King Fahd University of Petroleum & Minerals

Electrical Engineering Department

Control Engineering I

EE – 380

Term Project

Passive and Active Car Suspension Systems

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Section : 7

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Abstract: this project is about passive and active suspension systems. It will simulate passive system first and show its limitation. Then it will introduce active system model it and simulate it and show how it improve the ride quality.

Introduction

Car suspension system is a system connect the car body to the wheel. Suspension systems allow the distance between the car body and the wheel to change isolating the car body from any irregularity in the road. The suspension system must increase the comfort of the ride and must be able to hold the car wight and have stability when the car is moving.

This project will look at passive and active suspension systems. It will start by looking at passive suspension system showing that it cannot meet the requirement of comfort and stability. Then it will introduce active suspension system with hydraulic actuator placed between the car and the wheel that is controlled using PID controller. That will improve the system performance. In this project both systems will be studied and simulated using MATLAB.

Literature review

The quarter-car model is a model that is commonly used in car suspension system as the one shown in figure 1. Although this model takes only one wheel into account, it shows all the important characteristic of the system. The equation of motion of the system can be found using newton second law for the vertical forces. The mass of the car shown as the sprung mass M. The mass of the axis and the wheel is the un-sprung mass m. With shock absorber place between the two.

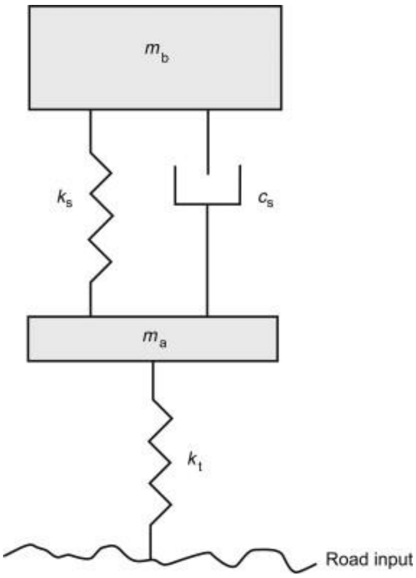

Figure 1 passive suspension system

The passive suspension system shown in the figure 1 uses fixed spring constant and damping. In this system the car do not need to use energy for the suspension system. But the drawback for that system that it is fixed. If the system is highly damped it will reduce the comfort of the car making the car body feel the road. And if its lightly damped it will decrease the stability of the car at the turn [1] .

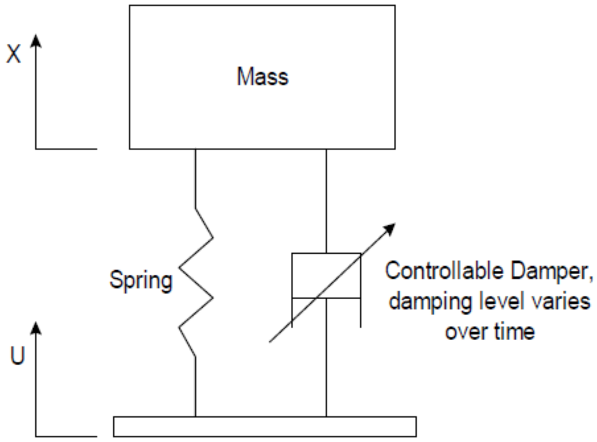

Figure 2 suspension system with adaptive damping

Due to this fact active suspension systems were developed for example adaptive damping system were developed figure 1. where unlike the passive system the damping could be controlled to increase the comfort or the stability of the car. Another way was developed is to put an actuator to exert force in the sprung and un-sprung masses to improve the system [2]. Many models were examined to control the actuator. for example, In [2] a comparison were made between adaptive linear quadratic gaussian and nonlinear control.

Passive suspension

The model that will be used in this project for the suspension system is quarter car model it will be used to derive the differential equation for the system. The figure 3 show the model with the symbol description

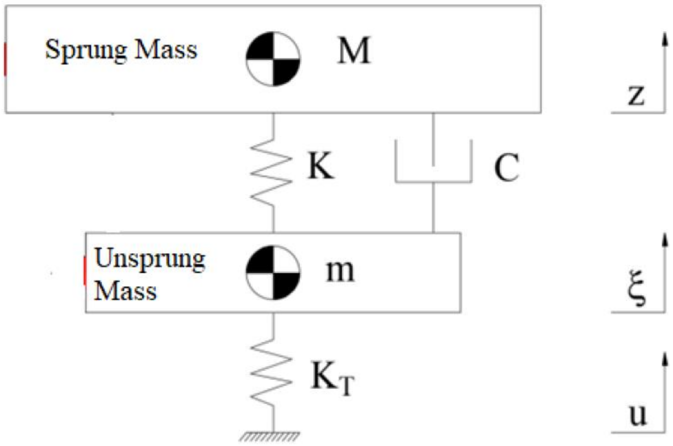

Figure 3 model used for passive system

Differential equation and state space models

From figure 3, using newton second law in the vertical forces

$$M\ddot{z} = K(\zeta - z) + C(\dot{\zeta} - \dot{z})$$

$$m\ddot{\zeta} = K_T(u - \zeta) + K(z - \zeta) + C(\dot{z} - \dot{\zeta})$$

From the differential equation we can find the state space model for the output $z, \ddot{z}$

$$\dot{\begin{bmatrix} z \\ \dot{z} \\ \zeta \\ \dot{\zeta} \end{bmatrix}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{K}{M} & -\frac{C}{M} & \frac{K}{M} & \frac{C}{M} \\ 0 & 0 & 0 & 1 \\ \frac{K}{m} & \frac{C}{m} & -\frac{K+K_T}{m} & -\frac{C}{m} \end{bmatrix} \begin{bmatrix} z \\ \dot{z} \\ \zeta \\ \dot{\zeta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{K_T}{m} \end{bmatrix} u$$

$$y = \begin{bmatrix} z \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{K}{M} & -\frac{C}{M} & \frac{K}{M} & \frac{C}{M} \end{bmatrix} \begin{bmatrix} z \\ \dot{z} \\ \zeta \\ \dot{\zeta} \end{bmatrix}$$

Simulation result

System will be simulated for the following input

$$u_1(t) = \begin{cases} 0 & , \quad t < 1 \\ 0.05 & , \quad 1 \le t \le 2 \\ 0 & , \quad t > 2 \end{cases} \qquad u_2(t) = \begin{cases} 0 & , \quad t < 1 \\ 0.05 & , \quad t > 2 \end{cases}$$

Unit of u_1 *and* u_2 are in meter

The other specification used for simulation are shown in the table 1

Table 1 car specifications

symbol	description	value	Unit
M	Sprung mass	350	kg
m	Un-sprung mass	50	kg
C	Damping coefficient	3000	Ns/m
K	Stiffness of the spring	35000	N/m
K_T	Stiffness of the tire	100000	N/m

Result for $u_1(t)$

Position z acceleration $\ddot{z}$

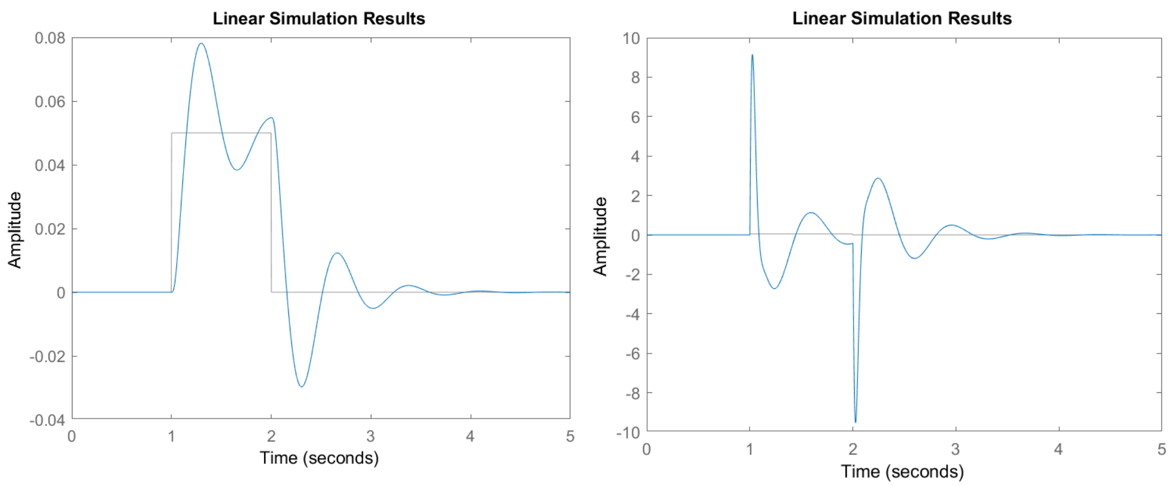

Figure 4 position and acceleration of the sprung mass for first input u1

Result for $u_2(t)$

Position z acceleration $\ddot{z}$

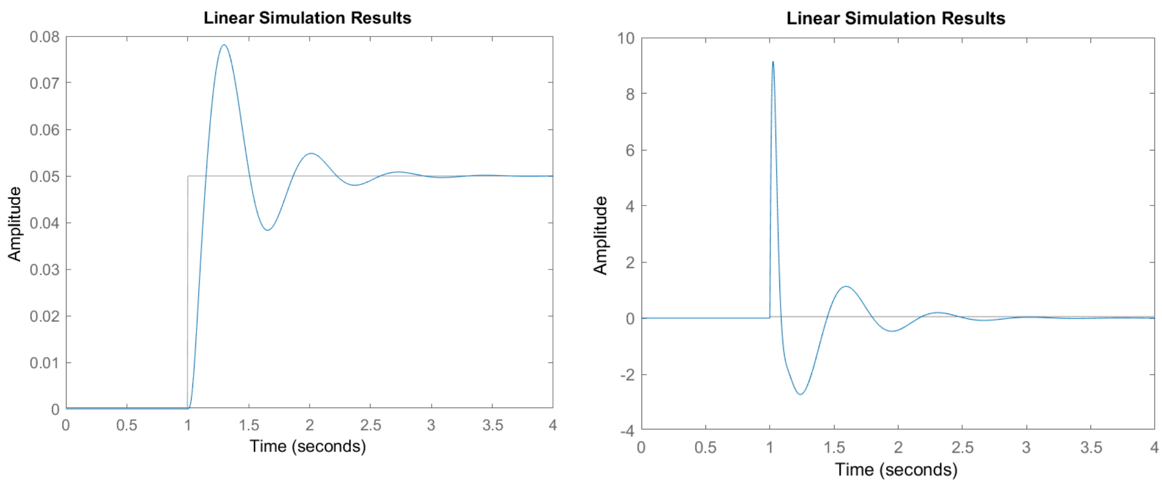

Figure 5 position and acceleration of the sprung mass forsecondt input u2

Varying the parameter

In this part the stiffness of the spring and tire K, K_T will be change $\pm 20\%$ and will see the effect of it in system performance. The input $u_1(t)$ will be used In all figures.

Fixing K_T and varying K

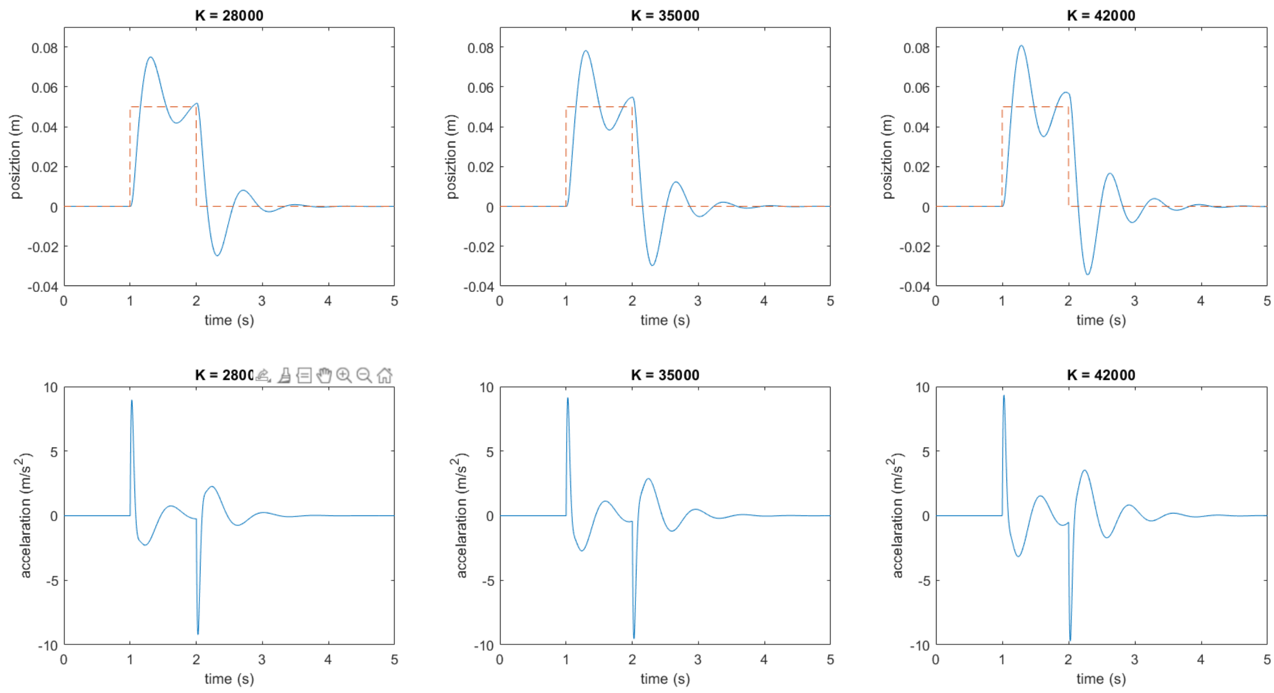

Figure 6 position and acceleration of the sprung mass varying spring stiffness by 20% keeping tire stiffness fixed

Fixing K and varying K_T

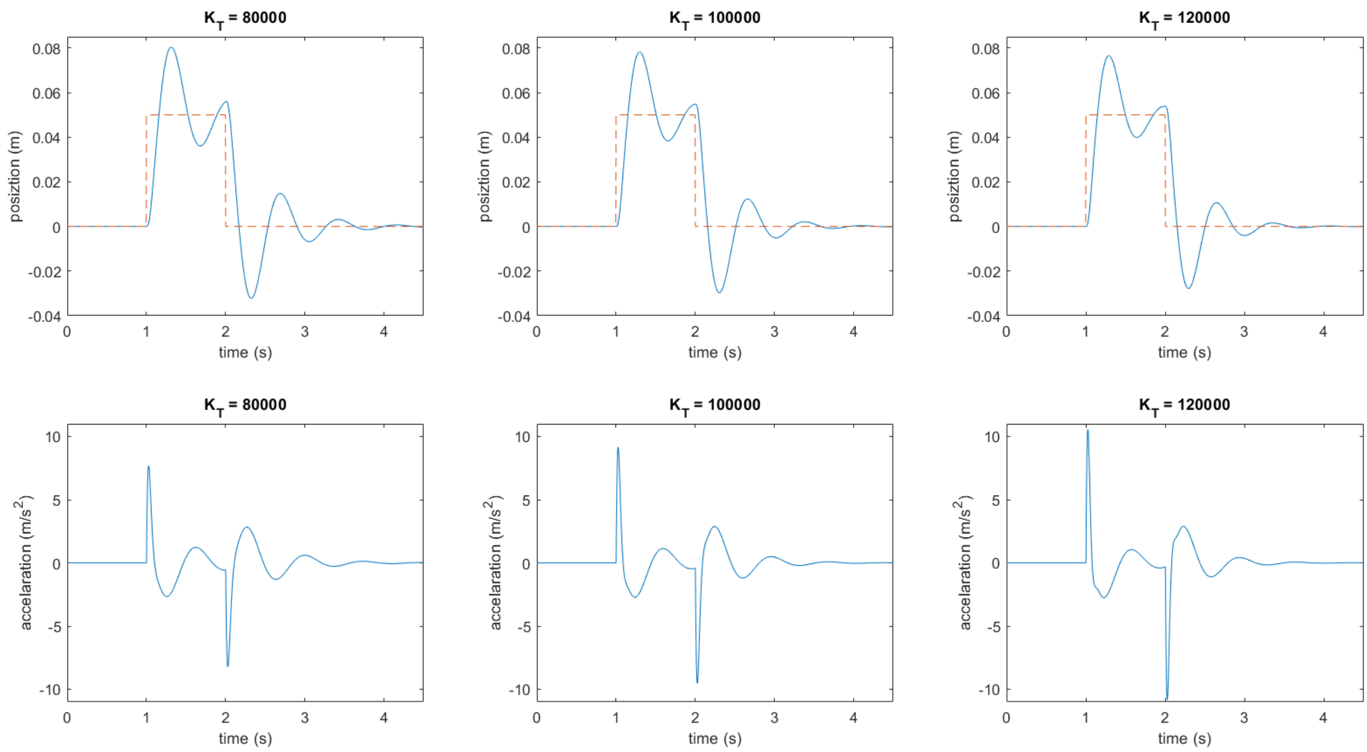

Figure 7 position and acceleration of the sprung mass varying tire stiffness by 20% keeping spring stiffness fixed

From figure 6 it is clear that changing K does not have much effect in the performance of system increasing K slightly increase the over shot. In figure 7 K was fixed and K_T was changed. As K_T increase the over shot decrease slightly but the acceleration increase. In both figures the parameters K, K_T was tuned by 20%. The best behavior we can get will be by decreasing K and K_T .From the result above the passive suspension systems cannot meet the required comfort and stability even if the parameter is tuned to improve system performance. Therefore, a hydraulic actuator will be added in the next section to improve the performance.

The code used in the simulation is in the appendix.

Active suspension system

Passive suspension system cannot meet both requirement of comfort and stability. Therefor, active suspension system was introduced. A hydraulic actuator is placed between the sprung and the un-sprung masses as in figure x. the actuator is controlled by Engine Control Unit (ECU) that is connected to sensor to measure the acceleration of the car body.

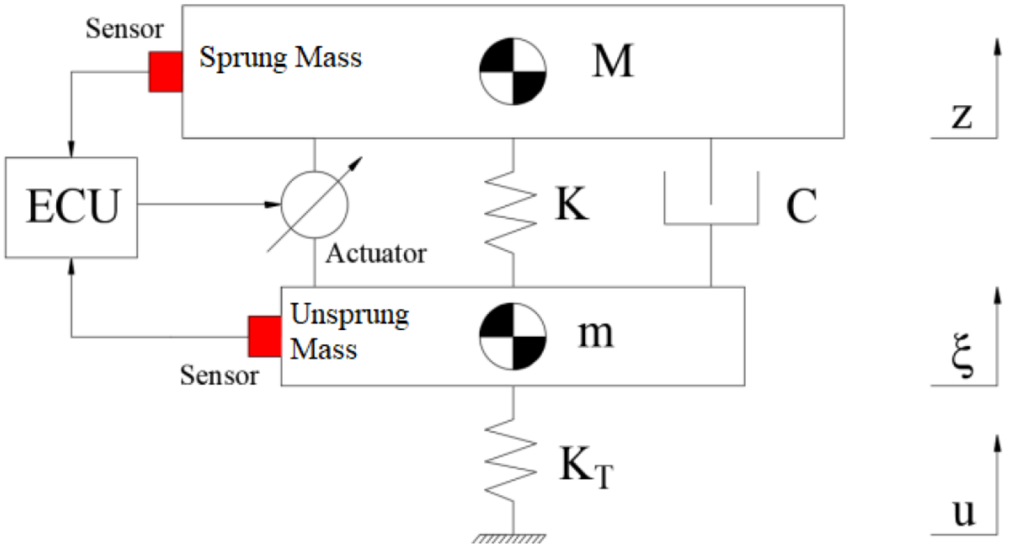

Figure 8 active suspension system model

The ECU uses a PID controller to control the actuator figure 9

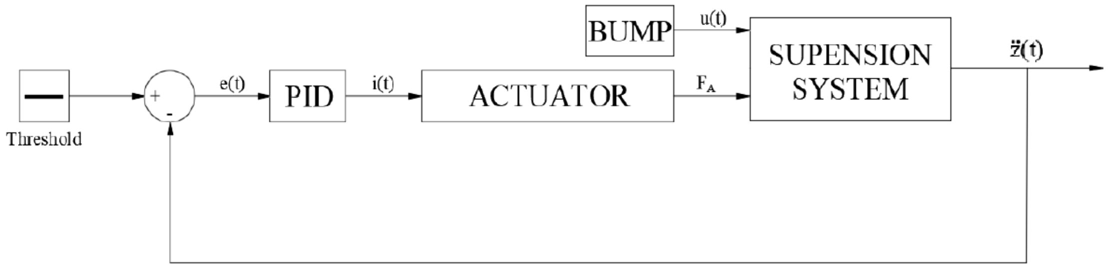

Figure 9 active suspension system block diagram

This block diagram will be used for simulation of the active car suspension system. The threshold is zero the suspension system block is composed of two blocks in parallel one is same as the passive system and the other is the active. The actuator is assumed to have first order transfer function.

Mathematical model

Firstly, suspension system block will be analyzed. From figure x using newton second law

$$M\ddot{z} = K(\zeta - z) + C\left(\dot{\zeta} - \dot{z}\right) + F_A$$

$$m\ddot{\zeta} = K_T(u - \zeta) + K(z - \zeta) + C\left(\dot{z} - \dot{\zeta}\right) - F_A$$

Where F_A is the thrust force coming from the hydraulic actuator that is controlled by $i(t)$ where $i(t)$ is the signal coming from the PID controller. Writing the state space for the system

$$\dot{\begin{bmatrix} z \\ \dot{z} \\ \zeta \\ \dot{\zeta} \end{bmatrix}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{K}{M} & -\frac{C}{M} & \frac{K}{M} & \frac{C}{M} \\ 0 & 0 & 0 & 1 \\ \frac{K}{m} & \frac{C}{m} & -\frac{K+K_T}{m} & -\frac{C}{m} \end{bmatrix} \begin{bmatrix} z \\ \dot{z} \\ \zeta \\ \dot{\zeta} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{M} \\ 0 & 0 \\ \frac{K_T}{m} & -\frac{1}{m} \end{bmatrix} \begin{bmatrix} u \\ F_A \end{bmatrix}$$

$$y = [\ddot{z}] = \begin{bmatrix} -\frac{K}{M} & -\frac{C}{M} & \frac{K}{M} & \frac{C}{M} \end{bmatrix} \begin{bmatrix} z \\ \dot{z} \\ \zeta \\ \dot{\zeta} \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{M} \end{bmatrix} \begin{bmatrix} u \\ F_A \end{bmatrix}$$

Now the suspension system has two inputs one from the road and the other is the force from the hydraulic actuator. The transfer function of the system for each input can be found using MATLAB and can be used in simulation. The specification used for simulation are in table 1.

The transfer function used for the hydraulic system is

$$H(s) = \frac{K_A}{\tau s + 1}$$

τ is set to $0.05s$ and $K_A = 4000\ N/V$

The PID controller transfer function is:

$$K_P + \frac{K_I}{s} + K_D s$$

where K_P, K_I and K_D are gains that will be tuned to have the best response by trial and error.

Simulation results

By trial and error, it is found that $K_P = 0.5\,V\frac{s^2}{m}$ $\quad K_I = 1.02\,V\frac{s}{m}$ $\quad K_D = 0.01\,V\frac{s^3}{m}$

These values of the gains in the PID controller will help improving the system performance

Simulating the system for input $u_1(t)$ and $u_2(t)$

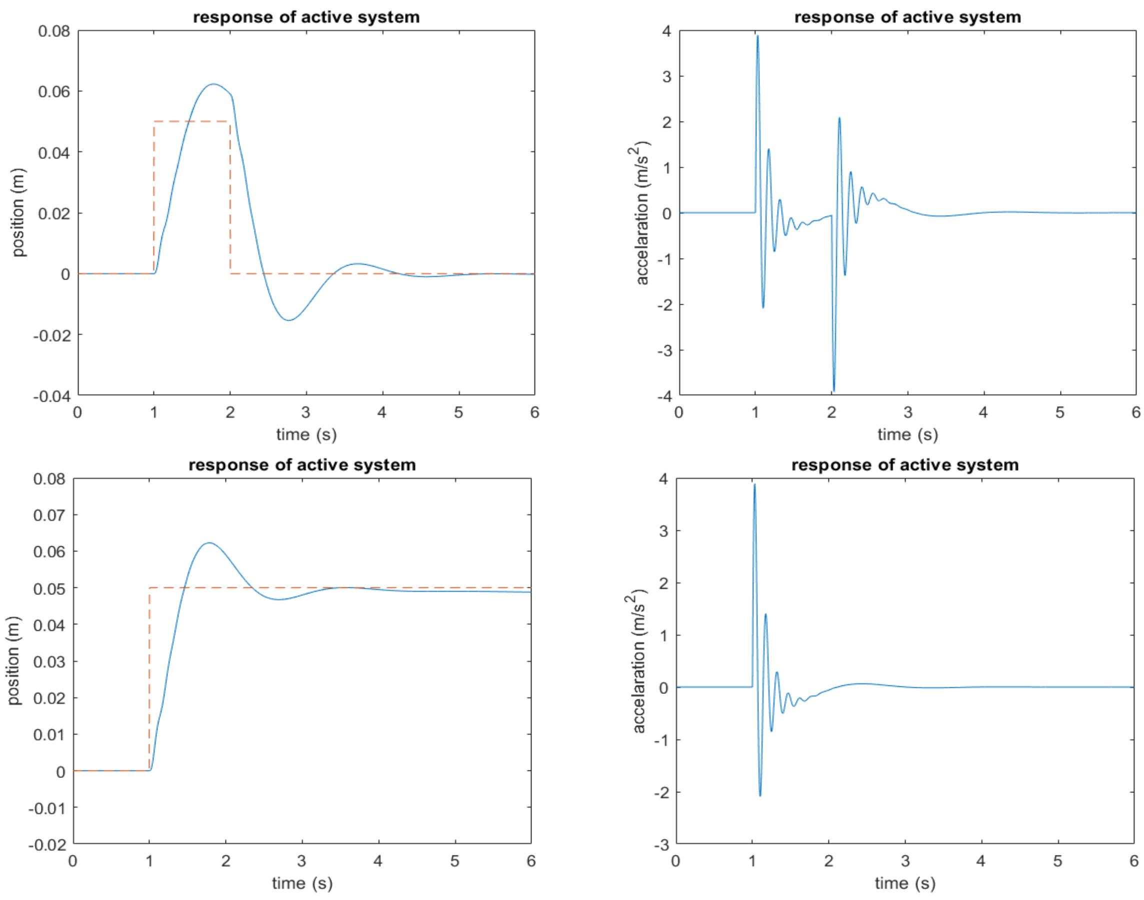

Figure 10 active systems response for inputs u1 and u2

figure 10 shows the response of the active suspensions system. It clear from those figures that active system has more comfort than the passive system. It has less acceleration therefore the body of the care will feel less. And it has less overshot and oscillation in the position which have similar effect in ride comfort.

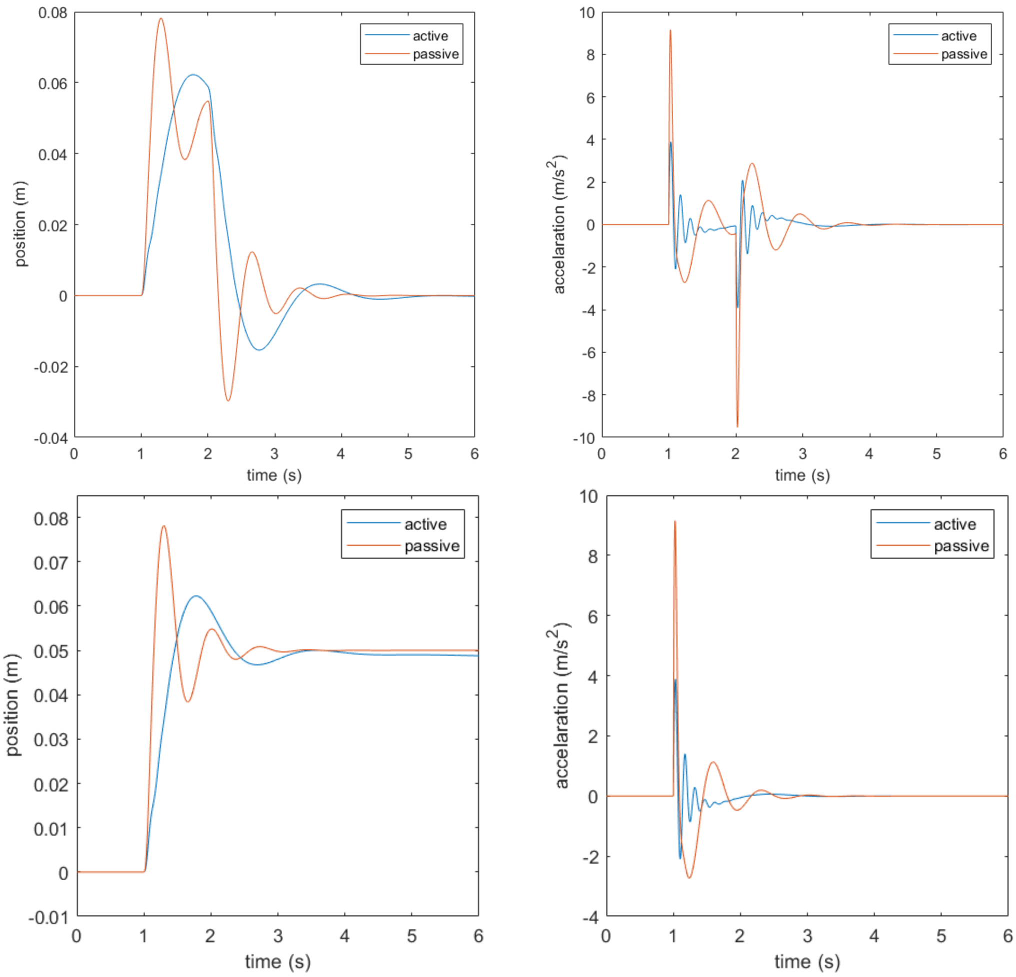

Figure 11 active passive comparison

The comparison between the active and the passive suspension systems in figure 11 support the discussion about the ride comfort. In that figure we can see the difference between active and passive in the acceleration and the overshot.in addition, the settling time decrease consequently the oscillation will take end in less time increasing the comfort. Adding hydraulic actuator improve the system performance.

The code used in the simulation is in the appendix.

Conclusion

This project investigated car suspension systems. It started with passive suspension systems by finding the differential equation then use it to write the state space model. From the state space, the transfer function was found by MATLAB and it used for the simulation. Then varying the parameter of the passive system led to the conclusion that passive suspension systems cannot meet the riding comfort and stability demand which led to the active systems.

The second part studied the active suspension system. Where a hydraulic actuator is placed between the car body and the wheels. The hydraulic was controlled by ECU which has PID controller. The controller gains were tuned to improve the system performance and the system was simulated by MATLAB. The results show improvement in the performance of the system more than what can be achieved by the passive system as the acceleration and the over shot decrease.

Appendix (code)

Passive

```
dt=6/1799;
K=35000;
KT=100000;
C=3000;
M=350;
m=50;
A=[0 1 0 0;
    -K/M -C/M K/M C/M;
    0 0 0 1;
    K/m C/m -(K+KT)/m -C/m];
B=[0;0;0;KT/m];
C_a=[-K/M -C/M K/M C/M];
C_p=[1 0 0 0];
D=0;
[num,den]=ss2tf(A,B,C_a,D);
acctf = tf(num,den);
[num,den]=ss2tf(A,B,C_p,D);
ptf = tf(num,den);
t=0:dt:6;
u1=0.05*(heaviside(t-1)-heaviside(t-2));
u2=0.05*heaviside(t-1);
pasv_p=lsim(ptf,u1,t);
pasv_a=lsim(acctf,u1,t);
```

```
%% passive part 5
j=1;
C_p=[1 0 0 0];
t=0:6/1799:6;
%KT =100000;
K=35000;
figure
%for K=[28000 35000 42000]
    for KT = [80000 100000 120000]
A=[0 1 0 0;
    -K/M -C/M K/M C/M;
    0 0 0 1;
    K/m C/m -(K+KT)/m -C/m];
B=[0;0;0;KT/m];
 C_a=[-K/M -C/M K/M C/M];
 [num,den]=ss2tf(A,B,C_a,D);
 acctf = tf(num,den);
[num,den]=ss2tf(A,B,C_p,D);
 ptf = tf(num,den);
z=lsim(ptf,u1,t);
a=lsim(acctf,u1,t);
subplot(2,3,j)
plot(t,z,t,u1,"--")
title("K_T = "+KT);xlabel("time (s)");xlim([0 4.5])
ylabel("posiztion (m)");ylim([-0.04 0.085])
subplot(2,3,j+3)
plot(t,a)
title("K_T = "+KT);xlabel("time (s)");xlim([0 4.5])
ylabel("accelaration (m/s^2)");ylim([-11 11])
j=j+1;
end
```

Active

```
%% simulate active
A=[0 1 0 0;
    -100 -8.5714 100 8.5714;
    0 0 0 1;
    700 60 -2700 -60];
B=[0 0;0 1/350;0 0;2000 -1/50];
C=[-100 -8.5714 100 8.5714];
D=[0 1/350];
[num,den]=ss2tf(A,B,C,D,1);
acctfu = tf(num,den);
[num,den]=ss2tf(A,B,C,D,2);
acctfi = tf(num,den);
p=0.5;
I=1.02;
D=0.01;
ka= 4000;
De = tf([D 0],1);
PI = tf([p I],[1 0]);
PID = parallel(PI,De);
hyd=tf(ka,[0.05 1]);
FA=series(PID,hyd);
L1 = series(FA,acctfi);
G = feedback(1,L1);
sys = series(G,acctfu);
y=lsim(sys,u1,t); % get the accelaration
z=cumsum(cumsum(y)*dt)*dt; % integrate to get the position

subplot(1,2,1)
plot(t,z,t,pasv_p);legend("active","passive")%;title("response of active system")
ylim([-0.04 0.08]);ylabel("position (m)");xlabel("time (s)");
subplot(1,2,2)
plot(t,y,t,pasv_a);legend("active","passive")%;title("response of active system")
ylabel("accelaration (m/s^2)");xlabel("time (s)");
```

References

[1]- R. Darus and Y. M. Sam, "Modeling and control active suspension system for a full car model," *2009 5th International Colloquium on Signal Processing & Its Applications*, 2009, pp. 13-18, doi: 10.1109/CSPA.2009.5069178.

[2]- T.J. GORDON , C. MARSH & M.G. MILSTED (1991): A Comparison of Adaptive LQG and Nonlinear Controllers for Vehicle Suspension Systems, Vehicle System Dynamics: International Journal of Vehicle Mechanics and Mobility 20:6, 321-340