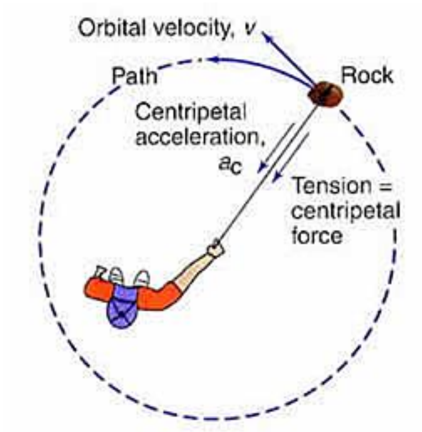
Circular Motion

In physics, circular motion is a movement of an object along the circumference of a circle or rotation along a circular path. Circular motion is one type of rotational motion and the axis through which the object rotates is called axis of rotation.

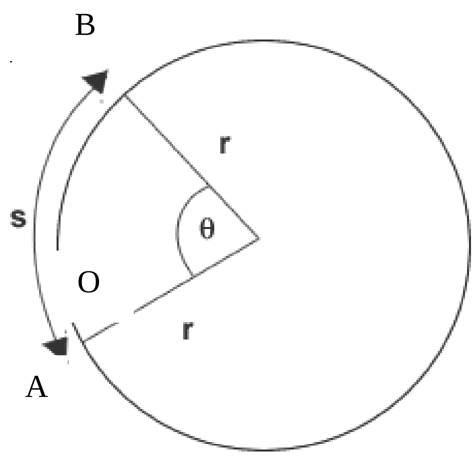
Examples of circular motion include: an artificial satellite orbiting the Earth at constant height, a stone which is tied to a rope and is being swung in circles, a car turning through a curve in a race track, an electron moving perpendicular to a uniform magnetic field.



Angular Displacement

An object or a particle when moves round a point as center, the angular distance it travels is called angular displacement of that object or particle.It can be denoted by θ. Its unit is radian. Sometimes θ is expressed in degree or radian.

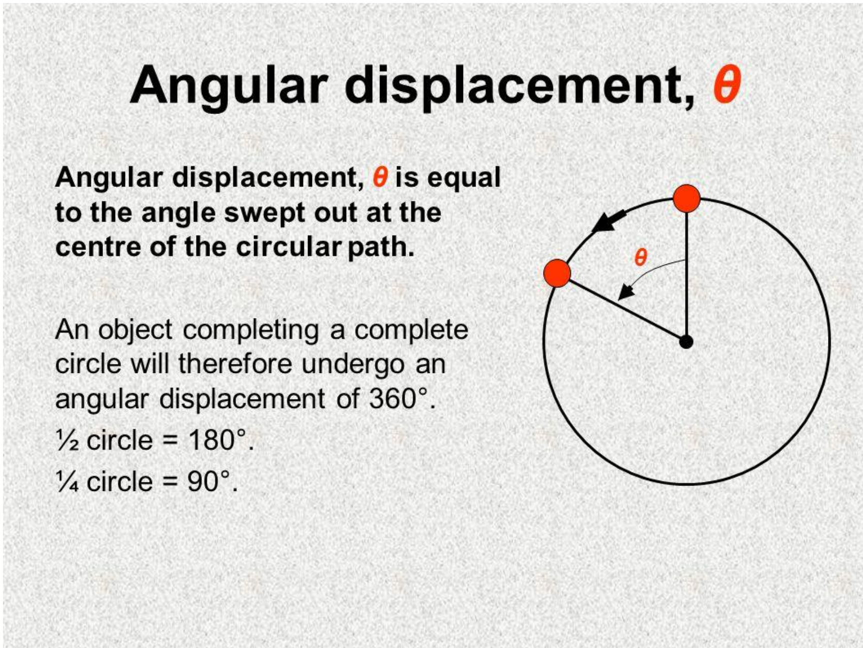
Let a particle move from A to B at time t, then AOB = θ , is the angular displacement of the particle during that time.



1 radian: Let a particle at time t moves from A to B position. Thus, the particle travels a distance s along the circumference and it makes angle θ at the centre. Now according to the definition of radian

$$\theta = \frac{\text{arc}}{\text{radius}} = \frac{s}{r}$$

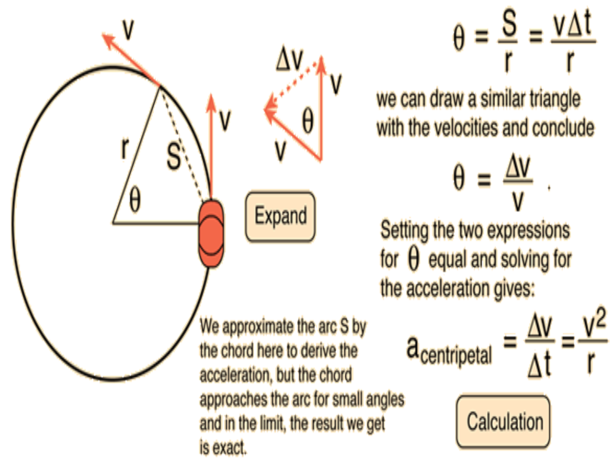
When the length of an arc is equal to the radius of a circle, the angle which the arc makes at the centre is called 1 radian.

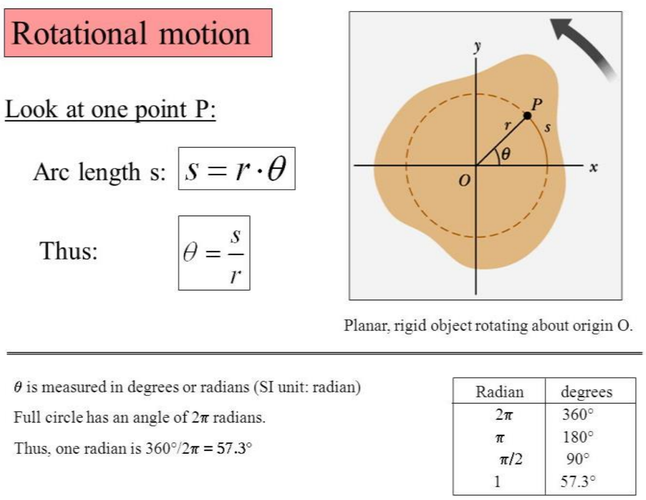


If the rotating particle completes one revolution of a circle, then

$$\theta = \frac{\text{circumference of the circle}}{\text{radius of the circle}} = \frac{2\pi r}{r} = 2\pi \text{ radian} = 360^0$$

1 revolution = 2π radian = 360^0





Radian	degrees
2π	360°
π	180°
$\pi/2$	90°
1	57.3°

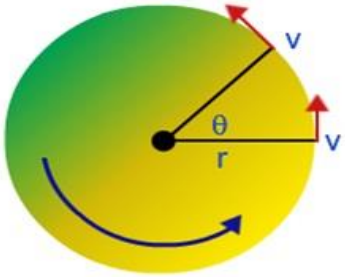
Angular Velocity

In physics, the angular velocity is defined as the rate of change of angular displacement and is a vector quantity (more precisely, a pseudo vector) which specifies the angular speed (rotational speed) of an object and the axis about which the object is rotating.

The SI unit of angular velocity is radians per second. It is denoted by ω.

Mathematically, Angular velocity can be defined as

$\omega = \frac{d\theta}{dt}$



$$\omega = \frac{d\theta}{dt}$$

Relation between angular velocity and time period

We know, the circumference makes angle 2π at the centre. Now, if a particle takes time T to travel the circumference once then from the definition of angular velocity

$$\omega = \frac{\text{Angular displacement}}{\text{time}} = \frac{2\pi}{T}$$

This is the relation between angular velocity and time period.

Relation between angular velocity and frequency

Number of rotations per second is called frequency. It is denoted by f.

We know

$$f = \frac{1}{T}$$

Multiplying both side by 2π

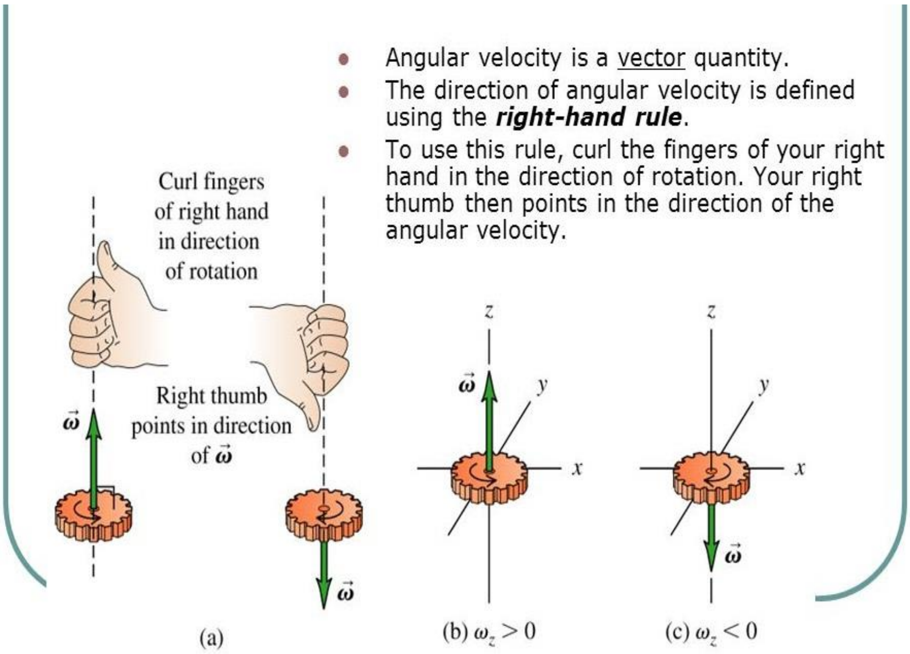
$$2\pi f = \frac{2\pi}{T} = \omega \qquad (\text{Since, } \omega = \frac{2\pi}{T})$$

Finally

$$\omega = 2\pi f$$

This is the relation between angular velocity and frequency.

Direction of Angular Velocity



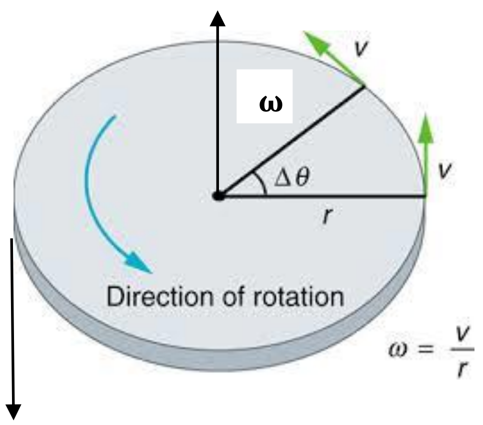
Relation between linear velocity and angular velocity

Let an object moving with uniform circular motion along a circular path of radius r. Further, let the linear velocity be v, angular velocity be ω and time period be T.

Now, in T sec the object completes one complete rotation along the circular path. So, angular displacement in T sec = 2π

Angular velocity, $\omega = \frac{2\pi}{T}$

Or, $T = \frac{2\pi}{\omega}$ ……………… (1)



Now in T sec the object travels $2\pi r$ linear distance.

Then the linear velocity $v = \frac{2\pi r}{T}$

$$\text{Or, } T = \frac{2\pi r}{v} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(2)}$$

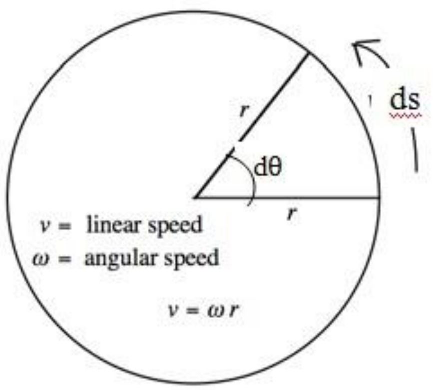
From equation number (1) and (2)

$$\frac{2\pi r}{v} = \frac{2\pi}{\omega}$$

$$v = \omega r$$

<u>Alternative Method</u>

Let an object moving in a circular path of radius r. In time t it travels s arc length of the circle and makes an angle θ radian at the centre.



According to the definition of radian

$$d\theta = \frac{ds}{r}$$

$$ds = r\, d\theta$$

Differentiating this equation with respect to t, we get

$$\frac{ds}{dt} = r\,\frac{d\theta}{dt}$$

But, $\frac{ds}{dt}$= v, linear velocity and $\frac{d\theta}{dt}$= ω = angular velocity

This equation becomes

$$v = \omega\, r$$

Vector form of this equation

We know, $\vec{\omega}$ is a vector quantity and radius vector is also a vector quantity. So, the cross product of two vectors will also be a vector.

Let the resultant of the cross product be $\vec{c}$.

$\vec{c} = \vec{\omega} \times \vec{r}$ …………………… (1)

The magnitude of $\vec{c}$ is

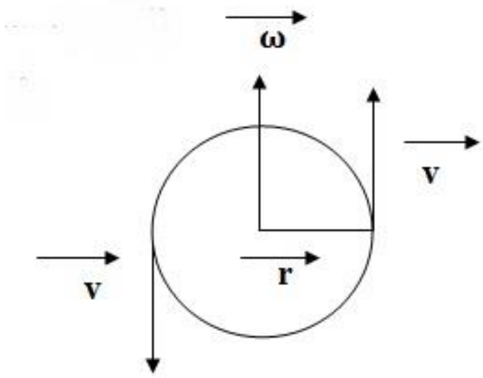
$c = \omega\, r \sin 90^0$

$c = \omega\, r$

We know that

$v = \omega\, r$

$v = c$



Also the direction $\vec{v}$ and $\vec{c}$ are the same (According to the rule of cross- product).

$\vec{v} = \vec{c}$ …………….. (2)

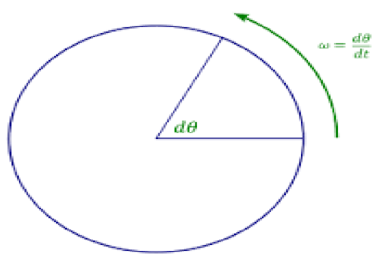
From equation number (1) and (2)

$\vec{v} = \vec{\omega} \times \vec{r}$

Angular Acceleration

Angular acceleration is the rate of change of angular velocity.

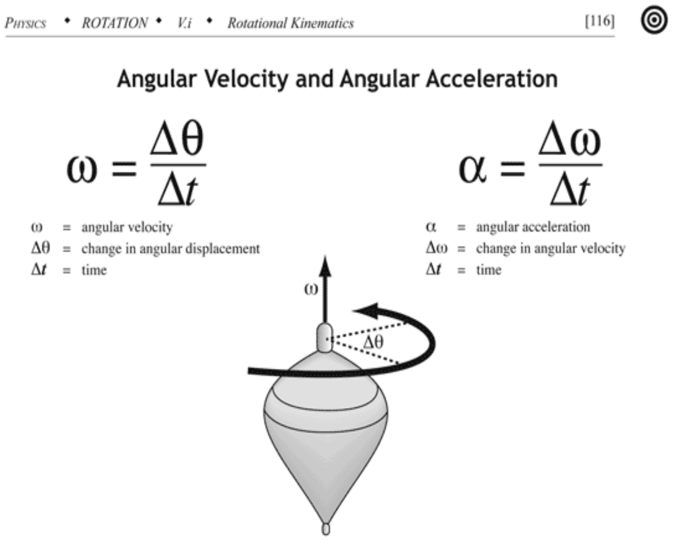
It can be denoted by ω . In SI units, it is measured in radians per second squared (rad / s ^{2}), and is usually denoted by the Greek letter alpha (α).



Mathematically, it can be written as

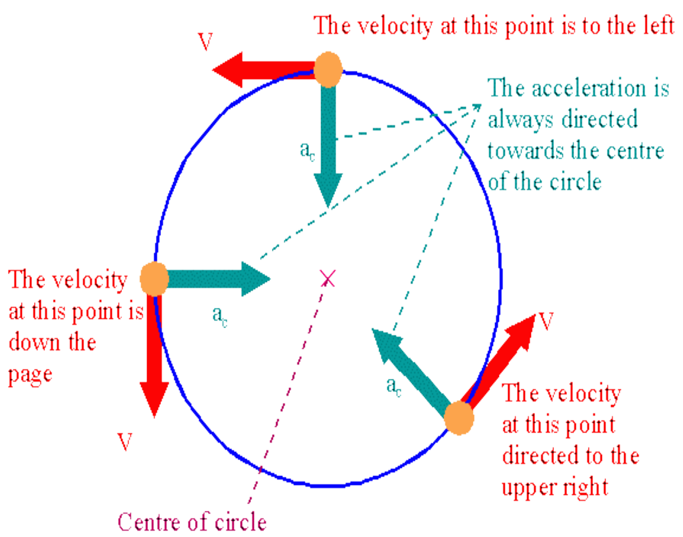
$$\omega = \frac{\omega_f - \omega_i}{dt}$$

Where, ω_f is the final angular velocity and ω_i is the initial angular velocity in time dt.



Centripetal Acceleration

An object moving in a circular path of radius r with a constant speed v has an acceleration called centripetal acceleration. The acceleration directed towards the centre of the circle.

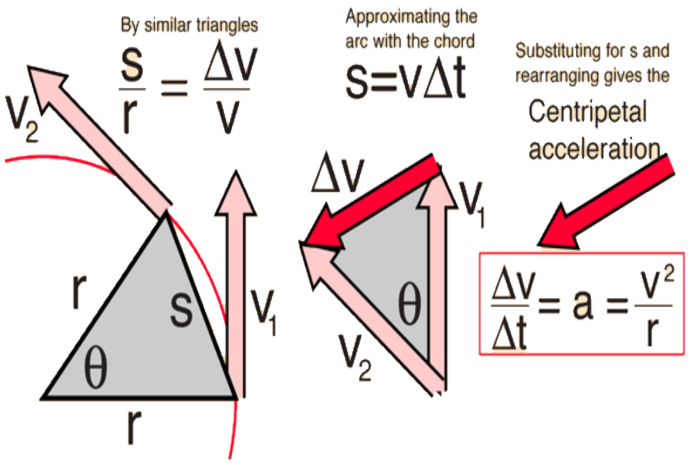


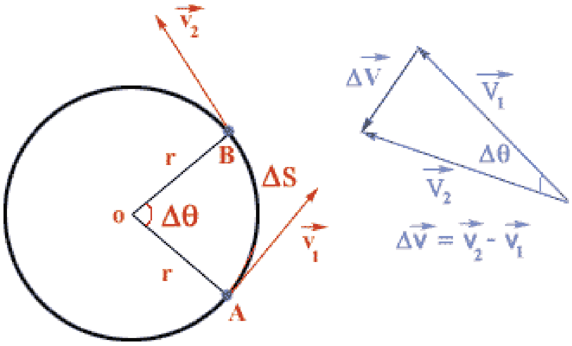
In other words

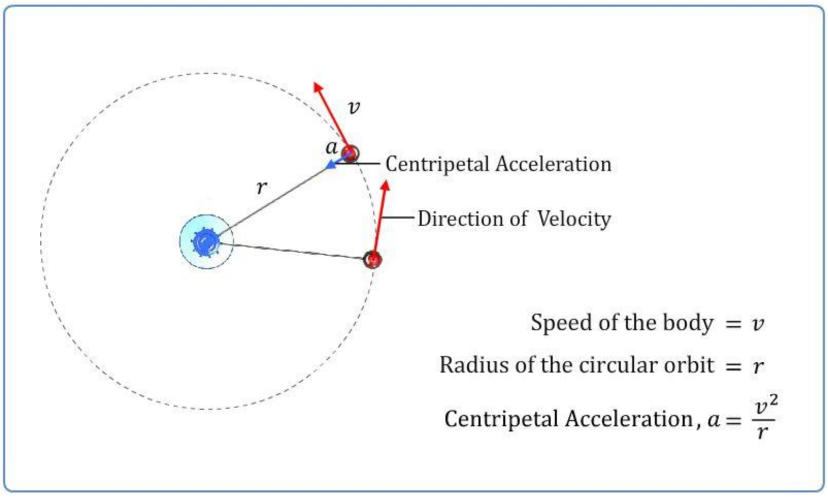
Centripetal acceleration is the rate of change of tangential velocity. The direction of the centripetal acceleration is always inwards along the radius vector of the circular motion.

It can be denoted by $\boldsymbol{a_c}$ and mathematically

$$\boldsymbol{a_c} = \frac{v_t^2}{r} = \omega^2 r$$

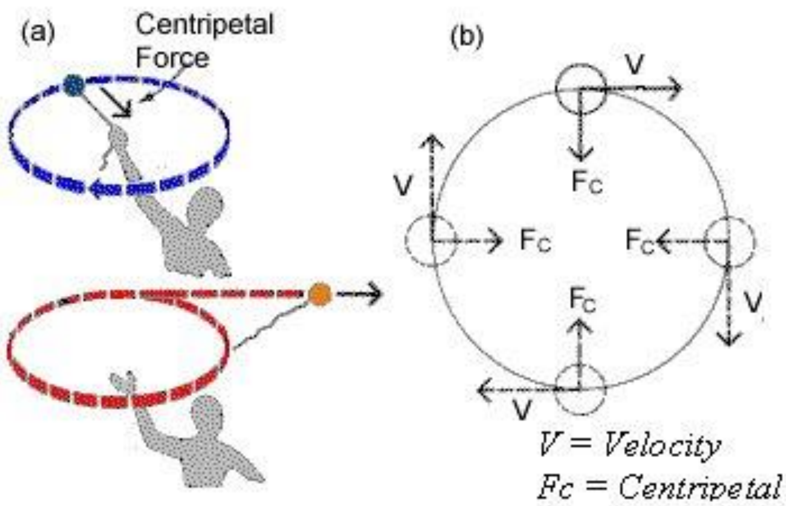






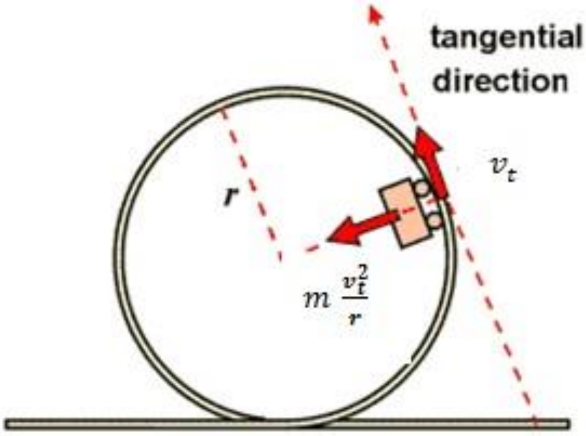
Centripetal Force

Centripetal force is defined as the force which acts towards the center along the radius of a circular path on which the body is moving with a uniform velocity.



It can be denoted by F_c . Mathematically

$$F_c = m\,a_c = m\,\frac{v_t^2}{r}$$



<u>Relation between Centripetal force and Acceleration</u>

Let A and B be two positions of the body after an interval of time t.

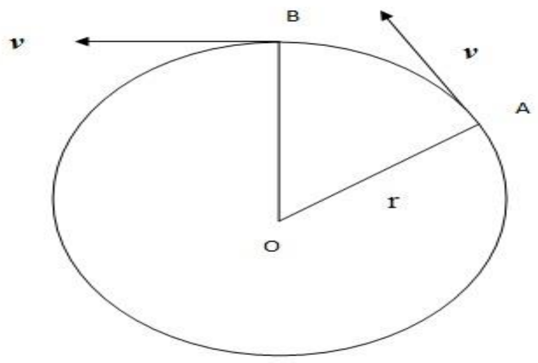
Then

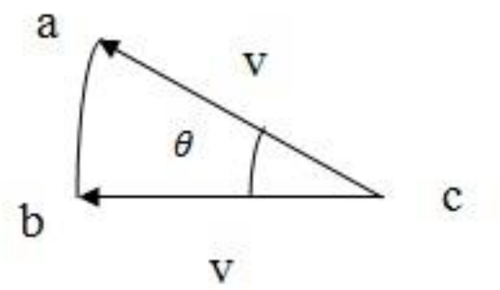
$$AB = \text{velocity} \times \text{time} = v\,t \quad \ldots\ldots\ldots\ldots\ldots (1)$$

Let **ca** and **cb** are the vectors representing the velocities at A and B respectively. Then **acb** = θ (The angle between the tangents is equal to the angle between the radii), **ab** is the change in velocity from A and B.

$$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time}} = \frac{ab}{t}$$

$$\theta = \frac{\text{Arc AB}}{r} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$





Also

$$\theta = \frac{\text{arc ab}}{v} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots \;(3)$$

From equation number (2) and (3)

$$\frac{vt}{r} = \frac{ab}{v}$$

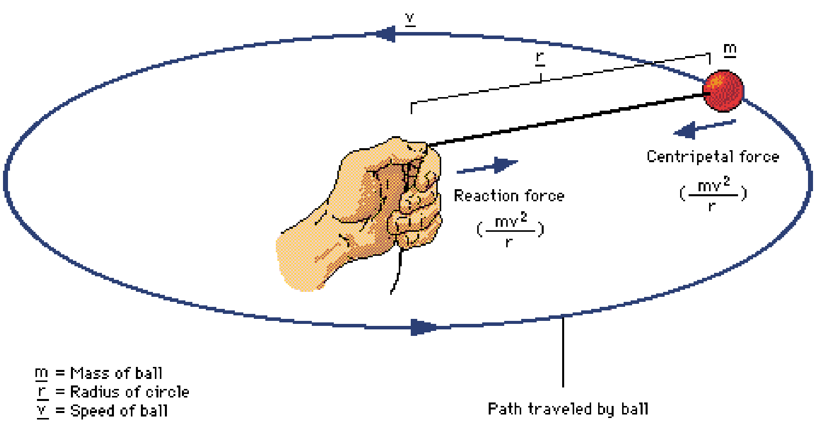
$$\frac{ab}{t} = \frac{v^2}{r}$$

Acceleration $= \frac{ab}{t} = \frac{v^2}{r}$

The acceleration on the body acting towards the center of the circular path $= \frac{v^2}{r}$

So, the centripetal force = Mass × Acceleration

$$F_{centripetal} = m\,\frac{v^2}{r}$$

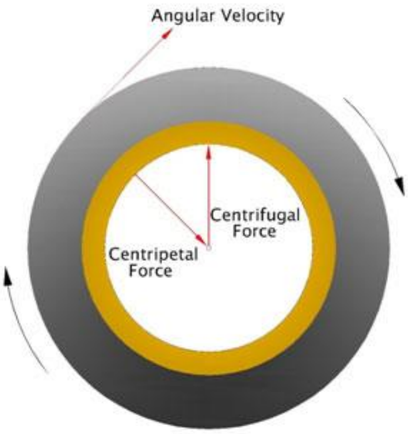


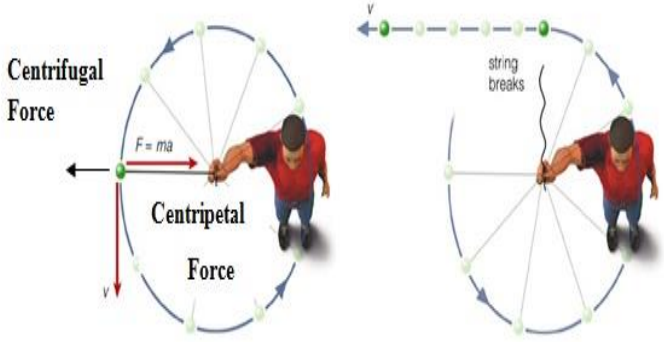
Centrifugal Force

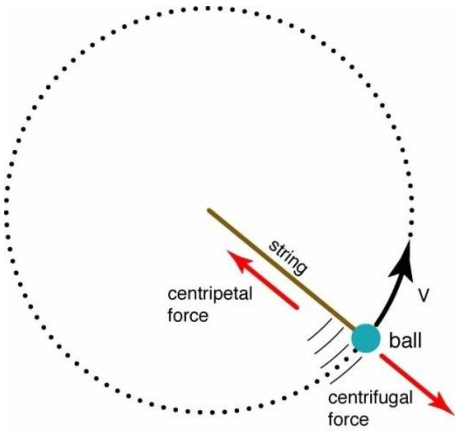
In Newtonian mechanics, the term **centrifugal force** is used to refer to an inertial force (also called a 'fictitious' force) directed away from the axis of rotation that appears to act on all objects when viewed in a rotating reference frame.

The concept of centrifugal force can be applied in rotating devices such as centrifuges, centrifugal pumps, centrifugal governors, centrifugal clutches, etc., as well as in centrifugal railways, planetary orbits, banked curves, etc. when they are analyzed in a rotating coordinate system.

The name has historically sometimes also been used to refer to the **reaction force to the centripetal force**.







	Linear Motion	Rotational Motion	
Position	x	θ	Angular position
Velocity	v	ω	Angular velocity
Acceleration	a	α	Angular acceleration
Motion equations	$x = \bar{v}t$	$\theta = \bar{\omega}t$	Motion equations
	$v = v_0 + at$	$\omega = \omega_0 + \alpha t$	
	$x = v_0 t + \frac{1}{2}at^2$	$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$	
	$v^2 = v_0^2 + 2ax$	$\omega^2 = \omega_0^2 + 2\alpha\theta$	
Mass (linear inertia)	m	I	Moment of inertia
Newton's second law	$F = ma$	$\tau = I\alpha$	Newton's second law
Momentum	$p = mv$	$L = I\omega$	Angular momentum
Work	Fd	$\tau\theta$	Work
Kinetic energy	$\frac{1}{2}mv^2$	$\frac{1}{2}I\omega^2$	Kinetic energy
Power	Fv	$\tau\omega$	Power

Constant Angular Acceleration

The equations of motion for constant angular acceleration are the same as those for linear motion, with the substitution of the angular quantities for the linear ones.

Angular	Linear
$\omega = \omega_0 + \alpha t$	$v = v_0 + at$
$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$	$x = v_0 t + \frac{1}{2}at^2$
$\omega^2 = \omega_0^2 + 2\alpha\theta$	$v^2 = v_0^2 + 2ax$
$\bar{\omega} = \frac{\omega + \omega_0}{2}$	$\bar{v} = \frac{v + v_0}{2}$

Rotational	Translational	
$\theta = \overline{\omega} t$	$x = \overline{v} t$	
$\omega = \omega_0 + \alpha t$	$v = v_0 + at$	(constant α, a)
$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$	$x = v_0 t + \frac{1}{2}at^2$	(constant α, a)
$\omega^2 = {\omega_0}^2 + 2\alpha\theta$	$v^2 = {v_0}^2 + 2ax$	(constant α, a)

Table 1: **Rotational Kinematic Equations**