Work, Energy and Power

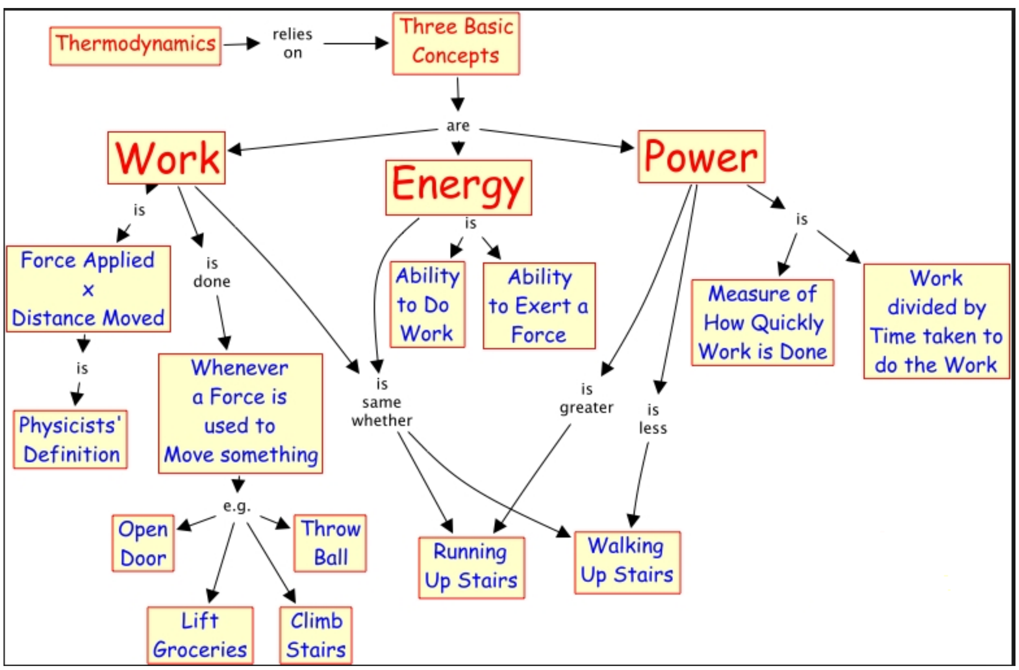
The term **work** was introduced in 1826 by the French mathematician Gaspard-Gustave Coriolis as "weight lifted through a height", which is based on the use of early steam engines to lift buckets of water out of flooded ore mines.

Work refers to an activity involving a force and movement in the direction of the force. A force of 20 Newton's pushing an object 5 meters in the direction of the force does 100 joules of work.

Energy is the capacity for doing work. You must have energy to accomplish work - it is like the "currency" for performing work. To do 100 joules of work, you must expend 100 joules of energy.

Power is the rate of doing work or the rate of using energy, which are numerically the same. If you do 100 joules of work in one second (using 100 joules of energy), the power is 100 watts.

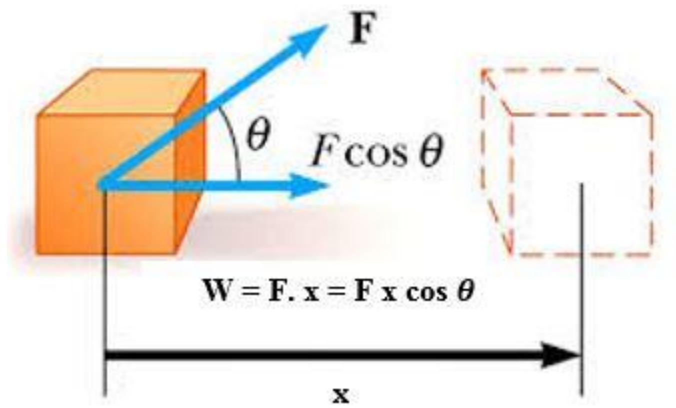
The following diagram explains the practical example and concepts of work, energy and power.



Work

If force is applied on an object and if there is a displacement of the object, then the product of the force and the component of displacement along the direction of force is called work.

The SI unit of work is the newton-meter or joule (J). It is denoted by W.



Work Done by Gravity

Gravitational force is all pervading force that is acting! Let's assume that a particle is falling, the particle points in the direction of gravity. Its magnitude depends on how much is the mass m, gravitational constant g and from what height it is falling h!

The Work done by gravity formula is given by,

$$\boxed{W = mg \times h}$$

m = mass of the object
g = gravitational acceleration
h = height of the object from its original position

Here, the direction of gravitational force and displacement are both downward, hence work is positive.

- **Work Done in Expanding a Spring**

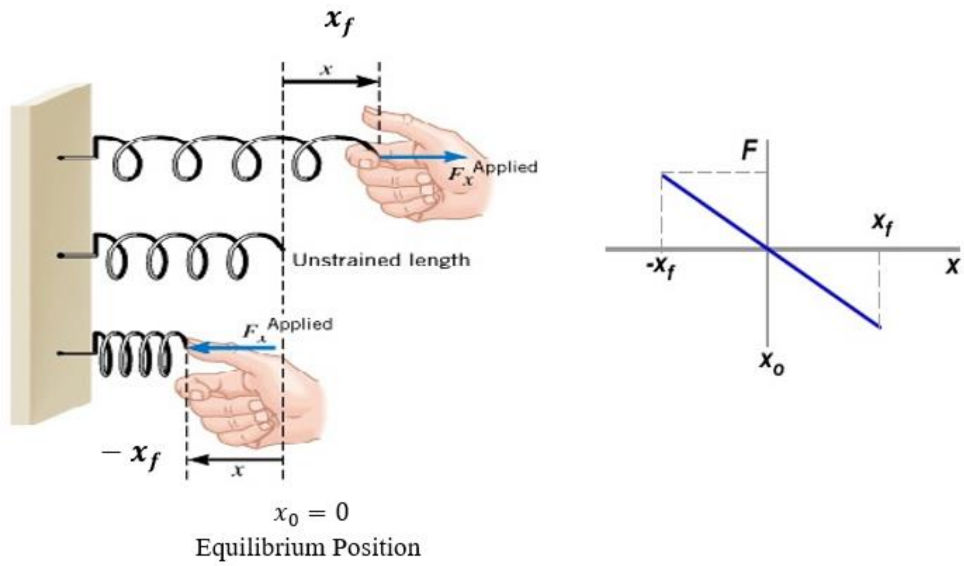
Let one end of a horizontal ideal spring be fastened with a wall. Now, if we apply force then the spring will be strained along its length.

According to Hooke's law, within elastic limit, we can write

$F \propto -x$

$F = -k\,x$

Here, k is a constant. This is called spring constant. (Since, restoring force is against the displacement, negative sign is introduced.)



In order to expand the spring, equal amount of external force is to be applied in the spring. Let F be the applied force,

$$F_{applied} = F_{restoring}$$

$$F = -(-k\,x)$$

$$F = k\,x \ldots\ldots\ldots\ldots\ldots(1)$$

In expanding the spring from position x_0 to x_f , work done is given by

$$W = \int_{x_0}^{x_f} \vec{F}\,.\,d\,\vec{x}$$

$$W = \int_{x_0}^{x_f} F\,dx$$

$$W = \int_{x_0}^{x_f} kx\,dx \qquad \because \text{From equation number (1)}$$

$$W = k \int_{x_0}^{x_f} x\, dx$$

$$W = k\, [x^2]_{x_0}^{x_f}$$

$$W = \frac{1}{2}\, k\, (x_f^2 - x_0^2)$$

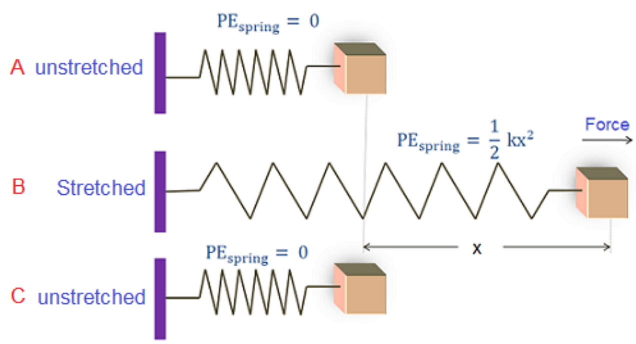
$$W = \frac{1}{2}\, k\, x_f^2 - \frac{1}{2}\, k\, x_0^2$$

This work is positive, which remains stored as potential energy in the spring.

Since, $x_0 = 0$ and $x_f = x$ then

$$W = \frac{1}{2}\, k\, x^2$$

If displacement is x then this is stored as potential energy or elastic potential energy ($U = \frac{1}{2}\, k\, x^2$).



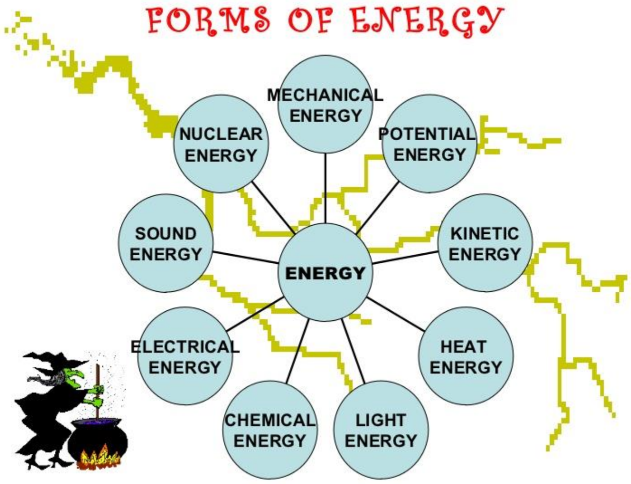
Energy

> **Energy of a body is the capacity or ability to do work. It is measured by the amount of work the body can do.**

Energy can exist in many different forms. All forms of energy are either kinetic or potential.

> **The energy associated with motion is called kinetic energy. The energy associated with position is called potential energy.**

Potential energy is not "stored energy". Energy can be stored in motion just as well as it can be stored in position.

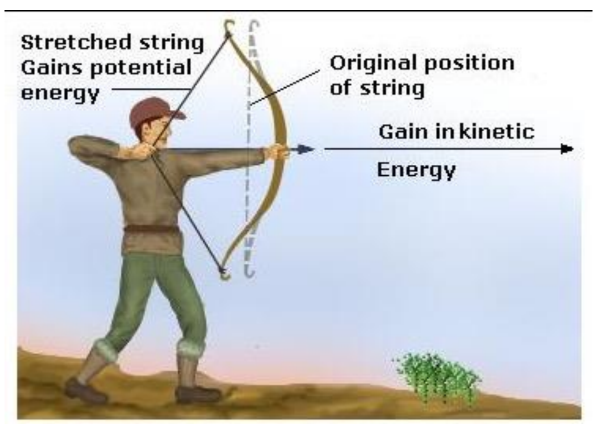


Potential Energy

> **In physics, potential energy is the energy that an object has due to its position in a force field.**

Common types include the gravitational potential energy of an object that depends on its mass and its distance from the center of mass of another object, the elastic potential energy of an extended spring, and the electric potential energy of an electric charge in an electric field.

The unit for energy in the International System of Units (SI) is the joule, which has the symbol J.



Gravitational Potential Energy

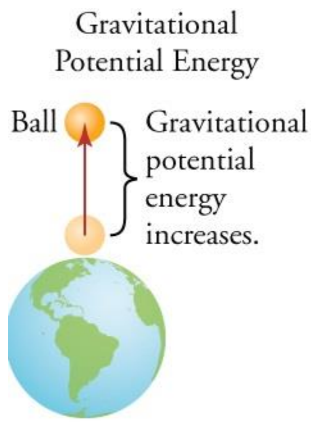
> **On Earth, we always have the force of gravity acting on us. When we're above the Earth's surface we have potential (stored) energy. This is called gravitational potential energy.**

The amount of gravitational potential energy an object on Earth has depends on its mass and height above the ground. Mathematically, it can be defined as

$$P.E = mgh$$

$$\boxed{PE = mgh}$$

PE = gravitational potential energy (J)
m = mass (kg)
g = 9.8 m/s^2 [on Earth]
h = height above reference level (m)



Kinetic Energy

> **In physics, the kinetic energy of an object is the energy that it possesses due to its motion. It is defined as the work needed to accelerate a body of a given mass from rest to its stated velocity.**

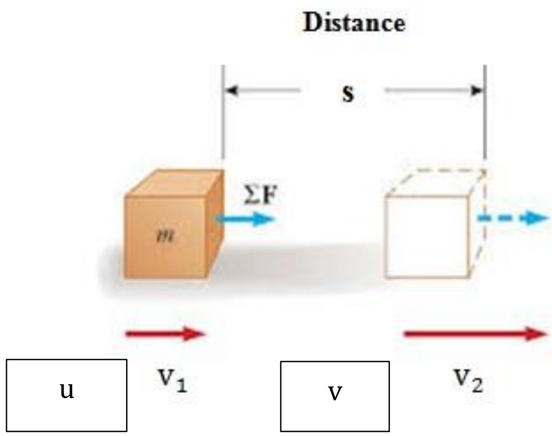
In classical mechanics, the kinetic energy of a non-rotating object of mass m traveling at a speed v is (K.E) or $\mathbf{E_k = \frac{1}{2}\, m\, v^{2}}$ **.** The standard unit of kinetic energy is the **joule**.

Work – Energy Theorem

Let a force F be applied on an object of mass m and the velocity of the object changes from u to v , when the object travels a distance s , then according to the equation of motion

$v^2 = u^2 + 2as$

$$s = \frac{v^2 - u^2}{2a} \ldots\ldots\ldots\ldots\ldots..(1)$$



So, the work done by the force

$$W = F.s$$

$$W = ma \times \frac{v^2 - u^2}{2a}$$

$$W = \frac{1}{2}\, m\, (v^2 - u^2)$$

$$W = \frac{1}{2}\, m\, v^2 - \frac{1}{2}\, m\, u^2$$

Work done $=$ Final kinetic energy $-$ Initial kinetic energy

$\qquad\qquad = $ Increase in kinetic energy

Hence, change in kinetic energy of an object is equal to the net work done by the applied force. It is the work - energy theorem.

Measurement of Kinetic Energy

- **In Case of Translational Energy**

According to the work-energy theorem

Change in kinteic energy $=$ Work done

$$\Delta K.E = W$$
$$= F.s$$
$$= \int F.\ ds$$
$$= \int ma\ ds$$
$$= m \int \frac{dv}{dt}\ ds$$

Rearrange the differential terms to get the integral and the function into agreement.

$$\Delta K.E = m \int \frac{dv}{dt}\ ds$$
$$= m \int \frac{ds}{dt}\ dv$$
$$= m \int v\ dv$$

The integral of which is quite simple to evaluate over the limits initial speed (u) to final speed (v)

$$\Delta K.E = \frac{1}{2}\ m\ v^2 - \frac{1}{2}\ m\ u^2$$

Naturally, the kinetic energy of an object at rest should be zero. Thus, an object's kinetic energy is defined mathematically by the following equation

$$\mathbf{E_k = \frac{1}{2}\ m\ v^{2}}$$