S	iST	i	\$,c,a
	e	e	
T	cS	c	\$,c,a
	a	a	

	a	c	i	\$
S				
T				

Example Grammar 2 - Parsing Table

LHS	RHS	First	Follow
S	iST	i	\$,c,a
	e	e	
T	cS	c	\$,c,a
	a	a	

	a	c	i	\$
S			iST	
T				

Example Grammar 2 - Parsing Table

LHS	RHS	First	Follow
S	iST	i	\$,c,a
	e	e	
T	cS	c	\$,c,a
	a	a	

	a	c	i	\$
S	e	e	iST	e
T				

Example Grammar 2 - Parsing Table

LHS	RHS	First	Follow
S	iST	i	\$,c,a
	e	e	
T	cS	c	\$,c,a
	a	a	

	a	c	i	\$
S	e	e	iST	e
T		cS		

Example Grammar 2 - Parsing Table

LHS	RHS	First	Follow
S	iST	i	\$,c,a
	e	e	
T	cS	c	\$,c,a
	a	a	

	a	c	i	\$
S	e	e	iST	e
T	a	cS		

Example 4

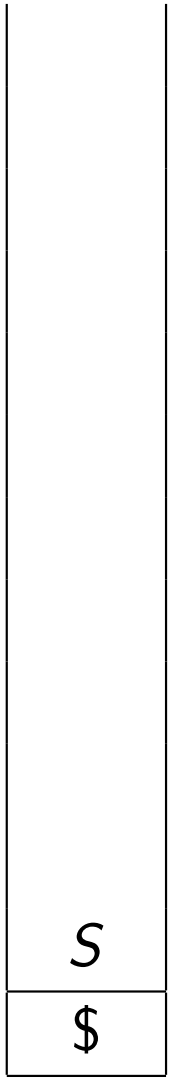
	a	c	i	\$
S	e	e	iST	e
T	a	cS		

Derivation:
S

Initialized

i	i	a	c	\$

↑



Example 4

	a	c	i	\$
S	e	e	iST	e
T	a	cS		

Derivation:
S; iST

$M(S, i) = iST$. The stack content is updated and a new step in the derivation is added.

i	i	a	c	\$

↑

Stack
i
S
T
\$

Example 4

	a	c	i	$
S	e	e	iST	e
T	a	cS		

Derivation:
S; iST

Matching the top-of-the-stack terminal with input symbol

i	i	a	c	$

↑ (under the second i)

Stack:

S
T
$

Example 4

	a	c	i	$\$$
S	e	e	iST	e
T	a	cS		

Derivation:
S; iST; iiSTT

> $M(S, i) = iST$. The stack content is updated and a new step in the derivation is added.

i	i	a	c	$
	↑			

i
S
T
T
$\$$

Example 4

	a	c	i	$\$$
S	e	e	iST	e
T	a	cS		

Derivation:
S; iST; iiSTT

> Matching the top-of-the-stack terminal with input symbol

i	i	a	c	$

↑ (under a)

Stack:

S
T
T
$\$$

Example 4

	a	c	i	$\$$
S	e	e	iST	e
T	a	cS		

Derivation:
S; iST; iiSTT; iiTT

$M(S, a) = e$. The stack content is updated and a new step in the derivation is added.

i	i	a	c	$

↑ (under a)

Stack
T
T
$\$$

Example 4

	a	c	i	\$
S	e	e	iST	e
T	a	cS		

Derivation:
S; iST; iiSTT; iiTT; iiaT

$M(T, a) = a$. The stack content is updated and a new step in the derivation is added.

i	i	a	c	\$

↑

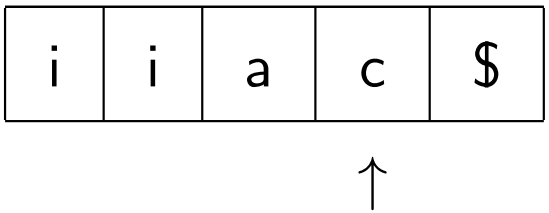
a
T
\$

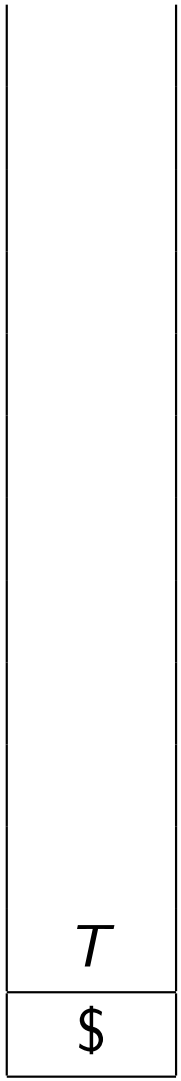
Example 4

	a	c	i	\$
S	e	e	iST	e
T	a	cS		

Derivation:
S; iST; iiSTT; iiTT; iiaT

Matching the top-of-the-stack terminal with input symbol





Example 4

	a	c	i	$\$$
S	e	e	iST	e
T	a	cS		

Derivation:
S; iST; iiSTT; iiTT; iiaT; iiacS

$M(T, c) = cS$. The stack content is updated and a new step in the derivation is added.

i	i	a	c	$

↑ (pointing at c)

Stack:

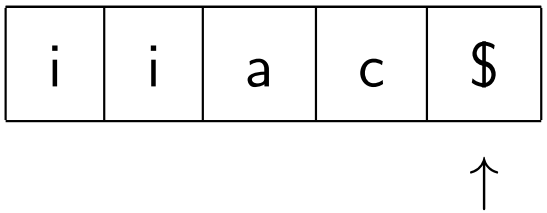
c
S
$\$$

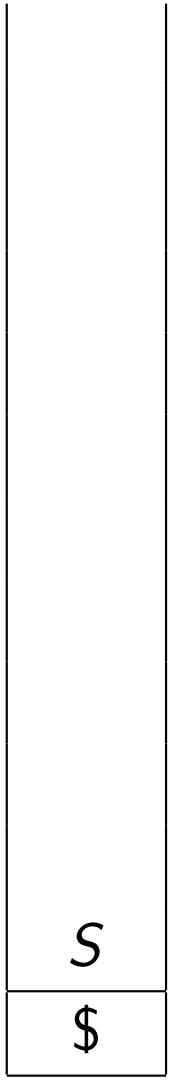
Example 4

	a	c	i	\$
S	e	e	iST	e
T	a	cS		

Derivation:
S; iST; iiSTT; iiTT; iiaT; iiacS

Matching the top-of-the-stack terminal with input symbol



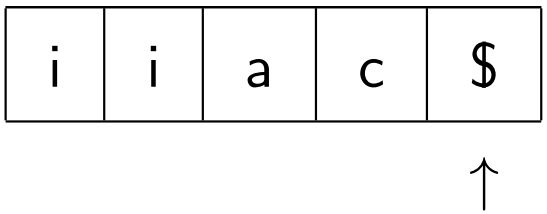


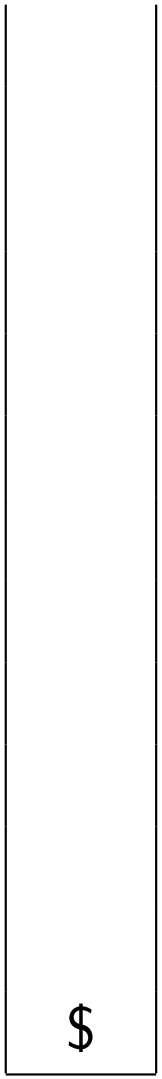
Example 4

	a	c	i	\$
S	e	e	iST	e
T	a	cS		

Derivation:
S; iST; iiSTT; iiTT; iiaT; iiacS; iiac

$M(S, \$) = e$. The stack content is updated and a new step in the derivation is added.





Example 4

	a	c	i	\$
S	e	e	iST	e
T	a	cS		

Derivation:
S; iST; iiSTT; iiTT; iiaT; iiacS; iiac

Parsed

i	i	a	c	\$

Example 5

	a	c	i	\$
S	e	e	iST	e
T	a	cS		

Derivation:
S

Initialized

i	i	a	\$
↑			

S
\$

Example 5

	a	c	i	$\$$
S	e	e	iST	e
T	a	cS		

Derivation:
S; iST

> $M(S, i) = iST$. The stack content is updated and a new step in the derivation is added.

i	i	a	$

↑

i
S
T
$\$$

Example 5

	a	c	i	$\$$
S	e	e	iST	e
T	a	cS		

Derivation:
S; iST

> Matching the top-of-the-stack terminal with input symbol

i	i	a	$
	↑		

S
T
$\$$

Example 5

	a	c	i	$
S	e	e	iST	e
T	a	cS		

Derivation:
S; iST; iiSTT

$M(S, i) = iST$. The stack content is updated and a new step in the derivation is added.

i	i	a	$

↑ (under the second i)

Stack (top to bottom):

i
S
T
T
$

Example 5

	a	c	i	$
S	e	e	iST	e
T	a	cS		

Derivation:
S; iST; iiSTT

Matching the top-of-the-stack terminal with input symbol

i	i	a	$
		↑	

Stack
S
T
T
$

Example 5

	a	*c*	*i*	$
S	*e*	*e*	*iST*	*e*
T	*a*	*cS*		

Derivation:
S; iST; iiSTT; iiTT

$M(S, a) = e$. The stack content is updated and a new step in the derivation is added.

i	i	a	$

↑ (under a)

Stack:

T
T
$

Example 5

	a	c	i	$\$$
S	e	e	iST	e
T	a	cS		

Derivation:
S; iST; iiSTT; iiTT; iiaT

$M(T, a) = a$. The stack content is updated and a new step in the derivation is added.

i	i	a	$

↑ (under a)

Stack:

a
T
$\$$

Example 5

	a	c	i	\$
S	e	e	iST	e
T	a	cS		

Derivation:
S; iST; iiSTT; iiTT; iiaT

Matching the top-of-the-stack terminal with input symbol

i	i	a	\$

↑ (under \$)

Stack:

T
\$

Example 5

	a	c	i	\$
S	e	e	iST	e
T	a	cS		

Derivation:
S; iST; iiSTT; iiTT; iiaT; ERROR

ERROR as $M(T, \$)$ is $\varnothing$.