PARTIAL FRACTION

PART 2

PROBLEM 1: SEPARATE $\frac{5x-11}{2x^2+x-6}$ INTO PARTIAL FRACTIONS.

We have $$\frac{5x-11}{2x^2+x-6}$$

$$=\frac{5x-11}{2x^2+4x-3x-6}$$

$$=\frac{5x-11}{2x(x+2)-3(x+2)}$$

$$=\frac{5x-11}{(x+2)(2x-3)}$$

$$=\frac{3}{x+2}+\frac{-1}{2x-3}$$

PROBLEM 2: SEPARATE $\frac{3x^2+x-2}{(x-2)^2(1-2x)}$ INTO PARTIAL FRACTIONS.

We have $$\frac{3x^2+x-2}{(x-2)^2(1-2x)} = \frac{-4}{(x-2)^2} + \frac{-\frac{1}{3}}{(1-2x)} + \frac{A}{(x-2)} \cdots\cdots (1)$$

Putting $x=0$ in (1):

$$\frac{3(0)^2+0-2}{(0-2)^2(1-2\times 0)} = \frac{-4}{(0-2)^2} + \frac{-\frac{1}{3}}{(1-2\times 0)} + \frac{A}{(0-2)}$$

$$\text{or, } \frac{-2}{4} = \frac{-4}{4} + \frac{-\frac{1}{3}}{1} + \frac{A}{-2}$$

$$or, -\frac{1}{2} = -1 - \frac{1}{3} - \frac{A}{2}$$

$$\text{or, } \frac{A}{2} = -1 - \frac{1}{3} + \frac{1}{2}$$

$$\text{or, } \frac{A}{2} = \frac{-6-2+3}{6} \quad \text{or, } A = -\frac{5}{3}$$

From (1):

$$\frac{3x^2+x-2}{(x-2)^2(1-2x)} = \frac{-4}{(x-2)^2} + \frac{-\frac{1}{3}}{(1-2x)} + \frac{-\frac{5}{3}}{(x-2)}$$

$$= -\frac{4}{(x-2)^2} - \frac{1}{3}\cdot\frac{1}{(1-2x)} - \frac{5}{3}\cdot\frac{1}{(x-2)}$$

Daffodil International University

PROBLEM 3: SEPARATE $\frac{7+x}{(1+x)(1+x^2)}$ INTO PARTIAL FRACTIONS.

We have $$\frac{7+x}{(1+x)(1+x^2)}=\frac{3}{(1+x)}+\frac{Ax+B}{(1+x^2)}\cdots\cdots(1)$$

Putting $x=0$ in (1):

$$\frac{7+0}{(1+0)(1+0)}=\frac{3}{(1+0)}+\frac{A(0)+B}{(1+0)}$$

or, $7=3+B$ $\quad\therefore\ B=4$

Putting $x=1$ in (1):

$$\frac{7+1}{(1+1)(1+1)}=\frac{3}{(1+1)}+\frac{A(1)+B}{(1+1)}$$

or, $\frac{8}{2\times 2}=\frac{3}{2}+\frac{A+4}{2}$

or, $2=\frac{3}{2}+\frac{A+4}{2}$

or, $\frac{A+4}{2}=2-\frac{3}{2}$

or, $\frac{A+4}{2}=\frac{1}{2}$

or, $A+4=1$ $\quad\therefore\ A=-3$

From (1):

$$\frac{7+x}{(1+x)(1+x^2)}=\frac{3}{(1+x)}+\frac{-3x+4}{(1+x^2)}$$

$$=\frac{3}{(1+x)}-\frac{3x-4}{(1+x^2)}$$

Daffodil International University

PROBLEM 4: DECOMPOSE $\frac{2x^2+x+1}{x^3+x}$ INTO PARTIAL FRACTIONS.

We have $$\frac{2x^2+x+1}{x^3+x} = \frac{2x^2+x+1}{x(x^2+1)} = \frac{1}{x} + \frac{Ax+B}{(x^2+1)} \cdots\cdots (1)$$

Putting $x = 1$ in (1):

$$\frac{2+1+1}{1+1} = \frac{1}{1} + \frac{A+B}{(1+1)}$$

$$\text{or, } \frac{4}{2} = 1 + \frac{A+B}{2}$$

$$\text{or, } 2 - 1 = \frac{A+B}{2}$$

$$\text{or, } 1 = \frac{A+B}{2}$$

$$\text{or, } A + B = 2 \cdots\cdots (2)$$

Putting $x = -1$ in (1):

$$\frac{2-1+1}{-1-1} = \frac{1}{-1} + \frac{-A+B}{(1+1)}$$

$$\text{or } \frac{2}{-2} = -1 + \frac{-A+B}{2}$$

$$\text{or, } -1 + 1 = \frac{-A+B}{2}$$

$$\text{or, } 0 = \frac{-A+B}{2}$$

$$\text{or, } -A + B = 0 \cdots\cdots (3)$$

Adding equations (2) & (3):

$$2B = 2 \qquad \therefore\ B = 1$$

Putting the value of B in (1):

$$A = 1$$

From (1):

$$\frac{2x^2+x+1}{x^3+x} = \frac{1}{x} + \frac{x+1}{(x^2+1)}$$

PROBLEM 5: SEPARATE $\frac{x+1}{(x^2+5)(x^2-3)}$ INTO PARTIAL FRACTIONS.

We have $$\frac{x+1}{(x^2+5)(x^2-3)}=\frac{Ax+B}{(x^2+5)}+\frac{Cx+D}{(x^2-3)}\cdots\cdots(1)$$

Multiplying both sides of (1) by $(x^2+5)(x^2-3)$:

$$x+1=(Ax+B)(x^2-3)+(Cx+D)(x^2+5)$$

or, $x+1=Ax^3-3Ax+Bx^2-3B+Cx^3+5Cx+Dx^2+5D$

or, $x+1=(A+C)x^3+(B+D)x^2+(5C-3A)x-3B+5D$

Equating the coefficient of like term

$$A+C=0;\ B+D=0;\ 5C-3A=1;\ -3B+5D=1$$

$A+C=0\Longrightarrow A=-C$

$B+D=0\Longrightarrow B=-D$

$5C-3A=1\Longrightarrow 5C-3(-C)=1$

or , $5C+3C=1$

or , $8C=1$ $\quad\therefore C=\frac{1}{8}$

$$\therefore A=-\frac{1}{8}$$

$-3B+5D=1\Longrightarrow -3(-D)+5D=1$

or, $8D=1$

$$\therefore D=\frac{1}{8}$$

$$\therefore B=-\frac{1}{8}$$

From (1):

$$\frac{x+1}{(x^2+5)(x^2-3)}=\frac{-\frac{1}{8}x-\frac{1}{8}}{(x^2+5)}+\frac{\frac{1}{8}x+\frac{1}{8}}{(x^2-3)}$$

$$=\frac{1}{8}\cdot\frac{x+1}{(x^2-3)}-\frac{1}{8}\cdot\frac{x+1}{(x^2+5)}$$

Daffodil International University

PROBLEM 6: SEPARATE $\frac{2x^2+x+1}{x^2+2x-3}$ INTO PARTIAL FRACTIONS.

We have $\dfrac{2x^2+x+1}{x^2+2x-3} = \dfrac{2(x^2+2x-3)+7-3x}{x^2+2x-3}$

$$= 2 + \frac{7-3x}{x^2+2x-3}$$

$$= 2 + \frac{7-3x}{x^2+3x-x-3}$$

$$= 2 + \frac{7-3x}{x(x+3)-1(x+3)}$$

$$= 2 + \frac{7-3x}{(x+3)(x-1)}$$

$$= 2 + \frac{1}{x-1} + \frac{-4}{x+3} \qquad = 2 + \frac{1}{x-1} - \frac{4}{x+3}$$

PROBLEM 7: RESOLVE $\frac{6x^3+5x^2-7}{3x^2-2x-1}$ INTO PARTIAL FRACTIONS.

We have $\dfrac{6x^3+5x^2-7}{3x^2-2x-1}$

$$= \frac{2x(3x^2-2x-1)+9x^2+2x-7}{3x^2-2x-1}$$

$$= \frac{2x(3x^2-2x-1)}{3x^2-2x-1} + \frac{9x^2+2x-7}{3x^2-2x-1}$$

$$= 2x + \frac{3(3x^2-2x-1)+8x-4}{3x^2-2x-1}$$

$$= 2x + \frac{3(3x^2-2x-1)}{3x^2-2x-1} + \frac{8x-4}{3x^2-2x-1}$$

$$= 2x + 3 + \frac{8x-4}{3x^2-3x+x-1}$$

$$= 2x + 3 + \frac{8x-4}{(x-1)(3x+1)} = 2x + 3 + \frac{1}{x-1} + \frac{5}{3x+1}$$

EXERCISE

1. Resolve $\frac{x+2}{(x-1)(x+3)}$ into partial fractions.
2. Resolve $\frac{1}{(x+2)(x+1)}$ into partial fractions.
3. Separate $\frac{x}{(x-2)(x+1)^2}$ into partial fractions.
4. Find the decomposition of $\frac{1}{(x^2+5)(x^2-3)}$.
5. Resolve $\frac{x^2+5x-7}{x^2-x-2}$ into partial fractions.
6. Resolve $\frac{x^4+5x^3-7}{x^2+5x+6}$ into partial fractions.