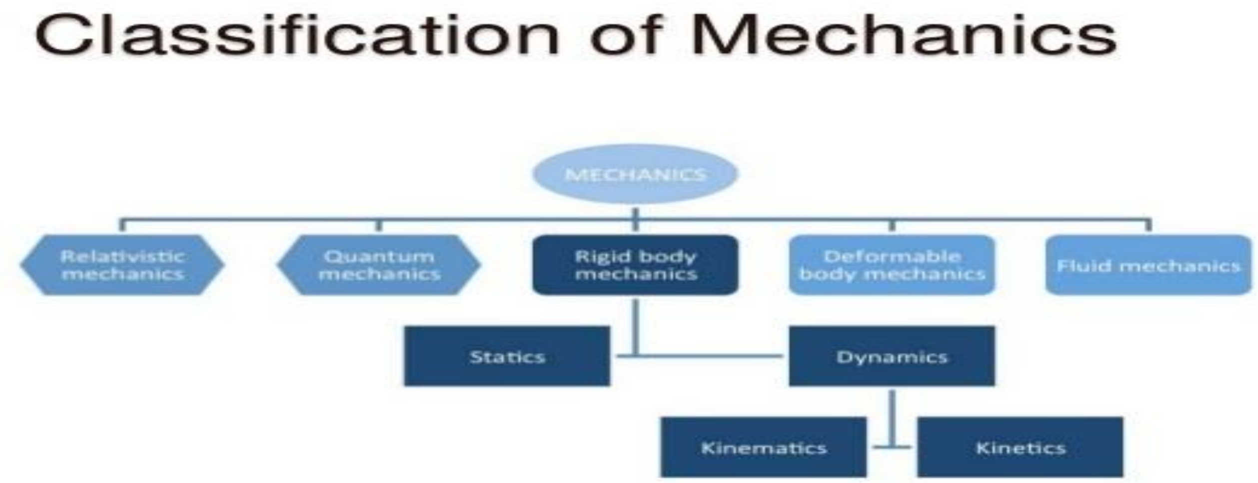
Mechanics

Mechanics is an area of science concerned with the behavior of physical bodies when subjected to forces or displacements, and the subsequent effects of the bodies on their environment. The scientific discipline has its origins in Ancient Greece with the writings of Aristotle and Archimedes.

Mechanics is the branch of Physics dealing with the study of motion. No matter what your interest in science or engineering, mechanics will be important for you - motion is a fundamental idea in all of science.

Mechanics can be divided into 2 areas - **kinematics, dealing with describing motions, and dynamics, dealing with the causes of motion.**



During the early modern period, scientists such as Khayaam, Galileo, Kepler, and Newton, laid the foundation for what is now known as classical mechanics. It is a branch of classical physics that deals with particles that are either at rest or are moving with velocities significantly less than the speed of light. It can also be defined as a branch of science which deals with the motion of and forces on objects.

In physics, classical mechanics and quantum mechanics are the two major sub-fields of mechanics. Classical mechanics is concerned with the set of physical laws describing the motion of bodies under the action of a system of forces. The study of the motion of bodies is an ancient one, making classical mechanics one of the oldest and largest subjects in science, engineering and technology. It is also widely known as **Newtonian mechanics**.

Classical mechanics describes the motion of macroscopic objects, from projectiles to parts of machinery, as well as astronomical objects, such as spacecraft, planets, stars, and galaxies. Besides this, many specializations within the subject deal with solids, liquids and gases and other specific sub-topics. **Classical mechanics also provides extremely accurate results as long as the**

domain of study is restricted to large objects and the speeds involved do not approach the speed of light.

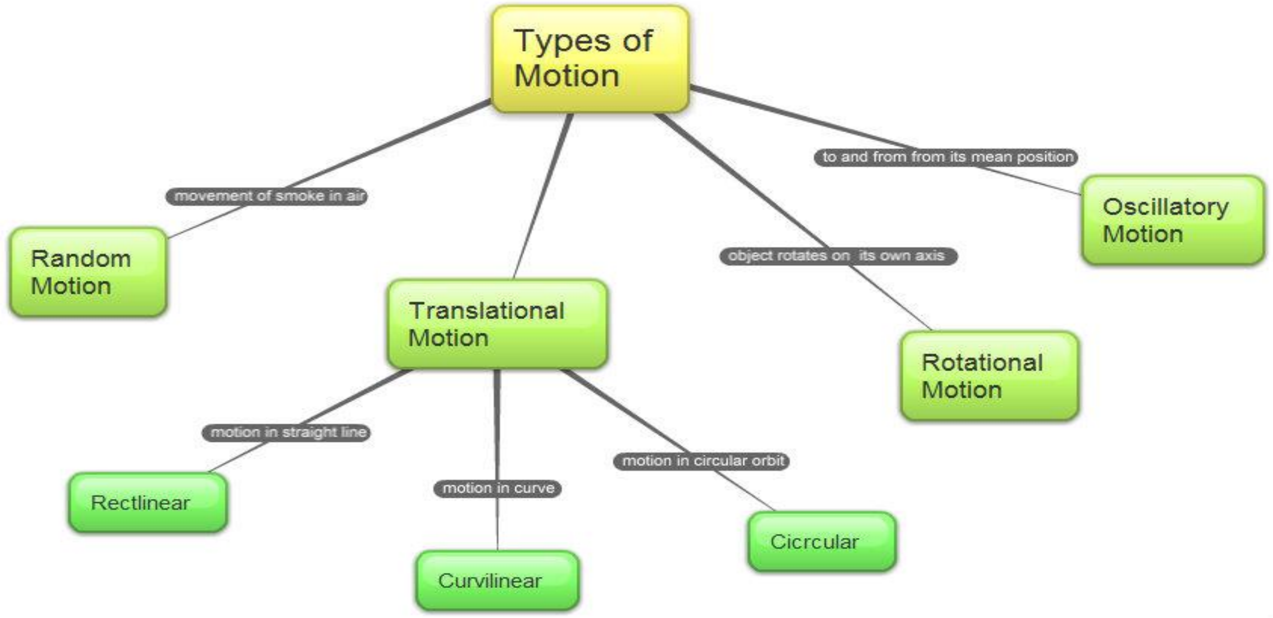
When the objects being dealt with become **sufficiently small**, it becomes necessary to introduce the other major sub-field of mechanics, **quantum mechanics**, which reconciles the macroscopic laws of physics with the atomic nature of matter and handles the wave–particle duality of atoms and molecules.

When both quantum mechanics and classical mechanics cannot apply, such as at the quantum level with high speeds, quantum field theory (QFT) becomes applicable.

Motion in One Dimension

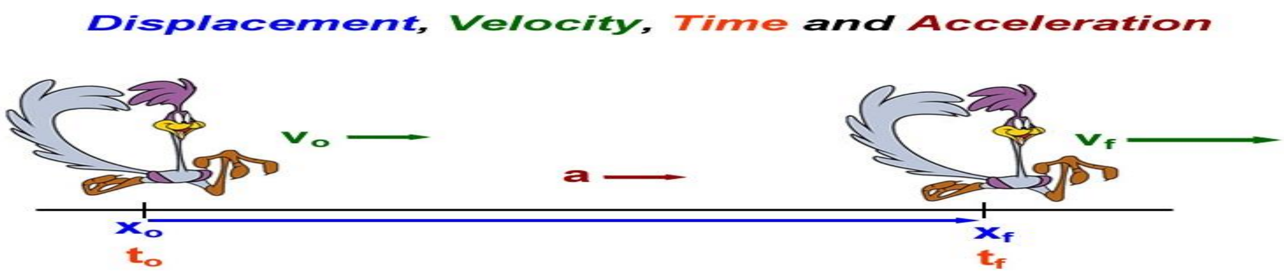
Motion

Motion is defined as change in position of a body over time with respect to the surrounding. Motion may be divided into three basic types — translational, rotational, and oscillatory.



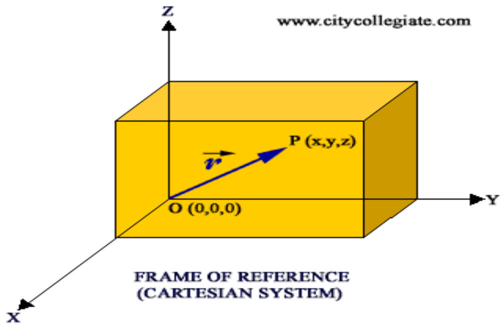
For the concept of motion in one direction, consider a motion of a body along a straight track. The subject of motion in physics is called a, "body".

One dimensional motion is motion along a straight line. The displacement (x), velocity (v), and acceleration (a). All of these three are linked together.



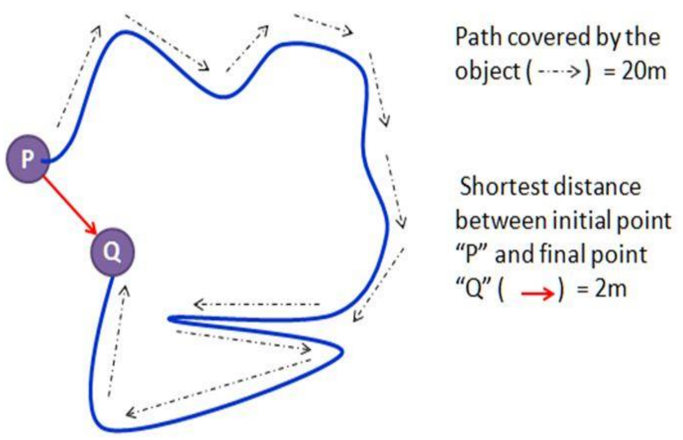
Reference Frame

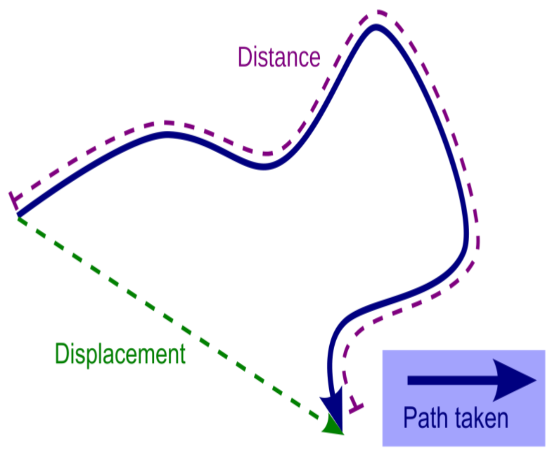
A framework that is used for the observation and mathematical description of physical phenomena and the formulation of physical laws, usually consisting of an observer, a coordinate system, and a clock or clocks assigning times at positions with respect to the coordinate system.



Displacement

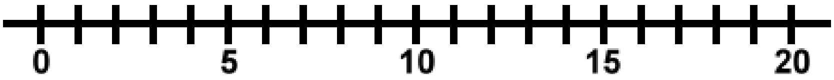
Displacement is defined as the **shortest distance (x)** of a one-dimensional object from a center point, or an origin. Displacement is plotted against time in a curved graph. A body, in motion in one dimension, can only move left and right.



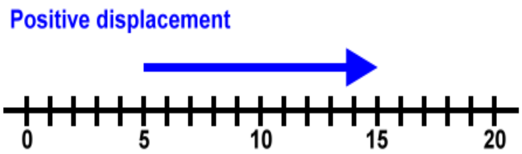


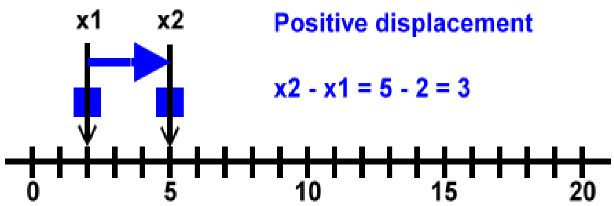
Positive and Negative Displacement

The line used for this motion is often the familiar x-axis, or x number line. The object may move forward or backward along this line:

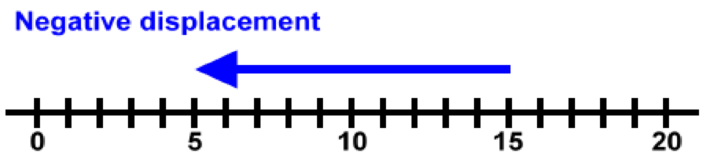


Forward is usually considered positive movement, and this movement is usually considered to be to the right. So, as an object moves forward down the x-axis, it is heading toward larger and larger x coordinates, and we say that it has a positive displacement and a positive velocity:





Backward is usually considered negative movement to the left. As an object moves backward along the x-axis, it is heading toward smaller and smaller x coordinates, and we say that it has a negative displacement and a negative velocity:



Speed

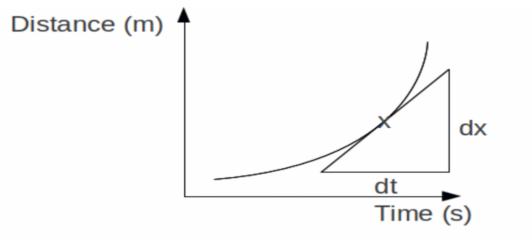
Speed is the rate of change of distance with time. In order to calculate the speed of an object we must know how far it's gone and how long it took to get there.

$$v = \frac{d}{t}$$

Where, v = speed, d = distance travelled and t= time taken.

Let's say you drove a car from New York to Boston. The distance by road is roughly 300 km (200 miles). If the trip takes four hours, what was your speed? Applying the formula gives…

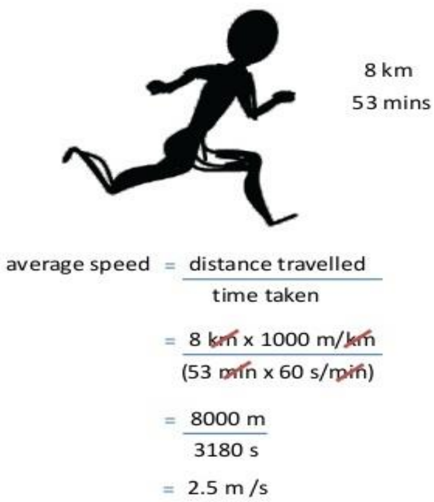
$v = \frac{s}{t} = \frac{300 \text{ Km}}{4 \text{ hour}}$ = 75 Km/h



Average Speed

The average speed (s) of an object is the quotient between the total distance (d) travelled and the total time (t) taken to complete the travel.

$$S = \frac{total\ distance}{total\ time}$$



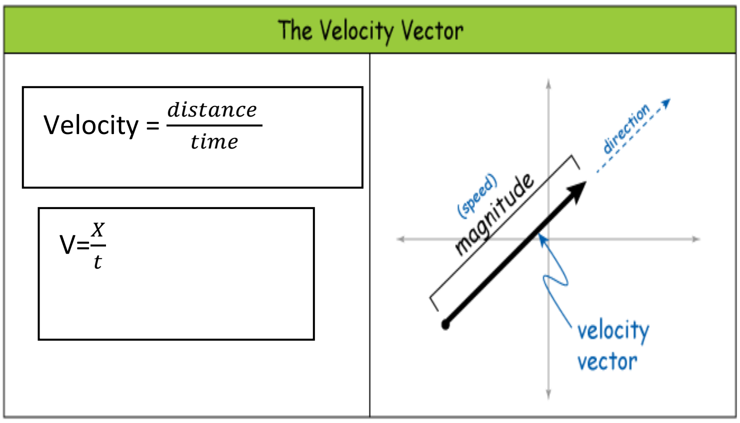
Instantaneous speed, that is, the speed determined over a very small interval of time — an instant. Ideally this interval should be as close to zero as possible, but in reality, we are limited by the sensitivity of our measuring devices.

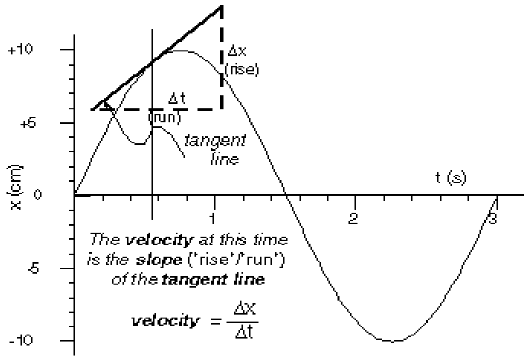
Mentally, however, it is possible imagine calculating average speed over ever smaller time intervals until we have effectively calculated instantaneous speed. This idea is written symbolically as

$$v = lim_{\Delta t \to 0} \frac{\Delta S}{\Delta t} = \frac{dS}{dt}$$

Velocity

Velocity is defined as the rate of which displacement changes over time. The higher the velocity, the faster a body is moving. It is a vector quantity i.e. it requires both magnitude and direction.Velocity can be zero also if the total displacement is zero, and this is only when the body after travelling a certain distance in any direction comes to rest at the same point where it started. The instantaneous velocity can also be zero when the sign of its magnitude changes; for example, a body experiencing constant acceleration against its direction of travel will eventually switch directions and move in the direction of the acceleration, and at that instant, its velocity is zero.





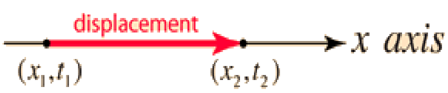
In terms of mathematics, the most general definition of velocity is, v = $\frac{dx}{dt}$. What this means is that velocity is the derivative of displacement (x) with respect to time.

Average Velocity

The average speed of an object is defined as the distance traveled divided by the time elapsed.

Velocity is a vector quantity, and **average velocity** can be defined as the displacement divided by the time. For the special case of straight line motion in the x direction,

The average velocity takes the form:



$$v_{average} = \bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

The units for velocity can be implied from the definition to be meters/second or in general any distance unit over any time unit.

You can approach an expression for the **instantaneous velocity** at any point on the path by taking the limit as the time interval gets smaller and smaller. Such a **limiting process** is called a **derivative** and the instantaneous velocity can be defined as

$$v_{\text{instantaneous}} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

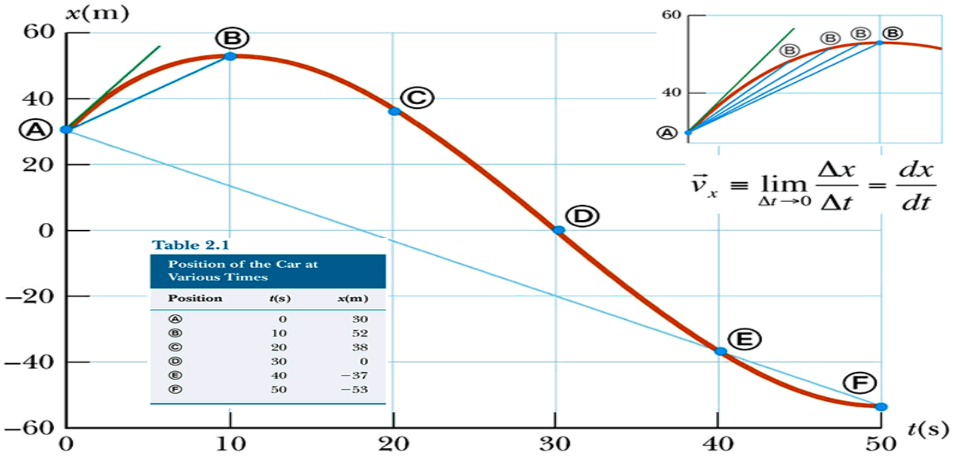


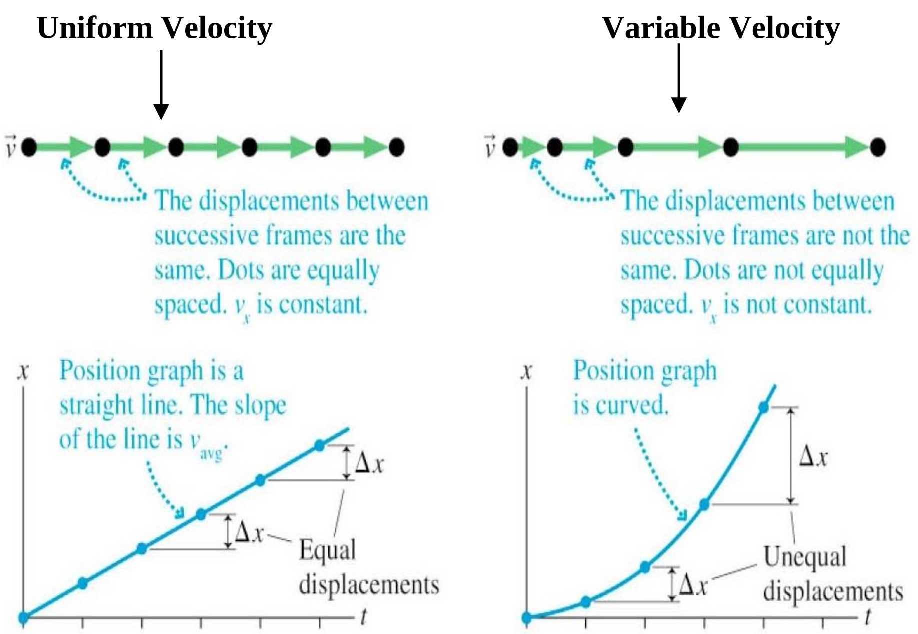
Table 2.1

Position of the Car at Various Times

Position	t(s)	x(m)
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	−37
Ⓕ	50	−53

Instantaneous Velocity

Uniform Velocity and variable Velocity



Acceleration

Acceleration is the rate of change of velocity with time. A body with a positive acceleration is gaining velocity over time. A body with a negative acceleration is losing velocity over time.

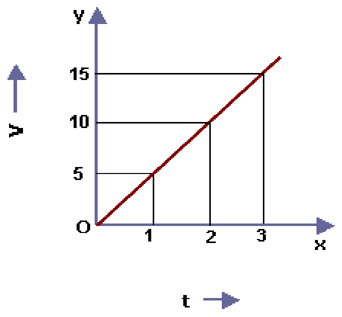
It is denoted by a.

$$a = \frac{Change\ in\ Velocity}{Time\ taken} = \frac{dv}{dt}$$

Uniform Acceleration

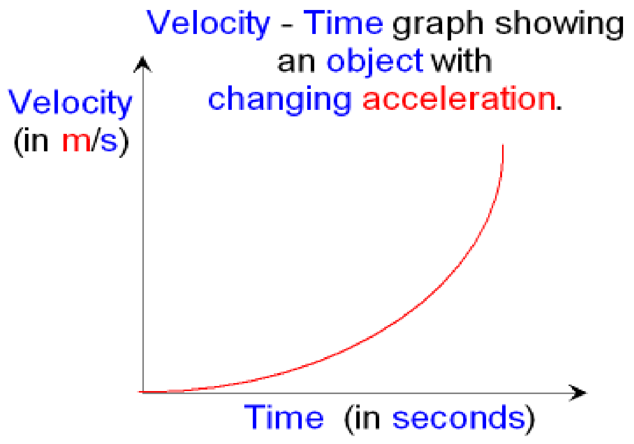
If the acceleration remains always constant, then that acceleration is called uniform acceleration

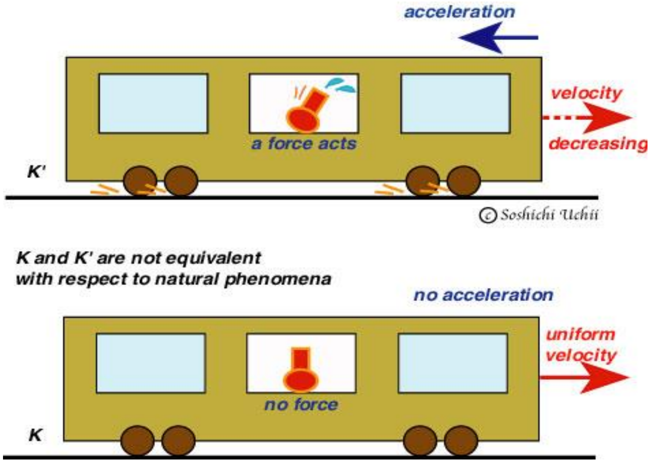
The uniform acceleration of a body is 10 ms^{-2} means that the velocity of the body changes in each second by 10 ms^{-1} in the same direction.



Variable Acceleration

When the acceleration of a body changes with time, the acceleration is called variable acceleration. The acceleration of bus, train, car etc is examples of variable acceleration.





Average acceleration

Average acceleration is determined over a "long" time interval. The word long in this context means finite — something with a beginning and an end. The velocity at the beginning of this interval is called the initial velocity, represented by the symbol $\mathbf{v_0}$, and the velocity at the end is called the final velocity, represented by the symbol $\mathbf{v}$.

Average acceleration is a quantity calculated from two velocity measurements.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{\Delta t}$$

Instantaneous acceleration

Instantaneous acceleration is measured over a "short" time interval. The word short in this context means infinitely small or infinitesimal — having no duration or extent whatsoever. It's a mathematical ideal that can only be realized as a limit.

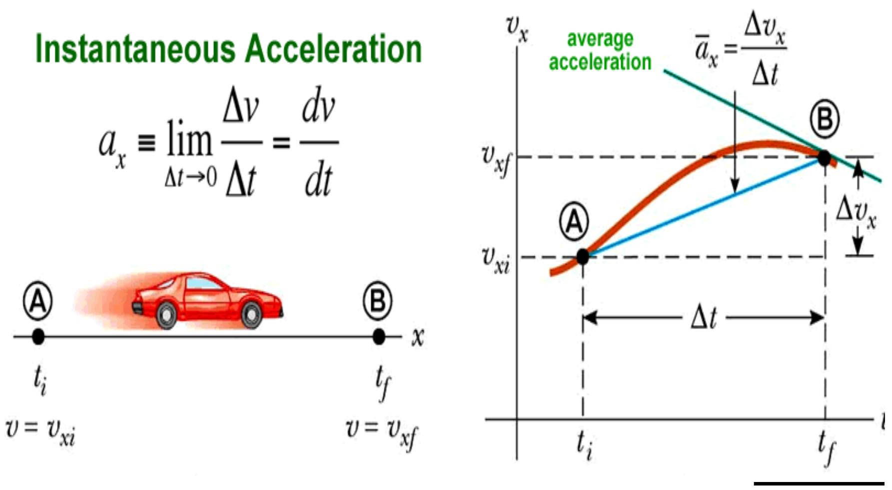
The limit of a rate as the denominator approaches zero is called a derivative.

Instantaneous acceleration is then the limit of average acceleration as the time interval approaches zero — or alternatively, acceleration is the derivative of velocity.

$$a = lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

Acceleration is the derivative of velocity with time, but velocity is itself the derivative of displacement with time. The derivative is a mathematical operation that can be applied multiple times to a pair of changing quantities. Doing it once gives you a first derivative. Doing it twice (the derivative of a derivative) gives you a second derivative. That makes acceleration the first derivative of velocity with time and the second derivative of displacement with time.

$$a = \frac{dv}{dt} = \frac{d}{dt}\frac{dx}{dt} = \frac{d^2x}{dt^2}$$



Equations of motion

(i) $\mathbf{v = v_o + at}$

Consider a particles moving with uniform acceleration a. From definition

$a = \frac{dv}{dt}$

$dv = adt$ ……………… (1)

Integrating both sides

$\int dv = a \int dt$

$v = at + k$ ………….. (2)

Let v_o be the initial velocity at time t = 0 from equation (2), we get

$v_o = k$

Submitting this value in equation number (2)

$v = v_o + at$

(ii) $\mathbf{s = v_o t + \frac{1}{2} at^2}$

Suppose at any instant of time, the velocity of the particle is v. The distance covered by the particle in an interval of time is dt, then

$ds = v\,dt$ ……………. (1)

But, $v = v_o + at$

$ds = (v_o + at)dt$

$ds = v_o dt + at\,dt$

Integrating this equation

$\int ds = v_o \int dt + at \int dt$

$s = v_o t + \frac{1}{2} at^2 + k$

When, t = 0, s = 0, we get k = 0

$s = v_o t + \frac{1}{2} at^2$

(iii) $\mathbf{v^2 = v_o^2 + 2as}$

We know that

$v = v_o + at$

$at = v - v_o$ …………. (1)

The distance travelled

$s = \frac{v + v_o}{2} t$

$\frac{2s}{t} = v + v_o$ …………… (2)

Multiplying (1) and (2)

$at \times \frac{2s}{t} = (v - v_o)(v + v_o)$

$2as = v^2 - v^2_o$

$v^2 = v_o^2 + 2as$

In case of free falling bodies

$v = v_o + gt$

$v^2 = v_o^2 + 2gh$

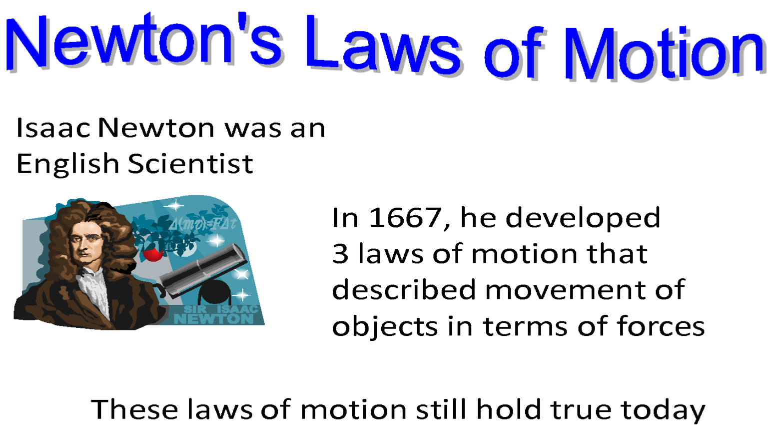
$h = v_o t + \frac{1}{2} g t^2$

$$\boxed{\begin{array}{ll} v = u + at & s = ut + \frac{1}{2}at^2 \\ s = \frac{1}{2}(u+v)t & v^2 = u^2 + 2as \end{array}}$$

a = acceleration
v = final velocity
u = initial velocity
t = time taken
s = displacement

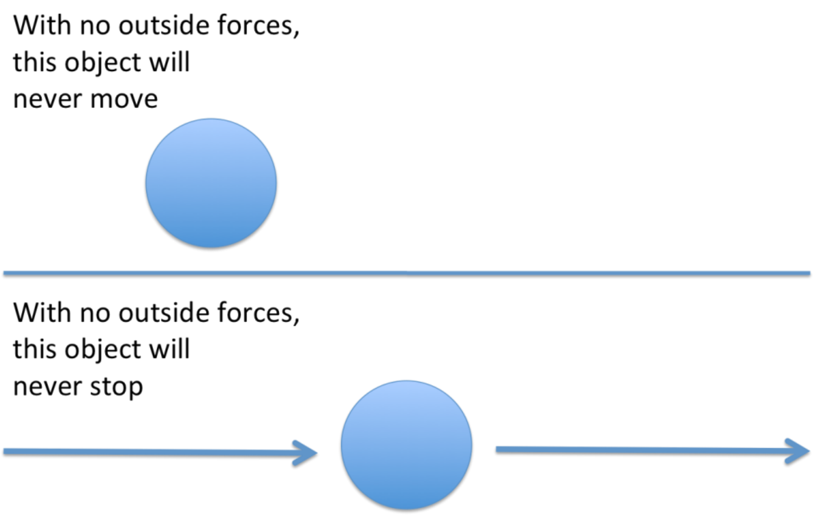
Newton's Laws of Motion

Newton's laws of motion are three physical laws that, together, laid the foundation for classical mechanics. They describe the relationship between a body and the forces acting upon it, and its motion in response to those forces.



First law

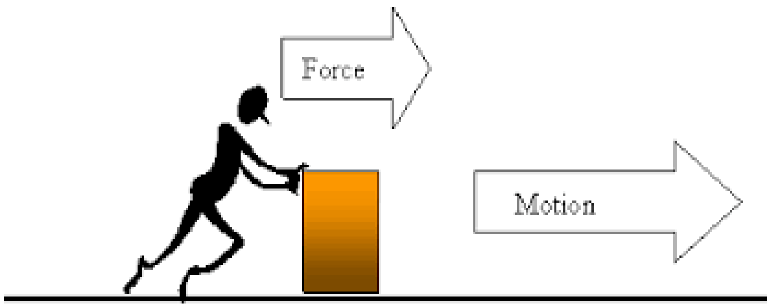
"Each body, in this universe continues to be in its state of rest or uniform motion in a straight line, unless it is compelled to change that state by forces impressed on it."



Second law

"The rate of change of momentum of a body is directly proportional to the impressed force and takes place in the direction of the force."

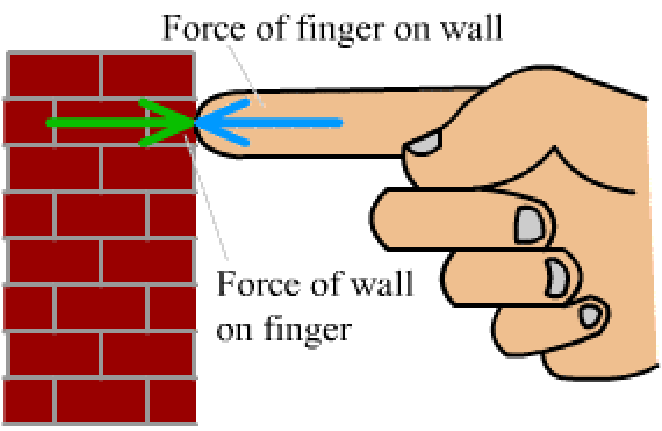
The vector sum of the external forces F on an object is equal to the mass m of that object multiplied by the acceleration vector a of the object: F = ma.

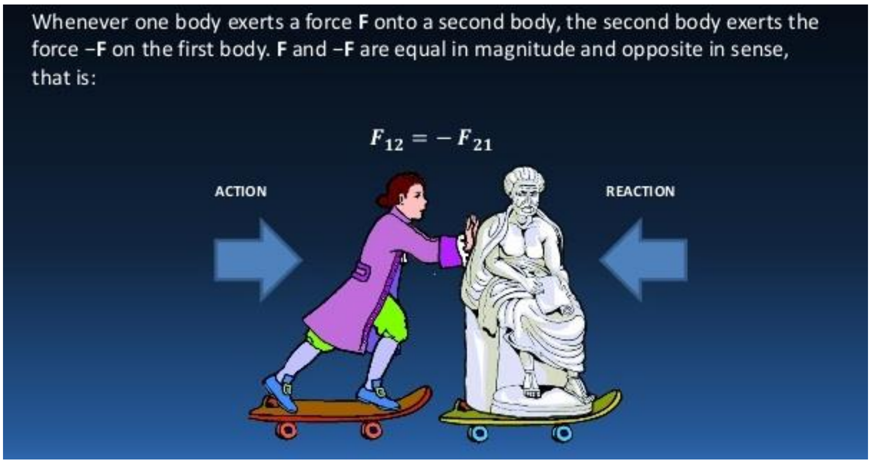


Third law

“To every action there is always an equal and opposite reaction.”

When one body exerts a force on a second body, the second body simultaneously exerts a force equal in magnitude and opposite in direction on the first body.





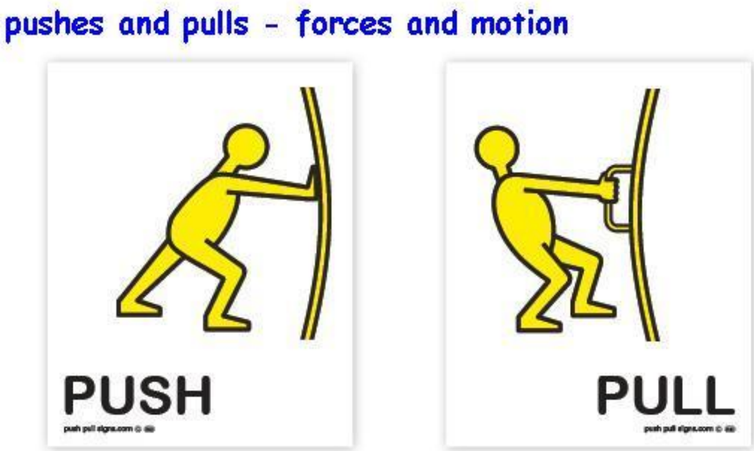
The three laws of motion were first compiled by Isaac Newton in his Philosophiæ Naturalis Principia Mathematica (Mathematical Principles of Natural Philosophy), first published in 1687. Newton used them to explain and investigate the motion of many physical objects and systems.

For example, in the third volume of the text, Newton showed that these laws of motion, combined with his law of universal gravitation, explained Kepler's laws of planetary motion.

The first law gives the definition of force, the second law gives a measure of the force and third law specifies the property of force.

Force

Force is defined as that, external agency that changes or tends to change the state of rest or uniform motion of a body in a straight line. The first law of motion is also called the "law of inertia."



Momentum

The momentum of a particle is defined as the product of its mass times its velocity. It is a vector quantity. The momentum of a system is the vector sum of the momenta of the objects which make up the system.

Like velocity, linear momentum is a vector quantity, possessing a direction as well as a magnitude:

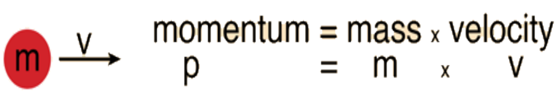
$$\mathbf{p} = m\mathbf{v},$$

Where p is the three-dimensional vector stating the object's momentum in the three directions of three-dimensional space, v is the three-dimensional velocity vector giving the object's rate of movement in each direction, and *m* is the object's mass.

If the system is an isolated system, then the momentum of the system is a constant of the motion and subject to the principle of conservation of momentum.

The basic definition of momentum applies even at relativistic velocities but then the mass is taken to be the relativistic mass.

The most common symbol for momentum is p. The SI unit for momentum is kg m/s.

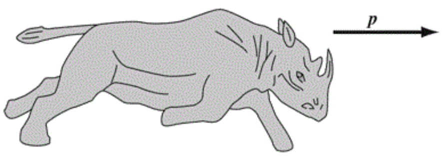


Linear momentum is also a *conserved* quantity, meaning that if a closed system is not affected by external forces, its total linear momentum cannot change.

Momentum

$$p = mv$$

p = momentum
m = mass
v = velocity



The charging rhinoceros has a great deal of momentum because of its large mass and high velocity.

Impulse

The product of average force and the time it is exerted is called the impulse of force. From Newton's second law

$$F_{average} = ma_{average} = m\frac{\Delta v}{\Delta t}$$

Newton's Second Law can be rearranged to define the impulse, *J*, delivered by a constant force, *F*.

Impulse is a vector quantity defined as the product of the force acting on a body and the time interval during which the force is exerted. If the force changes during the time interval, *F* is the average net force over that time interval. The impulse caused by a force during a specific time interval is equal to the body's change of momentum during that time interval: impulse, effectively, is a measure of change in momentum.

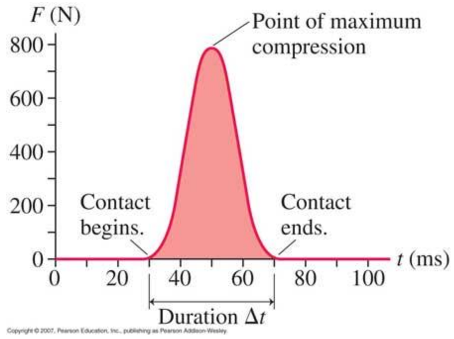
$$\boldsymbol{J} = \boldsymbol{F}\Delta t = \Delta \boldsymbol{p}$$

a small force applied for a long time produces the same change in momentum—the same impulse—as a larger force applied briefly.

$$J = F_{average}(t_2 - t_1)$$

The impulse is the integral of the resultant force (*F*) with respect to time:

$$J = \int F dt$$



Impulse

Newton's Form of Second Law:

$$\vec{F} = \frac{d\,\vec{p}}{d\,t} = \frac{\Delta\vec{p}}{\Delta t}$$

Impulse Form:

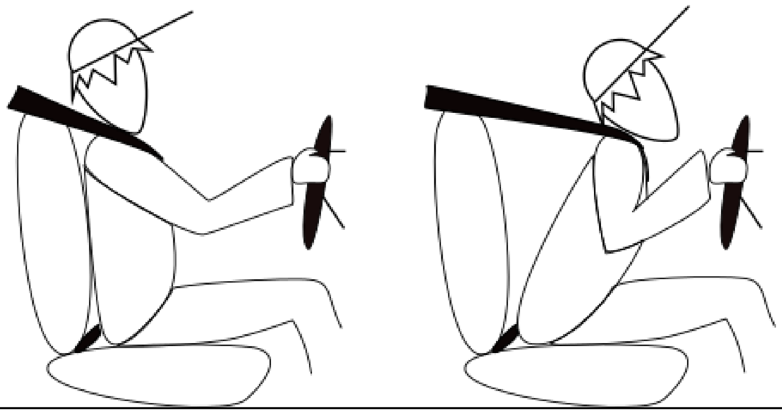
$$\Delta\vec{p} = \int \vec{F}\,dt$$

$$\Delta\vec{p} = F\,\Delta t$$

Inertia

Inertia is the resistance of any physical object to any change in its state of motion (this includes changes to its speed, direction or state of rest). It is the tendency of objects to keep moving in a straight line at constant velocity.

The principle of inertia is one of the fundamental principles of classical physics that are used to describe the motion of objects and how they are affected by applied forces. Inertia comes from the Latin word, iners, meaning idle, sluggish. Inertia is one of the primary manifestations of mass, which is a quantitative property of physical systems. Isaac Newton defined inertia as his first law in his Philosophiæ Naturalis Principia Mathematica.



Car is in motion	**Car Stopping**

Derivation of F = ma

If a body of mass m is moving with a velocity v, it possesses momentum

p = mv

Here, p and v are vector quantities. The direction of momentum is the same as the direction of velocity.

According to Newton's second law of motion, the rate of change of momentum of a body is directly proportional to the impressed force and takes place in the direction of the force.

$F \propto \frac{dp}{dt}$

$F \propto \frac{d(mv)}{dt}$

$F \propto v\frac{d}{dt}(m) + m\frac{d}{dt}(v)$

$\frac{dm}{dt} = 0$ as the mass of the body is constant.

$F \propto m\frac{d}{dt}(v)$

F = k ma

From the definition of unit force k = 1. Suppose, m = 1. a = 1 and F = 1, then

1 = k × 1 × 1

Then we have

F = ma

I Newton = 1 kg × 1 ms^{-2} = 1000 gm × 100 cm^{-2} = 10^5 dynes.

F = m a

This is certainly the most familar equation
in all of Physics.

Perhaps it is the most important.

F = ma

forms the basis of all of Mechancis.

Principle of Conservation of Linear Momentum

Consider two particles in an isolated system. These two particles interact with each other and no external forces are acting on the system. In such a case, the momentum of the system will remain constant but the momentum of each particle may change due to the interaction.

"The vector sum of the linear momentum of all the particles in an isolated system remains constant in the absence of any external forces."

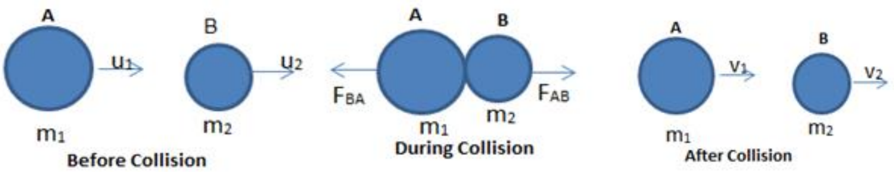
Proof

Let two particles A and B of mass m_1 and m_2 move in the same direction in straight line with velocity u_1 and u_2 respectively. Here ($u_1 > u_2$).

At one time the first particle (A) will hit the second particle (B) and then two particles will continue moving in the same direction and along the same line with velocities v_1 and v_2 respectively.

Let the time of action and reaction due to collision is t. Thus, the resultant of initial momentum of the two particles

$$P_1 = (m_1u_1 + m_2u_2)$$



The resultant of the final momentum of the particles

$$P_2 = m_1v_1 + m_2v_2$$

Rate of change of momentum of the first particle = $\frac{m_1v_1 - m_1u_1}{t}$

Rate of change of momentum of the second particle = $\frac{m_2v_2 - m_2u_2}{t}$

From Newton's third law

Applied Force = - Reaction Force

$$F_{AB} = - F_{BA}$$

$$\frac{dP_{AB}}{dt} = - \frac{dP_{BA}}{dt}$$

$$\frac{dP_{AB}}{dt} + \frac{dP_{BA}}{dt} = 0$$

$$\frac{dP}{dt} = 0$$

where, $P=P_{AB}+P_{BA}$

$$\text{so, } P=\text{constant}$$

that means initial momentum and final momentum of the particles are same

$$P_1=P_2$$

$$\text{i.e., } m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

Summation or resultant of the initial momentum of the two particles = Summation or resultant of the final momentum of the particles. Hence the conservation principle is proved.

<u>Newton's third law of motion</u>

Consider two particles in an isolated system. Their masses are m_1 and $m_{2.}$ Their velocities are v_1 and $v_{2.}$ Consider that the two bodies are moving along the same line and they interact.

Due to interaction, their velocities change. Consequently, there will be change in their momentum in time Δt. supposes, change in momentum of the first particle = Δp_1 and change in momentum of the second particle = $\Delta p_{2.}$

From the law of conservation of linear momentum, in an isolated system, having external forces zero, we get

$$\Delta p_1 + \Delta p_2 = 0$$

$$\Delta p_1 = - \Delta p_2$$

Dividing by Δt

$$\frac{\Delta p_1}{\Delta t} = \frac{\Delta p_2}{\Delta t}$$

In the limit, when $\Delta t \rightarrow 0$

$$\frac{dp_1}{dt} = \frac{dp_2}{dt}$$

$$F_2 = - F_1$$

$$\text{Force acting on } m_2 = - \text{Force acting on } m_1$$

$$\text{Action} = - \text{Reaction}$$

This represents Newton's third law of motion. According to this law, "Action and reaction are equal in magnitude and opposite in direction and act on different bodies in an isolated system"

Deduction of Newton's first law form its second law

Newton's second law states that the rate of change of momentum of a body is directly proportional to the impressed force and takes place in the direction of the force.

If a particle has mass m and velocity v due to a applied force F on the particle then according to this law we can write

$$\frac{dp}{dt} \propto F$$

$$or,\ \frac{dp}{dt} = K\ F\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

If the impressed force is zero then F=0. So from equation (1) we get that

$$\frac{dp}{dt} = 0$$

$$or, \quad dp = 0$$

$$or,\ \ P = constant \qquad \text{(integrating)}$$

$$or, mv = constant \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

As the mass of the particle is constant so from equation (2) we can say

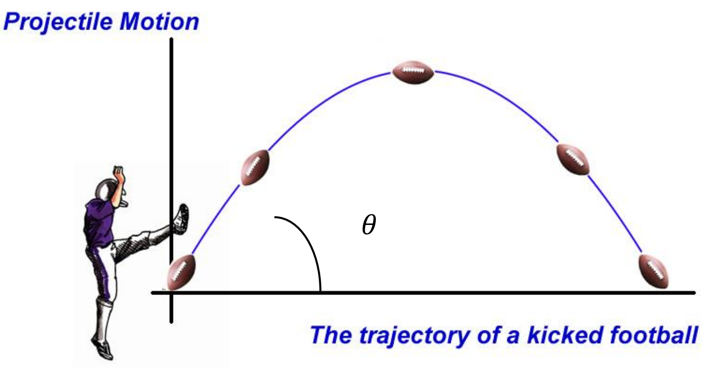
v = constant

that means, if the applied force is zero then the velocity of the particle will be constant, which is the statement of the Newton's first law.

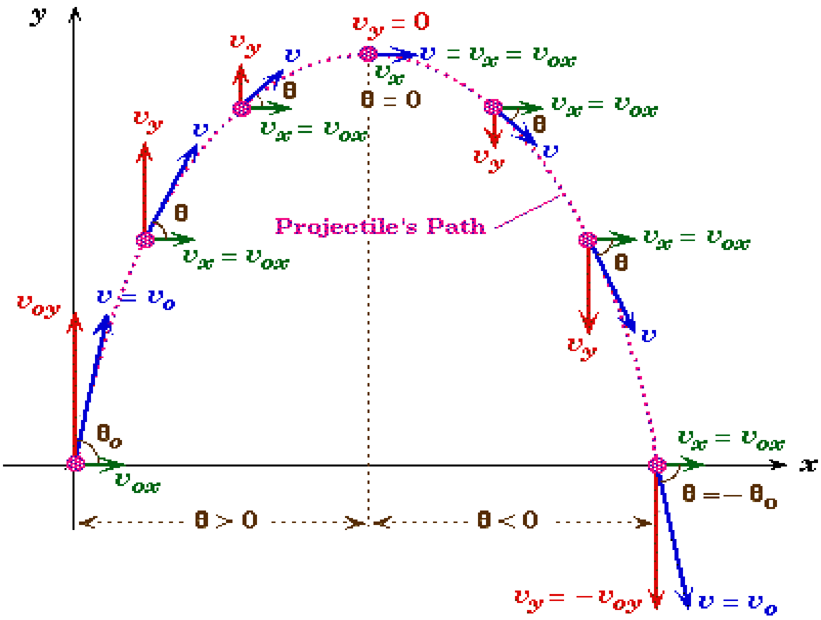
Motion in Two Dimensions

Projectile Motion

A body projected with a uniform velocity at an angle θ with the horizontal in the vertical plane of the earth, is called a **projectile**. The path traversed by the projectile is called its **trajectory.**



Projectile motion is a form of motion in which an object or particle (called a projectile) is thrown near the earth's surface, and it moves along a curved path under the action of gravity only. The only force of significance that acts on the object is gravity, which acts downward to cause a downward acceleration. There are no horizontal forces needed to maintain the horizontal motion – consistent with the concept of inertia.

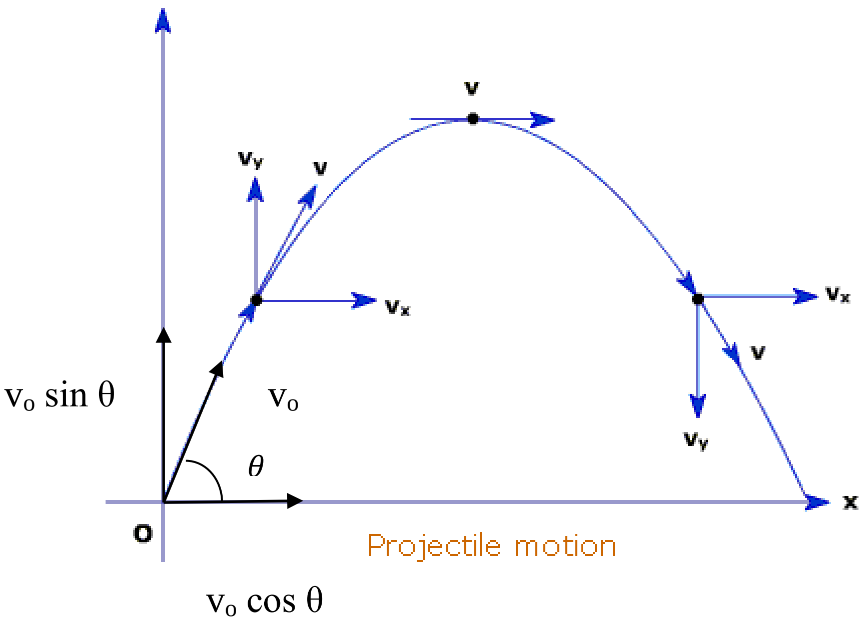


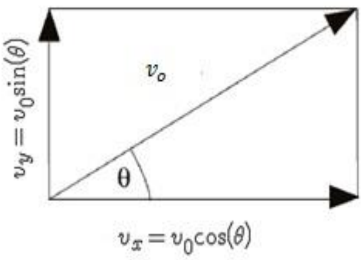
Consider a plane X-Y in this figure and the origin at the point O. The body is projected from the point O with a velocity vo making an angle θ with the horizontal. The two components of this velocity at the point P are

$v_x = v_o \cos\theta$ in the horizontal direction

$v_y = v_o \sin\theta$ in the vertical direction

Components of velocity





General Equation of a Parabola

Let a projectile begin its flight from a point O with initial velocity v_o and making an angle α with the horizontal direction. Taking O as origin let the horizontal and vertical direction be considered along X and Y axis. So, at t=0, the horizontal component of initial velocity is

$v_{xo} = v_o \cos\alpha$ and

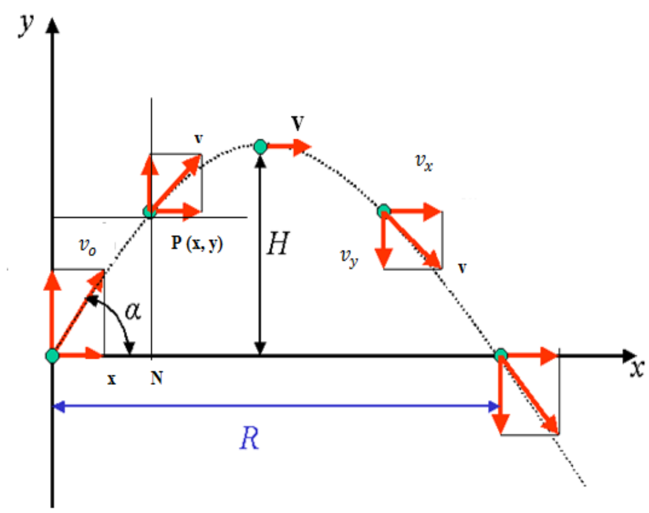
The vertical component

$v_{yo} = v_o \sin\alpha$

Now, from the equation of motion

$v_x = v_{x_0} + a_x t$

$v_x = v_o \cos\alpha$ (since, $a_x = 0$, acceleration works only in the vertical direction which is gravitational acceleration)



Let at t = t, the projectile reaches the point P, whose co-ordinate is (x, y) and where its velocity is v.

So, the displacement of the projectile parallel to the ground along X axis is

$x = ON = v_o \cos\alpha \times t$

$t = \frac{x}{v_0 \cos\alpha}$ …………………… (1)

Now, the vertical component of the velocity

$v_y = v_{yo} + a_y t$

$v_y = v_o \sin\alpha - gt$ (Since, $a_y = -g$)

So, the vertical displacement at t = t

y = PN

We know that, $y = y_o + v_{yo} t + \frac{1}{2} a_y t^2$

$y = 0 + v_o \sin\alpha\, t - \frac{1}{2} g t^2$

$y = v_o \sin\alpha\, t - \frac{1}{2} g t^2$ ………………… (2)

Putting the value of t, from equation number (1) into equation number (2)

$y = v_o \sin\alpha \frac{x}{v_0 \cos\alpha} - \frac{1}{2} g \left(\frac{x}{v_0 \cos\alpha} \right)^2$

$= x \tan\alpha - \frac{1}{2} g x^2 \frac{1}{v_0^2 \cos^2\alpha}$ ……………….. (3)

In this equation α, g and v_o are constants. So, taking

$\tan\alpha = b$ and $\frac{g}{2 v_0^2 \cos^2\alpha} = c$ as constants

$y = bx - cx^2$

This is an equation of a parabola. Hence, the path of motion (called trajectory) of a projectile is parabola.

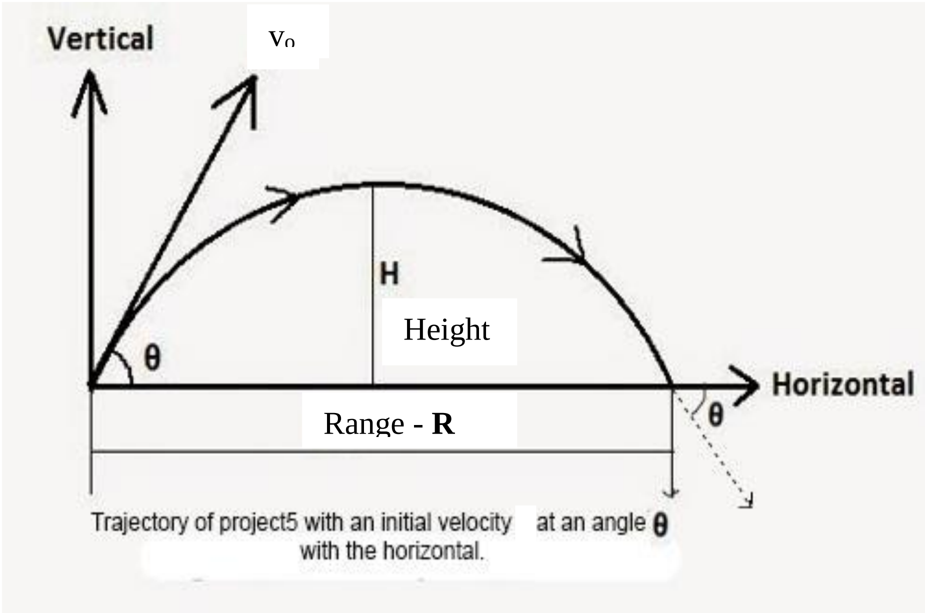
<u>Maximum height of the path of a projectile</u>

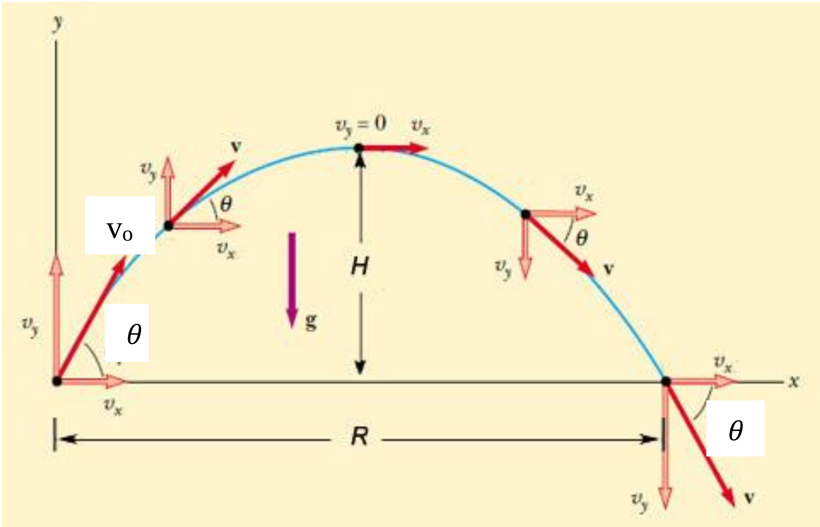
We, know from the equation of motion

$v^2 = v_0^2 + 2as$

Since, the vertical component of the projectile is along Y-axis. So, the above equation becomes

$v_y^2 = v_{yo}^2 + 2 a_y y$





Let, the height reached by the projectile be H where final velocity = 0. That means $v_y = 0$. So the equations becomes

$0 = v_{yo}^2 - 2\,g\,H$ (Since, $a_y = -\,g$)

But $v_{yo} = v_o \sin\theta$

$0 = v_o^2 \sin^2\theta - 2gH$

$H = \frac{v_o^2 \sin^2\theta}{2g}$

When $\theta = 90^0$, the height will be maximum. Then, this equation will be

$H = \frac{v_o^2}{2g}$

Knowing the value of v_o and g, H can be calculated.

Time to reach maximum height

From the equation of motion

$v = v_o + at$

Along Y axis (vertical direction) this equation becomes

$v_y = v_{yo} + a_y t$ ………….. (1)

Vertical component of initial velocity is $v_{yo} = v_o \sin\theta$ and at maximum height final velocity $v_y = 0$. So, from equation number (1)

$0 = v_o \sin\theta - gt$

$t = \frac{v_0 \sin\theta}{g}$

By knowing the value of v_o, θ and g, we can calculate t.

Time of flight

Let the time of flight be T. Now, we know

The time of ascend to the maximum height = time of descend to the ground

$T = t + t = 2\,t$

$T = 2 \times \frac{v_0 \sin\theta}{g}$

$T = \frac{2\,v_0 \sin\theta}{g}$

Horizontal Range

The linear distance from the point of projection to the end of flight is called the **horizontal range.** Alternatively, the distance travelled along the horizontal direction in the time of flight is called the horizontal range.

This is denoted by R.

R = horizontal component of the initial velocity × time of flight

$R = v_o \cos\theta \times T$

$R = v_o \cos\theta \times \frac{2\,v_0\,\sin\theta}{g}$

$R = v_o{}^2 \;\; \frac{2\sin\theta\;\cos\theta}{g}$

$R = v_o{}^2 \ \frac{\sin 2\theta}{g}$

Knowing the value of v_o, θ and g, we can find the value of R.

Maximum Horizontal Range

The equation of horizontal range

$R = v_o{}^2 \ \frac{\sin 2\theta}{g}$

It is evident that R will be maximum, when $\sin 2\theta = 1$

$\sin 2\theta = 90^0$

$2\theta = 90^0$

$\theta = 45^0$. In that case $R_{maximum} = \frac{v_0^2}{g}$

Conclusion, if an object is thrown at an angle 45^0 with the horizontal direction, the horizontal range will be maximum.