NUMBER SYSTEM

MAT111

Number, Real, complex, Rational, Irrational, Fraction, Integer, Classification

Factor, Divisor and Multiple, Find prime factorization & all factor of a number

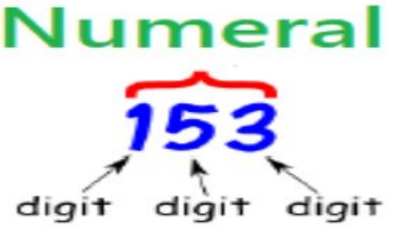
LCM, HCF/GCD, Find LCM & HCF/GCD of Integers or Fractions

Complex number, Modulus, Argument and its various forms

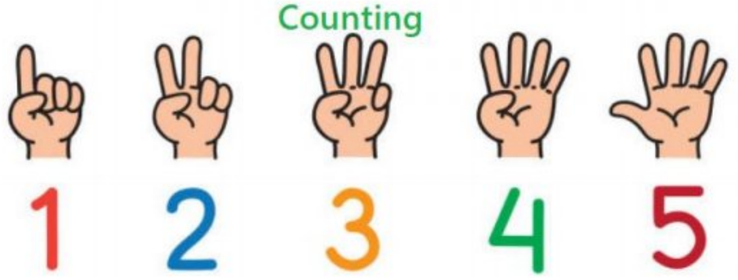
TOPIC-1

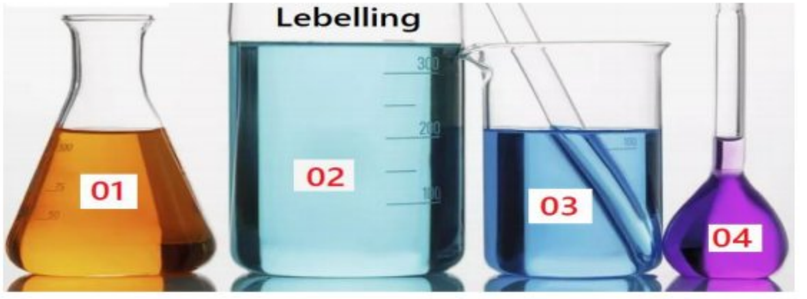
Number:

A number is a numeral which represents a quantity and also used for counting, measurement & labelling.



For example: The weight of a man is 153 kg





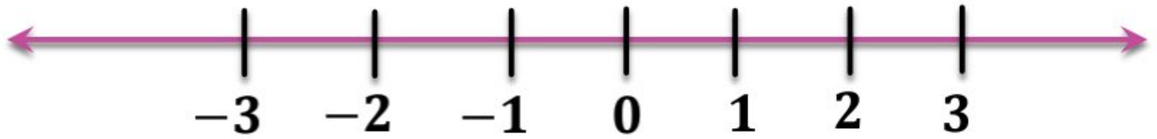
Real Number :

A number is said to be real if the square of it is non-negative otherwise imaginary.

A real number geometrically represents a point on X-axis.

$$\left(\sqrt{2}\right)^2 = 2$$ → Non-negative

$$\left(\sqrt{-2}\right)^2 = -2$$ → Negative



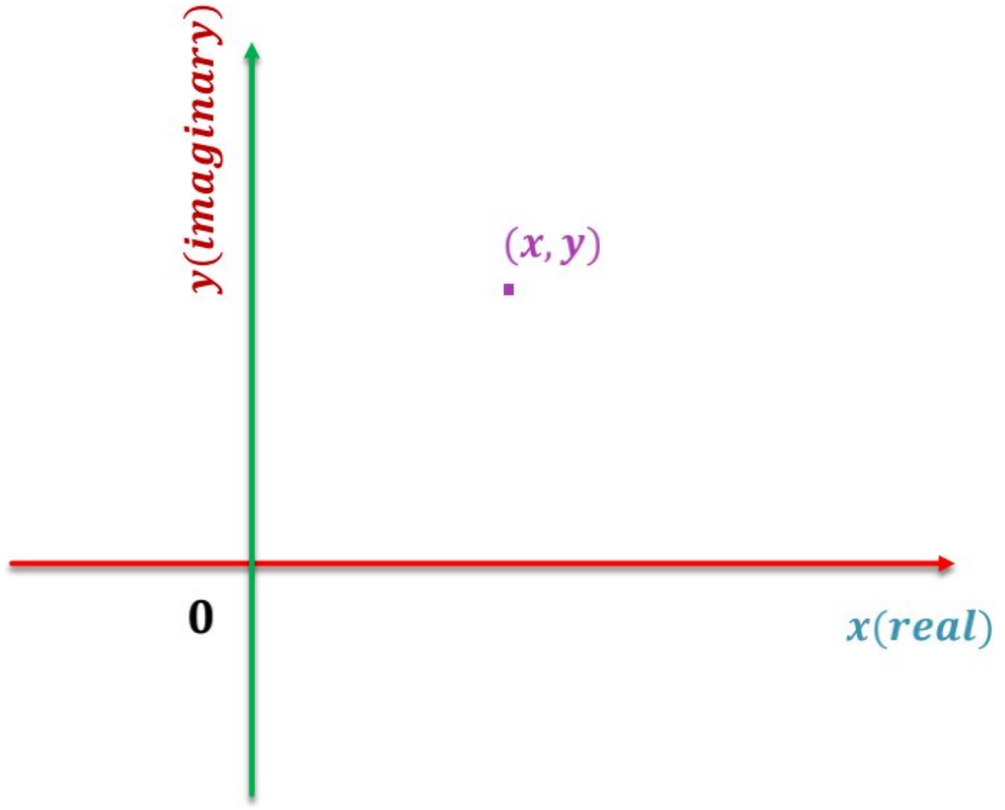
Complex Number :

A complex number z is the linear combination of two real numbers x & y with a special sign i.

A complex number geometrically represents a point on Complex plane or Argand plane.

$$z = x + iy$$

where: x and y real number & $(\pm i)^2 = -1$



Rational Number:

A number of the form $\frac{p}{q}, q \neq 0$ where p and q are integers.

$$\text{Fraction} = \frac{\text{Numerator}}{\text{Denominator}}$$

For example:

01 Integer / Whole number

$$2 = \frac{2}{1}, 3 = \frac{3}{1} \& -4 = \frac{-4}{1} \ etc.$$

02 Terminating decimal number

$$1.5 = \frac{15}{10} = \frac{3}{2}, 1.3 = \frac{13}{10} \ etc.$$

03 Recurrence decimal number /Infinite decimal point number

$$0.\dot{3} = \frac{1}{3}, 1.\dot{3} = \frac{13-1}{9} = \frac{4}{3} \ etc.$$

04 Fraction

$$\frac{1}{3}, \frac{4}{3} \& \frac{9}{5} \ etc.$$

Is $\sqrt{4}$ a rational number ?

Irrational Number:

A number which is not expressible as $\frac{p}{q}, q \neq 0$ where p and q are integers.

For example:

01 **Inexpressible as fraction**

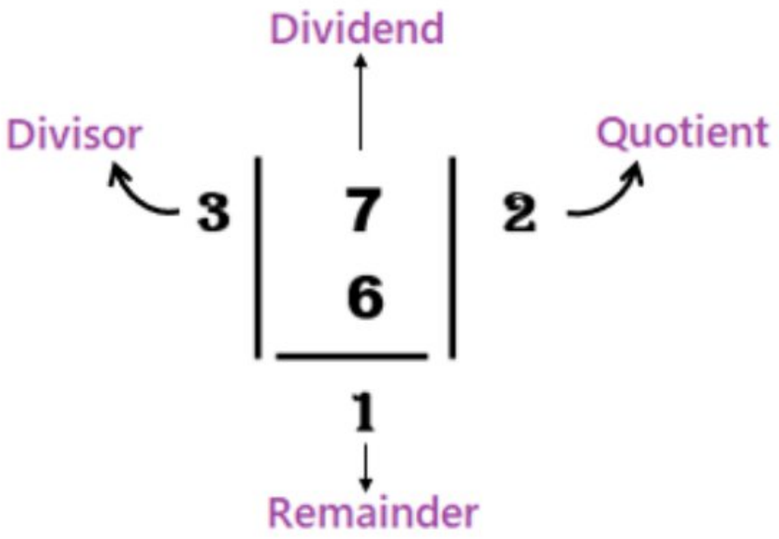
$$\sqrt{2}\ ,\sqrt{3}\ \&\left(\sqrt{2}+\sqrt{3}\right)\ etc.$$

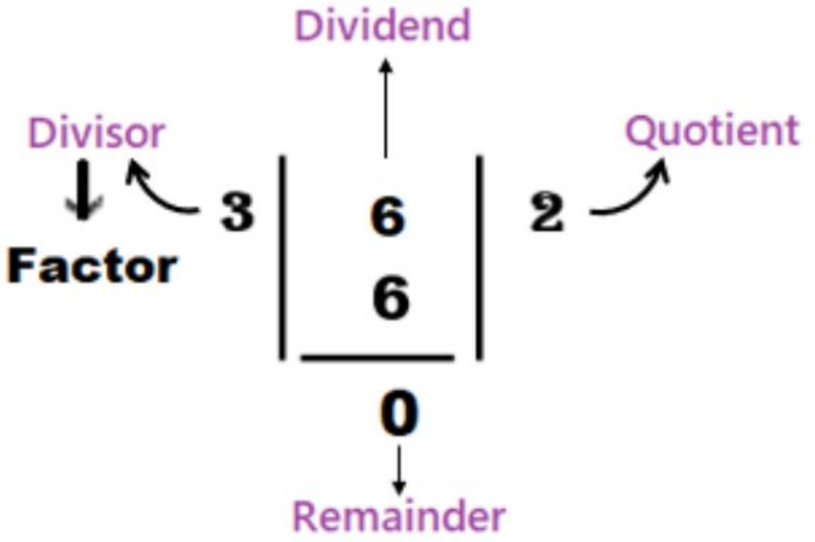
02 Non-terminating decimal number/Infinite nonrepeating decimal number

$$1.32495879\ldots$$

$$2.78997824\ldots$$

Division





TRY IT: What are the factors of the number 6 ?

DEFINITION

- **Product** – An answer to a multiplication problem.

$$7 \times 8 = 56$$

Product

- **Factor** – a number that is multiplied by another to give a product.

$$7 \times 8 = 56$$

Factors

Factor:

Factors of a number are all those numbers which can exactly divide the given number.

For example: $1, 2$ and 4 are the factors of 4.

Multiple:

Multiples of a number are those numbers which are exactly divisible by the given number.

For example: The multiples of 4 are $4, 8, 12, 16, 20, \ldots$

Factors

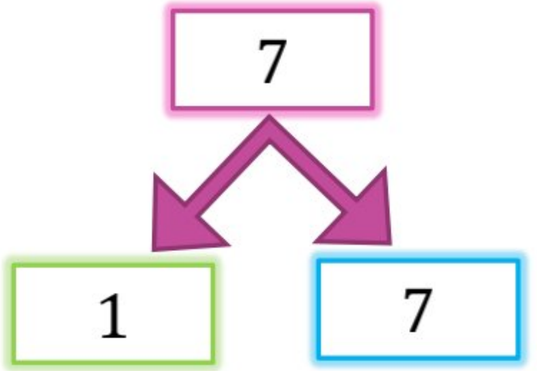
5	**6**	**7**	**8**
1	1	1	1
5	2	7	2
	3		4
	6		8

Multiples

5	**6**	**7**	**8**
5	6	7	8
10	12	14	16
15	18	21	24
20	24	28	32
25	30	35	40
.	.	.	.
.	.	.	.
.	.	.	.

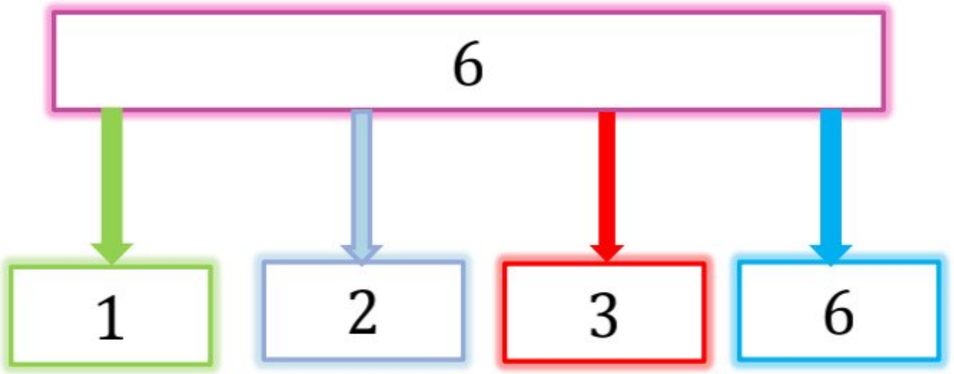
Prime Number :

A prime number is a natural number greater than 1, that has exactly two factors 1(one) and itself.



Composite Number :

A composite number is a natural number greater than 1, that has more than 2 factors.




ONE IS SPECIAL BECAUSE . . .

One is <u>not</u> prime.

(because it does not have exactly two different factors).

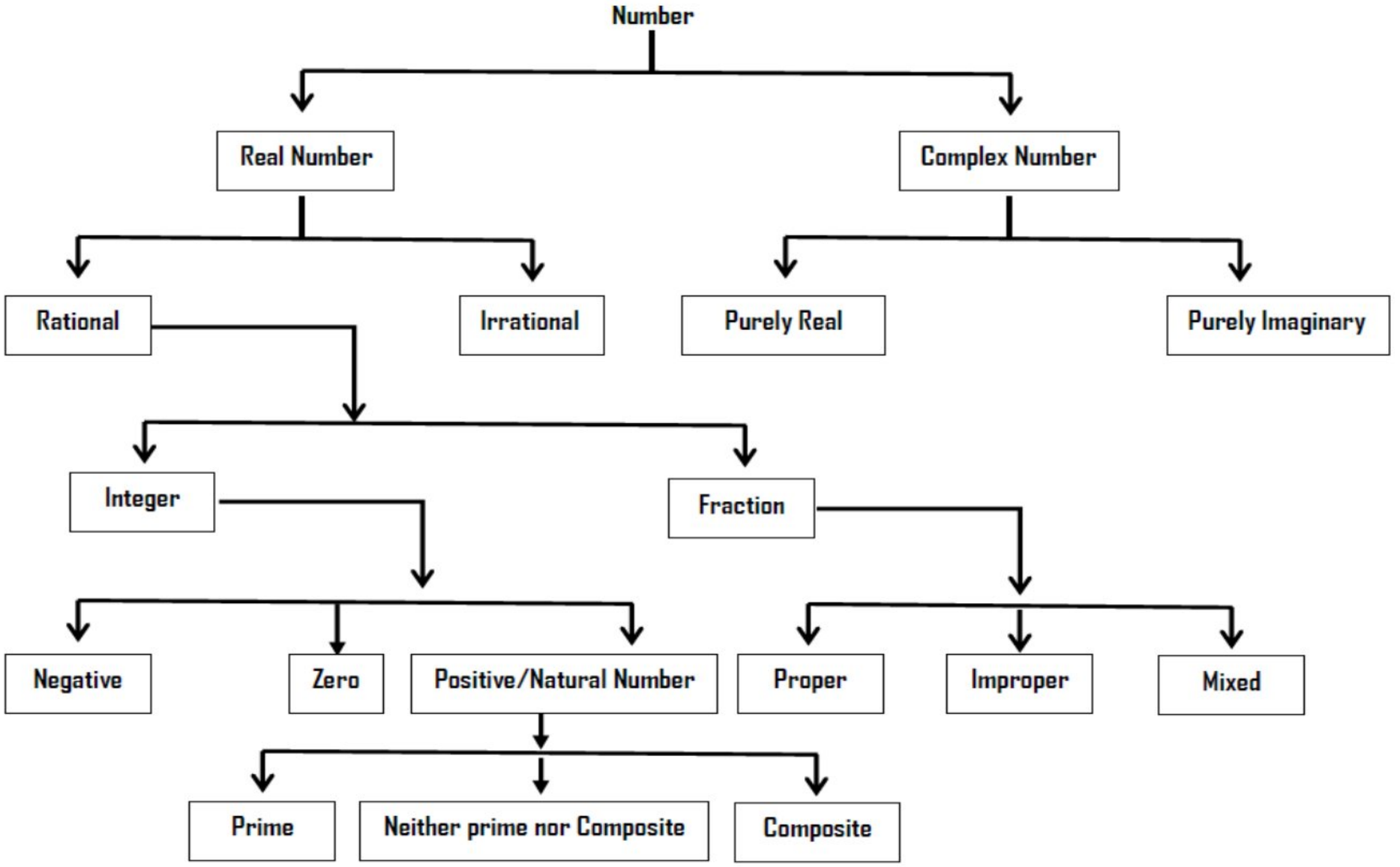
1

One is <u>not</u> Composite.

(because it does not have more than 2 factors).

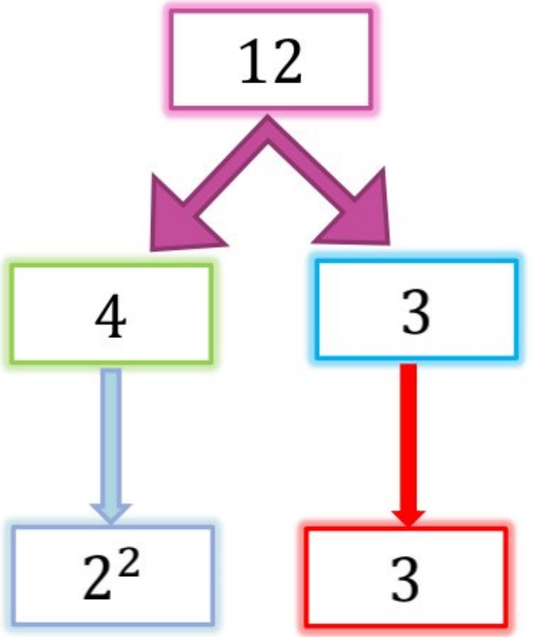


Classification:



Prime Factorization

Prime factorization of a number is the representation of the number by its prime factors.

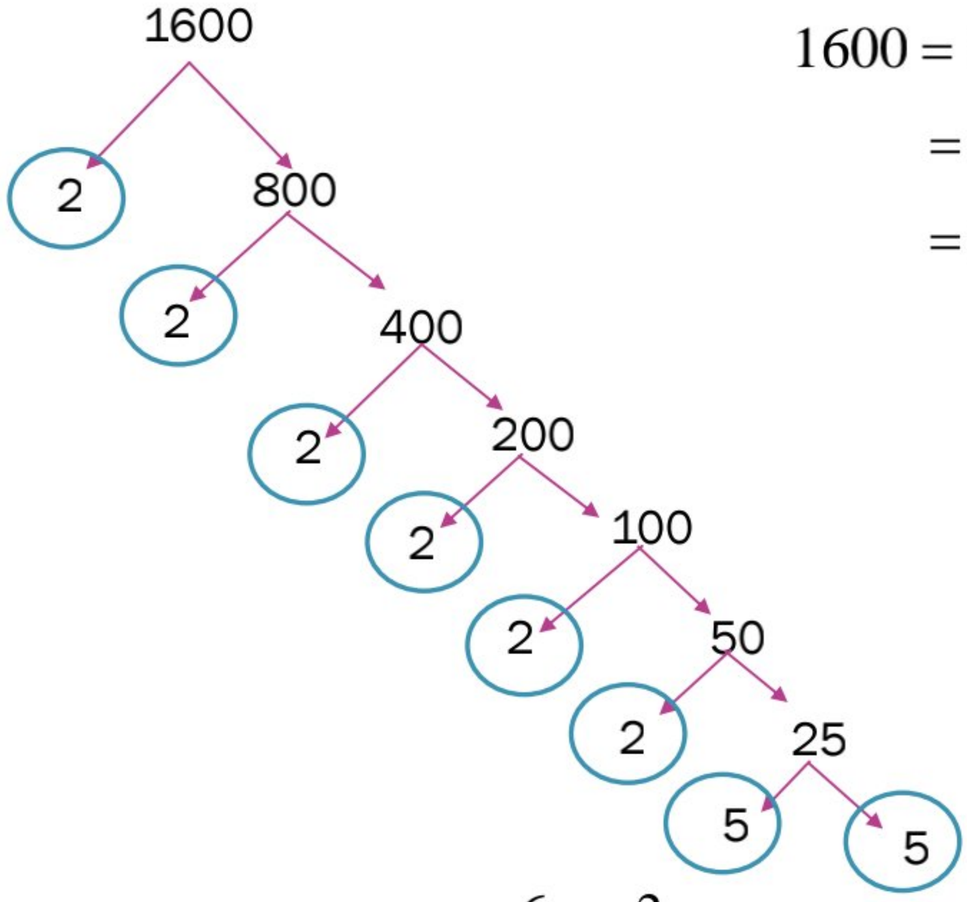


Problem 01: Find the Prime Factorization of 1600.

Division method:

$$
\begin{array}{r|l}
2 & 1600 \\
2 & 800 \\
2 & 400 \\
2 & 200 \\
2 & 100 \\
2 & 50 \\
5 & 25 \\
 & 5
\end{array}
$$

Tree Diagram:



Multiplication Method :

$$1600 = 2 \times 800 = 2 \times 2 \times 400 = 2^2 \times 2 \times 200$$
$$= 2^3 \times 2 \times 100 = 2^4 \times 2 \times 50$$
$$= 2^6 \times 5 \times 5 = 2^6 . 5^2$$

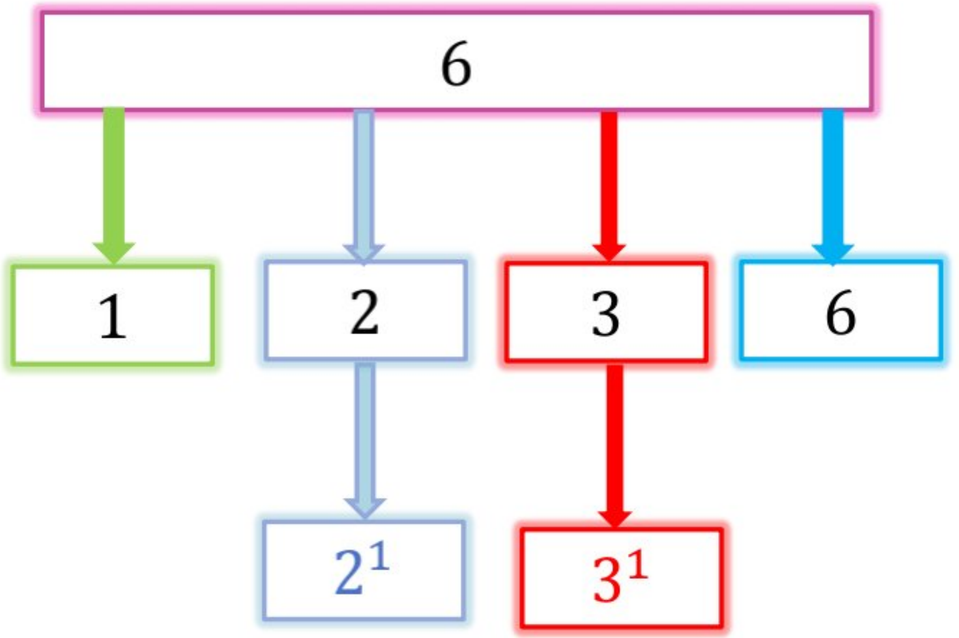
Therefore ,the prime factorization of 1600 is $= 2^6 . 5^2$

Formula:

$$Q = p^l q^m r^n$$

Number of factors

$$(l+1)(m+1)(n+1)$$



$$(1+1)(1+1) = 4$$

Try it. How many factors of 520?

Problem 02: Find the all factors of 1600.

$$
\begin{array}{r|l}
2 & 1600 \\
2 & 800 \\
2 & 400 \\
2 & 200 \\
2 & 100 \\
2 & 50 \\
5 & 25 \\
 & 5
\end{array}
$$

Therefore ,the prime factorization of 1600 is $= 2^6 . 5^2$

So ,the total number of factors of 1600 is

$$(6+1)(2+1) = 7 \cdot 3 = 21$$

Calculation for all factors

$$
\begin{aligned}
1600 &= 1 \times 1600 \\
&= 2 \times 800 \\
&= 4 \times 400 \\
&= 5 \times 320 \\
&= 8 \times 200 \\
&= 10 \times 160 \\
&= 16 \times 100 \\
&= 20 \times 80 \\
&= 25 \times 64 \\
&= 32 \times 50 \\
&= 40 \times 40
\end{aligned}
$$

The factors of 1600 are

1, 2, 4, 5, 8, 10, 16, 20, 25, 32, 40, 50, 64, 80, 100, 160, 200, 320, 400, 800 & 1600.

Exercise :

1. Find the prime factorization of 1240 using three different methods.
2. Find the all factors of 1240 using tree diagram.
3. Find the all prime factors of 1240.
4. Find the all composite factors of 1240.
5. In how many ways can the number 7056 be resolved into two factors?
6. Find the prime factors of 1280 using tree diagram and also find the sum of its composite factors.

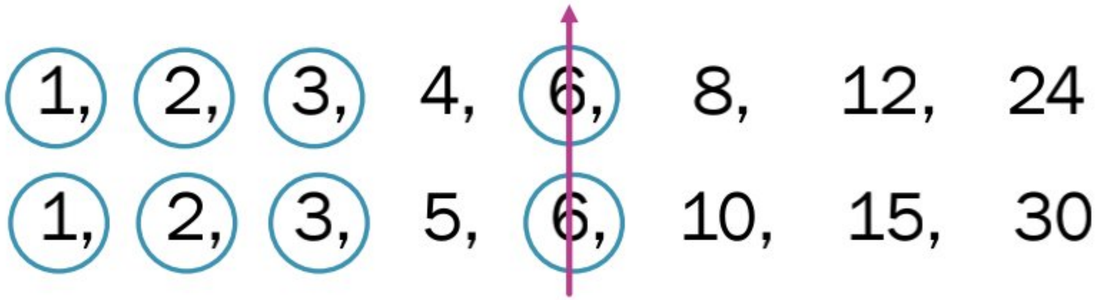
DEFINITION

HCF – The **Highest Common Factor** of two numbers is the largest number that is a common factor of both numbers. We call it the **HCF**.

- List all factors of each number
- Circle all common factors
- The highest circled number is the HCF

Example

Find the HCF of 24 and 30



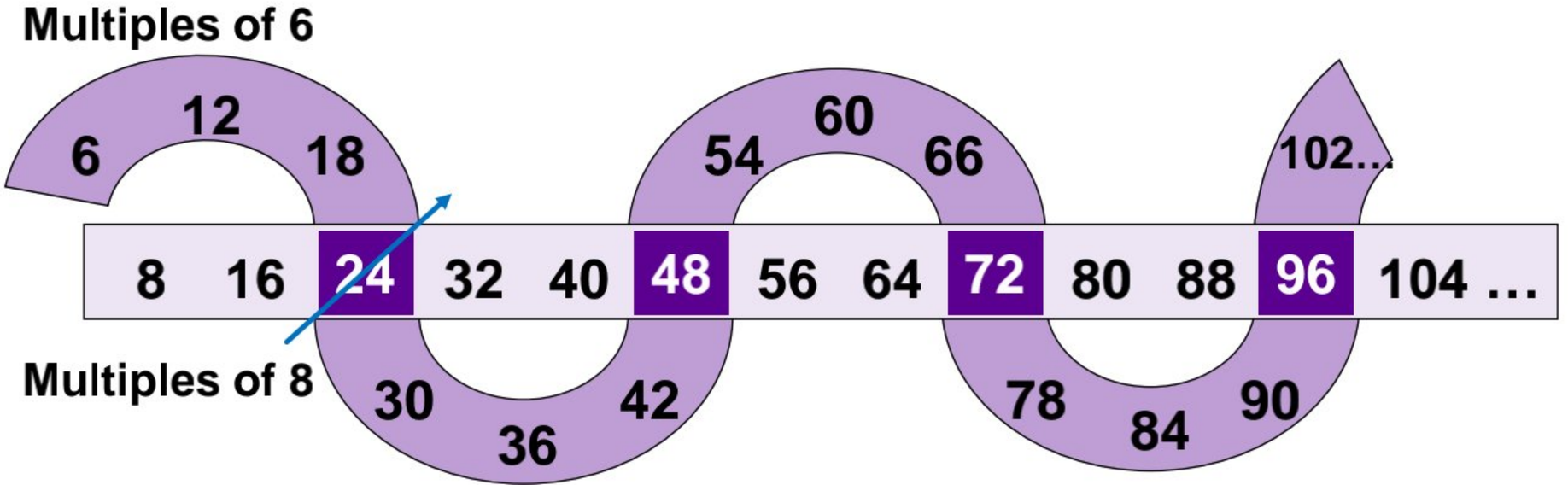
So the HCF of 24 and 30 is: 6

DEFINITION

LCM – The **Least Common Multiple** of two numbers is the smallest number that is a common multiple of both numbers. We call it the LCM.

- List the first few multiples of each number
- Circle the common multiples
- The lowest circled number is the LCM

Example: Find the LCM of 6 and 8

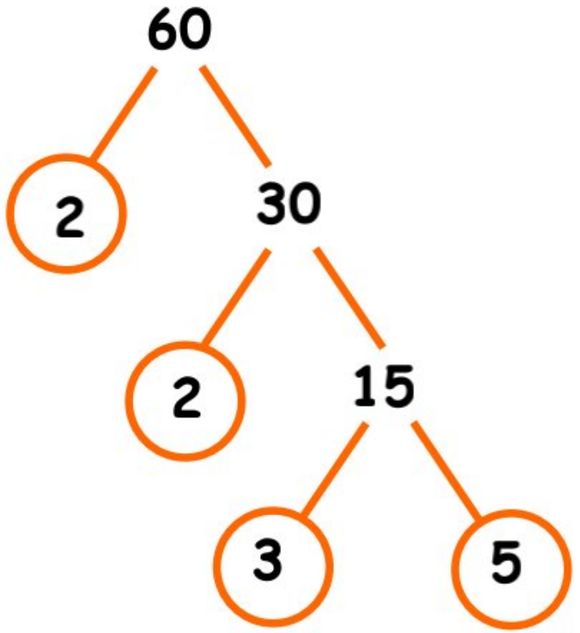


Calculation of HCF and LCM by using Prime Factorization.

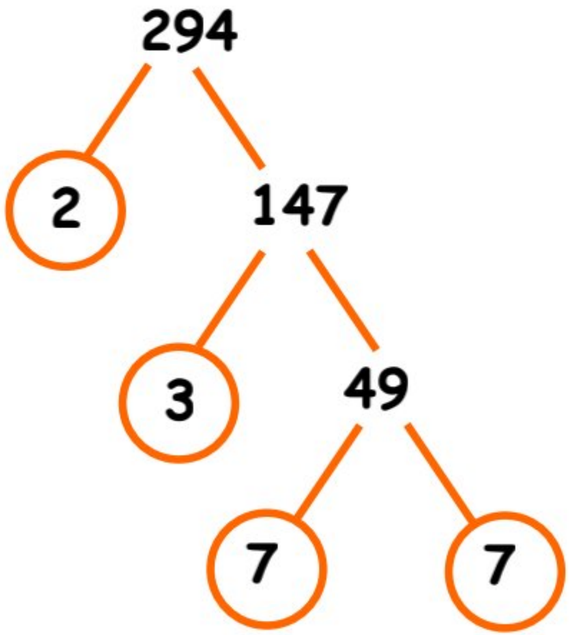
Problem 03: Find the HCF & LCM of the numbers 60 & 294.

Solution: Given numbers are 60 & 294

Therefore, the prime factorization of 60 & 294 are



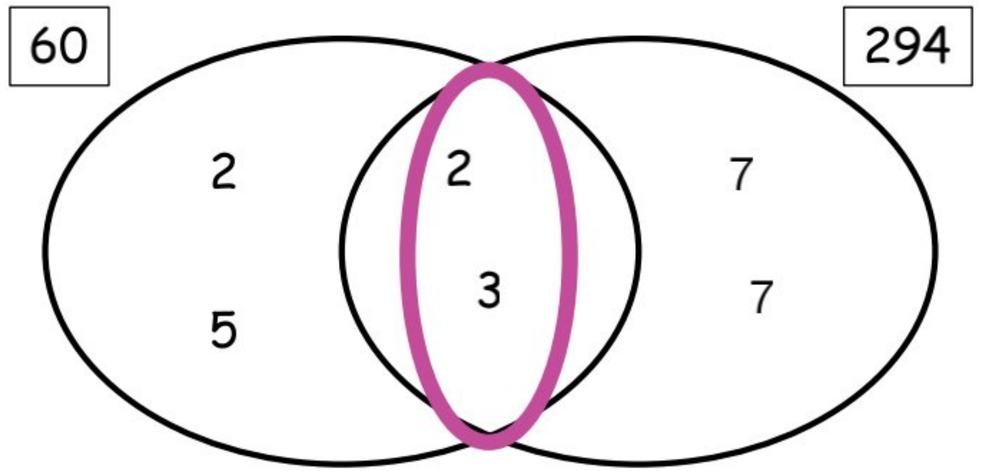
$60 = 2 \times 2 \times 3 \times 5$



$294 = 2 \times 3 \times 7 \times 7$

HCF/GCD = Common factors

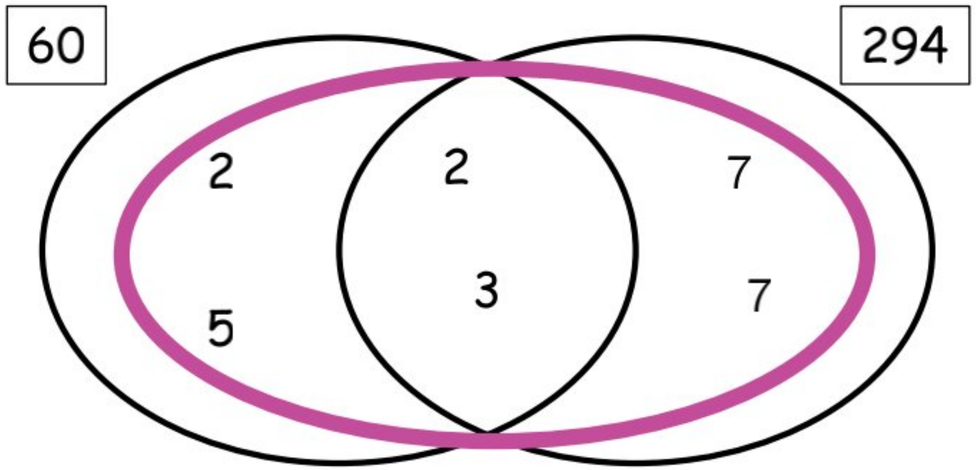
60 = ~~2 × 2 × 3 × 5~~ 294 = ~~2 × 3 × 7 × 7~~



HCF of 60 and 294 = 2 x 3 = **6**

LCM = Common/Uncommon factors

60 = ~~2 × 2 × 3 × 5~~ 294 = ~~2 × 3 × 7 × 7~~



LCM of 60 and 294 = 2 x 5 x 2 x 3 x 7 x 7 = **2940**

Problem 04 : Find the LCM and HCF of $\mathbf{50}$, $\mathbf{140}$ & $\mathbf{240}$.

Solution: Given numbers are 50 , 140 & 240.

$$50 = 2.25 = 2.5.5 = 2.5^2$$

$$140 = 2.70 = 2.2.35 = 2^2.5.7$$

$$240 = 2.120 = 2.2.60 = 2^2.2.30 = 2^3.2.15 = 2^4.3.5$$

$$\therefore LCM\,(50,140,240) = 2^4.5^2.3.7 = 8400$$

$$\&\ HCF\,(50,140,240) = 2.5 = 10.$$

LCM & HCF of Fractions

$$\therefore \text{LCM of Fractions} = \frac{\text{LCM of Numerators}}{\text{HCF of Denominators}}$$

$$\therefore \text{HCF of Fractions} = \frac{\text{HCF of Numerators}}{\text{LCM of Denominators}}$$

Problem 05: Find the LCM and HCF of $\frac{2}{3}, \frac{8}{9}$ and $\frac{16}{81}$.

Calculation for Numerators

$$2 = 2^1$$

$$8 = 2^3$$

$$16 = 2^4$$

$\text{LCM}\ (2, 8, 16) = 2^4 = 16$

$\text{HCF}(2, 8, 16) = 2^1 = 2$

Calculation for Denominators

$$3 = 3^1$$

$$9 = 3^2$$

$$81 = 3^4$$

$\text{LCM}\ (3, 9, 81) = 3^4 = 81$

$\text{HCF}\ (3, 9, 81) = 3^1 = 3$

$$\text{LCM of } \frac{2}{3}, \frac{8}{9} \text{ and } \frac{16}{81} = \frac{\text{LCM of } (2,8,16)}{\text{HCF of } (3,9,81)} = \frac{16}{3}$$

$$HCF \text{ of } \frac{2}{3}, \frac{8}{9} \text{ and } \frac{16}{81} = \frac{\text{HCF of } (2,8,16)}{LCM \text{ of } (3,9,81)} = \frac{2}{81}$$

Problem 06: Find the HCF and LCM of $0.63, 10.5$ and 2.1.

Solution: Given, $0.63, 10.5, 2.1$

Converting each of the decimal numbers into fractions,

$$0.63 = \frac{63}{100}, \qquad 10.5 = \frac{105}{10} = \frac{21}{2}, \qquad 2.1 = \frac{21}{10}$$

Calculation for Numerators

$$63 = 3^2 \times 7$$

$$21 = 3 \times 7$$

$$21 = 3 \times 7$$

$$\text{HCF}\ (63, 21, 21) = 3 \times 7 = 21$$

$$\text{LCM}\ (63, 21, 21) = 3^2 \times 7 = 63$$

Calculation for Denominators

$$100 = 2^2 \times 5^2$$

$$2 = 2^1$$

$$10 = 2 \times 5$$

$$\text{HCF}\ (100, 10, 10) = 2^1 = 2$$

$$\text{LCM}\ (100, 10, 10) = 2^2 \times 5^2 = 100$$

$$\text{HCF of } 0.63, 10.5 \text{ and } 2.1 = \frac{\text{HCF of } (63, 21, 21)}{\text{LCM of } (100, 10, 10)} = \frac{21}{100}$$

$$\text{LCM of } 0.63, 10.5 \text{ and } 2.1 = \frac{\text{LCM of } (63, 21, 21)}{\text{HCF of } (100, 10, 10)} = \frac{63}{2}$$

Relationship between HCF and LCM

HCF and LCM relationship states that the product of two numbers is equal to the product of their HCF and LCM

$$1\text{st number} \times 2\text{nd number} = \text{HCF} \times \text{LCM}$$

Problem 06: The H.C. F of two numbers is 11 and their L.C.M is 693. If one of the numbers is 77, find the other.

Solution: Let x be the other number.

We know that,

$$\text{Product of the two numbers} = \text{LCM} \times \text{HCF}$$

Then,

$$77(x) = 693 \times 11$$

Divide each side by 77.

$$x = \frac{693 \times 11}{77}$$

$$x = 99$$

The other number is 99.

Exercise:

1. Find the H.C.F & L.C.M of $42, 63$ & 140 .
2. Find the H.C.F & L.C.M of $16, 24, 36, 54$.
3. Find the H.C.F & L.C.M of $\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ & $\frac{10}{27}$.
4. Find the H.C.F & L.C.M of $1.2, 0.24$ & 6 .
5. The H.C. F of two numbers is 36 and their L.C.M is 432. If one of the numbers is 108, find the other.
6. The product of two numbers is 1320 and their HCF is 6. Find the LCM of the numbers.
7. Two numbers are in the ratio of 3:4. If their L.C.M is 84. Find the numbers.


ANY
QUESTIONS