كلية الهندسة بحلوان

Dept/Division: Electronics, Communication, and Computer Department
Course code & title: Digital Signal Processing
Academic level: 4th Year
Total mark: 100 marks
Semester: Fall 2022
Time allowed: 3 hrs
Instructor: Prof. Elsayed M. Saad & Assoc. Prof. Amr E. Mohamed

Answer all questions. Write each question number and part number ahead of your answer.

$(-\alpha)^n u(-n-1) = \frac{z}{z-\alpha}$

Question 1: [15-marks]

When the input to an LTI system is $x(n) = (0.5)^n u(n) + 2^n u(-n-1)$

and the output is $y(n) = 6\,(0.5)^n u(n) - 6\,(0.75)^n u(n)$

a) Find the system function $H(z)$ of the system.

b) Plot the poles and zeros of $H(z)$ and indicate the ROC.

c) Find the impulse response $h(n)$ of the system.

d) Write the difference equation that characterizes the system.

Question 2: [20-marks]

a) **For each of the following systems, determine whether it is (1) linear, (2) time-invariant, (3) causal, and (4) stable. Briefly explain your answer.**

5 i) $y(n) = x(n) + n x(n+1)$ non linear

5 ii) $y(n) = \begin{cases} x(n) & if\ x(n) \geq 0 \\ 0 & if\ x(n) < 0 \end{cases}$ non linear

y $\quad a x + b x$ $\quad X(e^{j\omega}) = \sum_{n=1}^{\infty} x(n) e^{-j\omega n}$ $\quad (\frac{1}{4} e^{-j\omega 4}$

b) **Consider an LTI system with a frequency response $H(e^{j\omega}) = e^{j2\omega} + e^{-j6\omega}$** $= z^2 + z^{-6}$

i) Calculate the DTFT of $x[n] = \left(\frac{1}{4}\right)^n u[n+1]$. $\quad \frac{1}{4}^{-1} (\frac{1}{4})^{n+1} u(n+1) = \frac{z}{z-0.25} \times z_2 \quad \boxed{\frac{z^2 4}{z-0.25}}$

ii) Find the DTFT $Y(e^{j\omega})$ of the output $y[n]$ when $x[n]$ is the input to the system. $= \frac{e^{j\omega 2}}{e^{j\omega} - 0.25}$

iii) Find the output y[n]. $\quad \frac{4z^4 + 4z^{-4}}{z - 0.25}$ $\quad = \frac{1}{e^{j\omega} - 0.25 e}$

iv) Calculate the group delay of the system. $\quad 4 (\frac{1}{4})^n$

-2ω $\quad \frac{d}{d\omega} = -2$ $\quad \frac{1}{2\pi} \int_{-\pi}^{\pi} X e^{j\omega} / e^{j\omega n}$ $\quad \to$ Filter Sampling

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Question 3: [15-marks]

a) **The response, $y(n)$ of an LTI 3rd-order causal non-recursive digital filter to the excitation, $x(n)$ is given in the following table:**

N	0	1	2	3	4	5	6	7	……
y(n)	1	2.5	3	4	4	4	4	4	……

$z^{-5} X(z) = z^{-4}$

If the input $x(n)$ is: $x(n) = U(n)$, do the following:

i) Find the impulse response of the filter, using the convolution method.

ii) Discuss the system stability.

b) **A system is specified by its transfer function H(z) given:**

$$H(z) = \frac{z(z+0.25)(z-1)(z-2)}{(z-0.5-j0.5)(z-0.5+j0.5)(z-0.25-j0.45)(z-0.25+j0.45)}$$

Realize the system in cascade of two sections.

$(z - \frac{1}{2})^2$

$z^2 + z + \frac{1}{2}$ $\quad z^2 - z(\frac{1}{2} + \frac{1}{2}i) - z(\frac{1}{2} - \frac{1}{2}i) + \frac{1}{2}$

$z^2 - \frac{1}{2} z + 0.265$

Question 4: [15-marks]

a) Indicate which of the following statements are "True" or "False".

i) FIR filters always have generalized linear phase $\boxed{T}$

ii) The bilinear transform $S = a\,\frac{z-1}{z+1}$, here a is an appropriate constant that can map a high-pass analog filter to a Low-pass digital filter. 0.5 25 $\boxed{F}$

iii) In windowing design of FIR filters, the rectangular window gives a shorter transition band than the Hamming window. 1 $\boxed{T}$

iv) In windowing design of FIR filters, the rectangular window gives lower ripples than the Hamming window. $\boxed{F}$

$W_N^{nk} = e^{-j2\pi nk/N}$

v) Let $X(k)$ and $X(e^{jw})$ be 32-DFT and DTFT of a real-valued sequence $x(n)$ of length 32 samples. $X(29) = X^*_{\omega}\left(\frac{3\pi}{16}\right)$ Then where * denotes conjugate operation. $\frac{3\pi}{16}\frac{32}{2\pi}$ $W = \frac{2\pi k}{N}$

$32-29$ $X(29) = X^*(3)$ $\boxed{T}$

b) An analog Butterworth filter of order $N = 2$ with a 3-dB cut-off at Ω_c has the following transfer function:

$X(k) = \sum_{0}^{N-1} x(n) W_N^{nk}$ $\rightarrow W_d T_s$ $\rightarrow$ analog $S = \frac{2}{T}\,\frac{z-1}{z+1}$

$$H_a(s) = \frac{\Omega_c}{s^2 + \sqrt{2}\,\Omega_c\, s + \Omega_c{}^2}$$

$\Omega = \frac{2}{T}\tan\frac{w}{2}$

The goal is to use the bilinear transform technique to design a digital lowpass filter having a 3 dB cut-off at $\omega_c = 2\tan^{-1}\{\sqrt{2}\}$ rads/sec (this cut-off has been chosen so that the numbers work out nicely).

ω_c $-\omega_c$ $\frac{\Omega T}{2}$

i) Pre-warp the frequency $\omega_c = 2\tan^{-1}\{\sqrt{2}\}$ based on the bilinear transformation $S = \frac{z-1}{z+1}$, to determine the analog 3-dB cut-off frequency, Ωc, required.

ii) Determine the transfer function, $H(z)$, of the digital filter obtained by applying the bilinear transform into the given analog filter with the Ωc determined in part (a). Simplify as much as possible.

iii) Is the resulting digital filter stable?

~~iv)~~ Determine the difference equation for implementing the resulting digital lowpass filter.

Question 5: [25-marks] $\boxed{5}$

Given $x[n] = \{1, 2, 3\}$ **and** $h[n] = \{1, 0, -1\}$.

i) Compute the 4-point Circular convolution $y[n] = x[n] \otimes h[n]$, **using DFT transform.**

ii) Compute the Linear convolution $y[n] = x[n] * h[n]$, **using DFT transform.**

Question 6: [10-marks]

The following system is to change the sampling rate by a non-integer factor. Express $v(n)$ and $y(n)$ in terms of $x(n)$ as well as $V(e^{j\omega})$ and $Y(e^{j\omega})$ in terms of $X(e^{j\omega})$.

$$x(n) \longrightarrow \boxed{\uparrow 5} \xrightarrow{v(n)} \boxed{\downarrow 6} \longrightarrow y(n)$$

--- ***Good Luck***-------