



كلية الهندسة بحلوان

Dept/Division: Electronics, Communication, and Computer Department  
Course code & title: Digital Signal Processing  
Academic level: 4<sup>th</sup> Year  
Total mark: 100 marks  
Semester: Fall 2022  
Time allowed: 3 hrs  
Instructor: Prof. Elsayed M. Saad & Assoc. Prof. Amr E. Mohamed



**Answer all questions. Write each question number and part number ahead of your answer.**

**Question 1: [15-marks]**

When the input to an LTI system is  $x(n] = (0.5)^n u(n) + 2^n u(-n-1)$  and the output is  $y(n] = 6(0.5)^n u(n) - 6(0.75)^n u(n)$

- Find the system function  $H(z)$  of the system.
- Plot the poles and zeros of  $H(z)$  and indicate the ROC.
- Find the impulse response  $h(n]$  of the system.
- Write the difference equation that characterizes the system.

**Question 2: [20-marks]**

For each of the following systems, determine whether it is (1) linear, (2) time-invariant, (3) causal, and (4) stable. Briefly explain your answer.

i)  $y(n] = x(n] + nx(n+1)$

ii)  $y(n] = \begin{cases} x(n] & \text{if } x(n] \geq 0 \\ 0 & \text{if } x(n] < 0 \end{cases}$

b) Consider an LTI system with a frequency response  $H(e^{j\omega}) = e^{j2\omega} + e^{-j6\omega} = z^2 + z^{-6}$

- Calculate the DTFT of  $x[n] = \left(\frac{1}{4}\right)^n u[n+1]$ .
- Find the DTFT  $Y(e^{j\omega})$  of the output  $y[n]$  when  $x[n]$  is the input to the system.
- Find the output  $y[n]$ .
- Calculate the group delay of the system.

**Question 3: [15-marks]**

a) The response,  $y(n]$  of an LTI 3<sup>rd</sup>-order causal non-recursive digital filter to the excitation,  $x(n]$  is given in the following table:

N	0	1	2	3	4	5	6	7	.....
y(n]	1	2.5	3	4	4	4	4	4	.....

If the input  $x(n]$  is:  $x(n] = U(n)$ , do the following:

- Find the impulse response of the filter, using the convolution method.
- Discuss the system stability.

b) A system is specified by its transfer function  $H(z)$  given:

$$H(z) = \frac{z(z+0.25)(z-1)(z-2)}{(z-0.5-j0.5)(z-0.5+j0.5)(z-0.25-j0.45)(z-0.25+j0.45)}$$

Realize the system in cascade of two sections.



**Question 4: [15-marks]**

a) Indicate which of the following statements are "True" or "False".

- FIR filters always have generalized linear phase ☒ T
- The bilinear transform  $S = a \frac{z-1}{z+1}$ , here  $a$  is an appropriate constant that can map a high-pass analog filter to a Low-pass digital filter. ☒ F
- In windowing design of FIR filters, the rectangular window gives a shorter transition band than the Hamming window. ☒ T
- In windowing design of FIR filters, the rectangular window gives lower ripples than the Hamming window. ☒ F
- Let  $X(k)$  and  $X(e^{j\omega})$  be 32-DFT and DTFT of a real-valued sequence  $x(n)$  of length 32 samples.  $X(29) = X^* \left( \frac{3\pi}{16} \right)$  Then where  $*$  denotes conjugate operation.  $X(29) = X^*(3)$  ☒ T

b) An analog Butterworth filter of order  $N = 2$  with a 3-dB cut-off at  $\Omega_c$  has the following transfer function:

$$H_a(s) = \frac{\Omega_c}{s^2 + \sqrt{2}\Omega_c s + \Omega_c^2}$$

The goal is to use the bilinear transform technique to design a digital lowpass filter having a 3 dB cut-off at  $\omega_c = 2 \tan^{-1}\{\sqrt{2}\}$  rads/sec (this cut-off has been chosen so that the numbers work out nicely).

- Pre-warp the frequency  $\omega_c = 2 \tan^{-1}\{\sqrt{2}\}$  based on the bilinear transformation  $S = \frac{z-1}{z+1}$ , to determine the analog 3-dB cut-off frequency,  $\Omega_c$ , required.
- Determine the transfer function,  $H(z)$ , of the digital filter obtained by applying the bilinear transform into the given analog filter with the  $\Omega_c$  determined in part (a). Simplify as much as possible.
- Is the resulting digital filter stable?
- Determine the difference equation for implementing the resulting digital lowpass filter.

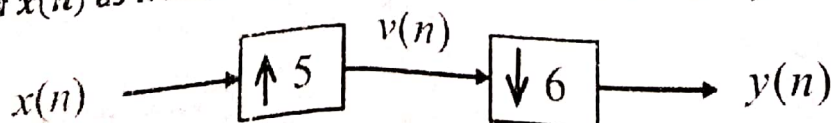
**Question 5: [25-marks]**

Given  $x[n] = \{1, 2, 3\}$  and  $h[n] = \{1, 0, -1\}$ .

- Compute the 4-point Circular convolution  $y[n] = x[n] \otimes h[n]$ , using DFT transform.
- Compute the Linear convolution  $y[n] = x[n] * h[n]$ , using DFT transform.

**Question 6: [10-marks]**

The following system is to change the sampling rate by a non-integer factor. Express  $v(n)$  and  $y(n)$  in terms of  $x(n)$  as well as  $V(e^{j\omega})$  and  $Y(e^{j\omega})$  in terms of  $X(e^{j\omega})$ .



Good Luck